Factorization approach for QED+QCD in semi-inclusive DIS

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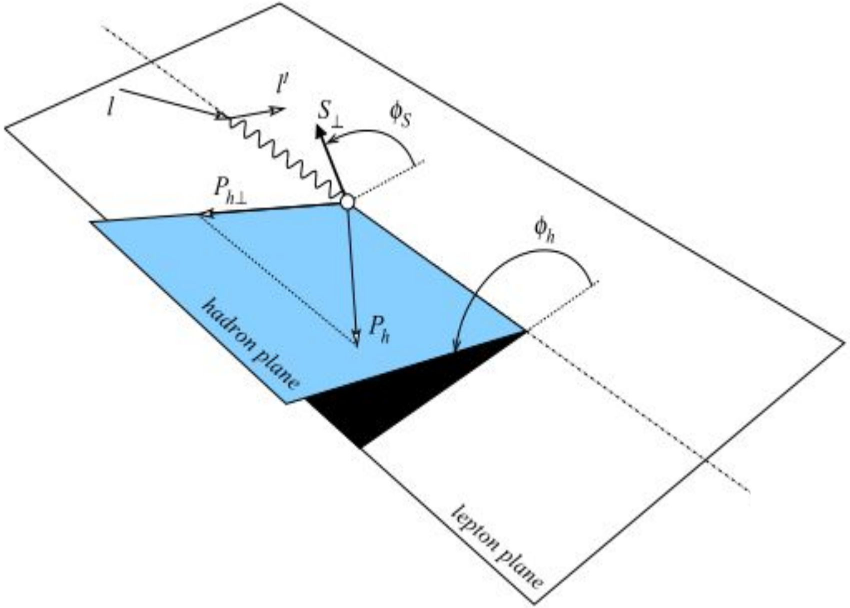
CPHI 2022

Duke UNIVERSITY

Jefferson Lab

SIDIS - the current frontier of hadron structure studies



hermes

COMPASS

Jefferson Lab

EIC^2

$$
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =
$$

$$
\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}
$$

$$
+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}
$$

$$
+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]
$$

$$
+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]
$$

$$
+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)
$$

$$
+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}
$$

$$
+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]
$$

$$
+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}
$$

$$
+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\},
$$

Physics goals

Name	Symbol	meaning
upol. PDF	f_1^q	U. pol. quarks in U. pol. nucleon
pol. PDF	g_1^q	L. pol. quarks in L. pol. nucleon
Transversity	h_1^q	T. pol. quarks in T. pol. nucleon
Sivers	$f_{1T}^{\perp(1)q}$	U. pol. quarks in T. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
⋮	⋮	⋮
FF	D_1^q	U. pol. quarks to U. pol. hadron
Collins	$H_1^{\perp(1)q}$	T. pol. quarks to U. pol. hadron
⋮	⋮	⋮

But.. can we access to all structure functions from experiments uniquely?

The trick

$$\int d\phi_h d\phi_S \; \sin(\phi_h - \phi_S) \; \sin(\phi_h + \phi_S) = 0$$

$$\int \mathrm{d}\phi_h \, \mathrm{d}\phi_S \, \sin(\phi_h - \phi_S) \frac{\mathrm{d}^6 \sigma_{\ell P(S_T) \to \ell' P_h X}}{\mathrm{d}x_B \mathrm{d}y \, \mathrm{d}\psi \, \mathrm{d}z_h \, \mathrm{d}\phi_h \mathrm{d}P_{hT}^2},$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \doteq \int \mathrm{d}\phi_h \, \mathrm{d}\phi_S \, \sin(\phi_h - \phi_S) \Big[\sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h - \phi_S)} + \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \Big].$$

Kinematics

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

Kinematics

□ = depends on leptons momenta

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

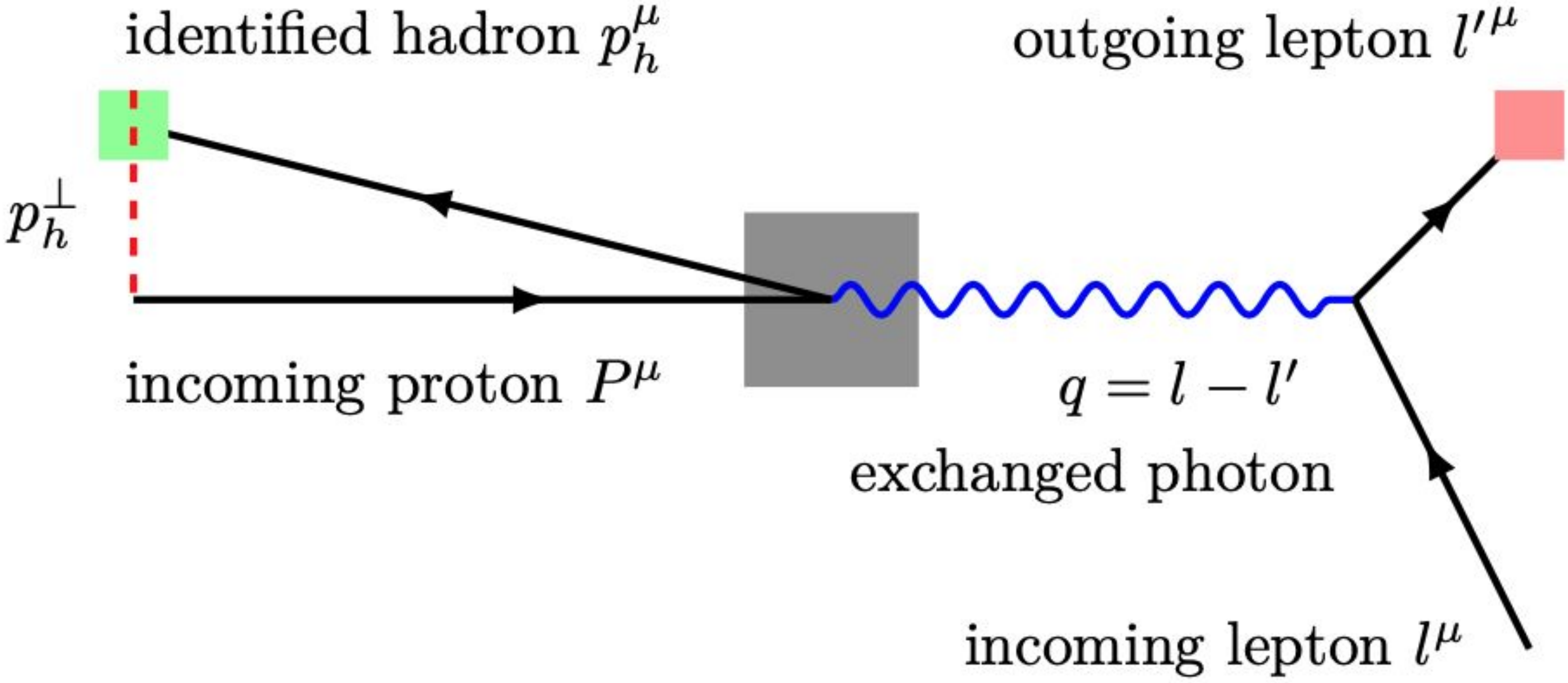
$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

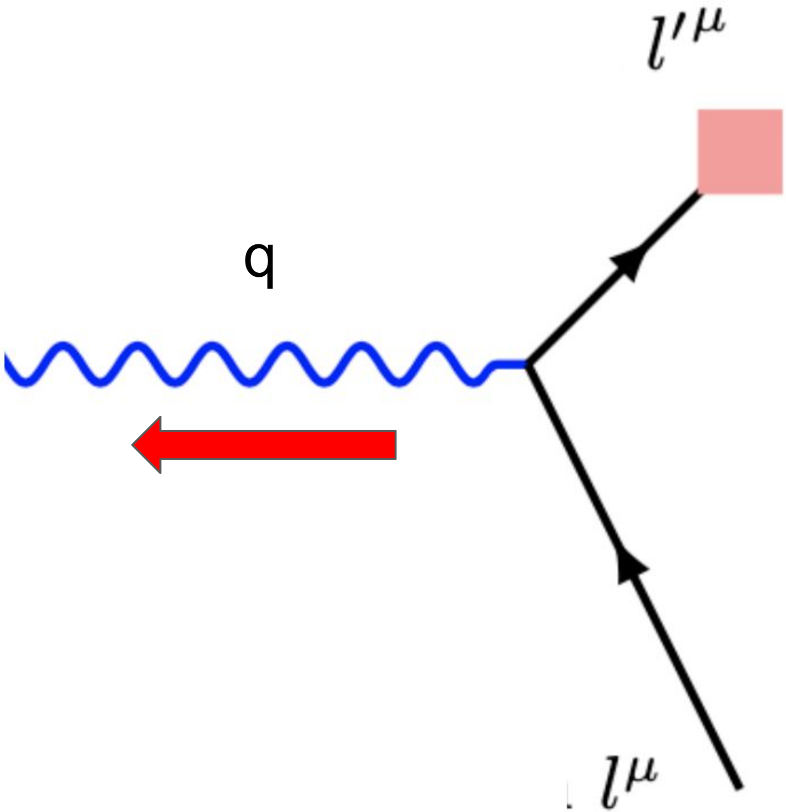
$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

The Breit Frame



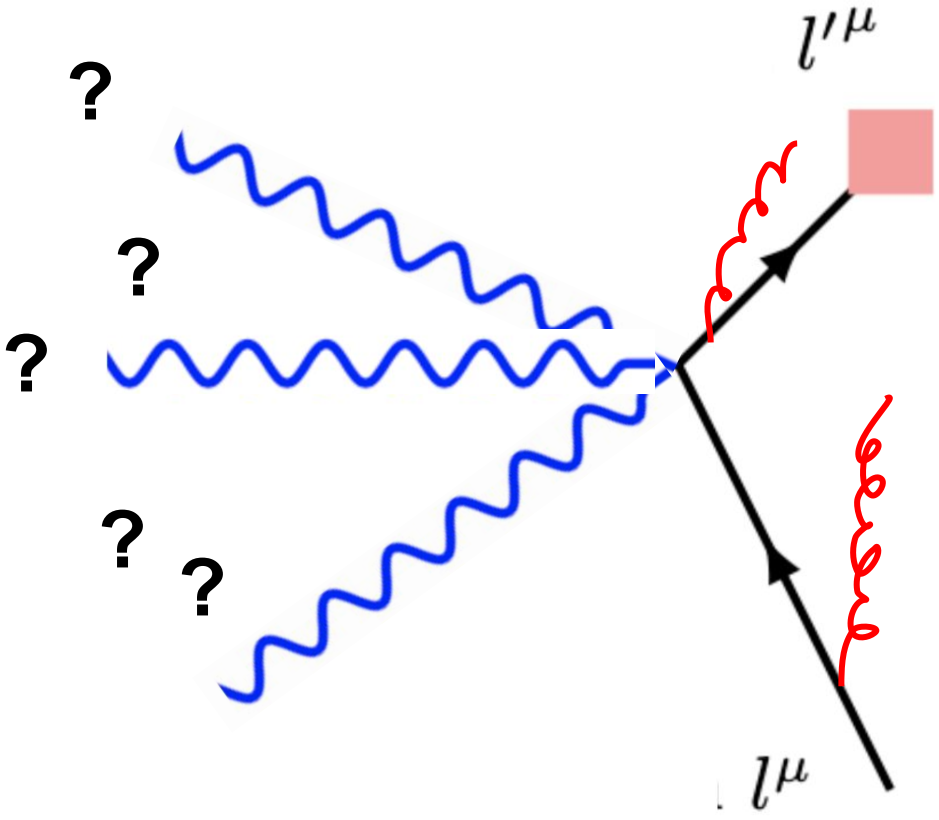
But ....

