Radiative correction for SIDIS

Factorized vs Traditional approach

Correlations in Partonic and Hadronic Interactions (CPHI-2022)

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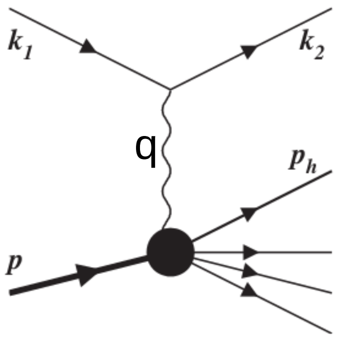
Bishnu Karki

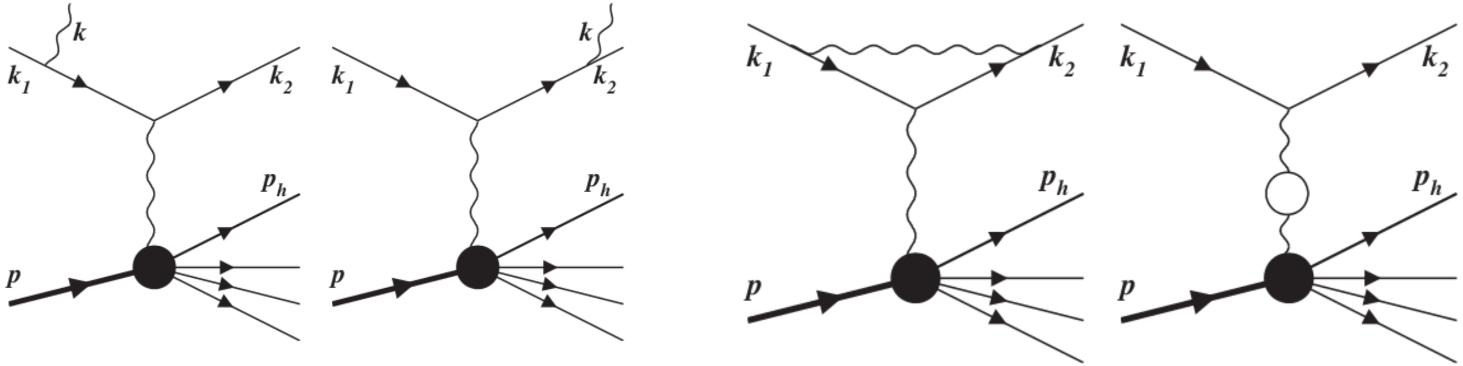
Duke University, NC

RC in SIDIS

Born process



RC for SIDIS



- Transverse Momentum Dependent PDFs (TMDs) provide a new way to understand 3-D structure of the nucleon
- TMDs can be accessed via Semi-Inclusive Deep Inelastic Scattering (SIDIS) measurements
- In such reactions, with a large momentum transfer, photons are radiated from incoming, outgoing leptons
- Radiation alters momentum transfer and angular modulation between leptonic and hadronic planes
- Collision induced QED radiation must be taken into account to reliably extract TMDs
- RC more important in future EIC and SoLID kinematics, high q allows more phase space to shower

Radiative correction vs radiative contribution

- **Traditional approach (QED radiative correction)** **I. Akushevich and A. Ilychev Physics Review D100 (2019)**

$$\sigma_{obs}(x_B, Q^2) = R_{QED}(x_B, Q^2; x_{B,true}, Q^2_{true}) \times \sigma_{Born}(x_{B,true}, Q^2_{true}) + \sigma_X(x_B, Q^2)$$

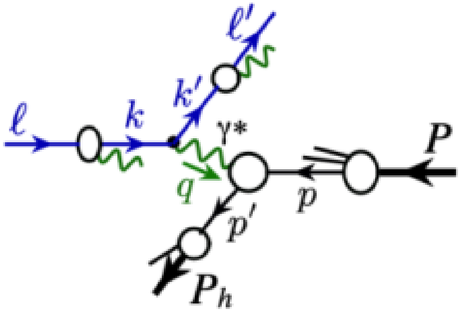
- R_{QED} and σ_x are computed theoretically
- Prescription of matching Born cross-section by removing radioactive effects becomes increasingly difficult for SIDIS or exclusive processes
- Analytical expression for the lowest order RC to SIDIS

Factorized approach (QED radiative contribution) **T. Liu et al. Journal of High energy Physics 157, (2021)**

$$\sigma_{obs}(x_B, Q^2) = \sigma^{univ}_{lep}(\mu^2; m_e^2) \otimes \sigma^{univ}_{had}(\mu^2; \Lambda^2_{QCD}) \otimes \sigma_{IR-safe}(x_{B,true}, Q^2_{true}, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}, \frac{m_e^2}{Q^2}\right)$$

- Simultaneously treats QED and QCD effects on the same footing
- Unify QED and QCD contribution to the lepton-nucleon scattering cross-section in a consistent factorization formalism
- All infrared sensitive QED contribution absorbed to the universal lepton and hadronic distribution
- Infrared safe contribution are calculated order by order

Factorized approach



$$\frac{d^6\sigma_{SIDIS}}{dx_B dy d\phi_s dz_h d\phi_h dP_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{min}}^{1} \frac{d\xi}{\xi} f_{e/e}(\xi) D_{e/e}(\zeta) \times \frac{\hat{x}_B}{x_B \xi \zeta} \boxed{\left[\frac{\alpha^2 \hat{y}}{\hat{x}_B \hat{Q}^2 2(1-\hat{\epsilon})} \left(1 + \frac{\hat{\gamma}^2}{2\hat{x}_B}\right) \sum_n \hat{w}_n F_n^h(\hat{x}_B, \hat{Q}^2, \hat{z}_h, \hat{P}_{hT}^2) \right]}$$

