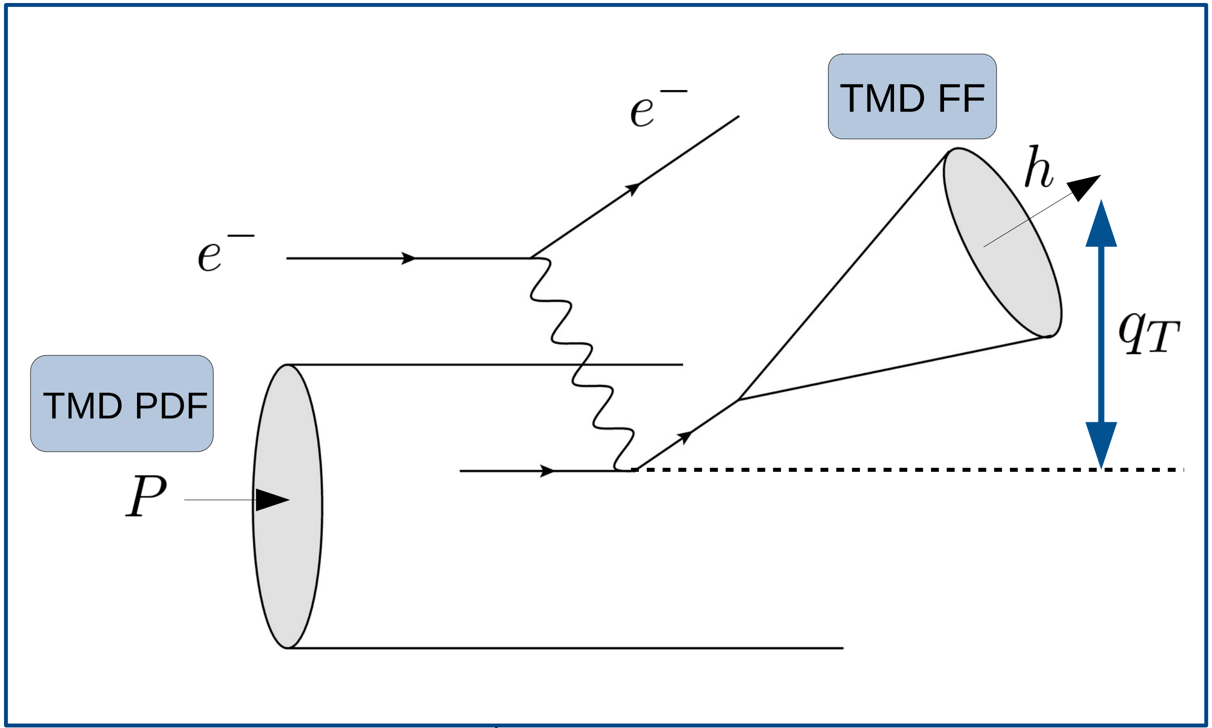
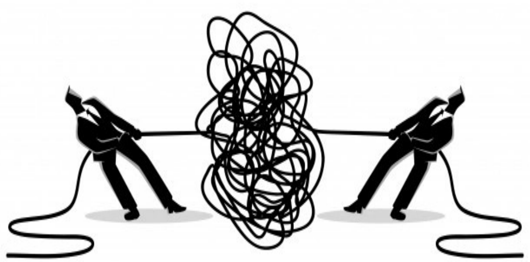
Phenomenological extraction of a universal TMD fragmentation function from BELLE data

M. Boglione
In collaboration with O. Gonzalez and A. Simonelli

SIDIS: $e\,p \to h\,X$



In e^+e^- cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?

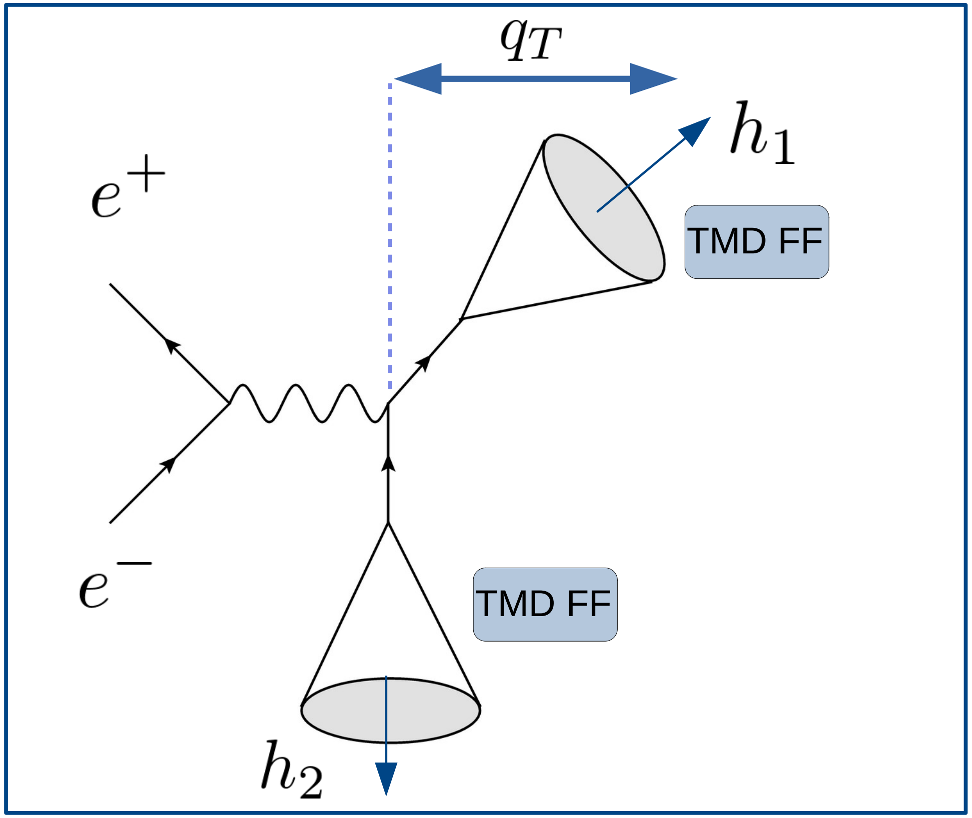


$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

3D-picture of partons inside the target hadron

3D-picture of partons hadronizing into the detected hadron

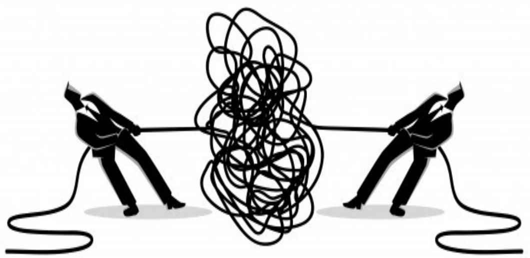
e^+e^- annihilations in two hadrons: $e^+ e^- \to h_1 h_2 X$



