Correlations in Partonic and Hadronic Interactions (CPHI-2022)

Duke University, March 7-12, 2022

Energy momentum tensor form factors, and long-range forces

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Outline

- Introduction: GPDs, EMT form factors, D-term
- Review: EMT properties from short-range (strong) forces
- New results: impact of long-range forces on EMT properties
- Open questions and conclusions

based on: M. Varma and P. Schweitzer, Phys. Rev. D **102** (2020) 014047

with Mira Varma (UConn undergrad, now Yale grad school)

supported by NSF (1812423, 2111490)

dedicated to

Maxim Polyakov

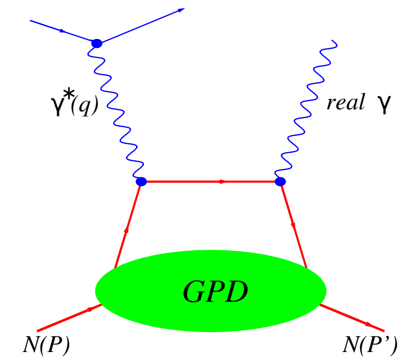
1966 – 2021

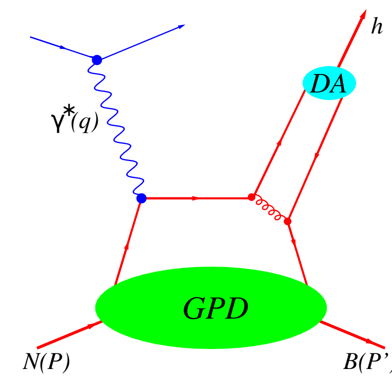
Introduction

- **GPDs and hard-exclusive reactions**

Müller et al Fortsch. Phys. **42**, 101 (1994)
Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997)
Radyushkin, PLB **380**, 417, PLB **385**, 333 (1996)
Collins, Frankfurt, Strikman, PRD **56**, 2982 (1997)

$$\int \frac{\mathrm{d}\lambda}{2\pi}\, e^{i\lambda x}\, \langle N'(p')|\overline{\psi}_q(-\tfrac{\lambda n}{2})\, n_\mu\gamma^\mu\, \mathcal{W}(-\tfrac{\lambda n}{2},\tfrac{\lambda n}{2})\, \psi_q(\tfrac{\lambda n}{2})\,|N(p)\rangle$$
$$= \overline{u}(p')\left[n_\mu\gamma^\mu\, H^q(x,\xi,t) + \overline{u}(p')\,\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}\, E^q(x,\xi,t)\right] u(p)$$





- **GPDs introduced to access EMT form factors**

polynomiality (Ji 1997)

$$\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$$
$$\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$$

- **applications**

mass decomposition Ji, PRL 74, 1071 (1995), ...
spin decomposition Ji, PRL **78**, 610 (1997), ...
mechanical forces Polyakov, PLB **555**, 57 (2003), ...
D-term Polyakov, Weiss, PRD60, 114017 (1999)

- **Definition EMT form factors of nucleon**

$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[\, A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M} + J^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu}\Bigg]u(p)$$

- conserved external current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu} \quad (a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, $B(t)$, $D(t)$ scale invariant, $\sum_a \bar{c}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $J(0) = \frac{1}{2}$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon spin *

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\rightarrow$ unconstrained! **Last global unknown!** **

$$2P = (p' + p) \qquad \Delta = (p' - p) \qquad t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

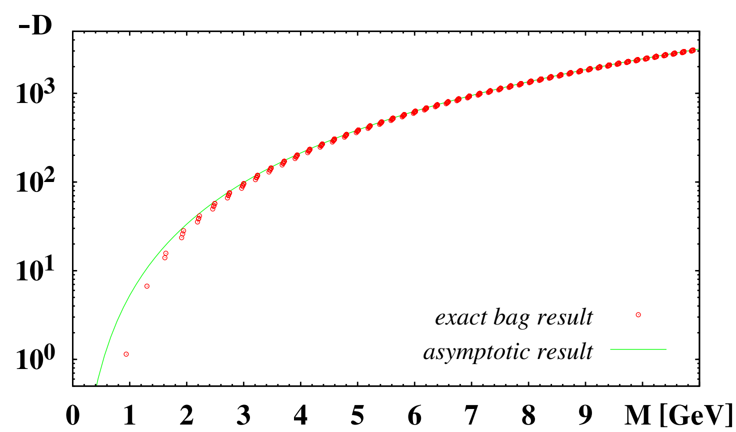
$A^q(t) = M_2^q(t)$

* equivalent to: nucleons total anomalous gravitomagnetic moment vanishes (Gordon identity)

** meanwhile something is known: Kumano et al (2018), Burkert et al (2018)

What we know about D-term from theory (selected results)

- **free spin-0 particle** $D = -1$ Pagels 1966; Hudson, PS 2017
- **free spin $\frac{1}{2}$ particle** $D = 0$ Donoghue et al (2002), Hudson, PS (2018)
- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- **nuclei** (liquid drop model, Walecka model) $D \propto A^{7/3}$ $\rightarrow$ DVCS with nuclei!
  Polyakov (2002), Guzey, Siddikov (2006); Liuti, Taneja (2005)
- **Q-balls** N^{th} excited Q-ball state: $M \propto N^3$ but $D \propto N^8$ Mai, PS (2012)
- **nucleon, bag model** $D = -1.15 < 0$ Ji, Melnitchouk, Song (1997)
- **chiral quark soliton** Goeke et al, PRD75 (2007) (see next slides)
- **χPT** Belitsky, Ji (2002), Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023
- **lattice QCD** Göckeler et al, PRL92 (2004), ...
  Shanahan, Detmold (2019)
- **dispersion relations**
  Pasquini, Polyakov, Vanderhaeghen (2014)
- **excited states**
  in bag model Neubelt et al (2019)
- see review Polyakov and PS, (2018)



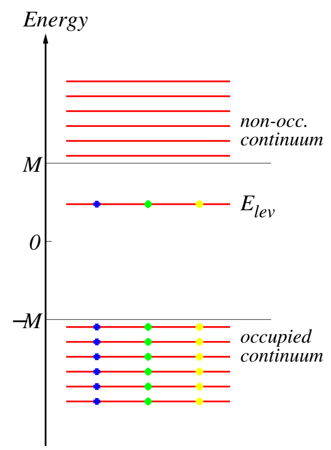
of all properties, D-term most sensitive
$\Rightarrow$ **dynamics!**

