Status in the Lattice QCD Calculation of TMDs from Quasi-TMDs

Correlations in Partonic and Hadronic Interactions (CPHI-2022)

Duke University, Durham

Mar. 7-10, 2022

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MAR 10, 2022

Argonne NATIONAL LABORATORY

Outline

- Relation between Quasi TMD and TMD
- Lattice QCD calculation of the Collins-Soper kernel
- Soft function from lattice QCD
- Outlook

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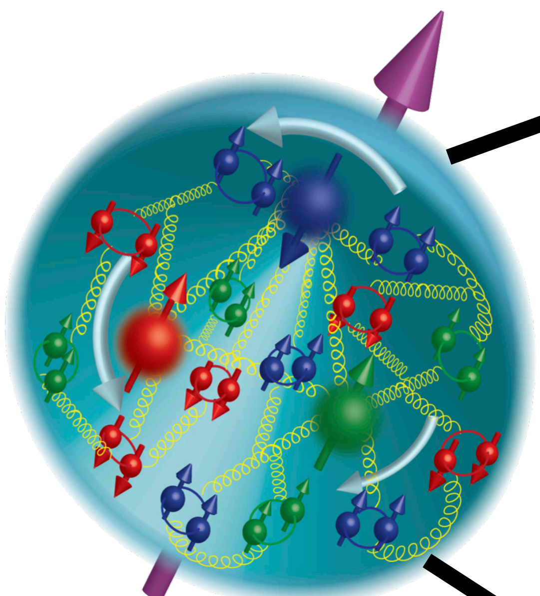
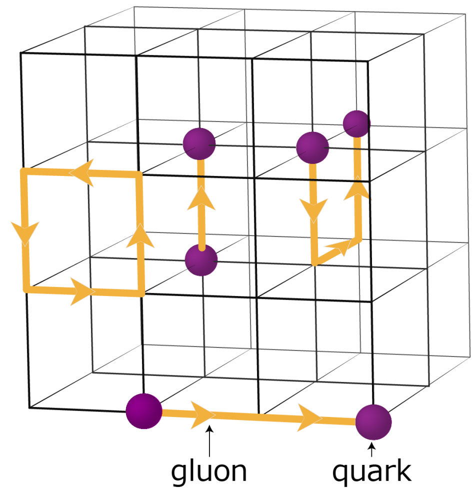
3D Tomography of the Proton (Hadrons)

