Collinear fragmentation function extractions: MAPFF2.0 at NNLO (approximate)

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from the **MAP collaboration**

Wednesday 03/09/2022 9:25AM

Outline

- **A** Recap of MAPFF1.0
- **B** Experimental data
- **C** Theoretical setup
- **D** Methodology
- **E** Results

Recap of MAPFF1.0

Main results [R. A. Khalek, V. Bertone, and E. R. Nocera, Phys. Rev. D 104, 034007 (2021).]

- The global χ^2 as well as the χ^2 of all the single data sets included in the fits are fully satisfactory.
- the resulting FFs are almost insensitive to the treatment of PDFs uncertainties and central values.
- The inclusion of SIDIS data worsened the description of SLD charm-tagged data, which was removed.
- COMPASS plays a vital role in constraining and separating FFs flavours. However, HERMES, despite the limited number of points, has a noticeable impact on FFs.
- The $Q_{\text{cut}} = 2$ on SIDIS, guarantees a reliable applicability of NLO accurate predictions to SIDIS data

Public code	https://github.com/MapCollaboration/MontBlanc
LHAPDF sets	https://lhapdf.hepforge.org/

Since then, used in 3 independent analyses (2022):

- Simultaneous QCD analysis for identified and unidentified light charged hadrons.
- Fragmentation Functions for $\Xi^-/\bar{\Xi}^+$ Using Neural Networks.
- SHK22.h: Neural Network QCD analysis of charged hadron Fragmentation Functions in the presence of SIDIS data.

By M. Soleymaninia, H. Hashamipour, H. Khanpour, N. Maynooth, H. Spiesberger

TODO list

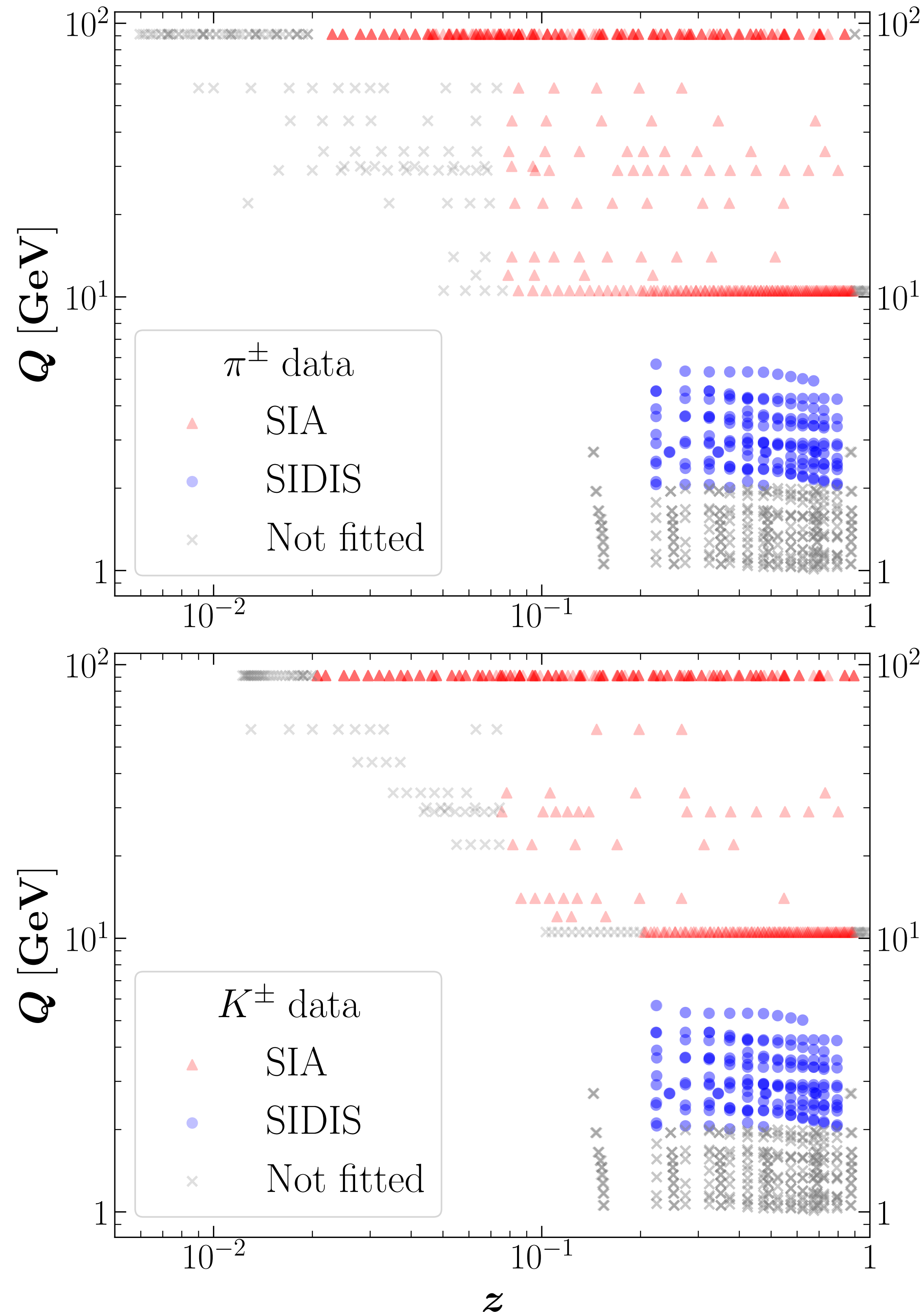
- **Extension to charged Kaons (this presentation)**
- Extension to proton/anti-proton and charged unidentified hadrons FFs. (See above)
- **Study the impact of NNLO corrections (this presentation)**
- Reliable determination of transverse-momentum-dependent (TMD) distributions in the light of MAPFF (MAP22, see Andrea Signori's presentation on Monday)

Experimental data

Single inclusive annihilation (SIA)
Semi-inclusive DIS (SIDIS)
Pions and Kaons (New)

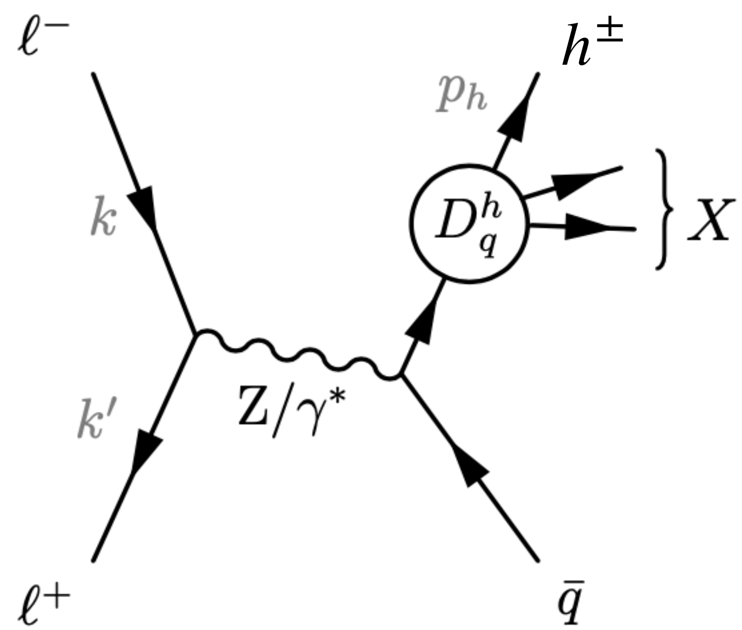
Experiment	Prc	Obs
ALEPH	SIA	M
BABAR	SIA	M
BELLE	SIA	X
DELPHI	SIA	M and TM
OPAL	SIA	M
SLD	SIA	M and TM
TASSO	SIA	M and X
TOPAZ	SIA	M
TPC	SIA	M
COMPASS	SIDIS	M
HERMES	SIDIS	M

X: Cross section
M: Multiplicity
TM: Tagged Multiplicity



Single-Inclusive annihilation

$$e^+(k_1) + e^-(k_2) \rightarrow h^\pm(p_h) + X\,.$$



SIA LO Kinematics

$$Q^2 = q^2$$

$$z = \frac{2p_h \cdot q}{Q^2}$$

$$s = Q^2$$

- **Cross section:**

$$\frac{d\sigma^h}{dz}(z,Q) = \frac{4\pi\alpha(Q)}{Q^2} F_2^h(z,Q)$$

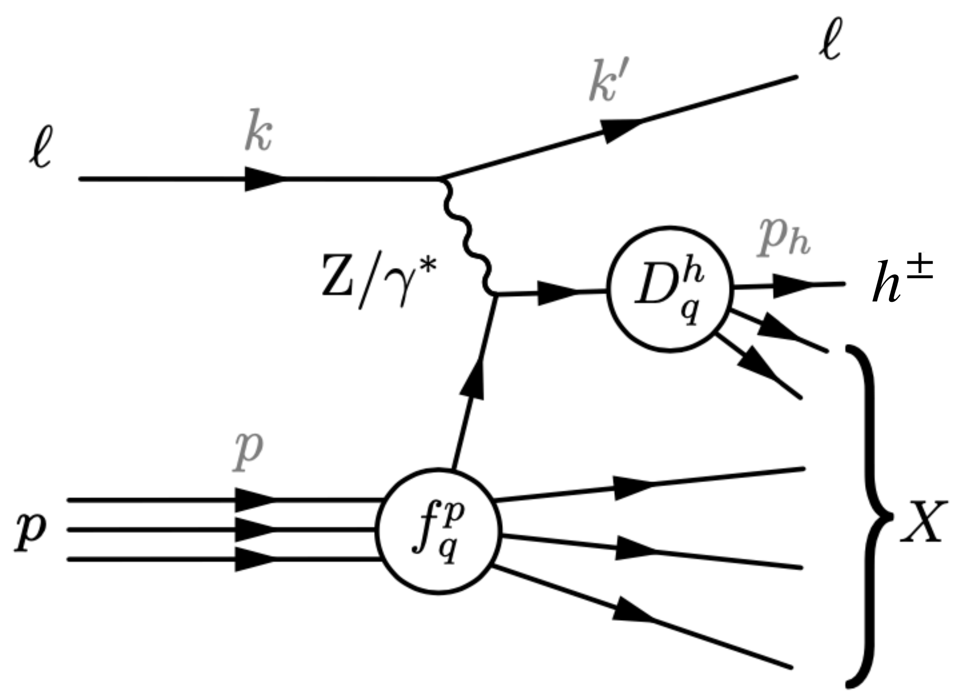
- **Structure functions:**

$$F_2^{\mathrm{NC},\,h^\pm} = \frac{1}{n_f}\left(\sum_j^{n_f} \hat{e}_{q_j}^2\right) F_{2,\mathrm{S}}^{\mathrm{NC},\,h^\pm} + F_{2,\mathrm{NS}}^{\mathrm{NC},\,h^\pm}$$

$$F_{2,\mathrm{S}}^{\mathrm{SIA},h^\pm} = C_{2,\mathrm{S}}^{\mathrm{SIA}} \otimes D_{\underline{\Sigma}}^h + C_{2,g}^{\mathrm{SIA}} \otimes D_{\underline{g}}^h, \qquad F_{2,\mathrm{NS}}^{\mathrm{SIA},h^\pm} = C_{2,\mathrm{NS}}^{\mathrm{SIA}} \otimes \sum_j^{n_f} \hat{e}_{q_j} D_{j,\mathrm{NS}}^h$$