Evaluated in a “virtual photon-hadron” frame

$$D_{e/e}^{(1)}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+ \qquad f_{e/e}^{(1)}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat{q}, P, \widehat{P}_h\} \xrightarrow[(\xi,\zeta)]{} \{q, P, P_h\}$$

In a frame to compare with exp. measurement

- Leading power IR sensitive contribution factorized into LDFs and LFFs
- IR safe contribution are calculated order-by-order in power of α
- Collinear QED factorization for both inclusive DIS and SIDIS

Radiative correction (Traditional approach)

- Radiative part σ^R is divided into soft and hard parts by cutoff in photon energy $\bar{k}_0$.

$$\sigma = \underbrace{\sigma^B + \frac{\alpha}{\pi}(\delta_{\text{vert}} + \delta_{\text{vac}})\sigma^B + \sigma^{AMM} + \overbrace{\int_0^{\bar{k}_0} \sigma^R \, d^3\boldsymbol{k}}^{\text{difficult}}}_{\text{soft part}} + \underbrace{\int_{\bar{k}_0}^{\infty} \sigma^R \, d^3\boldsymbol{k}}_{\text{hard part}}$$

- To handle soft part of σ^R, split into components σ_{IR}^R (σ_F^R), with (without) infrared divergence, in such a way that

$$\sigma^R = \sigma_{IR}^R + \sigma_F^R, \quad \int_0^{\bar{k}_0} \sigma_{IR}^R \, d^3\boldsymbol{k} = \frac{\alpha}{\pi}\delta_S \sigma^B$$

- Difficult term becomes

$$\int_0^{\bar{k}_0} \sigma^R \, d^3\boldsymbol{k} = \frac{\alpha}{\pi}\delta_S \sigma^B + \overset{\approx 0}{\cancel{\int_0^{\bar{k}_0} \sigma_F^R \, d^3\boldsymbol{k}}}$$

- Infrared divergence in δ_S adds with δ_{vert} to form δ_{VS} (without infrared divergence).

$$\sigma \approx \underbrace{\sigma^B + \frac{\alpha}{\pi}(\delta_{VS} + \delta_{\text{vac}})\sigma^B + \sigma^{AMM}}_{\text{soft part}} + \underbrace{\int_{\bar{k}_0}^{\infty} \sigma^R \, d^3\boldsymbol{k}}_{\text{hard part}}, \quad \delta_{VS} = \delta_S + \delta_{\text{vert}}$$

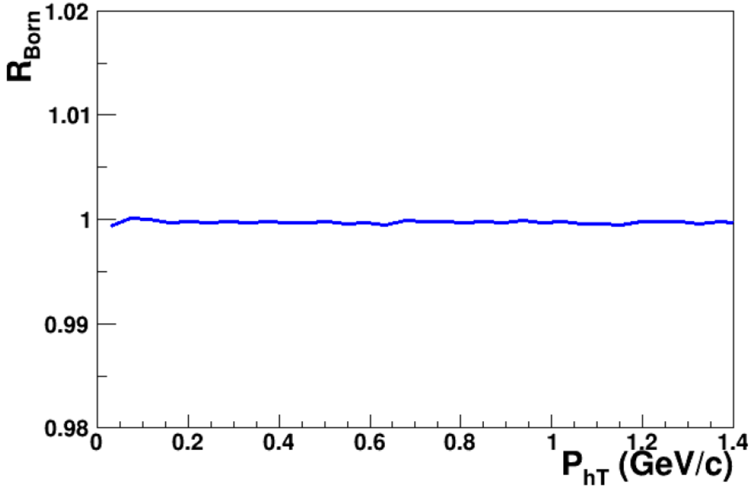
SIDIS Unpolarized cross-section without radiation

Unpolarized beam and transversely polarized target

$$\frac{d^6\sigma_{SIDIS}}{dx_B dy d\phi_s dz_h d\phi_h dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{\boxed{F_{UU}} + \boxed{|S_\perp|\Big[sin(\phi_h-\phi_s)F_{UT}^{Siv} + \epsilon sin(\phi_h+\phi_s)F_{UT}^{Col}} \right.$$
$$\left. \boxed{+\epsilon sin(3\phi_h-\phi_s)F_{UT}^{Pret}\Big]}\right\}$$

- H(e,e’π^+)X reaction
- Case Study F_{UU}
- SF from JAM3D20 global analysis J.Cammarota et al. Phys. Rev. D, 102:054002