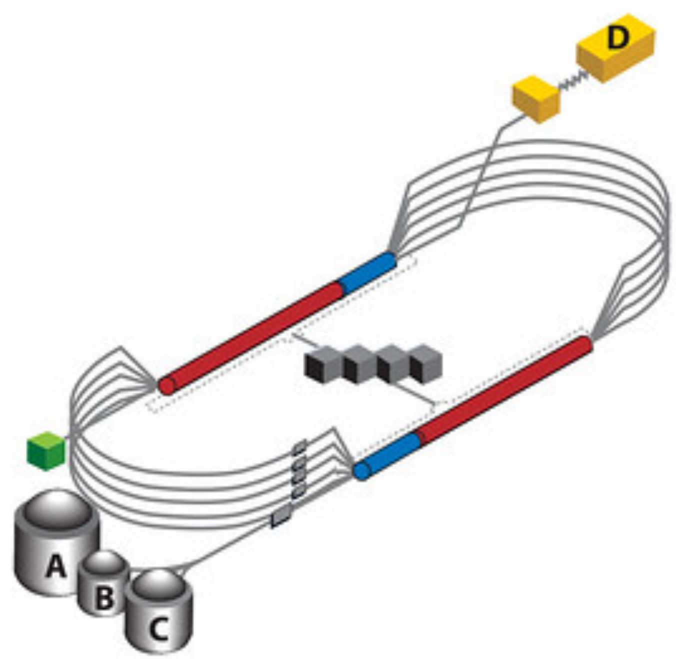


Lattice QCD

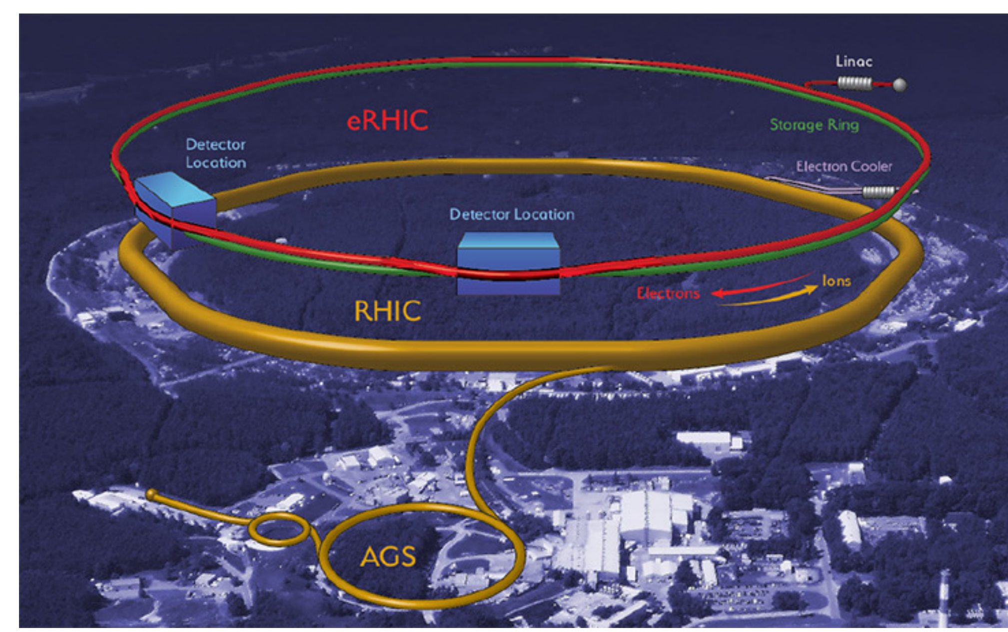


+

Hard Scattering



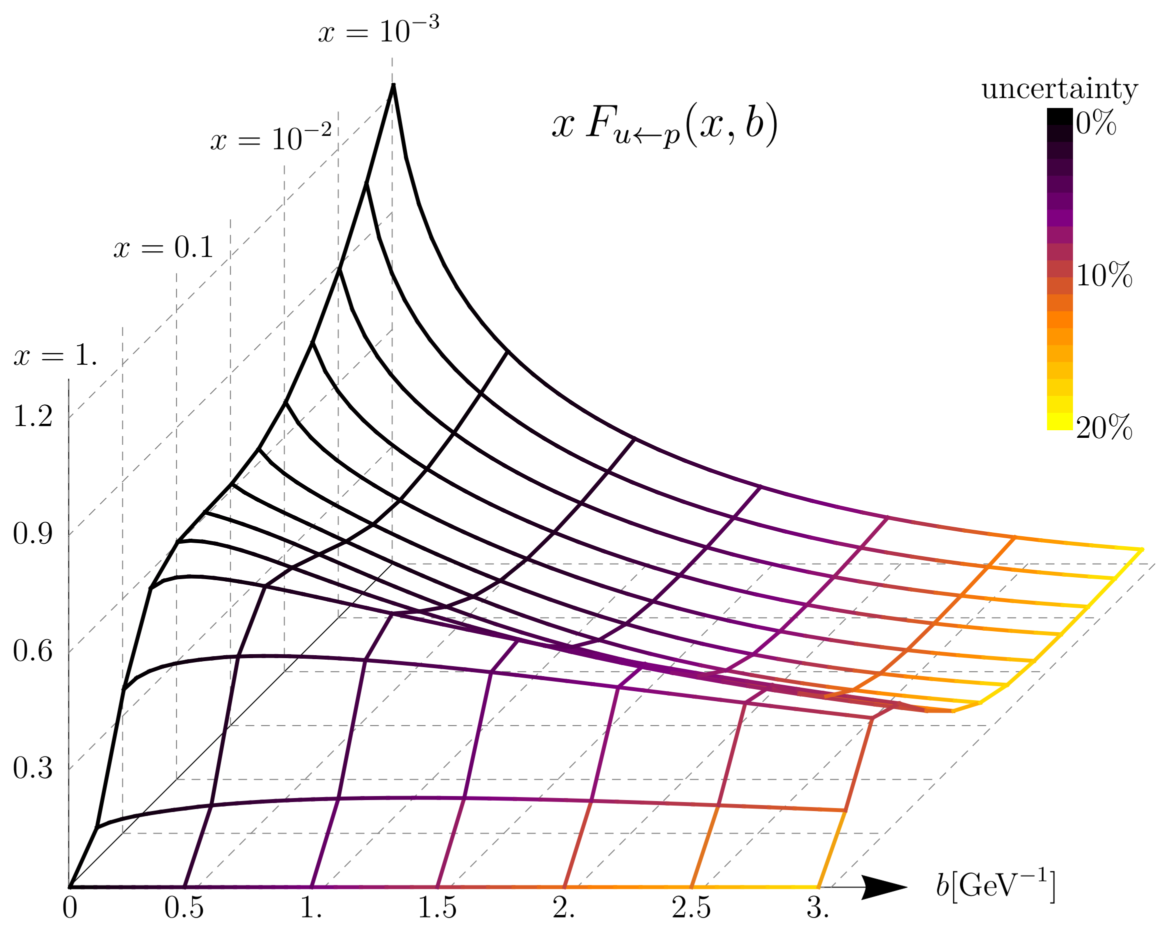
Jefferson Lab 12 GeV



The Electron-Ion Collider

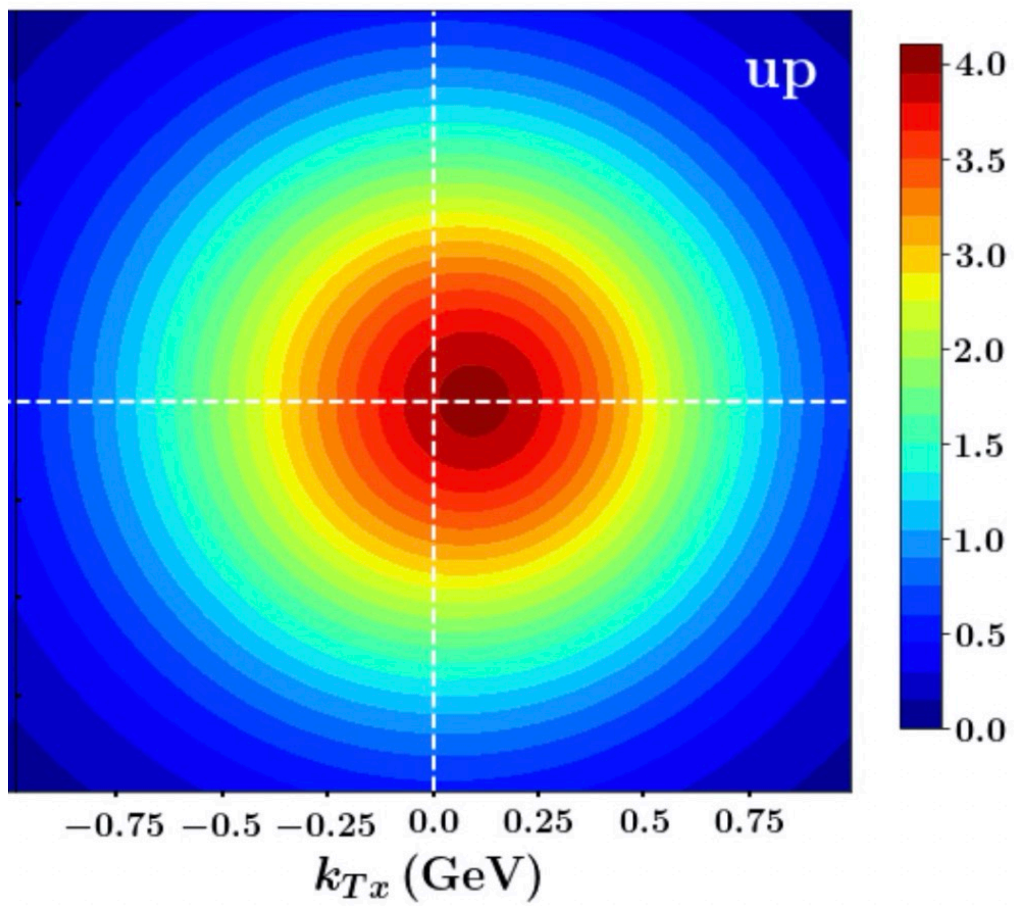
Tomography in the 3D momentum space

Unpolarized quark TMD



I. Scimemi and A. Vladimirov, JHEP **06** (2020).

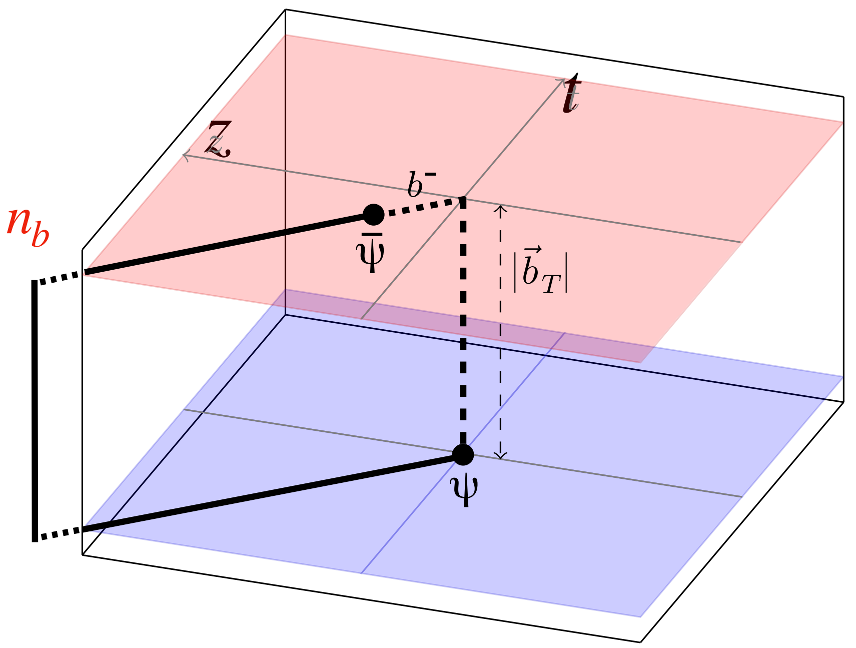
Quark Sivers function



Cammarota, Gamberg, Kang et al. (JAM Collaboration), PRD **102** (2020).

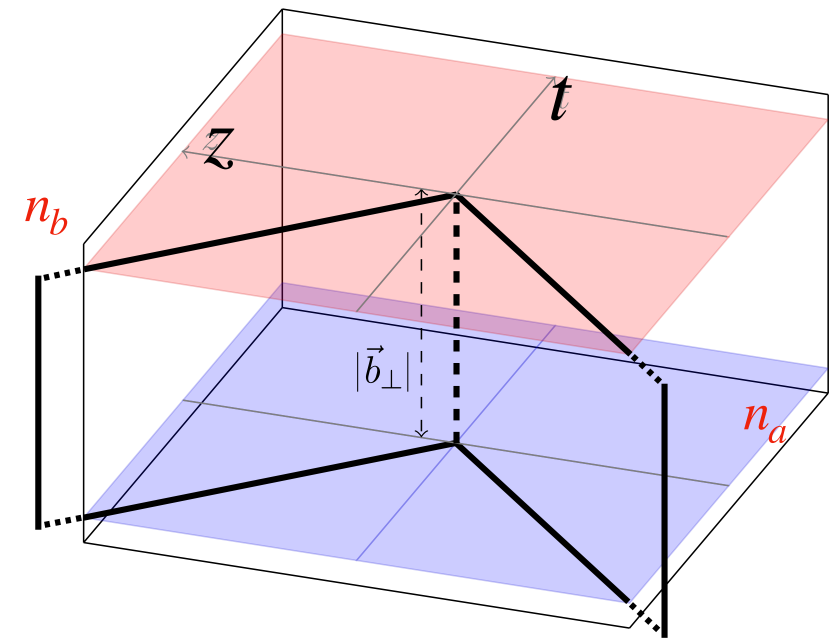
TMD definition

- Beam function:



Hadronic matrix element

- Soft function :



