Gravitational form factors on the lattice

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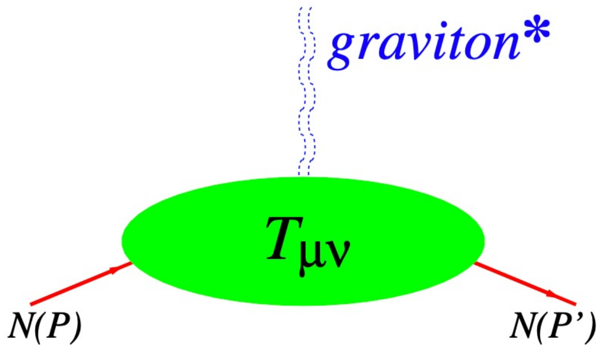
Outline



Gravitational form factors (GFFs)

- Physics motivation(s)
  - Fundamental properties of hadrons
  - Relation to GPDs
- Previous work

(Gluon) GFFs on the lattice [Pefkou DH Shanahan 2022]

- Pion, nucleon, ρ meson, Δ baryon at $M_\pi = 450\ \mathrm{MeV}$

Spatial densities of energy, pressure, shear forces

[cf. talks from Tuesday morning GPD session]

Motivation 1: Gravitational/EMT form factors (GFFs)

For (symmetric) EMT, $T^{\{\mu\nu\}} = T_g^{\{\mu\nu\}} + \sum_q T_q^{\{\mu\nu\}}$ ← $\{\ \} \equiv$ symmetrize e.g. $a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{\psi}\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\psi$ ← $\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

GFFs decompose hadronic matrix elements of T, e.g. for nucleon:

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

$$\langle N(p',s')|T_{g,q}^{\{\mu\nu\}}|N(p,s)\rangle = \bar{u}(p',s')\left[A_{g,q}(t)\,\gamma^{\{\mu}P^{\nu\}} + B_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)Mg^{\mu\nu}\right]u(p,s)$$

$u, \bar{u}$ = Dirac spinors $P = (p'+p)/2$ $\Delta = p' - p$ $t = \Delta^2$

Physics:

$A_{q,g}(t) \sim$ momentum of constituents

→ Momentum fraction $A_{q,g}(0) = \langle x\rangle_{q,g}$

$J_{q,g}(t) = \frac{1}{2}(A_{q,g}(t) + B_{q,g}(t)) \sim$ angular momentum

→ Total $J(0) = \frac{1}{2}$

$D_{q,g}(t) \sim$ pressure and shear forces

Total $D(0)$: "the last global unknown"

[Polyakov Schweitzer 2018]

em: $\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$Q =$	$1.602176487(40) \times 10^{-19}$C
			$\mu =$	$2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$g_A =$	$1.2694(28)$
			$g_p =$	$8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$m =$	$938.272013(23)\,\mathrm{MeV}/c^2$
			$J =$	$\frac{1}{2}$
			$\boldsymbol{D} =$	**?**

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

Motivation 2: Generalized Parton Distributions (GPDs)

Lattice approaches:

e.g. LaMET/SDF

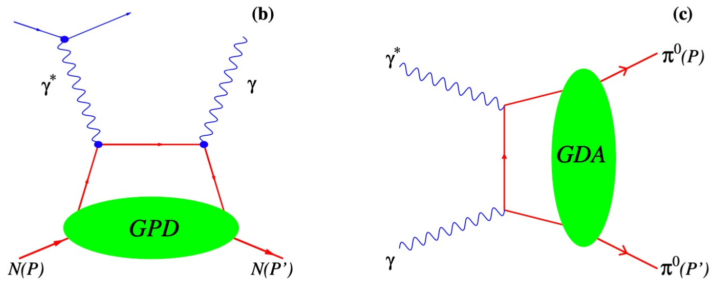
e.g. Compton amplitudes

This work: GFFs = lowest Mellin moments of (unpolarized) GPDs

e.g. nucleon

$$\int dx\, x\, H_{q,g}(x,\xi,t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$

$$\int dx\, x\, E_{q,g}(x,\xi,t) = B_{q,g}(t) - \xi^2 D_{q,g}(t)$$



[Polyakov Schweitzer 2018]

Experimental access:

Quarks:

JLAB: proton D term extracted from DVCS

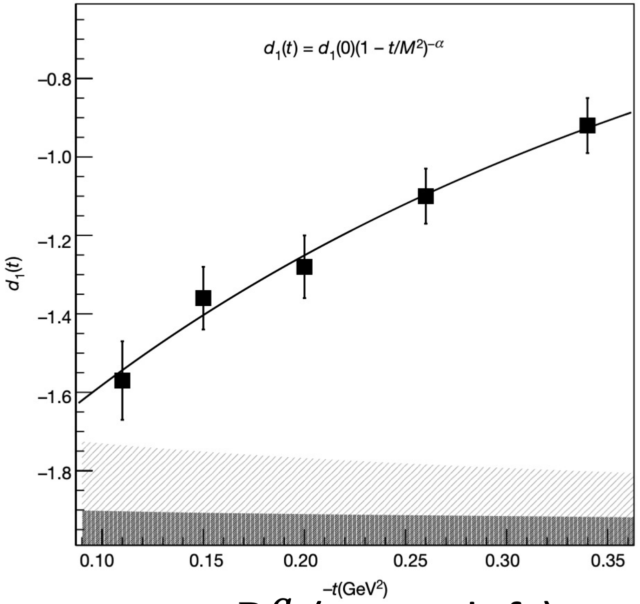
[Burkert Elouadrhiri Girod 2018]

Belle: pion GFFs extracted from $\gamma^*\gamma \to \pi^0\pi^0$

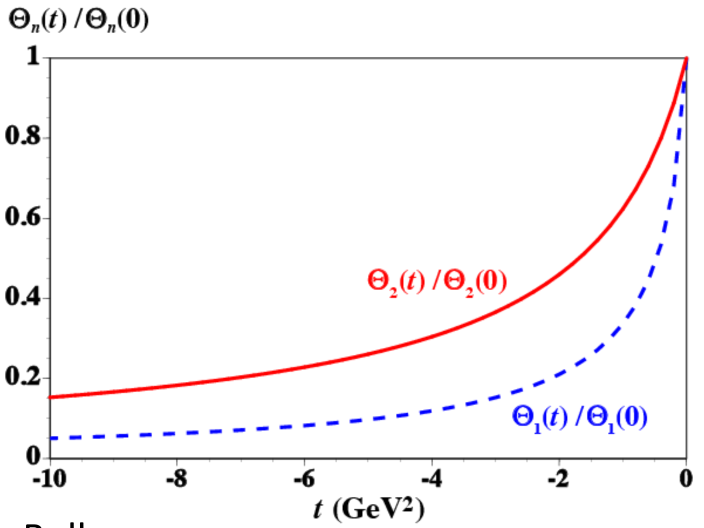
[Kumano Song Teryaev 2017]

Glue:

Future: gluon GFFs from e.g. J/ψ and Υ leptoproduction



Proton D^q (up to defs)



Belle

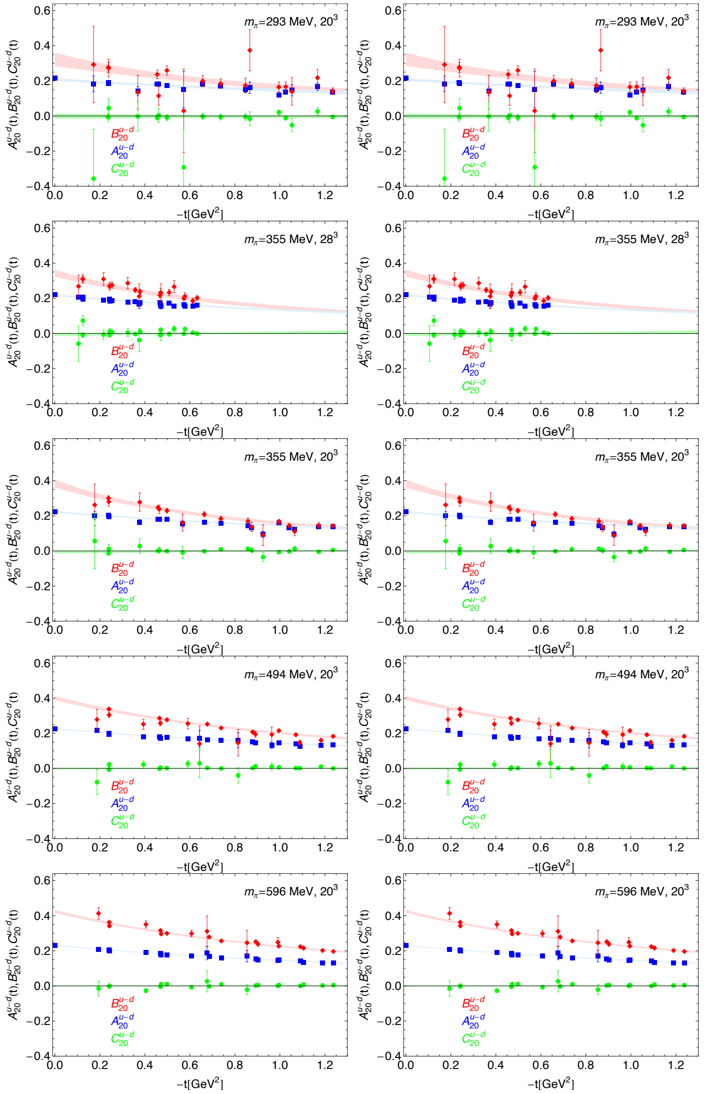
Pion D^q (up to defs)

