Generalised Partons Distributions: a review of PARTONS team results

Cédric Mezrag

CEA Saclay, Irfu DPhN

March 8^{th}, 2022

On behalf of the PARTONS team

Introduction

- Generalised Parton Distributions (GPDs):

Generalised Parton Distributions

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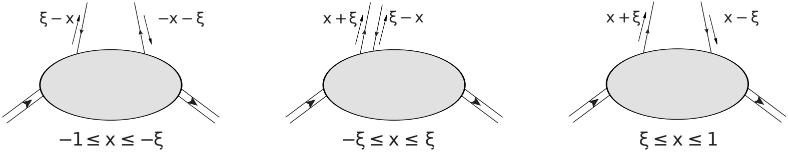
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,



- ★ x: average momentum fraction carried by the active parton
- ★ ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
- ★ t: the Mandelstam variable

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+ u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

- Generalised Parton Distributions (GPDs):
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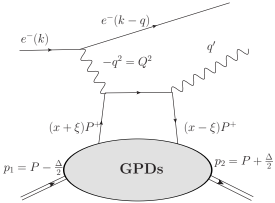
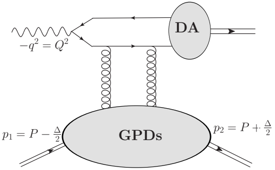
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





Properties

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t;\mu) + mod(m,2)\xi^{m+1} C^q_{m+1}(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\; H^q(x,\xi,t;\mu) = F_1^q(t)$$

Lorentz Covariance

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1;1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

Properties

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines
  $\rightarrow$ GPDs are continuous albeit non analytical at $x = \pm\xi$

  J. Collins and A. Freund, PRD 59 074009 (1999)

  Factorisation theorem

Properties

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Scale evolution property
  $\rightarrow$ generalization of DGLAP and ERBL evolution equations

  D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

  Renormalization

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

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- Support property:

Relativistic quantum mechanics

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Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

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- Possibility to extract density from experimental data

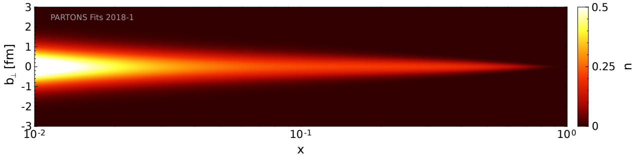

figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

Interpretation of GPDs I

2+1D structure of the nucleon

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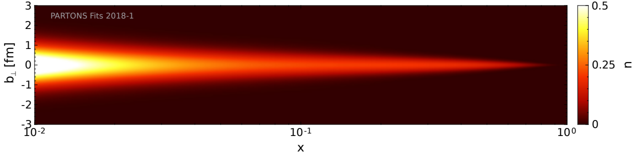

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- Correlation between x and $b_\perp$ $\to$ going beyond PDF and FF.

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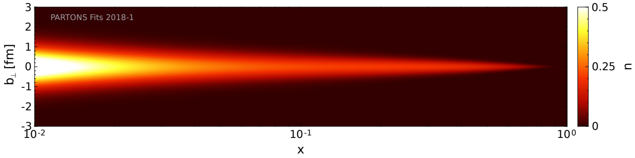

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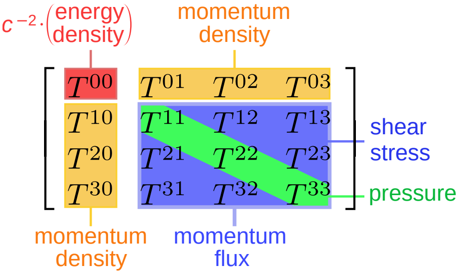
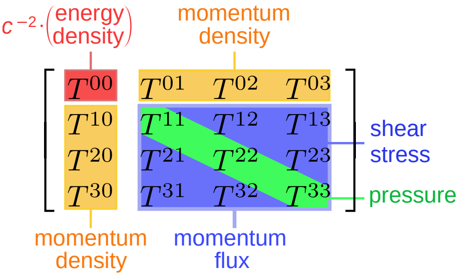
- Correlation between x and $b_\perp \to$ going beyond PDF and FF.
- Caveat: no experimental data at $\xi = 0$
  $\to$ extrapolations (and thus model-dependence) are necessary



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89



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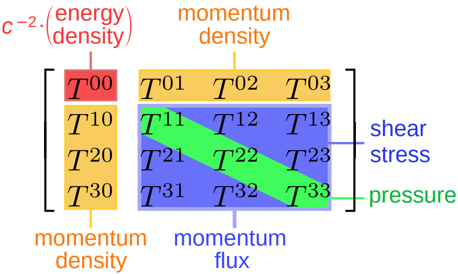
$$\langle p', s'|T^{\mu\nu}_{q,g}|p, s\rangle = \bar{u}\left[P^{\{\mu}\gamma^{\nu\}}A_{q,g}(t;\mu) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_{q,g}(t;\mu)\right.$$
$$\left. + Mg^{\mu\nu}\bar{C}_{q,g}(t;\mu) + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{2M}B_{q,g}(t;\mu) + \frac{P^{[\mu}i\sigma^{\nu]\Delta}}{2M}D_{q,g}(t;\mu)\right]u$$



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$$\langle p', s' | T^{\mu\nu}_{q,g} | p, s\rangle = \bar{u}\Bigg[P^{\{\mu}\gamma^{\nu\}} A_{q,g}(t;\mu) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_{q,g}(t;\mu) + M g^{\mu\nu}\bar{C}_{q,g}(t;\mu) + \frac{P^{\{\mu} i\sigma^{\nu\}\Delta}}{2M} B_{q,g}(t;\mu) + \frac{P^{[\mu} i\sigma^{\nu]\Delta}}{2M} D_{q,g}(t;\mu)\Bigg] u$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,t;\mu) = A_q(t;\mu) + (2\xi)^2 C_q(t;\mu)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,t;\mu) = B_q(t;\mu) - (2\xi)^2 C_q(t;\mu)$$