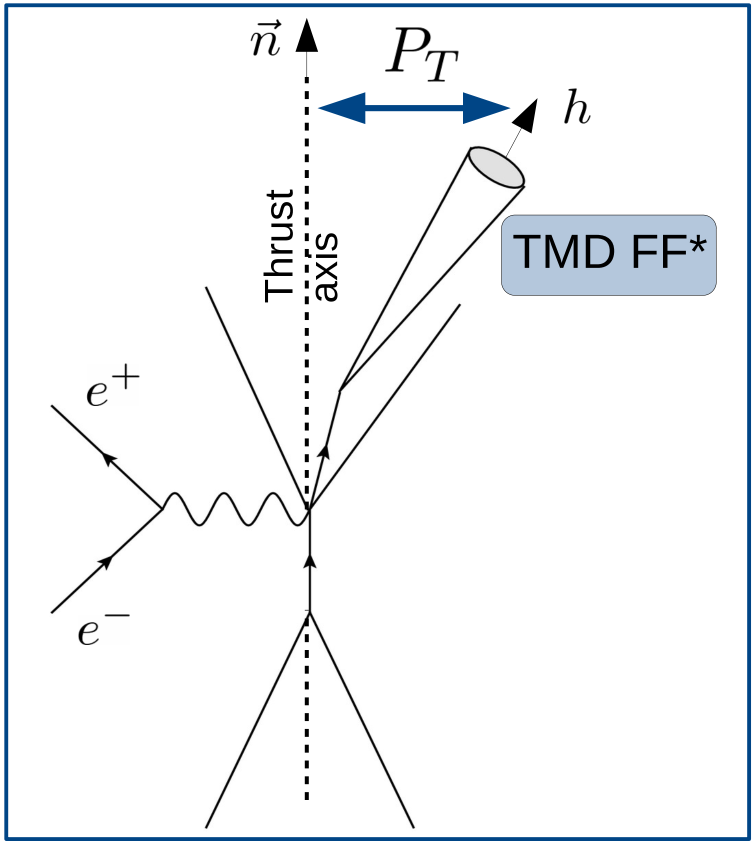
$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T} D_1(b_T)\, D_2(b_T)$$

3D-picture of the **hadronization** of partons into hadrons

In e^+e^- cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?



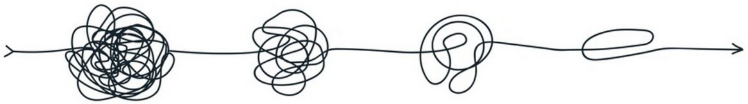
e^+e^- annihilations in one hadron: $e^+\,e^- \to h\,X$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^\star(P_T)$$

3D-picture of the **hadronization** of partons into hadrons

In $e^+\,e^- \to h\,X$ cross sections, only one fragmentation TMD appears



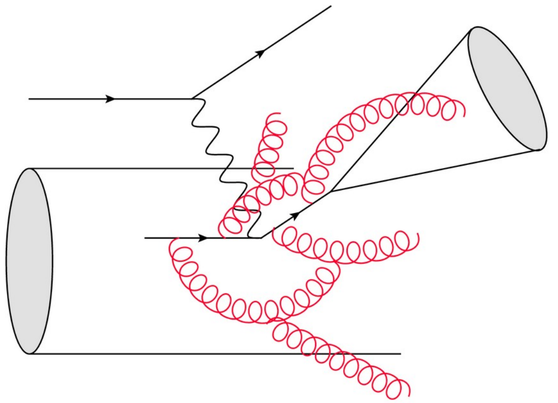
One of the cleanest ways to access TMD Fragmentation Functions***...**

BUT

D*(P_T) is not the same as D(P_T) !!!

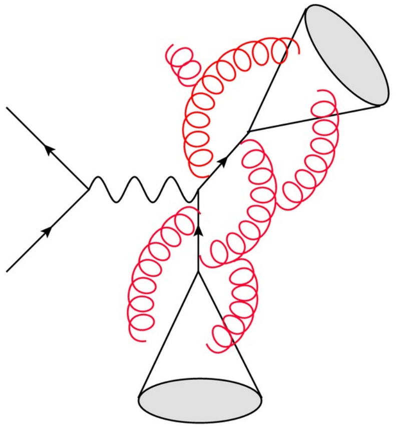
Soft Gluon contribution

SIDIS



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{sidis}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

Double hadron production



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, D_1(b_T)\, D_2(b_T)$$

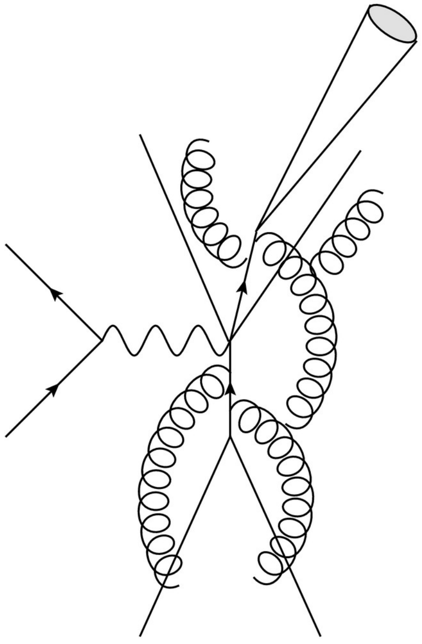
Soft Gluon Factor:

- Non-Perturbative contribution
- Evenly shared by the TMDs

Soft Gluons

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^\star(P_T)$$

Soft Gluon Factor:

- Perturbative contribution
- The TMD FF***** is **free** from any soft gluon contributions

D(P_T) and D*(P_T) are different,
BUT
the relation between D and D* is known!

We can perform combined analyses and disentangle non-perturbative terms.

Relation between FF and FF*

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$D = D^{\star}\sqrt{M_S}$$

SQUARE ROOT DEFINITION

Usual definition of TMDs.
Soft Gluon Factor contributing to the cross section are included in the two TMDS and equally shared between them.

FACTORIZATION DEFINITION

Purely collinear TMD, totally free from any soft gluon contribution.

SOFT MODEL

The Soft Gluon Factor appearing in the cross section (process dependent) is **not** included in the TMD

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).