Previous work

Many results for quarks, few for glue at $t \neq 0$

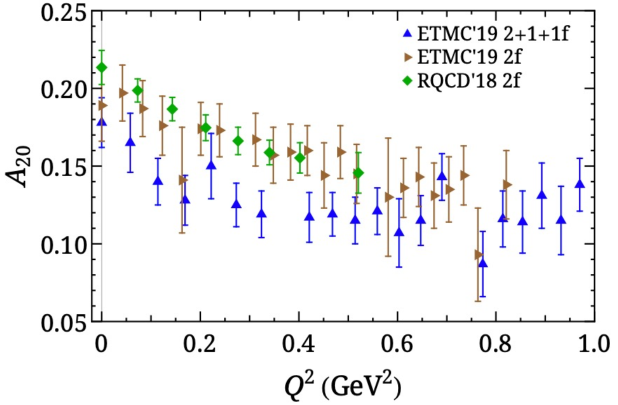
Nucleon quark

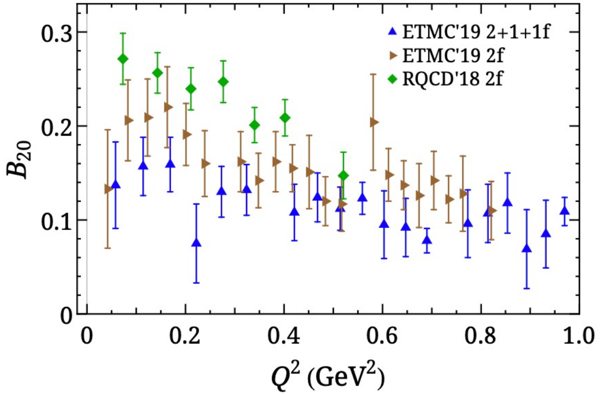
[LHPC 2010]



Nucleon quark

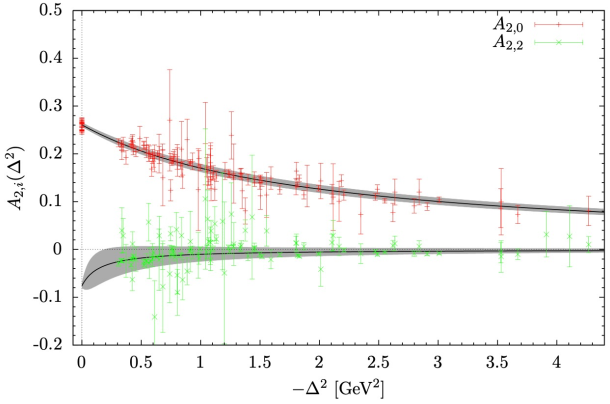
[ETMC 2020, RQCD 2019 (fig: 2006.08636)]





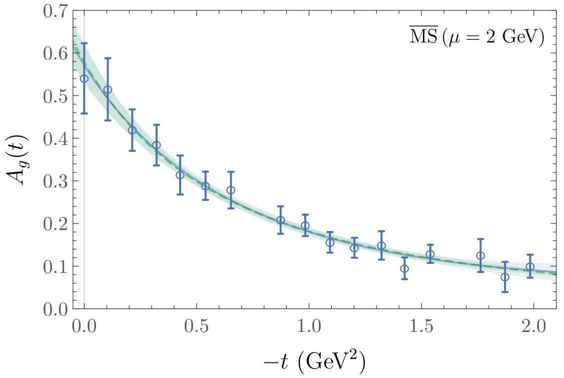
Pion quark

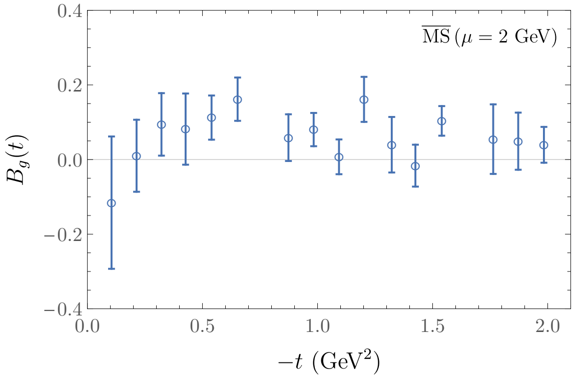
[Brommel 2007]

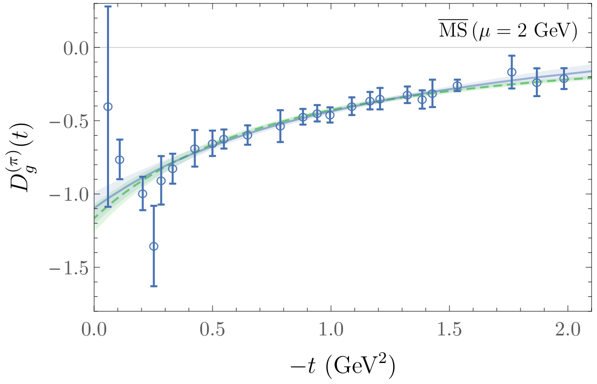


Nucleon glue

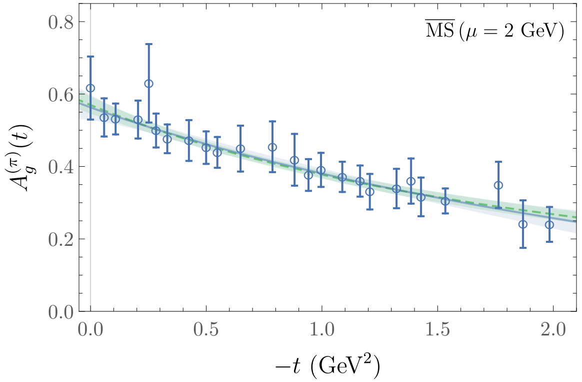
[Shanahan Detmold 2019]

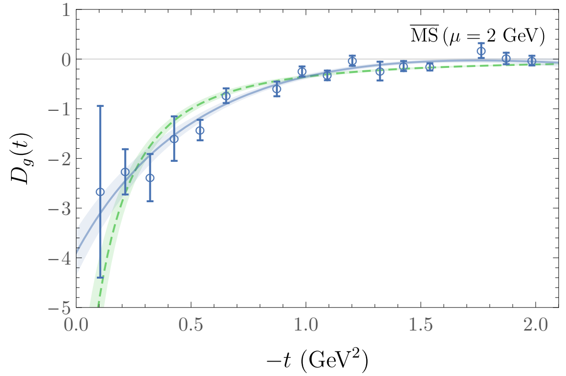






Pion glue





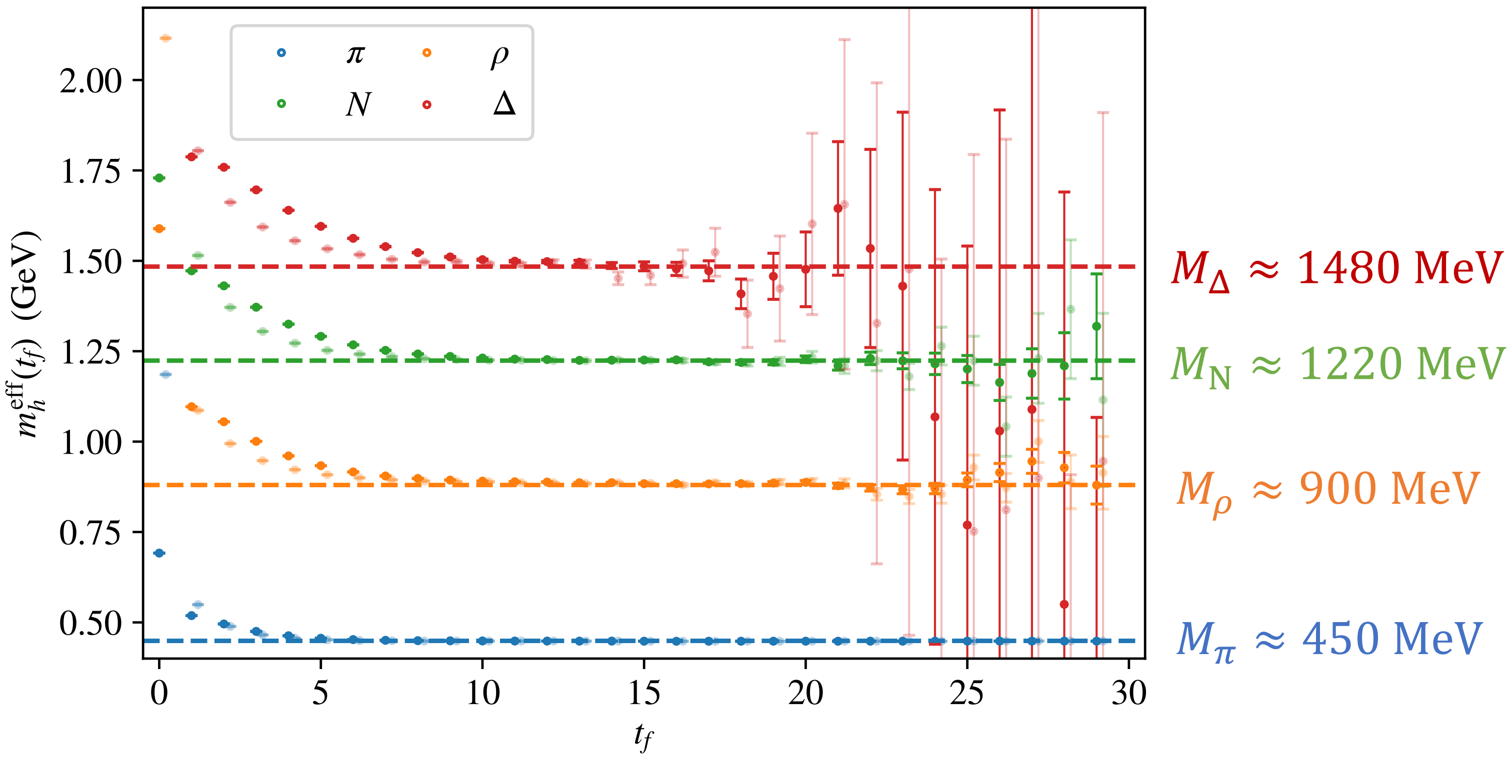
Lattice calculation

Ensemble:

- $32^3 \times 96$ lattice, $M_\pi L \sim 8.5$
- Gauge action: Lüscher-Weisz
- Fermion action: 2+1 Wilson clover
  - Stout links, tree-level tadpole c_{sw}
- $a = 0.1167(16)$ fm
- $M_\pi = 450(5)$ MeV
  - $\rightarrow \rho, \Delta$ are stable
- 2820 configs, $\approx$ 235 sources/config
- Two source/sink smearings