- Ji sum rule
- Fluid mechanics analogy

X. Ji, PRL 78, 610-613 (1997)
M.V. Polyakov PLB 555, 57-62 (2003)

Phenomenology of GPDs

A selection of recent results obtained with PARTONS

PARTONS and Gepard

Integrated softwares as a mandatory step for phenomenology

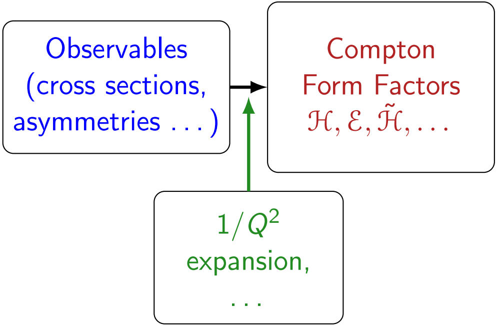
PARTONS	Gepard
partons.cea.fr	gepard.phy.hr
B. Berthou *et al.*, EPJC 78 (2018) 478	K. Kumericki, EPJ Web Conf. 112 (2016) 01012

- Similarities : NLO computations, BM formalism, ANN, ...
- Differences : models, evolution, ...

Physics impact

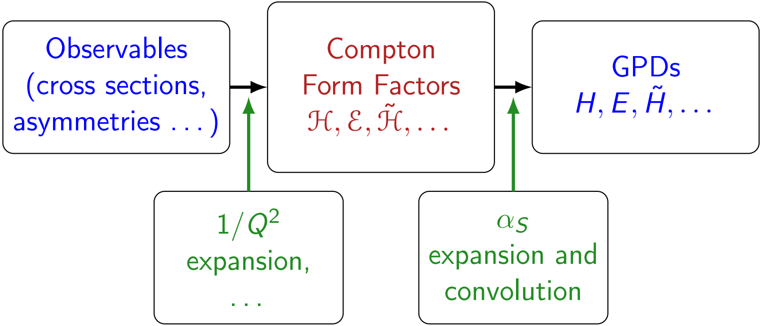
These integrated softwares are the mandatory path toward reliable multichannel analyses.

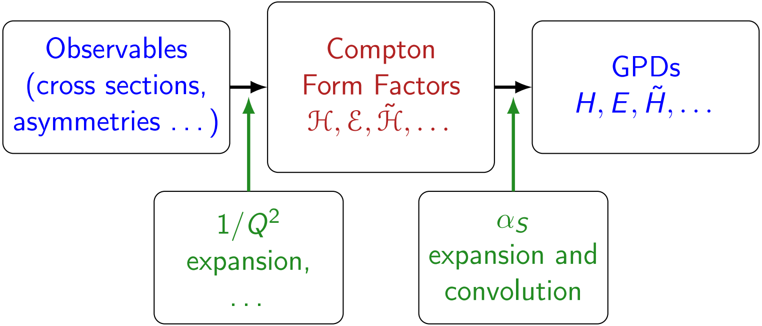
Observables
(cross sections,
asymmetries . . .)



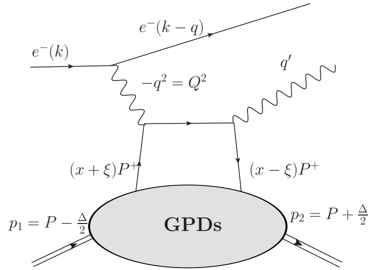
Observables
(cross sections,
asymmetries ...)
Compton
Form Factors
$\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \ldots$
$1/Q^2$
expansion,
...

Experimental connection to GPDs



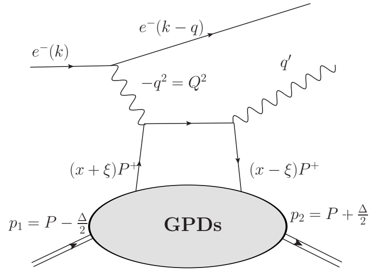


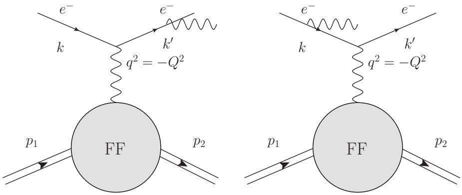
- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs



- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12

Deep Virtual Compton Scattering

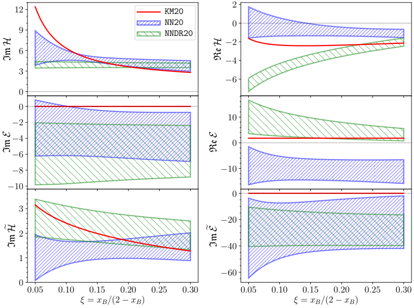




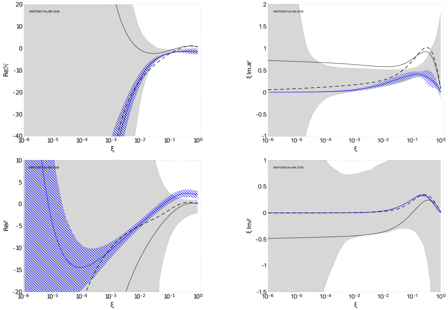
- Best studied experimental process connected to GPDs
  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