Semi-inclusive DIS

$$\ell(k) + N(p) \to \ell(k') + h^{\pm}(p_h) + X\,.$$



SIDIS LO Kinematics

$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p\cdot q}$$

$$z = \frac{p\cdot p_h}{p\cdot q}$$

$$y = \frac{Q^2}{xs}$$

$$s = (k^2 + p^2)$$

- **Cross section ($Q \ll M_Z$):** $$\frac{d^3\sigma}{dx dQ dz} = \frac{4\pi\alpha^2(Q)}{xQ^3}\left[\left(1+(1-y)^2\right)F_2(x,z,Q^2) - y^2 F_L(x,z,Q^2)\right]$$

$$F_i(x,z,Q) = x\sum_{q\bar{q}} e_q^2 \Big\{ [C_{i,qq}(x,z,Q)\otimes \underline{f_q(x,Q)} + C_{i,qg}(x,z,Q)\otimes \underline{f_g(x,Q)}]\otimes D_q^{h^{\pm}}(z,Q)$$
$$+ \; [C_{i,gq}(x,z,Q)\otimes \underline{f_q(x,Q)}]\otimes D_g^{h^{\pm}}(z,Q)\Big\}\,, \qquad i = 2, L\,.$$

$$C(x,z)\otimes f(x)\otimes D(z) = \int_x^1 \frac{dx'}{x'}\int_z^1 \frac{dz'}{z'} C(x',z') f\left(\frac{x}{x'}\right) D\left(\frac{z}{z'}\right)$$

Theoretical setup

- **Coefficient functions:** $$C(x,z,Q) = \sum_{n=0} \left(\frac{\alpha_s(Q)}{4\pi}\right)^n C^{(n)}(x,z)$$

SIA: NNLO known since 2006
[1] A. Mitov and S. O. Moch, Nucl. Phys. B 751, 18 (2006)
[2] J. Blumlein, and V. Ravindran, Nucl. Phys. B 749, 1 (2006)

SIDIS: Approximate NNLO became available in 2021
[3] M. Abele, D. de Florian, and W. Vogelsang, Phys. Rev. D 104, 094046 (2021)

Threshold resummation for SIDIS up to NNLL (expanded to approximate NNLO in the threshold limit $\hat{x} \to 1,\ \hat{z} \to 1$) including LP and some NLP

- **Integrated multiplicities:**

$$\frac{dM}{dz} = \left[\int_{Q_{\min}}^{Q_{\max}} dQ \int_{x_{\min}}^{x_{\max}} dx \int_{z_{\min}}^{z_{\max}} dz \frac{d^3\sigma}{dxdQdz}\right] \Big/ \left[\Delta z \int_{Q_{\min}}^{Q_{\max}} dQ \int_{x_{\min}}^{x_{\max}} dx \frac{d^2\sigma}{dxdQ}\right]$$

- **Exact charge conjugation:** $$D_{q(\bar{q})}^{h^-}(x,Q) \underset{h=\pi,K}{=} D_{\bar{q}(q)}^{h^+}(x,Q)\,, \quad D_g^{h^-}(x,Q) \underset{h=\pi,K}{=} D_g^{h^+}(x,Q)$$

- **ZM-VFNS with inactive flavours NOT set to zero below their threshold (inactive = do not evolve with DGLAP below their threshold):** $m_c = 1.51$ **GeV and** $m_b = 4.92$ **GeV**

- **NNPDF31_n(n)lo_pch_as_0118 PDF set used for SIDIS predictions at NLO(NNLO)**

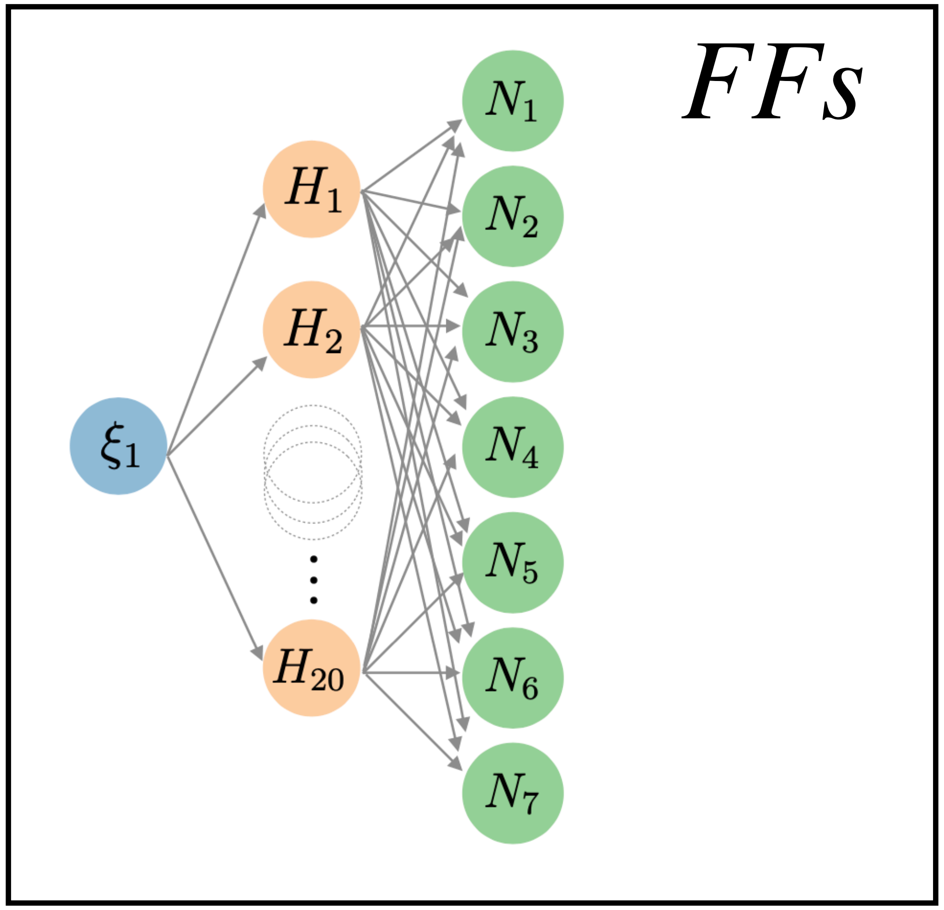
Methodology

NangaParbat
(chi-square, covariance, sampling, etc,)