Compute glue GFFs only

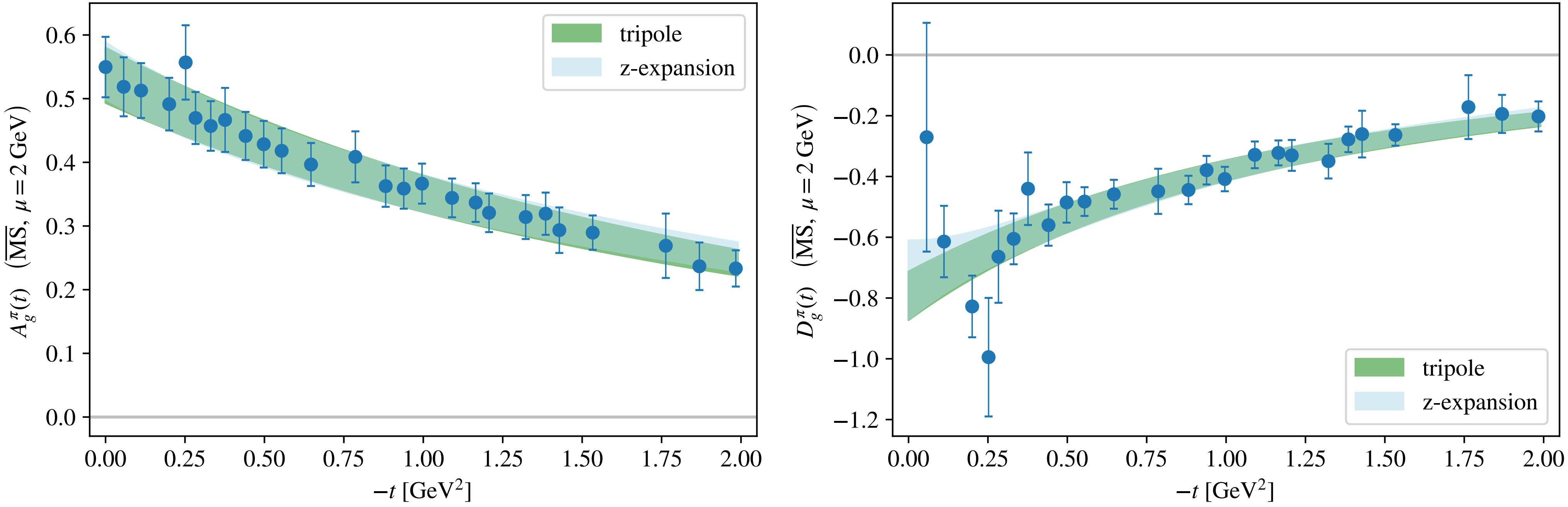
- Neglect mixing w/ quark GFFs under renormalization – expected around few % level $\ll$ stat uncertainties



Results: pion

$\sim \langle x \rangle_g \; [\sum_q A_q(0) + A_g(0) = 1]$

$$\left\langle \pi(p') \middle| T_g^{\{\mu\nu\}} \middle| \pi(p) \right\rangle = A_g(t)\, 2P^\mu P^\nu + D_g(t) \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) + \bar{c}_g(t) 2M^2 g^{\mu\nu}$$



Tripole:

$$G(t) = \frac{\alpha}{\left(1 - \frac{t}{\Lambda^2}\right)^3}$$

(Modified) *z*-expansion:

$$G(t) = \frac{1}{\left(1 - \frac{t}{\Lambda^2}\right)^3} \sum_{k=0}^{2} \alpha_k \, [z(t)]^k$$

$$z(t) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut} - t_0}}$$

$$t_0 = t_{cut}(1 - \sqrt{1 + (2\text{ GeV})^2 / t_{cut}})$$

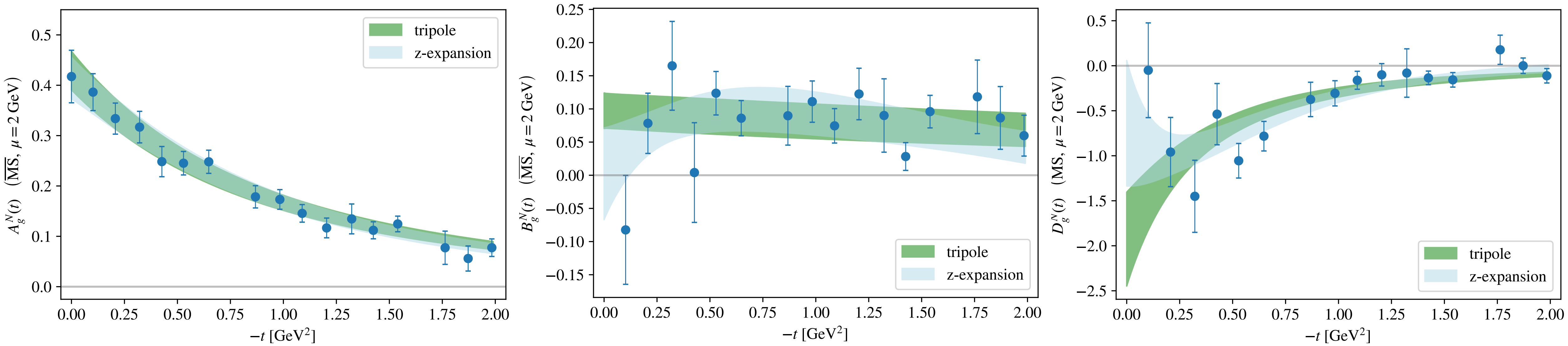
$$t_{cut} = 4M_\pi^2$$

Results: nucleon

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

$\sim \langle x \rangle_g \ [\sum_q A_q(0) + A_g(0) = 1]$

$$\left\langle N(p',s') \middle| T_g^{\{\mu\nu\}} \middle| N(p,s) \right\rangle = \bar{u}(p',s') \left[A_g(t)\, \gamma^{\{\mu} P^{\nu\}} + B_g(t) \frac{i\, P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} + D_g(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_g(t) M g^{\mu\nu} \right] u(p,s)$$



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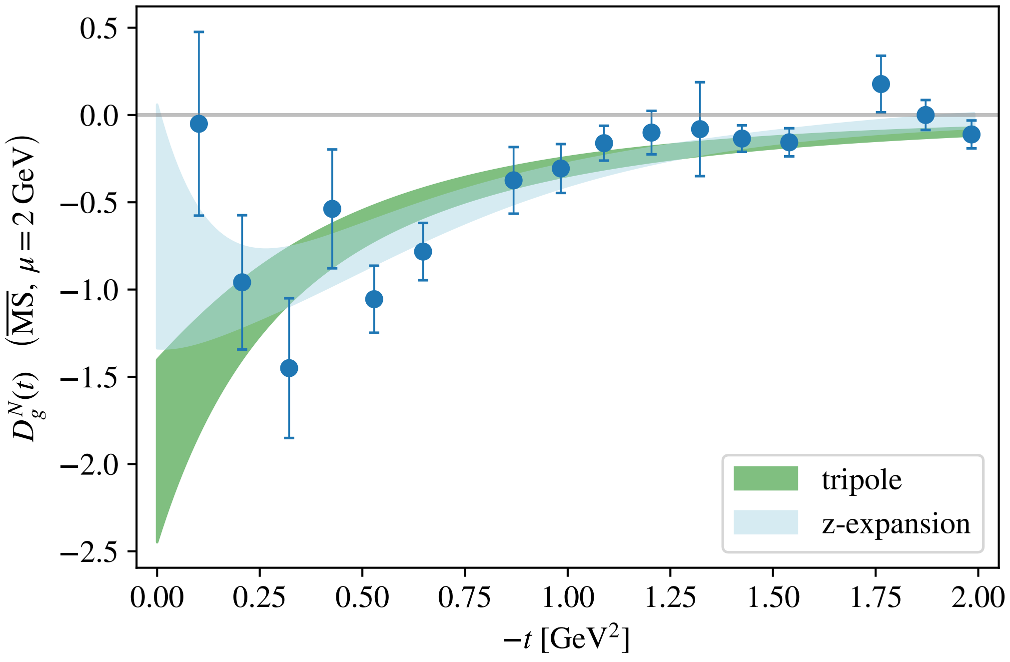
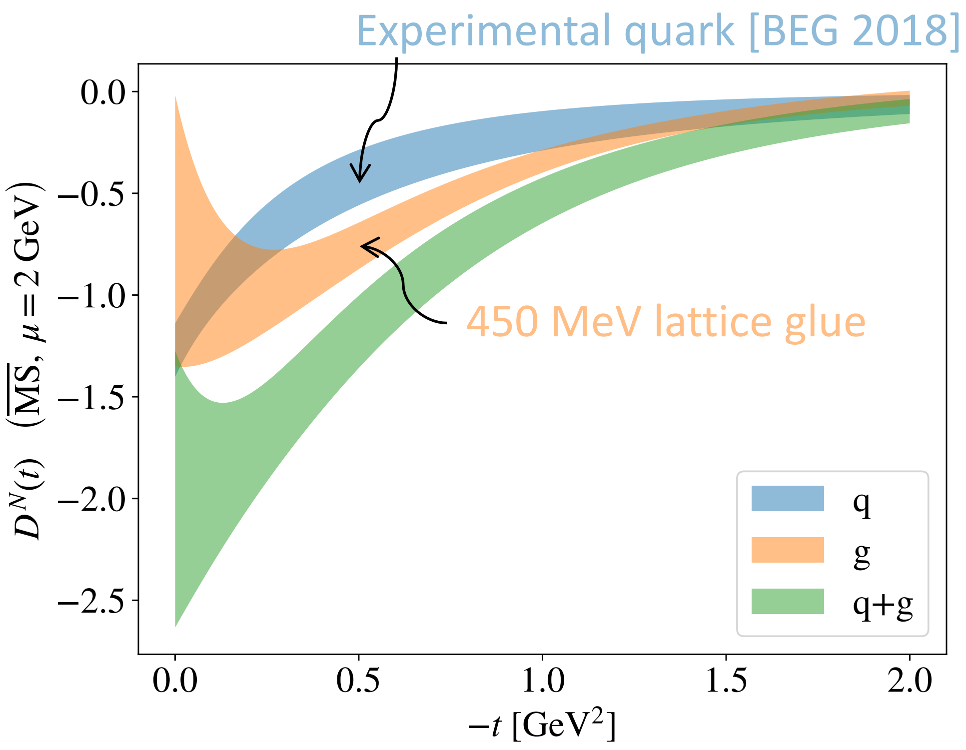
$$t_0 = t_{\text{cut}}\left(1 - \sqrt{1 + (2\text{ GeV})^2 / t_{\text{cut}}}\right)$$

$$t_{\text{cut}} = 4M_\pi^2$$

Glue vs total monotonicity?