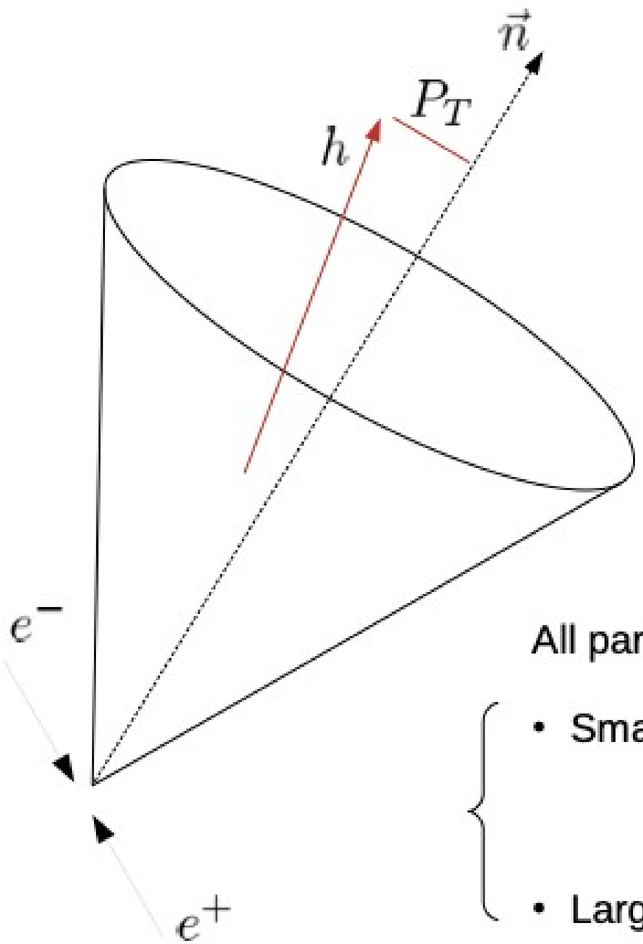
The $e^+e^- \to hX$ process

The cross section is differential in: $z_h = \frac{E}{Q/2}$, $\underbrace{T = \frac{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i}|}}$, P_T w.r.t $\vec{n}$

$0.5 \leq T \leq 1$

Spherical distribution ◄ ► 2-jet limit

2-jet final state is the most probable topology configuration



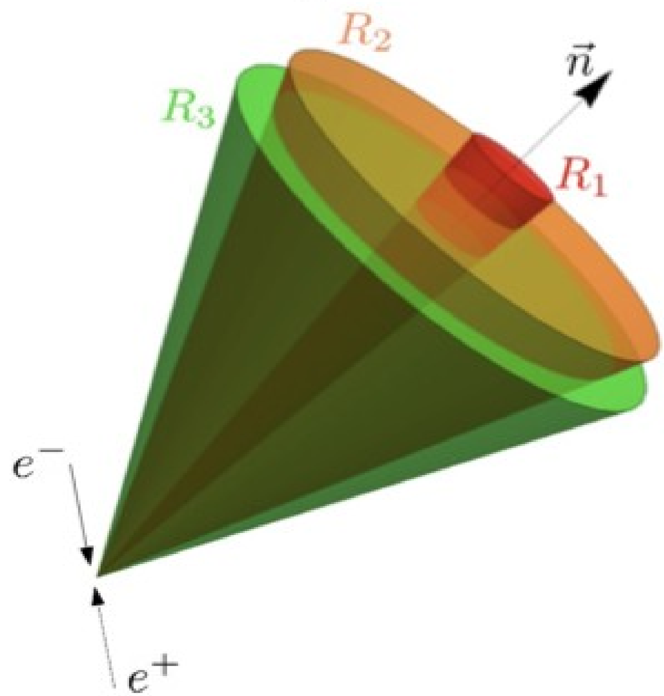
All particles inside the jet in which h is detected must have:

- Small transverse momentum $P_T \ll P^+ = z_h \frac{Q}{\sqrt{2}}$
- Large rapidity $y_P = \frac{1}{2} \log \frac{2(P^+)^2}{P_T^2 + M_h^2} \gg 0$

Kinematic Regions

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems

Three Regions:



M. Boglione and A. Simonelli, JHEP 02, 013 (2022)

The hadron is detected very close to the axis of the jet:
- Extremely small P_T
- Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

TMD FF + non-pert. SOFT contribution

The hadron is detected in the central region of the jet:
- Most common scenario
- Majority of experimental data fall into this case

TMD FF

The hadron is detected near the boundary of the jet:
- Moderately small P_T
- The hadron P_T causes the spread of the jet affecting the topology of the final state (i.e. the value of thrust)

Generalized FJF

Everything discussed above refers to Region 2

Rapidity divergencies and thrust in Region 2

ISSUES FROM TREATMENT OF RAPIDITY DIVERGENCES

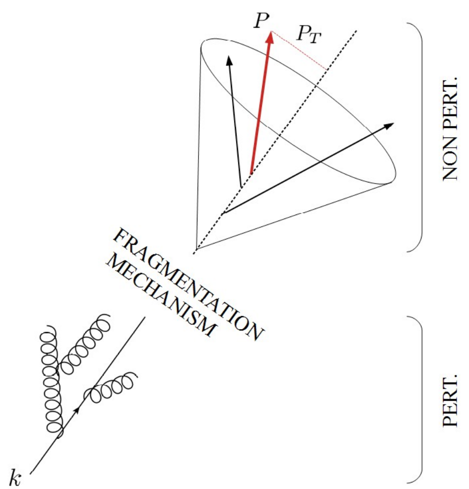
- Peculiar interplay between soft and collinear contributions ⇒ some of the rapidity divergences are naturally regulated by the thrust, T, but those associated with terms which are strictly TMD parts of the cross section need an extra artificial regulator, which is a rapidity cut- off.
- This induces a redundancy, which generates an additional relation between the regulator, the transverse momentum and thrust.
- This relation inevitably spoils the picture in which the cross section factorizes into the convolution of a partonic cross section (encoding the whole T dependence) with a TMD FF (which encapsulates the whole P_T dependence).
- Thrust resummation is intertwined with the transverse momentum dependence, making the treatment of the large T behavior highly non-trivial.
- A proper phenomenological analysis of Region 2 must rely on a factorized cross section where the regularization of rapidity divergences is properly taken into ac- count. All difficulties encountered in the theoretical treatment get magnified in the phenomenological applications.
- In this analysis we adopt some approximations, in order to simplify the structure of the factorization theorem without altering its main architecture.

$e^+e^- \to hX$ cross section

M. Boglione, A. Simonelli, JHEP 02 (2021) 076

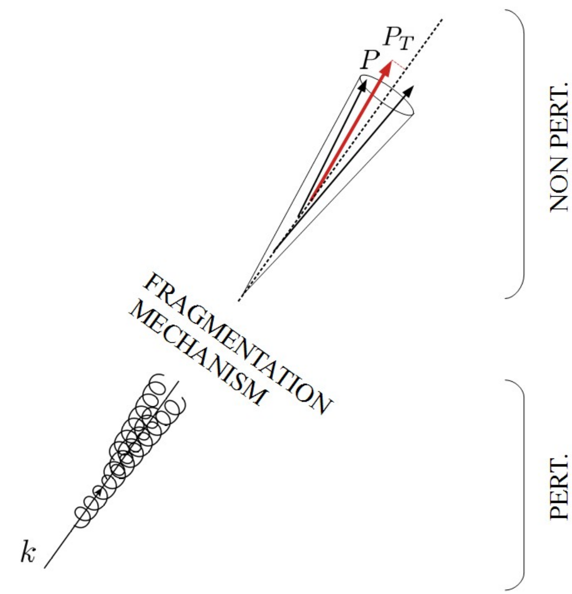
$$\frac{d\sigma}{dz_h\,dT\,dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z}\,\frac{d\widehat{\sigma}_f}{dz_h/z\,dT}\,D_{1,\pi^\pm/f}(z,\,P_T,\,Q,\,(1-T)\,Q^2)$$

The TMD FF acquires a dependence on thrust through its rapidity cut-off.