D-term in strongly interacting theories with short-range forces

- chiral quark-soliton model (χQSM) $\mathcal{L}_{\rm eff} = \overline{\Psi}\,(i\,\partial\!\!\!/ - M\,U^{\gamma_5})\Psi,\;\; U = \exp(i\tau^a\pi^a/f_\pi)$
  Diakonov, Petrov, Pobylitsa, NPB 306, 809 (1988)

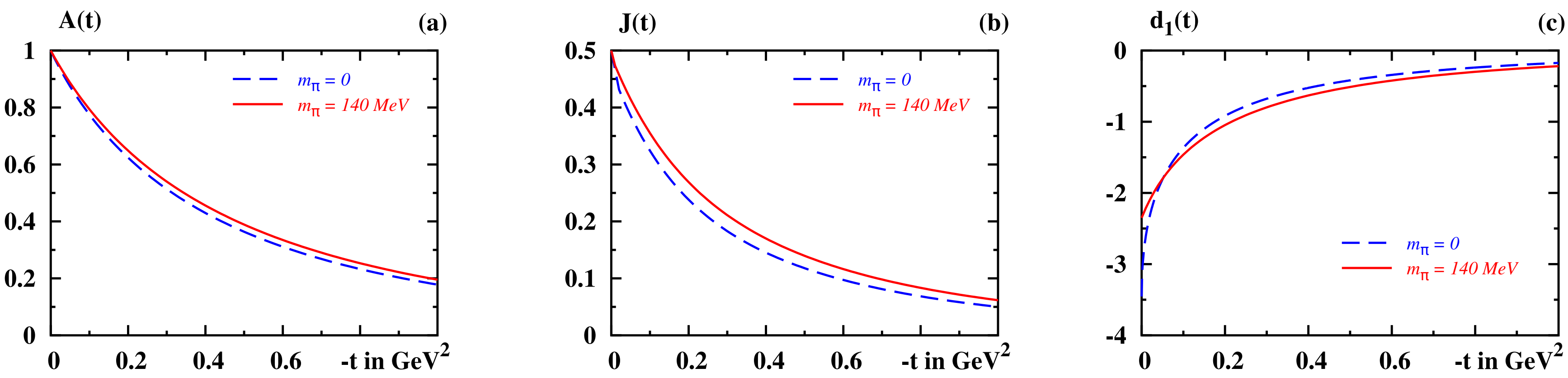
  solve in large-N_c limit, where $U(x) \to U(\vec{x})$ static mean field
  Witten NPB 223 (1983) 433

  Hamiltonian $H = -i\gamma^0\gamma^i\nabla^i + \gamma^0 M\,U^{\gamma_5}$ with $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$
  spectrum discrete level and continua



- EMT form factors
  Goeke et al, PRD75 (2007) 094021

$$
\begin{aligned}
\langle N'|\hat{T}_{\mu\nu}|N\rangle &= \bar{u}(\vec{p}\,')\left[\frac{P_\mu P_\nu}{M_N}\,A(t) + \frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N}\,J(t) + \frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N}\,D(t)\right]u(\vec{p}) \\
&= \lim_{T\to\infty}\frac{\displaystyle\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;\hat{T}_{\mu\nu}\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}}{\displaystyle\int \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}\,\mathcal{D}U\;J_N(\tfrac{T}{2})\;J_N^\dagger(-\tfrac{T}{2})e^{-\int d^4x_E\,\mathcal{L}_{\rm eff}}} \\
&= 2M_N\int d^3x\;e^{i(\vec{p}\,'-\vec{p})\vec{x}}\,N_c\sum_n \overline{\Phi}_n(\vec{x})(i\gamma^\mu\partial^\nu + i\gamma^\nu\partial^\mu)\Phi_n(\vec{x}) + \mathrm{NLO} + \dots \\
&= 2M_N\int d^3r\;e^{i\vec{\Delta}\,\vec{r}}\;T_{\mu\nu}(\vec{r}) + \dots
\end{aligned}
$$



- form factors are the physical quantities
- in large N_c limit, naturally expressed in terms of Fourier transforms of 3D densities

$$\langle N'|\hat{T}_{\mu\nu}|N\rangle \;=\; 2M_N\int d^3r\; e^{i\vec{\Delta}\,\vec{r}}\,T_{\mu\nu}(\vec{r}) + \mathcal{O}(1/N_c)$$

  - this brings us to the interpretation

interpretation as 3D-distributions

M.V.Polyakov, PLB 555 (2003) 57

- **static EMT** in Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: $\quad \boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$

$$\int d^3r\, T_{00}(\vec{r}) = M \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r}, \vec{s}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5}\, M \int d^3r \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(\vec{r}) \equiv \boldsymbol{D} \qquad \textbf{new!}$$

with: $\quad \boldsymbol{T_{ij}(\vec{r})} = \boldsymbol{s(r)}\left(\frac{\boldsymbol{r_i r_j}}{\boldsymbol{r^2}} - \frac{\boldsymbol{1}}{\boldsymbol{3}}\,\boldsymbol{\delta_{ij}}\right) + \boldsymbol{p(r)}\,\boldsymbol{\delta_{ij}}$ **stress tensor**

$s(r)$ related to distribution of *shear forces*
$p(r)$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **"mechanical properties"**

- **relation to stability**: EMT conservation $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

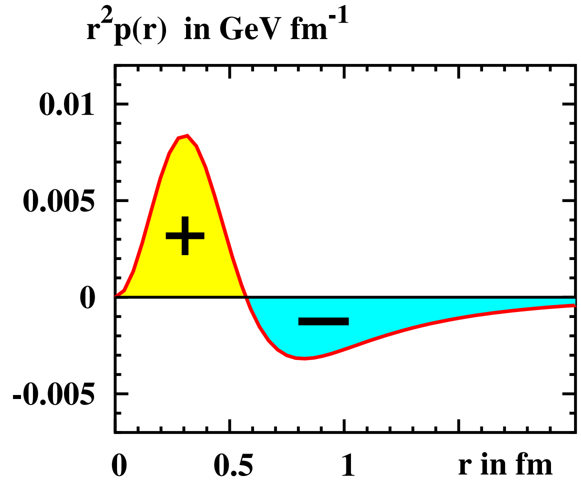
$\hookrightarrow$ necessary condition for stability $\int_0^\infty dr\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$\boldsymbol{D} = -\frac{16\pi}{15}\, M \int_0^\infty dr\, r^4 s(r) = 4\pi M \int_0^\infty dr\, \boldsymbol{r^4\, p(r)} \rightarrow \text{balance of internal forces}$$

