Factorization for TMDs in SIDIS at Subleading Power

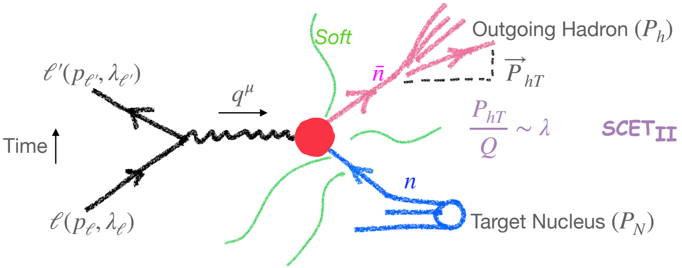
Anjie Gao

w/ Markus Ebert, Iain Stewart

arXiv: 2112.07680

CPHI 2022

Basics

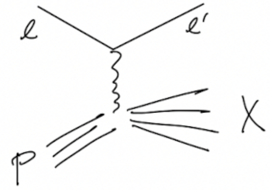


- Lorentz invariants $Q = \sqrt{-q^2}\,, \quad x = \frac{Q^2}{2P_N\cdot q}\,, \quad y = \frac{P_N\cdot q}{P_N\cdot p_\ell}\,, \quad z = \frac{P_N\cdot P_h}{P_N\cdot q}$
- $$\frac{d\sigma}{dx\,dy\,dz\,d^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4}\frac{y}{z}L_{\mu\nu}(p_\ell, p_{\ell'})W^{\mu\nu}(q, P_N, P_h)$$
- $$W^{\mu\nu}(q, P_N, P_h) = \sum\!\!\!\!\!\!\!\int_X \int \frac{d^4b}{(2\pi)^4}\, e^{ib\cdot q}\, \langle N|J^{\dagger\,\mu}(b)|h, X\rangle\, \langle h, X|J^\nu(0)|N\rangle$$

$$L^{\mu\nu}(p_\ell, p_{\ell'}) = \langle \ell|J_e^{\dagger\mu}|\ell'\rangle\,\langle \ell'|J_e^\nu|\ell\rangle$$
$$= 2\delta_{\lambda_\ell\lambda_{\ell'}}\left[\left(p_\ell^\mu p_{\ell'}^\nu + p_\ell^\nu p_{\ell'}^\mu - p_\ell\cdot p_{\ell'}\, g^{\mu\nu}\right) + i\lambda_\ell\, \epsilon^{\mu\nu\rho\sigma} p_{\ell\rho} p_{\ell'\sigma}\right]$$

$$J^\mu = \sum_f \bar{q}_f\gamma^\mu q_f\,, \qquad J^\mu_{\bar{\ell}\ell} = \bar{\ell}\gamma^\mu\ell$$

Tensor Decomposition for (Unporlarized) Inclusive DIS



- Summing over final states

$$W^{\mu\nu}(q, P_N) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} \, e^{ib\cdot q} \, \langle N | J^{\dagger\,\mu}(b) | X \rangle \, \langle X | J^{\nu}(0) | N \rangle$$

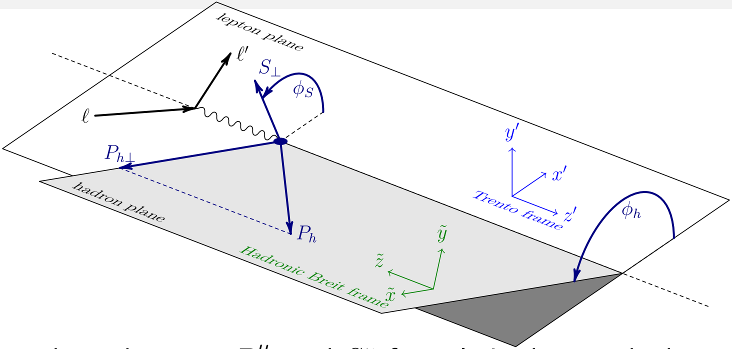
$$= \int \frac{\mathrm{d}^4 b}{(2\pi)^4} \, e^{ib\cdot q} \, \langle N | J^{\dagger\,\mu}(b) J^{\nu}(0) | N \rangle$$