2-jet limit

$T \to 1$



Partonic cross section (NLO)

M. Boglione, A. Simonelli, JHEP 02 (2021) 076

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \underbrace{\frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT}} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)$$

$$\boxed{\frac{d\widehat{\sigma}_f}{dz\, dT} = \left[-\sigma_B e_f^2 N_C \frac{\alpha_S(Q)}{4\pi} C_F \delta(1-z) \left[\frac{3+8\log\tau}{\tau}\right] + \mathcal{O}\left(\alpha_S(Q)^2\right)\right] e^{-\frac{\alpha_S(Q)}{4\pi}\, 3C_F(\log\tau)^2 + \mathcal{O}\left(\alpha_S(Q)^2\right)}}$$

TMD Fragmentation Function

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z}\, \frac{d\widehat{\sigma}_f}{dz_h/z\, dT}\, D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\,Q^2)$$

Fourier Transform of:

Collinear FFs

$$\widetilde{D}_{1,\pi^\pm/f}(z,\, b_T;\, Q,\, \tau Q^2) = \frac{1}{z^2} \sum_k \left[\boxed{d_{\pi^\pm/k}} \otimes \mathcal{C}_{k/f}\right](\mu_b) \times$$
$$\times \exp\left\{\frac{1}{4}\,\widetilde{K}\,\log\frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[\gamma_D - \frac{1}{4}\,\gamma_K\,\log\frac{\tau Q^2}{\mu'^2}\right]\right\} \times$$
$$\times\, \underbrace{(M_D)_{f,\pi^\pm}(z,\, b_T)}\, \exp\left\{-\frac{1}{4}\, \underbrace{g_K(b_T)}\, \log\left(\tau \frac{Q^2}{M_H^2}\right)\right\}$$

Perturbative part (NLL)

Non-Perturbative part Pheno Model

Embeds the non-perturbative, long-range behavior of the TMD FF

Universal, independent of the TMD definition used

Phenomenological parametrization: M_D

$$M_D = \underbrace{\frac{2^{2-p}(b_T M)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_T M)}_{\text{Power-law model}} \times \underbrace{F(b_T, z_h)}_{\text{Multiplicative function modulating the z dependece}}$$

Power-law model

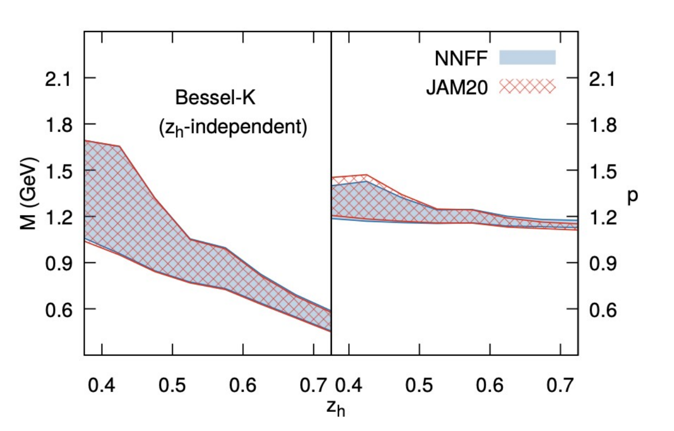
Multiplicative function modulating the z dependece

$\mathcal{FT}\{M_D\}$ reminiscent of a propagator in k_T space $\frac{1}{(k_T^2 + M^2)^p}$

Exponential behaviour at $b_T \to \infty$

Preliminary fits at fixed z show that

- the M and p parameters are VERY strongly correlated
- M requires some z-dependence while p does not vary much with z



Phenomenological parametrization: M_D

$$\underbrace{M_{\mathrm{D}} = \frac{2^{2-p}(b_{\mathrm{T}} M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_{\mathrm{T}} M_0)}_{\text{BK parameters do not depend on z}} \times \underbrace{F(b_{\mathrm{T}}, z_h)}_{\text{z-dependence controlled by F}}$$

M_D MODEL 1

ID	M_{D} model	parameters
I	$F = \left(\frac{1 + \log\left(1 + (b_{\mathrm{T}} M_z)^2\right)}{1 + (b_{\mathrm{T}} M_z)^2} \right)^q$ $M_z = -M_1 \log(z_h)$	$M_0,\ M_1$ $p = 1.51,\ q = 8$

Phenomenological parametrization: M_D

$$M_{\mathrm{D}} = \underbrace{\frac{2^{2-p}(b_{\mathrm{T}}M_0)^{p-1}}{\Gamma(p-1)}K_{p-1}(b_{\mathrm{T}}M_0)}_{\text{BK parameters do not depend on z}} \times F(b_{\mathrm{T}}, z_h)$$

M_D MODEL 1

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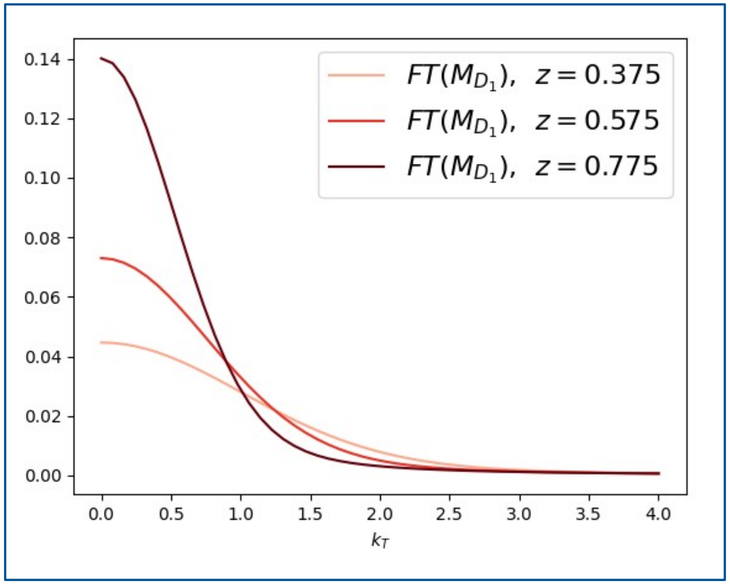
z-dependence controlled by the function F, through M_z

z-dependence controlled by F



Phenomenological parametrization: M_D

$$M_D = \underbrace{\frac{2^{2-p_z}(b_T M_z)^{p_z-1}}{\Gamma(p_z-1)} K_{p_z-1}(b_T M_z)}_{\text{BK parameters depend on z}} \times \underbrace{F(b_T, z_h)}_{F\,=\,1}$$

