Accessing fracture functions in back-to-back dihadron production at CLAS12

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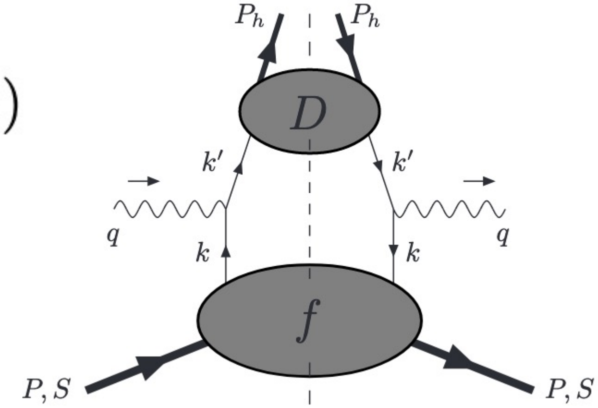
Accessing 3D nucleonic structure

- Decades of study have led to detailed mappings of the momentum distribution of the nucleon in terms of 1-D parton distribution functions (PDFs).
- Theoretical advances have led to a framework in which information on the confined motion of the partons inside a nucleon are matched to 3-D transverse momentum dependent parton distribution functions (TMDs).
- Cross section factorized as a convolution of PDFs and Fragmentation Functions (FFs)[1].

$$\frac{d\sigma^{\mathrm{CFR}}}{dx_B\,dy\,dz_h} = \sum_a e_a^2\, f_a(x_B)\, \frac{d\hat{\sigma}}{dy}\, D_a(z_h)$$

- TMDs/PDFs
  - Confined motion of quarks and gluons inside the nucleus
  - Orbital motion of quarks, correlations between quarks and gluons
- Fragmentation Functions
  - Probability for a quark to form particular final state particles
  - Insight into transverse momenta and polarization



M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

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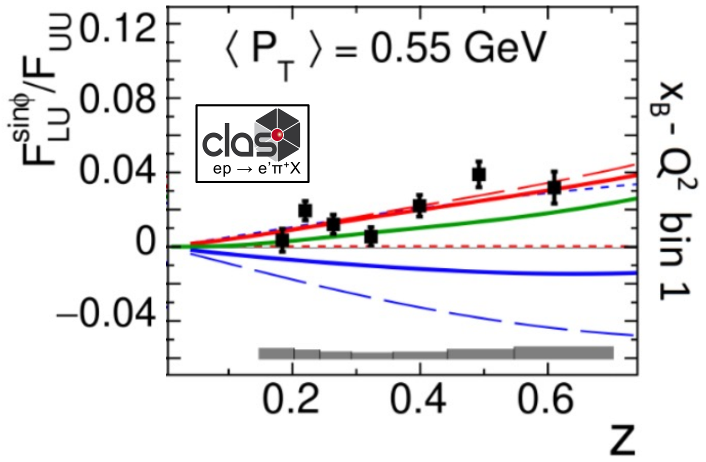
1. A. Bacchetta et al., JHEP 02 (2007) 093 [hep-ph] 0611265,

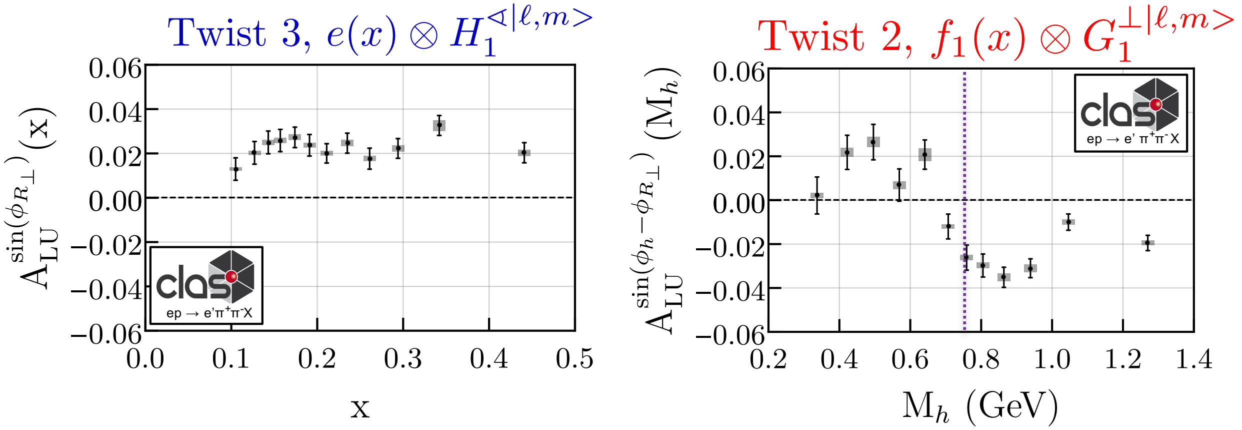
PDF Sensitive CLAS12 Measurements

- Measurements traditionally focus on factorization theorems and assumption that **hadrons are produced in current fragmentation**.

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{h}\cdot k_T}{M_h}\left(xeH_1^\perp + \frac{M_h}{M}f_1\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{h}\cdot p_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}h_1^\perp\frac{\tilde{E}}{z}\right)\right]$$



S. Diehl et al., *Phys. Rev. Lett.*, 128, 062005, (2022), [hep:ex] 2101.03544



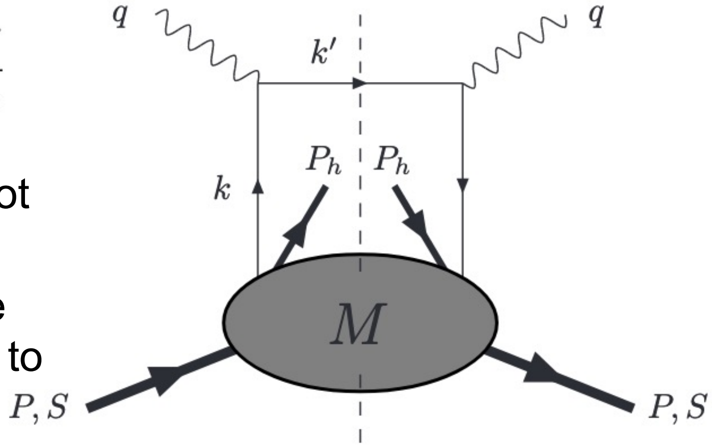
T. B. Hayward et al., *Phys. Rev. Lett.*, 126, 152501, (2021), [hep:ex] 2101.04842

The Neglected Hemisphere – Target Fragmentation

- Final state hadrons also form from the left-over target remnant (TFR) whose partonic structure is defined by "fracture functions"[1,2]: the probability to form a certain hadron given a particular ejected quark.
- In the TFR, factorization into x and z does not hold because it is not possible to separate quark emission from hadron production.

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z} = \sum_a e_a^2\,(1-x_B)\,M_a(x_B,(1-x_B)z)\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}$$

- Possible to kinematically separate CFR and TFR … but not always clear.
- Studying the TFR tests our complete understanding of the SIDIS production mechanism while also providing access to information not available in the CFR.



M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

UCONN

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1. L. Trentadue and G. Veneziano, Phys. Lett. B323 (1994) 201,
2. M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214

Azimuthal Modulations

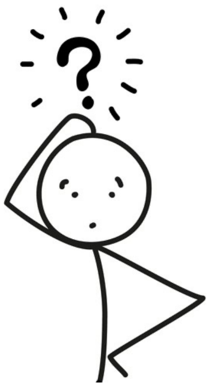
$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\Bigg\{\left(1-y+\frac{y^2}{2}\right)$$
$$\times \sum_a e_a^2 \left[\hat{u}_1(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\boxed{\sin(\phi_h-\phi_S)}\right]$$
$$+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel\;\hat{l}_{1L}\;(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\right.$$
$$\left.+\;|\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\;\hat{l}_{1T}^{\perp}\;\;(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\boxed{\cos(\phi_h-\phi_S)}\right]\Bigg\}\,.$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214

$$\boxed{\begin{aligned}\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} &= -\sum_a e_a^2\,x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\\ \left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} &= \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,f_{1T}^{\perp}\,D_1\right]\end{aligned}}$$

$$\boxed{\begin{aligned}\left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} &= \sum_a e_a^2\,x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\;\hat{l}_{1T}^{\perp}\;(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\\ \left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} &= \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,g_{1T}\,D_1\right]\end{aligned}}$$

The same azimuthal asymmetries can appear in both the CFR and TFR complicating their interpretation…



… six more azimuthal asymmetries appear in the CFR at leading twist which are absent in the TFR.