Recent CFF extractions



M. Cuič *et al.*, PRL 125, (2020), 232005



H. Moutarde *et al.*, EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)

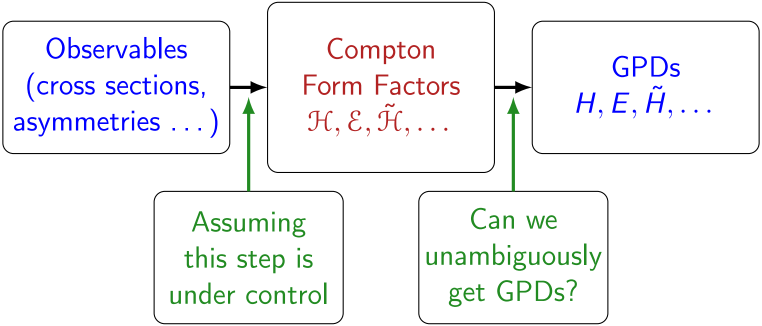
  additional ongoing studies, J. Grigsby *et al.*, PRD 104 (2021) 016001

- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,. . .)
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)

  see *e.g.* H. Dutrieux *et al.*, EPJA 57 8 250 (2021)

The DVCS deconvolution problem I

From CFF to GPDs



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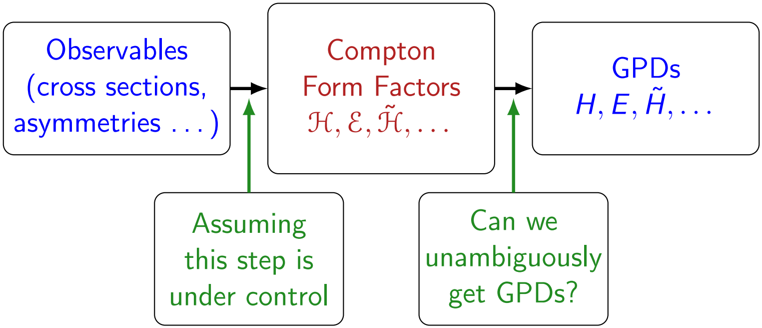

- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).

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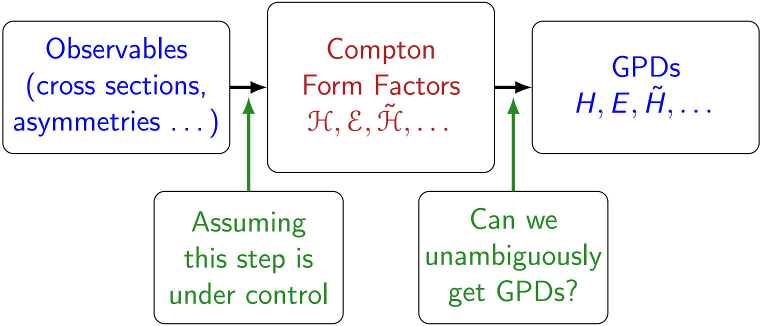

- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).
- Are QCD corrections improving the situation?

CFF Definition

$$\underbrace{\mathcal{H}(\xi, t, Q^2)}_{\text{Observable}} = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \underbrace{T\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)}_{\text{Perturbative DVCS kernel}} H(x, \xi, t, \mu^2)$$

Introducing shadow GPDs

CFF Definition

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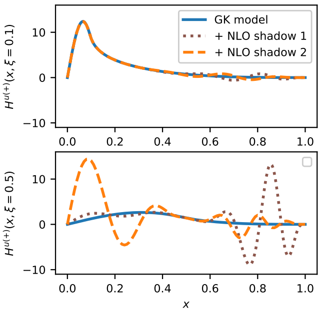
Shadow GPD definition

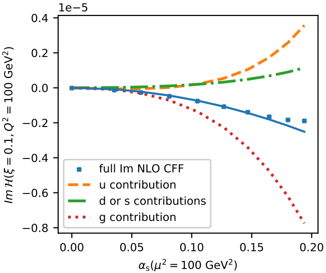
We define shadow GPD $H^{(n)}$ of order n such that when T is expanded in powers of α_s up to n one has:

$$0 = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^{(n)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu_0^2}, \alpha_s(\mu_0^2)\right) H^{(n)}(x, \xi, t, \mu_0^2) \quad \text{invisible in DVCS}$$

$$0 = H^{(n)}(x, 0, 0) \quad \text{invisible in DIS}$$

A part of the GPD functional space is invisible to DVCS and DIS combined

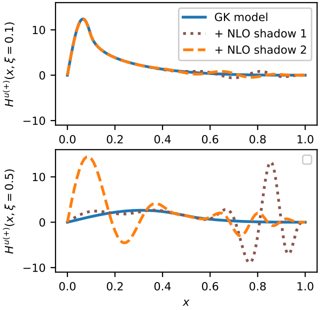


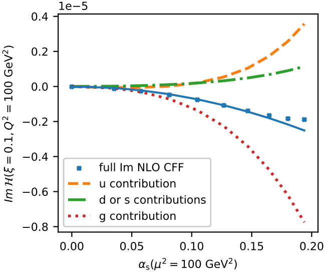


- NLO analysis of shadow GPDs:
  - Cancelling the line $x = \xi$ is necessary but **no longer** sufficient
  - Additional conditions brought by NLO corrections reduce the size of the “shadow space”...
  - ... but do not reduce it to 0 $\rightarrow$ NLO shadow GPDs

H. Dutrieux *et al.*, PRD 103 114019 (2021)

The DVCS deconvolution problem II





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    H. Dutrieux *et al.*, PRD 103 114019 (2021)
- Evolution
  - it was argued that evolution would solve this issue