- $q_\mu W^{\mu\nu} = 0$, $W^{\mu\nu} = W^{\nu\mu}$, dependence on only two vectors q^μ and P_N^μ

$\Rightarrow$ Two structure functions

$$W^{\mu\nu}(q, P_N) = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \left(P_N^\mu - \frac{P_N \cdot q}{q^2} q^\mu \right) \left(P_N^\nu - \frac{P_N \cdot q}{q^2} q^\nu \right)$$

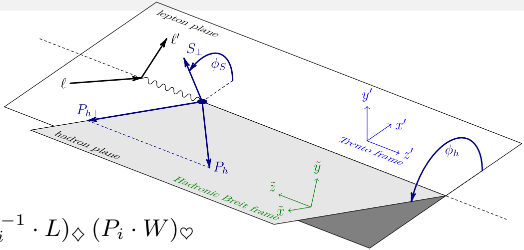
Kinematics and Tensor Decomposition for SIDIS



- Extra dependence on P_h^μ, and S^μ for polarized target hadron
- $S^\mu = \Big(0\,,\, S_T\cos\phi_S\,,\, S_T\sin\phi_S\,,\, -S_L\Big)_T$
- $W^{\mu\nu} = W_U^{\mu\nu} + S_L W_L^{\mu\nu} + S_T\cos(\phi_h-\phi_S)W_{T\tilde{x}}^{\mu\nu} + S_T\sin(\phi_h-\phi_S)W_{T\tilde{y}}^{\mu\nu}$
- Different polarization contributions of lepton/hadron $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}\right)$

$$\frac{d\sigma}{dx\,dy\,dz\,d^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L\cdot W)_{UU} + \lambda_\ell(L\cdot W)_{LU}$$
$$+ S_L(L\cdot W)_{UL} + \lambda_\ell S_L(L\cdot W)_{LL} + S_T(L\cdot W)_{UT} + \lambda_\ell S_T(L\cdot W)_{LT}\Big]$$

Kinematics and Tensor Decomposition for SIDIS



- Projection

$$(L \cdot W)_{\diamondsuit\heartsuit} = \sum_{i=-1}^{7} (P_i^{-1} \cdot L)_\diamondsuit \, (P_i \cdot W)_\heartsuit$$

- Projectors defined in the hadronic Breit frame

$$P_{-1}^{\mu\nu} = (\tilde{x}^\mu \tilde{x}^\nu + \tilde{y}^\mu \tilde{y}^\nu) \,, \; P_0^{\mu\nu} = \tilde{t}^\mu \tilde{t}^\nu \,, \; P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu) \,, \dots \,, \; P_7^{\mu\nu}$$

- $q \cdot L = q \cdot W = 0 \Rightarrow$ no $\tilde{z} \Rightarrow 3 \times 3 = 9$ projectors
- Parity and hermiticity constraints reduce # of structure functions

$\Rightarrow$ In total 18 structure functions [Bacchetta et al '06]

$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)} \,,$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)} \sin(\phi_h) W_{LU}^{\sin(\phi_h)} \,,$$

$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2} \cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h-\phi_S)}\right] \,,$$

......

Power Expansion in $\lambda = P_{hT}/Q \ll 1$

- Standard structure function decomposition [Bacchetta et al '06]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\delta_{\lambda_\ell\lambda_{\ell'}}}{1-\epsilon}\Big[\big(W_{-1}+\epsilon W_0\big)+\epsilon\cos(2\phi_h)W_3 + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h W_1 + \lambda\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h W_2\Big]+\dots\,.$$

- $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$
- $W_i = P_i^{\mu\nu}W_{\mu\nu}$ with projectors $P_i^{\mu\nu}$ (defined in the hadronic Breit frame)
- $P_{-1}^{\mu\nu} = (\tilde{x}^\mu\tilde{x}^\nu + \tilde{y}^\mu\tilde{y}^\nu)\,, \quad P_3^{\mu\nu} = \tilde{x}^\mu\tilde{x}^\nu - \tilde{y}^\mu\tilde{y}^\nu\,,$