- Factorization theorem relies on q moving along -z
- How do we know that q is exactly along -z even-by-event?
- Role of QED radiation?



- In the presence of QED radiation, **the q direction is not fixed**

- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization



arXiv.org > hep-ph > arXiv:2008.02895

High Energy Physics – Phenomenology

[Submitted on 6 Aug 2020 (v1), last revised 17 Mar 2021 (this version, v3)]

Factorized approach to radiative corrections for inelastic lepton-hadron collisions

Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, N. Sato

arXiv.org > hep-ph > arXiv:2108.13371

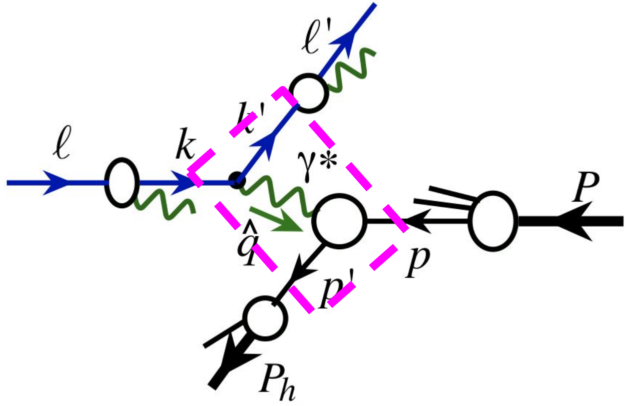
High Energy Physics – Phenomenology

[Submitted on 30 Aug 2021]

A new approach to semi-inclusive deep-inelastic scattering with QED and QCD factorization

Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, N. Sato

Inclusive DIS



Pure collinear framework

$$E'\frac{d\sigma_{\rm DIS}}{d^3\ell'} = \frac{1}{2s}\sum_{ija}\int_{z_L}^1 \frac{d\zeta}{\zeta^2}\int_{x_L}^1 \frac{d\xi}{\xi}\, D_{e/j}(\zeta)\, f_{i/e}(\xi) \times \int_{x_h}^1 \frac{dx}{x} f_{a/N}(x)\, \widehat{H}_{ia\to j}(\xi,\zeta,x) + \mathcal{O}\Big(\frac{1}{\ell_T'^2}\Big),$$

Collinear LDFs and LFFs

Collinear PDFs

Short distance hard part

Lepton Distribution Function (**LDF**)
Lepton Fragmentation Function (**LFF**)

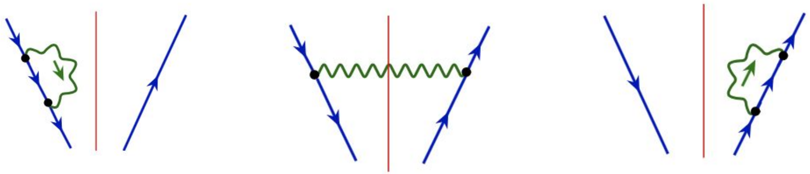
Collinear LDFs and LFFs

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi\ell^+ z^-} \langle e| \overline{\psi}_i(0)\gamma^+ \Phi_{[0,z^-]}\, \psi_i(z^-)|e\rangle$$

$$D_{e/j}(\zeta) = \frac{\zeta}{2} \sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^-/\zeta}\, \mathrm{Tr}\big[\gamma^+ \langle 0| \overline{\psi}_j(0)\Phi_{[0,\infty]}\, |e, X\rangle\langle e, X|\psi_j(z^-)\Phi_{[z^-,\infty]}|0\rangle\big]$$

perturbatively calculable if we
neglect hadronic components

RGE



DGLAP

μ^2

$$\frac{\partial f_{i/l}(\xi,\mu^2)}{\partial \ln \mu^2} = \sum_j \int_\xi^1 \frac{dz}{z} P_{ij}(z) f_{i/l}(z/\xi,\mu^2)$$

μ_0^2

ξ

$$f^{(0)}_{e/e}(\xi) = \delta(\xi - 1)$$

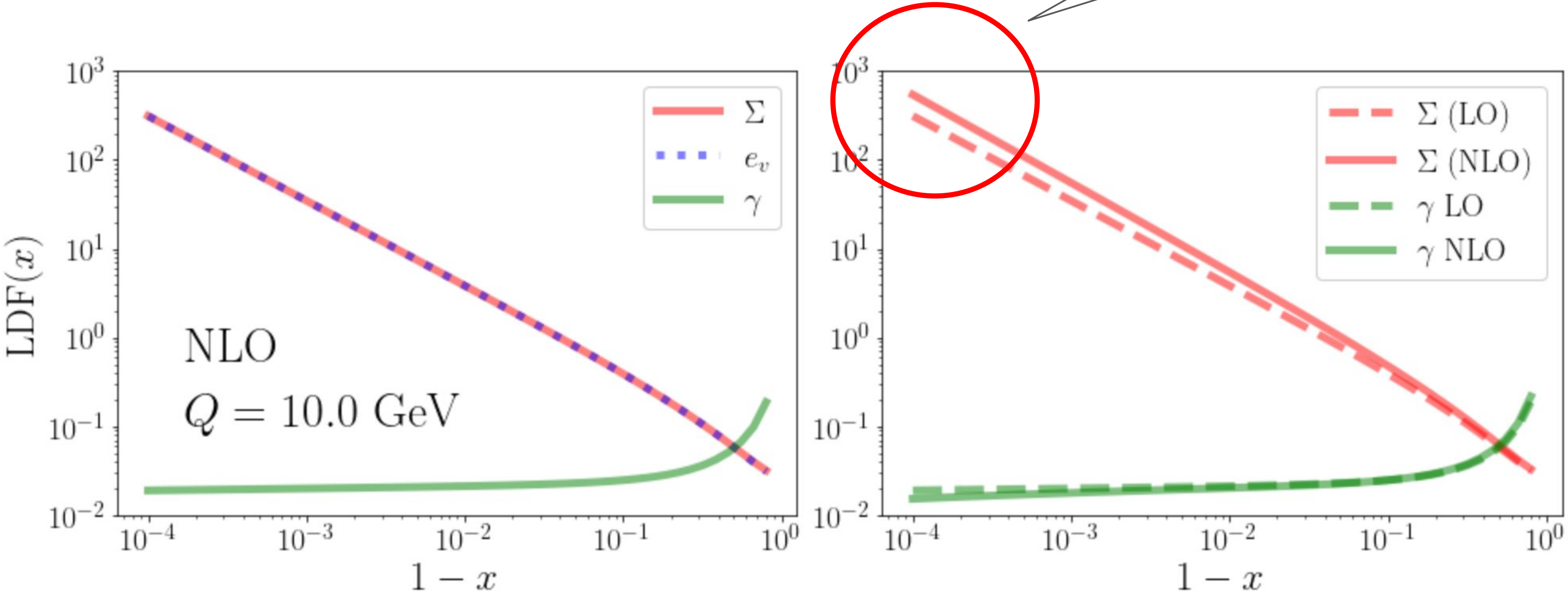
$$f^{(1)}_{e/e}(\xi,\mu_0^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu_0^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D^{(0)}_{e/e}(\zeta) = \delta(\zeta - 1)$$

$$D^{(1)}_{e/e}(\zeta,\mu_0) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu_0^2}{(1-\zeta)^2 m_e^2}\right]_+$$

Evolved LDFs

LDFs peaks at the endpoint



Endpoint issues

Endpoint regions are difficult for LDF and LFF RGE

$$E' \frac{d\sigma_{\mathrm{DIS}}}{d^3\ell'} = \frac{1}{2s} \sum_{ija} \int_{z_L}^{1} \frac{d\zeta}{\zeta^2} \int_{x_L}^{1} \frac{d\xi}{\xi} D_{e/j}(\zeta)\, f_{i/e}(\xi)$$

$$\times \int_{x_h}^{1} \frac{dx}{x} f_{a/N}(x)\, \widehat{H}_{ia\to j}(\xi, \zeta, x) + \mathcal{O}\Big(\frac{1}{\ell_T'^2}\Big),$$