Categorizing Fracture Functions

- At leading twist 16 fracture functions exist that can be organized into tables of quark and nucleon polarizations just like the more familiar PDFs.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

CFR ↔ TFR

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Analogs to PDFs

- A direct relationship exists to the eight leading twist PDFs after the fracture functions are integrated over the fractional longitudinal nucleon momentum.

$$\sum_h \int d\zeta M_a(x_B)(x_B, k_\perp^2, \zeta) = (1 - x_B) f_a(x_B, k_\perp^2)$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-ph] 1102.4214

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Boer-Mulders analog

etc.etc.

Sivers analog

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Accessing longitudinal polarization

- TFR studies provide unique access to longitudinally polarized quarks in unpolarized nucleons and unpolarized quarks in longitudinally polarized nucleons.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1	☹	$h_1^\perp$
L	☹	g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Single hadron limitations

- FrFs describing transversely polarized quarks are chiral odd and inaccessible in single hadron production.
- Functions with double superscripts containing h and ⊥ have no analog and disappear after integration over either momentum.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

CFR ⟷ TFR

Quark polarization

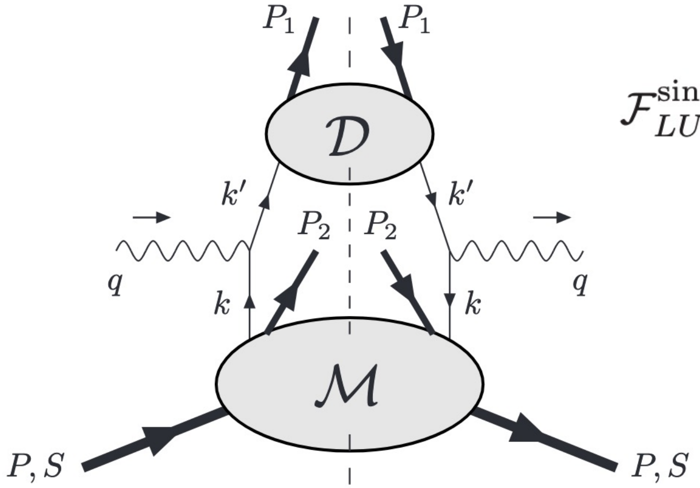
Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Back-to-back Formalism

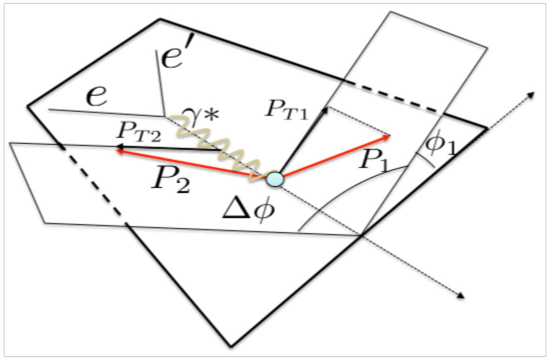
- When two hadrons are produced “back-to-back”[1,2] with one in the CFR and one in the TFR the structure function contains a convolution of a fracture function (l_1) and a fragmentation function (D_1).



Handbag diagram for dihadron production; lower blob contains FrFs and upper blob contains the FFs.

$$\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)} = \frac{|\mathbf{P}_{1\perp}||\mathbf{P}_{2\perp}|}{m_N m_2} \boxed{\mathcal{C}\left[w_5\, \hat{l}_1^{\perp h} D_1\right]},$$

$$\boxed{\mathcal{A}_{LU} = -\frac{y\left(1-\frac{y}{2}\right)}{\left(1-y+\frac{y^2}{2}\right)} \frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}} \sin\Delta\phi}$$



Kinematic plane for b2b hadron production.

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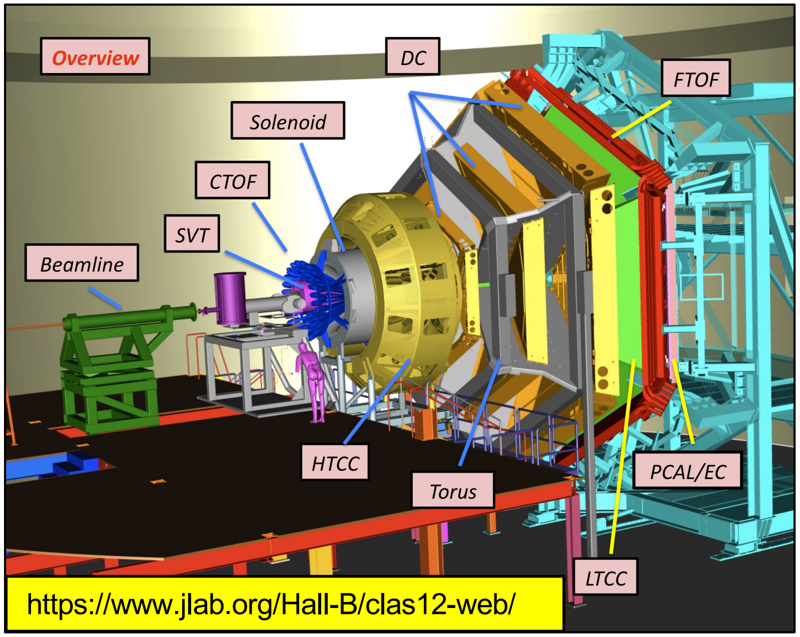
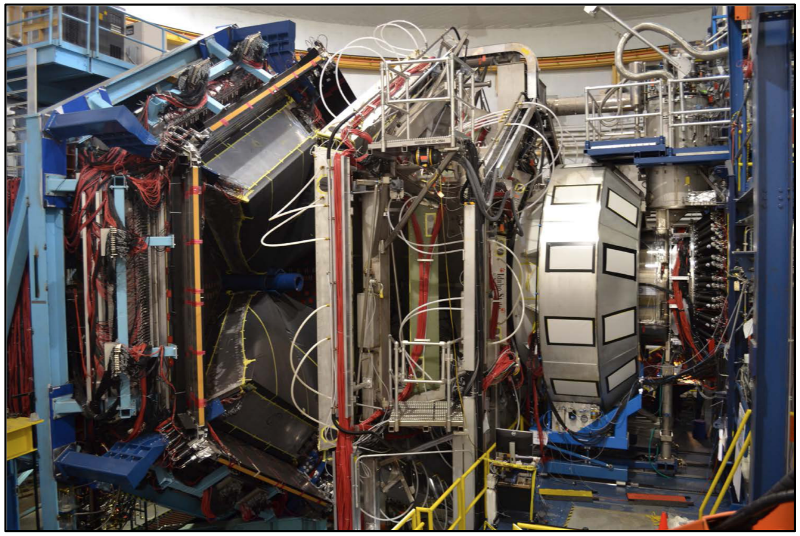
1. M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132
2. M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