    A. Freund PLB 472, 412 (2000)
  - but in practice it is not the case

    H. Dutrieux *et al.*, PRD 103 114019 (2021)

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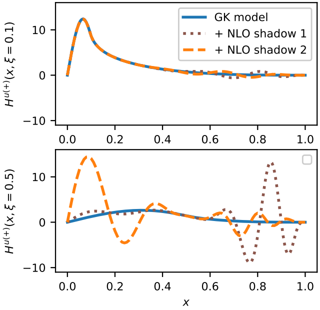

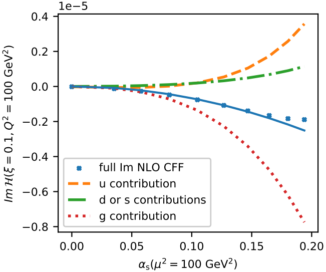

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H. Dutrieux *et al.*, PRD 103 114019 (2021)

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H. Dutrieux *et al.*, PRD 103 114019 (2021)

Theoretical uncertainties promoted to main source of GPDs uncertainties

GPD properties and unbiased techniques

Model $H = H_{\text{visible}} + H_{\text{shadow}}$ with two different neural network fulfilling by construction all the properties but one, the positivity property.

GPD properties and unbiased techniques

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The positivity property

$$\left| H^q(x,\xi,t) - \cancel{\frac{\xi^2}{1-\xi^2} E^q(x,\xi,t)} \right| \leq \sqrt{\frac{1}{1-\xi^2} q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

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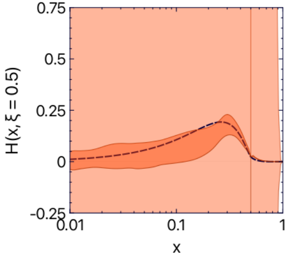

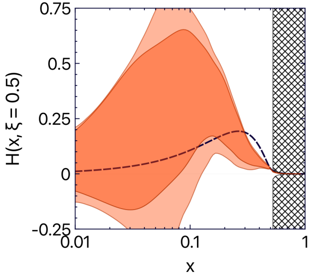

H. Dutrieux *et al.*, accepted in EPJC, arXiv:2112.10528

Modelling GPDs

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  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Scale evolution property

  Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- GPDs are related to Double Distributions (DDs) through:

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta\left(x-\beta-\xi\alpha\right)$$

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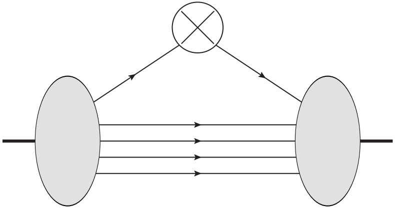
- They also appear naturally in covariant modelling attempts

Positivity property is not guaranteed, and may be violated.

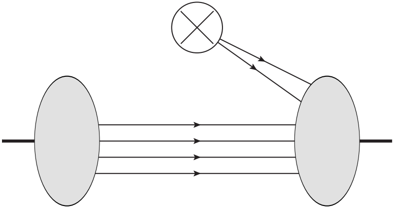
- On the light front, hadronic states can be expanded on a Fock basis

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- No ambiguity

ERBL: $|x| < |\xi|$



- N and $N + 2$ partons LFWFs
- Ambiguity

M. Diehl *et al.*, Nucl.Phys. B596 (2001) 33-65

Fulfilling positivity
LFWFs approach to GPDs

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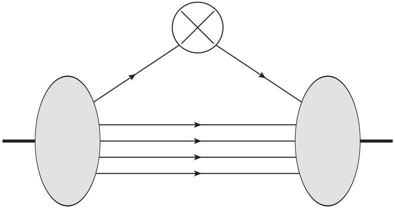

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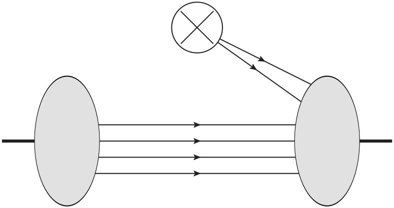

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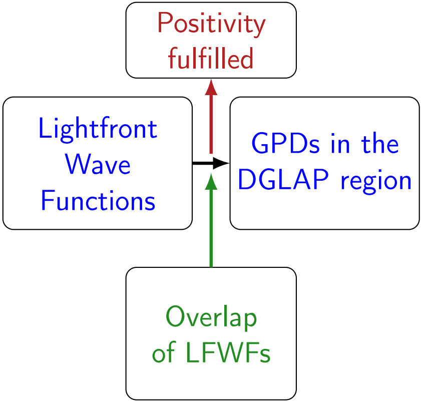
M. Diehl *et al.*, Nucl.Phys. B596 (2001) 33-65

LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

A new modelling procedure

Lightfront
Wave
Functions

A new modelling procedure



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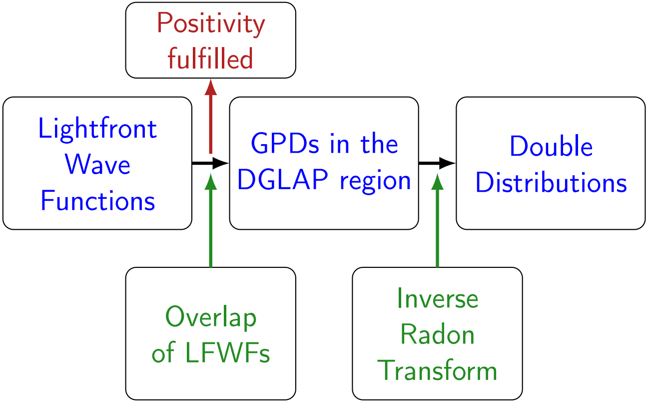
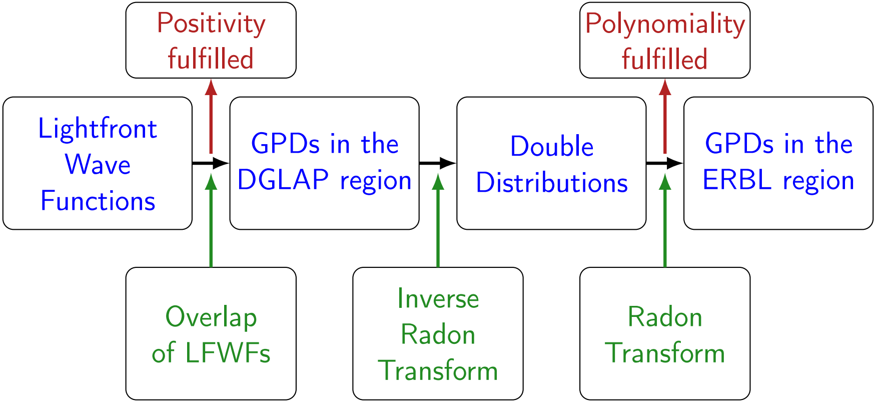
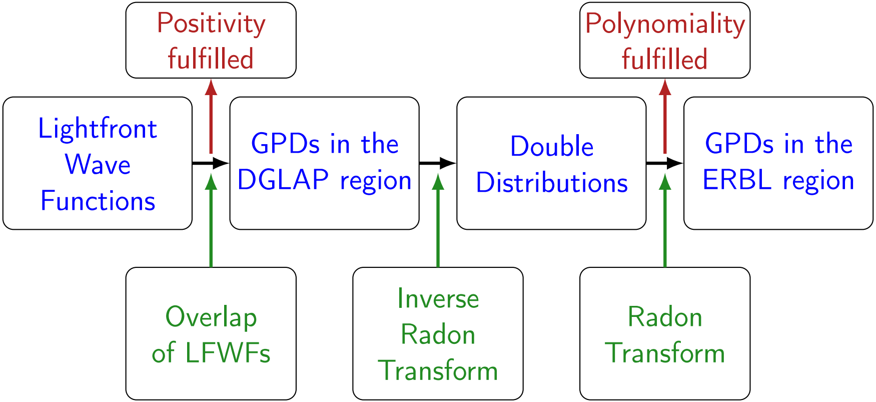



Positivity fulfilled
Polynomiality fulfilled
Lightfront Wave Functions
GPDs in the DGLAP region
Double Distributions
GPDs in the ERBL region
Overlap of LFWFs
Inverse Radon Transform
Radon Transform

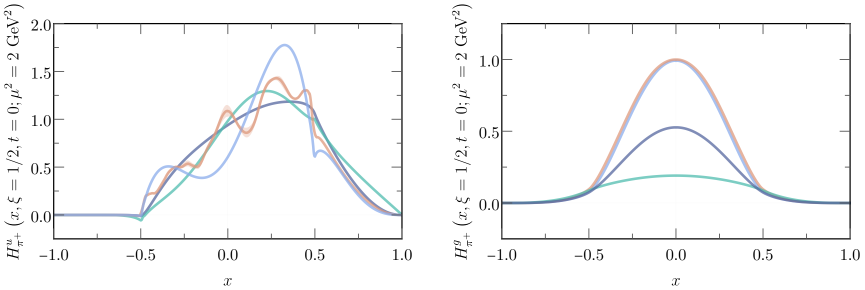
A new modelling procedure



Not necessary to start from LFWFs
$\rightarrow$ Fulfilling the positivity and forward limit properties is enough

An example on the pion

J. M. Morgado Chavez *et al.*, accepted in PRD



- Blue: GPD based on algebraic PDFs model
- Orange: GPD based on refine numerical PDF model
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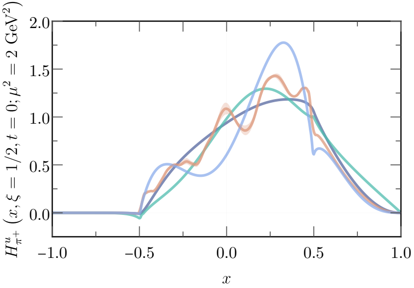

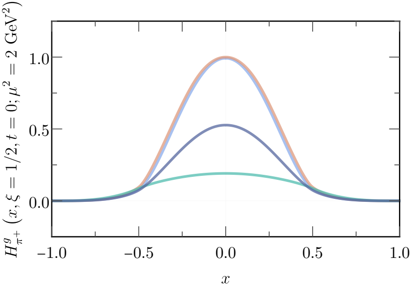

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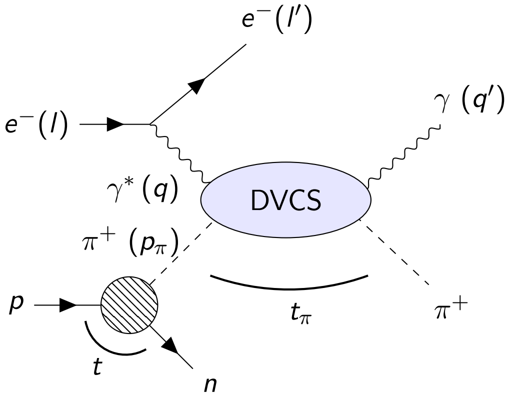
All theoretical constraints are fulfilled by construction !

J. M. Morgado Chavez *et al.*, accepted in PRL

Can we measure DVCS on a virtual pion ?