$$\left\langle N(p',s')\middle| T_g^{\{\mu\nu\}} \middle| N(p,s)\right\rangle = \bar{u}(p',s')\left[A_g(t)\,\gamma^{\{\mu}P^{\nu\}} + B_g(t)\,\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_g(t)\,\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_g(t) M g^{\mu\nu}\right]u(p,s)$$





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$$z(t) = \frac{\sqrt{t_{\mathrm{cut}} - t} - \sqrt{t_{\mathrm{cut}} - t_0}}{\sqrt{t_{\mathrm{cut}} - t} + \sqrt{t_{\mathrm{cut}} - t_0}}$$

$$t_0 = t_{\mathrm{cut}}(1 - \sqrt{1 + (2\ \mathrm{GeV})^2/t_{\mathrm{cut}}})$$

$$t_{\mathrm{cut}} = 4M_\pi^2$$

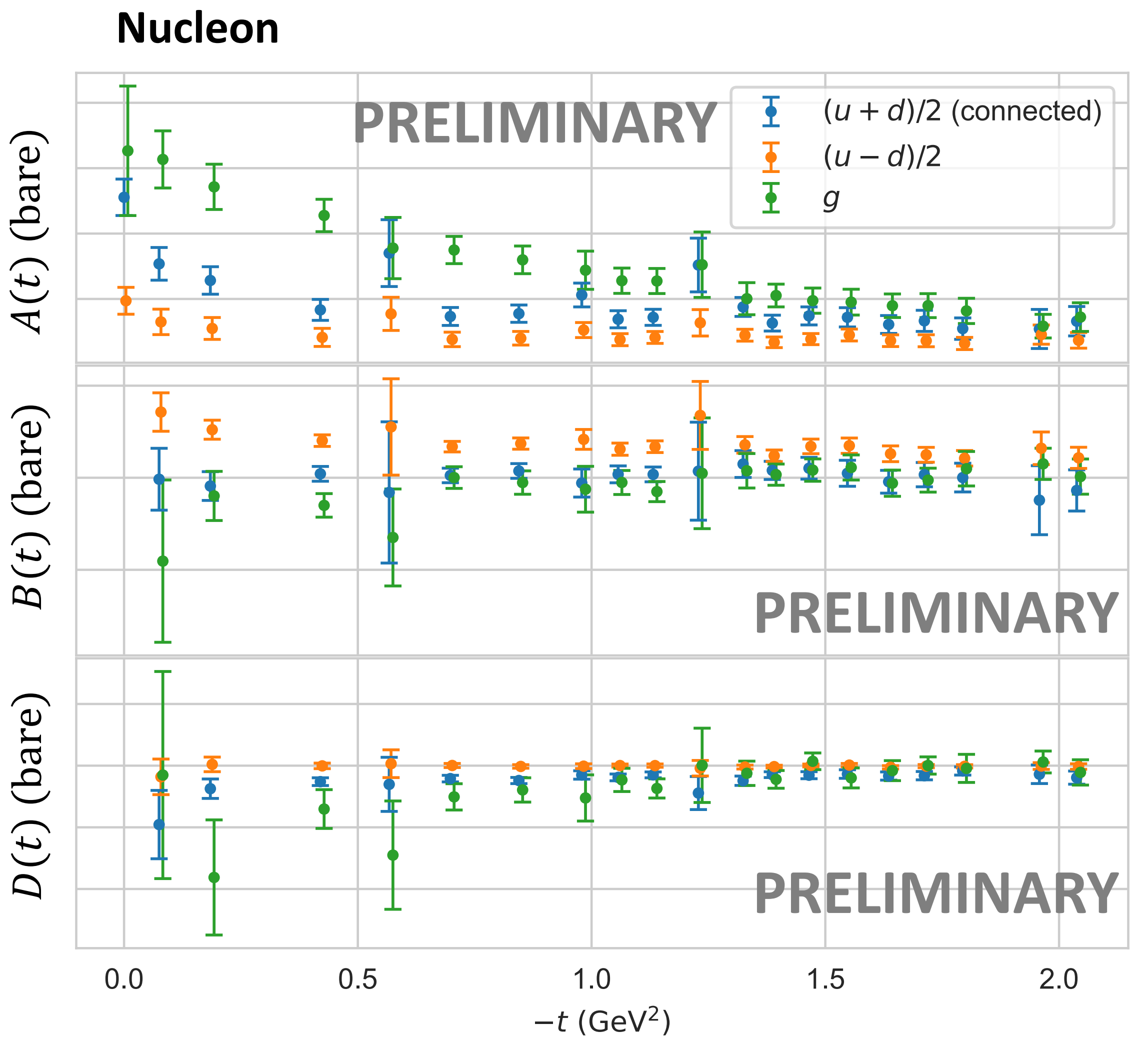
In progress

Compute both quark and glue GFFs on a different ensemble

- Quantify systematics in glue GFFs due to $a \neq 0$, unphysical M_π, mixing with quark GFFs
- Compute total GFFs → non-conserved trace GFFs cancel

Ensemble [“a091m170”]

- Gauge action: Lüscher-Weisz
- Fermion action: 2+1 Wilson clover, stout links
- $M_\pi = 170\ \mathrm{MeV}$
- $a = 0.091\ \mathrm{fm}$ (from w_0)
- $48^3 \times 96$
- ρ, Δ unstable → N, π only



Results: ρ meson

$A(0) \sim \langle x \rangle_g$ i.e. $\sum_q A_{0q}(0) + A_{0g}(0) = 1$

$$\left\langle \rho(p',\lambda') \middle| T_g^{\{\mu\nu\}} \middle| \rho(p,\lambda) \right\rangle = E^*_{\alpha'}(p',\lambda') \Big\{ 2P^{\{\mu}P^{\nu\}} \left[-g^{\alpha'\alpha} A_{0g}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2} A_{1g}(t) \right] + \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) \left[g^{\alpha'\alpha} D_{0g}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2} D_{1g}(t) \right]$$

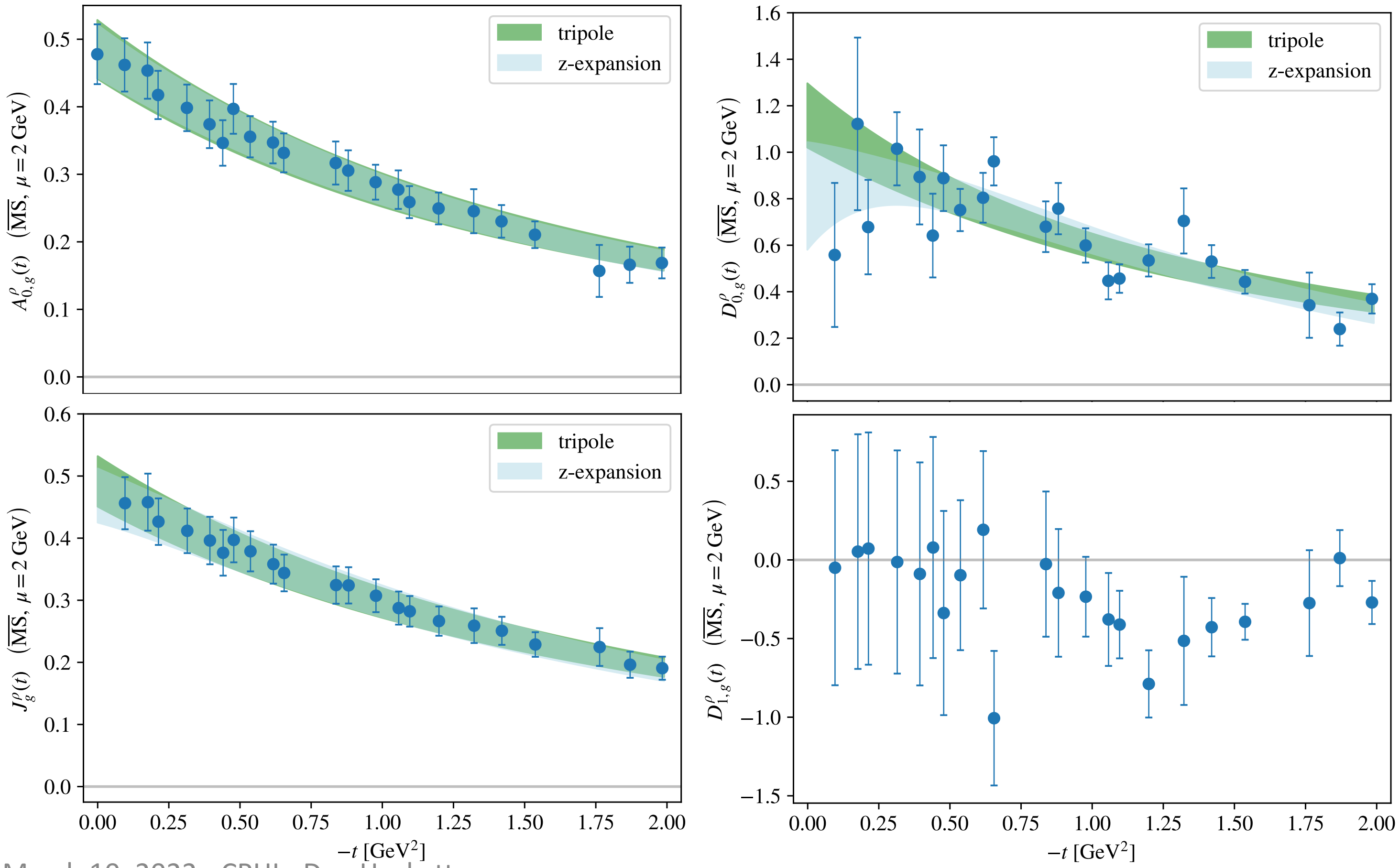
$$+ J_g(t)\, 8P^{\{\mu} g^{\nu\}\{\alpha'} P^{\alpha\}} + E_g(t) \left(g^{\alpha\{\mu} g^{\nu\}\alpha'} \Delta^2 - 2\, g^{\alpha'\{\mu} \Delta^{\nu\}} P^\alpha + 2\, g^{\alpha\{\mu} \Delta^{\nu\}} P^{\alpha'} - 4 g^{\mu\nu} P^\alpha P^{\alpha'} \right)$$

