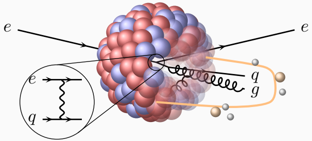
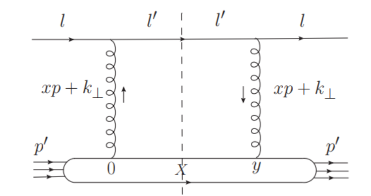
Monte-Carlo Event Generator for *e*-*A* / eHIJING

Correlations in Partonic and Hadronic Interactions, CPHI-2022
March 08, 2022, Duke University

Weiyao Ke (LANL)
Collaborate with Yayun He & Hongxi Xing (SCNU), Yuanyuan Zhang (CUHK-SZ), Xin-Nian Wang (LBNL)

Jet broadening in e-A & TMD gluon distribution



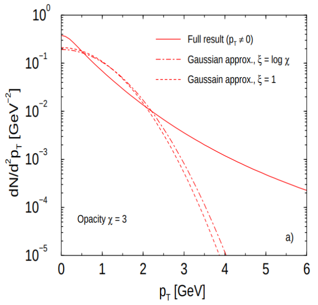


- In large nuclei, multiple interactions are enhanced $\frac{A^{1/3}}{Q^2}$.
- Jet p_T distribution broadens when propagates in target

$$\frac{\partial}{\partial t^+} f(\vec{\mathbf{p}}_\perp) = \rho_0 \int d^2\mathbf{k}_\perp \underbrace{\left[\frac{C_R}{d_A}\frac{\alpha_s \phi_g(x, \mathbf{k}_\perp)}{\mathbf{k}_\perp^2}\right]}_{d\sigma/d\mathbf{k}_\perp^2}{}_+ f(\vec{\mathbf{p}}_\perp - \vec{\mathbf{k}_\perp})$$

- A channel to probe nuclear TMD distribution of gluon

$$\phi_g(x, \mathbf{k}_\perp) = \int \frac{d\xi^+ d\vec{\xi}_T^2}{2\pi P^-} e^{-ixP^-\xi^+ - i\mathbf{k}_\perp \cdot \vec{xi}_T} \langle F^{i-}(0, \vec{0}) F_i^-(\xi^+, \vec{\xi}_T)\rangle.$$



[M Gyulassy, P Levai, I Vitev 2002]

Medium-modified fragmentation function in cold nuclear matter

- Momentum broadening correlates with modified fragmentation function.
- One can simultaneously fit of both nuclear TMD PDF and FF [M Alrashed, D Anderle, ZB Kang, J Terry, HX Xing 2107.12401] $R_A = D_A(z, p_T)/D(z, p_T)$.
- It is also possible to build models to study the nuclear size dependence of $D(z, p_T)$ and $\phi_g(x, \mathrm{k}_\perp)$.

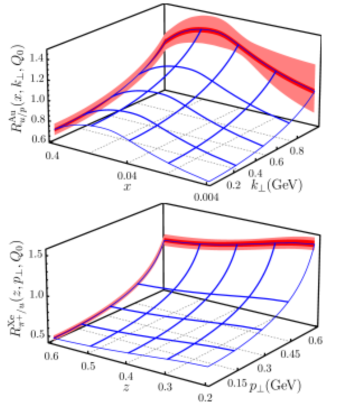


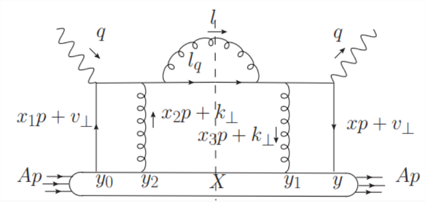
FIG. 3. The extracted nuclear ratio for the TMDPDF (top) and the TMDFF (bottom).

eHIJING (electron-Heavy-Ion-Jet-INteraction-Generator)

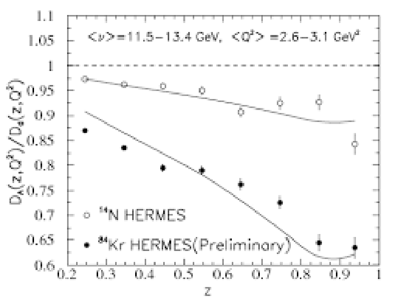
1. Nuclear gluon distribution:

Typical gluon $x = \frac{k_\perp^2}{Q^2} x_B \ll 1$ motivates saturation-based models of $\phi_g(x, k_\perp^2)$.

$$\phi_g \propto \begin{cases} \frac{1}{\alpha_s Q_s^2}, k_\perp^2 \ll Q_s^2 \\ \frac{1}{\alpha_s k_\perp}, k_\perp^2 \gg Q_s^2 \end{cases}$$



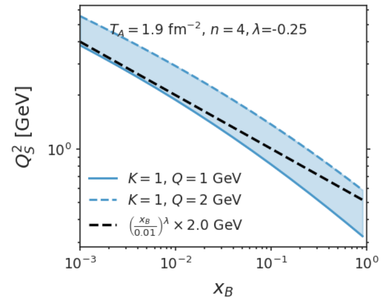
2. In-medium fragmentation from (generalzied) higher-twist in-medium QCD splitting function:
$D(z) = [1 + \alpha_s(P + \Delta P)_{ij} \otimes + \cdots]D(z)$



[E. Wang, X-N Wang, PRL 89, 162301]

A simple model of saturation & multiple collisions

$$\phi_g(x, k_\perp^2; Q_s^2) = \frac{N}{\alpha_s}(1-x)^n x^\lambda \frac{1}{\mathbf{k}_\perp^2 + Q_s^s}$$



- ϕ_g saturates for $\mathbf{k}_\perp \lesssim Q_s$.
- Q_s is determined self-consistently [Y-Y Zhang and X-N Wang, 2104.04520, A. Mueller NPB 558 (1999) 285-303]

$$Q_s^2(x_B, Q^2; T_A) \quad = \quad T_A \frac{C_A}{d_A} \int^{\frac{Q^2}{x_B}} d^2\mathbf{k}_\perp \alpha_s \phi_g(x_B \frac{\mathbf{k}_\perp^2}{Q^2}, \mathbf{k}_\perp^2; Q_s^2)$$

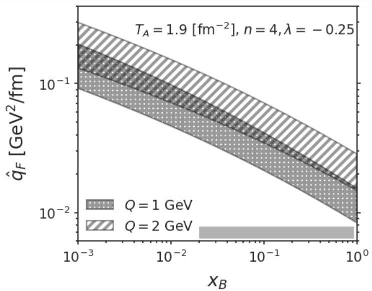
- $\Delta\langle p_T^2\rangle = C_F/C_A Q_s^2$ as probed by a quark.
- In eHIJING: stochastically sample $\frac{dN}{dt^+ d^2\mathbf{k}_\perp} = \rho_0 \frac{C_R}{d_A} \frac{\alpha_s \phi_g}{\mathbf{k}_\perp^2}$ $\rightarrow$ for each parton $(t_1, k_{\perp,1}), (t_2, k_{\perp,2}), \cdots, (t_n, k_{\perp,n})$.

Jet transport parameter $\hat{q}$: a "local" quantity of the medium

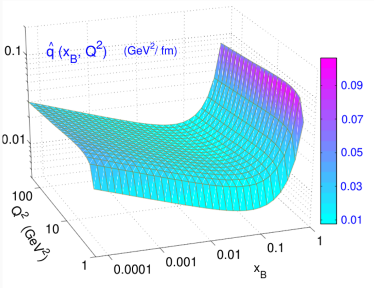
- (Quark) jet transport parameter of collisional broadening

$$\hat{q}_F(x, Q^2, T_A) \equiv \frac{d}{dt^+}\Delta\langle p_\perp^2\rangle = \frac{C_F}{C_A}\frac{Q_s^2(x, Q^2, T_A)}{L}$$

- Compare to values from global analysis $x \sim 0.01 - 0.1$

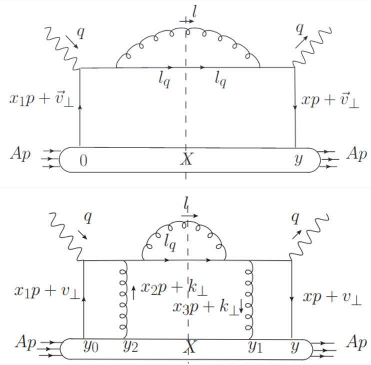


[$\hat{q}$ of the simple model.]



[P Ru et al PRD 103, 031901]

Modified QCD splitting functions at twist-4 (double scattering)

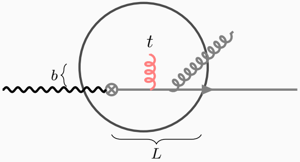


. . .