- 2D interpretation Lorcé et al (2019); Freese, Miller (2021), Panteleeva, Polyakov (2021)

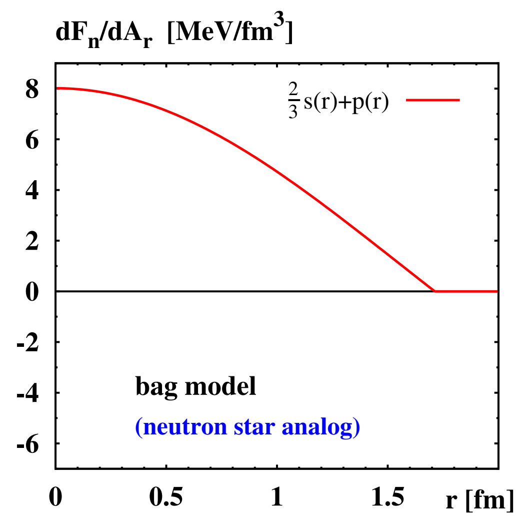
results from selected models

chiral quark soliton model
Goeke et al, PRD75 (2007) 094021



$p(r) > 0$ for $r > r_0$,
$p(r) < 0$ for $r < r_0$,
single node at $r_0 = 0.57\,\text{fm}$

bag model
Neubelt et al PRD101 (2020) 034013



normal force directed towards outside

$$dF_n^i = T^{ij} e_r^j \, dA_r = \underbrace{[\tfrac{2}{3}\, s(r) + p(r)]}_{>0} e_r^i \, dA_r$$

same picture in all models $p(r)$ has a single node and normal force > 0

Common in all models used so far: governed by short-range forces
What about long-range (Coulomb) force?

Strong forces $\sim \mathcal{O}(10)$ vs em effects $\sim \mathcal{O}(\alpha)$ with $\alpha = \frac{1}{137}$, i.e. negligible?

D-term in the presence of long-range forces

Simple relativistic classical model of a finite size particle Białynicki-Birula, Phys. Lett. A 182 (1993) 346

non-interacting "dust particles" within R described by phase-space distribution $\Gamma(\vec{r},\vec{p},t)$ feel 3 forces:

- massive scalar field force (attractive, mass m_S, short range $\sim \frac{1}{r}\,e^{-m_s r}$)
- massive vector field force (repulsive, mass $m_V > m_S$, even shorter range $\sim \frac{1}{r}\,e^{-m_V r}$)
- massless vector field force (repulsive, Coulomb force, infinite range $\sim \frac{1}{r}$)

$$\begin{aligned}
\Big[(m-g_S\phi)(\partial_t+\vec{v}\cdot\vec{\nabla}_r)+m\,\vec{F}\cdot\vec{\nabla}_p\Big]\,\Gamma(\vec{r},\vec{p},t) &= 0\,,\\
\partial_\alpha G^{\alpha\beta}+m_V^2V^\beta &= g_V\,j^\beta,\\
(\Box+m_S^2)\phi &= g_S\,\rho\,,\\
\partial_\alpha F^{\alpha\beta} &= e\,j^\beta,
\end{aligned}$$

with $j^\alpha(\vec{r},t)=\displaystyle\int\frac{d^3p}{E_p}\,p^\alpha\,\Gamma(\vec{r},\vec{p},t),\quad \rho(\vec{r},t)=\int\frac{d^3p}{E_p}\,m\,\Gamma(\vec{r},\vec{p},t).$ relativistically invariant.

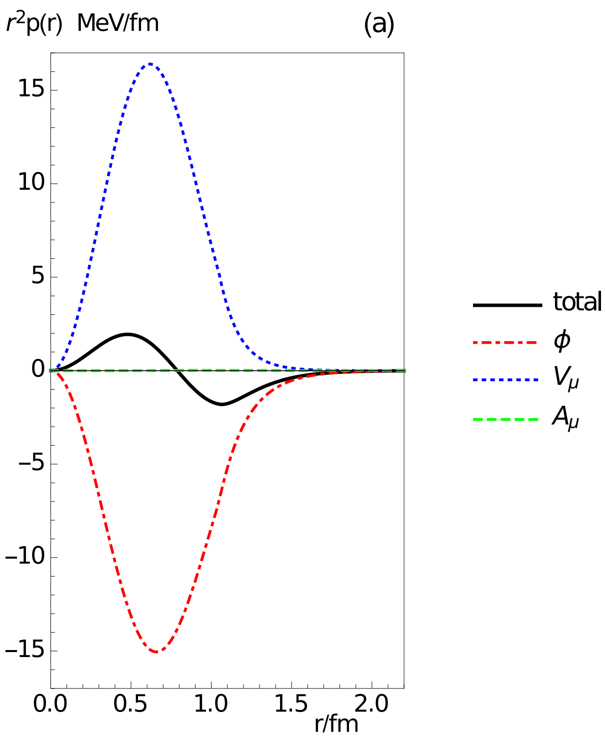
parameters from model QHD-I of the mean field theory of nuclear matter Serot, Walecka (1986)

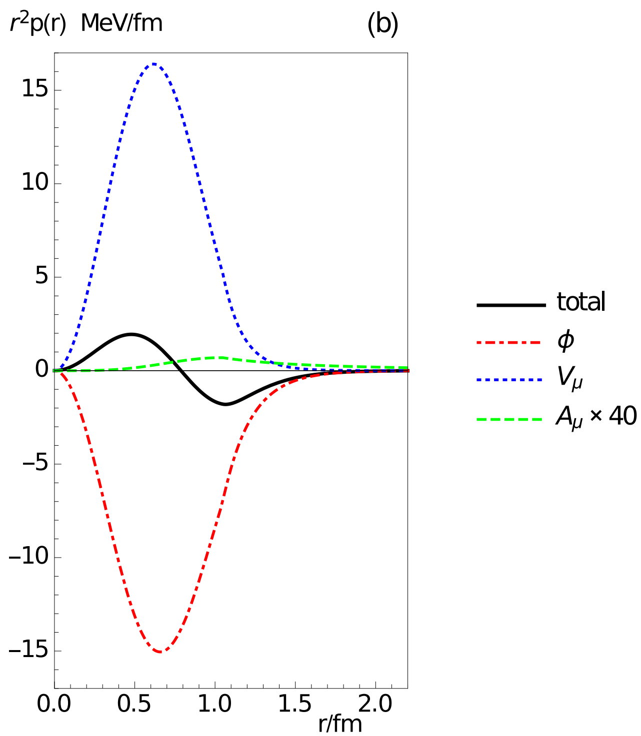
$$m_S=550\,\text{MeV},\quad m_V=783\,\text{MeV},\quad \frac{g_S^2}{4\pi\hbar c}=7.29\,,\quad \frac{g_V^2}{4\pi\hbar c}=10.84\,,\quad \alpha=\frac{e^2}{4\pi\hbar c}=\frac{1}{137},$$