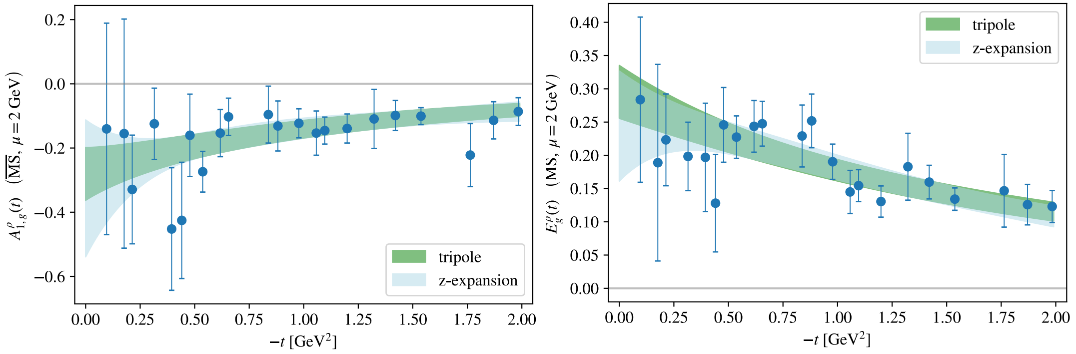
$$+ M^2 \left(2 g^{\alpha'\{\mu} g^{\nu\}\alpha} - \frac{1}{2} g^{\alpha\alpha'} g^{\mu\nu} \right) \bar{f}_g(t) + g^{\mu\nu} \left[g^{\alpha\alpha'} M^2 \bar{c}_{0g}(t) + P^\alpha P^{\alpha'} \bar{c}_{1g}(t) \right] \Big\} E^*_\alpha(p,\lambda)$$

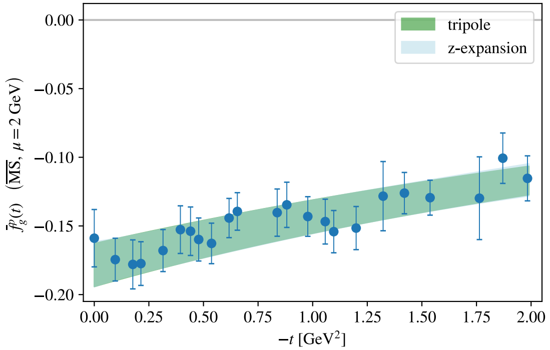
[Polarization vector]

$\sum_q J_q(0) + J_g(0) = 1$

[Not conserved, $\sum_q \bar{f}_q + f_g = 0$]







[conventions: Polyakov, Sun 1912.08749]

Results: Δ baryon

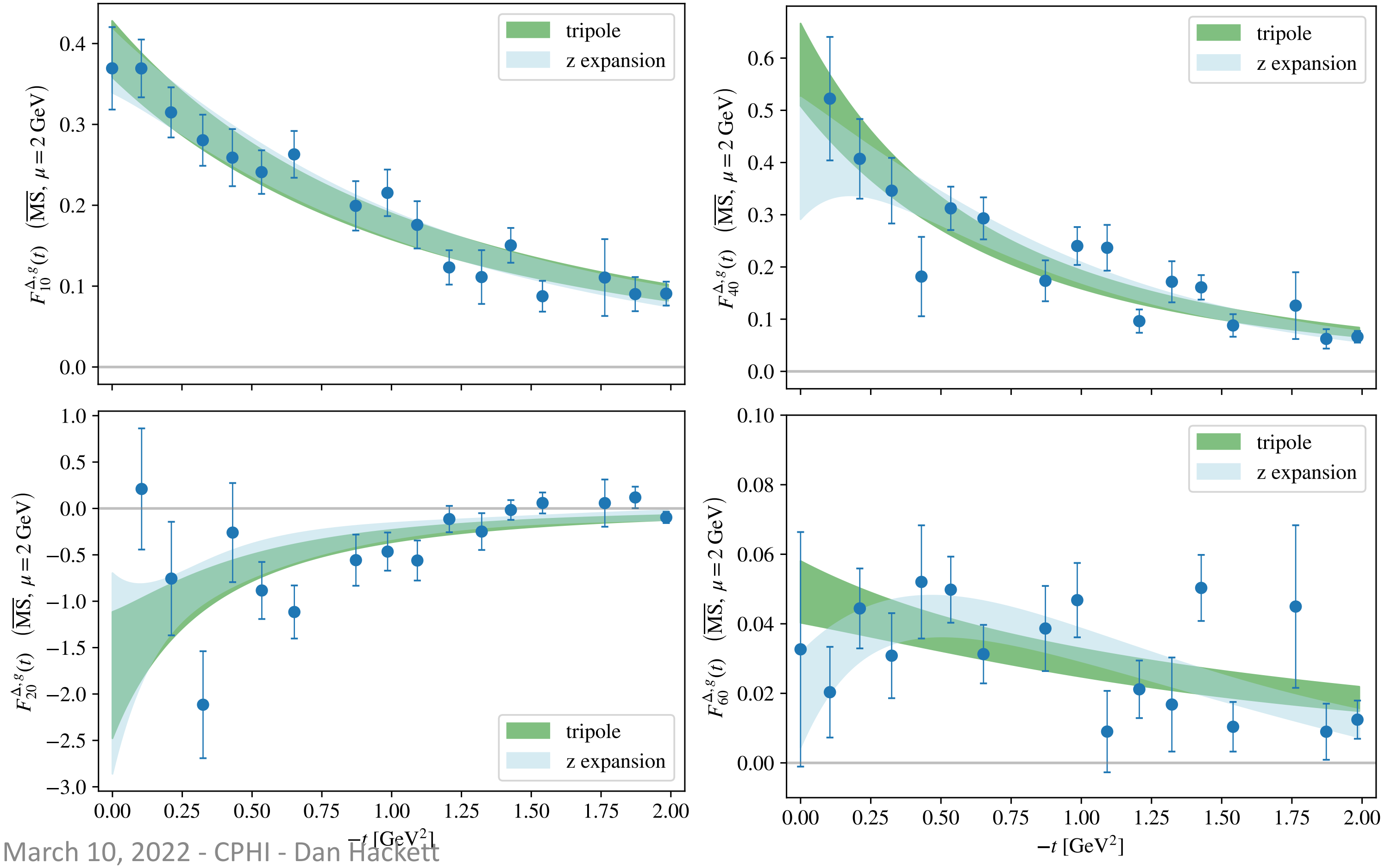
“$A(t)$” i.e. $A(0) \sim \langle x \rangle_g$

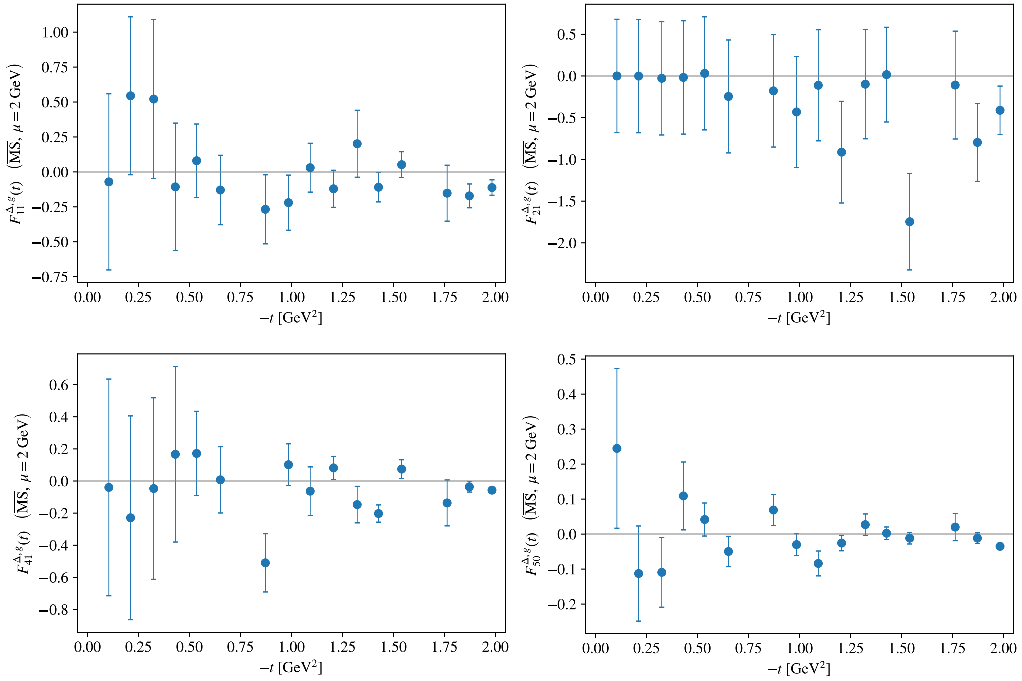
$$\langle \Delta(p',\xi') | T_g^{\mu\nu} | \Delta(p,\xi) \rangle = \bar{u}_{\alpha'}(p',\xi') \Bigg[\frac{P^\mu P^\nu}{M} \left(-g^{\alpha\alpha'} F_{10}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{11}^g(t) \right) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} \left(-g^{\alpha\alpha'} F_{20}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{21}^g(t) \right)$$
$$+ M g^{\mu\nu} \left(-g^{\alpha\alpha'} F_{30}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{31}^g(t) \right) + \frac{i P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{M} \left(-g^{\alpha'\alpha} F_{40}^g(t) + \frac{\Delta^{\alpha'}\Delta^\alpha}{2M^2} F_{41}^g(t) \right)$$
$$+ \frac{2}{M} \left(\Delta^{\{\mu} g^{\nu\}\{\alpha'} \Delta^{\alpha\}} - g^{\mu\nu}\Delta^\alpha\Delta^{\alpha'} - g^{\alpha'\{\mu} g^{\nu\}\alpha} \Delta^2 \right) F_{50}^g(t) - 2 g^{\alpha'\{\mu} g^{\nu\}\alpha} M F_{60}^g(t) \Bigg] u_\alpha(p,\xi)$$