  $P_1^{\mu\nu} = -(\tilde{t}^\mu\tilde{x}^\nu + \tilde{x}^\mu\tilde{t}^\nu)\,, \quad P_2^{\mu\nu} = \mathrm{i}\big(\tilde{t}^\mu\tilde{x}^\nu - \tilde{x}^\mu\tilde{t}^\nu\big)\,, \quad P_0^{\mu\nu} = \tilde{t}^\mu\tilde{t}^\nu\,,$
- $\lambda = P_{hT}/Q \ll 1$
- $W_{-1}\,, W_3 \sim \mathcal{O}\big((\frac{P_{hT}}{Q})^0\big)$, standard factorization theorems (CSS, SCET)
- $W_1\,, W_2 \sim \mathcal{O}\big(\frac{P_{hT}}{Q}\big)$
  - First treated in parton model (tree level matching) [Mulders, Tangerman '95]
  - Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - Conjecture: Resolved by adding a LP soft function [Bacchetta et al '19]
- $W_0 \sim \mathcal{O}\big((\frac{P_{hT}}{Q})^2\big)$, not considered in this talk

$\Rightarrow$ *Use SCET to derive all-order factorization at subleading power*

Factorization for Structure Functions: General Procedure

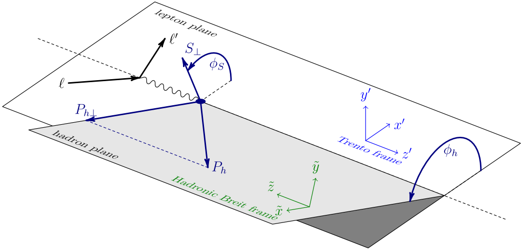
$$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{\mathrm{i}b\cdot q} \langle N | J^{\dagger\,\mu}(b) | h, X\rangle \langle h, X | J^\nu(0) | N\rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at next-to-leading power

- Match SCET currents (operators) with QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \ldots$

  (in the factorization frame: $P_N^\mu = P_N^- \frac{n^\mu}{2}$, $P_h^\mu = P_h^+ \frac{\bar{n}^\mu}{2}$, $g_{F\perp}^{\mu\nu} = g_{B\perp}^{\mu\nu} + \mathcal{O}(P_{hT}/Q)$)
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $\quad W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand projectors in the factorization frame: $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \ldots$

$$P_1^{\mu\nu} = \frac{1}{2}(t^\mu x^\nu + x^\mu t^\nu) - \frac{q_T}{Q} x^\mu x^\nu + \ldots , \quad P_2^{\mu\nu} = \frac{1}{2}(t^\mu x^\nu - x^\mu t^\nu) + \ldots$$



Factorization for Structure Functions: General Procedure

$$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q} \langle N|J^{\dagger\,\mu}(b)|h, X\rangle \langle h, X|J^{\nu}(0)|N\rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at next-to-leading power

- Match SCET currents (operators) with QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \ldots$
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $\qquad W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand projectors in the factorization frame: $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \ldots$

Categories of power corrections

1) Subleading current contributions, $P_i^{(0)} \cdot W^{(1)}$
2) Kinematic correction, $P_i^{(1)} \cdot W^{(0)}$
3) Subleading soft contributions including SCET$_{\text{II}}$ subleading Lagrangians $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1/2)} + \mathcal{L}^{(1)} + \ldots$

- Assumption: Glauber Lagrangian $\mathcal{L}_G^{(0)}$ doesn't spoil factorization

Factorization at Leading Power

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta} C_f^{(0)}(Q)\, \bar{\chi}^\alpha_{\bar{n},\omega_b} [S^\dagger_{\bar{n}} S_n]\, \chi^\beta_{n,\omega_a} \sim C_f^{(0)}(Q)$ 