MAP_FF1,0: NNAD & ceres-solver

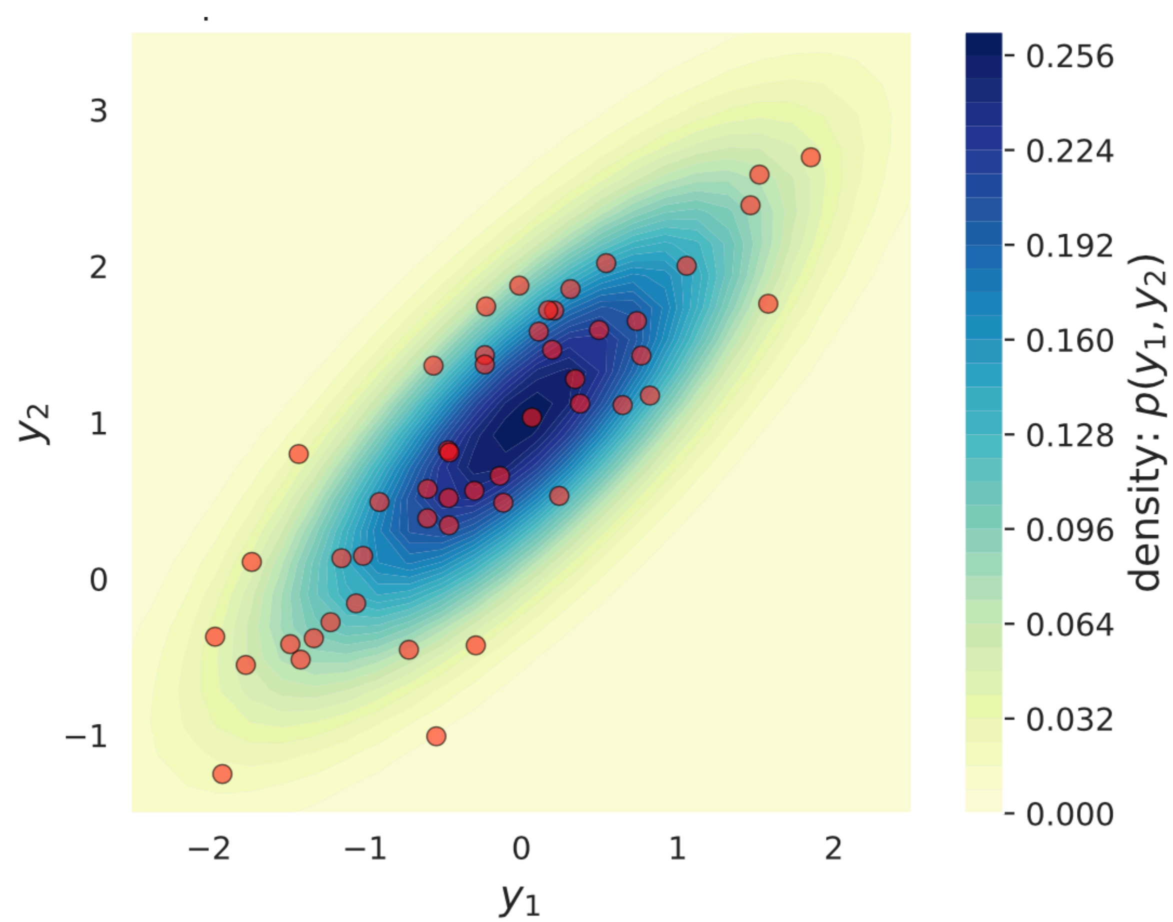
APFEL++

$$\textbf{Preds} = \hat{\sigma} \otimes PDF \otimes$$



Methodology

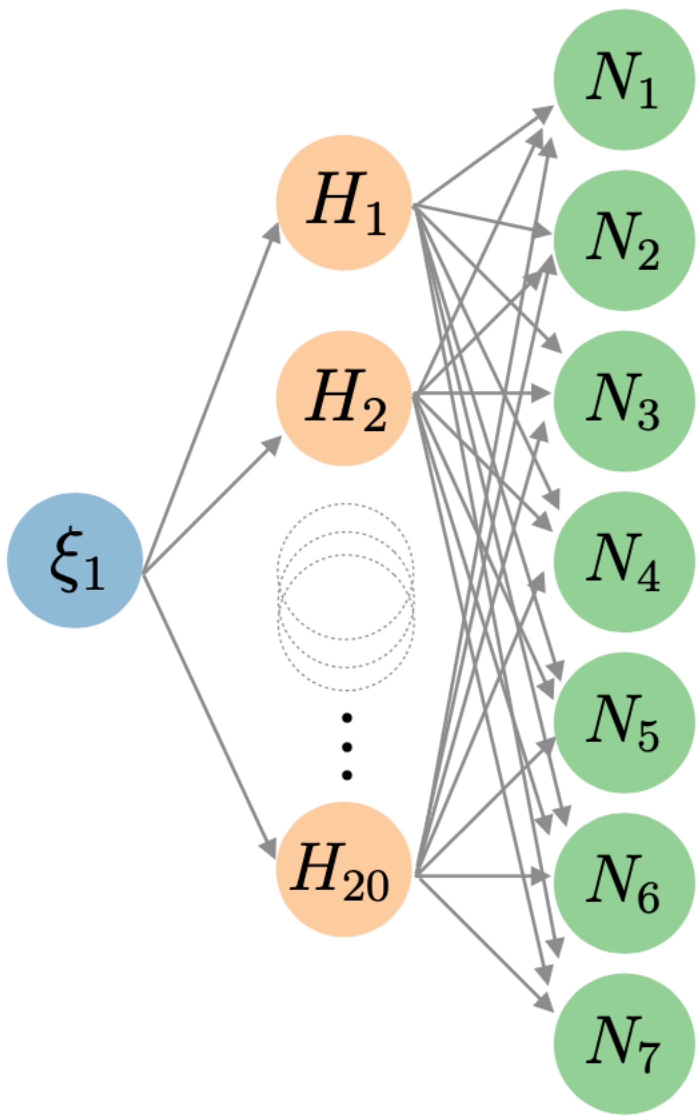
- **The Gaussian assumption:** $$\mathcal{G}\left(\boldsymbol{x}^{(k)}\right) \propto \exp\left[\left(\boldsymbol{x}^{(k)}-\boldsymbol{\mu}\right)^{T}\cdot \boldsymbol{C}^{-1}\cdot\left(\boldsymbol{x}^{(k)}-\boldsymbol{\mu}\right)\right]$$
- **Covariance Matrix:** $$C_{ij} = \delta_{ij}\sigma^2_{i,\,\mathrm{unc}} + \sum_{\beta}\sigma^{(\beta)}_{i,\,\mathrm{corr}}\sigma^{(\beta)}_{j,\,\mathrm{corr}}$$
- **Replica generation:** $$\boldsymbol{x}^{(k)} = \boldsymbol{\mu} + \boldsymbol{L}\cdot\boldsymbol{r}^{(\boldsymbol{k})} \longrightarrow \frac{1}{N_{\mathrm{rep}}}\sum_{k}^{N_{\mathrm{rep}}} x_i^{(k)} \simeq \mu_i\,, \qquad \frac{1}{N_{\mathrm{rep}}}\sum_{k}^{N_{\mathrm{rep}}} x_i^{(k)}x_j^{(k)} \simeq \mu_i\mu_j + C_{ij}$$



Methodology

◆ **Parameterisation:**

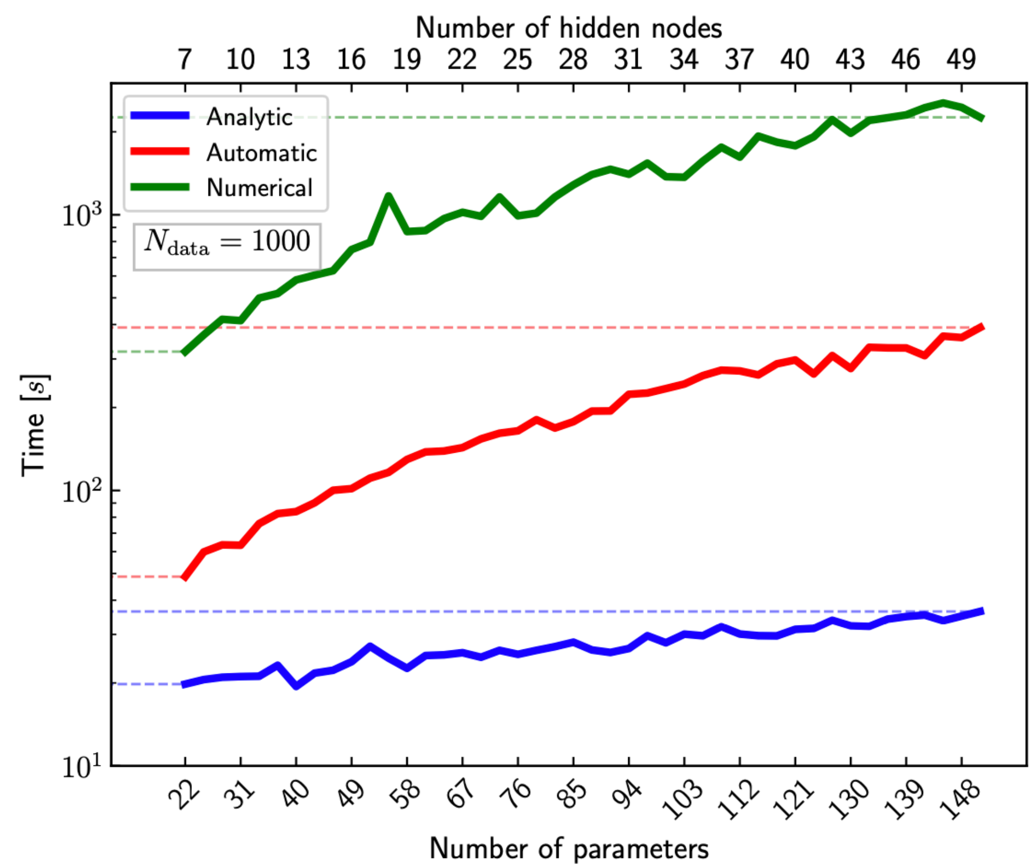
$$zD_i^h(z, \mu_0 = 5\ \textbf{GeV}) = \left(N_i(z;\theta) - N_i(1;\theta)\right)^2$$



7 flavours assuming a partially symmetric sea

$$\pi^+: \{u,\quad \bar{d},\quad d = \bar{u},\quad s^+,\quad c^+,\quad b^+,\quad g\}$$
$$K^+: \{u,\quad \bar{s},\quad s = \bar{u},\quad d^+,\quad c^+,\quad b^+,\quad g\}$$

Analytic derivatives for minimisation (NNAD)



◆ **6 flavours [SU(2) isospin symmetry + partially symmetric sea] → deterioration of the fit quality**

◆ **11 flavours [no symmetries] → overly redundant**

Methodology

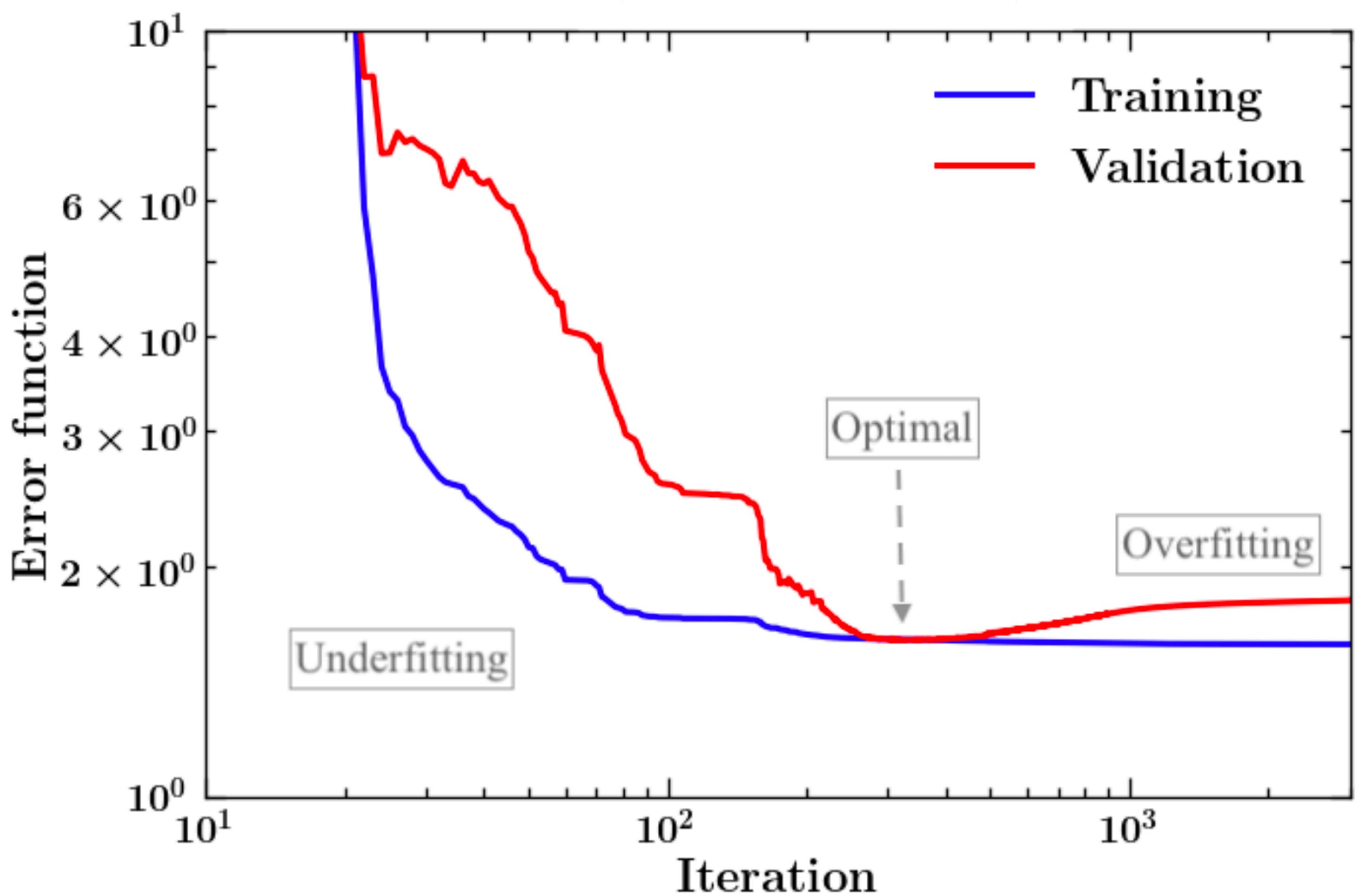
- **Maximum Log-likelihood:**

$$\chi^{2(k)} \equiv \left(\boldsymbol{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)^T \cdot \boldsymbol{C}^{-1} \cdot \left(\boldsymbol{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)$$