Subtraction trick

$$\sigma = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min}(\zeta)}^{1} d\xi f(\xi) D(\zeta) H(\xi,\zeta)$$

$$D_N = \int_0^1 d\zeta \zeta^{N-1} D(\zeta)$$

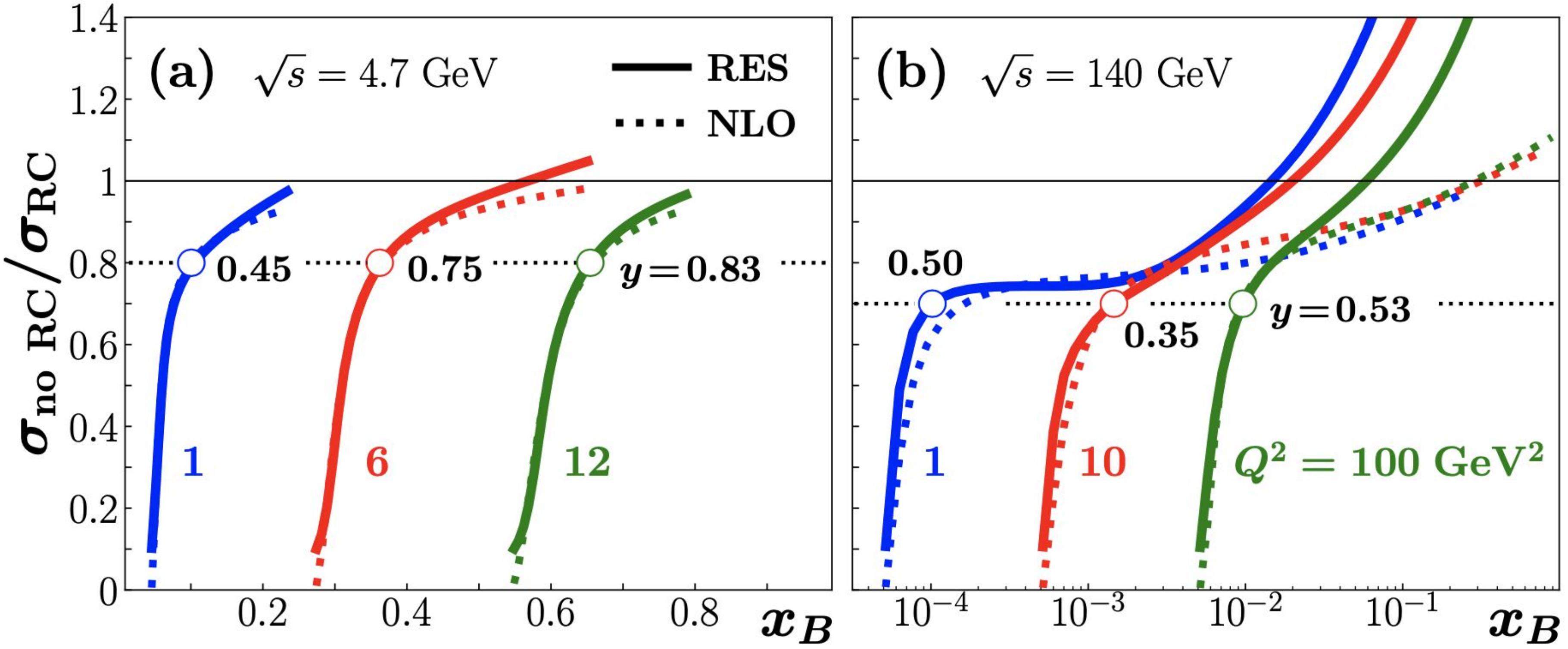
$$\sigma = \int_{\zeta_{\min}}^{1} d\zeta\ d(\zeta)[g(\zeta) - g(1)] + g(1)\frac{\zeta_{\min}}{2\pi i}\int dN \zeta_{\min}^{-N}\frac{D_N}{N-1}$$

$$F_N = \int_0^1 d\xi \xi^{N-1} f(\xi)$$

$$g(\zeta) = \int_{\xi_{\min}(\zeta)}^{1} d\xi\ f(\xi)[H(\xi,\zeta) - H(1,\zeta)] + H(1,\zeta)\frac{\xi_{\min}(\zeta)}{2\pi i}\int dN \xi_{\min}(\zeta)^{-N}\frac{F_N}{N-1}$$

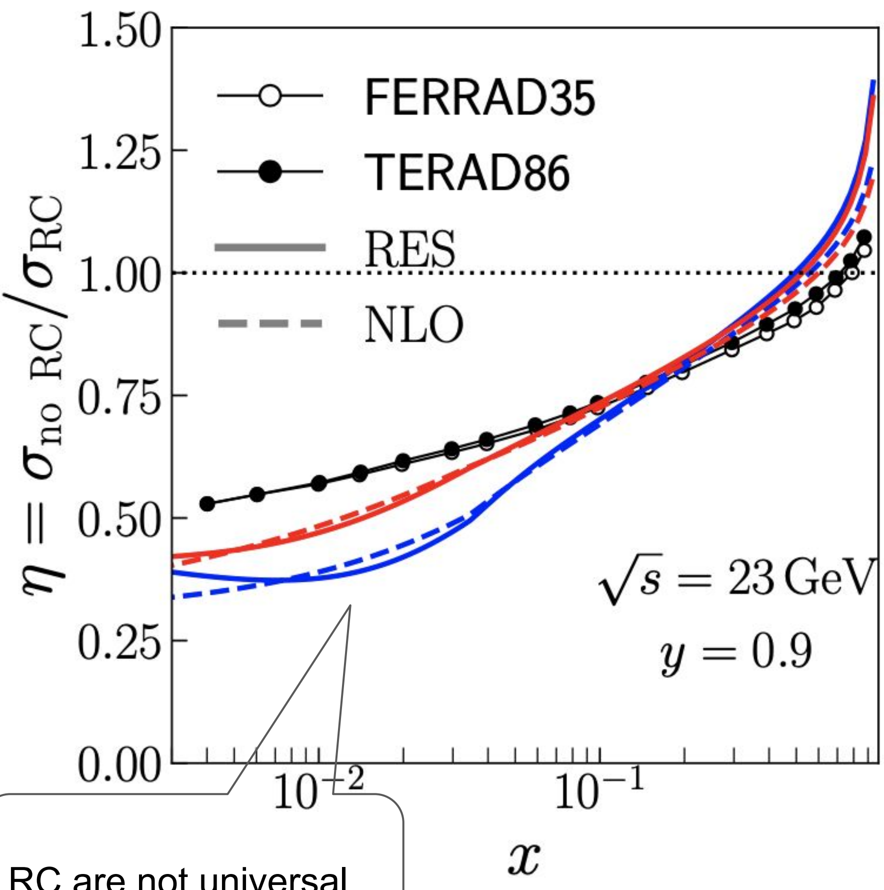
We remove the numerically problematic region and compute the difference accurately in Mellin space

Pheno



(a) $\sqrt{s} = 4.7$ GeV
RES
NLO
0.45
0.75
$y = 0.83$
1
6
12
(b) $\sqrt{s} = 140$ GeV
0.50
0.35
$y = 0.53$
1
10
$Q^2 = 100$ GeV^2
$\sigma_{\text{no RC}}/\sigma_{\text{RC}}$
x_B

Comparison With existing literature...



RC are not universal

Radiative correction schemes in deep inelastic muon scattering

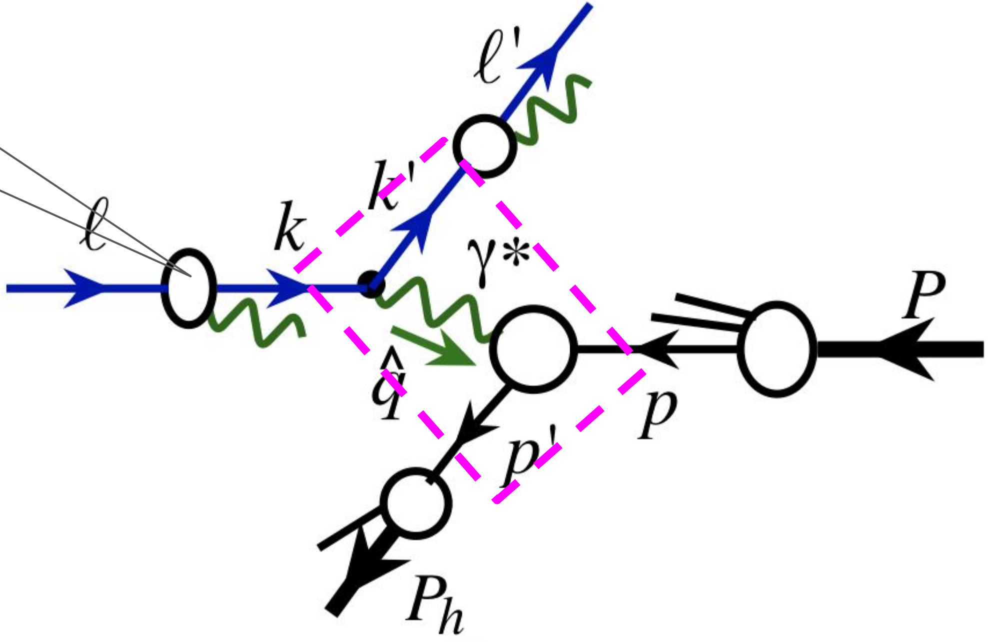
B. Badelek (Uppsala U. and Warsaw U.), Dmitri Yu. Bardin (CERN and Dubna, JINR), Scholz (Heidelberg, Max Planck Inst.) (Feb, 1994)

Published in: *Z.Phys.C* 66 (1995) 591-600 • e-Print: hep-ph/9403238 [hep-ph]

pdf DOI cite

Semi-Inclusive DIS

Do we need to worry about TMD LDFs and LFFs ?



