## **Extraction of e(x), other higher twist topics**

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Dirección General de Asuntos del Personal Académico



## Higher-twist parton distribution functions

Gluon at low energy, "the glue that binds us all"?

- What are higher-twist distribution functions?
- What information do they encapsulate?
- From low-energy experiments to higher Q<sup>2</sup>.



### PDF kinematics coverage: collinear PDFs



One possible definition for higher-twist contributions:

terms effectively suppressed like (M/Q)<sup>t-2</sup>

[Prog.Part.Nucl.Phys. 121 (2021) 103908]

Fixed Target DIS & SIDIS: M/Q is not so small

- Spurious contaminations
- Spin asymmetries can be defined to get sensitive to twist-3
- Present data: Hermes, COMPASS, JLab.

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### Higher-twist in observables

#### From spurious contaminations...

CJ15 global analysis includes lower cuts on W<sup>2</sup>. [Accardi et al., PRD93]

HT's role in fulfilling duality [e.g. Melnitchouk et al., Phys.Rept.406]

#### …to genuine effects

JAM analysis of the helicity PDF  $g_1$  extends to  $g_T$ , with  $g_T=g_1+g_2$ . [Sato et al., PRD93]

 $g_2^{(\tau 3)}(x, Q^2) = D(x, Q^2) - \int_x^1 \frac{dz}{z} D(z, Q^2)$ 



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$$g_2^{(\tau 3)}(x, Q^2) = D(x, Q^2)$$

#### $g_T$ is the only twist-3 PDF accessible through inclusive DIS

Exploratory studies suggest that quark-gluon-quark correlations are non-zero.

[Accardi et al, JHEP11 (2009)]



1. Parametrizing the proton matrix element, relations between scalars and moments can be found: Wandzura-Wilczek relation.

 $g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) +$ twist-2 PDF

see talks by Shohini Bhattacharya (TMDs), Simonetta Liuti (GPDs)

 $\tilde{g}_2(x)$ 

[e.g. Jaffe, eprint/9602236]

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 $\tilde{g}_2(x)$ [e.g. Jaffe, eprint/9602236] related to genuine qgq correlation through the equations of motion

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$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) + \tilde{g}_2(x) +$$

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 $-g_{2,\delta}(x)|_{\mathrm{model}}$ 

[Jaffe & Ji, PRD43] [Burkardt & Koike, NPB632] [Aslan & Burkardt, PRD101] [Ji, NPB960]

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3. Mass terms: 
$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) + \tilde{g}_2(x) + g_2(x) + g_2$$

see talks by Shohini Bhattacharya (TMDs), Simonetta Liuti (GPDs)

## Scalar PDF

The composition of the scalar PDF is worked out through the EoM of QCD:

Only observable-related contribution to the proton mass: the singularity of e(x) is proportional to the pion-nucleon sigma term through sum rules [e.g. Kodaira & Tanaka, PTP, Vol. 101]

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#### Lots of interests for that function in the past years

[Schweitzer and Efremov, JHEP08006] [Burkardt & Koike, NPB632] [Ji, NPB960] [Lorcé, Pasquini, Schweitzer, JHEP01 (2015)] [Pasquini & Rodini, PLB788] [Hatta & Zhao, PRD102] [Bhattacharya et al., PRD102]

 $(x) + e^q_{\text{mass}}(x)$ 

gq correlation

rom covariant derivative

quark mass term

originates from kinetic+mass

 $(0)|P\rangle$ 

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### Scalar PDF and the proton mass

#### **QCD** mass decomposition



<u>Sigma terms</u>

$$\left\langle P|m_u\bar{u}u + m_d\bar{d}d|P\right\rangle = \sigma_{\pi N}$$

<sup>a</sup> have been determined from theoretical analysis of  $\pi N$  data [Meissner et al.]