CLAS12 Experimental Setup



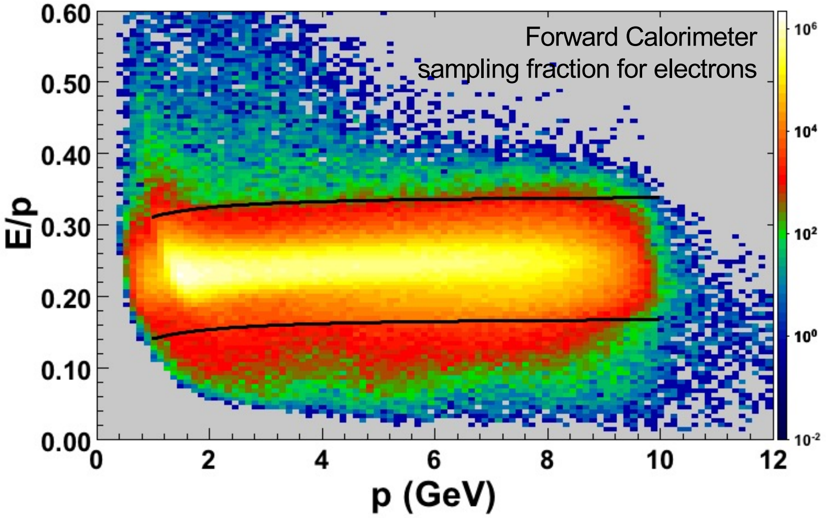


V. Burkert et al., Nucl. Instrum. Meth. A 959 (2020) 163419

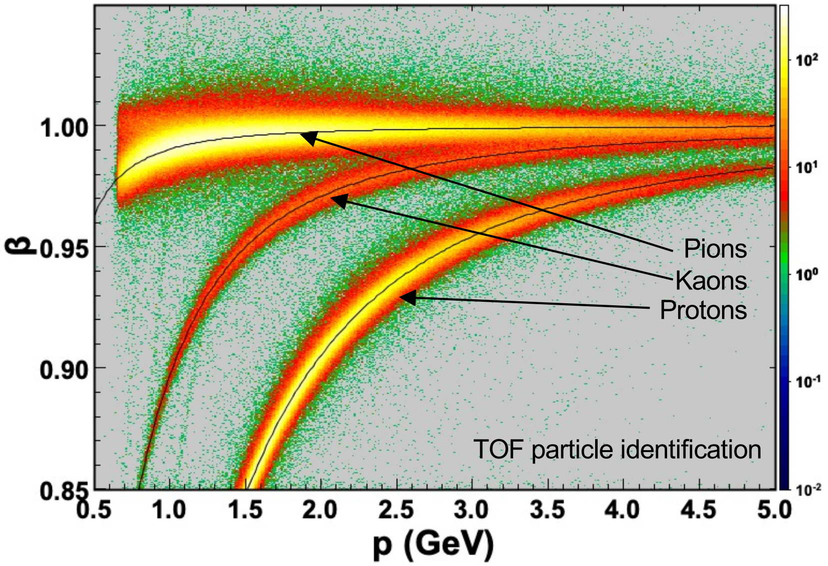
- CLAS12: very high luminosity, wide acceptance, low Q^2 (higher twist measurements)
- Began data taking in Spring 2018 – many “run periods” now available.
- 10.6 (2018) and 10.2 (2019) GeV electron beam, longitudinally polarized beam, liquid H_2 target.

Particle ID

- Electron
  - Electromagnetic calorimeter.
  - Cherenkov detector.
  - Vertex and fiducial cuts.



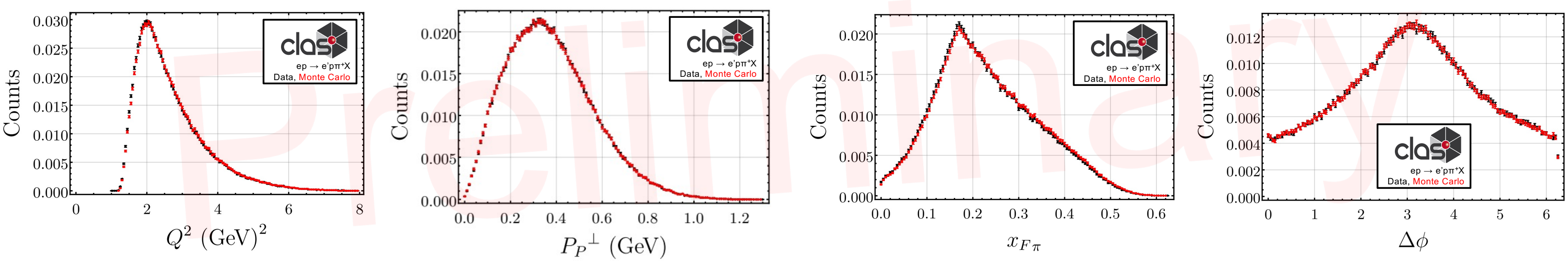
- Hadron
  - β vs p comparison between vertex timing and event start time.
  - Vertex and fiducial cuts.
  - Pion momentum limited to < 4 GeV.



UCONN

Monte Carlo

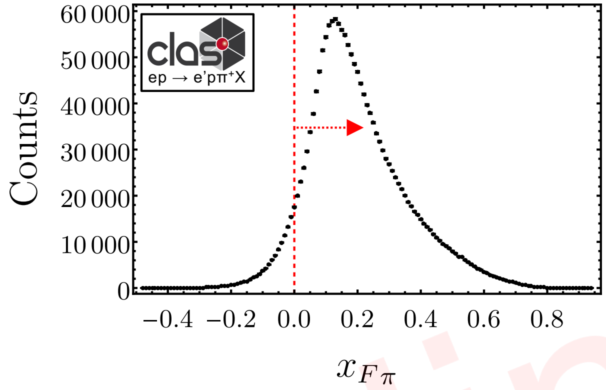
- SIDIS MC “clasdis”[1] based on PEPSI[2] generator, the polarized version of the well-known LEPTO[3] generator.
- Parameters changed to reproduce observed distributions include average transverse momentum, fraction of spin-1 light mesons and fraction of spin-1 strange mesons.
- CLAS12 detector system described in “GEMC”[4], a detailed GEANT4 simulation package.
- Excellent agreement between data and MC!

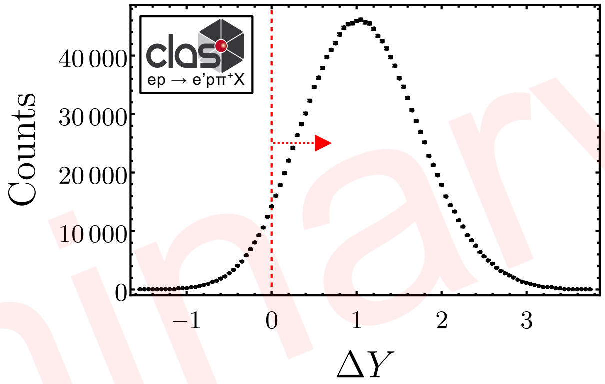


1. H. Avakian, “clasdis.” https://github.com/JeffersonLab/clasdis, 2020.
2. L. Mankiewicz, A. Schafer, and M. Veltri, “Pepsi: A monte carlo generator for polarized leptoproduction,” Comput. Phys. Commun., vol. 71, pp. 305–318, 1992.
3. G. Ingelman, A. Edin, and J. Rathsman, “LEPTO 6.5: A Monte Carlo generator for deep inelastic 912 lepton - nucleon scattering,” Comput. Phys. Commun., vol. 101, pp. 108–134, 1997.
4. M. Ungaro et al., “The CLAS12 Geant4 simulation,” Nucl. Instrum. Meth. A, vol. 959, p. 163422, 2020.

Selecting back-to-back events

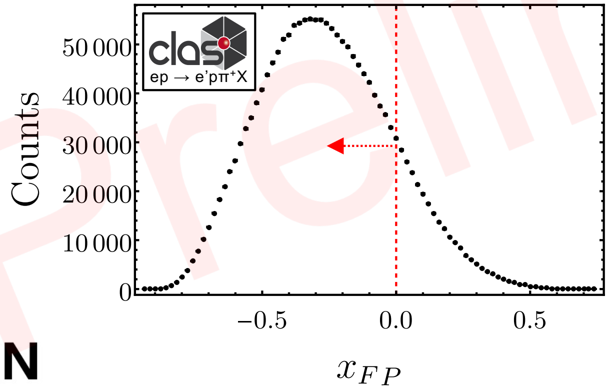
- A natural choice for a first analysis are events with a pion (CFR biased) and proton (TFR biased).