Every fit of a data replica is performed with a different PDF MC member in the SIDIS theory prediction
Ensuring the propagation of PDF correlated uncertainties,

However, the contribution of these uncertainties to the fit turned out to be very mild (see appendix)

- **cross-validation and stopping:**

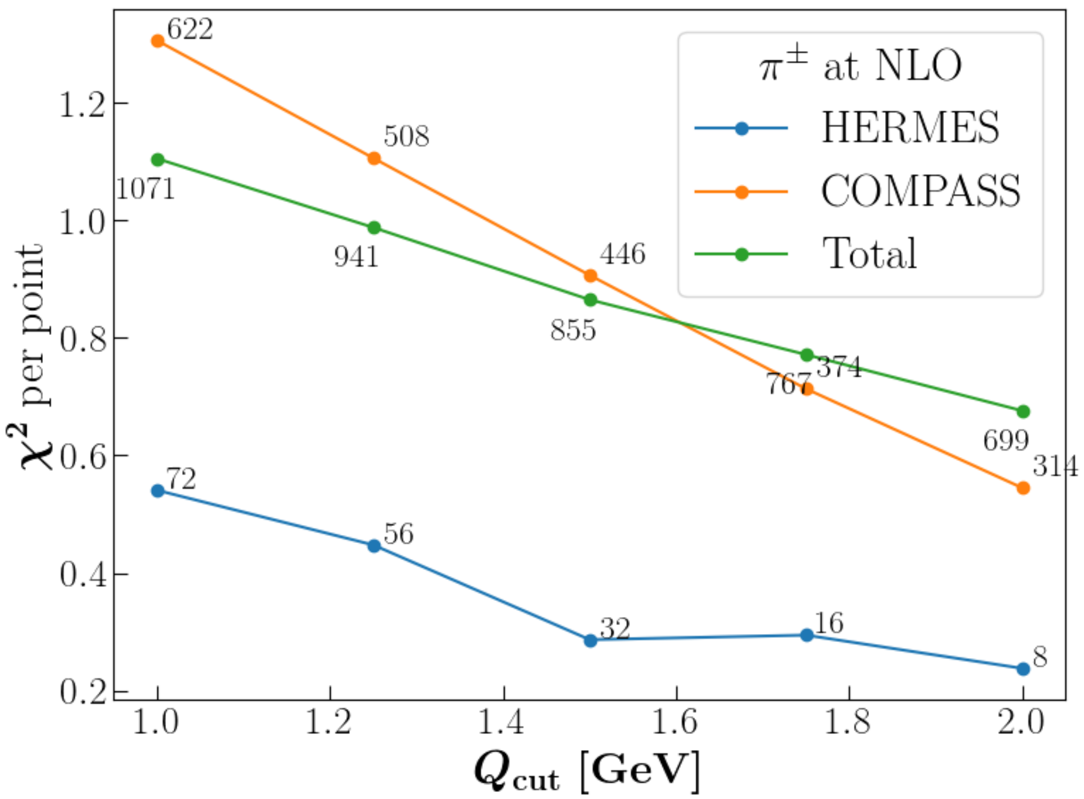


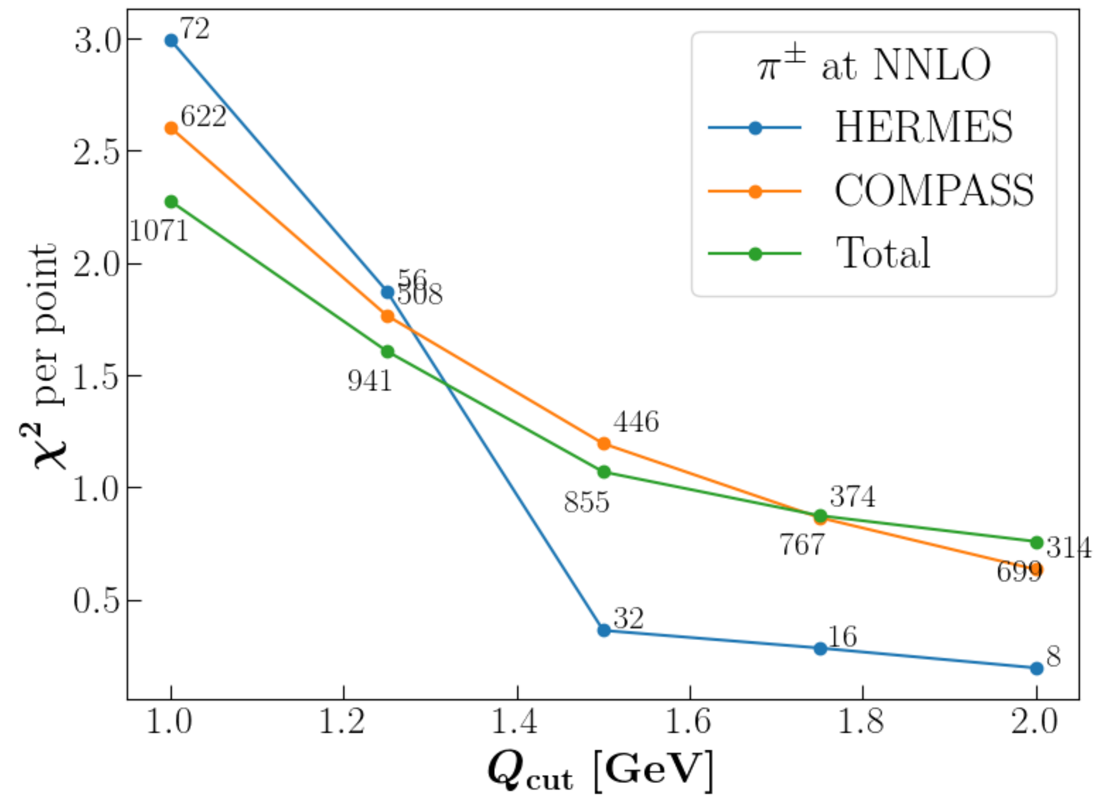
The minimisation is carried out by means of *ceres-solver*

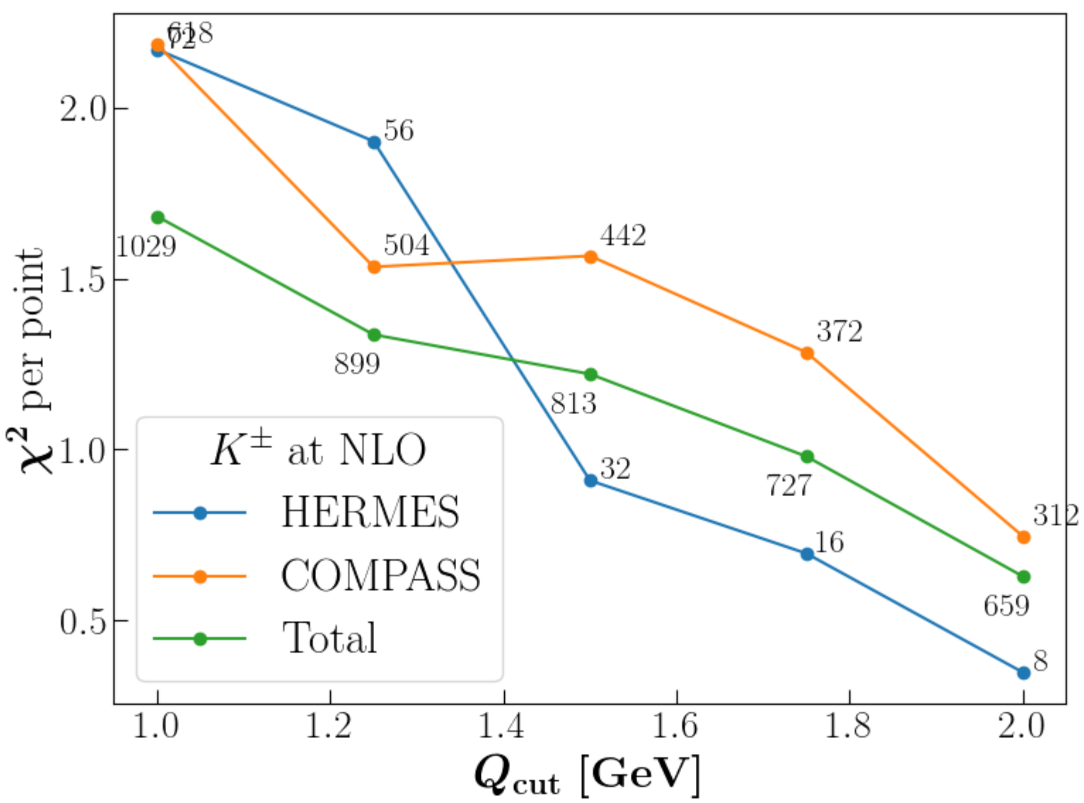
in particular the trust region Levenberg–Marquardt algorithm