TMDs in leptonic tensor

$$L^{(0)}_{\mu\nu}(\ell,\ell',\lambda_\ell) = \mathrm{Tr}\Big[\gamma_\nu\,\frac{1}{2}(1+\lambda_\ell\gamma_5)\,\gamma\cdot\ell\,\gamma_\mu\,\gamma\cdot\ell'\Big]$$
$$= 2\big(\ell_\mu\,\ell'_\nu + \ell_\nu\,\ell'_\mu - \ell\cdot\ell' g_{\mu\nu} + i\lambda_\ell\,\epsilon_{\mu\nu\alpha\beta}\,\ell^\alpha\,\ell'^\beta\big)\,;$$

qT of the photon in the back to back lepton frame

Small qT approximation to lepton tensor

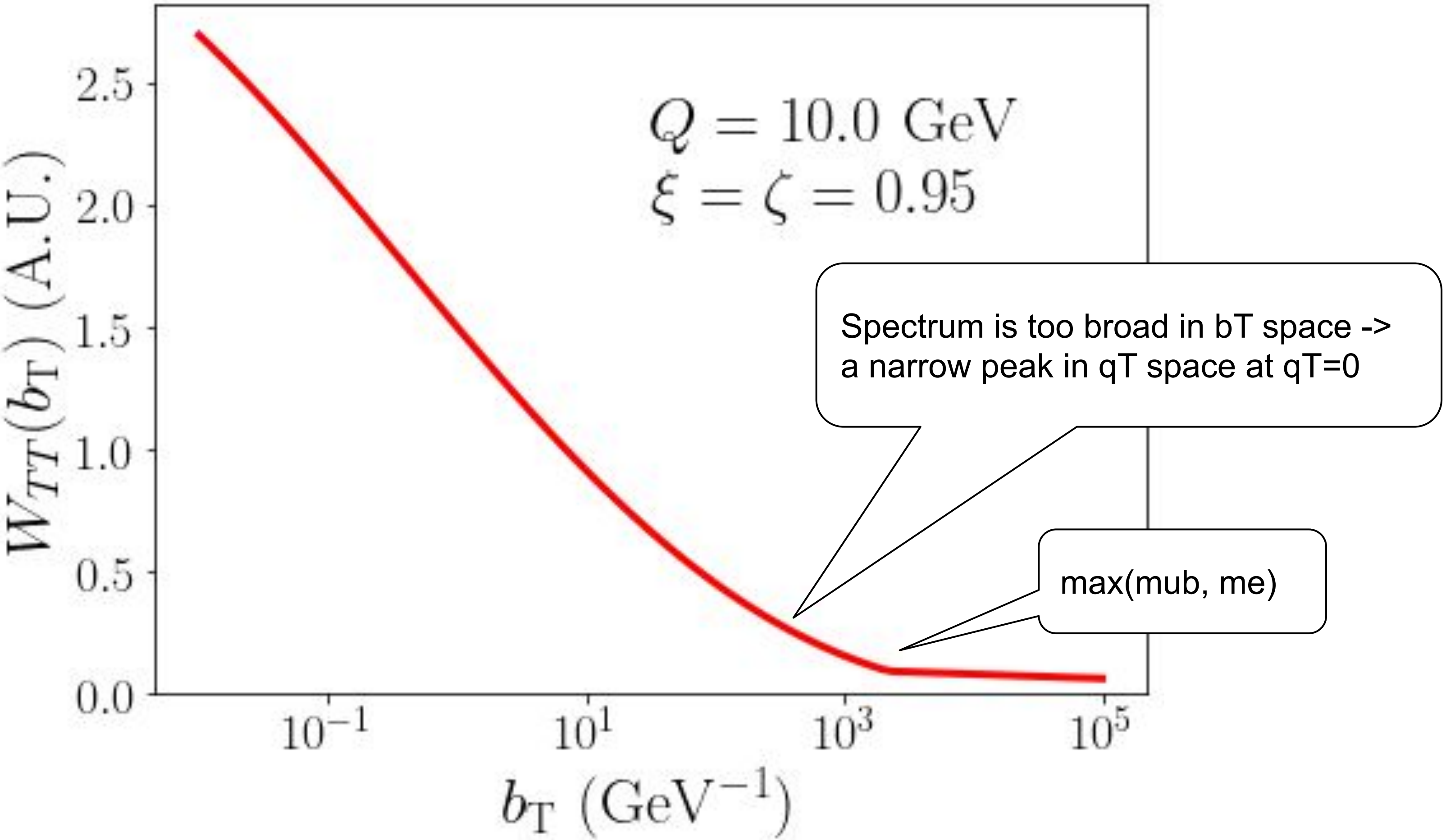
$$L_{\rho\sigma}(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}}\,\underline{\widetilde{W}_{\rho\sigma}(\xi_B,\zeta_B,Q^2,b)} + Y_{\rho\sigma}(\xi_B,\zeta_B,Q^2,\hat{\boldsymbol{q}}_T^2),$$

TMDs in lepton tensor

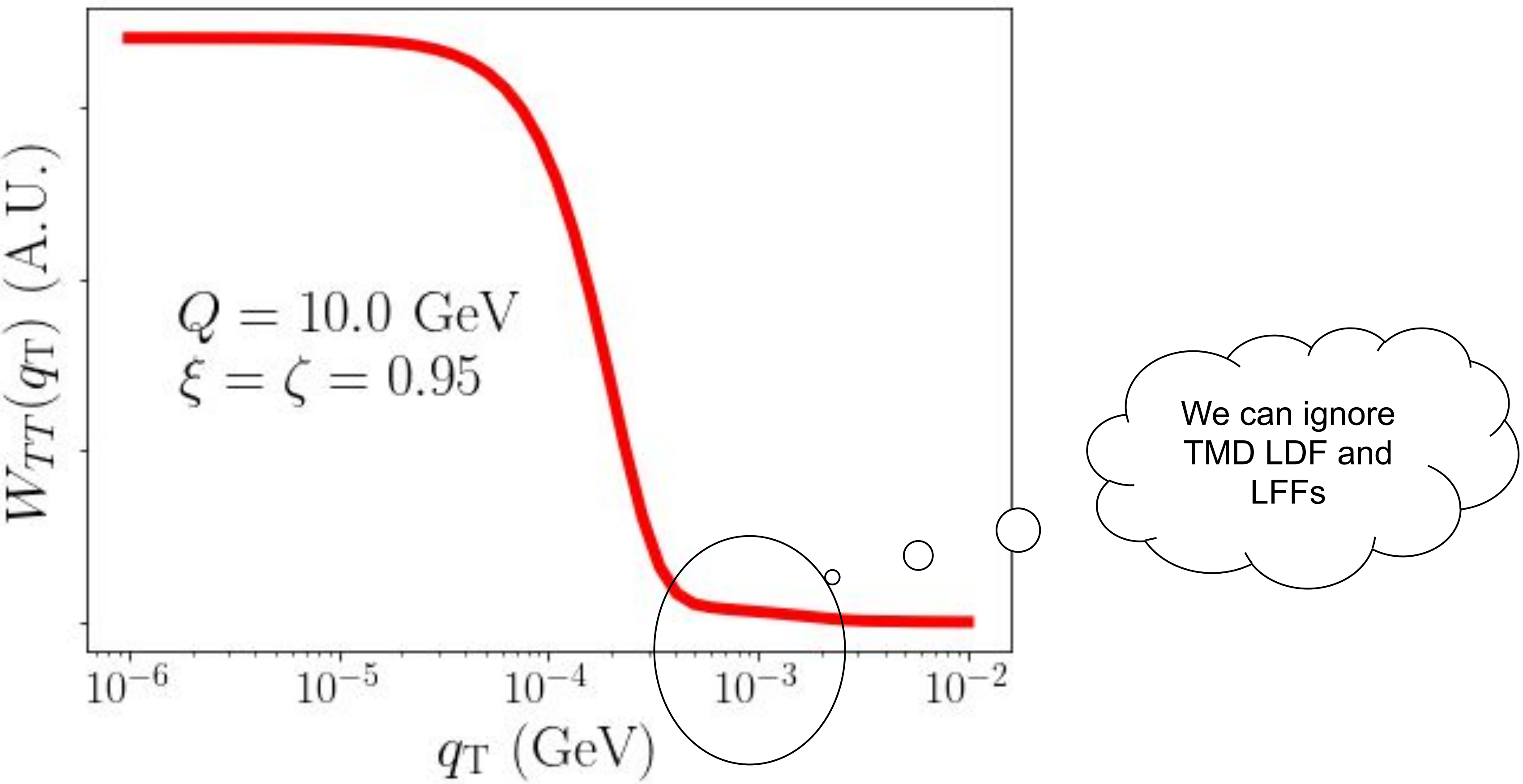
$$\widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) = 2\int_{\xi_B}^{1} \frac{d\xi}{\xi} \int_{\zeta_B}^{1} \frac{d\zeta}{\zeta^2} \underline{f(\xi)D(\zeta)} C_f(\lambda) C_D(\eta)$$
$$\times \exp\left\{ -\int_{\mu_b^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A(\alpha(\mu')) \ln \frac{\mu_Q^2}{\mu'^2} + B(\alpha(\mu')) \right] \right\}$$

Collinear LDF and LFFs

The QED Sudakov varies very slowly



$Q = 10.0$ GeV
$\xi = \zeta = 0.95$
$\widetilde{W}_{TT}(b_\mathrm{T})$ (A.U.)
b_T (GeV^{-1})
0.0
0.5
1.0
1.5
2.0
2.5
10^{-1}
10^{1}
10^{3}
10^{5}
Spectrum is too broad in bT space -> a narrow peak in qT space at qT=0
max(mub, me)

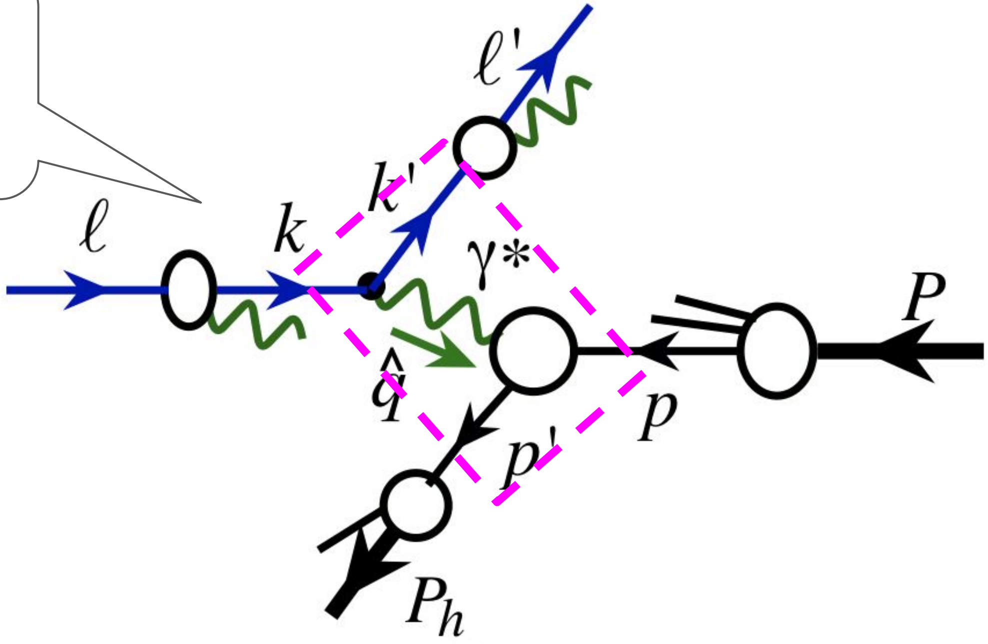


$Q = 10.0$ GeV
$\xi = \zeta = 0.95$
$W_{TT}(q_T)$ (A.U.)
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
q_T (GeV)
We can ignore TMD LDF and LFFs