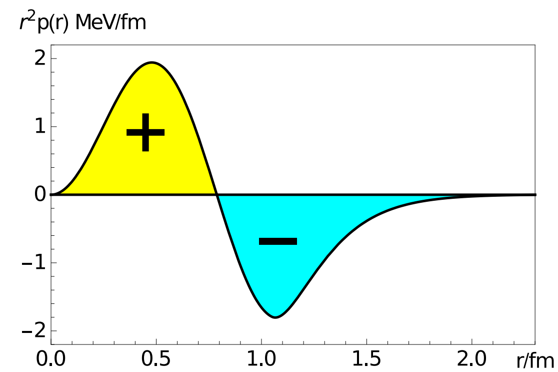
Can be solved analytically, describes particle of charge radius 0.71 fm ("proton") Białynicki-Birula (1993)

We use it to investigate in consistent framework effects of long-range forces Varma, PS (2020)

- **usual features in inner region $r < 2\,\mathrm{fm}$**







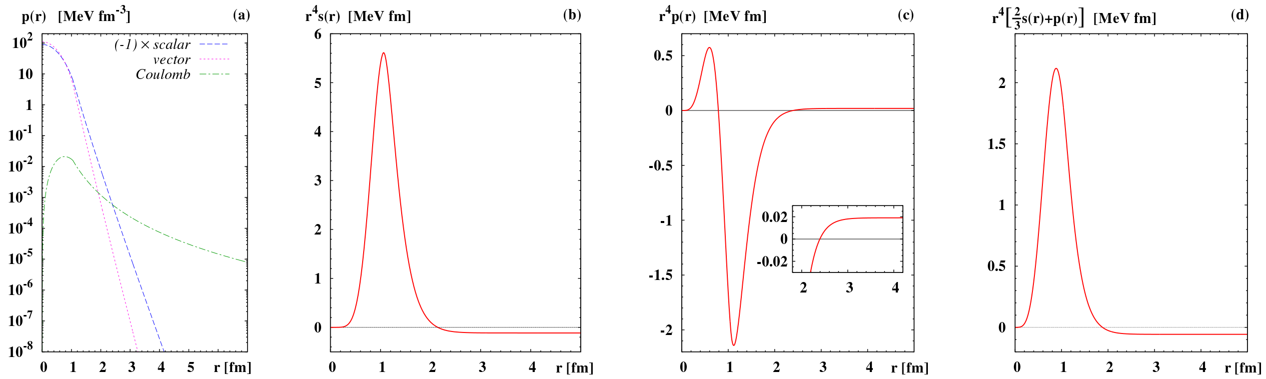
strong forces (scalar and vector fields ϕ and V^μ) make large contributions
about $10\times$ smaller than in chiral quark soliton ("residual nuclear forces")
Coulomb field minuscule contribution, hardly visible

$p(r)$ exhibits node at $r = 0.788\,\mathrm{fm}$, balance of forces:

$$\int dr\; r^2 p_i(r) = \begin{cases} -10.916\,\text{MeV} & \text{for} \quad i = \text{scalar}, \\ +10.891\,\text{MeV} & \text{for} \quad i = \text{vector}, \\ +\;\; 0.025\,\text{MeV} & \text{for} \quad i = \text{Coulomb}. \end{cases}$$

So far, same picture as in systems with short-range forces.
But we are looking at the region of $r < 2\,\mathrm{fm}$. Let's look more closely at larger r ...

- **unusual features in outer region $r < 2\,\mathrm{fm}$**



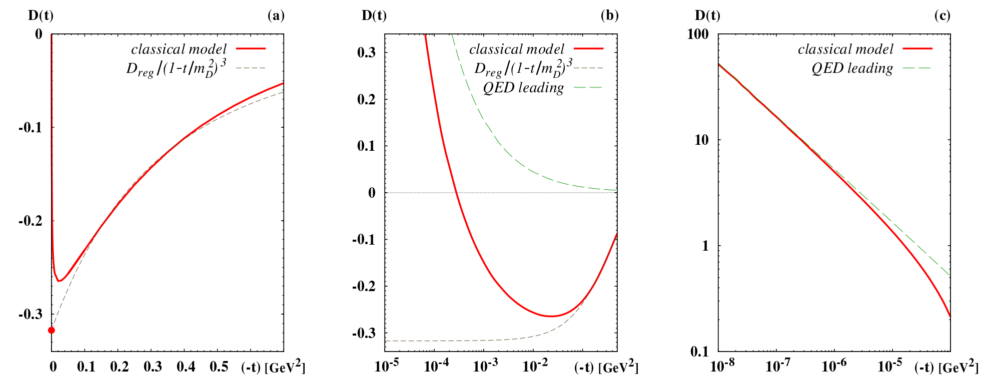
- at large $r > 2$ fm, Coulomb contribution takes over! Consequences!!

- shear forces $s(r)$ exhibit a node (in short-range systems $s(r) > 0$)
  $p(r)$ has 2nd node at 2.4 fm (short-range systems one node)
  normal force turns negative (in short range systems $>$ 0)

- model is still mechanically stable:
  dust particles within $R = 1.05$ fm
  where features "as usual"

- D-term is affected by that ...
  (most sensitive to dynamics!!)