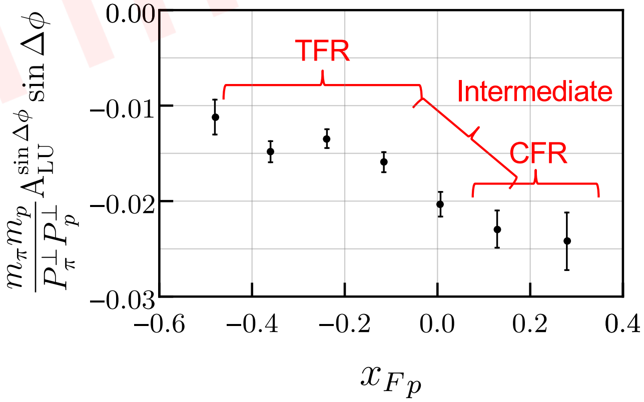




$$x_F = \frac{2p \cdot q}{|q|W}$$

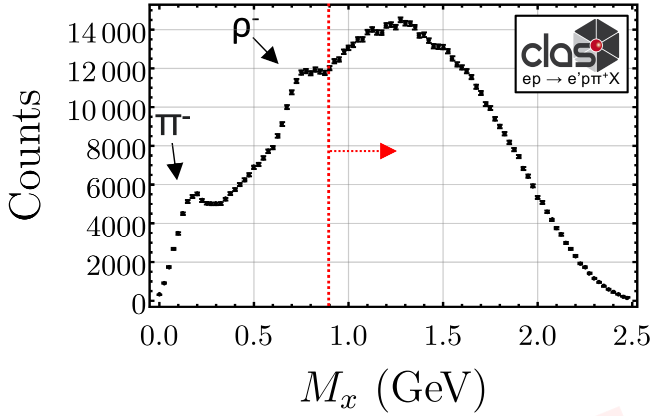
$$Y = \frac{1}{2}\log\left[\frac{E_h + p_z}{E_h - p_z}\right]$$



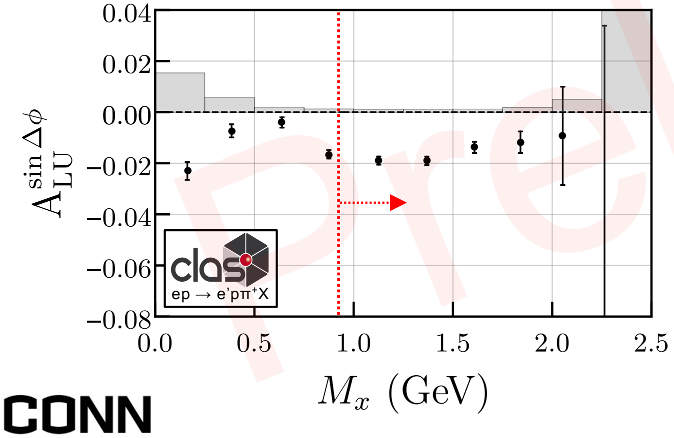


Early signs of separate signatures in both interaction regions.

Removing background

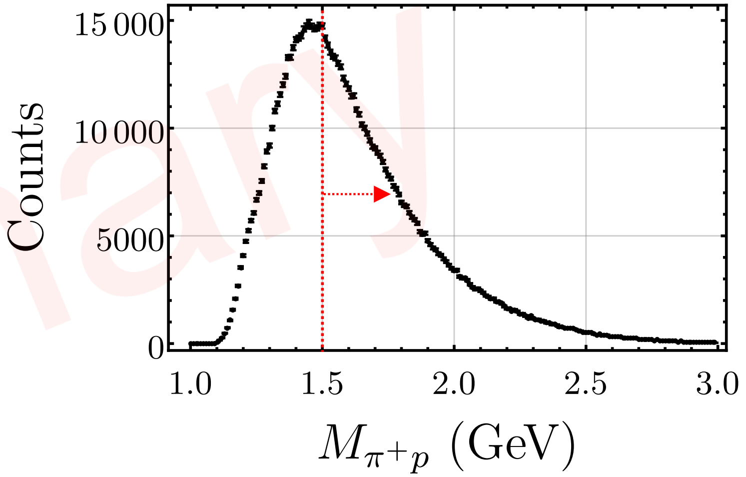


- Exclusive pion and rho production clearly visible.



- Different amplitudes at low M_x are generated from separate physics than our signal.

$\Delta^{++} \rightarrow p\pi^+$ Contamination?



- Little sign of Δs; cut on mass > 1.5 GeV for safety.
- Estimate remaining contribution from MC.

Extracting A_{LU}

- Select $ep \rightarrow e'P\,\pi^{+} + X$.
- Consider all possible hadron pairs.
- Amplitudes are extracted simultaneously via maximizing a likelihood function.
- Unbinned maximum likelihood method:

$$-\ln \mathcal{L}_{ML}(A) = N - \sum_{i}^{N} \ln\left[1 + h_i P_i \left(A_1 \sin\Delta\phi_i + A_2 \sin 2\Delta\phi_i\right)\right]$$

- Use MINUIT to minimize the -log likelihood.
- Include relevant beam polarization (~85% at JLab).

Channel selection
- Q^2>1.0 GeV^2
- W>2.0 GeV
- M_x>0.90 GeV
- y<0.75
- $\Delta Y > 0$
- $x_{F\pi} > 0$
- $x_{Fp} < 0$
- $z_{\pi} > 0.2$
- $M_h > 1.5$ GeV

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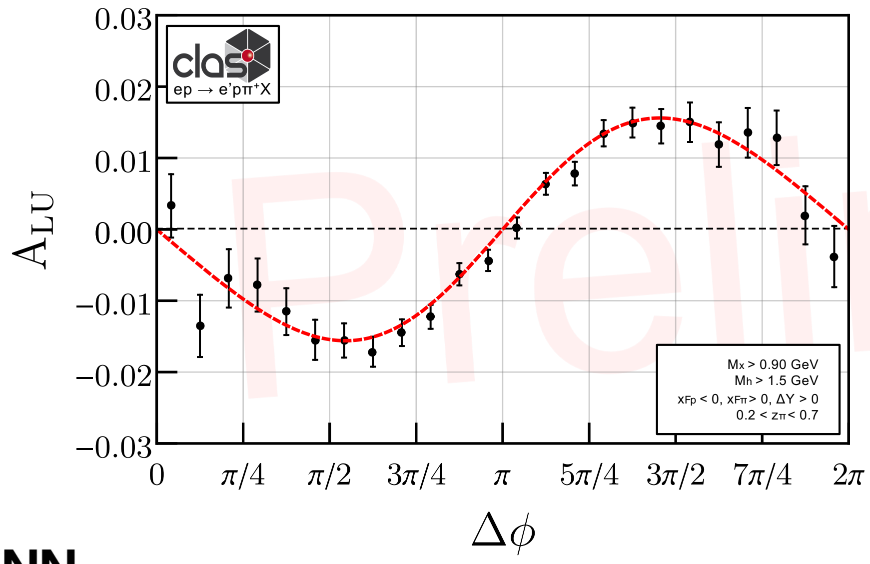
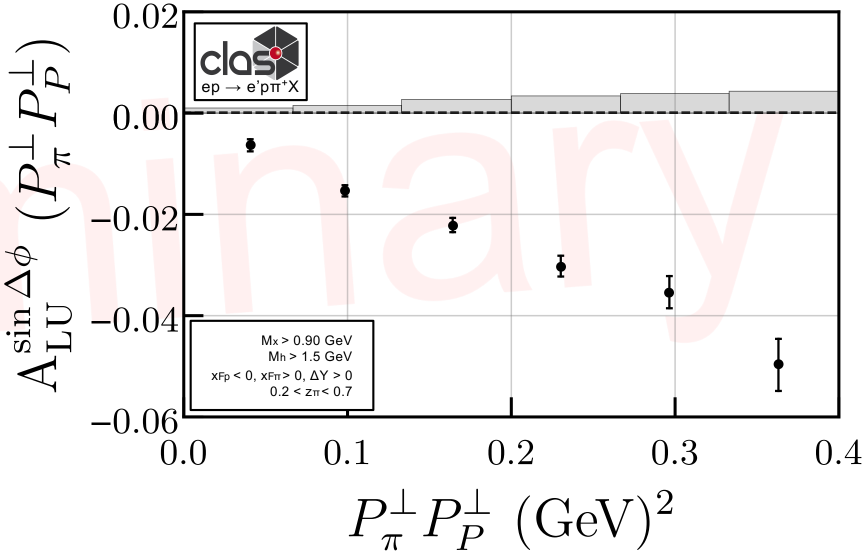
Initial Observation

- Observed linear dependence on the product of transverse momenta is consistent with expectations.
- Non-zero asymmetries are the first experimental observation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.