Semi-Inclusive DIS

Do we need to worry about TMD LDFs and LFFs ? -> **NO**

Hybrid TMD/collinear framework



SIDIS with QED+QCD

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\sum_i w_i F_i(x, Q^2, z, \mathbf{P}_{h\perp})$$

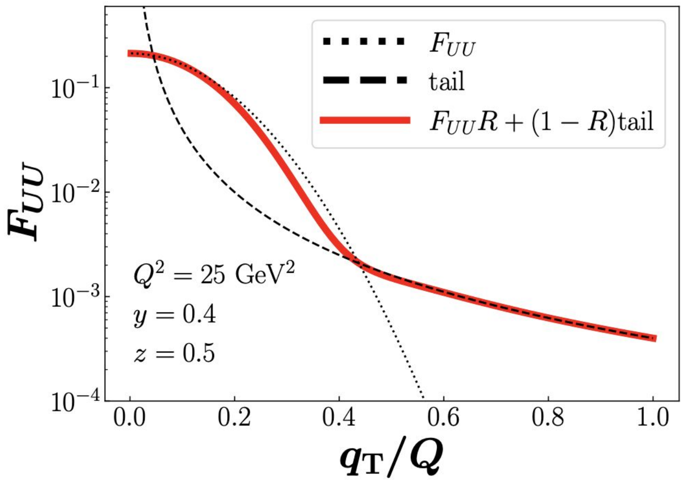
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min}(\zeta)}^{1} d\xi\, \underline{f_{k/l}(\xi) D_{k'/l'}(\zeta)}$$
$$\times \frac{\hat{x}}{x\xi\zeta}\left[\frac{\alpha^2}{\hat{x}\hat{y}\widehat{Q}^2}\frac{\hat{y}}{2(1-\hat{\varepsilon})}\left(1+\frac{\hat{\gamma}^2}{2\hat{x}}\right)\sum_i \hat{w}_i F_i(\hat{x}, \widehat{Q}^2, \hat{z}, \hat{\mathbf{P}}_{h\perp})\right]$$

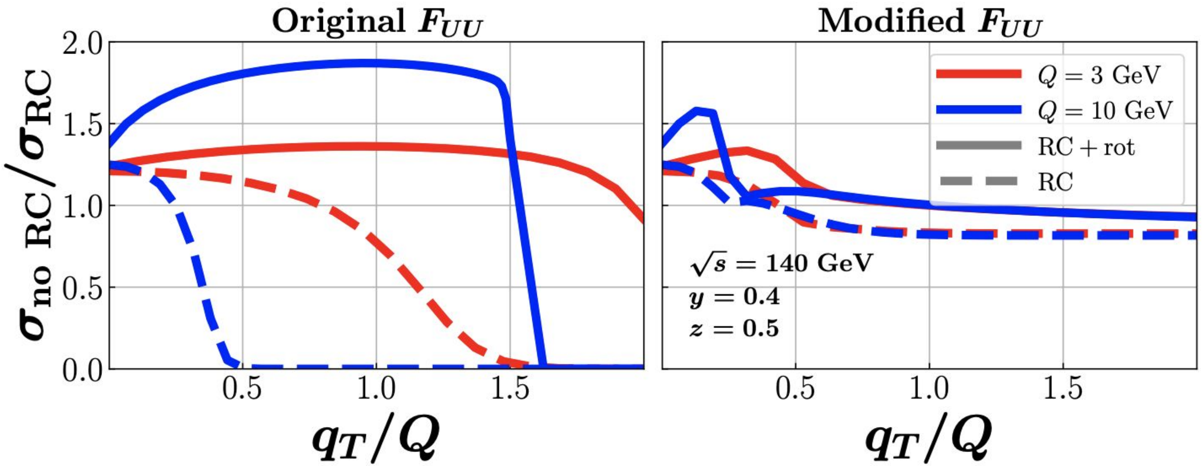
Collinear LDFs and LFFs

The inverse problem

$$F_{UU} \to F_{UU} R + (1 - R)\text{tail}$$

$$\text{tail} = \frac{0.01}{q_T^2}, \qquad R = \exp\left[-20\left(\frac{q_T}{Q}\right)^3\right]$$





QED RC depends significantly on the hadronic input

Radiative Effects in the Processes of Hadron Electroproduction

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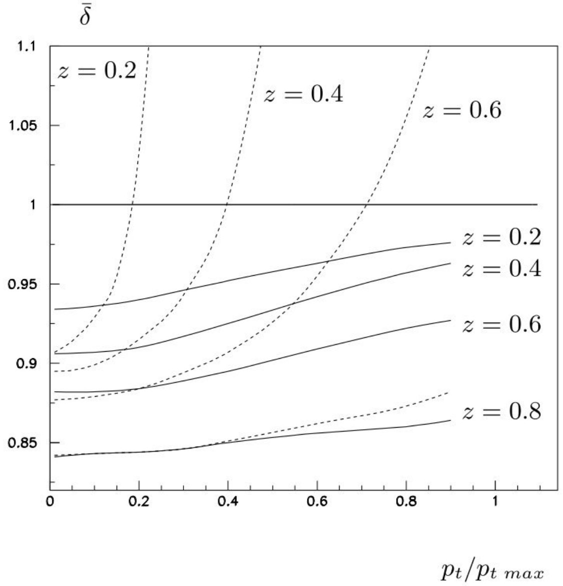
National Center of Particle and High Energy Physics, 220040 Minsk, Belarus

Received: date / Revised version: date

Abstract. An approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. An explicit formulae for the lowest order QED radiative correction are presented. Detailed numerical analysis is performed for the kinematics of experiments at the fixed targets.



Similar trends, e.g., RCs depend on hadronic input

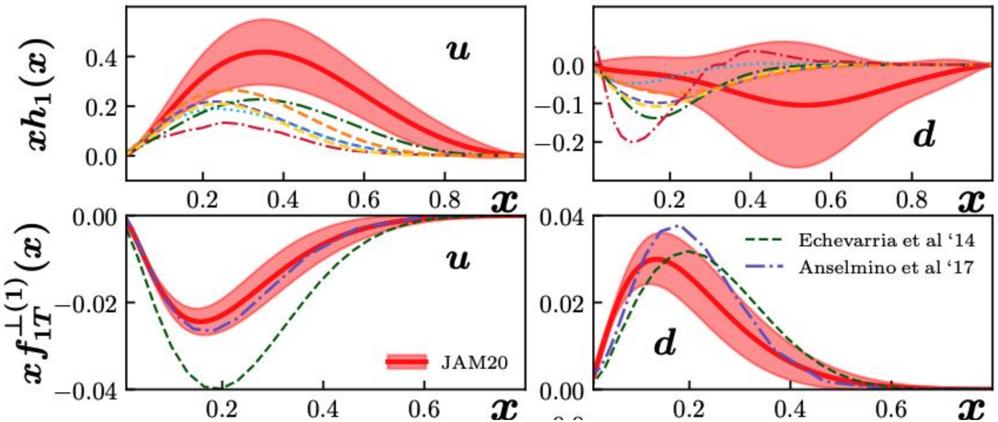
What about spin structures?

arXiv:2002.08384 (hep-ph)

[Submitted on 19 Feb 2020 (v1), last revised 2 Sep 2020 (this version, v2)]

Origin of single transverse-spin asymmetries in high-energy collisions

Justin Cammarota, Leonard Gamberg, Zhong-Bo Kang, Joshua A. Miller, Daniel Pitonyak, Alexei Prokudin, Ted C. Rogers, Nobuo Sato



Transversity

Sivers

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\, \frac{y^2}{2\,(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \Bigg\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$+ \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h}$$

$$+ S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \right]$$

$$+ S_\parallel \lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right]$$

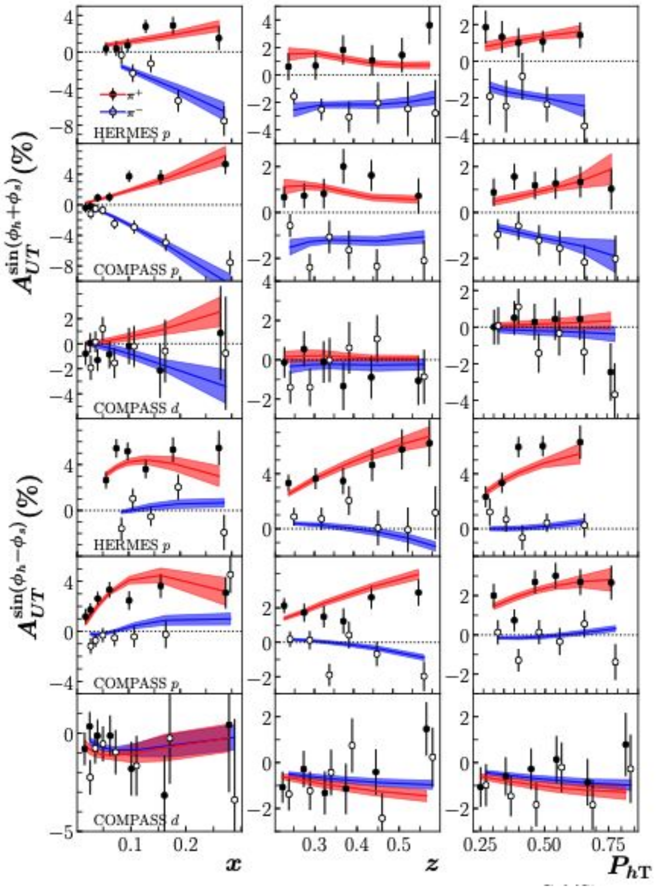
$$+ |\boldsymbol{S}_\perp| \Bigg[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right)$$

$$+ \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg]$$

$$+ |\boldsymbol{S}_\perp| \lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\},$$



Standard approach

$$\left.\frac{d^6\sigma_{\ell P(S_T)\to\ell' P_h X}}{dx_B dy\, d\psi\, dz_h\, dP_{hT}^2}\right|_{UT,T}^{\sin(\phi_h-\phi_S)} = \int d\phi_h\, d\phi_S\, \sin(\phi_h-\phi_S)\, \frac{d^6\sigma_{\ell P(S_T)\to\ell' P_h X}}{dx_B dy\, d\psi\, dz_h\, d\phi_h dP_{hT}^2},$$