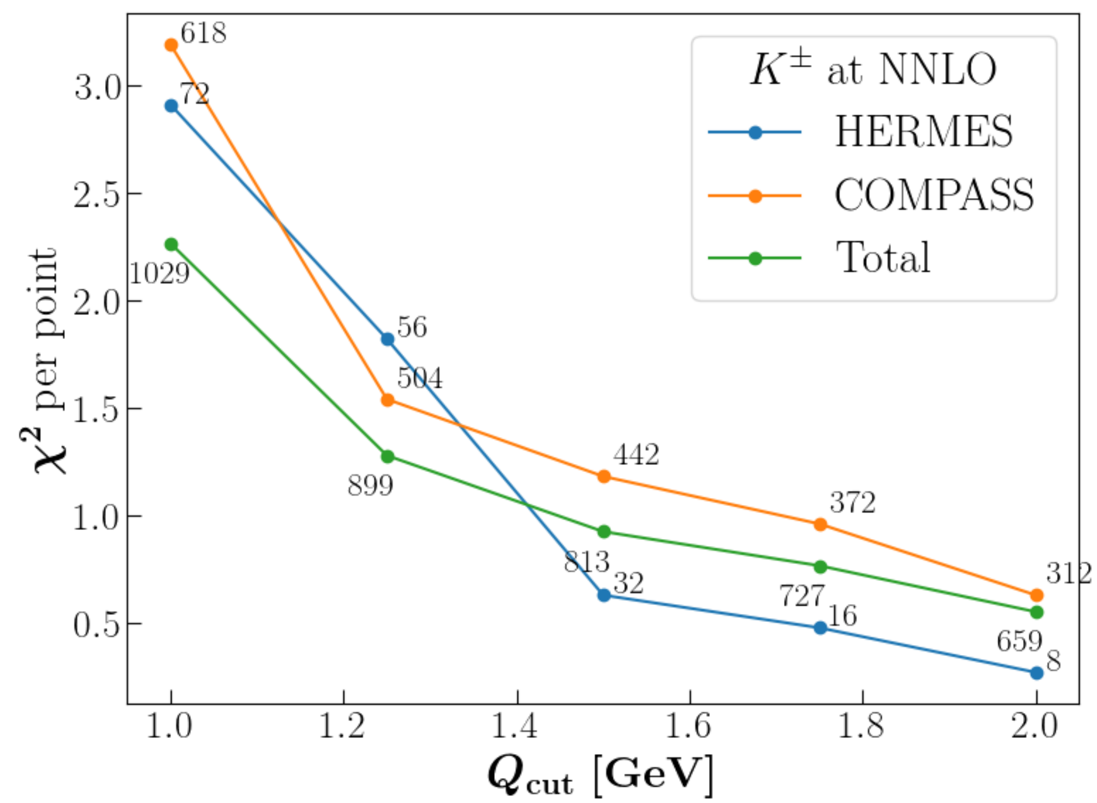
Relying on the knowledge of the analytic derivatives of the neural network itself via *NNAD*

Results — Fit Quality

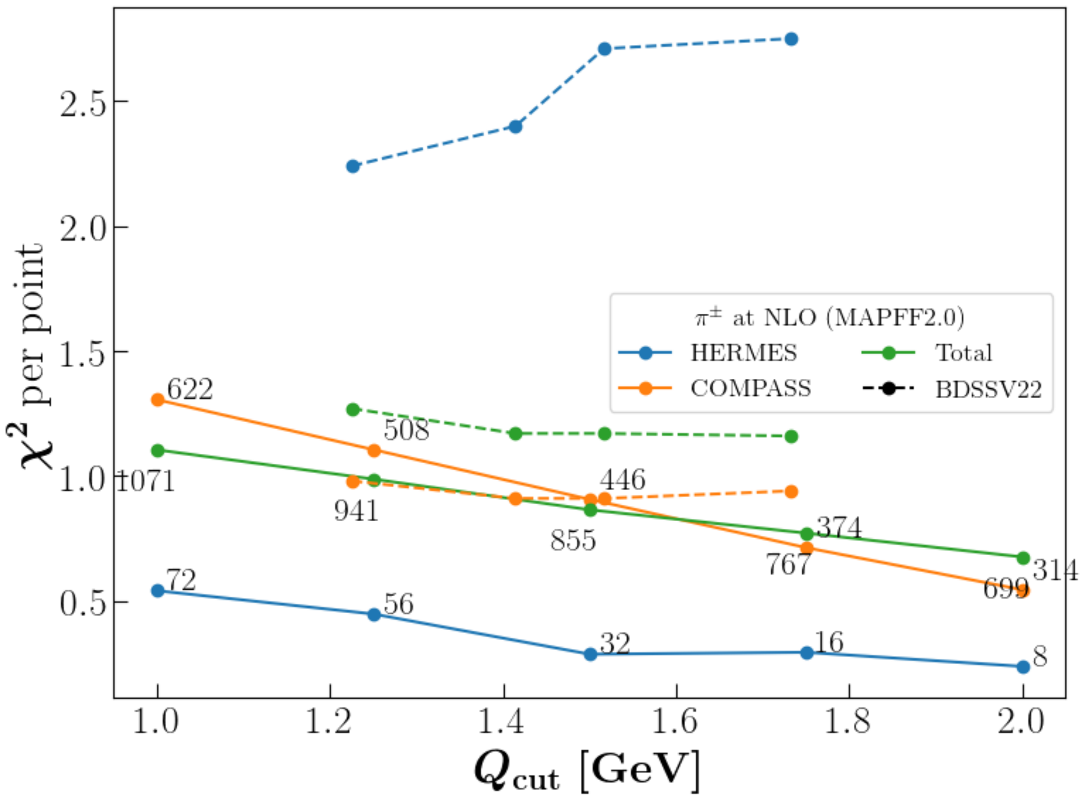


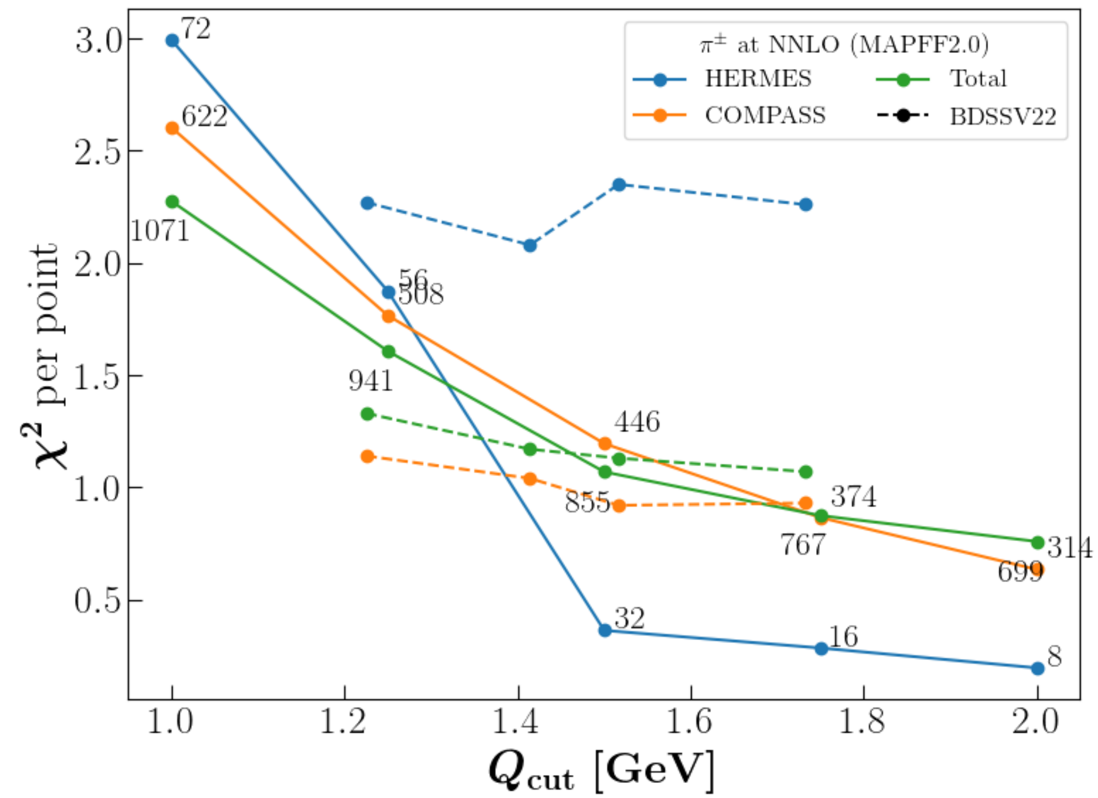






Results — Fit Quality





I. Borsa, D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, BDSSV22 Pion-only fit, arXiv:2202,05060

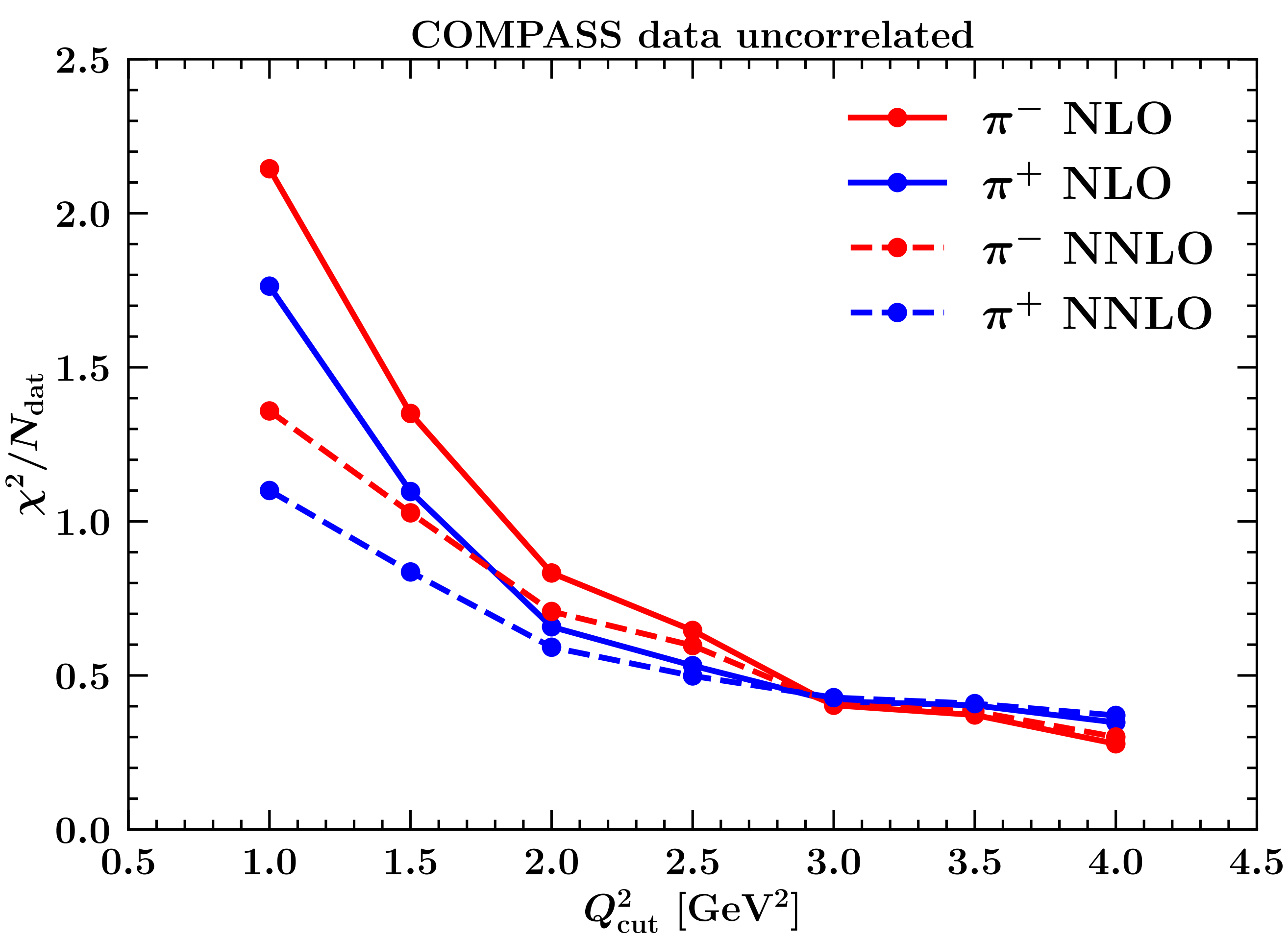
Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07