• **consequences for D-term**



$$D(t) = (\text{regular strong part}) + \frac{\alpha}{\pi}\Big(-\frac{11}{18} + \frac{\pi^2 M}{4\sqrt{-t}} + \frac{2}{3}\log\frac{(-t)}{M^2}\Big) \qquad \text{QED part model-independent!}$$

• from QED diagrams Donoghue, Holstein, Garbrecht, Konstandin,

• long-range tail of densities ⇔ small-t behavior of $D(t)$
due to exchange of massless photons (also the "classical Coulomb potential")

• model independent features, seen in
Kubis, Meissner, Nucl. Phys. A **671**, 332 (2000)
Metz, Pasquini, Rodini, PLB **820**, 136501 (2021)
X. Ji and Y. Liu, arXiv:2110.14781 [hep-ph]

Deeper reason:
$T^{ij}(r) = -E^i\,E^j + \frac{1}{2}\,\delta^{ij}\,\vec{E}^{\,2} = -\,\sigma^{ij}$
(σ^{ij} Maxwell stress tensor, with $\vec{E} \sim \frac{1}{r^2}$ for $r > R$)

$$\rightarrow \qquad \begin{aligned} T_{00}(r)_{\rm QED} &= \frac{1}{2}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} \\ s(r)_{\rm QED} &= -\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} \\ p(r)_{\rm QED} &= \frac{1}{6}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} \end{aligned}$$

Important: in classical model **consistently** incorporated!
balance of forces: von Laue condition $\boldsymbol{\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0}$
consistent nonperturbative solution, proton stable!

Summary and open questions

- **EMT form factors, and D-term**: important to know
  D-term of fermions: generated dynamically (free Dirac theory $D = 0$)
  theory: D negative (Goldstone bosons, models, lattice, dispersion relations)

- generic features of EMT properties in all short-range systems
  ($s(r) > 0$, $p(r)$ single node, normal force $\frac{2}{3}s(r) + p(r) > 0$)

- presence of long-range (Coulomb force) forces essential: proton charged!
  features changed (at large r: $s(r) < 0$, $p(r)$ second node, $\frac{2}{3}s(r) + p(r) < 0$)

- consequence: $D(t) \propto \alpha/\sqrt{-t}$ model independent feature
  (classical model of proton, χPT for charged pions, 1-loop QED for electron)

- effect for $(-t) \ll 0.1\,\mathrm{GeV}^2$ only for $D(t)$ (most sensitive!)
  ($A(t)$, $J(t)$ finite for $t \to 0$, slopes affected, harder to see)

- can this be seen in experiment? (at least, in principle?)
  do we need to include QED effects in factorization? (radiative corrections?)
  what is the definition of $D(t)$ in long-range systems? (proton, charged pion, atoms, photon, ... ?)

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Thank you!

Support slides

What means “last global unknown”?

$|N\rangle$ = **strong**–interaction particle. Use other forces to probe it! Simplest observables:

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$Q,\ \mu,$	$\ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$g_A,\ g_p,$	$\ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$A(t),\ B(t),\ D(t)$	$\longrightarrow$	$M,\ J,\ D,$	$\ldots$

global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40)\times 10^{-19}\text{C}\\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N\\
g_A &= 1.2694(28)\\
g_p &= 8.06(0.55)\\
M &= 938.272013(23)\,\text{MeV}\\
J &= \tfrac{1}{2}\\
\boldsymbol{D} &= \textbf{?}
\end{aligned}
$$

first insight D of π^0, nucleon
Kumano, Song, Teryaev, PRD97, 014020 (2018)
Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)

D-term in theory

- **free spin-0 particle** $D = -1$
  Pagels 1966; Hudson, PS 2017
- **free spin** $\frac{1}{2}$ **particle** $D = 0$
  Donoghue et al, (2002), Hudson, PS PRD97 (2018) 056003
- **Goldstone bosons** chiral symmetry breaking $D = -1$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

$$
\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$
Donoghue, Leutwyler (1991)
estimates: Hudson, PS (2017)

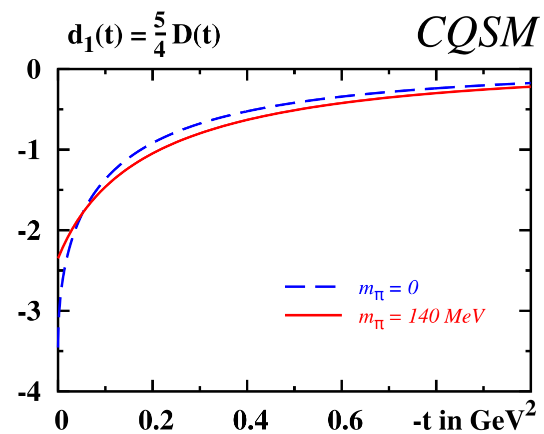
- **nuclei** (liquid drop model, Walecka model) $\boldsymbol{D \approx -0.2 \times A^{7/3}}$ $\to$ DVCS with nuclei!
  Polyakov (2002),
  Guzey, Siddikov (2006);
  Liuti, Taneja (2005)

$$
\begin{aligned}
{}^{12}\text{C}: &\quad D = -6.2 \\
{}^{16}\text{O}: &\quad D = -115 \\
{}^{40}\text{Ca}: &\quad D = -1220 \\
{}^{90}\text{Zr}: &\quad D = -6600 \\
{}^{208}\text{Pb}: &\quad D = -39000
\end{aligned}
$$

- **Q-balls** N^{th} excited Q-ball state: mass $M \propto N^3$ but $\boldsymbol{D \propto N^8}$
  Mai, PS PRD**86**, 096002 (2012)

- **nucleon, bag model** $\boldsymbol{D = -1.15 < 0}$
  Ji, Melnitchouk, Song (1997)

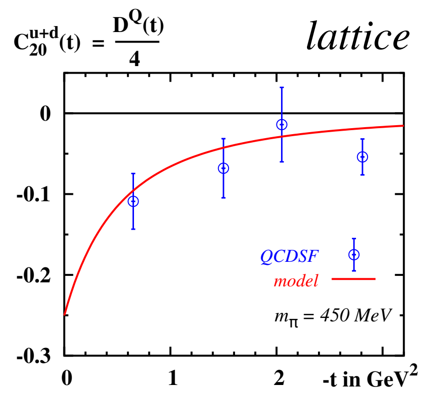
- **chiral quark soliton**
  Goeke et al, PRD75 (2007)
  $d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5\,k\,g_A^2 M}{64\,\pi\,f_\pi^2}\, m_\pi + \ldots$
  $\overset{\circ}{d}{}_1'(0) = -\frac{k\,g_A^2\,M}{32\,\pi f_\pi^2\,m_\pi} + \ldots \quad k = \begin{cases} 1, & N_c \text{ finite} \\ 3, & N_c \to \infty \end{cases}$