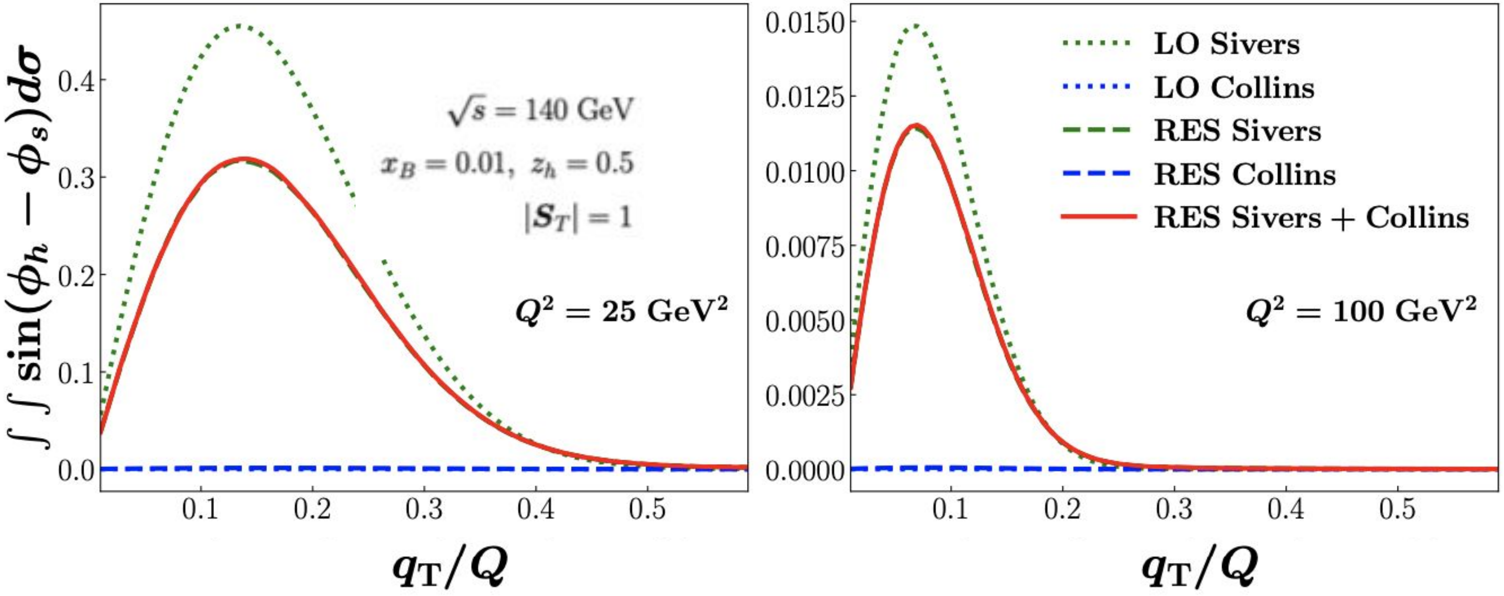
$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \overset{\text{no QED}}{=} \int d\phi_h\, d\phi_S\, \sin(\phi_h-\phi_S)\Big[\sin(\phi_h-\phi_S) F_{UT,T}^{\sin(\phi_h-\phi_S)} + \sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)}\Big].$$

$$\int d\phi_h d\phi_S \;\; \sin(\phi_h-\phi_S) \;\; \sin(\phi_h+\phi_S) = 0$$

BUT with QED this does not hold!

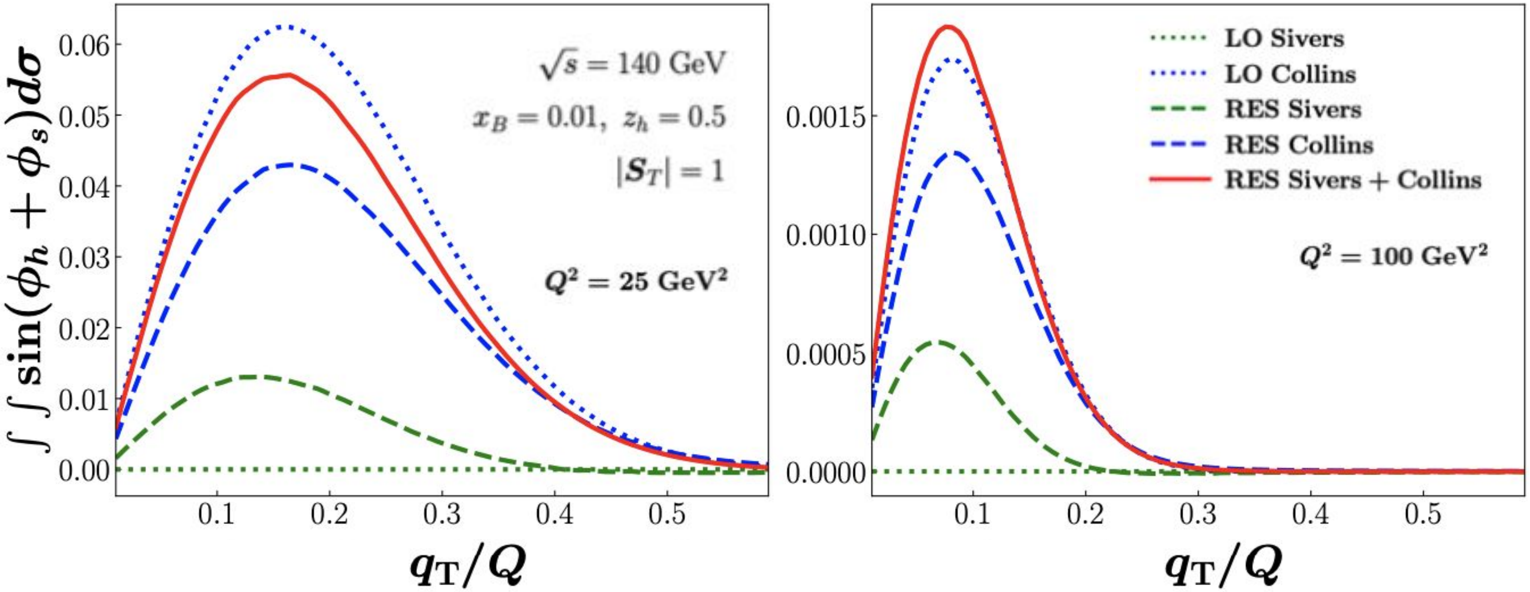
$$\int d\phi_h\, d\phi_S\, \sin(\phi_h-\phi_S) \sin(\hat{\phi}_h+\hat{\phi}_S) \neq 0$$

Sivers



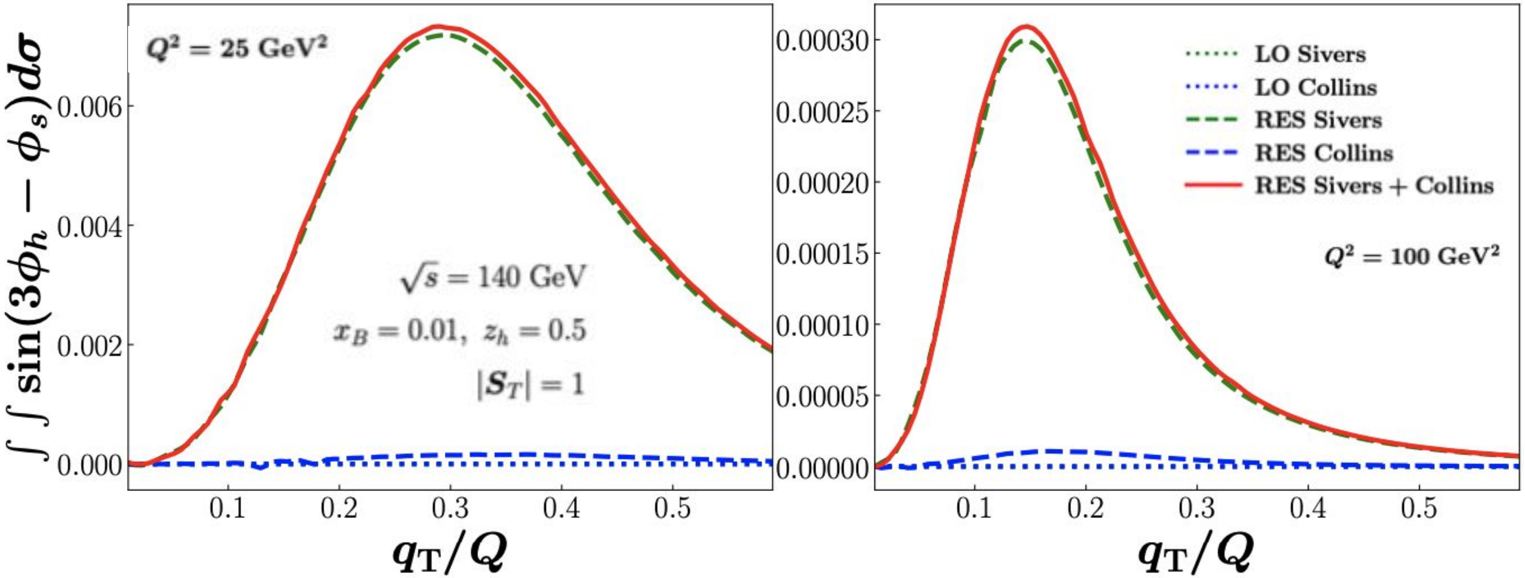
$\int\int \sin(\phi_h - \phi_s) d\sigma$
0.0
0.1
0.2
0.3
0.4
$\sqrt{s} = 140$ GeV
$x_B = 0.01,\ z_h = 0.5$
$|S_T| = 1$
$Q^2 = 25$ GeV^2
0.1
0.2
0.3
0.4
0.5
q_T/Q
0.0000
0.0025
0.0050
0.0075
0.0100
0.0125
0.0150
LO Sivers
LO Collins
RES Sivers
RES Collins
RES Sivers + Collins
$Q^2 = 100$ GeV^2
0.1
0.2
0.3
0.4
0.5
q_T/Q