$$R_{Born} = \frac{d\sigma_{Born}^{Factorized}}{d\sigma_{Born}^{Traditional}}$$

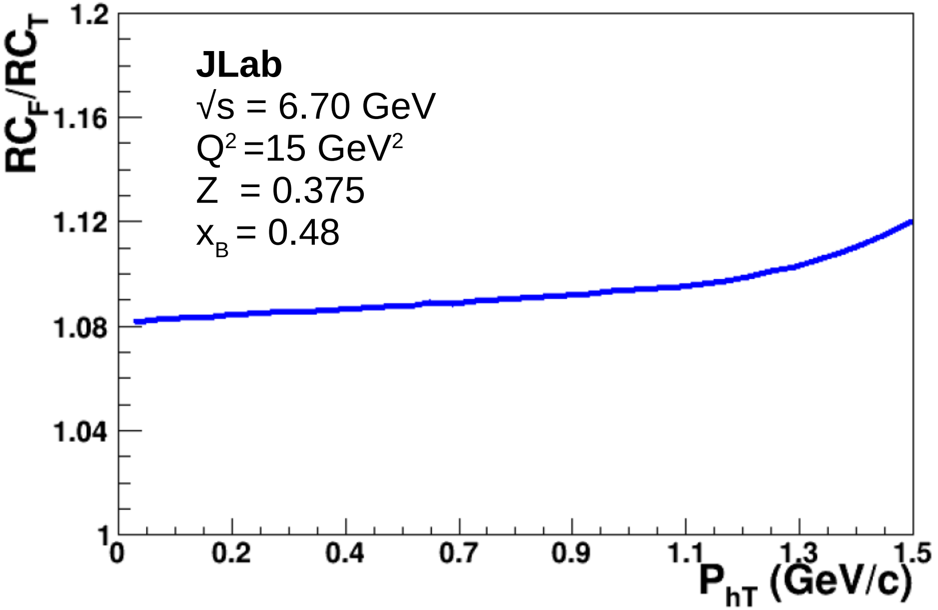


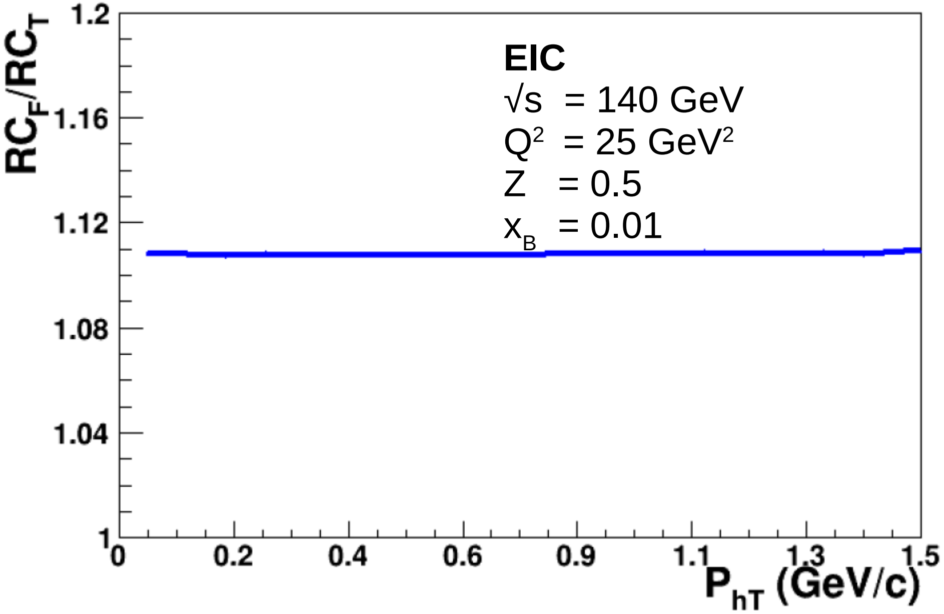
- Used two different codes for cross-section extraction
- Born cross-section (without radiation) agrees between two codes
- Consistency in structure function and kinematics between two codes

Traditional vs Factorized approach

Unpolarized cross-section

- $H(e,e'\pi^+)X$ reaction
- Case Study F_{UU}





$$\mathrm{RC} = \frac{d\sigma_{\text{no RC}}}{d\sigma_{RC}}$$

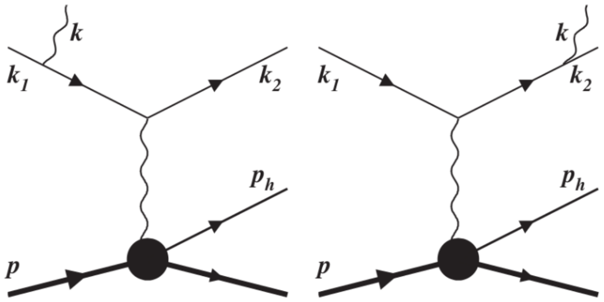
Traditional vs Factorized approach

Unpolarized cross-section

- **H(e,e’π⁺)X reaction**
- **Case Study F_{UU}**

$\sqrt{s}$ (GeV)	x_B	Q^2 (GeV^2)	z_h	RC ratio
Jefferson Lab Kinematics				
3.2	0.32	2.3	0.55	1.06
4.9	0.48	8	0.375	1.08
6.7	0.48	15	0.375	1.09
EIC Kinematics				
140	0.01	9	0.5	1.08
140	0.01	25	0.5	1.10
140	0.01	100	0.5	1.10

Exclusive process not included



6-10 % difference in RC factor between two approaches

Single transverse spin asymmetry

UT cross-section

$$\frac{d^6\sigma_{SIDIS}}{dx_B dy d\phi_s dz_h d\phi_h dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ \quad \boxed{+|S_\perp|\Big[sin(\phi_h-\phi_s)F_{UT}^{Siv}+\epsilon sin(\phi_h+\phi_s)F_{UT}^{Col}}$$

$$\boxed{+\epsilon sin(3\phi_h-\phi_s)F_{UT}^{Pret}\Big]}\Big\}$$

Integrating the UT cross-section provides:
Sivers $sin(\phi_h-\phi_s)$ and Collins $sin(\phi_h+\phi_s)$ modulation

For instance,

$$\left.\frac{d^6\sigma_{SIDIS}}{dx_B dy d\phi_s dz_h d\phi_h dP_{hT}^2}\right|_{UT}^{sin(\phi_h-\phi_s)} = \int d\phi_h d\phi_s sin(\phi_h-\phi_s)\frac{d^6\sigma_{SIDIS}}{dx_B dy d\phi_s dz_h d\phi_h dP_{hT}^2}$$