In-medium splitting function

$$\frac{\alpha_s}{\mathbf{l}_\perp^2}P(z) \to \frac{\alpha_s}{\mathbf{l}_\perp^2}P(z)[1 + \Delta_1(z, \mathbf{l}_\perp; x_g, \mathbf{k}_\perp; x_q, \mathbf{v}_\perp; t^+)]$$

depends on kinematics of quark, gluons & phases $\Delta p^- t^+$ (Landau-Pomeranchuk-Migdal interference).



TMD gluon: t^+, x_g, $\mathbf{k}_\perp$

Emitted gluon: z, $\mathbf{l}_\perp$

Formation time: $\tau_f = \frac{2(1-z)zE}{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2}$

Path length L, $0 < t^+ < L$

$$D(z, Q) = D_0(z) + \int^{Q^2} \frac{\alpha_s}{2\pi}\frac{d\mathbf{l}_\perp^2}{\mathbf{l}_\perp^2}\int_x^1 \frac{dz}{z} D_0(\frac{x}{z})[P(z)(1 + \Delta_1(z, \mathbf{l}_\perp)]_+ + \cdots .$$

In-medium QCD splittings: model choice I) generalized formula

- Full results at twist-4, see [Y-Y Zhang & X-N Wang, 2104.04520] for derivation for very general cases: no hierarchy between $\mathbf{k}_\perp, \mathbf{l}_\perp$ and z, x_g.
- In the current stage of eHIJING, we take the formula in the soft gluon limit: $z \ll 1$.

$$\Delta_1 \approx \int_0^L dt^+ \rho_0(t) \int d^2\mathbf{k}_\perp \frac{C_F}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2} \frac{C_A}{C_F} \frac{2\mathbf{k}_\perp \cdot \mathbf{l}_\perp}{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2} \left(1 - \cos\frac{t}{\tau_f}\right)$$

$$\to \sum_i \frac{C_A}{C_F} \frac{2(\mathbf{k}_\perp)_i \cdot \mathbf{l}_\perp}{[\mathbf{l}_\perp - (\mathbf{k}_\perp)_i]^2} \left(1 - \cos\frac{t_i}{(\tau_f)_i}\right)$$

- In eHIJING: collision integral replaced by a sum over the stochastic samples $(t_i, k_{\perp,i}), \cdots$. Easier to generalize to the full formula in the future.

In-medium QCD splittings: model choice II) collinear limit

$$\Delta_1^{\mathrm{Gen}} = \int_0^L dt \int \frac{d^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2} \alpha_s \frac{C_A \rho_0 \phi_g(x_g, \mathbf{k}_\perp^2)}{d_A} \frac{2\mathbf{k}_\perp \cdot \mathbf{l}_\perp}{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2} \left(1 - \cos \frac{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2 t}{2(1-z)zE}\right)$$

Expand the generalized formula in powers of $|\mathbf{k}_\perp|/|\mathbf{l}_\perp|$

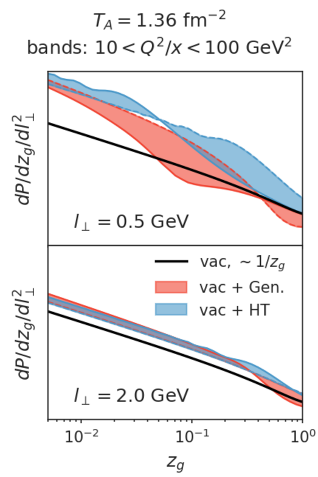
[X-N Wang, X Guo, A. Majumder, etc].

$$\Delta_1^{\mathrm{Gen}} \to \Delta_1^{\mathrm{HT}} = \int_0^L dt \frac{2\hat{q}_A^{\mathrm{rad}}}{\mathbf{l}_\perp^2} \left[1 - \cos \frac{\mathbf{k}_\perp^2 t}{2(1-z)zE}\right]$$

$$\hat{q}_A^{\mathrm{rad}} = \int_0^{\mathbf{l}_\perp^2} d\mathbf{k}_\perp^2 \alpha_s \frac{\pi C_A \rho_0 \phi_g(x_g, \mathbf{k}_\perp^2)}{d_A}$$

▷ Generalized v.s. collinearly-expanded formula.