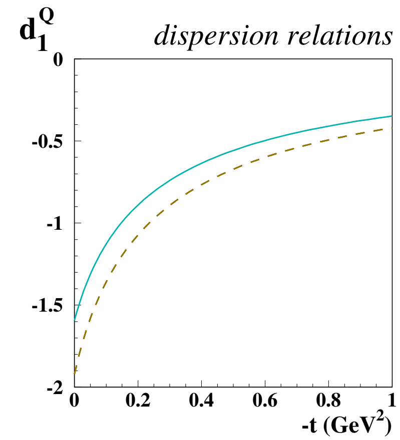


- **χPT**
  Belitsky, Ji (2002), Diehl et al (2006),
  Alharazin, Djukanovic, Gegelia, Polyakov PRD102 (2020) 7, 076023

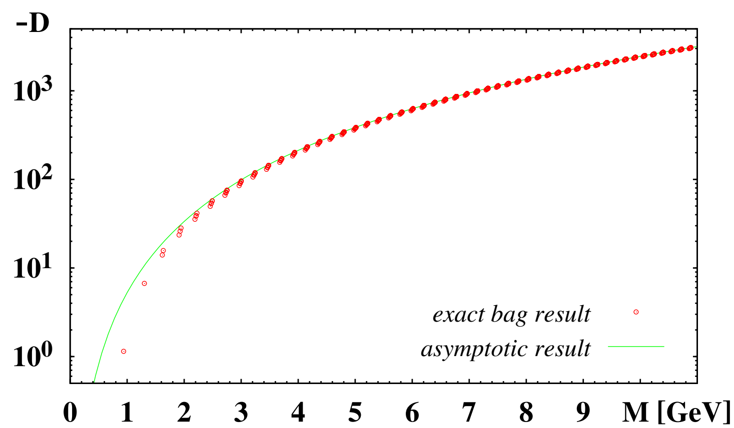
- **non-relativistic limit** $\boldsymbol{D = -\,N_c^2\,\frac{4\,\pi^2 - 15}{45}} = -\,4.89$
  Neubelt et al (2019) (in bag)

- **lattice:** QCDSF
  Göckeler et al, PRL92 (2004)
  $\mu = 2\,\text{GeV}$, $m_\pi = 450\,\text{MeV}$
  disconnected diagrams neglected
  recently:
  $D^g(t) < 0$ with $|D^g(t)| > |D^Q(t)|$
  Shanahan, Detmold, PRD99 (2019)





- **dispersion relations** $d_1^Q(t) = \frac{5}{4}\,D^Q(t)$
  Pasquini, Polyakov, Vanderhaeghen (2014)
  pion PDFs are input, scale $\mu^2 = 4\,\text{GeV}^2$



- **excitated stats**

  in bag model Neubelt et al (2019)
  M over 1 order of magnitude
  D over 3 orders of magnitude

of all properties, D-term most sensitive to variations (parameters, excitations)

$\Rightarrow$ dynamics!

keep in mind: free spin $\frac{1}{2}$ theory $\to D = 0$;
i.e. D-term of nucleon due to dynamics!

What will we learn?

comment on 2D vs 3D interpretation

- **3D density** not exact, "relativistic corrections" for $r \lesssim \lambda_{\text{Compt}} = \frac{\hbar}{mc}$
  2D densities exact partonic probability densities

  known since earliest days:
  - Yennie, Levy, Ravenhall, Rev. Mod. Phys. 29 (1957) 144
  - Sachs, Physical Review 126 (1962) 2256
  - Belitsky, Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
  - X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
  - G. Miller, PRC80 (2009) 045210 (toy model, very dramatic effect)
  - Lorcé, PRL 125 (2020) 232002 (for $r \leq 1/(2m)$ quasi-probabilistic phase-space average á la Wigner)
  - Jaffe, arXiv:2010.15887 (not possible to measure spatial dependence of nucleon matrix elements)
  - Freese, Miller, 2102.01683 (expectation value of a local operator within spatially-localized state)
  - Julia Panteleeva, Maxim Polyakov, arXiv:2102.10902 (unique Abel transformations, 3D $\leftrightarrow$ 2D)
  - Epelbaum, Gegelia, Lange, Meißner, Polyakov, arXiv:2201.02565

- important distinction:

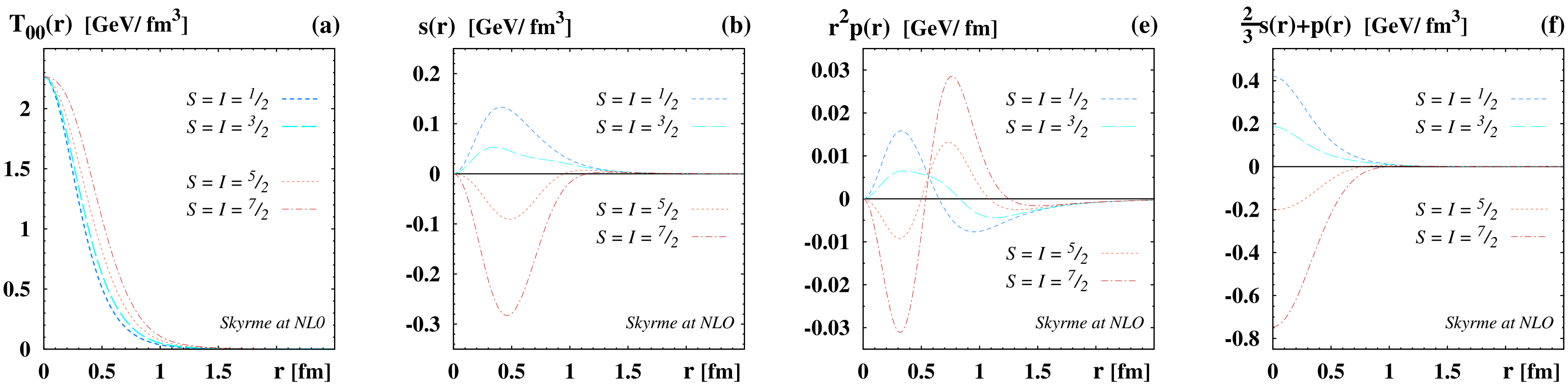
  2D densities = **partonic probability densities** (unitarity)
  must (and better be) exact! $\rightarrow$ M. Burkardt (2000)
  apply to any particle (including the light pion)

  vs

  3D densities = **mechanical response functions**
  correlation functions ($\neq$ probabilities!)
  if corrections "reasonably small" $\rightarrow$ we do not need to worry