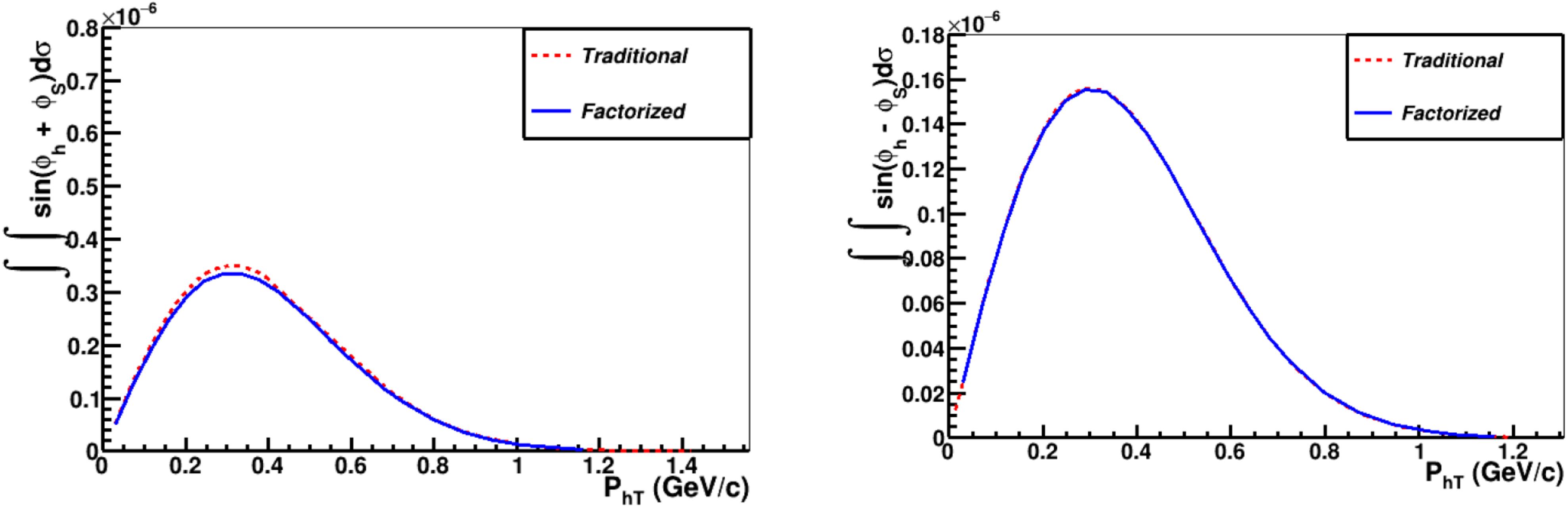
At structure function level

$$F_{UT}^{Siv} = \int d\phi_h d\phi_s sin(\phi_h-\phi_s)\Big[sin(\phi_h-\phi_s)F_{UT}^{Siv}+\epsilon sin(\phi_h+\phi_s)F_{UT}^{Col}$$
$$+\epsilon sin(3\phi_h-\phi_s)F_{UT}^{Pret}\Big]$$

Single transverse spin asymmetry

Preliminary results

JLab: $\sqrt{s}$ = 6.70 GeV Q^2 =15 GeV^2 Z = 0.375 x_B = 0.48

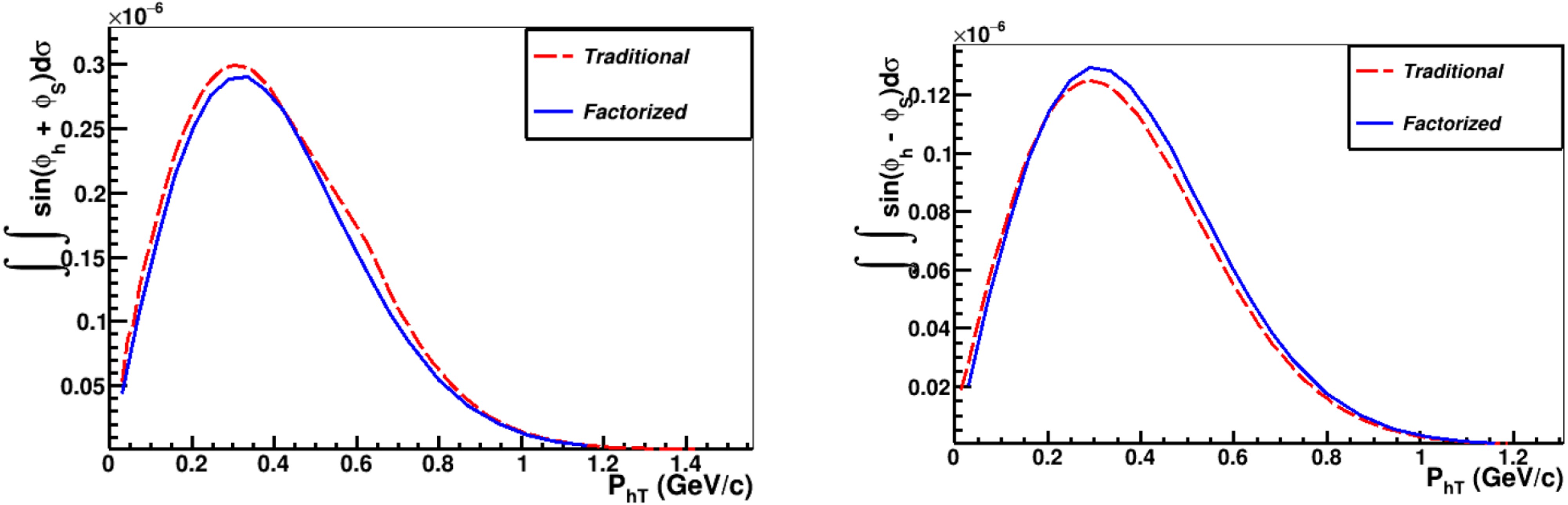


Without radiation Sivers Sin(ϕ_h - ϕ_s) and Collins Sin(ϕ_h + ϕ_s) modulation agrees well