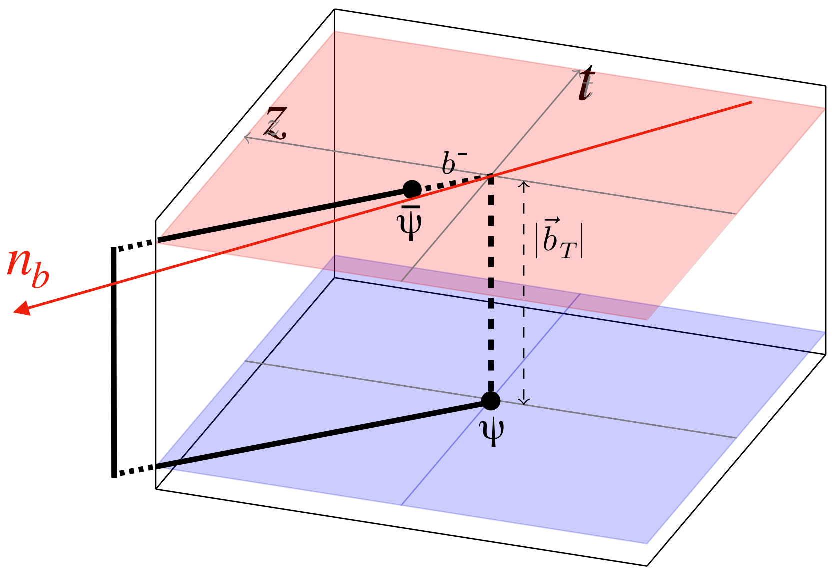
Vacuum matrix element

$$f_i^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

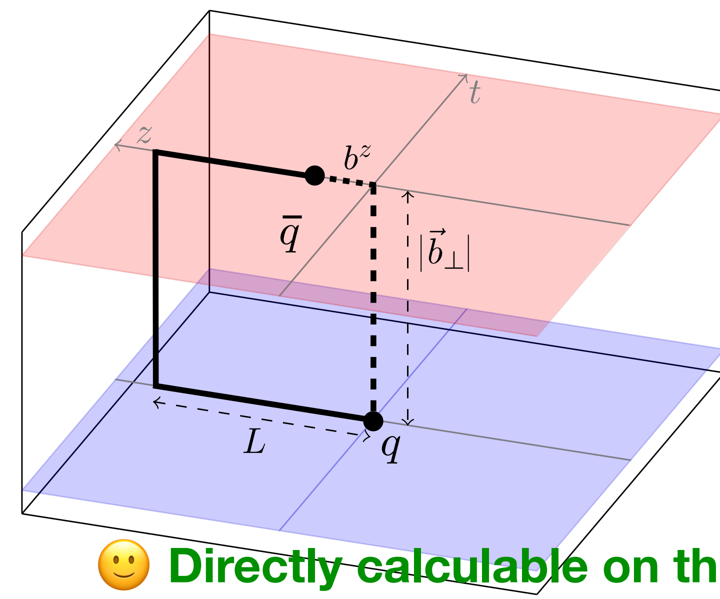
Collins-Soper scale

Quasi TMD in the LaMET formalism

- **(Collins) Beam function:**



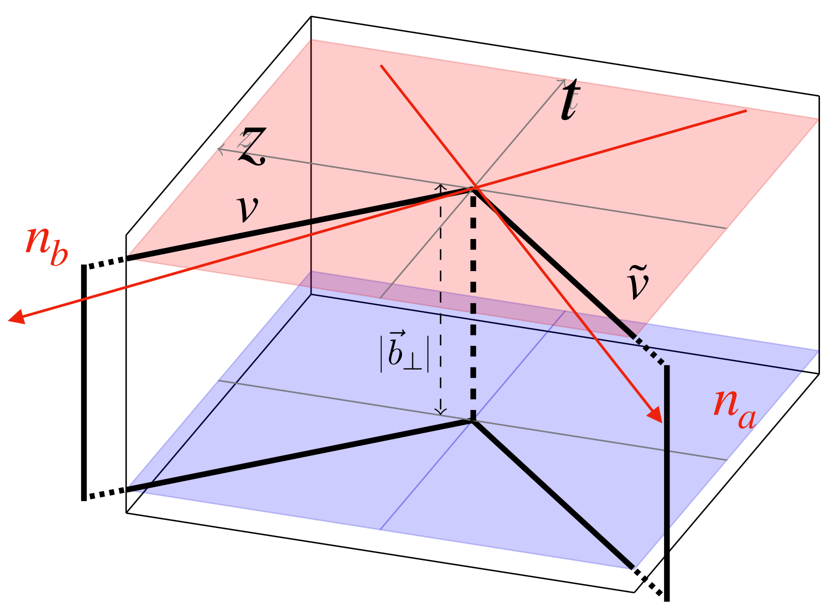
- **Quasi beam function :**



Lorentz boost and $L \to \infty$

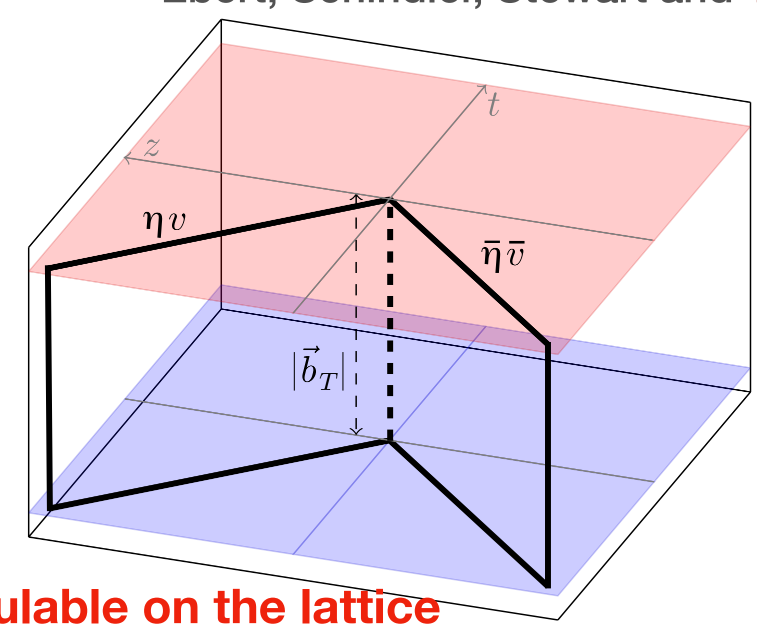
🙂 Directly calculable on the lattice

- **(Collins) Soft function:**



- **Quasi soft function**

Ebert, Schindler, Stewart and YZ, 2201.08401.



$L \to \infty$

! Not directly calculable on the lattice
due to explicit time dependence

Relation between quasi TMD and TMD

- Quasi TMD:

$$\tilde{\zeta} = x^2 m_N^2 e^{2(y_{\tilde{P}}+y_B-y_n)}$$

$$\tilde{f}_{i/h}(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z, \tilde{\eta}) = \lim_{\tilde{P}^z \gg m_N} \lim_{a\to 0} Z_{\rm uv} \frac{\tilde{B}_{i/h}}{\sqrt{\tilde{S}^q(a, 2y_n, 2y_B \ldots)}}$$

- Factorization relation:

$$\lim_{\tilde{\eta}\to\infty} \tilde{f}_{q/h}(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z, \tilde{\eta}) = C(x\tilde{P}^z, \mu)\exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right] f_{i/h}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}(y_{\tilde{P}}^{-k} e^{-y_{\tilde{P}}})$$

Matching coefficient **Collins-Soper kernel**

See I. Stewart's talk on the proof.

🙂 **No mixing between quarks of different flavors and between gluon and singlet quarks;**

🙂 **Matching is spin independent.**

- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
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Matching coefficient

Collins-Soper kernel

☹️ **Not directly calculable due to the soft function**

See I. Stewart's talk on the proof.