Skyrme model nucleon, Δ vs large-N_c artifacts Witten 1979

- in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

stability requires:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

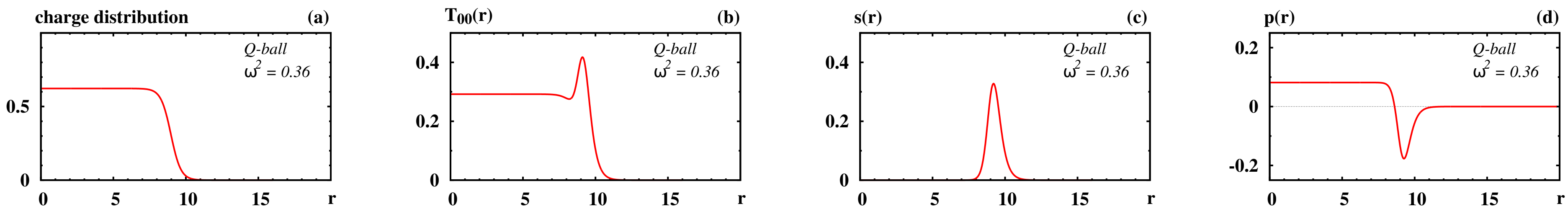
mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow$ $\frac{2}{3}s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ have positive D-term!
So do not exist!
dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Q-balls $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$, $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$

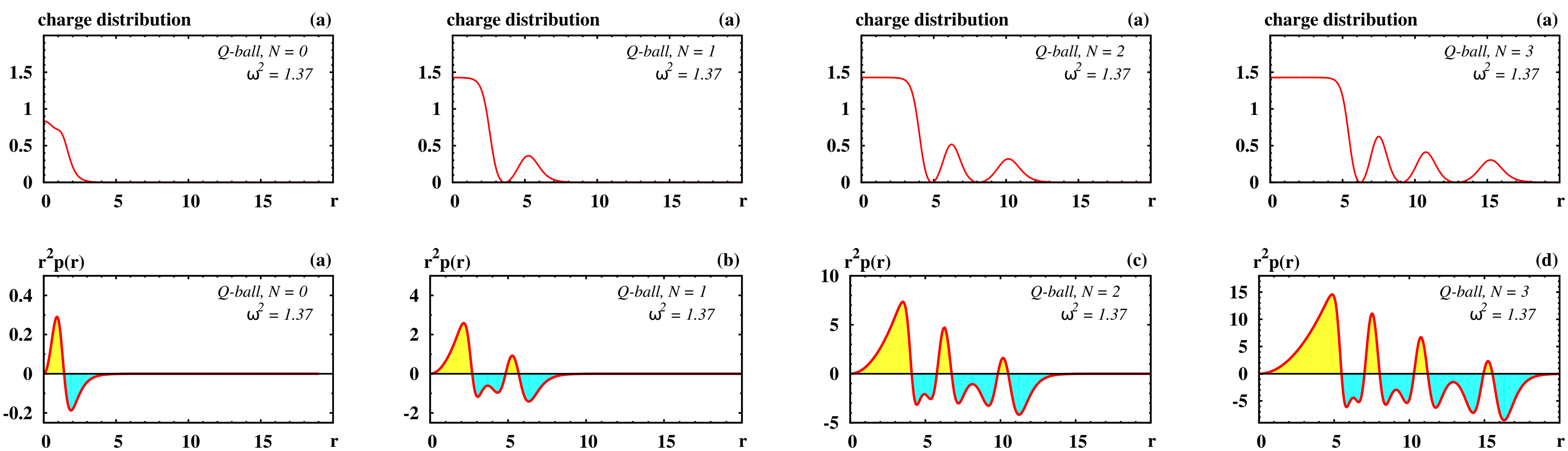
global U(1) symmetry, solution $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

- ground state properties for large Q-ball



- excitations: $N = 0$ ground state, $N = 1$ first excited state, etc Volkov, Wohnert 2002; Mai, PS 2012

  charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros

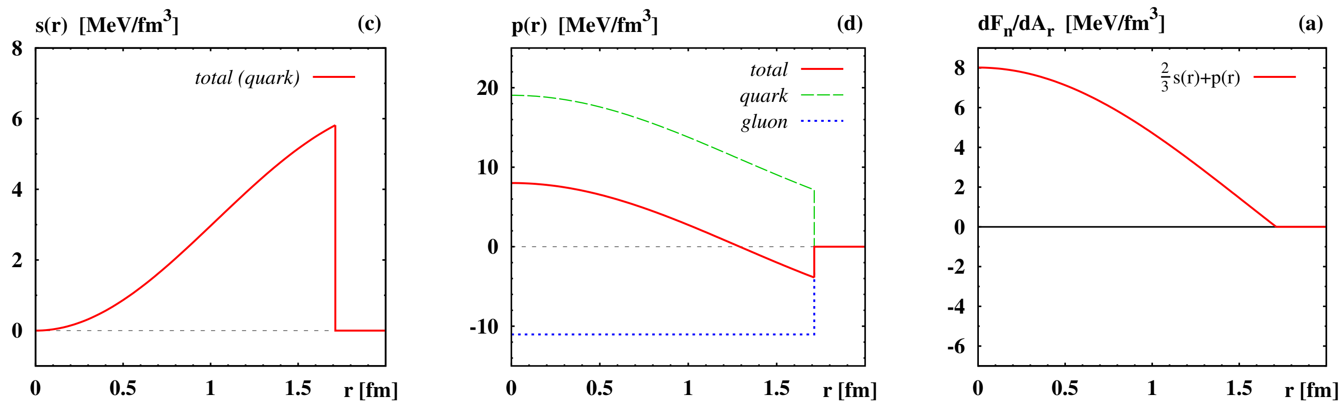


excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

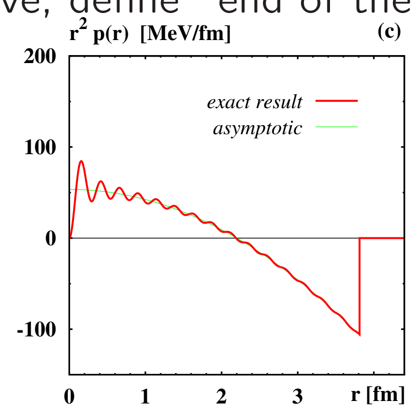
bag model Neubelt, Sampino, Hudson, Tezgin, PS, PRD101 (2020) 034013