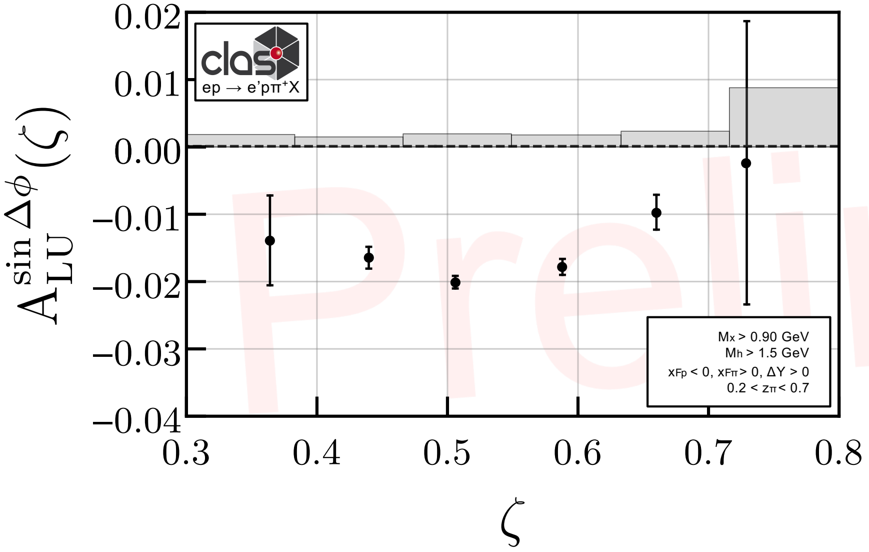
$$\mathcal{F}_{LU} = \frac{|p_{\pi^+}^{\perp}||p_P^{\perp}|}{m_p m_\pi}\mathcal{C}\left[w_5 \hat{l}_1^{\perp h} D_1\right]$$

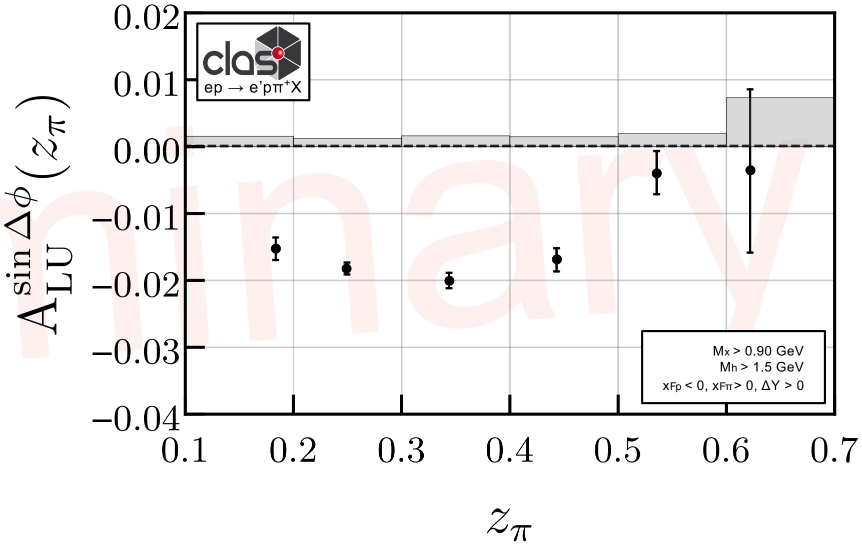




Access to unmeasured fracture functions

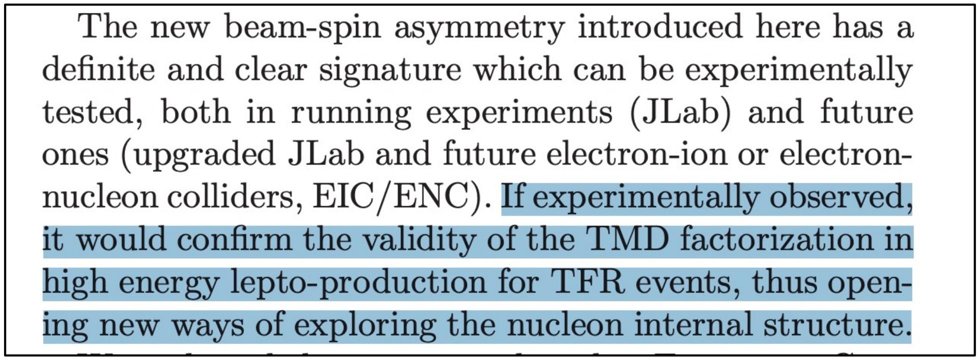
- Unmeasured fracture function $\hat{l}_1^{\perp h}$ depends on ζ. $\mathcal{F}_{LU}^{\sin(\Delta\phi)} = \frac{|p_{\pi^+}^{\perp}||p_P^{\perp}|}{m_P m_{\pi^+}} \mathcal{C}\left[w_5 \hat{l}_1^{\perp h} D_1\right]$
- Measured ragmentation function D_1 depends on z_π.





Conclusions

- Significant leading twist single-spin asymmetries have been observed **for the first time** in back-to-back proton-pion electroproduction.
- Beam-spin asymmetry amplitudes indicate that spin-orbit correlations may exist between hadrons produced simultaneously in the target and current fragmentation regions.
- Publication coming soon, follow up analysis with extended statistics (multidimensional binning) and additional pion flavors already in the works.

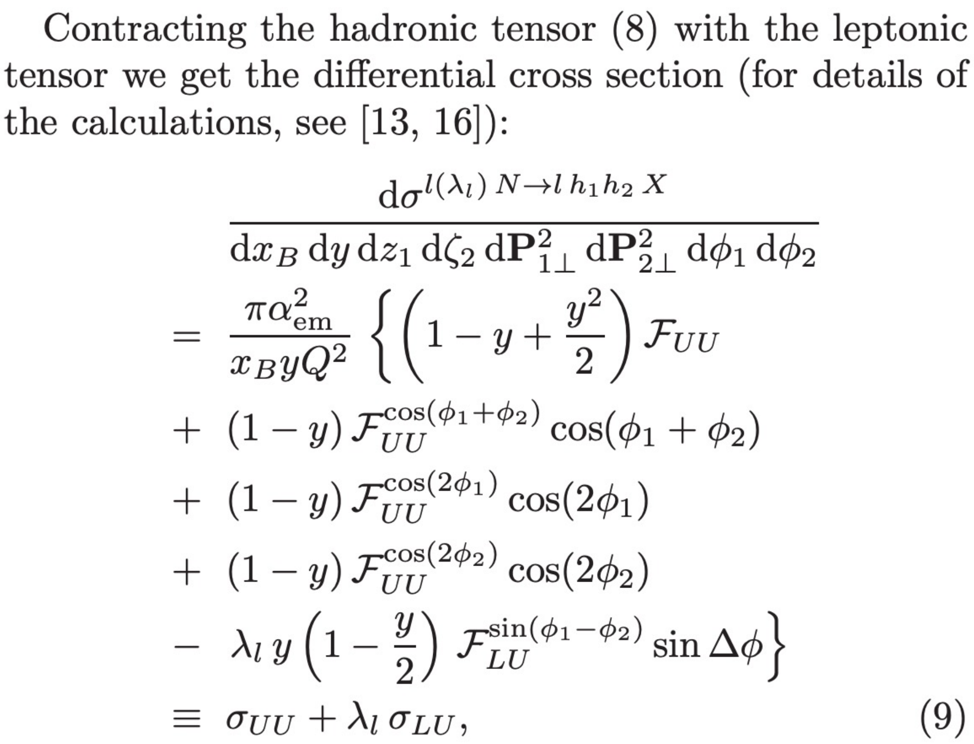


The new beam-spin asymmetry introduced here has a definite and clear signature which can be experimentally tested, both in running experiments (JLab) and future ones (upgraded JLab and future electron-ion or electron-nucleon colliders, EIC/ENC). If experimentally observed, it would confirm the validity of the TMD factorization in high energy lepto-production for TFR events, thus opening new ways of exploring the nucleon internal structure.