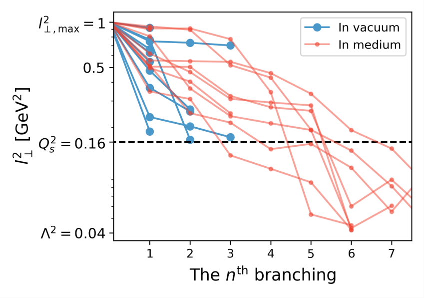
Both are implemented in eHIJING

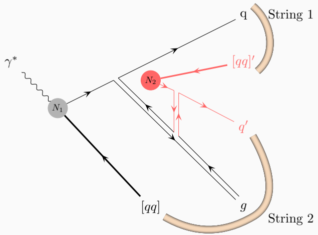


Multiple emissions & hadronization in the medium

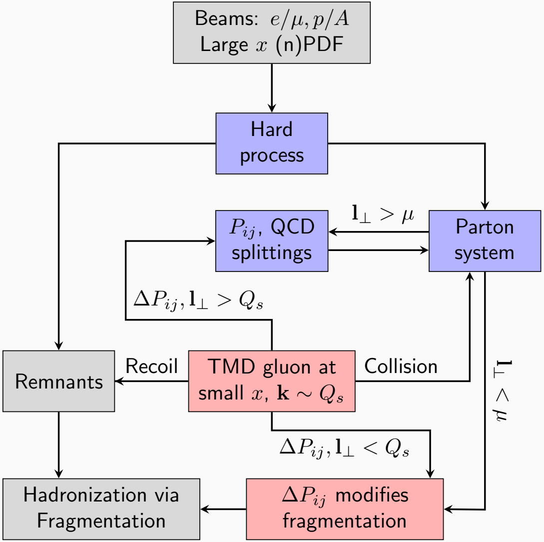
Δ_1 is split into two parts $\Delta_1(l_\perp)\Theta(l_\perp^2 - Q_s^2) + \Delta_1(l_\perp)\Theta(Q_s^2 - l_\perp^2)$:

- $\Delta_1(x, l_\perp)\Theta(l_\perp^2 - Q_s^2)$: modifies the vacuum-like p_T-ordered parton shower.
- $\Delta_1(x, l_\perp)\Theta(Q_s^2 - l_\perp^2)$: samples gluons right before hadronization, understood as modified DGLAP initial condition in A. **τ_f-ordered emission.**



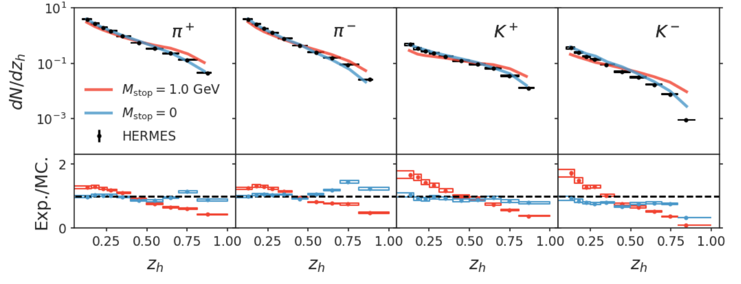


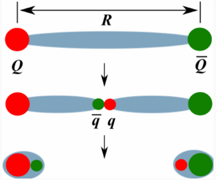
String fragmentation.



- Use Pythia8 for e-p, e-d.
- Red: medium corrections, inputs A, parameters of ϕ_g.

Fragmentation function in *e*-*d*: Pythia8 baseline

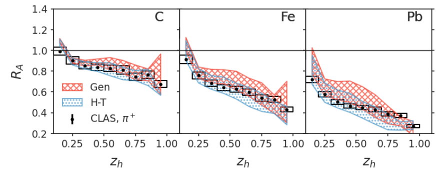




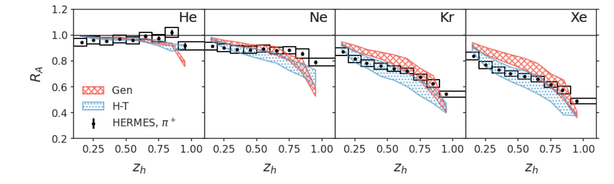
[HERMES, Phys Rev D 87, 074029 (2013)]

- Change a default Pythia8 fragmentation parameter M_{stop} from 1 GeV to 0 to fit π and K spectra in *e*-*d* collisions.
- M_{stop} controls the minimum mass of string to break $W > m_q + m_{\bar{q}'} + M_{\text{stop}}$.