D. Amrath *et al.*, EPJC 58 (2008) 179-192

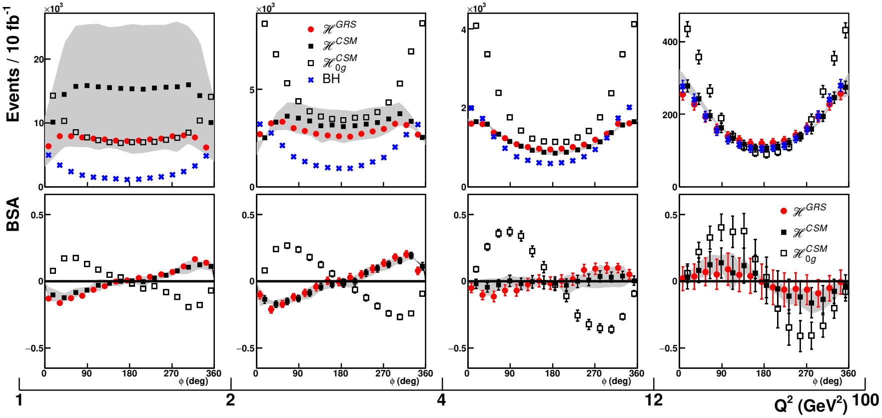


- $e^- p \rightarrow e^- \gamma \pi^+ n$
- kinematical cuts to avoid N^* resonances
- Already used to extract pion EFF at JLab
- Considered for pion structure function at EIC and EicC

EIC Yellow report, arXiv:2103.05419
EicC white paper, arXiv:2102.09222

An example on the pion

J. M. Morgado Chavez *et al.*, accepted in PRL



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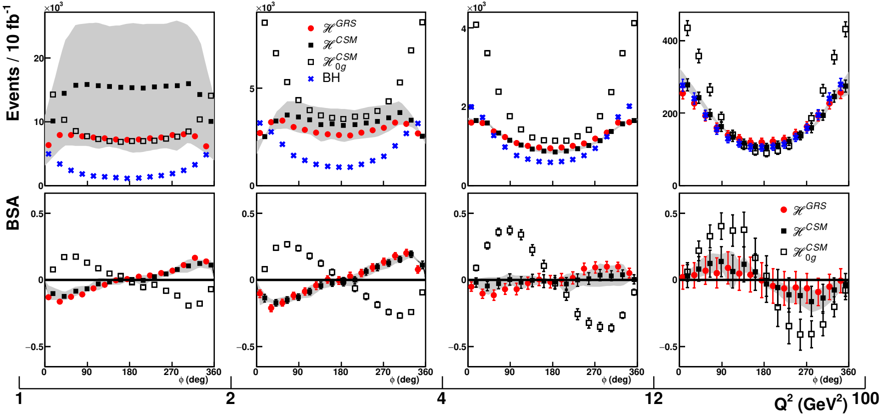
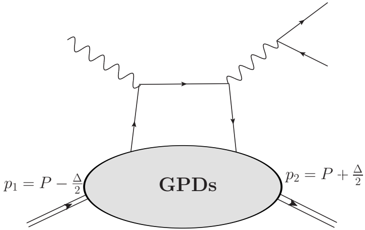


DVCS off virtual pion measurable at EIC and EicC

Results beyond DVCS

Timelike Compton Scattering



- Amplitude related to the DVCS one ($Q^2 \to -Q^2, \ldots$)
  $\to$ theoretical development for DVCS can be extended to TCS

  E. Berger *et al.*, EPJC 23 (2002) 675

- Excellent test of GPD universality but not the best option to solve the deconvolution problem

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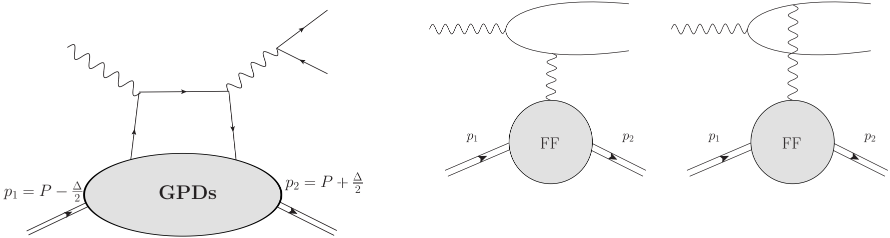

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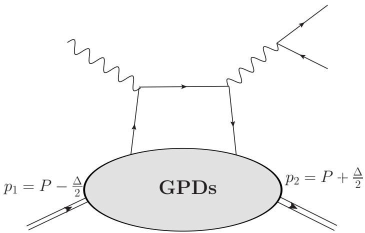

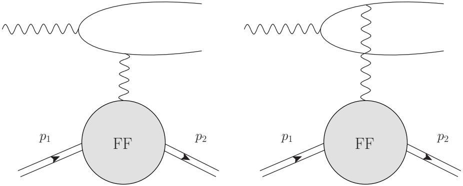

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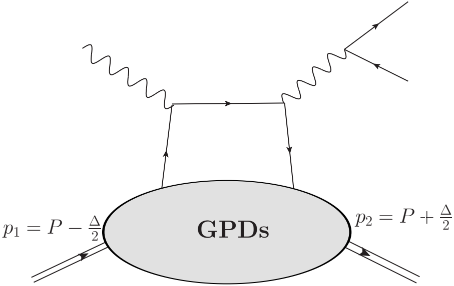
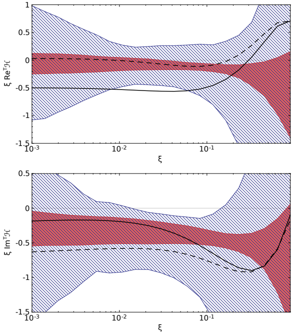
- Same type of final states as exclusive quarkonium production