M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

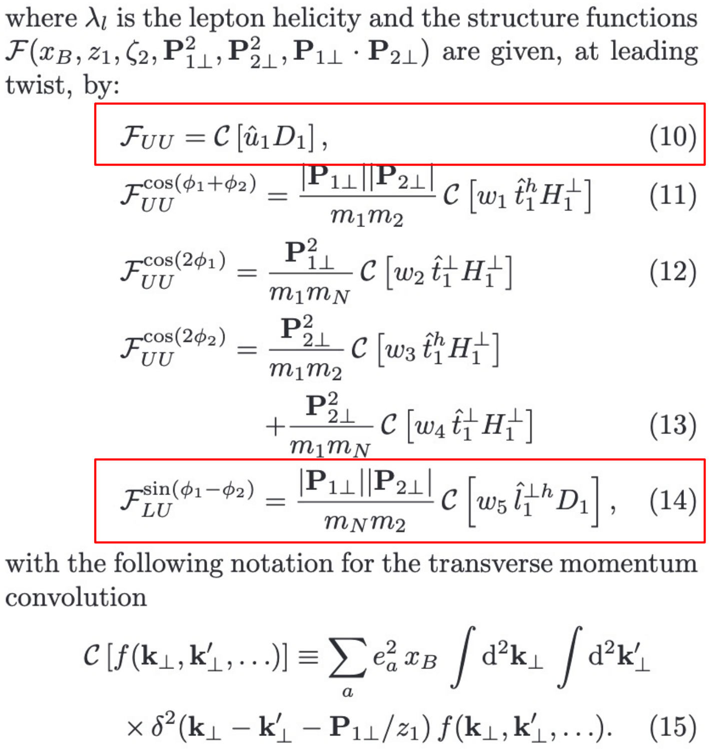
Backup Slides

Differential Cross Section



Contracting the hadronic tensor (8) with the leptonic tensor we get the differential cross section (for details of the calculations, see [13, 16]):

$$
\begin{aligned}
&\frac{d\sigma^{l(\lambda_l)\,N\to l\,h_1 h_2\,X}}{dx_B\,dy\,dz_1\,d\zeta_2\,d\mathbf{P}_{1\perp}^2\,d\mathbf{P}_{2\perp}^2\,d\phi_1\,d\phi_2} \\
=\;& \frac{\pi\alpha_{\rm em}^2}{x_B y Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)\mathcal{F}_{UU}\right. \\
+\;& (1-y)\,\mathcal{F}_{UU}^{\cos(\phi_1+\phi_2)}\cos(\phi_1+\phi_2) \\
+\;& (1-y)\,\mathcal{F}_{UU}^{\cos(2\phi_1)}\cos(2\phi_1) \\
+\;& (1-y)\,\mathcal{F}_{UU}^{\cos(2\phi_2)}\cos(2\phi_2) \\
-\;& \left.\lambda_l\,y\left(1-\frac{y}{2}\right)\,\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)}\sin\Delta\phi\right\} \\
\equiv\;& \sigma_{UU}+\lambda_l\,\sigma_{LU}\,, \qquad (9)
\end{aligned}
$$



where λ_l is the lepton helicity and the structure functions $\mathcal{F}(x_B, z_1, \zeta_2, \mathbf{P}_{1\perp}^2, \mathbf{P}_{2\perp}^2, \mathbf{P}_{1\perp}\cdot\mathbf{P}_{2\perp})$ are given, at leading twist, by:

$$\boxed{\mathcal{F}_{UU} = \mathcal{C}\left[\hat{u}_1 D_1\right], \qquad (10)}$$

$$\mathcal{F}_{UU}^{\cos(\phi_1+\phi_2)} = \frac{|\mathbf{P}_{1\perp}||\mathbf{P}_{2\perp}|}{m_1 m_2}\,\mathcal{C}\left[w_1\,\hat{t}_1^h H_1^\perp\right] \qquad (11)$$

$$\mathcal{F}_{UU}^{\cos(2\phi_1)} = \frac{\mathbf{P}_{1\perp}^2}{m_1 m_N}\,\mathcal{C}\left[w_2\,\hat{t}_1^\perp H_1^\perp\right] \qquad (12)$$

$$\mathcal{F}_{UU}^{\cos(2\phi_2)} = \frac{\mathbf{P}_{2\perp}^2}{m_1 m_2}\,\mathcal{C}\left[w_3\,\hat{t}_1^h H_1^\perp\right] + \frac{\mathbf{P}_{2\perp}^2}{m_1 m_N}\,\mathcal{C}\left[w_4\,\hat{t}_1^\perp H_1^\perp\right] \qquad (13)$$

$$\boxed{\mathcal{F}_{LU}^{\sin(\phi_1-\phi_2)} = \frac{|\mathbf{P}_{1\perp}||\mathbf{P}_{2\perp}|}{m_N m_2}\,\mathcal{C}\left[w_5\,\hat{l}_1^{\perp h} D_1\right], \qquad (14)}$$

with the following notation for the transverse momentum convolution

$$
\begin{aligned}
\mathcal{C}\left[f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \ldots)\right] \equiv \sum_a e_a^2\,x_B \int d^2\mathbf{k}_\perp \int d^2\mathbf{k}'_\perp \\
\times\,\delta^2(\mathbf{k}_\perp - \mathbf{k}'_\perp - \mathbf{P}_{1\perp}/z_1)\,f(\mathbf{k}_\perp, \mathbf{k}'_\perp, \ldots). \qquad (15)
\end{aligned}
$$

M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Additional Modulations

$$\mathcal{F}_{LU} = \frac{|p_\pi^\perp||p_P^\perp|}{m_p m_\pi}\mathcal{C}\left[w_5 \hat{l}_1^{\perp h} D_1\right]$$

Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Structure functions carry a dependence on $|p_\pi^\perp||p_P^\perp|$ which introduces a dependence on $\cos\Delta\phi$.

If the correlations are assumed small, the fracture functions can be expanded in powers of $k^\perp \cdot p_P^\perp$.

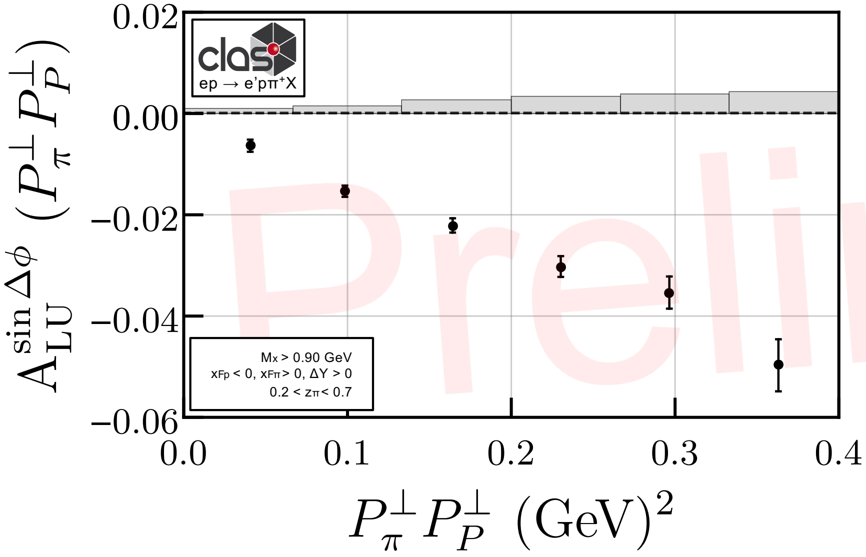
$$\hat{l}_1^{\perp h}(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2, \mathbf{k^\perp} \cdot \mathbf{p_P^\perp}) \\ \approx a(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2) \\ +b(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2)\mathbf{k^\perp} \cdot \mathbf{p_P}^\perp$$