- have been evaluated on the lattice [Constantinou et al.]
- <sup>(a)</sup> pheno analysis of e(x) could pave the way towards another possible determination



quark and gluon energy  $\propto \langle x \rangle_{q,g}$ 

[Ji, PRL 74; Ji, PRD 52] [Lorcé, EPJC78; Lorcé et al, 2109.11785]

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### Twist-3 in SIDIS dihadron production



Collinear framework — led to collinear transversity extraction [Radici, Jakob & Bianconi, PRD65].

modulations of spin asymmetries single out:

Scalar PDF from the beam spin asymmetry

$$A_{LU}^{\sin\phi_R}(x,z,m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z,m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z,m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z,m_{\pi\pi})}$$

$$[ \underline{\text{twist-3 PDF} \times \text{twist-2 FF}} + [ \underline{\text{twist-2 PDF} \times \text{twist-3 FF}} ] \\ \text{unpolarized}$$

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## Beam spin asymmetry at CLAS and CLAS12

dihadron SIDIS on proton target – sensitive to  $e^P \equiv \frac{1}{\Omega} (4 e^{u_V} - e^{d_V});$ 

> [Bacchetta & Radici, PRD69 (2004)] [Courtoy, 1405.7659]

non-vanishing twist-3 effects at CLAS12;

- projections of beam spin asymmetries on  $(x, z, M_h; Q^2, y)$ 
  - $\Rightarrow$  (x, z, M<sub>h</sub>) triptych from the parton distribution and fragmentation function.

Road map for e(x) extraction and (global) analysis.

#### see talk by Christopher Dilks



[CLAS Collaboration, PRL126 (2021) 6, 062002]

CLAS12:  $1.5 < Q^2 < 5.7 \,\text{GeV}^2$ 



Extraction of e(x)



leading-twist DiFFs

#### **Twist-2 Dihadron Fragmentation Functions**

Phenomenologically tested for the twist-2 transversity PDF [Bacchetta, Courtoy & Radici, PRL107 and follow-ups] Sextracted in e+e- at Belle here: [Radici, Courtoy, Bacchetta, JHEP 05 (2015)] we get the ratio R that is believed to be universal (portable) up to evolution effects

$$R(z, M_h) = \frac{|\mathbf{R}|}{M_h} \frac{H_1^{\triangleleft u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)} \xrightarrow{\qquad \text{ chiral-odd DiFF}} \text{ unpolarized DiFF}$$

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Extraction of e(x)

### Extraction of e

e(x) from CLAS data  

$$A_{LL}^{\sin(\phi_n)} = \frac{A}{A_{LL}} = \frac{A}{A} = \frac{A}{$$

#### **Twist-3 Dihadron Fragme**

- Unknown phenomenologically;
- Solutions for genuine twist-3 DiFF:  $\tilde{D}^{\triangleleft}$  [Luo et al., PRD100],  $\tilde{G}^{\triangleleft}$  [Yang et al., PRD99]
- Setimate of Interference FF through the asymmetries on longitudinally-polarized target at COMPASS [Sirtl, PhD thesis, 2017]

$$\begin{split} A_{UL}^{\sin(\phi_R)} &= -\frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x h_L^q(x) H_1^{\angle q, sp}(z, M_h^2) + \frac{M_h}{Mz} g_1^q(x) \tilde{G}^{\angle q, sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q, ss + pp}(z, M_h^2)} \\ A_{LL}^{\cos(\phi_R)} &= -\frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x e_L^q(x) H_1^{\angle q, sp}(z, M_h^2) - \frac{M_h}{Mz} g_1^q(x) \tilde{D}^{\angle q, sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q, ss + pp}(z, M_h^2)}. \end{split}$$

$$A_{UL}^{\sin(\phi_R)} = 0.0050 \pm 0.0010(\text{stat}) \pm 0.0007(\text{sys})$$
$$A_{LL}^{\cos(\phi_R)} = -0.0135 \pm 0.0064(\text{stat}) \pm 0.0046(\text{sys})$$
$$\Rightarrow |A_{LL}^{\cos\phi_R}| > A_{UL}^{\sin\phi_R}$$















### Our ansatz for the twist-3 DiFF contribution

- CLAS12: split invariant-mass regions M<sub>h</sub> > or < 0.63 GeV to pinpoint vector meson contributions</li>
   We assume the trend of all interference DiFFs in the invariant mass is similar for M<sub>h</sub> > 0.63 GeV (up to overall sign)
   supported by model evaluation of  $\tilde{D}^{\triangleleft}$  and  $\tilde{G}^{\triangleleft}$
- Reproducing A<sup>cos \phi\_R</sup><sub>LL</sub> in that range sets our upper bound to \kappa \Rightarrow \kappa\_{M\_h}
   \kappa\_{M\_h} reproduces the order of magnitude for A<sup>sin \phi\_R</sup><sub>UL</sub> adequately



 $\Rightarrow$  invariant-mass behavior is key, twist-2 DiFF alone not enough to interpret all  $M_h$  -projected twist-3 asymmetries.