TCS: Recent results





O. Grocholski *et al.*, EPJC 80, (2020) 61

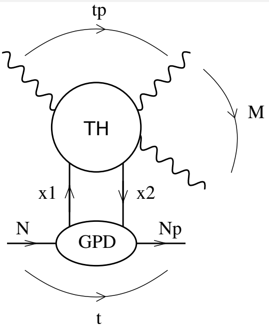
- DVCS Data-driven prediction for TCS at LO and NLO
- First experimental measurement at JLab through forward-backward asymmetry (interference term)

P. Chatagnon *et al.*,arXiv:2108.11746

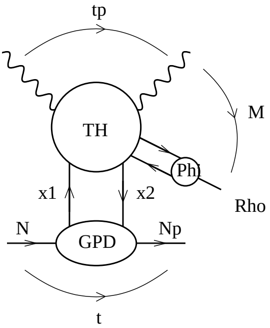
- Measurable at the LHC in UPC ?

On-going efforts

New channels: Multiparticle production



A. Pedrak *et al.*, PRD 96 (2017) 7, 074008
O. Grocholski *et al.*, PRD 104 (2021) 11, 114006



R. Boussarie *et al.*, JHEP 02 (2017) 054

- New combination of CFFs $\rightarrow$ welcome in global fits.
- LT access to chiral-odd GPDs in the (γ, ρ) case.
- Electroproduction done for $\gamma\gamma$.
- Additional particle in the final state
  - more difficult experimentally $\rightarrow$ need higher luminosity
  - more degrees of freedom $\rightarrow$ solution to the deconvolution problem?

Evolution of GPDs

- No publicly available and maintained evolution code for GPDs even at LO
  (Vinnikov code is not maintained anymore but available in PARTONS)

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  - LO splitting functions have been implemented and validated
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V. Bertone *et al.*, *in preparation*

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V. Bertone *et al.*, *in preparation*

Benchmarking

Benchmarking GPDs evolution code is a real topic today between PARTONS and GePard groups. But some difficulties need to be overcome (among them: choice of the model for benchmarking)

Experimental extraction
of the nucleon mechanical properties

Dispersion relation and the D-term

- At all orders in α_S, dispersion relations relate the real and imaginary parts of the CFF.

I. Anikin and O. Teryaev, PRD 76 056007
M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932

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- For instance at LO:

$$Re(\mathcal{H}(\xi,t)) = \frac{1}{\pi}\int_{-1}^{1}\mathrm{d}x\ Im(\mathcal{H}(x,t))\left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right] + \underbrace{2\int_{-1}^{1}\mathrm{d}\alpha\frac{D(\alpha,t)}{1-\alpha}}_{\text{Independent of }\xi}$$

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M.V. Polyakov PLB 555, 57-62 (2003)

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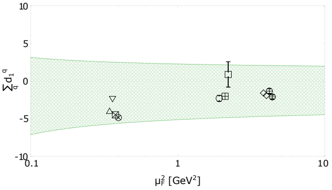

figure from H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- First attempt from JLab 6 *GeV* data

Burkert *et al.*, Nature 557 (2018) 7705, 396-399

- Tensions with other studies
  → uncontroled model-dependence

K. Kumericki, Nature 570 (2019) 7759, E1-E2
H. Moutarde *et al.*, Eur.Phys.J.C 79 (2019) 7, 614
H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- Scheme/scale dependence

D-term Expansion and Shadow D-term

$$\int_{-1}^{1} \mathrm{d}\alpha \frac{D^q(\alpha, t)}{1-\alpha} = 2d_1(t) + 2 \sum_{\text{n odd>1}} d_n^q(t)$$

- Fitting scenario with d_1 only:
  $d_1(\mu_F^2) = -0.5 \pm 1.2$

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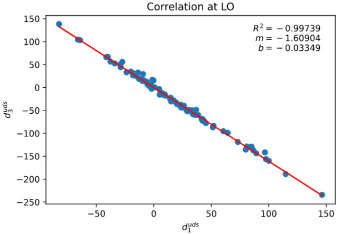

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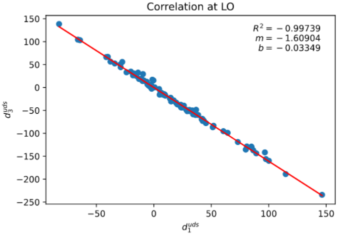

Shadow D-term

We extract shadow D-term yielding vanishing contribution to the subtraction constant

Impact of NLO corrections

$$\mathcal{C} = \int_{-1}^{1} \mathrm{d}\alpha\, M(\alpha)\, D(\alpha, t) = F(d_i^q, d_i^g)$$

CM *et al.*, in preparation

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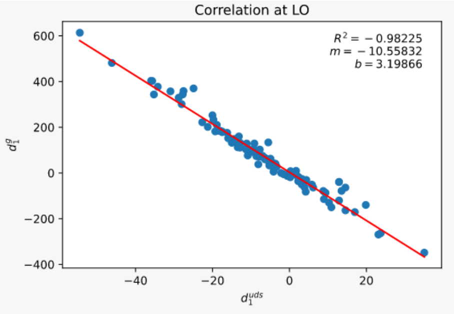