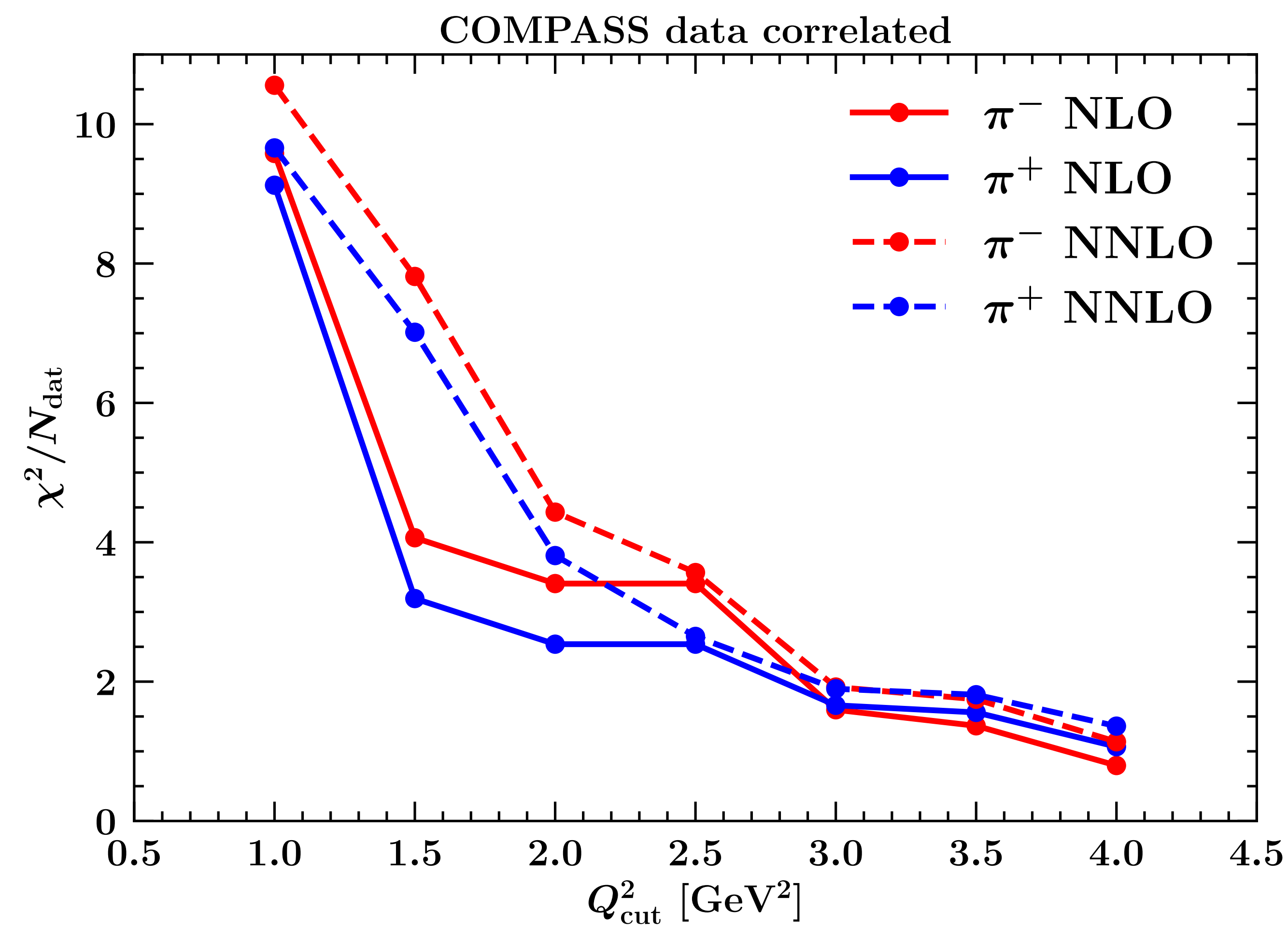
C, Adolph et al, [COMPASS], Phys, Lett, B 764, 1 (2017),

"From the total systematic uncertainty a large fraction, $0.8\sigma_{\rm syst}$, is estimated to be correlated, and the remaining, $\sqrt{(1-0.8^2)}\sigma_{\rm syst}$, is uncorrelated."

Results — COMPASS

Precise information and estimation of experimental correlated uncertainties, is becoming increasingly crucial for a reliable quantification of the impact of higher order corrections



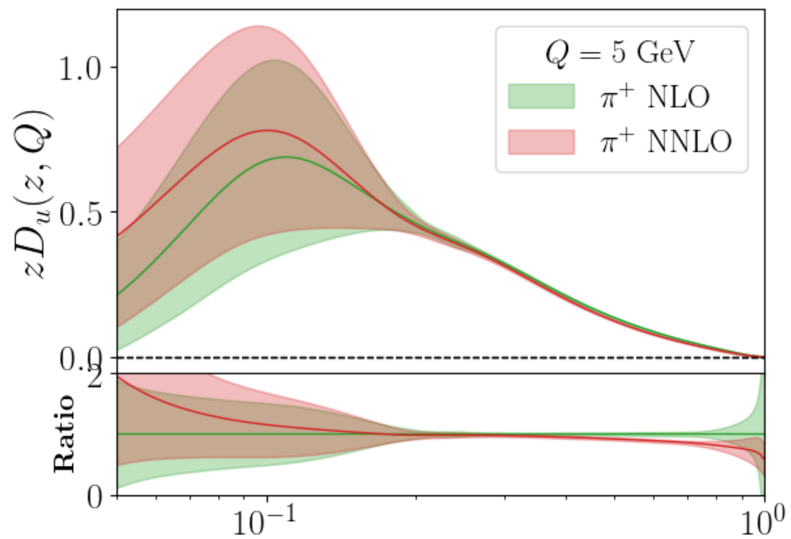


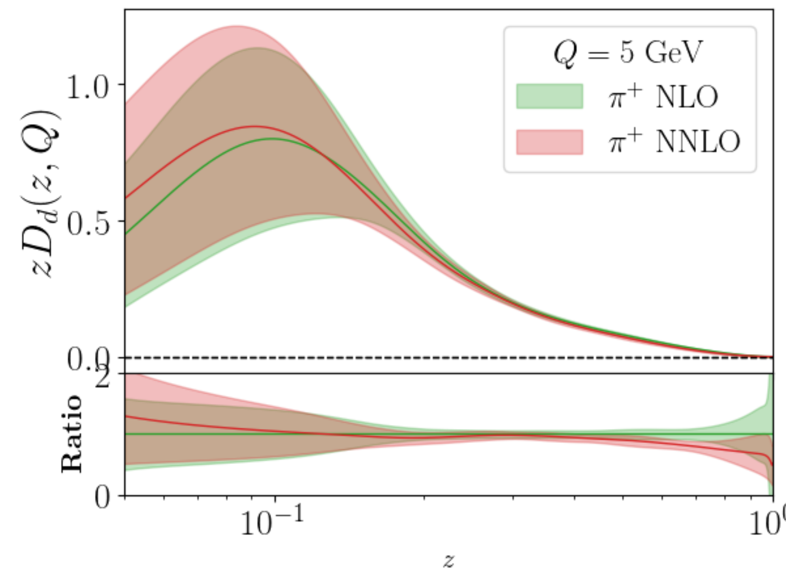
Results — Fit Quality $Q_{\text{cut}} > 2$ GeV

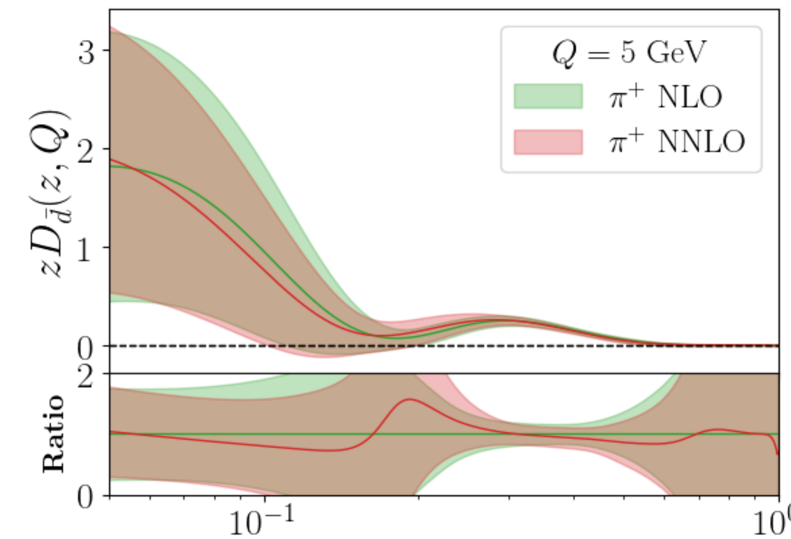
$\pi^{\pm}$	NLO	NNLO	# pts
HERMES \pi^- deuteron	0,42	0,33	2
HERMES \pi^- proton	0,01	0,02	2
HERMES \pi^+ deuteron	0,17	0,11	2
HERMES \pi^+ proton	0,35	0,32	2
COMPASS \pi^-	0,47	0,55	157
COMPASS \pi^+	0,61	0,72	157
BELLE \pi^\pm	0,09	0,10	70
BABAR prompt \pi^\pm	0,87	0,77	39
TASSO 12 GeV \pi^\pm	0,97	0,97	4
TASSO 14 GeV \pi^\pm	1,38	1,40	9
TASSO 22 GeV \pi^\pm	1,81	1,93	8
TPC \pi^\pm	0,23	0,26	13
TASSO 30 GeV \pi^\pm	0,33	0,36	2
TASSO 34 GeV \pi^\pm	1,07	1,48	9
TASSO 44 GeV \pi^\pm	1,12	1,37	6
TOPAZ \pi^\pm	0,24	0,37	5
ALEPH \pi^\pm	1,24	1,43	23
DELPHI total \pi^\pm	1,32	1,25	21
DELPHI uds \pi^\pm	2,68	2,93	21
DELPHI bottom \pi^\pm	1,61	1,74	21
OPAL \pi^\pm	1,62	1,78	24
SLD total \pi^\pm	1,02	1,11	34
SLD uds \pi^\pm	1,60	2,23	34
SLD bottom \pi^\pm	0,55	0,65	34
Total	**0,68**	**0,76**	**699**