M_D MODEL 2

II	$F = 1$ $M_z = M_h \frac{1}{z\, f(z)^2} \sqrt{\frac{3}{1-f(z)}}$ $p_z = 1 + \frac{3}{2} \frac{f(z)}{1-f(z)}$ $f(z) = 1 - (1-z)^\beta, \quad \beta = \frac{1-z_0}{z_0}$	β $M_h = 0.139$ GeV

z-dependence of M_D controlled by the M_z and p_z parameters

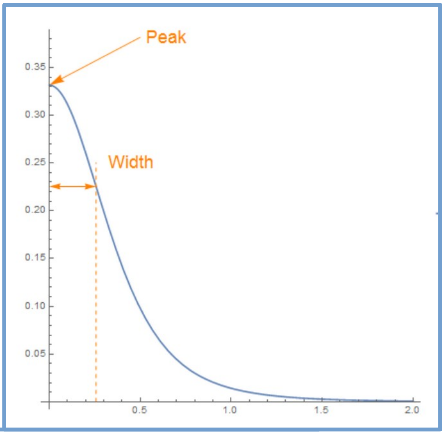
Phenomenological parametrization: M_D

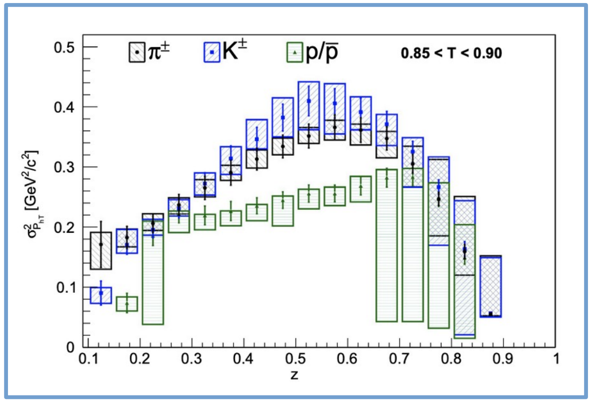
$$M_D = \underbrace{\frac{2^{2-p_z}(b_T M_z)^{p_z-1}}{\Gamma(p_z-1)} K_{p_z-1}(b_T M_z)}_{\text{BK parameters depend on z}} \times \underbrace{F(b_T, z_h)}_{F\,=\,1}$$

M_D MODEL 2

II	$F=1$ $M_z = M_h \dfrac{1}{z\, f(z)^2}\sqrt{\dfrac{3}{1-f(z)}}$ $p_z = 1 + \dfrac{3}{2}\dfrac{f(z)}{1-f(z)}$ $f(z) = 1-(1-z)^\beta, \quad \beta = \frac{1-z_0}{z_0}$

The z behaviour of M_D is constrained by requiring that the theory lines appropriately reproduce the peak and the width of the measured cross sections, at each value of z.





BELLE Phys. Rev. D99 (2019) 11 112006

Phenomenological parametrization: g_K

In this analysis we consider hypothesis for which, asymptotically, $g_K = o(b_T)$

g_K model		
A	$g_K = \log\left(1 + (b_T M_K)^{p_K}\right)$	$M_K, \; p_K$
B	$g_K = M_K b_T^{(1-2p_K)}$	$M_K, \; p_K$

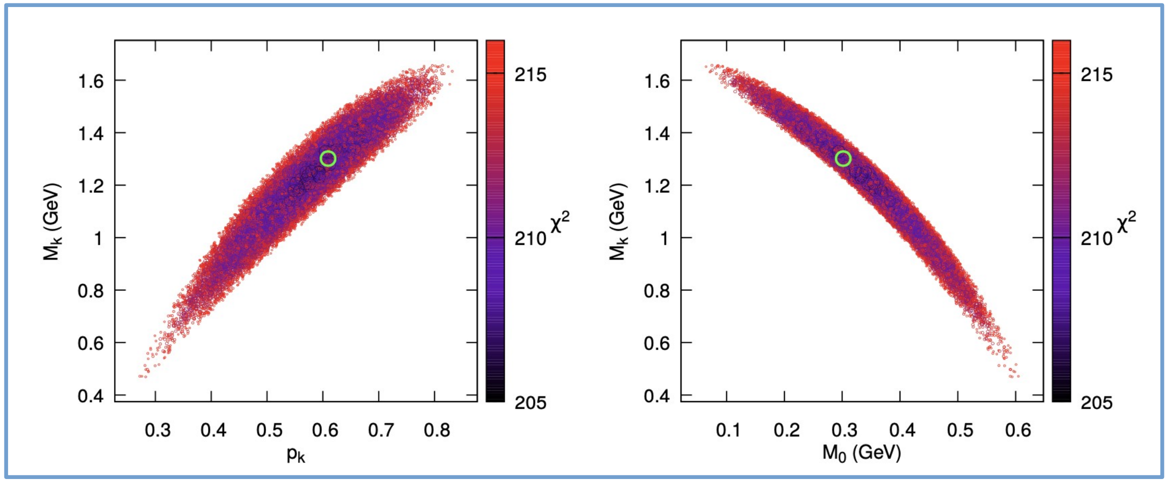
See talk by A. Vladimirov
and Phys. Rev. Lett. 125, 192002 (2020).

Testing different b_T behaviors of g_K
allows us to give a reliable estimate
of the uncertainties affecting our analysis