Single transverse spin asymmetry

Preliminary results

JLab: $\sqrt{s}$ = 6.70 GeV^2 Q^2 =15 GeV^2 Z = 0.375 X$_B$ = 0.48

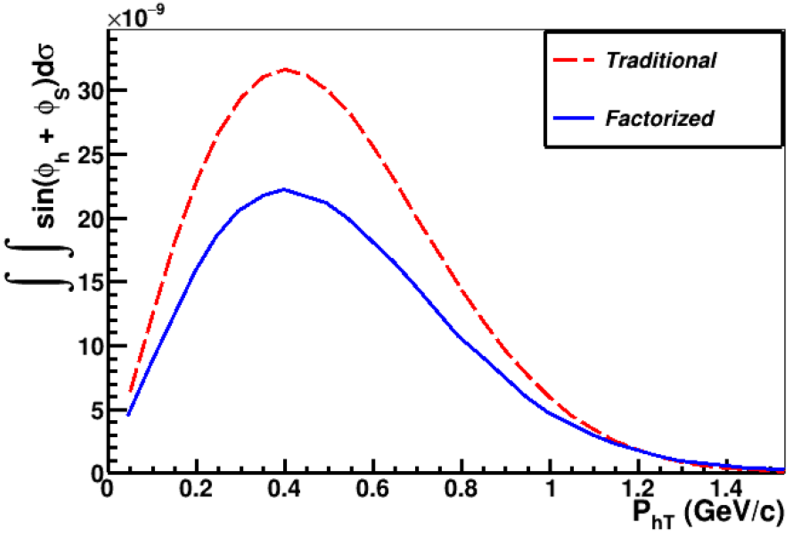


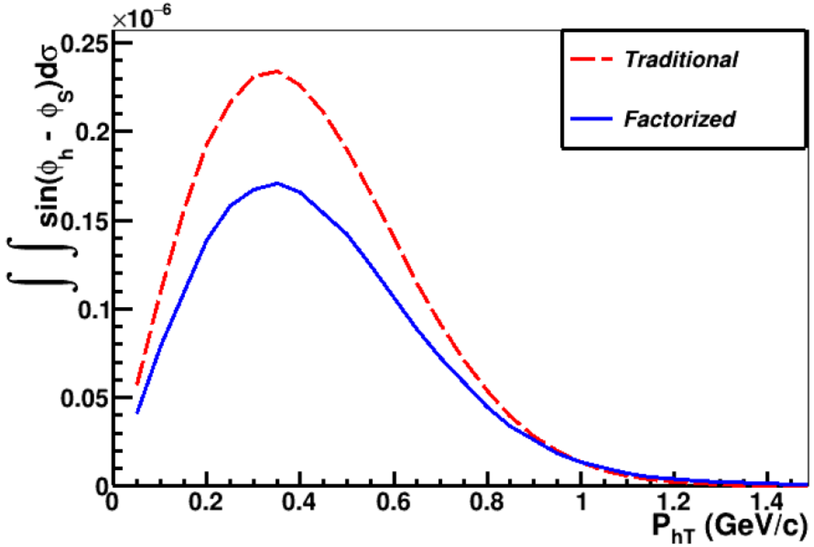
Radiative effects on Sivers Sin(ϕ_h - ϕ_s) and Collins Sin(ϕ_h + ϕ_s) modulation