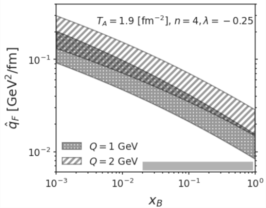
Nuclear modification factor of $D(z_h)$



[CLAS arXiv:2109.09951]



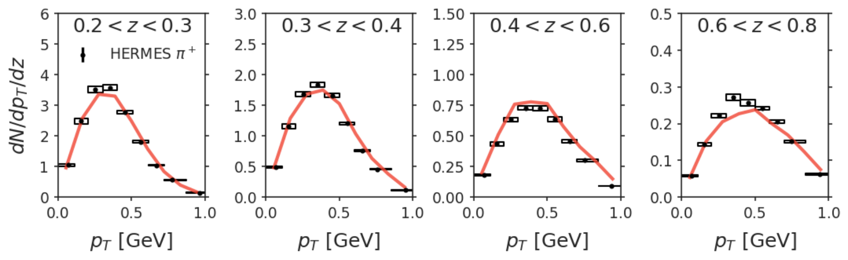
[HERMES, NPB 780, 24 (2007)]



- Nuclear modification
  $R_A = \left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{eA} / \left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{ed}.$
- HT (red) & generalized HT (blue). Bands: $\hat{q}_F$ variation ▷.
- Consistent with the A dependence of data.
- Nuclear PDF EPPS16 [EPJC 77, 163 (2017)] used for hard process.

Transverse momentum dependence $d^2N_h/dz_h/dp_T$: e-d

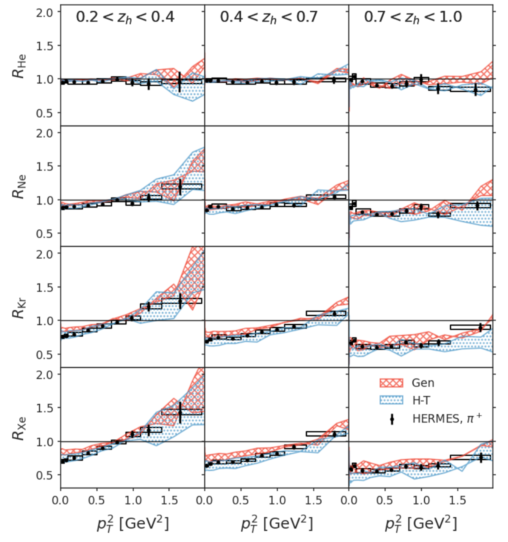
$d^2N_h/dz_h/dp_T$



[HERMES, Phys Rev D 87, 074029 (2013)]

- Reasonable agreement.
- Pythia primordial quark k_T, $k_T \sim e^{-k_T^2/2\sigma_1^2}$ with $\sigma_1 \propto (1 + Q_{1/2}/Q)^{-1}$ [T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053].
- k_T from Lund string fragmentation, $k_T \sim e^{-k_T^2/2\sigma_2^2}$ with $\sigma_2 = 0.335$ GeV as default.

p_T-dependent nuclear modification of $D(z_h, p_T)$

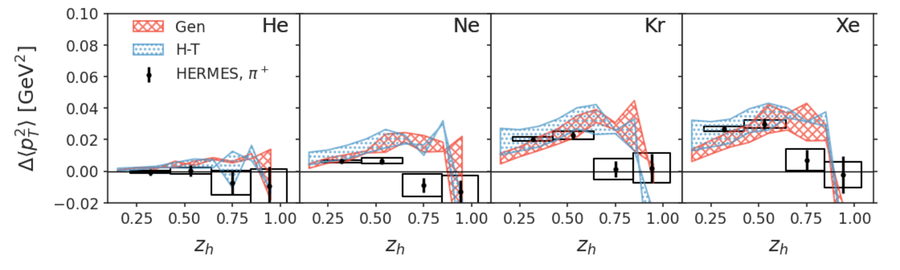


[HERMES, Nuclear Physics B 780, 24 (2007)]

$$R_A = \frac{\left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{eA}}{\left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{ed}}$$

- Interplay of parton energy loss and momentum broadening.
- Energy loss due to medium-induced emission: a suppression at large z_h.
- Broadening (change of shape) is weaker in the large z region.