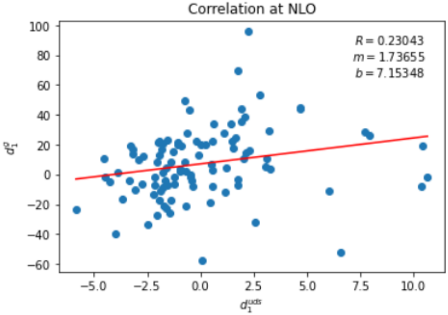

Data-driven extractions of (d_1^q, d_1^g) possible at EIC

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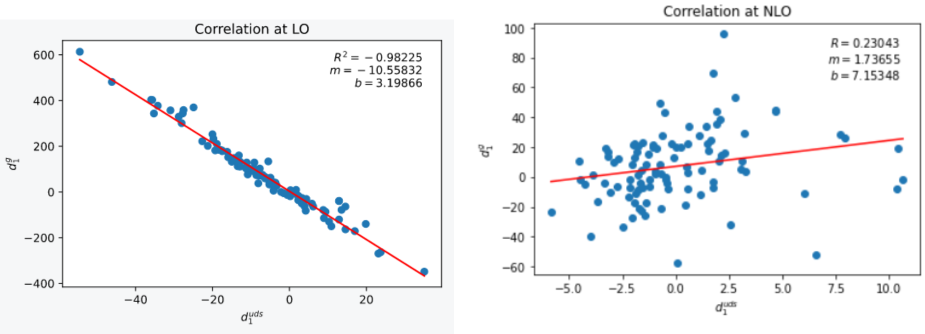


Data-driven extractions of (d_1^q, d_1^g) possible at EIC

However $(d_1^q, d_1^g, d_3^q, d_3^g)$ remains out of reach due to shadow D-terms

Unmentioned topics

- Event generator EpIC for EIC (K. Tezgin *et al.*)
- Impact studies at JLab 12 (O. Bessidskaia *et al.*)
- Small-x GPDs and Shuvaev Transform at LHC (H. Dutrieux *et al.*)
- Nucleon GPD modelling (M. Riberdy *et al.*)
- Double DVCS studies (V. Martínez-Fernández *et al.*)
- Transition GPDs (CM *et al.*)
- ...

Software Targets for 2022

- Release of PARTONSv3 (including APFEL++ interface)
- Update of Website and documentation
- Release of Python interface to PARTONS
- Release of the pion branch of PARTONS

Conclusions

Summary

- After 25 years, GPDs formalism is well established ...
- ... but the GPDs themselves remain poorly known
- the situation may change with JLAB 12, EIC and EicC

Perspectives

- Significant efforts in phenomenology remain be done (CFF and GPD)
- Multichannel analysis could help solving the deconvolution problem
- Ab-initio computations may provide insights in the next decade

In the perspective of EIC and EicC, a lot of work remains to be done to exploit the forthcoming data.

Thank you for your attention

Back up slides

Lattice and CSM distributions

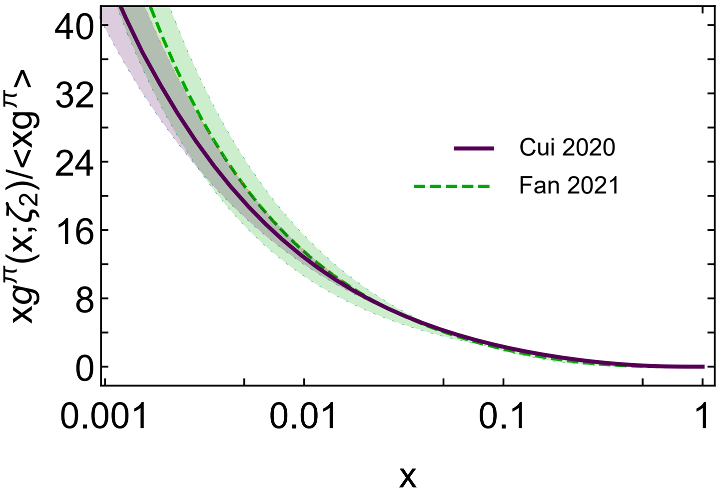
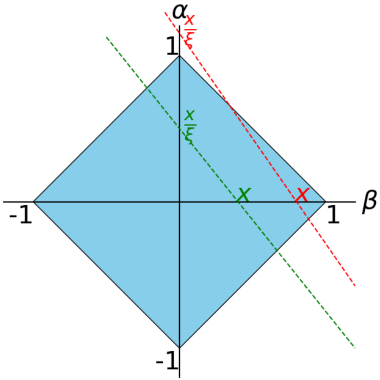


figure from L. Chang and C.D. Roberts, Chin.Phys.Lett. 38 (2021) 8, 081101

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) \left[F(\beta,\alpha) + \xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

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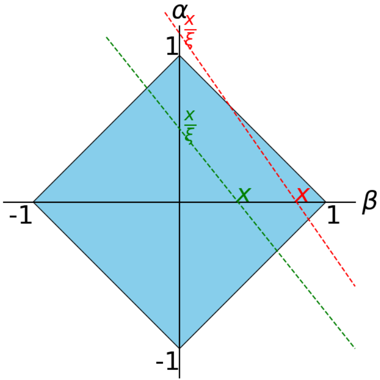

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Is it possible to recover the DDs from the DGLAP region only?

GPD properties from Radon Transform

- Double Distribution representation:

$$H(x,\xi) = D(x/\xi) + \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi) F_D(\beta,\alpha)$$

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$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \text{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

insuring the uniqueness of the extension up to D-term like terms.

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- F_D is the Radon transform of $H - D$.
- Since DD are compactly supported, we can use the **Boman and Todd-Quinto theorem** which tells us

$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \text{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

insuring the uniqueness of the extension up to D-term like terms.

New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**