Single transverse spin asymmetry

Preliminary results

EIC: √s = 140 GeV Q^2 =100 GeV^2 Z = 0.5 y = 0.4



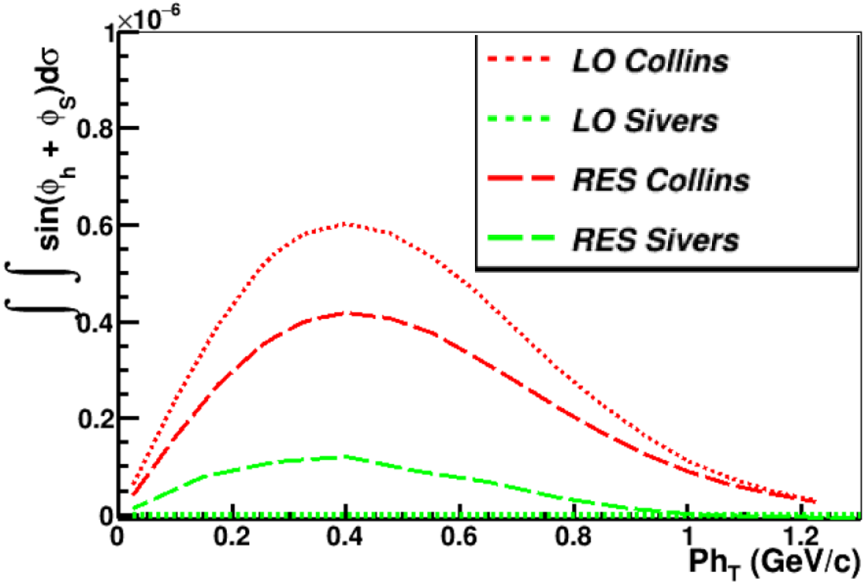


Radiative effects on Sivers Sin(ϕ_h - ϕ_s) and Collins Sin(ϕ_h + ϕ_s) modulation

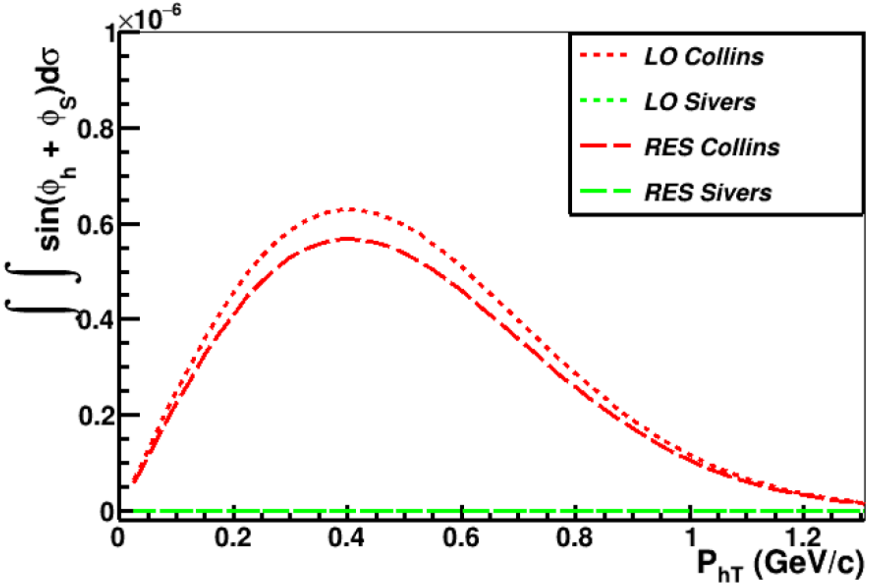
Single transverse spin asymmetry

EIC: $\sqrt{s}$ = 140 GeV Q^2 =25 GeV^2 Z = 0.5 y = 0.4

Factorized approach



Traditional approach



Leakage effect in factorized approach but not in traditional approach

Conclusion

- Compared the RC between two different approaches:
  - Factorized approach **T. Liu et al. Journal of High energy Physics 157, (2021)**
  - Traditional approach **I. Akushevich and A. Ilychev Physics Review D100 (2019)**
- In case of unpolarized F_{uu} about 6-10 % difference in RC between two approach

- For single transverse spin asymmetry difference is larger at EIC kinematics

- This comparison will provide an understanding of systematic due to RC which is one of the most important source of uncertainty in extracting TMD-PDFs

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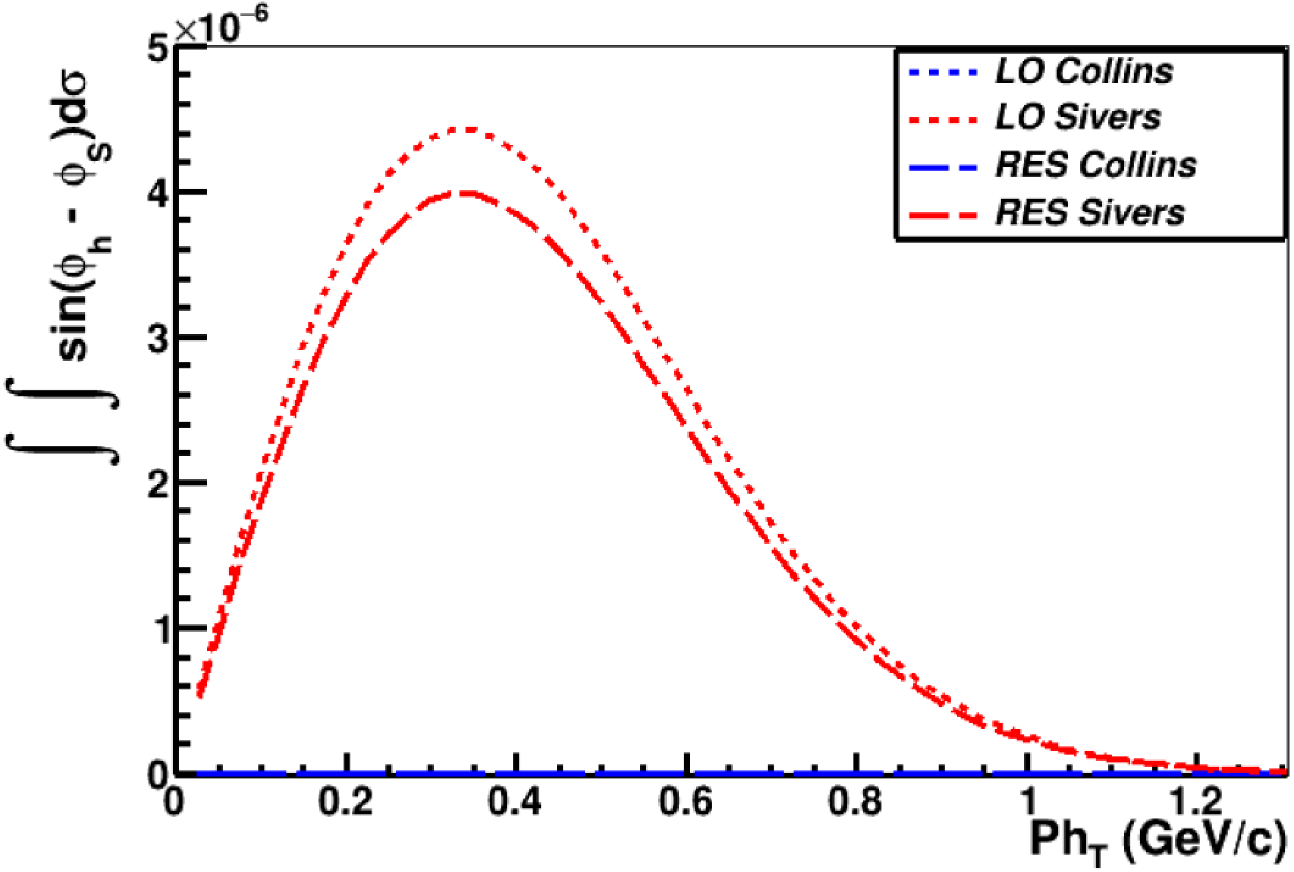
Thank you