[Rarita-Schwinger spinvector]

“$J(t)$”, $\sum_q F_{40}^q + F_{40}^g = 3/2$

[Not conserved]



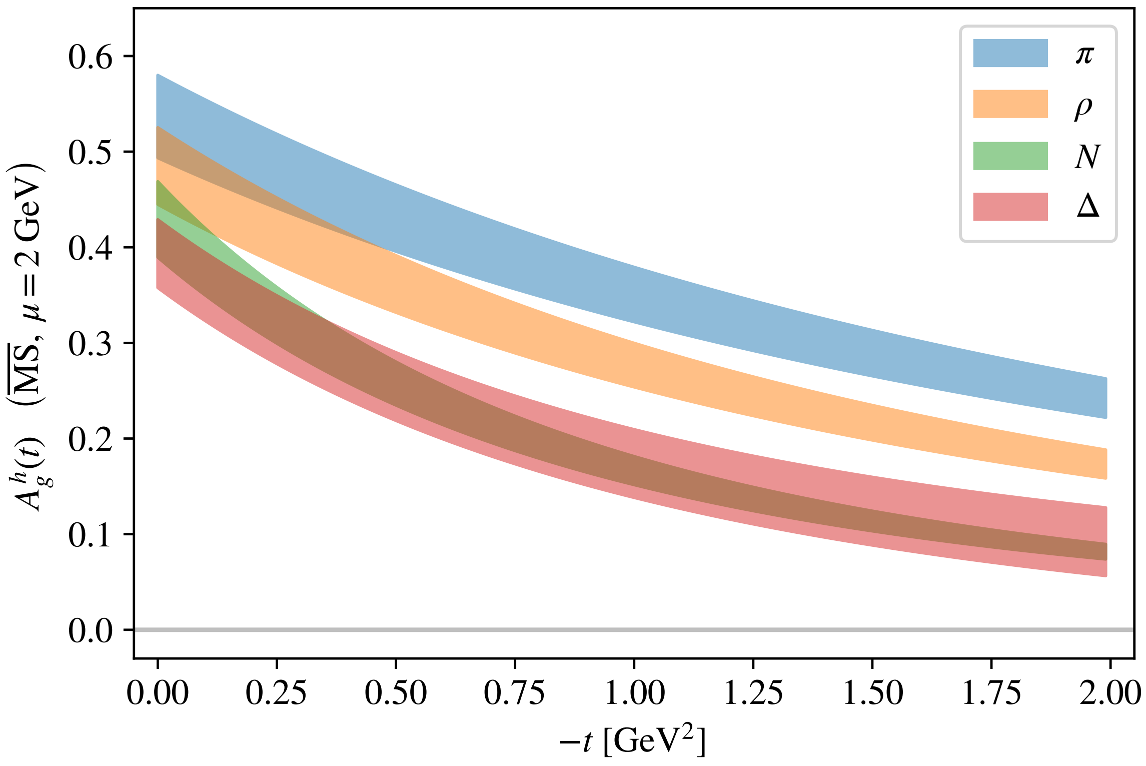


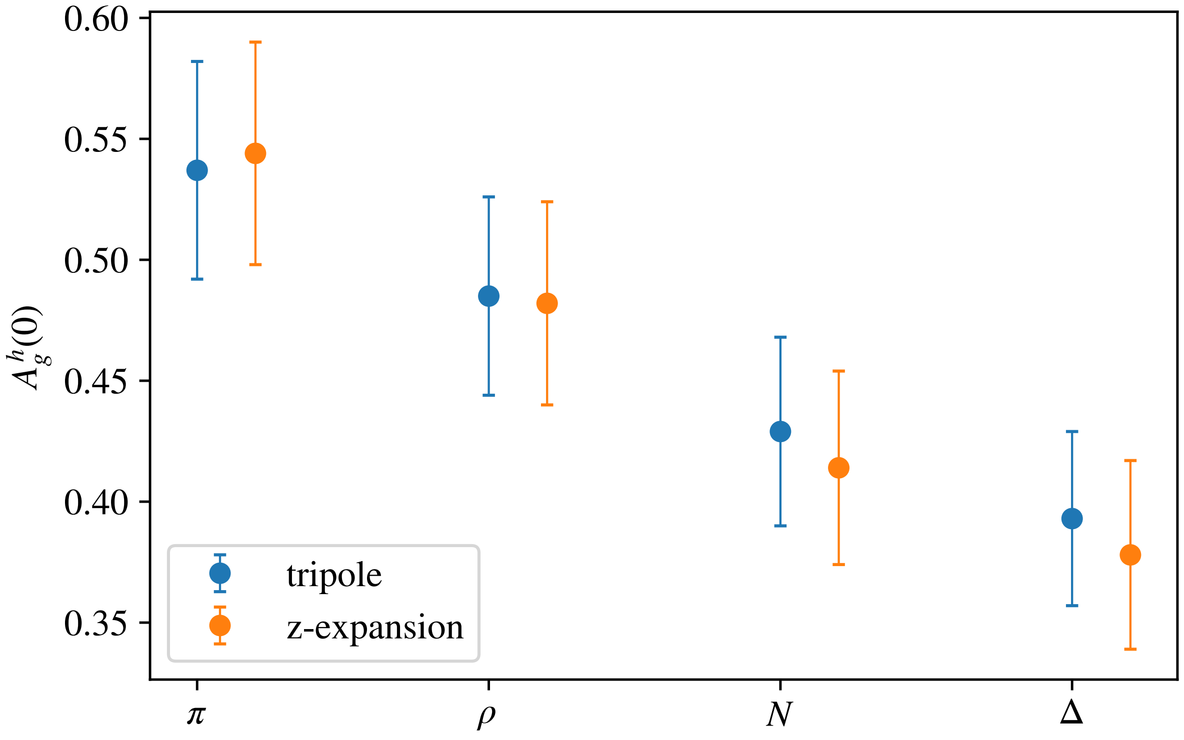
[conventions: Kim, Sun 2011.00292]

Comparison: glue momentum fraction

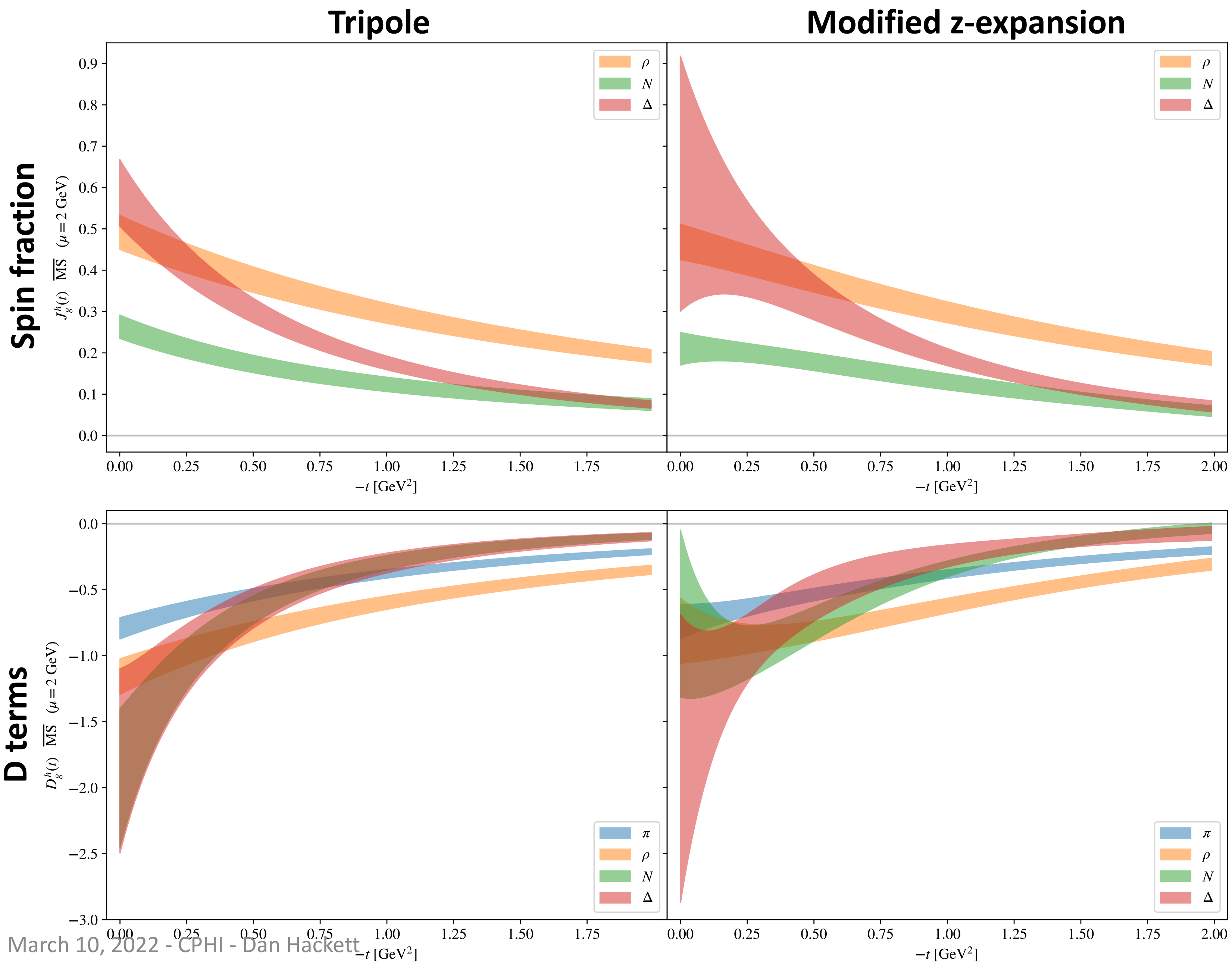
Tripole and z-expansion $A(t)$ same w/in error