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$$\tilde{\zeta} = x^2 m_N^2 e^{2(y_{\tilde{P}}+y_B-y_n)}$$

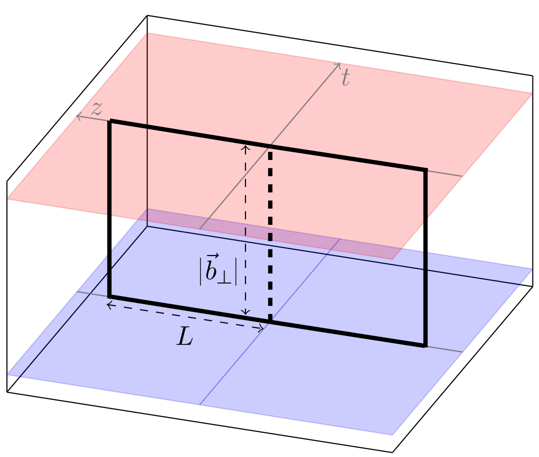
- Factorization relation:

$$\lim_{\tilde{\eta}\to\infty} \frac{\tilde{f}^{\rm naive}_{i/h}}{g_S^q(b_T,\mu)} = C(x\tilde{P}^z,\mu)\exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] f_{q/h}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}(y_{\tilde{P}}^{-k} e^{-y_{\tilde{P}}})$$

$$\mathcal{O}\left(\frac{b_T}{\tilde{\eta}}, \frac{1}{(x b_T \tilde{P}^z)^2}, \frac{1}{\tilde{P}^z\tilde{\eta}}, \frac{\Lambda^2_{\rm QCD}}{(x\tilde{P}^z)^2}\right)$$

$$\tilde{f}^{\rm naive}_{i/h} = \lim_{a\to 0} Z_{\rm uv}\tilde{B}_{i/h}\Big/\sqrt{\tilde{S}^q_{\rm naive}}$$

$\tilde{S}^q_{\rm naive}$:



🙂 Directly calculable on the lattice

Reduced soft function: $g_S^q(b_T,\mu) = \sqrt{S^r(b_T,\mu)}$

Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

Relation between quasi TMD and TMD

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①

②

③

$$\mathcal{O}\left(\frac{b_T}{\tilde{\eta}}, \frac{1}{(xb_T\tilde{P}^z)^2}, \frac{1}{\tilde{P}^z\tilde{\eta}}, \frac{\Lambda^2_{\rm QCD}}{(x\tilde{P}^z)^2}\right)$$

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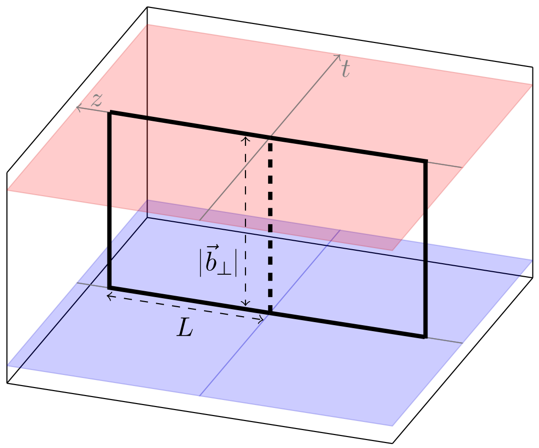

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Collins-Soper (CS) kernel from lattice QCD

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)} + \text{power corrections}$$

Studying CS kernel through quasi-TMDs suggested in

- Ji, Sun, Xiong and Yuan, PRD91 (2015);

The concrete formalism first derived in

- Ebert, Stewart and YZ, Phys.Rev.D 99 (2019).

- Does not depend on the external hadron state, could be calculated with pion TMD or Wave function (vacuum to pion amplitude) for simplicity;

  - Shanahan, Wagman and YZ, PRD 102 (2020);
  - Ebert, Stewart and YZ, Phys.Rev.D 99 (2019);
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- One can also calculate ratios of TMDs with different spin structures.

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- One can also calculate ratios of TMDs with different spin structures.

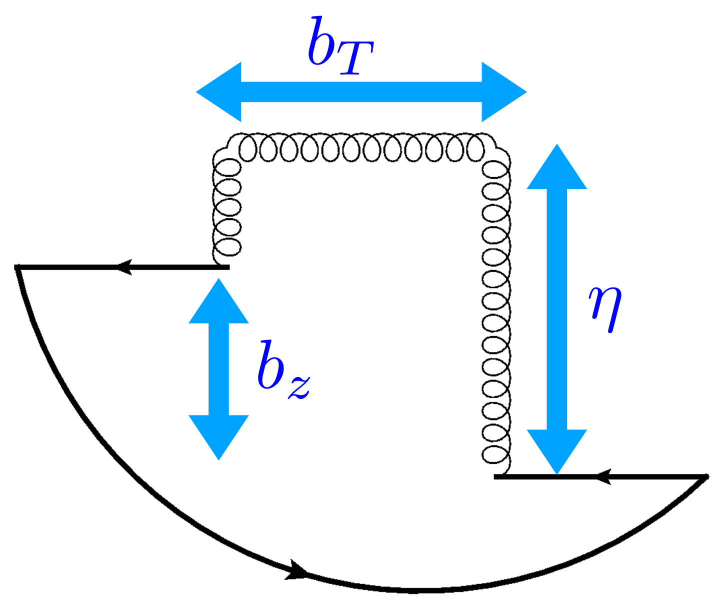
The idea of using ratios has been used in the calculation of *x*-moments of TMDs:

Musch, Hägler, Engelhardt, Negele, Schäfer, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

Collins-Soper (CS) kernel from lattice QCD

- Simulation of bare quasi beam function:

$\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, \eta, P_1^z)$



Collins-Soper (CS) kernel from lattice QCD

- Lattice renormalization and conversion to MSbar scheme

$$\overline{\tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,\eta,P_1^z)}$$

- Nonperturbative Renormalization: $\tilde{Z}_{\rm UV}$

Cancel linear divergences: $\propto e^{-(2\eta+|b_T|)/a}$

Operator mixing: $\tilde{Z}_{\rm UV}^{\Gamma\Gamma'}(b^z)\tilde{B}^{\Gamma'}(b^z)$

Shanahan, Wagman and YZ, PRD 101 (2020).

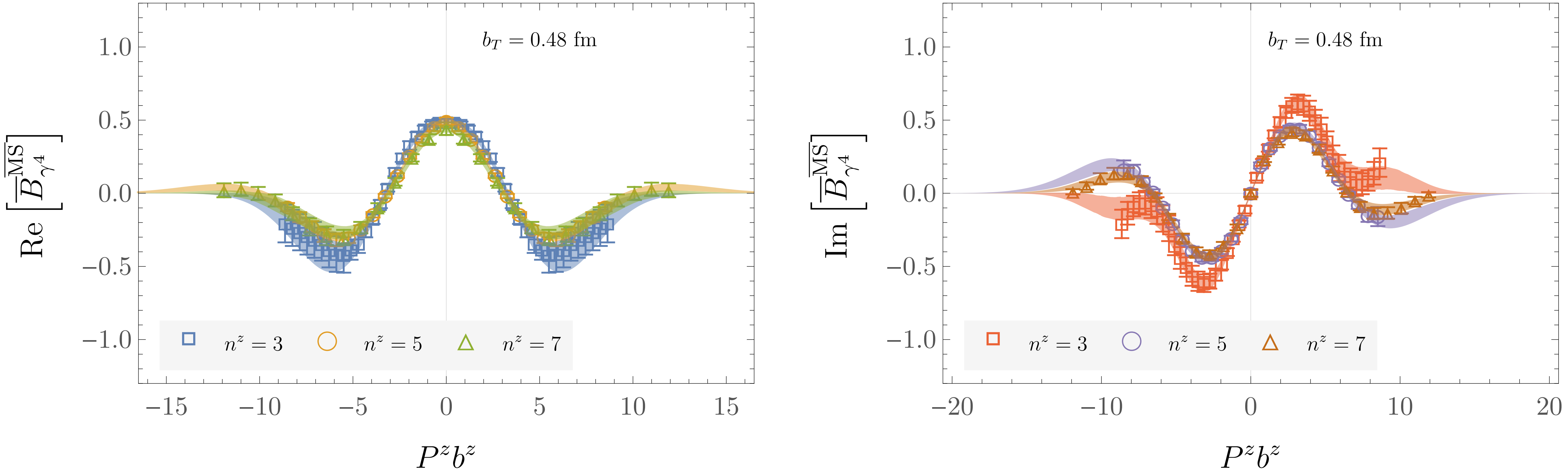
- Conversion to MSbar scheme: $\tilde{Z}'$

For $b^z = 0$,
Constantinou, Panagopoulos and Spanoudes, PRD99 (2019);
For arbitrary b^z,
Ebert, Stewart and YZ, JHEP 03 (2020).