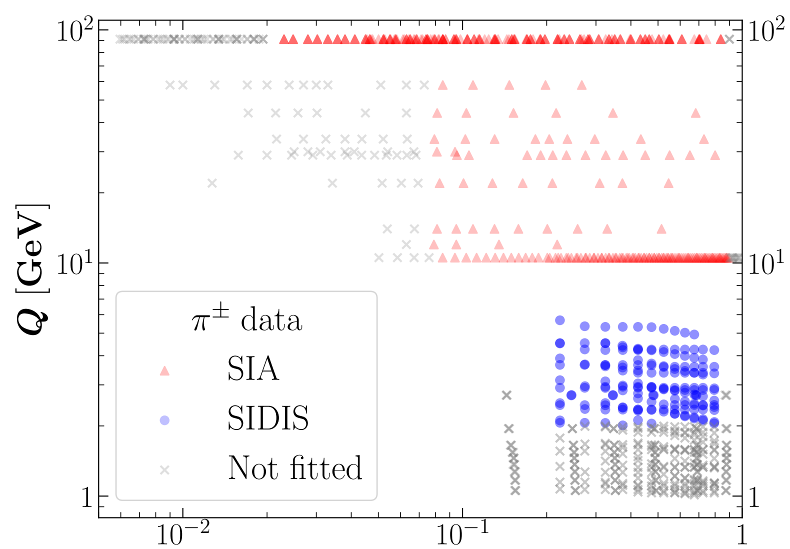
$K^{\pm}$	NLO	NNLO	# pts
HERMES K^- deuteron	0,18	0,13	2
HERMES K^- proton	0,04	0,04	2
HERMES K^+ deuteron	0,59	0,49	2
HERMES K^+ proton	0,57	0,43	2
COMPASS K^-	0,74	0,60	156
COMPASS K^+	0,75	0,67	156
BELLE K^\pm	0,49	0,47	70
BABAR K^\pm	0,37	0,26	28
TASSO 12 GeV K^\pm	0,83	0,86	3
TASSO 14 GeV K^\pm	1,24	1,22	9
TASSO 22 GeV K^\pm	0,88	0,89	6
TPC K^\pm	0,38	0,38	13
TASSO 34 GeV K^\pm	0,07	0,06	5
TOPAZ K^\pm	0,10	0,11	3
ALEPH K^\pm	0,50	0,48	18
DELPHI total K^\pm	0,99	0,99	23
DELPHI uds K^\pm	0,44	0,38	23
DELPHI bottom K^\pm	0,42	0,44	23
OPAL K^\pm	0,39	0,36	10
SLD total K^\pm	0,81	0,67	35
SLD uds K^\pm	1,37	1,52	35
SLD bottom K^\pm	0,76	0,78	35
Total	**0,63**	**0,55**	**659**

Results — $\pi^{\pm}$ FFs $Q_{\text{cut}} > 2$ GeV

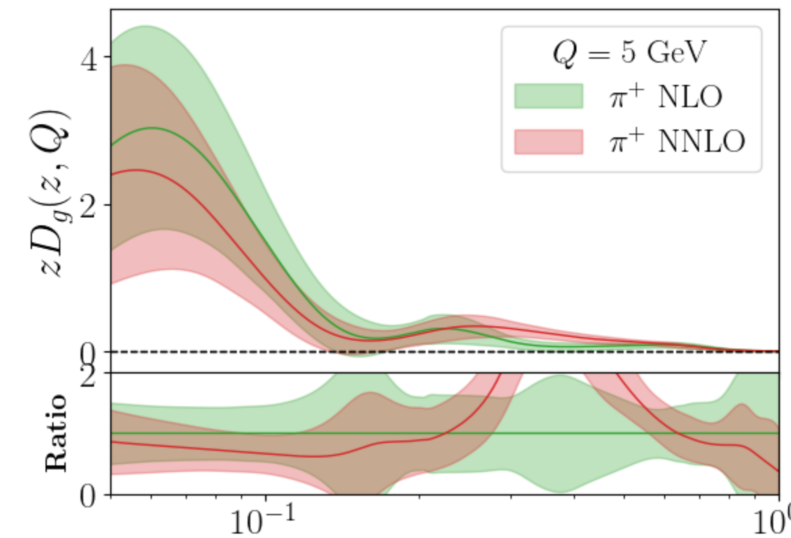


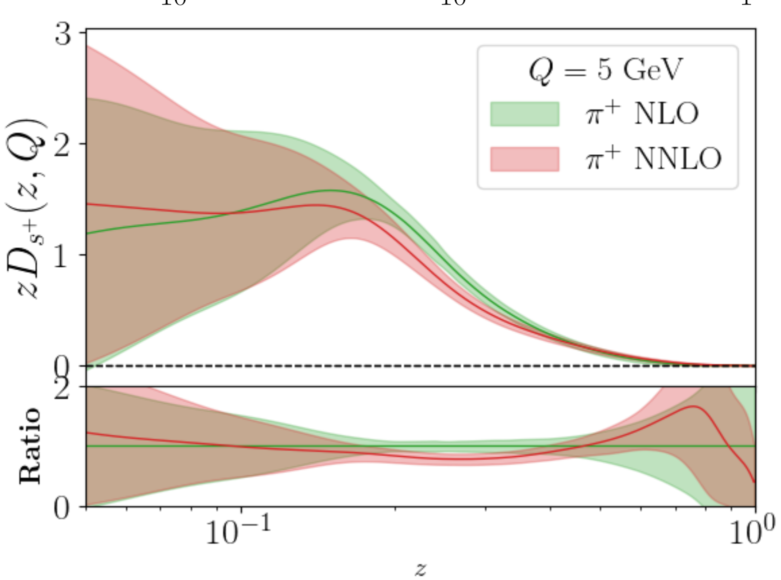


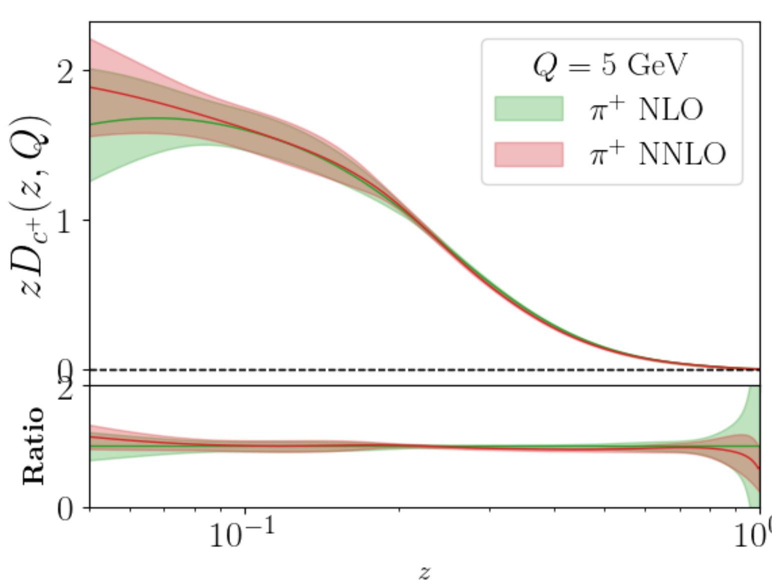


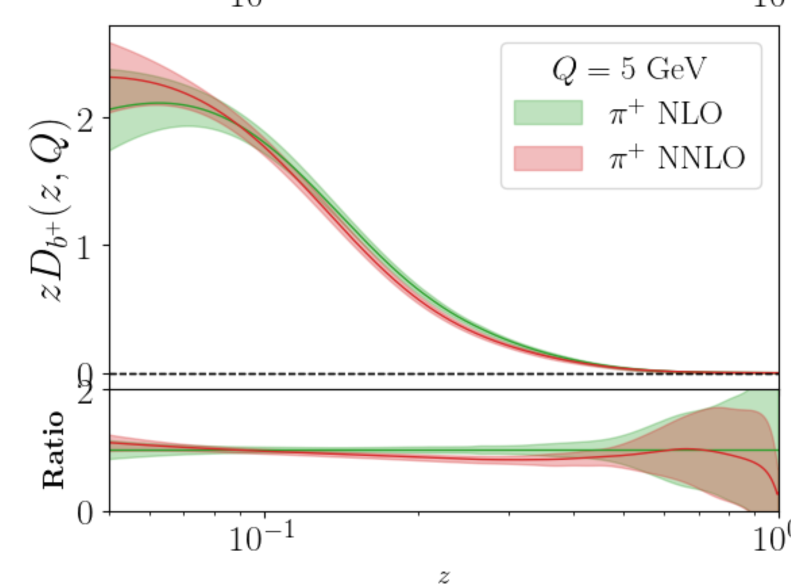


Noticeable impact in general, particularly on the gluon and the sea.