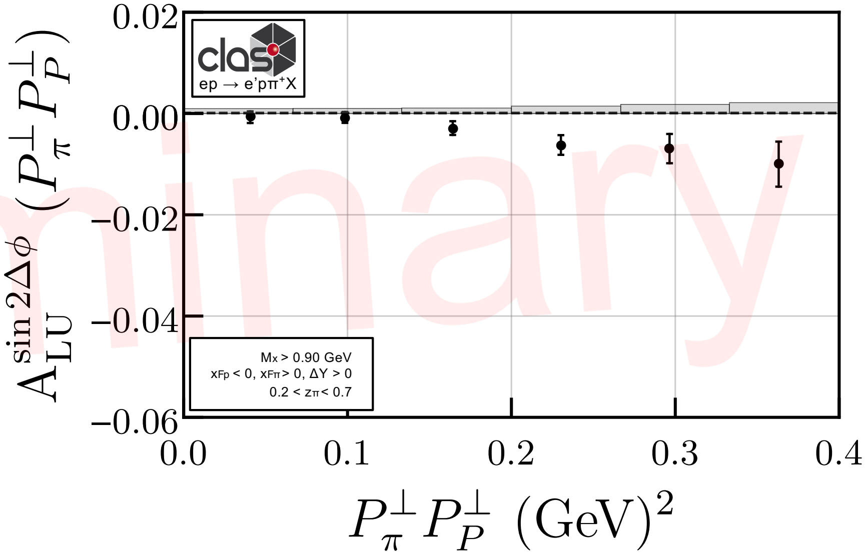
The term linear in $k^\perp \cdot p_P^\perp$ yields a $\cos\Delta\phi$ which when combined with the already existing $\sin\Delta\phi$ term results in a $\sin 2\Delta\phi$.

$$\boxed{\mathcal{A}_{LU}(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2, \Delta\phi) = A(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2)\sin\Delta\phi + B(x, \zeta, \mathbf{k^\perp}^2, \mathbf{p_P^\perp}^2)\sin(2\Delta\phi)}$$

Additional modulation is small

- The sin(2Δφ) is mostly (coincidentally) very small.
- Still important to extract simultaneously.



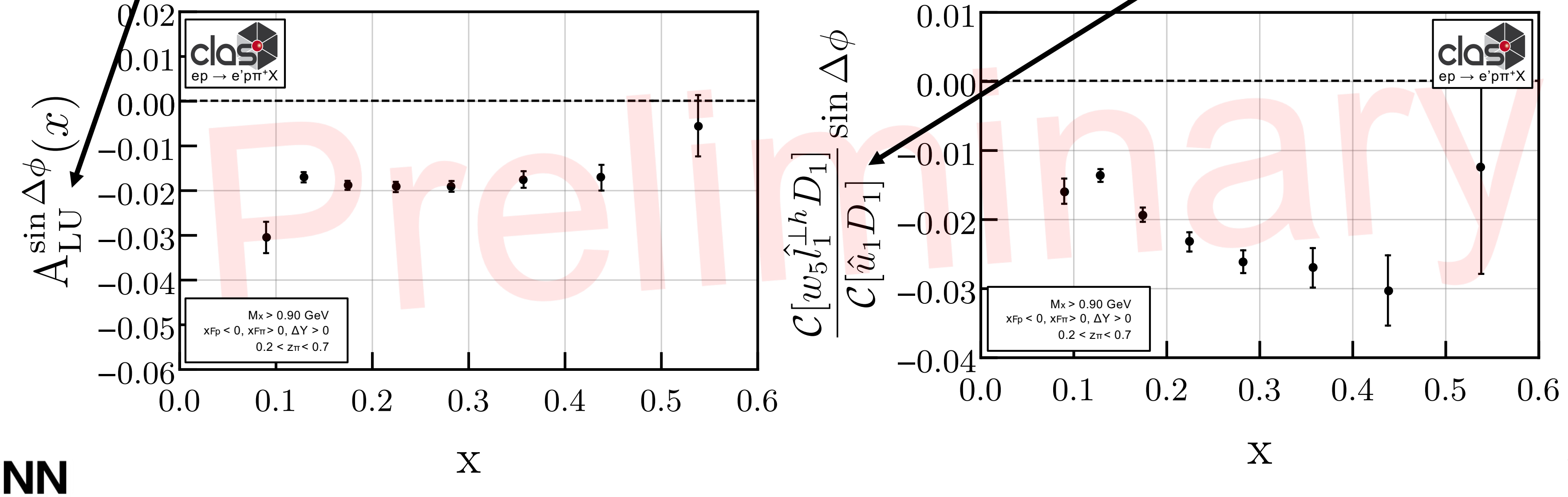


Directly plotting FrFs and FFs

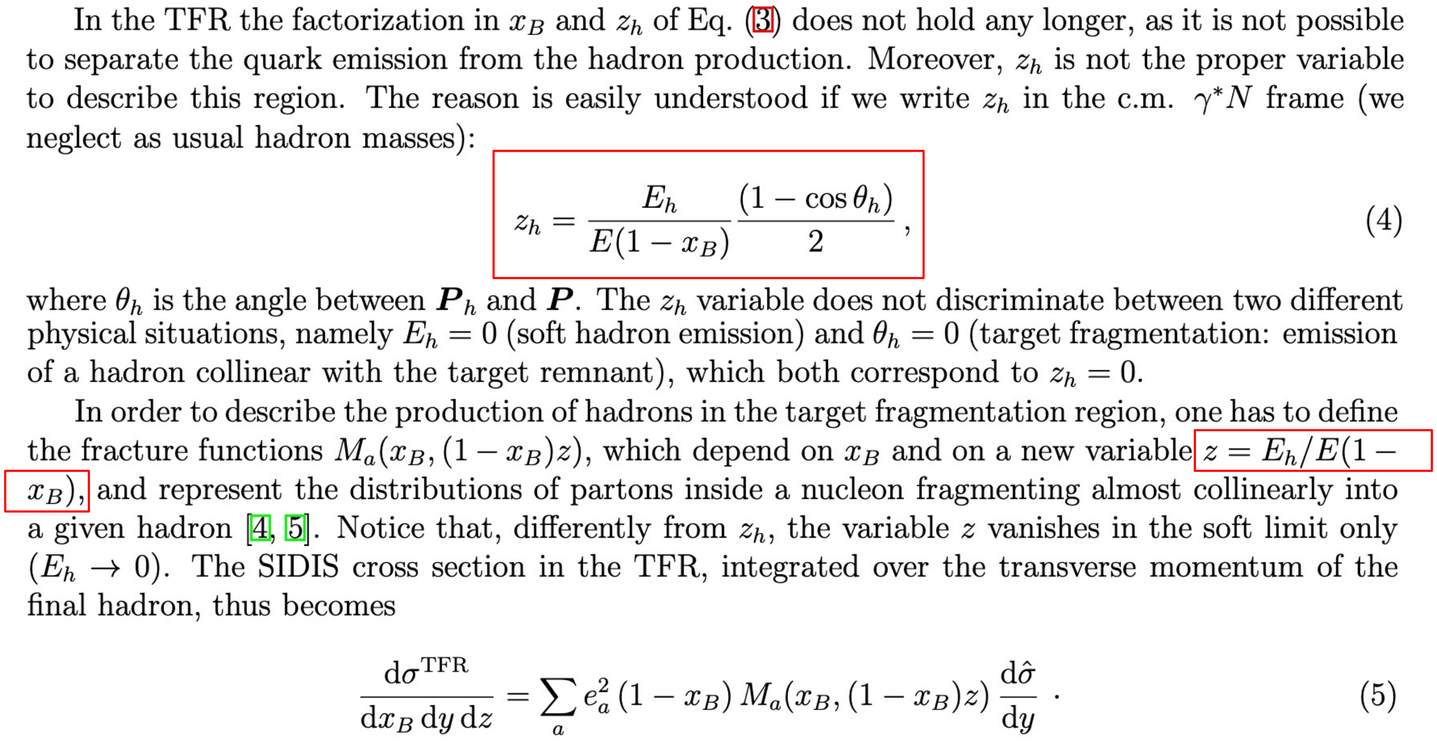
Depolarization Structure Functions

Depolarization Structure Functions

$$\mathcal{A}_{LU} = -\boxed{\frac{1-\frac{y}{2}}{1-y+\frac{y^2}{2}}}\boxed{\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}}\sin\Delta\phi, = -\boxed{\frac{y\left(1-\frac{y}{2}\right)}{\left(1-y+\frac{y^2}{2}\right)}}\boxed{\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N\, m_2}\frac{\mathcal{C}[w_5\,\hat{l}_1^{\perp h}D_1]}{\mathcal{C}[\hat{u}_1 D_1]}}\sin\Delta\phi\,.$$