Collins-Soper (CS) kernel from lattice QCD

- Fourier transform: extrapolation to ∞ necessary.

$$\int db^z\, e^{ib^z x P_1^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\overrightarrow{b}_T,a,\eta,P_1^z)$$



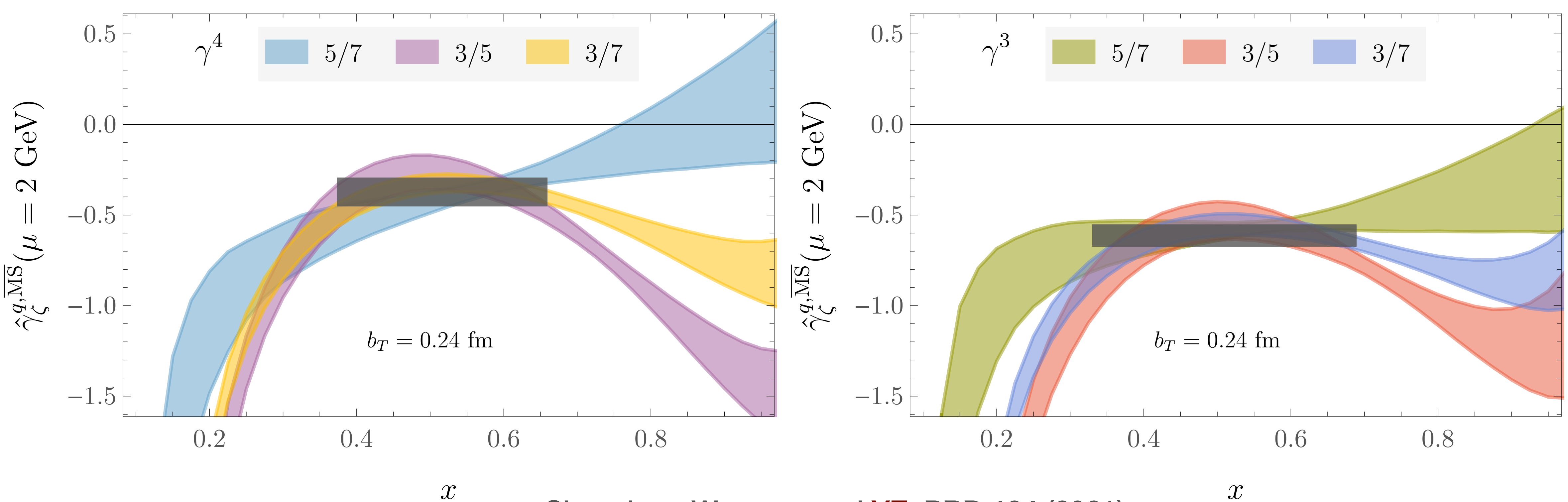
Shanahan, Wagman and YZ, PRD **104** (2021).

Collins-Soper (CS) kernel from lattice QCD

- Perturbative matching and forming ratios at different P^z:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\ e^{ib^z xP_1^z}\ \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,\eta,P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\ e^{ib^z xP_2^z}\ \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,\eta,P_2^z)}$$



Shanahan, Wagman and YZ, PRD **104** (2021).

Current status for the CS kernel calculation

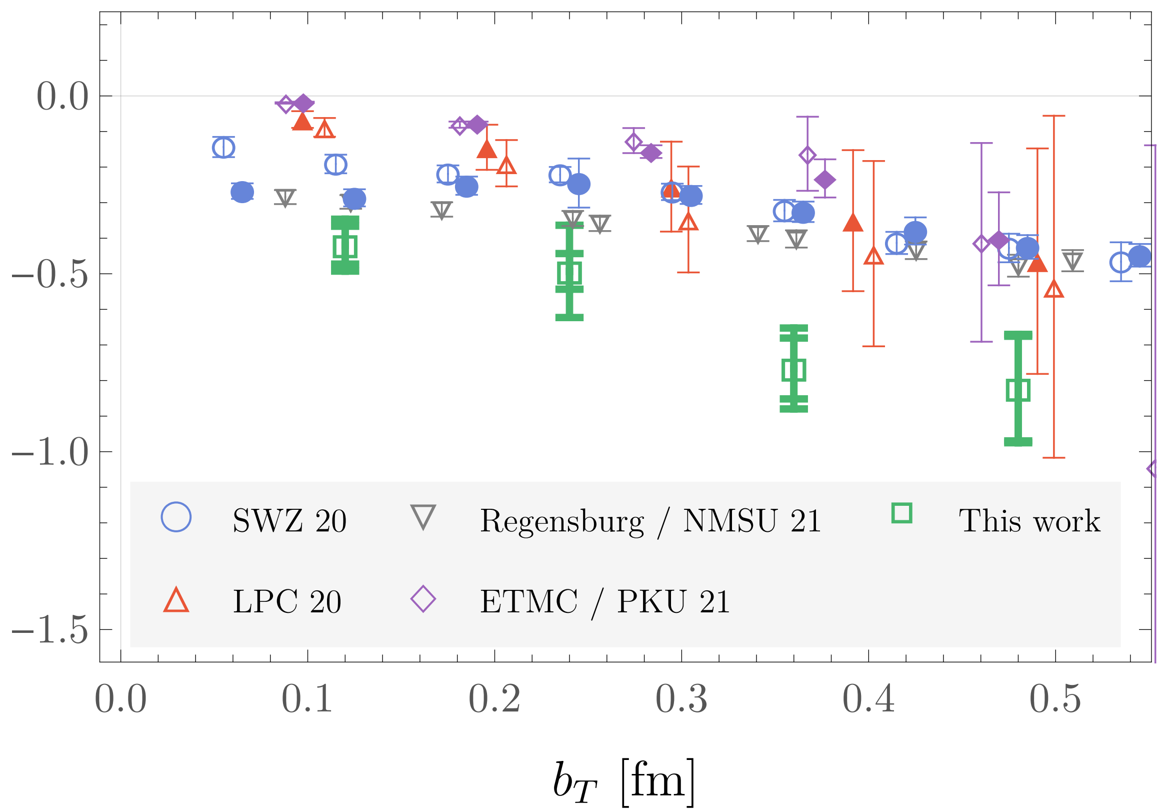
	Lattice setup	Renormalization	Operator mixing	Fourier transform	Matching	Plateau search
SWZ20 PRD 102 (2020) Quenched	$a = 0.06$ fm, $m_\pi = 1.2$ GeV, $P^z_{\max} = 2.6$ GeV	Yes	Yes	Yes	LO	Yes
LPC20 PRL 125 (2020)	$a = 0.10$ fm, $m_\pi = 547$ MeV, $P^z_{\max} = 2.11$ GeV	N/A	Yes	N/A	LO	N/A
Regensburg/ NMSU 21 JHEP 08 (2021)	$a = 0.09$ fm, $m_\pi = 422$ MeV, $P^+_{\max} = 2.27$ GeV	N/A	No	N/A	NLO	N/A
PKU/ETMC 21 PRL 128 (2022)	$a = 0.09$ fm, $m_\pi = 827$ MeV, $P^z_{\max} = 3.3$ GeV	N/A	No	N/A	LO	N/A
SWZ21 PRD 104 (2021)	$a = 0.12$ fm, $m_\pi = 580$ MeV, $P^z_{\max} = 1.5$ GeV	Yes	Yes	Yes	NLO	Yes