Collins

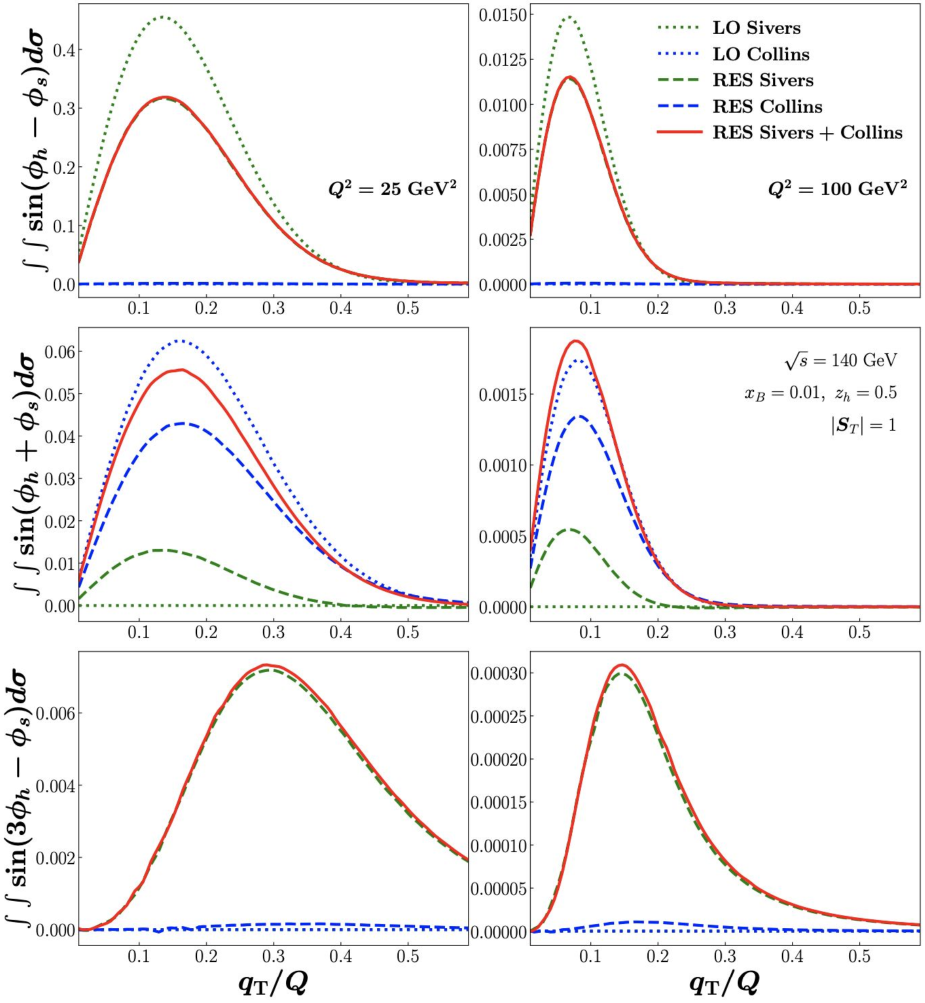


$\sqrt{s} = 140$ GeV
$x_B = 0.01,\ z_h = 0.5$
$|S_T| = 1$
$Q^2 = 25$ GeV^2
$Q^2 = 100$ GeV^2
LO Sivers
LO Collins
RES Sivers
RES Collins
RES Sivers + Collins
$\int\int \sin(\phi_h + \phi_s) d\sigma$
q_T/Q
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.0000
0.0005
0.0010
0.0015
0.1
0.2
0.3
0.4
0.5

3phi_h - phi_s



$Q^2 = 25$ GeV^2
$\sqrt{s} = 140$ GeV
$x_B = 0.01, \ z_h = 0.5$
$|S_T| = 1$
$\int\int \sin(3\phi_h - \phi_s) d\sigma$
0.000
0.002
0.004
0.006
0.1
0.2
0.3
0.4
0.5
q_T/Q
LO Sivers
LO Collins
RES Sivers
RES Collins
RES Sivers + Collins
$Q^2 = 100$ GeV^2
0.00000
0.00005
0.00010
0.00015
0.00020
0.00025
0.00030



$$|S_\perp| \Big[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)} \right)$$
$$+ \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)}$$

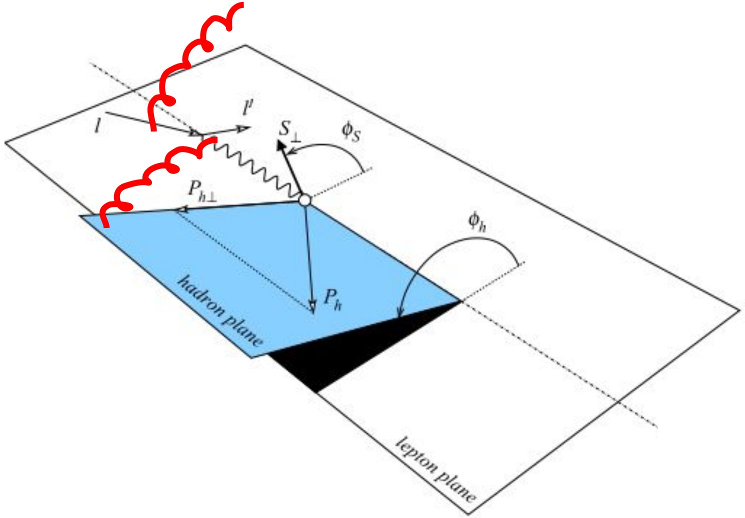
- Visible leaking effects from Sivers -> Collins
- Not possible to isolate QED-free individual signals
- Any QED corrections to data is model dependent

Summary/Outlook

- In the presence of QED radiation, **the q direction is not fixed**

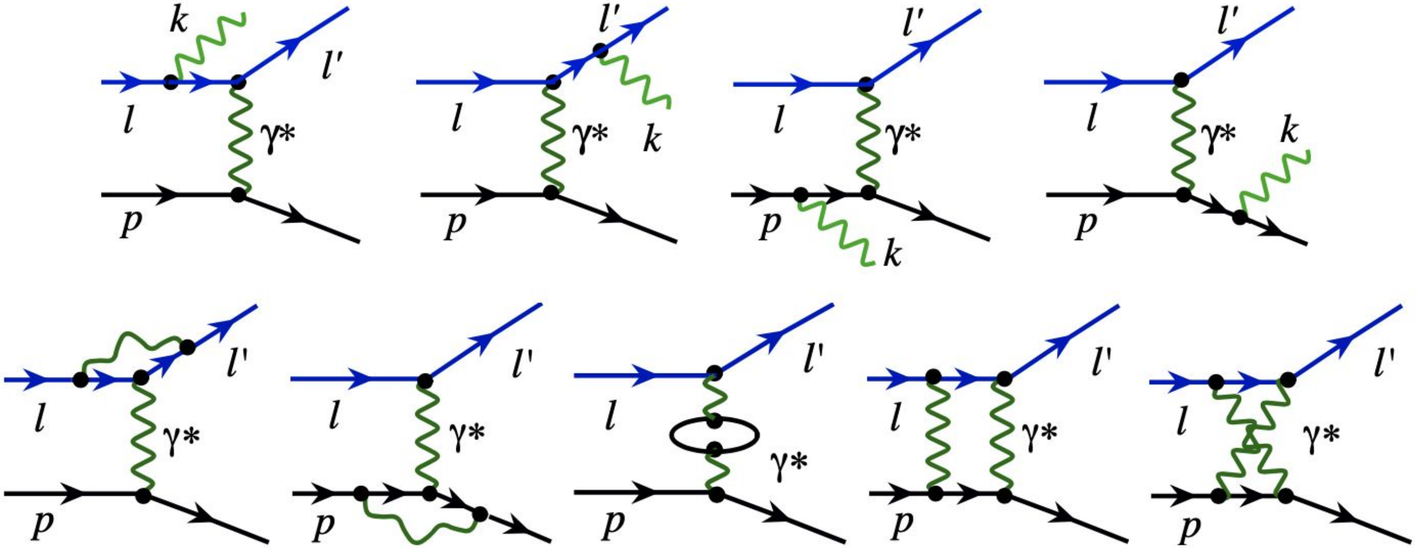
- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization

- QED effects **needs** to take into account for reliable TMD SIDIS analysis



BACKUP

Hard part in QED



Two photon exchange contributions

Kinematics

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P \cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q\sqrt{1+\gamma^2}}$$

Kinematics affected by QED

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \qquad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \qquad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

SIDIS with QED+QCD

$$\underbrace{\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2}} = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min}(\zeta)}^{1} d\xi\, f_{k/l}(\xi)\, D_{k'/l'}(\zeta) \times \frac{\hat{x}}{x\xi\zeta}\left[\frac{\alpha^2}{\hat{x}\hat{y}\widehat{Q}^2}\frac{\hat{y}}{2(1-\hat{\varepsilon})}\left(1+\frac{\hat{\gamma}^2}{2\hat{x}}\right)\sum_i \hat{w}_i F_i(\hat{x},\widehat{Q}^2,\hat{z},\hat{\mathbf{P}}_{h\perp})\right]$$

External kinematics

Internal kinematics

$$l \to \xi k$$

$$l' \to k'/\zeta$$

External Kinematics

$$[l_\perp]^2 = Q^2\left(\frac{Q^2 - M^2x^2y^2 - Q^2y}{y^2(4M^2x^2+Q^2)}\right) \tag{C1}$$

$$[q\cdot P_h] = \frac{Q}{(4M^2x^2)}\left(Q^3z - \sqrt{(4M^2x^2+Q^2)(Q^4z^2 - 4M^2M_h^2x^2 - 4M^2P_{h\perp}^2x^2)}\right) \tag{C2}$$

$$[l\cdot P_h] = \frac{1}{y(4M^2x^2+Q^2)}\left(-4M^2P_{h\perp}[l_\perp]x^2y\cos(\phi_h) + 2M^2x^2y\;[q\cdot P_h] - P_{h\perp}Q^2[l_\perp]y\cos(\phi_h)\right.$$

$$\left. -Q^4yz/2 + Q^4z + Q^2\;[q\cdot P_h]\right) \tag{C3}$$

$$[l'\cdot P_h] = [l\cdot P_h] - [q\cdot P_h] \tag{C4}$$

$$[q\cdot S] = -\frac{Q}{2Mx}\sqrt{-(|\mathbf{S}_\perp| - 1)(|\mathbf{S}_\perp| + 1)(4M^2x^2+Q^2)} \tag{C5}$$

$$[l\cdot S] = \frac{1}{y(4M^2x^2+Q^2)}\left(-4M^2|\mathbf{S}_\perp|l_\perp x^2y\cos(\phi_S) + 2M^2x^2y[q\cdot S]\right.$$

$$\left. -Q^2|\mathbf{S}_\perp|l_\perp y\cos(\phi_S) + Q^2[q\cdot S]\right) \tag{C6}$$

$$[l'\cdot S] = [l\cdot S] - [q\cdot S] \tag{C7}$$

$$[\epsilon_{\mu\nu\rho\sigma}P^\mu l^\nu l'^\rho P_h^\sigma] = -\frac{P_{h\perp}Q^2[l_\perp]}{2x}\sqrt{1+\frac{4M^2x^2}{Q^2}}\sin(\phi_h) \tag{C8}$$

$$[\epsilon_{\mu\nu\rho\sigma}P^\mu l^\nu l'^\rho S^\sigma] = -\frac{|\mathbf{S}_\perp|Q^2[l_\perp]}{2x}\sqrt{1+\frac{4M^2x^2}{Q^2}}\sin(\phi_S) \tag{C9}$$