Subtly different z defintions



In the TFR the factorization in x_B and z_h of Eq. (3) does not hold any longer, as it is not possible to separate the quark emission from the hadron production. Moreover, z_h is not the proper variable to describe this region. The reason is easily understood if we write z_h in the c.m. $\gamma^* N$ frame (we neglect as usual hadron masses):

$$\boxed{z_h = \frac{E_h}{E(1-x_B)} \frac{(1-\cos\theta_h)}{2} \,,} \tag{4}$$

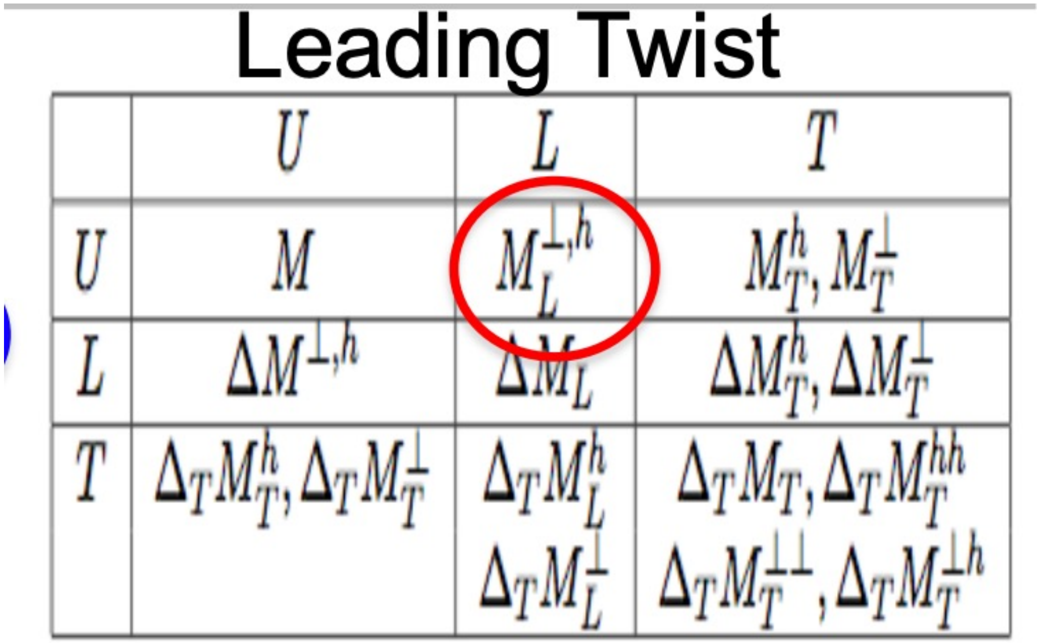
where θ_h is the angle between $\boldsymbol{P}_h$ and $\boldsymbol{P}$. The z_h variable does not discriminate between two different physical situations, namely $E_h = 0$ (soft hadron emission) and $\theta_h = 0$ (target fragmentation: emission of a hadron collinear with the target remnant), which both correspond to $z_h = 0$.

In order to describe the production of hadrons in the target fragmentation region, one has to define the fracture functions $M_a(x_B, (1-x_B)z)$, which depend on x_B and on a new variable $\boxed{z = E_h/E(1-x_B)}$, and represent the distributions of partons inside a nucleon fragmenting almost collinearly into a given hadron [4, 5]. Notice that, differently from z_h, the variable z vanishes in the soft limit only ($E_h \to 0$). The SIDIS cross section in the TFR, integrated over the transverse momentum of the final hadron, thus becomes

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\, dy\, dz} = \sum_a e_a^2\, (1-x_B)\, M_a(x_B, (1-x_B)z)\, \frac{d\hat{\sigma}}{dy} \,. \tag{5}$$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Fracture Functions Renamed



Leading Twist

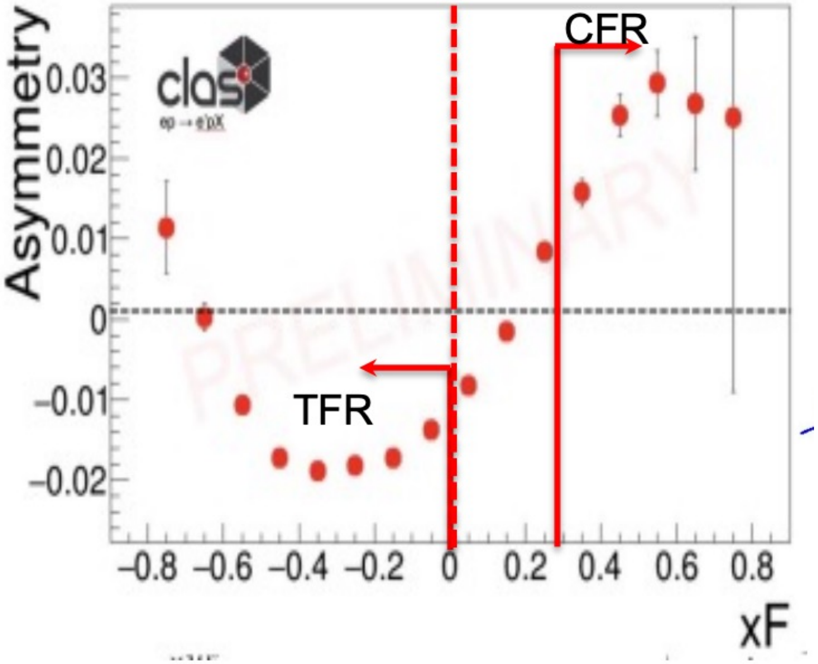
	U	L	T
U	M	$M_L^{\perp,h}$	$M_T^h, M_T^\perp$
L	$\Delta M^{\perp,h}$	ΔM_L	$\Delta M_T^h, \Delta M_T^\perp$
T	$\Delta_T M_T^h, \Delta_T M_T^\perp$	$\Delta_T M_L^h$ $\Delta_T M_L^\perp$	$\Delta_T M_T, \Delta_T M_T^{hh}$ $\Delta_T M_T^{\perp\perp}, \Delta_T M_T^{\perp h}$

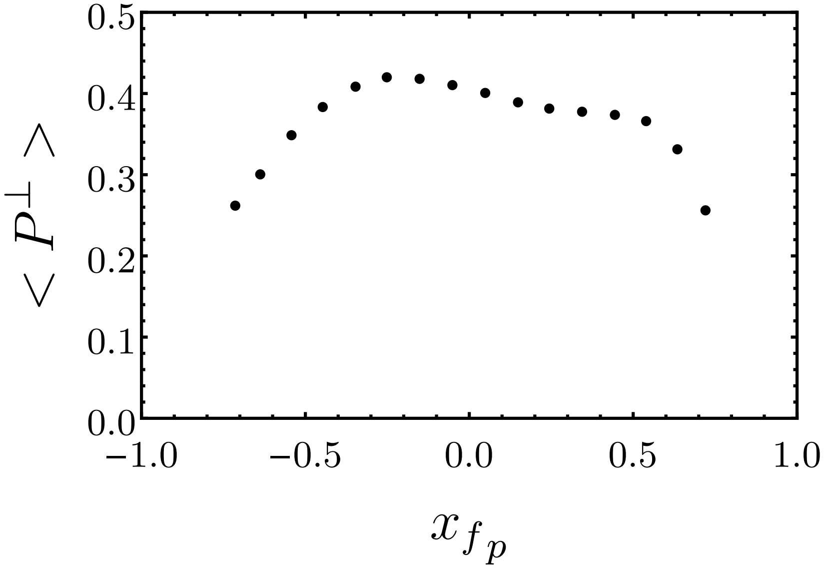
Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

ALU(xFp)





UCONN