Phenomenological results – correlations

Model I

4 parameter fit



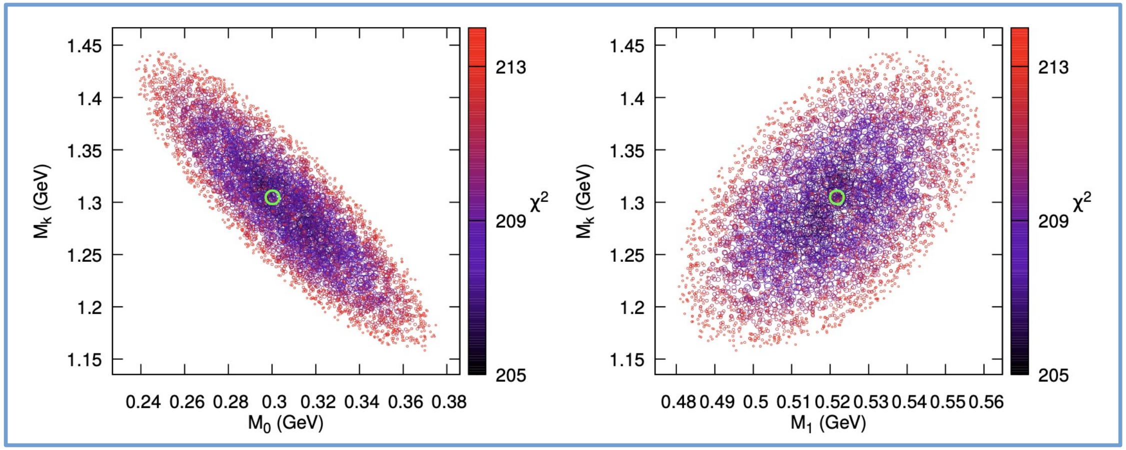
Data selection

$$0.375 \le z_h \le 0.725\,, \qquad 0.750 \le T \le 0.875\,,$$

$$q_T/Q \le 0.15$$

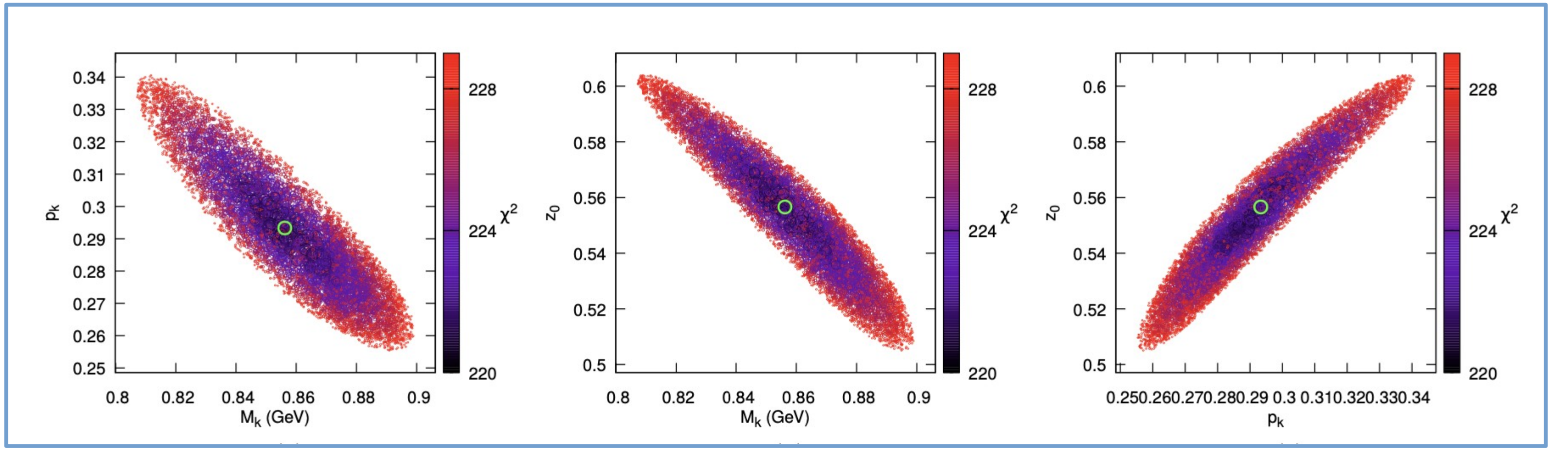
3 parameter fit
powers treated as
nuisance parameters

$q_T/Q < 0.15$ (pts = 168)		
	IA	IB
$\chi^2_{\rm d.o.f.}$	1.25	1.19
M_0(GeV)	$0.300^{+0.075}_{-0.062}$	$0.003^{+0.089}_{-0.003}$
M_1(GeV)	$0.522^{+0.037}_{-0.041}$	$0.520^{+0.027}_{-0.040}$
p^*	1.51	1.51
q^*	8	8
M_K(GeV)	$1.305^{+0.139}_{-0.146}$	$0.904^{+0.037}_{-0.086}$
p^*_K	0.609	0.229