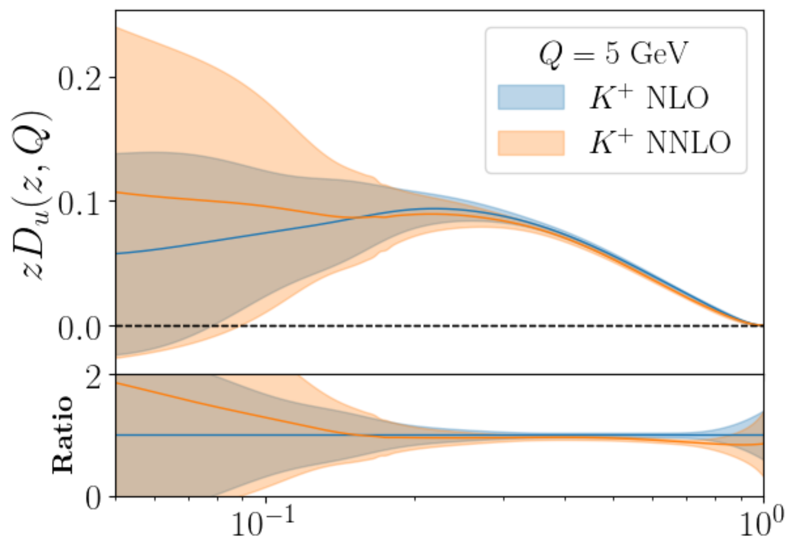


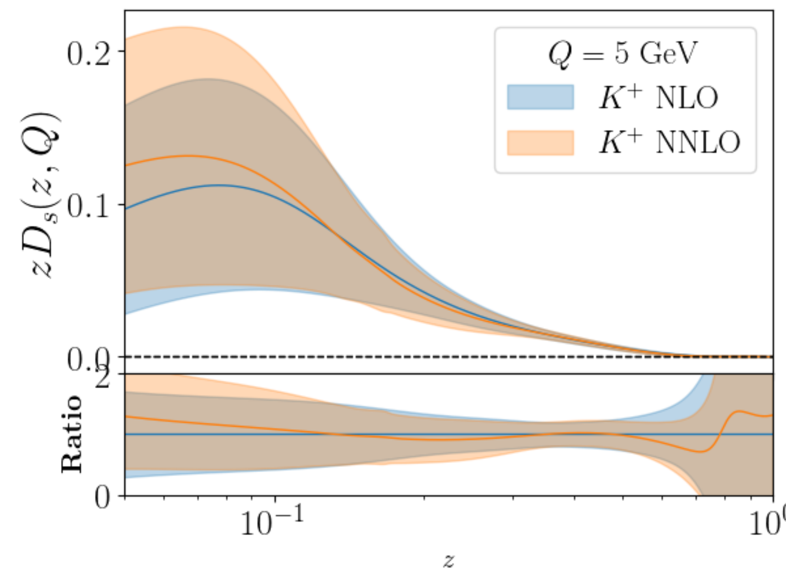


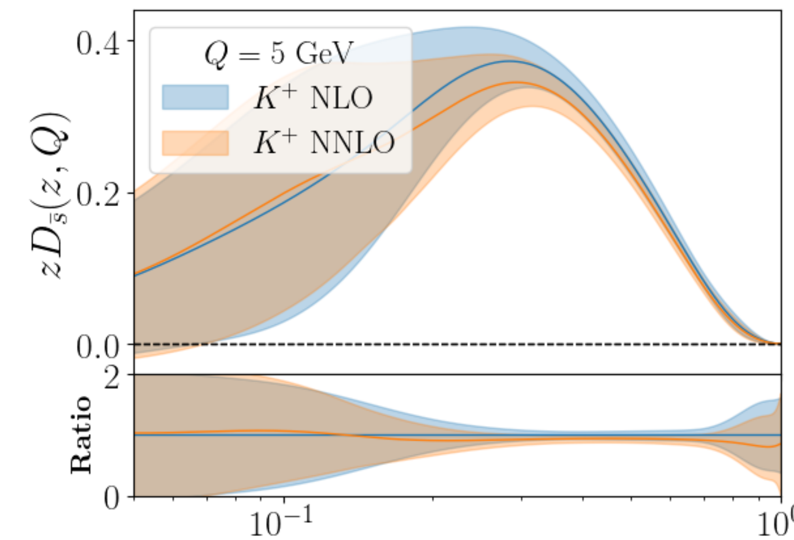


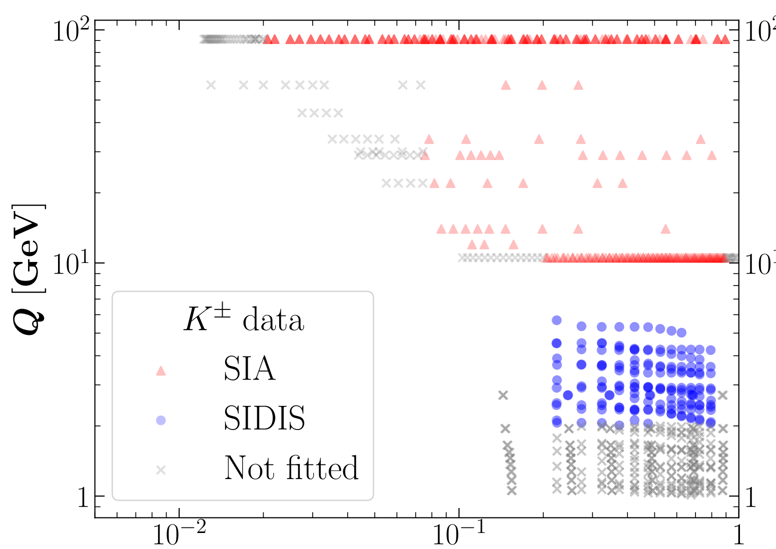


Results — $K^{\pm}$ FFs $\quad Q_{\rm cut} > 2$ GeV

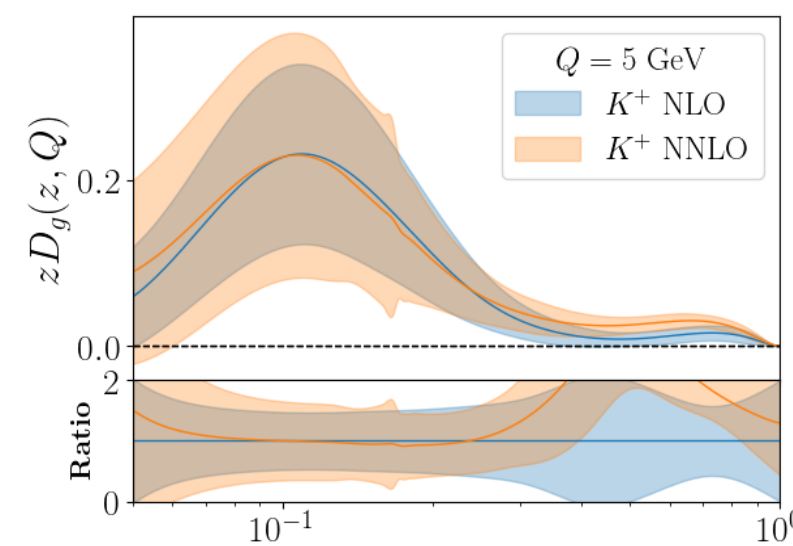


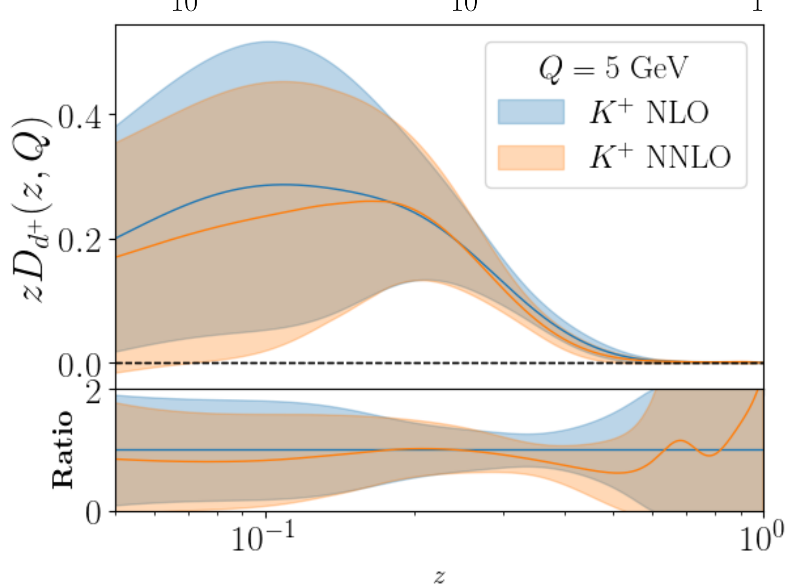


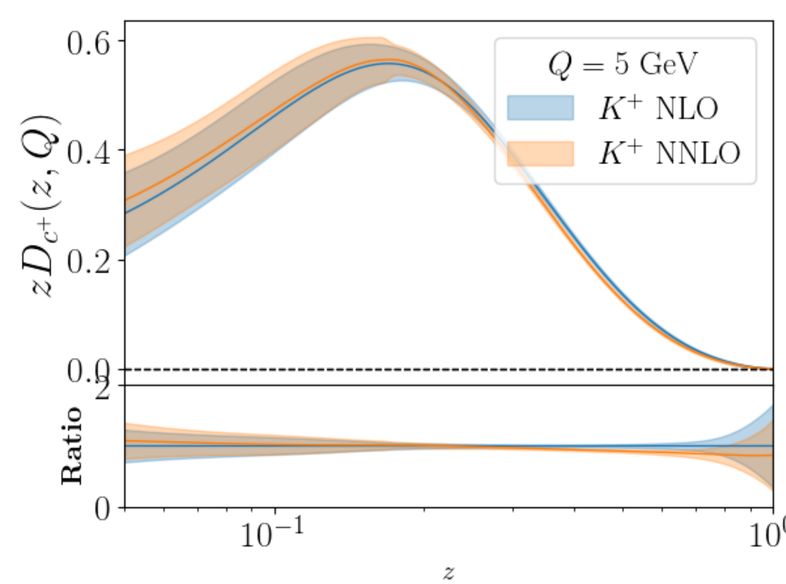


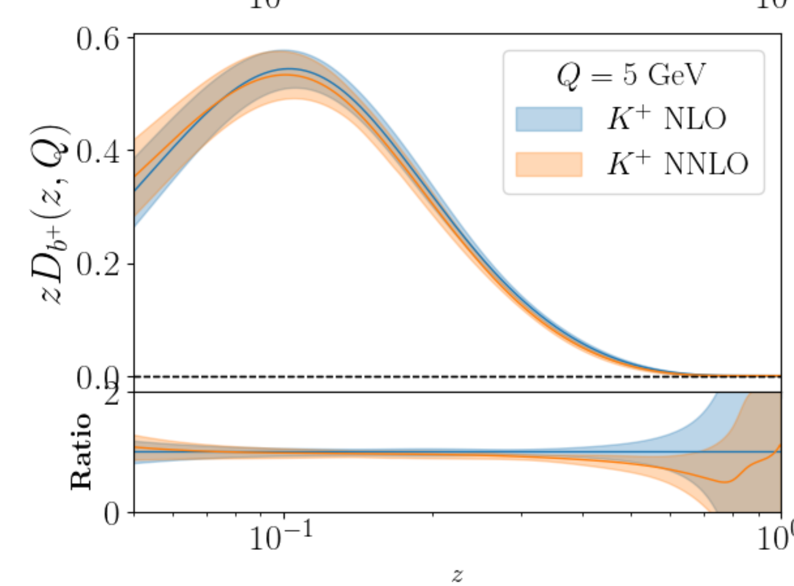


Noticeable impact in general, particularly on the gluon and the sea.









Thank you!

https://github.com/MapCollaboration