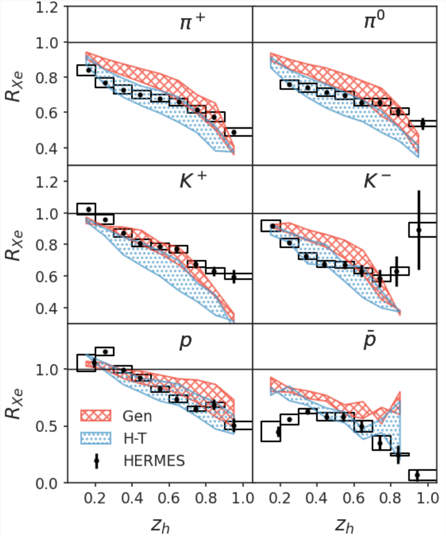
Momentum broadening $\Delta\langle p_T^2\rangle = \langle p_T^2\rangle_{eA} - \langle p_T^2\rangle_{ed}$



[HERMES, PLB 684 (2010) 114-118]

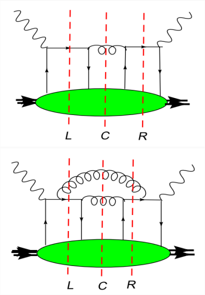
- Qualitatively similar z-dependence from simulation.
- Data drop more abruptly for $z_h > 0.7$.

Hadron specie dependence: $\pi^{\pm}$, π^0, $K^{\pm}$, p, $\bar{p}$

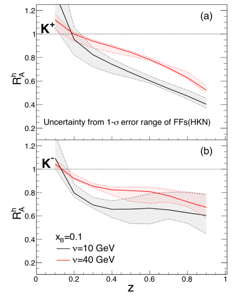


[HERMES, Nuclear Physics B 780, 24 (2007)]

- Notable difference between $R_A(K^+)$ vs $R_A(K^-)$, and $R_A(p)$ vs $R_A(\bar{p})$.
- Importance of medium-induced conversion of $g \to q$ and hadronic transport for future.



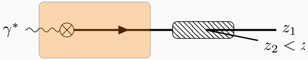
[BW Zhang, XN Wang, A Schaefer]



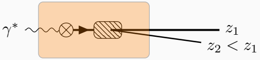
[NB Chang, WT Deng, XN Wang PRC 92 055207]

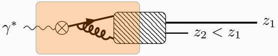
$$D_{2h} = \frac{d^2 N_h}{dz_1 dz_2}, z_1 > z_2; \qquad R_{2h} = \frac{D_{2h}^{eA}/D^{eA}}{D_{2h}^{ed}/D^{ed}}$$

Different space-time picture of hadronization:



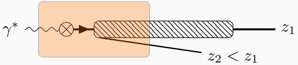
V.S.





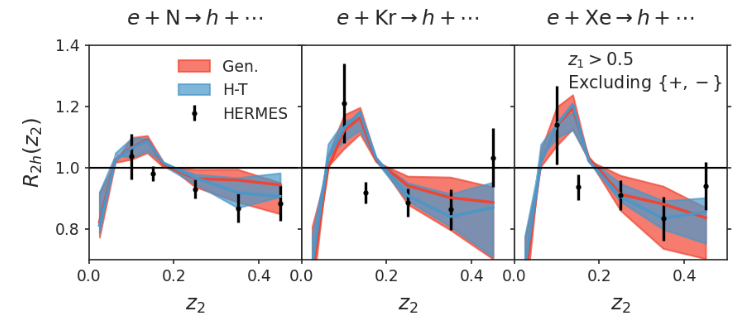
(Modified) radiation

[A. Majumder & X-N Wang]



Hadron formation time

Medium modified double hadron fragmentation



[HERMES, PRL 96 (2006) 162301]

- Reasonable agreement with double-hadron modifications.
- There is still room for hadronic transport at small z_2. ($z_2 = 0.1$, $E \sim 1$ GeV in target frame).