Phenomenological results – correlations

Model II



Data selection

$$0.375 \leq z_h \leq 0.725\,, \qquad 0.750 \leq T \leq 0.875\,,$$

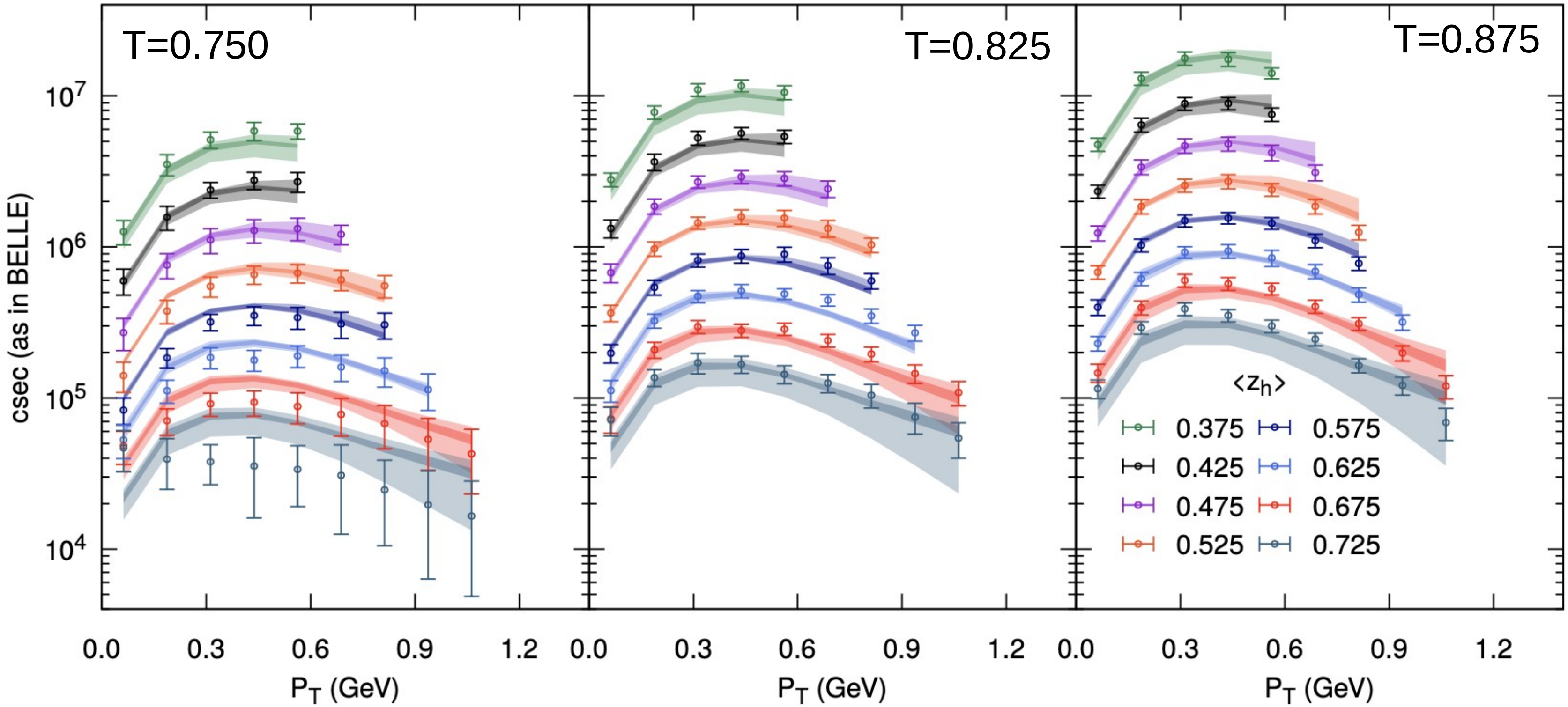
$$q_T/Q \leq 0.15$$

3 parameter fit
(z_0, M_K, p_K)

$q_T/Q < 0.15$ (pts = 168)		
	IA	IB
$\chi^2_{\text{d.o.f.}}$	1.35	1.33
$z0$	$0.574^{+0.039}_{-0.041}$	$0.556^{+0.047}_{-0.051}$
M_K(GeV)	$1.633^{+0.103}_{-0.105}$	$0.687^{+0.102}_{-0.106}$
p_k	$0.588^{+0.127}_{-0.141}$	$0.293^{+0.047}_{-0.038}$

Phenomenological results – T dependence

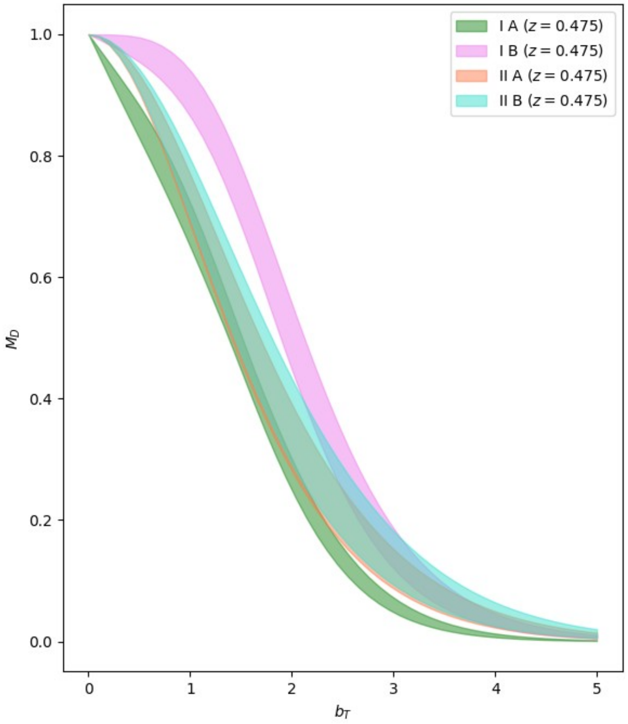
M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, work in progress



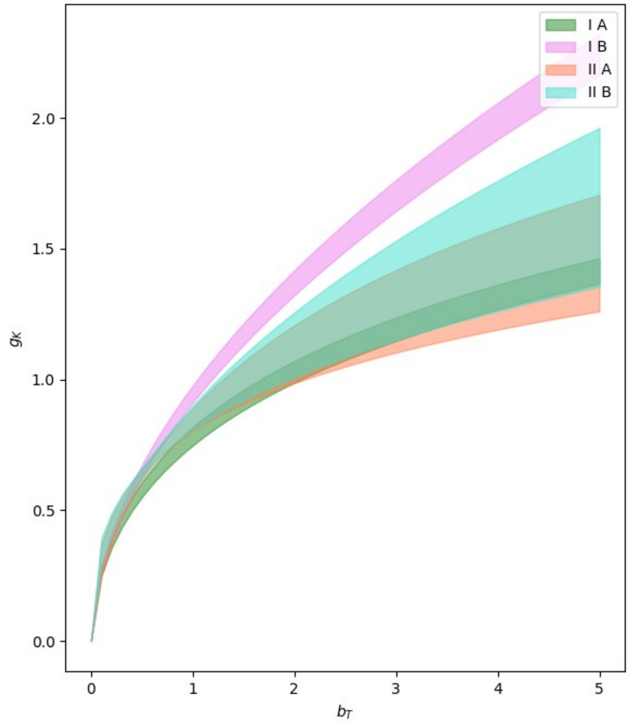
BELLE Collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