Internal Kinematics

$$l \to \xi k$$

$$l' \to k'/\zeta$$

$$[k \cdot P_h] = \xi[l \cdot P_h], \qquad [k' \cdot P_h] = \frac{1}{\zeta}[l' \cdot P_h] \tag{C12}$$

$$[k \cdot S] = \xi[l \cdot S] \qquad [k' \cdot S] = \frac{1}{\zeta}[l' \cdot S] \tag{C13}$$

$$[\epsilon_{\mu\nu\rho\sigma} P^\mu k^\nu k'^\rho P_h^\sigma] = \frac{\xi}{\zeta}[\epsilon_{\mu\nu\rho\sigma} P^\mu l^\nu l'^\rho P_h^\sigma], \qquad [\epsilon_{\mu\nu\rho\sigma} P^\mu k^\nu k'^\rho S^\sigma] = \frac{\xi}{\zeta}[\epsilon_{\mu\nu\rho\sigma} P^\mu l^\nu l'^\rho S^\sigma] \tag{C14}$$

$$[\hat{q} \cdot P_h] = [k \cdot P_h] - [k' \cdot P_h], \qquad [\hat{q} \cdot S] = [k \cdot S] - [k' \cdot S] \tag{C15}$$

$$[\hat{P}_{h\perp}]^2 = \frac{1}{\widehat{Q}^2(4M^2\hat{x}^2 + \widehat{Q}^2)} \left(-4M^2 M_h^2 \widehat{Q}^2 \hat{x}^2 - 4M^2\hat{x}^2[\hat{q} \cdot P_h]^2 - M_h^2 \widehat{Q}^4 + \widehat{Q}^6 \hat{z}^2 + 2\widehat{Q}^4 \hat{z}[\hat{q} \cdot P_h]\right) \tag{C16}$$

$$[|\hat{\mathbf{S}}_\perp|]^2 = \frac{1}{\widehat{Q}^2(4M^2\hat{x}^2 + \widehat{Q}^2)}(4M^2\widehat{Q}^2\hat{x}^2 - 4M^2\hat{x}^2[\hat{q} \cdot S]^2 + \widehat{Q}^4) \tag{C17}$$

$$[\hat{S}_{||}] = \frac{2M\hat{x}[\hat{q} \cdot S]}{\widehat{Q}^2\sqrt{4M^2\hat{x}^2/\widehat{Q}^2 + 1}} \tag{C18}$$

$$[\cos(\hat{\phi}_h)] = \frac{1}{2[\hat{P}_{h\perp}][k_\perp]\hat{y}(4M^2\hat{x}^2 + \widehat{Q}^2)} \Big(4M^2\hat{x}^2\hat{y}[\hat{q} \cdot P_h] - \widehat{Q}^4\hat{y}\hat{z} + 2\widehat{Q}^2(\widehat{Q}^2\hat{z} + [\hat{q} \cdot P_h]) - 2\hat{y}(4M^2\hat{x}^2 + \widehat{Q}^2)[k \cdot P_h]\Big) \tag{C19}$$

$$[\cos(\hat{\phi}_S)] = \frac{1}{[|\hat{\mathbf{S}}_\perp|][k_\perp]\hat{y}(4M^2\hat{x}^2 + \widehat{Q}^2)} \left(2M^2\hat{x}^2\hat{y}[\hat{q} \cdot S] + \widehat{Q}^2[\hat{q} \cdot S] - \hat{y}(4M^2\hat{x}^2 + \widehat{Q}^2)[k \cdot S]\right)$$

$$[\sin(\hat{\phi}_h)] = -2\hat{x}\frac{[\epsilon_{\mu\nu\rho\sigma} P^\mu k^\nu k'^\rho P_h^\sigma]}{[\hat{P}_{h\perp}]\widehat{Q}^2[k_\perp]\sqrt{4M^2\hat{x}^2/\widehat{Q}^2 + 1}}$$

$$[\sin(\hat{\phi}_S)] = -2\hat{x}\frac{[\epsilon_{\mu\nu\rho\sigma} P^\mu k^\nu k'^\rho S^\sigma]}{[|\hat{\mathbf{S}}_\perp|]\widehat{Q}^2[k_\perp]\sqrt{4M^2\hat{x}^2/\widehat{Q}^2 + 1}}$$

Case study: FUU

$$
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =
$$

$$
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}
$$

$$
+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}
$$

$$
+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]
$$

$$
+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]
$$

$$
+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)
$$

$$
+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}
$$

$$
+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]
$$

$$
+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}
$$

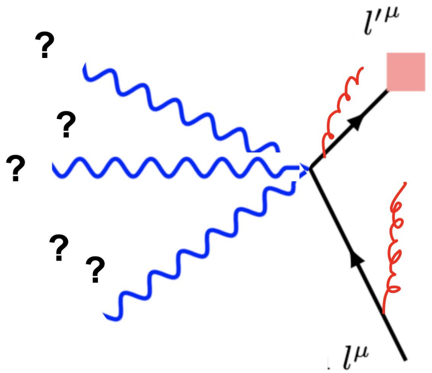
$$
+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}, \qquad (
$$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h,\mathrm{T}}^2} = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min(\zeta)}}^{1} d\xi\, f(\xi) D(\zeta) \frac{\hat{x}}{x\xi\zeta}$$

$$\times \left[\frac{\alpha_{\mathrm{EM}}^2}{\hat{x}\hat{y}\hat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\epsilon})} \left(1 + \frac{\hat{\gamma}}{2\hat{x}}\right) F_{UU}(\hat{x}, \hat{Q}^2, \hat{z}, \hat{P}_{h,\mathrm{T}}) \right]$$

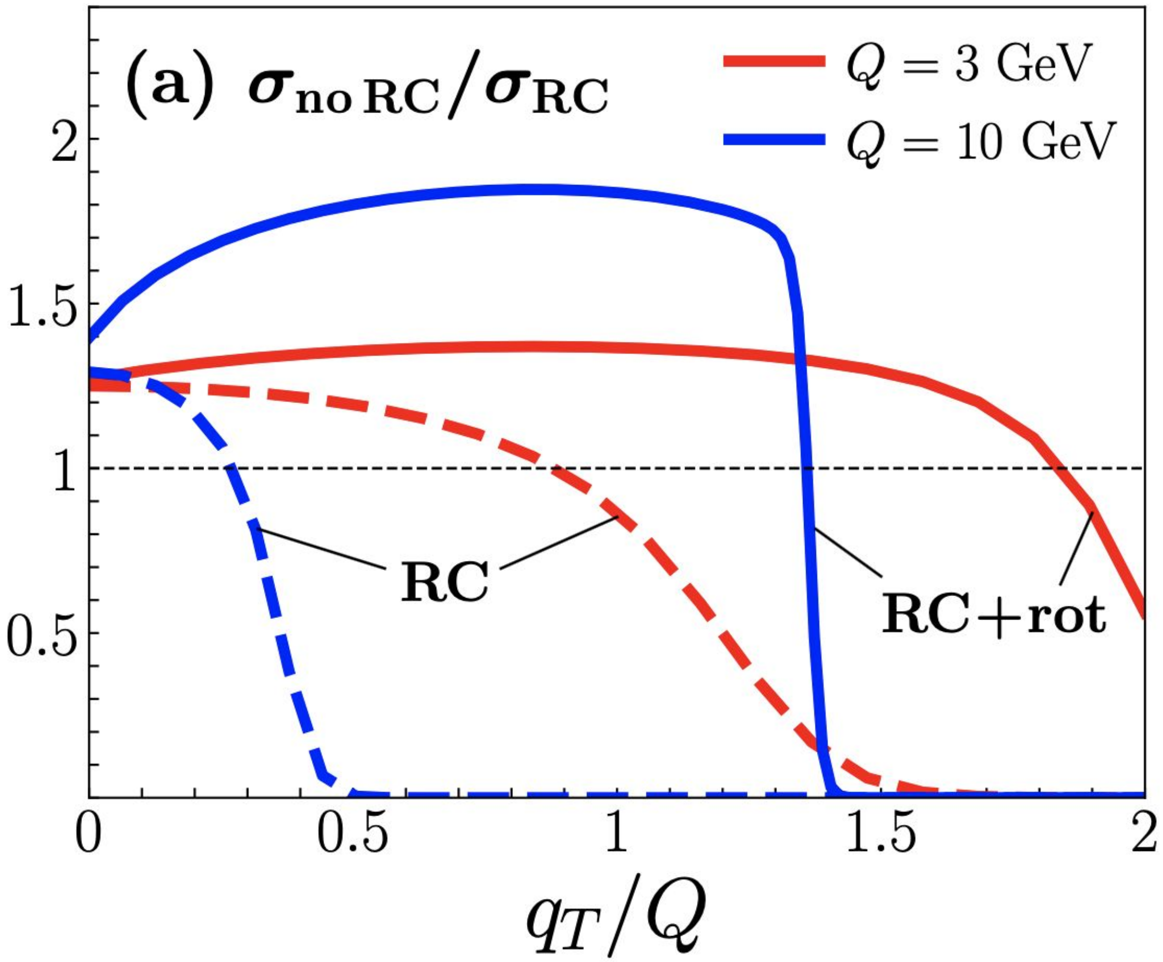
Collinear LDF and LFFs

Use simple parametrization fitted to data



QED rotational effects

Hadron pT in the true Breit frame



$\sqrt{s} = 140$ GeV
$y = 0.4$
$z = 0.5$