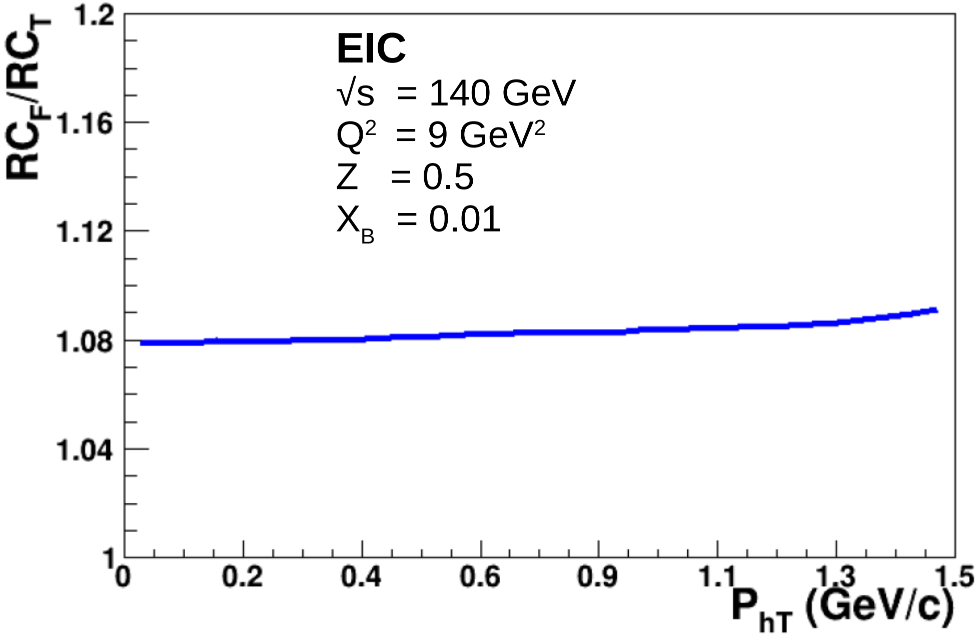
Backup

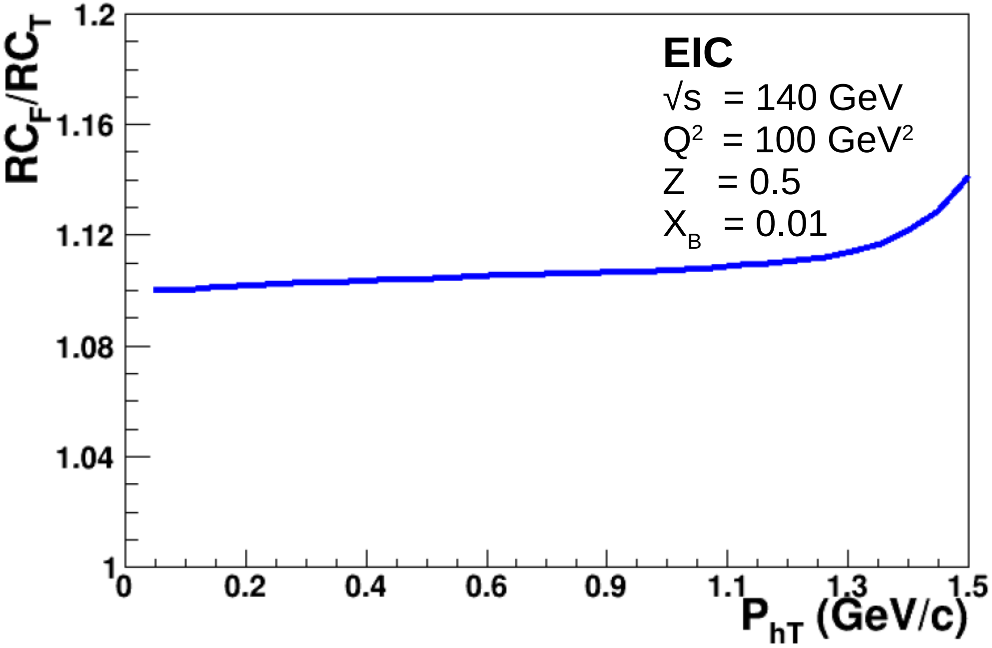
Single transverse spin asymmetry



Traditional vs Factorized approach

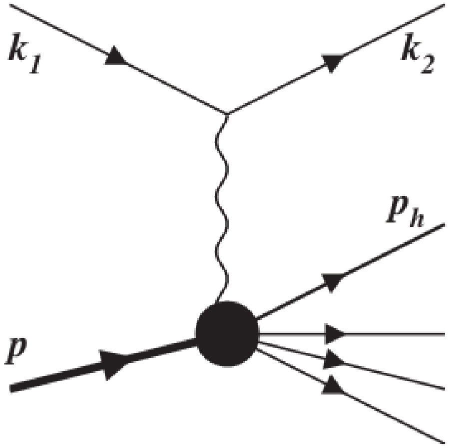
Unpolarized cross-section



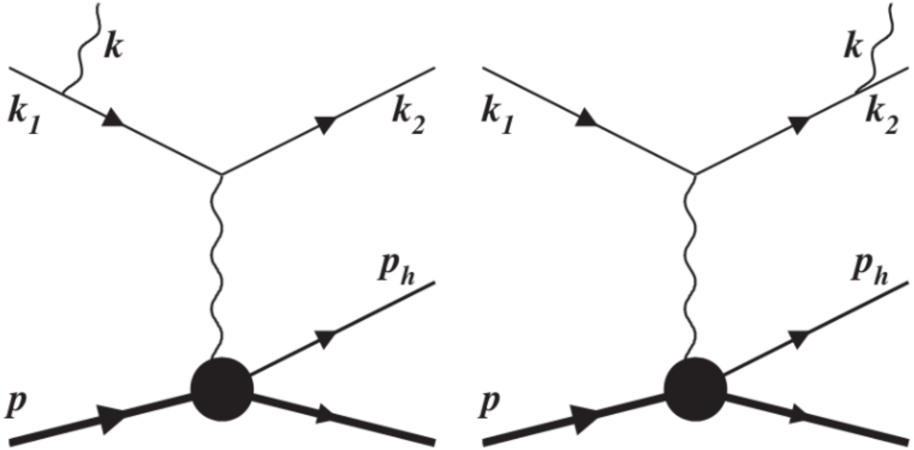


Contribution of exclusive tail

Born process (LO)



Exclusive processes



- Estimated to have significant contribution from exlcusive channels Akushevich, Ilyichev, Osipenko, Phys.Lett.B672(2009)35
- Due to limited knowledge of exclusive SF its hard to estimate
- From factorization point of view it is a power suppressed
- At high energy, exclusive tail does not enter, W_{min} exists

Radiative correction

- Total SIDIS cross-section with RC is

$$\sigma = \sigma^B + \frac{\alpha}{\pi}(\delta_{\mathrm{vert}} + \delta_{\mathrm{vac}})\sigma^B + \sigma^{AMM} + \int \sigma^R \, d^3\boldsymbol{k}$$