Lattice results on the CS kernel

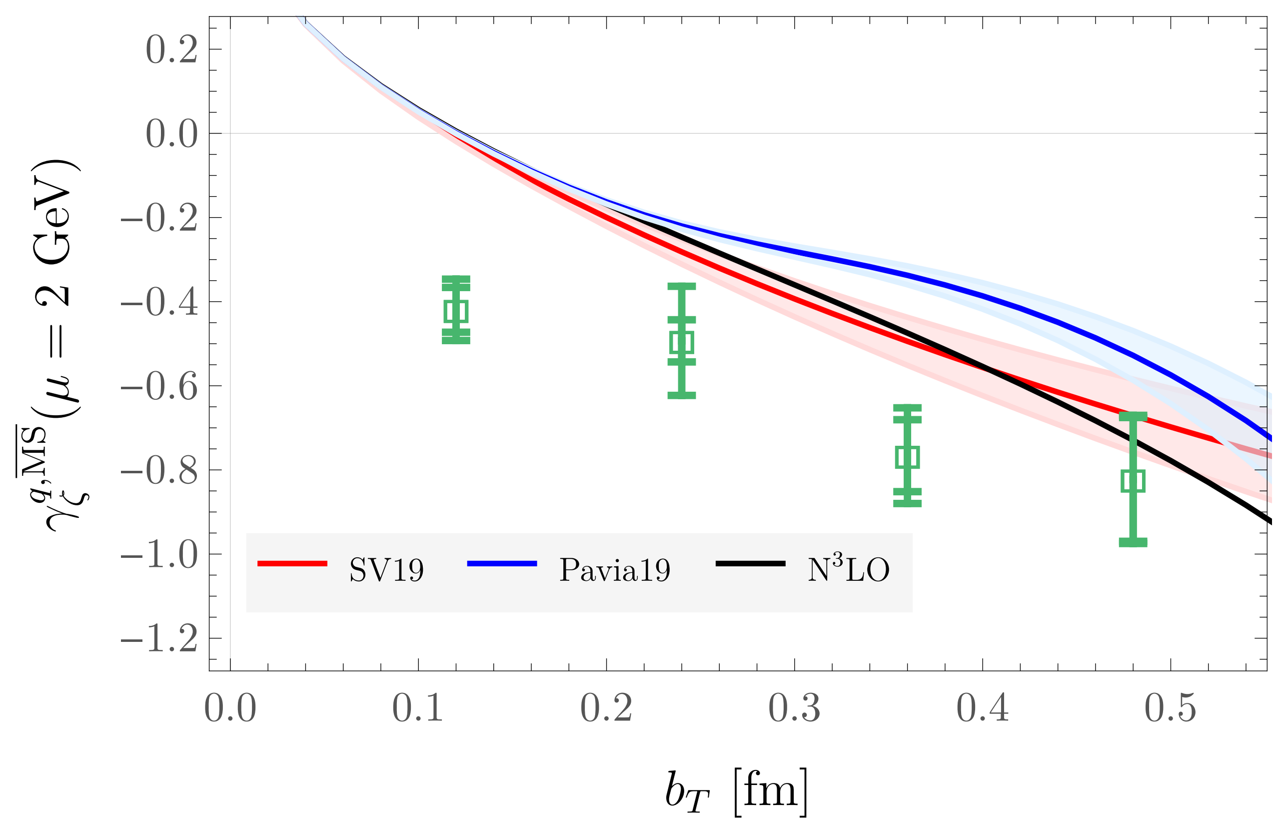
$a = 0.12$ fm, $m_\pi = 580$ MeV,

$P^z_{\max} = 1.5$ GeV

Shanahan, Wagman and YZ, PRD **104** (2021).



Comparison with phenomenology



SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020)
Pavia19: A. Bacchetta et al., JHEP 07 (2020)

Discretization and power corrections: $\mathcal{O}\left(\frac{b_T}{\tilde\eta}, \frac{1}{(xb_T\tilde P^z)^2}, \frac{1}{\tilde P^z\tilde\eta}, \frac{\Lambda^2_{\rm QCD}}{(x\tilde P^z)^2}\right)$

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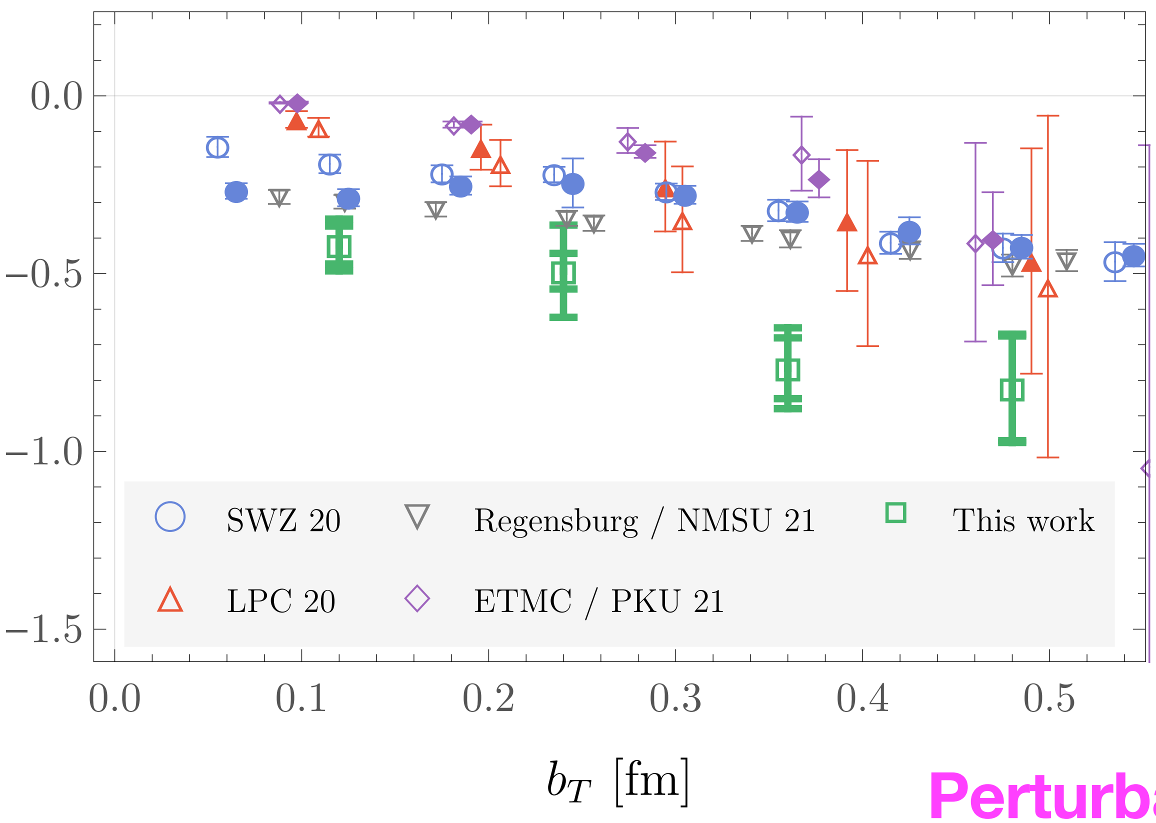

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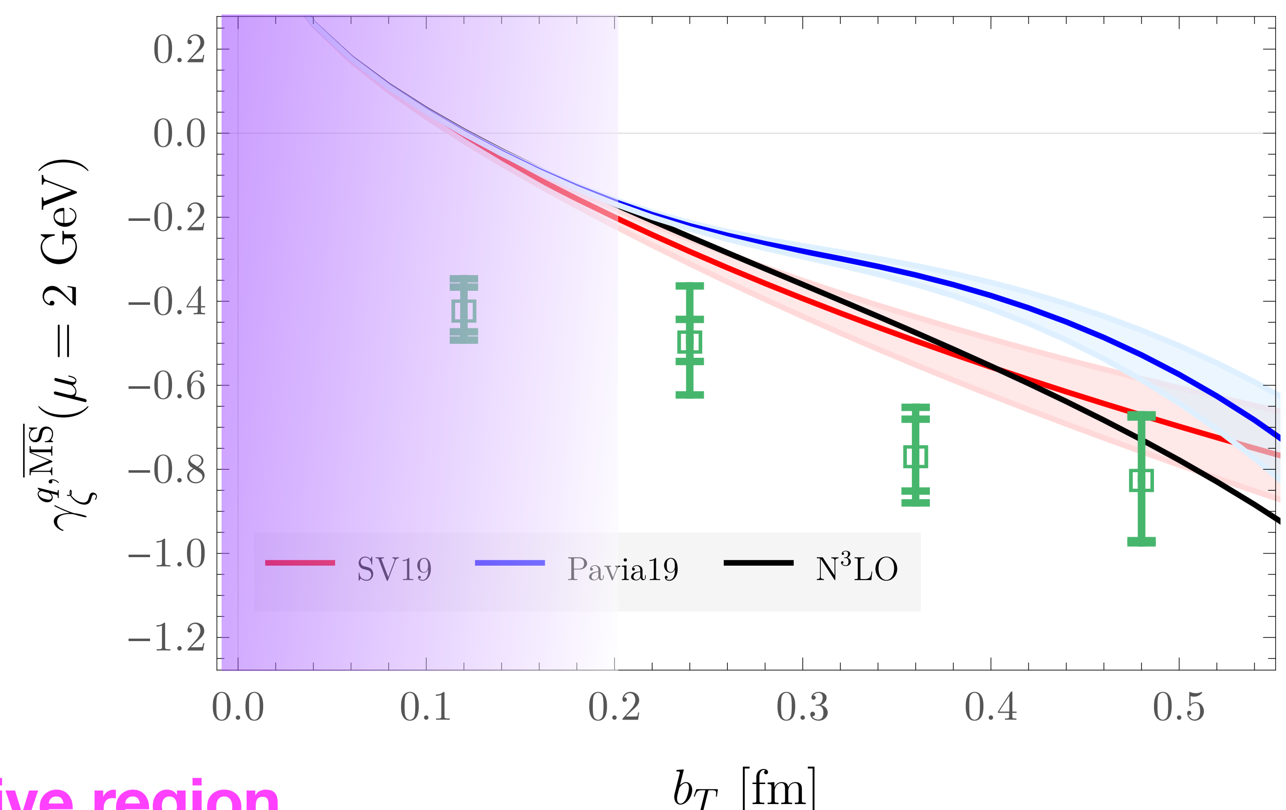