- free quarks + boundary condition, formulated in large-N_c
- $T^{\mu\nu}(r) = T^{\mu\nu}_{\text{quarks}}(r) + T^{\mu\nu}_{\text{bag}}(r)$

  $T^{\mu\nu}_{\text{bag}}(r) = B\,\Theta(R-r)\,g^{\mu\nu}$ binding effect (“mimics gluons” Jaffe & Ji 1991)
- all densities defined with Θ-functions, assume non-zero values at $r = R$



- only exception:
  the normal force $= \frac{2}{3}\,s(r) + p(r) > 0$ for $r < R$, becomes exactly zero at $r = R$
- this is how one determines the radius of a neutron star:
  solve Tolman-Oppenheimer-Volkoff equations with an “equation of state”
  where “radial pressure” $\frac{2}{3}\,s(r) + p(r)$ turns negative, define “end of the system”
- excitated states different pattern than Q-balls:
  $p(r)$ has one node (here 3163th excited state)
  but $D \sim \text{const} \times M^{8/3}$ bag & Q-balls
  deeper reason?



comment on "mass radius"

- energy density $T^{00}(r) \geq 0$ can also be used to define a mean square radius

$$\langle r_E^2 \rangle = \frac{\int d^3r\, r^2\, T^{00}(r)}{\int d^3r\, T^{00}(r)} = 6A'(0) - \frac{3D}{2M^2}$$ "energy" mean square radius

vs "mass radius" extracted by Dima Kharzeev $\langle r^2_{\text{trace}} \rangle = 6A'(0) - \frac{9D}{2M^2} = \langle r_E^2 \rangle - \frac{3D}{M^2}$

One way to look at nucleon mass: it's due to the trace anomaly.
Hence: name "mass radius" is reasonable. But keep in mind the different definitions.

all radii address *different* characteristics of nucleon! Important information!

prediction from $D < 0$ to be expected $\langle \boldsymbol{r_E^2} \rangle < \langle \boldsymbol{r^2_{\text{trace}}} \rangle$

$\sqrt{\langle r^2_{\text{trace}} \rangle} = 0.90\,\text{fm}$ vs $\sqrt{\langle r_E^2 \rangle} = 0.82\,\text{fm}$ chiral quark soliton model Goeke et al, PRD 2007
$\sqrt{\langle r^2_{\text{trace}} \rangle} = 0.96\,\text{fm}$ vs $\sqrt{\langle r_E^2 \rangle} = 0.73\,\text{fm}$ Skyrme model Cebulla et al, NPA 2007
$\sqrt{\langle r^2_{\text{trace}} \rangle} = 0.55 \pm 0.03$ fm from GlueX at $0.5 < (-t) < 1.4\,\text{GeV}^2$ op. cit.

mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}$ = symmetric 3×3 matrix $\rightarrow$ diagonalize:

$$\frac{2}{3}\,s(r) + p(r) = \text{ normal force (eigenvector } \vec{e}_r)$$

$$-\frac{1}{3}\,s(r) + p(r) = \text{ tangential force } (\vec{e}_\theta,\ \vec{e}_\phi, \text{ degenerate for spin 0 and } \tfrac{1}{2})$$

- **mechanical stability** $\Leftrightarrow$ normal force directed towards outside

$$\Leftrightarrow T^{ij} e_r^j\, dA = \underbrace{\left[\frac{2}{3}\,s(r) + p(r)\right]}_{>0} e_r^i\, dA \qquad \Rightarrow \qquad \boldsymbol{D < 0}$$

- define: $\langle \boldsymbol{r^2} \rangle_{\mathbf{mech}} = \dfrac{\int \boldsymbol{d^3r\ r^2}[\frac{2}{3}\,\boldsymbol{s(r)} + \boldsymbol{p(r)}]}{\int \boldsymbol{d^3r}\ [\frac{2}{3}\,\boldsymbol{s(r)} + \boldsymbol{p(r)}]} = \dfrac{\mathbf{6}\boldsymbol{D}(\mathbf{0})}{\int_{-\infty}^{0} \mathbf{dt}\,\boldsymbol{D(t)}}$ vs $\langle r^2_{\text{ch}} \rangle = \dfrac{6\,G_E'(0)}{G_E(0)}$ "anti-derivative" intuitive result for large nucleus $\frac{2}{3}\,s(r) + p(r) = p_0\,\Theta(R_A - r) \;\rightarrow\; \langle r^2\rangle_{\text{mech}} = \frac{3}{5}\,R_A^2$

M.Polyakov, PS arXiv:1801.05858

- proton: $\langle \boldsymbol{r^2}\rangle_{\textbf{mech}} \approx \mathbf{0.75}\,\langle \boldsymbol{r^2_{\textbf{ch}}}\rangle$ (chiral quark soliton model)

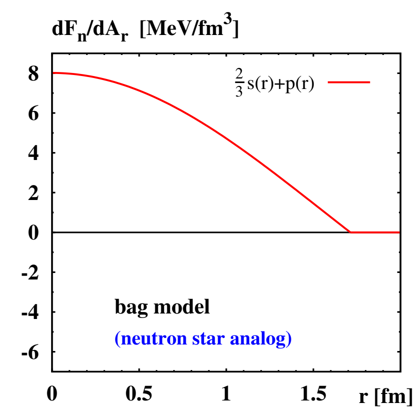
- in chiral limit $\langle r^2\rangle_{\text{mech}}$ finite!

  vs $\langle r^2_{\text{ch}}\rangle$ divergent (better concept)

- for neutron $\langle r^2\rangle_{\text{mech}}$ same as proton(!)

  neutron charge radius $\langle r^2_{\text{ch}}\rangle = -(0.11\,\text{fm})^2$

  insightful, but not particle size!



see also: Lorcé, Moutarde, Trawiński EPJC 79 (2019)