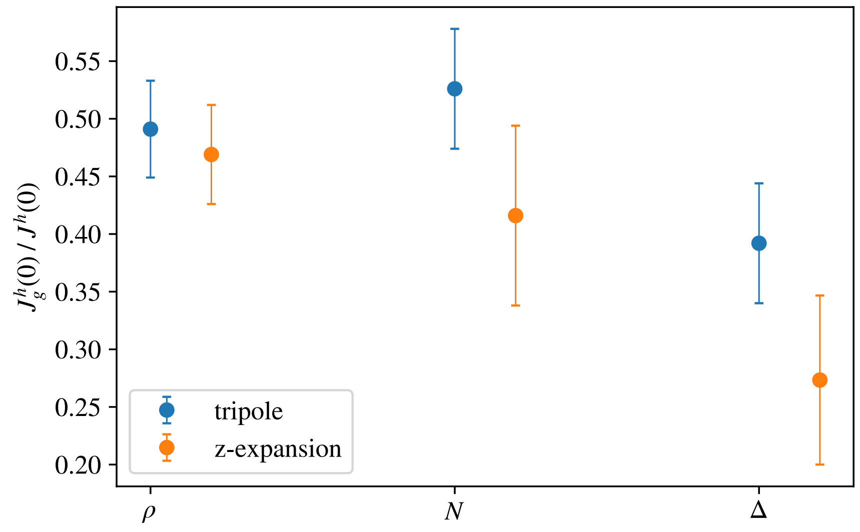
→ Little model dependence





Comparison: glue spin fraction, D terms





Strong model dependence:
nonmonotonic z-exp fits vs
monotonic-by-construction tripole fits

Energy, pressure, and shear force densities

Idea:

Fourier transform to get spatial EMT density $T_{\mu\nu}(r) = \mathrm{FT}[\, \mathrm{T}_{\mu\nu}(\Delta)\,]$

Identify e.g.: $T_{tt}(r) = \epsilon(r)$

$$T_{ij}(r) = \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r)$$

→ Spatial densities of energy $\epsilon(r)$, pressure $p(r)$, shear forces $s(r)$

Complication 1: physical interpretation?

Complication 2: frame dependence

3D Breit frame (BF3): $\Delta^0 = 0, \mathbf{P} = \mathbf{0}$

"Traditional" frame, but recent work questions interpretation as spatial density [see e.g. Panteleeva Polyakov 2102.10902, Freese Miller 2102.01683, Jaffe 2010.15887, Lorce 2007.05318, Lorce Moutarde Trawinski 1810.09837 etc.]

2D Breit frame (BF2)

Infinite momentum frame (IMF): $\mathbf{\Delta} \cdot \mathbf{P} = 0, P_z \to \infty$

$$T^{\mu\nu}_{i,\mathrm{BF3}}(r) = \int \frac{d^3\Delta e^{-i\mathbf{\Delta}\cdot\mathbf{r}}}{2P^0(2\pi)^3} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{BF2}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|_{\mathbf{P}=0}$$

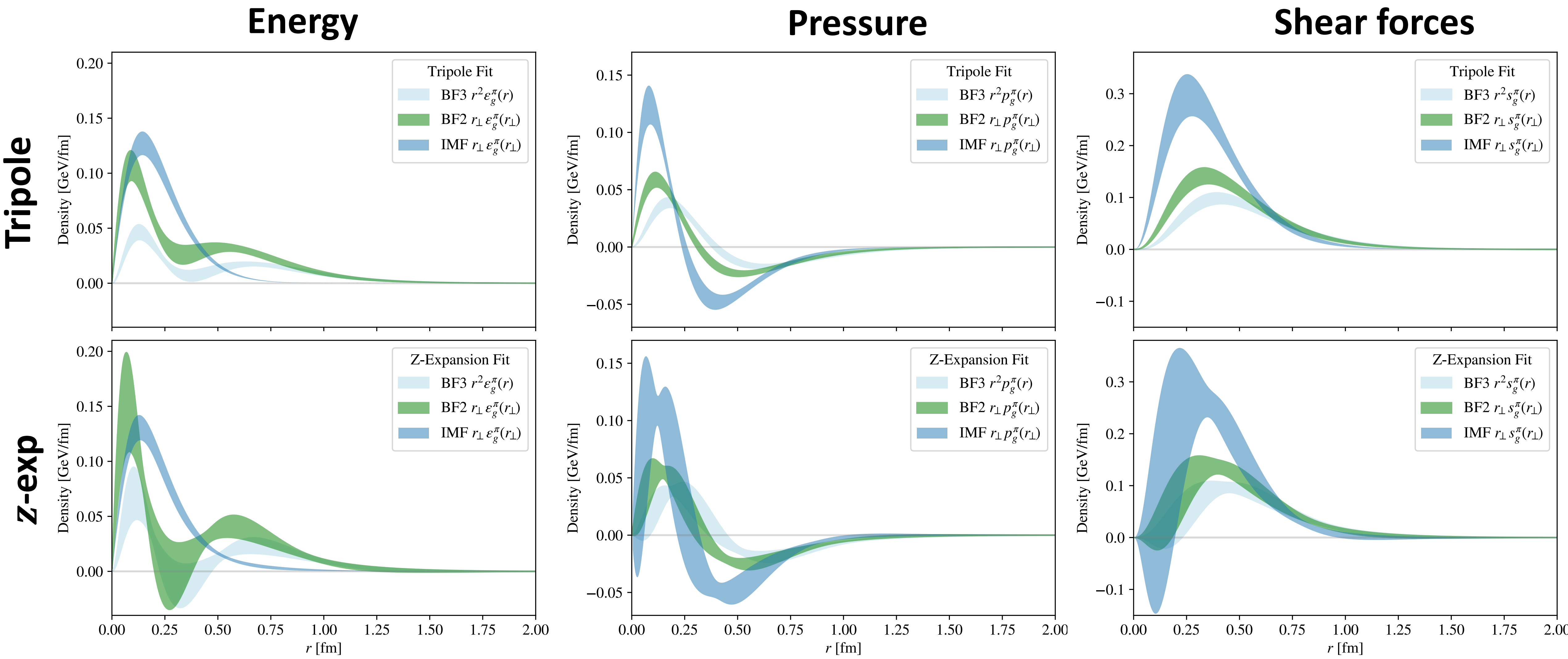
$$T^{\mu\nu}_{i,\mathrm{IMF}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|^{P_z\to\infty}_{\mathbf{P}\cdot\mathbf{\Delta}=0}$$

Different identifications of $\epsilon(r)$, $p(r)$, $s(r)$ with GFFs in each frame

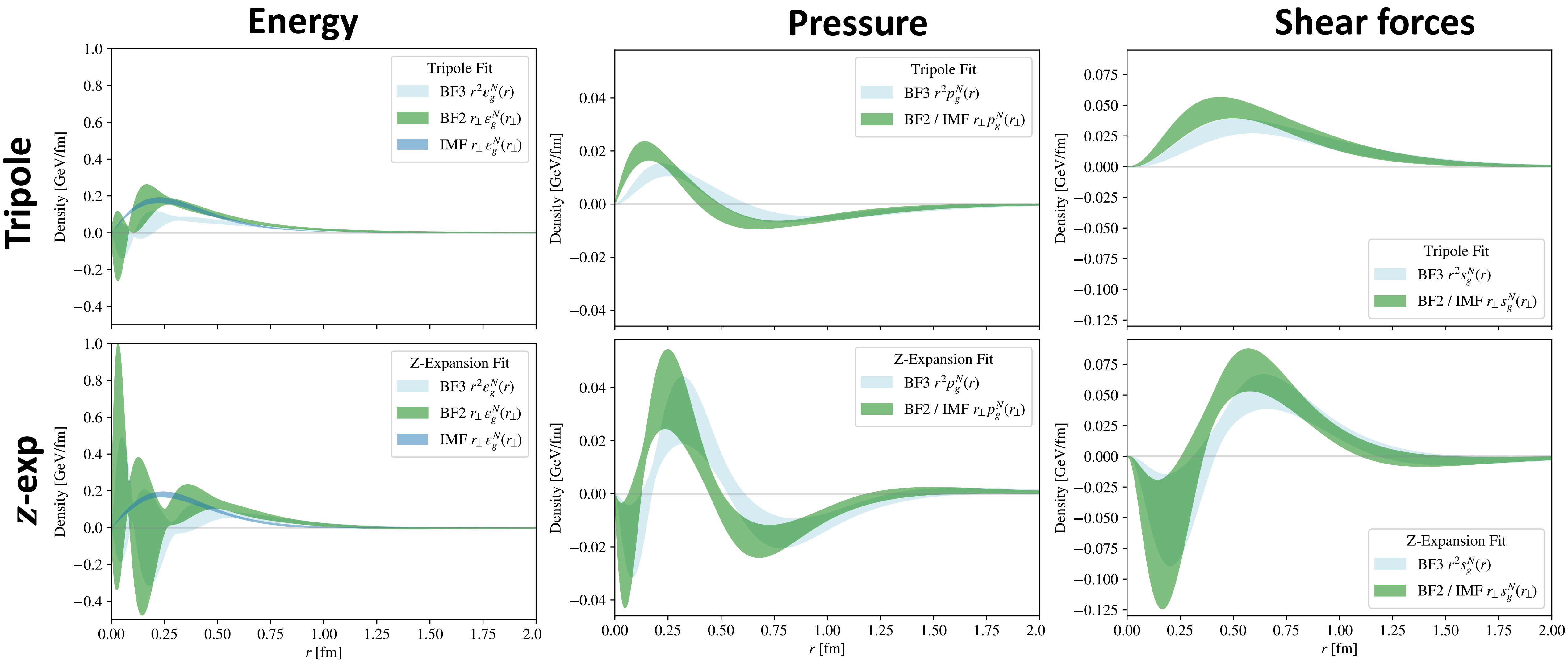
Complication 3: No trace GFFs $\sim \bar{c}$