Perturbative region

SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020)
Pavia19: A. Bacchetta et al., JHEP 07 (2020)

Discretization and power corrections: $\mathcal{O}\left(\frac{b_T}{\tilde{\eta}}, \frac{1}{(xb_T\tilde{P}^z)^2}, \frac{1}{\tilde{P}^z\tilde{\eta}}, \frac{\Lambda^2_{\rm QCD}}{(x\tilde{P}^z)^2}\right)$

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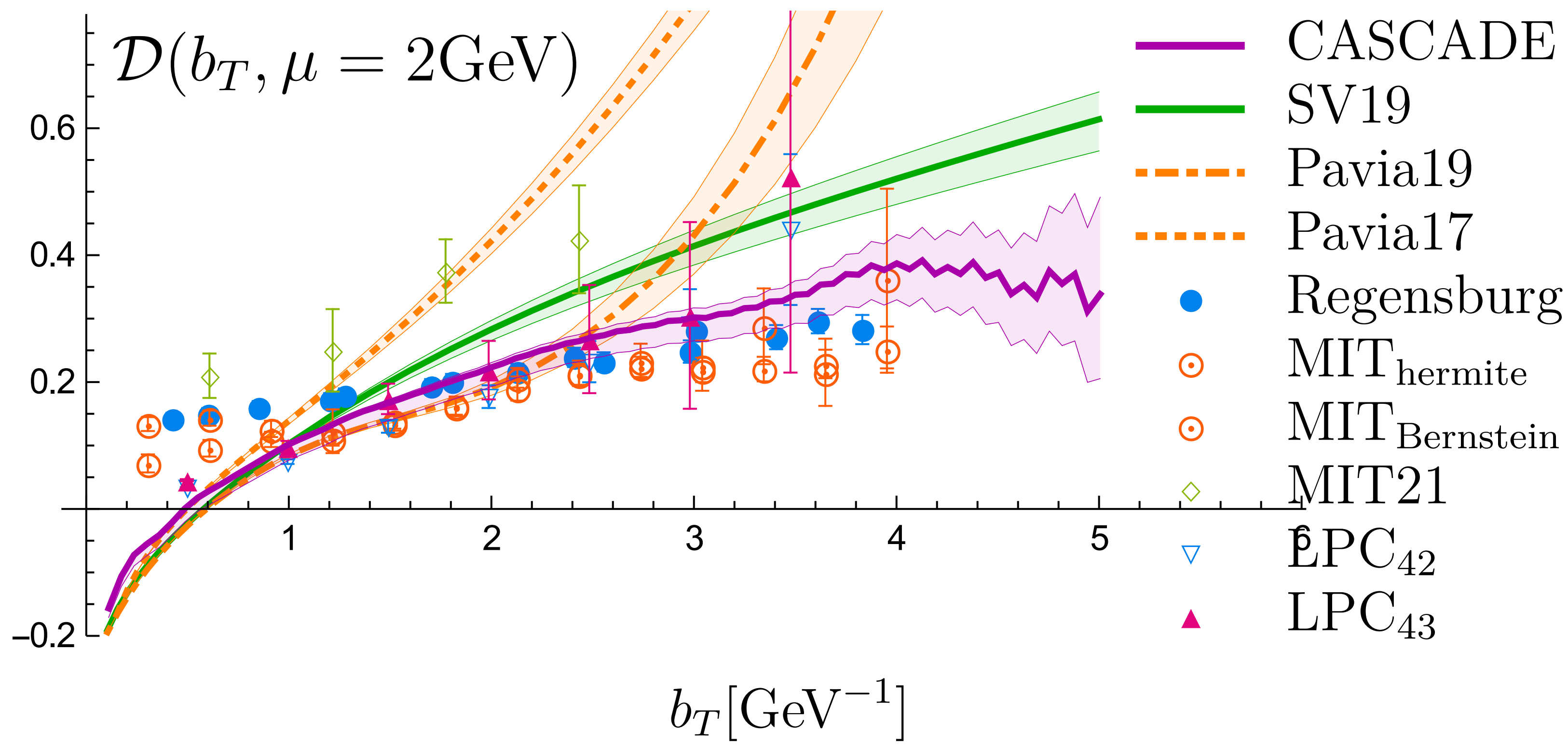

From A. Vladimirov's talk.

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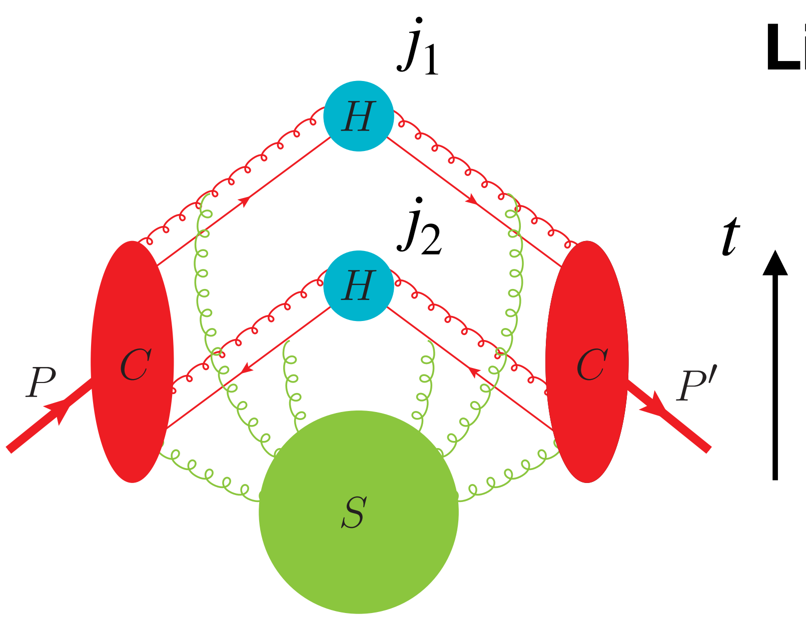
Reduced soft function from LaMET

$$\lim_{\tilde{\eta}\to\infty} \frac{\tilde{f}^{\text{naive}}_{i/h}}{g^q_S(b_T,\mu)} = C(x\tilde{P}^z,\mu)\exp\left[\frac{1}{2}\gamma^q_\zeta(\mu,b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] f_{q/h}(x,\vec{b}_T,\mu,\zeta) + \mathcal{O}(y_{\tilde{P}}^{-k}e^{-y_{\tilde{P}}})$$

Reduced soft function: $S^r(b_T,\mu) = [g^q_S(b_T,\mu)]^2$



Light-meson form factor:

$$F(b_T,P^z) = \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P)\rangle$$

$$\stackrel{P^z \gg m_N}{=} S^r_q(b_T,\mu)\int dx dx'\; H(x,x',\mu) \times \Phi^\dagger(x,b_T,P^z)\Phi(x',b_T,P^z)$$