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### Point-by-point e(x) from CLAS data

Scenario I: Wandzura-Wilczek approximation

$$\frac{e^{V}(x)}{f_{1}^{\Sigma}(x)}\frac{\tilde{H}_{1}^{\triangleleft}}{D_{1}} \propto \frac{Q}{M} A_{LU}^{\sin\phi_{R}}$$

Scenario II: beyond WW approximation

$$\frac{e^V(x)}{f_1^{\Sigma}(x)}\frac{\tilde{H}_1^{\triangleleft}}{D_1} \propto \frac{Q}{M} A_{LU}^{\sin\phi_R} \pm \kappa \frac{f_1^V(x)}{f_1^{\Sigma}(x)}\frac{\tilde{H}_1^{\triangleleft}}{D_1}$$



Sign of twist-3 DiFFs undetermined

[Courtoy, 1405.7659] [Courtoy, Miramontes, Avakian, Mirazita, in progress]

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6  $e^{P}(\mathbf{x})$ 2 -2 0.2

10

8

Combining the uncertainty at 90% CL⇒

0.1



Evolution omitted thanks to low-Q<sup>2</sup> values -Q=1GeV

Uncertainty on unpolarized PDF taken into account

Sign of twist-3 DiFFs undetermined



[Courtoy, 1405.7659] [Courtoy, Miramontes, Avakian, Mirazita, in progress]





## What is the probability for $e^{P}(x)$ to be non-zero?

Probability that the proton combination is <u>greater</u> than zero — not exactly "how incompatible with zero is it?" — is a useful information from the point-by-point extraction of a collinear twist-3 PDF with a minimum set of approximations.



#### Mostly far from zero!

### Universality of non-perturbative functions

**Dihadron fragmentation functions** 

- ➡ DiFF extracted in e<sup>+</sup>e<sup>-</sup>, to be tested against SIDIS multiplicities
- Consistency check on SIDIS  $(z, M_h)$  dependence at CLAS & CLAS12
- Determination of the integral of  $e^{P}(x)$  from reconstruction:  $n_{x}$

- Twist-2 and -3 PDFs
  - → Universality of transversity in pp and SIDIS [Radici et al, PRD94]
  - Global analysis of the transversity possible [Radici & Bacchetta, PRL120; JAM Coll., PRD102] see talk by Alexei Prokudin
  - ➡ Are twist-3 PDFs universal?

Yet to be answered.

Examples through TMD and dynamical twist-3 relations (e.g. Sivers and Qiu-Sterman) — see talk by Shohini Bhattacharya



Data

\_Extraction of e(x)\_

### Consequences of the extraction

- 1. Are twist-3 PDFs non-zero? Yes, to a certain CL.
- 2. Can we access qgq correlations and more non-perturbative information? Let's take the example of e(x).



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Schematic models for illustration purpose only!

Moments will matter.

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Other nonperturbative effects at not so small x



## Can we study qgq correlation at the EIC?



Future: EIC will cover low- to mid-Q<sup>2</sup> and smallish x values

- Yellow Report: access to multiparton correlations.
- Proposal for a 2nd interaction region IR2@EIC.
- Complementarity with present data.



Projections for IR2@EIC White Paper by A. Vossen

### Expectations for the EIC

- EIC error projections (from transversity studies)
- Proton target shown, but need for neutron
- Models × DiFFs predictions
  - → LC model [Pasquini & Rodini, PLB 788]
  - made-up mass-term contribution with mq=300MeV
- Non-negligible for lowest beam configurations

Archetype of observables for IR2@EIC

- Evolution equations for genuine qgq twist-3 known in most cases;
- Understanding of the various contributions to twist-3 PDFs;
- Especially "hot" for TMD studies.
- Require a second interaction region @EIC.