- Radiative part σ^R is divided into soft and hard parts by cutoff in photon energy $\bar{k}_0$.

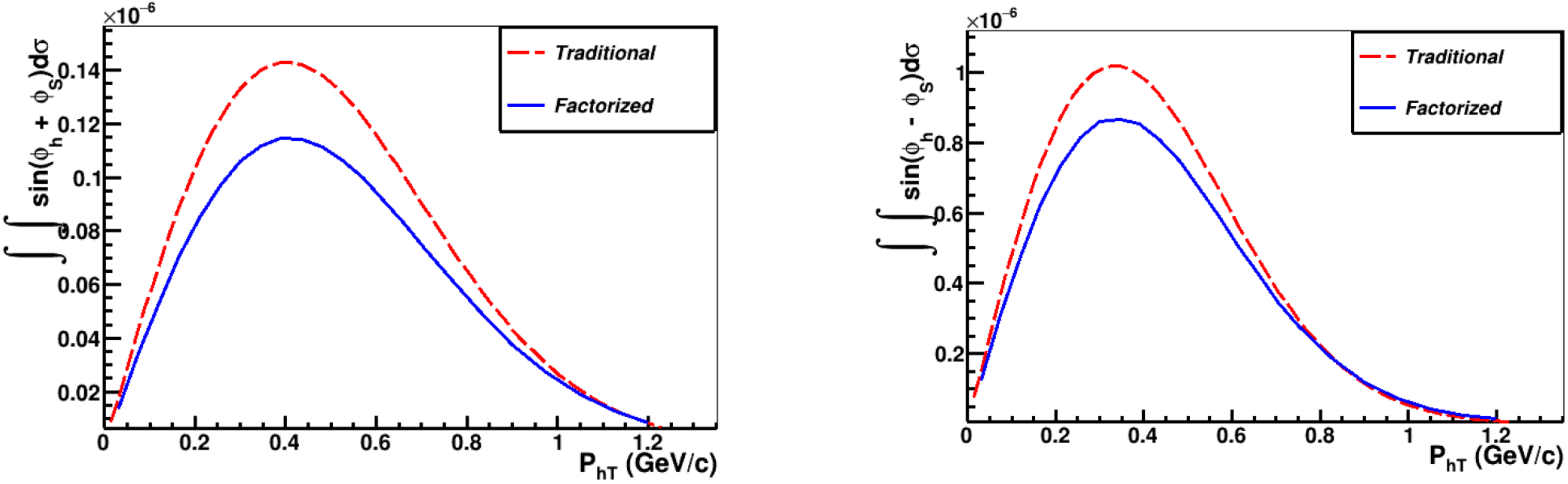
$$\sigma = \underbrace{\sigma^B + \frac{\alpha}{\pi}(\delta_{\mathrm{vert}} + \delta_{\mathrm{vac}})\sigma^B + \sigma^{AMM} + \overbrace{\int_0^{\bar{k}_0} \sigma^R \, d^3\boldsymbol{k}}^{\text{difficult}}}_{\text{soft part}} + \underbrace{\int_{\bar{k}_0}^{\infty} \sigma^R \, d^3\boldsymbol{k}}_{\text{hard part}}$$

- Events are randomly chosen to be soft or hard, based on total soft/hard cross-sections.
- Integral over soft part of σ^R is computationally expensive.

Single transverse spin asymmetry

Preliminary results

EIC: √s = 140 GeV Q^2 =9 GeV^2 Z = 0.5 y = 0.4



Radiative effects on Sivers Sin(ϕ_h - ϕ_s) and Collins Sin(ϕ_h + ϕ_s) modulation

Traditional vs Factorized approach

Unpolarized cross-section