Φ: Quasi-TMD wave function

$$\tilde{\Phi} = \frac{\langle 0 \,|\, \mathcal{O}(b^\mu) \,|\, \pi(P)\rangle}{\sqrt{\tilde{S}^q_{\text{naive}}}}$$

Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

Reduced soft function from LaMET

- Tree-level approximation:

$$H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s)$$

$$F(b_T, P^z) \overset{P^z \gg m_N}{=} S_q^r(b_T, \mu) \int dx dx' \ \Phi^\dagger(x, b_T, P^z)\Phi(x', b_T, P^z)$$

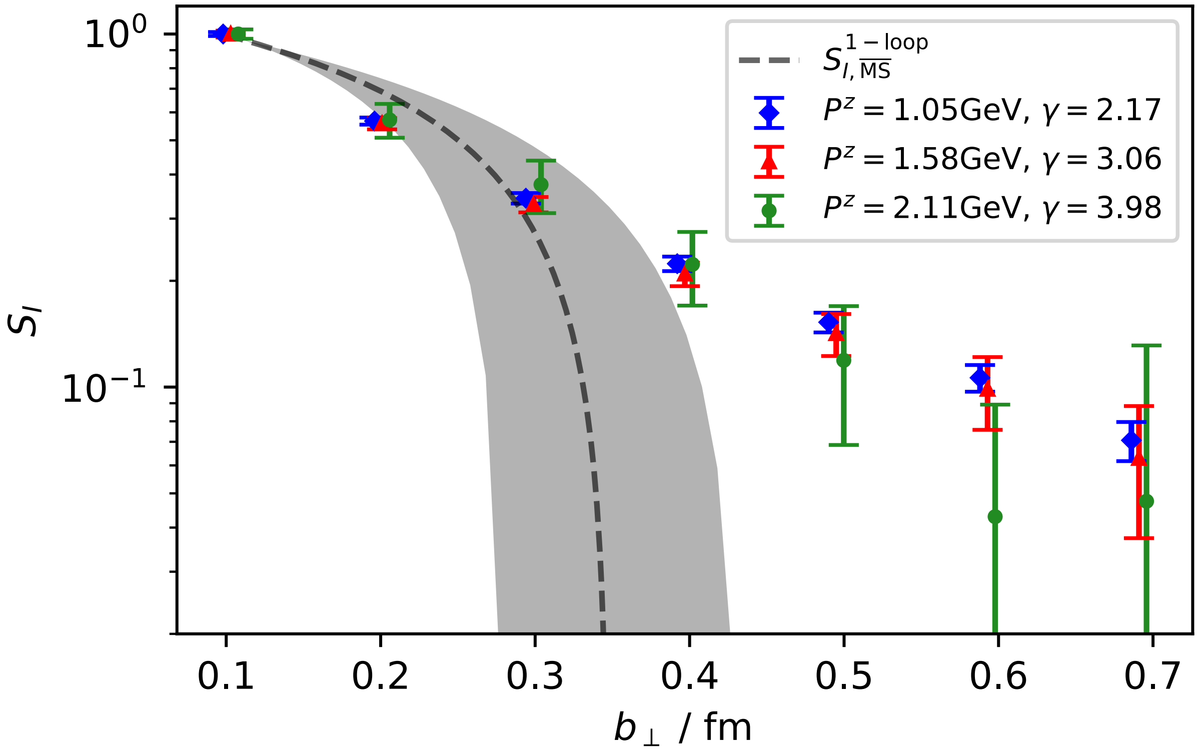
$$= S_q^r(b_T)[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2$$

$$\Rightarrow S_q^r(b_T) = \frac{F(b_T, P^z)}{[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2}$$

- Beyond tree-level, it is necessary to obtain *x*-dependence.

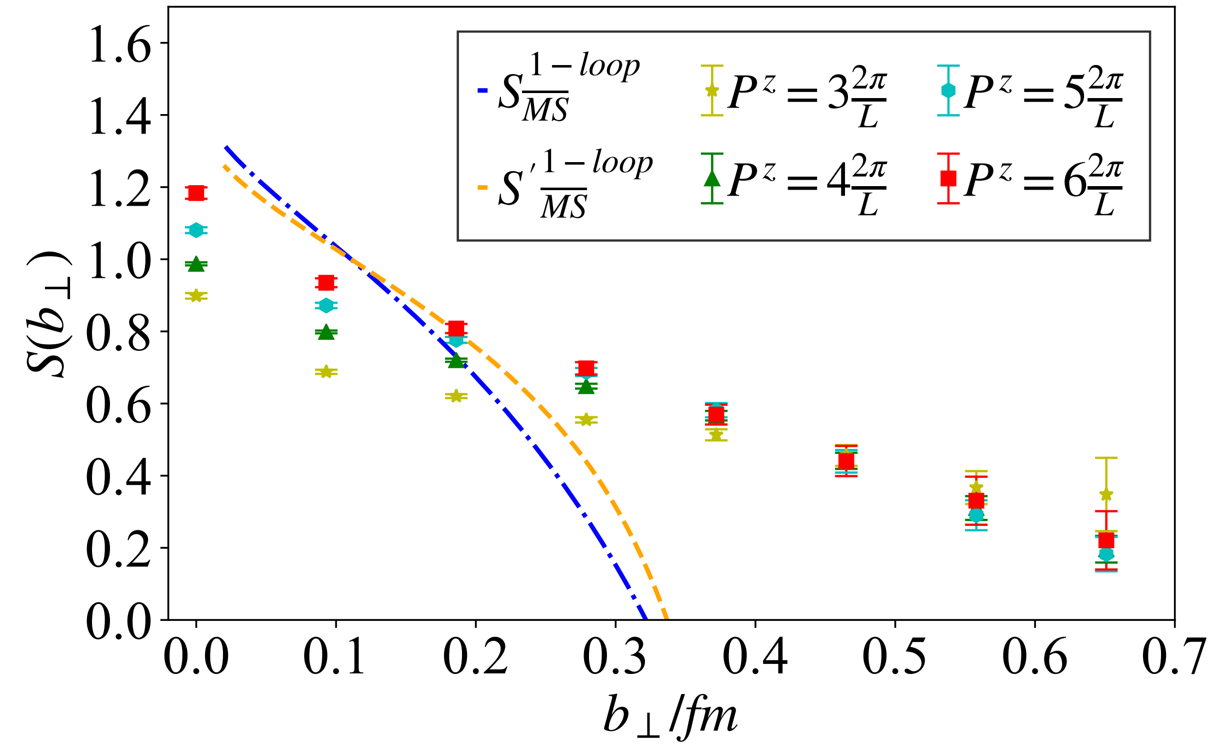
First lattice results with tree-level matching

$a = 0.10$ fm,
$m_\pi = 547$ MeV,
$P^z_{\max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).

$a = 0.09$ fm,
$m_\pi = 827$ MeV,
$P^z_{\max} = 3.3$ GeV



Y. Li et al., PRL 128 (2022).

Outlook

Targets for lattice QCD studies:

Observables	Status
Non-perturbative CS kernel	✔, improving the systematics
Soft factor	✔, to be under systematic control
Info on spin-dependent TMDs (in ratios)	In progress
Info on 3D structure, (x, b_T) (in ratios)	In progress
Proton v.s. pion TMDs, (x, b_T) (in ratios)	In progress
Flavor dependence of TMDs, (x, b_T) (in ratios)	to be studied
TMDs and TMD Wave functions, (x, b_T)	In progress
Gluon TMDs (x, b_T)	to be studied
Wigner distributions/GTMDs (x, b_T)	to be studied

Outlook

- **Systematic uncertainties:**
  - Pion mass dependence;
  - Lattice renormalization: operator mixing, continuum extrapolation;
  - Fourier transform (FT);
  - NLO (and higher order) matching, resummation;
  - Plateau in *x*-space affected by FT and the size of power corrections.
- **Milestones in 5 years:**
  - 5-10% level precision for the quark CS kernel for 0.4 GeV < b_T^{-1} < 2 GeV;
  - 5-10% level precision for ratios of spin-dependent (quark) TMDs;
  - Controlling the systematics in the soft function for obtaining the (x, b_T) dependence of (quark) TMDs;
  - Extend to the gluons.