#### EIC Yellow Report [2103.05419]



х



## Multi-parton distributions at the EIC



**Golden channel** 

fully inclusive DIS, access to g<sub>T</sub>

#### **Collinear observables.** 0

- Plethora of interesting TMD, GPD higher-twist observables to be considered too 0
- subWG: Avakian, Burkardt, AC, Gamberg, Pitonyak, Sato, Schweitzer, Vossen

#### Large range of Q<sup>2</sup> values, includes smallish x regions

Complementary to fixed-target experiments (HERMES, CLAS,...)

**Silver channel** 

semi-inclusive DIS, access to e(x)

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Seminar SMU

## Unpolarized PDFs at in the large x & low Q<sup>2</sup> regime

Denominator of asymmetries rely on first term in the expansion of their unpolarized structure function

Unpolarized PDFs:

(large x, low Q<sup>2</sup>) is either in the extrapolation region for high-energy global analyses -CT, MSHT, NNPDF or requires non-perturbative corrections related to the resonance region, e.g. TMC, HT -CJ, JAM.

Change in degrees of freedom.

Increased complexity on which most SIDIS-based extractions at low Q<sup>2</sup> will rely.

"We can tackle higher twist parts"—see talk by Andrea Signori Can we tackle the denominator?

Comparison of CTEQ-TEA (CT) and CTEQ-JLab (CJ) analyses [Accardi et al, EPJC81]

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 $\frac{\text{twist-3 PDF} \times \text{twist-2 FF}}{\text{unpolarized}}$ 





## Towards a (global) analysis of the scalar PDF

<u>Classes of first principle constraints for x-dependence of twist-3 PDFs</u>

- support in  $x \in [0,1]$
- end-point: f(x = 1) = 0
- sum rules as output:  $\langle x \rangle_n = \int_0^1 dx \, x^{n-1} f(x)$  some moments evaluated on the lattice.
- no QCD evolution for now -DGLAP eqs known for twist-3 starting from n>2.

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#### Huge family of functions that might describe the data.

The role of parametrization is important — neural-network approaches treat the problem differently.

[Kovarik et al, Rev.Mod.Phys. 92 (2020)]

Consequently, the PDF uncertainty is comprised of four categories of contributions:

- 1. Experimental uncertainties, including statistical and correlated and uncorrelated systematic uncertainties of each experimental data set;
- 2. Theoretical uncertainties, including the absent higher-order and power-suppressed radiative contributions, as well as uncertainties in using parton showering programs to correct the data in order to compare to fixed-order perturbative cross sections;
- 3. Parameterization uncertainties associated with the choice of the PDF functional form;
- 4. Methodological uncertainties, such as those associated with the selection of experimental data sets, fitting procedures, and goodness-of-fit criteria.



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The role of parametrization is important — neural-network approaches treat the problem differently.

Systematize the study of the parametrization for a faithfull analysis.

⇒ Fantômas4QCD

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Extraction of e(x)



### Fantômas4QCD

Uncertainty on the PDFs coming from the choice of functional form shown for the unpolarized PDFs of CT18.

Fantômas will propose an unbiased version of the analytic parametrizations for a variety of nonperturbative QCD functions.



[Courtoy, Nadolsky & students, in progress]

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[Hou et al, Phys.Rev.D 103 (2021)]

#### Fantômas team

Ryan Guess (SMU) Lucas Kotz (SMU) Maximiliano Ponce Chávez (UNAM) Varada Purohit (SMU)

> Pavel Nadolsky (SMU) AC (UNAM)



# Conclusions

We have discussed the role of higher-twist distributions in the understanding of hadron structure. We have presented a truly updated extraction of the scalar PDF, e(x). It is non-zero to more than 75% probability.

From the EIC wish list, higher twists contribute to, e.g.

- Precision 3D imaging of nucleons.
- Emergence of hadronic mass from the scalar PDF.
- Proton spin puzzle from GPDs.

Higher-twist distributions will unveil aspects of hadron dynamics.

Higher-twist distributions are accessible but require more statistics, phenomenological and theoretical developments.

Combine efforts with the lattice QCD? [e.g. Bhattacharya et al, PRD102; Braun & Vladimirov, JHEP10(2021)087]

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