Phenomenological results

$M_D\,(b_T)$

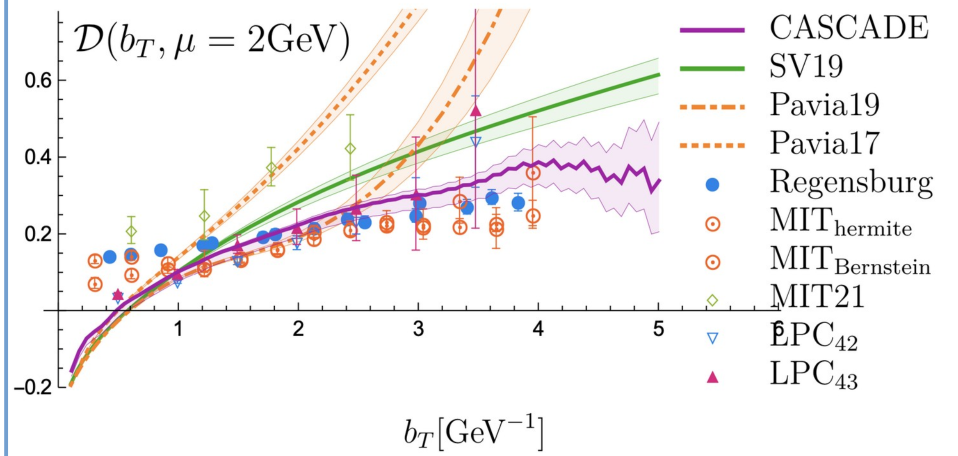


$g_K\,(b_T)$



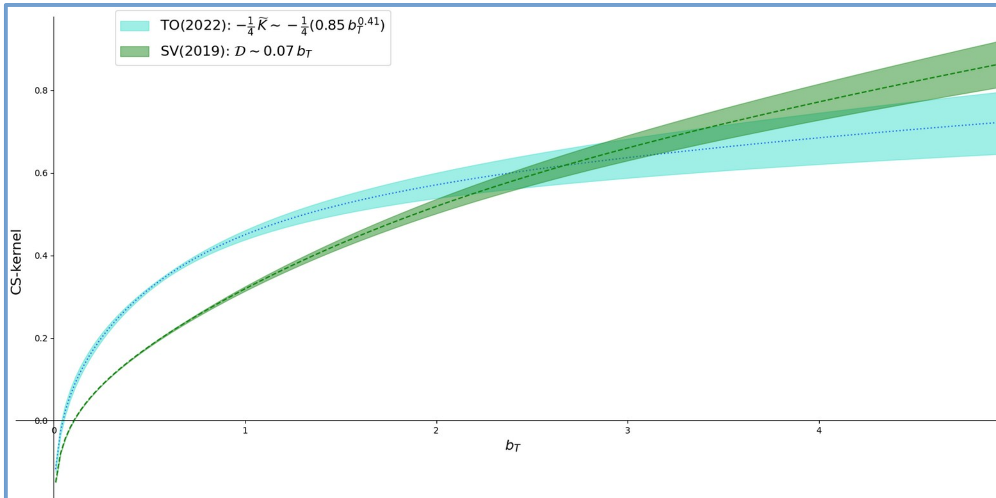
Collins-Soper kernel: comparison to other analyses

Plot from Vladimirov's talk



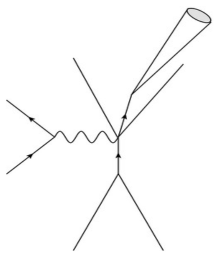
Vladimirov Phys. Rev. Lett. 125, 192002 (2020).

Comparison to our extraction



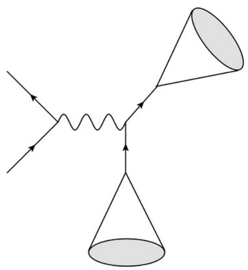
$$\mathcal{D}^{\text{Vlad}} = -\frac{1}{4}\mathcal{D}^{\text{our}}$$

Outlook



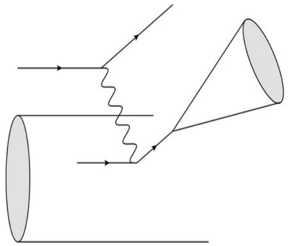
1. $e^+ e^- \to h\, X$

   Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. $e^+ e^- \to h_1 h_2\, X$

   Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_s, obtained as ratio: $M_S = D/D^\star$



3. $SIDIS$

   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_s, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, F*≠F).

Outlook

The Soft Factor acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the Soft Factor contribution (which encloses the full process dependent part of the TMD).

The Collins-Soper kernel acquires a central role

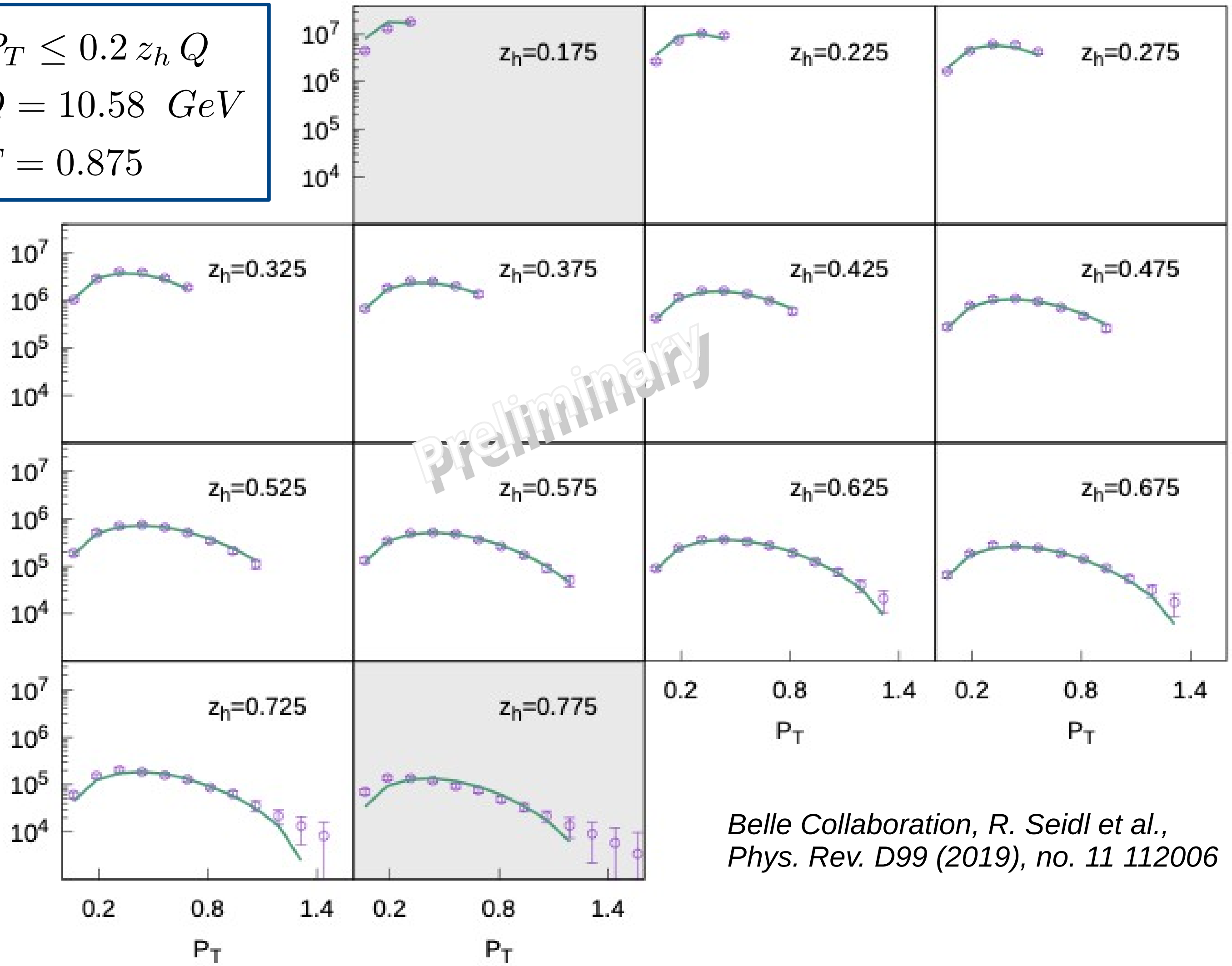
The focus of phenomenological analyses moves from the TMDs considered as a whole, to the gK function (which embeds the nonperturbative essence of the TMD evolution).

Back-up slides

Phenomenological results – z dependence

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, work in progress

$P_T \leq 0.2\, z_h\, Q$
$Q = 10.58 \;\; GeV$
$T = 0.875$



z-dependence is very well under control

$$\chi^2_{dof} = 0.9$$

n. of fitted data: 89

Data in the shaded boxes are not included in this fit

Belle Collaboration, R. Seidl et al., Phys. Rev. D99 (2019), no. 11 112006