See [2107.10368] for details, expressions

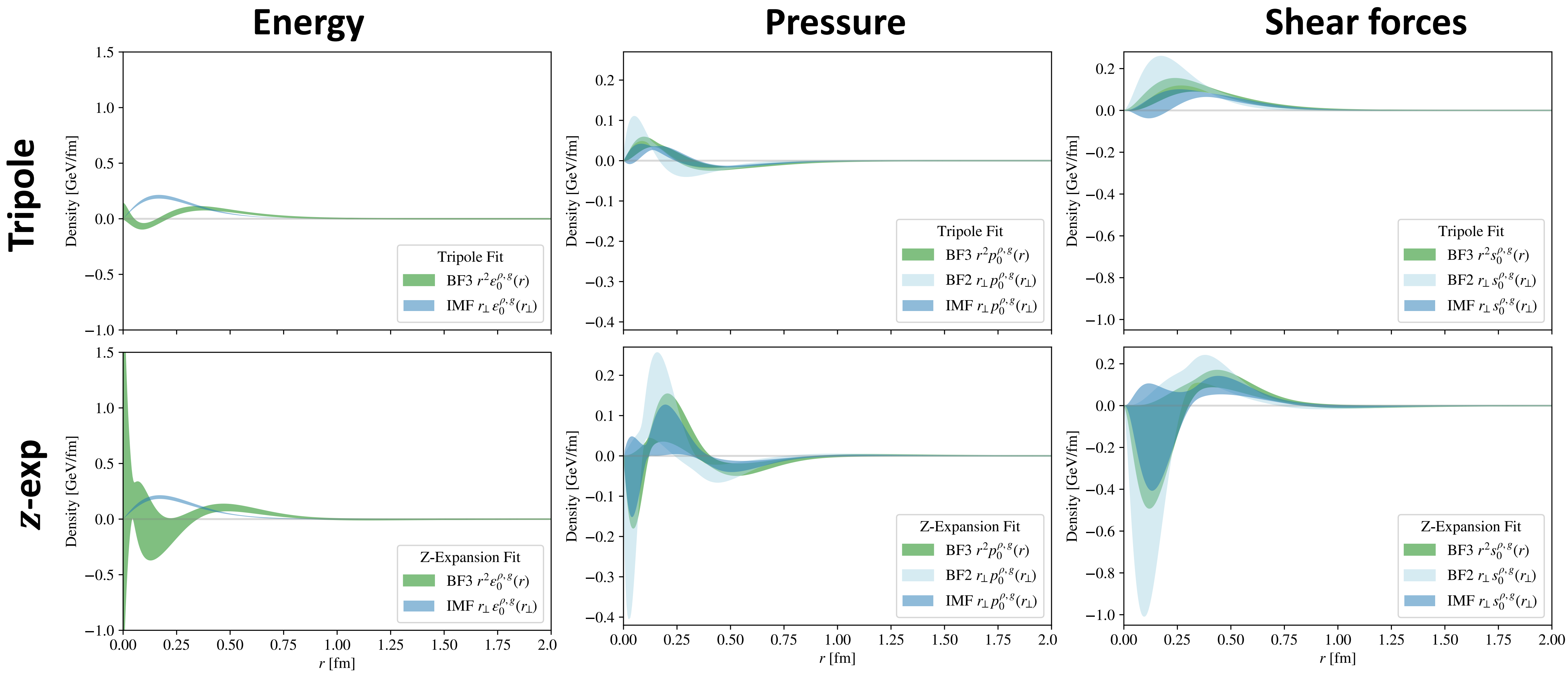
Results: pion densities



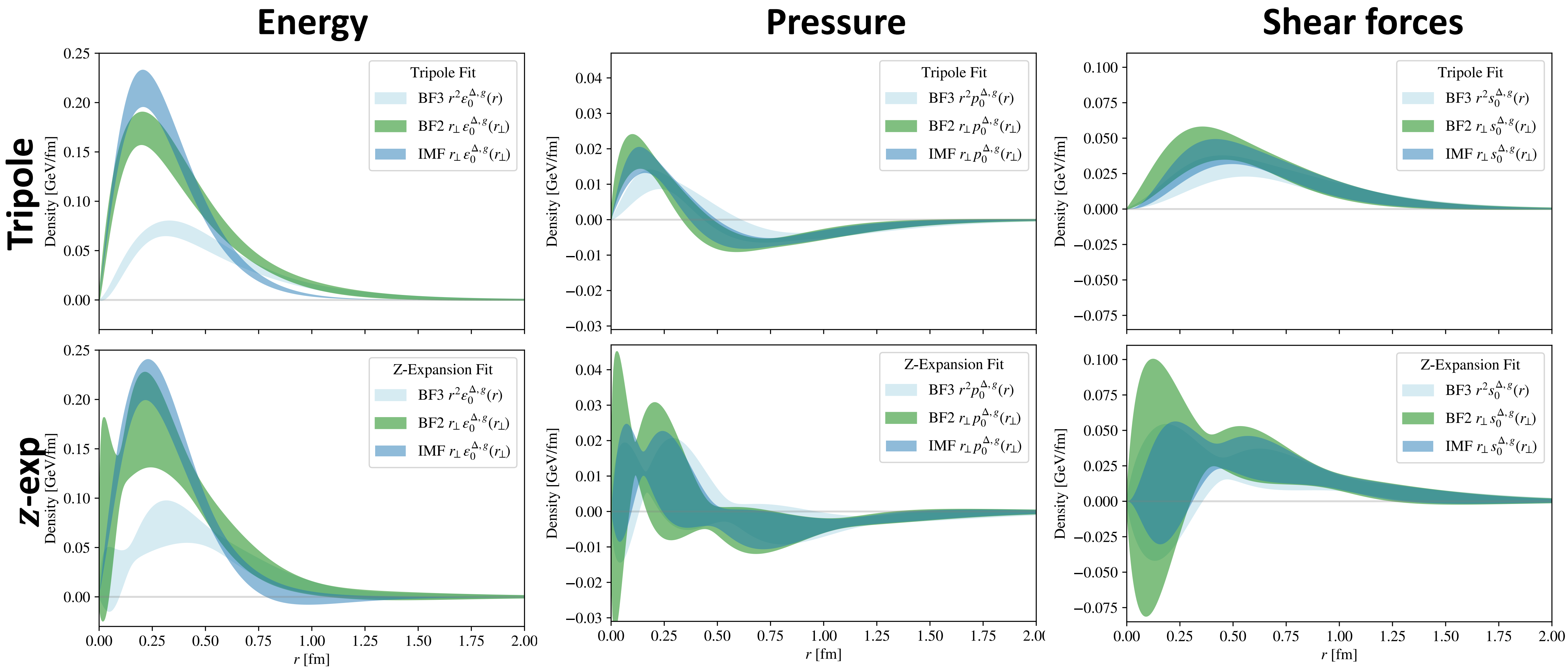
Results: nucleon densities



Results: (partial) ρ monopole densities



Results: (partial) Δ monopole densities



Summary

Physics:

- GFFs encode fundamental, global properties of hadrons
- GFFs are a window into GPD physics

Computed nucleon, π GFFs

- Experimental results → need lattice results to test against
- **Future:** complete (quark + glue) calculation ongoing

Computed ρ, Δ gluon GFFs

- First-of-kind results for ρ, Δ
- No experimental results → need lattice results to test against

Spatial densities (from first principles!)

Sketch of calculation [2107.10368]

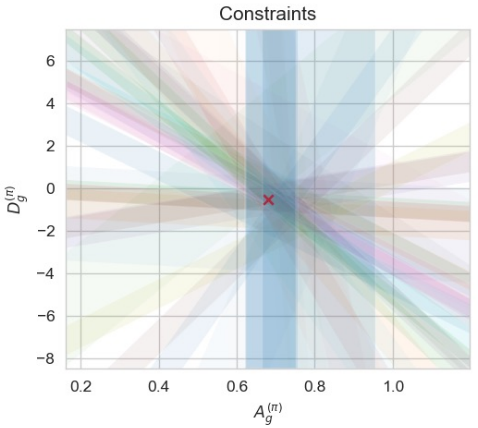
Compute hadronic two-point, three-point functions

Construct ratios of 3pts/2pts to isolate matrix element

$$R^{\mu\nu}_{ss'}(p,p';\tau,t_f) = \frac{C^{3\mathrm{pt}}_{ss'}(p,p';t_f,\tau)}{C^{2\mathrm{pt}}_{ss'}(p',t_f)} \sqrt{\frac{C^{2\mathrm{pt}}_{ss}(p,t_f-\tau)}{C^{2\mathrm{pt}}_{s's'}(p',t_f-\tau)} \frac{C^{2\mathrm{pt}}_{s's'}(p',t_f)}{C^{2\mathrm{pt}}_{ss}(p,t_f)} \frac{C^{2\mathrm{pt}}_{s's'}(p',\tau)}{C^{2\mathrm{pt}}_{ss}(p,\tau)}}$$

$$\xrightarrow{t_f \gg \tau \gg 0} (\text{extra kinematic factors}) \langle h(p',s') | T^{\mu\nu}_g | h(p,s) \rangle$$

$$= (\text{kinematic coeffs}) \cdot (\text{GFFs})(t)$$



Fit to extract GFFs

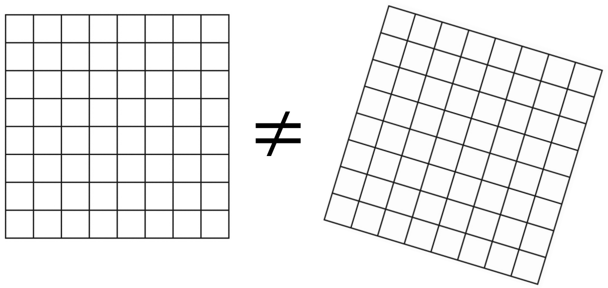
Result: GFFs for discrete values of t

Hypercubic Irreps

Lorentz symmetry broken → project $R^{\mu\nu}$ to hypercubic irreps

$$\tau_1^{(3)}: \quad \frac{1}{2}(O^{11} + O^{22} - O^{33} + O^{00}), \quad \frac{1}{\sqrt{2}}(O_{33} + O_{00}), \quad \frac{1}{\sqrt{2}}(O_{11} - O_{22})$$

$$\tau_3^{(6)}: \quad \left\{ \frac{i^{\delta_{\mu 0}}}{\sqrt{2}}(O^{\mu\nu} + O^{\nu\mu}), \quad 0 \le \mu \le \nu \le 3 \right\}$$



Example ratios

Note: plateaus