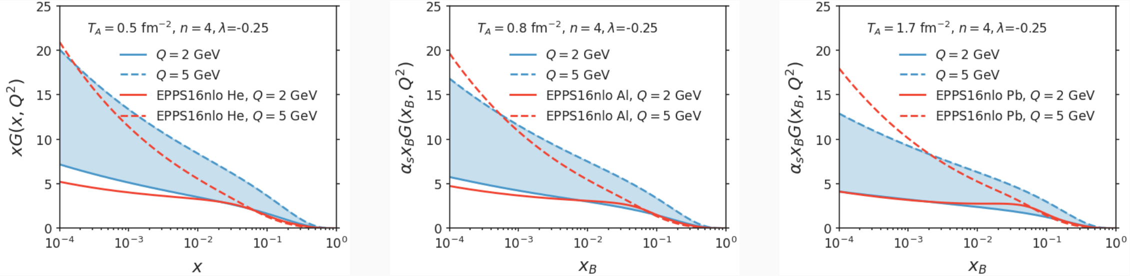
Summary and outlook

- eHIJING models jet broadening and fragmentation in *e*-*A* with
  - a simple model for nuclear gluon TMD motivated by saturation physics;
  - twist-four medium-modification to splitting in the soft limit.
- Systematic comparison to modified fragmentation at CLAS and HERMES.
  Good agreement with data using reasonable range of jet transport parameter $\hat{q}$.
- Open to more sophisticated gluon TMD model in the future.
- Move toward modified splitting function with full-x dependence, medium-induced flavor-conversion, hadronic interactions.

Questions?

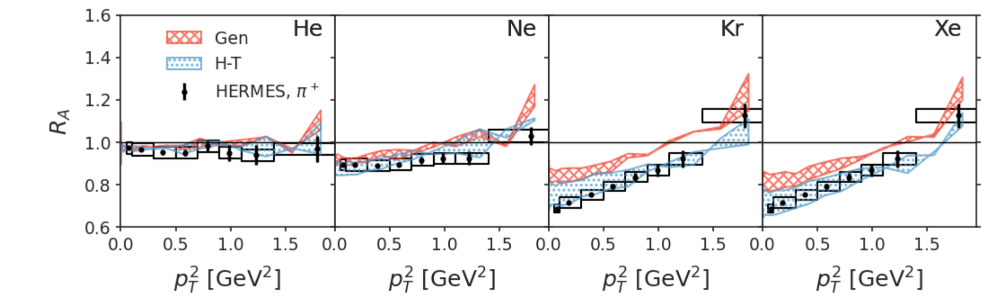
Backup slides: compared to phenomenological nuclear PDF

$$xG(x, Q^2) = \int_0^{Q^2} \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \phi_g(x, \mathbf{k}_\perp^2)$$



- Choice of parameters result in similar $xG(x, Q)$ at low Q^2.
- But lack proper evolution compared to realistic PDF.

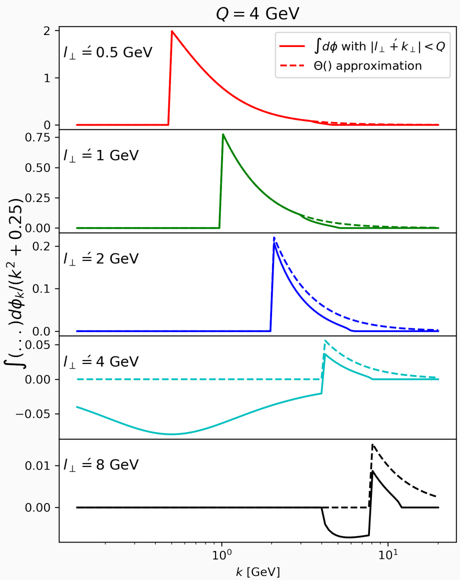
p_T-dependent nuclear modification factor



[HERMES, Nuclear Physics B 780, 24 (2007)]

- Nuclear modification $R_A = \left(N_h(\nu, Q^2; z_h, p_T)/N_\gamma\right)_{eA} / \left(N_h(\nu, Q^2; z_h, p_T)/N_\gamma\right)_{ed}$.
- HT (red) & generalized HT (blue). Bands: $\langle \hat{q}_q \rangle_{\text{eff}}$ varies from 0.01 to 0.04 GeV^2/fm.
- Consistent with the A-dependence of data from $A = 4$ to $A = 139$.

The Θ-function approximation of Δ_1^{Gen}



- Imposing the requirement that $|\mathbf{l}'_\perp + \mathbf{k}_\perp| = |\mathbf{l}_\perp| < Q$, the Θ-function approximation only works for small $\mathbf{l}'_\perp$.
- For large $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$, we had to tabulate the emission spectra for the generalized formula.