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat{B}_f^{\beta'\beta}(x, \vec{b}_T) = \Big\langle N \Big| \bar{\chi}_n^\beta(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi_n^{\beta'}(0) \Big| N \Big\rangle$$

$$\hat{\mathcal{G}}_f^{\alpha\alpha'}(z, \vec{b}_T) = \frac{1}{2z} \sum\!\!\!\!\!\!\int_X \Big\langle 0 \Big| \delta(\omega_b - \overline{\mathcal{P}}_{\bar{n}})\, \chi_{\bar{n}}^\alpha(b_\perp) \Big| h, X \Big\rangle \Big\langle h, X \Big| \bar{\chi}_{\bar{n}}^{\alpha'}(0) \Big| 0 \Big\rangle$$

- Soft Wilson lines yield the TMD soft function

$$\mathcal{S}(b_T) = \frac{1}{N_c} \operatorname{tr} \Big\langle 0 \Big| \big[S_n^\dagger(b_\perp)\, S_{\bar{n}}(b_\perp)\big]\, \big[S_{\bar{n}}^\dagger(0)\, S_n(0)\big] \Big| 0 \Big\rangle\,.$$

- Combine into the quark correctors

$$B_f^{\beta'\beta}(x, \vec{b}_T) = \hat{B}_f^{\beta'\beta}(x, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}\,, \qquad \mathcal{G}_f^{\alpha'\alpha}(z, \vec{b}_T) = \hat{\mathcal{G}}_f^{\alpha'\alpha}(z, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}$$

$\Rightarrow$ Factorized leading power hadronic tensor

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[B_f(x, \vec{b}_T)\, \gamma_\perp^\mu\, \mathcal{G}_f(z, \vec{b}_T)\, \gamma_\perp^\nu\Big]\,.$$

- Hard function: $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$

Structure Functions at Leading Power

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}(x, \vec{p}_T) = \frac{1}{4}\left\{ f_1 \not{n} + \mathrm{i} h_1^\perp \frac{[\not{p}_\perp, \not{n}]}{2M_N} \right\}^{\beta'\beta} + \ldots ,$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{k}_T) = \frac{1}{4}\left\{ D_1 \not{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\not{k}_\perp, \not{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha} + \ldots$$

- $h_1^\perp$ Boer-Mulders function, $H_1^\perp$ Collins function
- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{-1}^{(0)} = \mathcal{F}\left[\mathcal{H}^{(0)}\, f_1 D_1\right] ,$$

$$W_3^{(0)} = \mathcal{F}\left[-\frac{2\, p_{Tx}\, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \mathcal{H}^{(0)}\, h_1^\perp H_1^\perp\right] ,$$

[Bacchetta et al '06]

$$\mathcal{F}[\omega\, \mathcal{H}\, g\, D] = 2z \sum_f \int \mathrm{d}^2 p_T\, \mathrm{d}^2 k_T\, \delta^2\!\left(\vec{q}_T + \vec{p}_T - \vec{k}_T\right) \omega(\vec{p}_T, \vec{k}_T)\, \mathcal{H}_f(Q)\, g_f(x, p_T)\, D_f(z, k_T)$$

Kinematic Correction for W_1

- Taking

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right] ,$$

contract with $P_1^{(1)\mu\nu} = -\frac{q_T}{Q} x^\mu x^\nu$

$\Rightarrow$ kinematic corrections for W_1

$$\mathcal{F}\left\{ -\frac{P_{hT}}{zQ} \mathcal{H}^{(0)} \left[f_1 D_1 - \frac{2\, p_{Tx}\, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} h_1^\perp H_1^\perp \right] \right\} \in W_1$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

Unique hard operator to all orders [Feige et al '17]

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a}\bar{\chi}_{\bar{n},\omega_b}[S_{\bar{n}}^\dagger S_n]\,\gamma^\mu\,\not{\mathcal{P}}_\perp\,\not{\bar{n}}\chi_{n,\omega_a} + \text{h.c.} \sim$$

- Reparameterization of SCET relates it with the LP one

$\Rightarrow$ The Wilson coefficient is identical to the leading power one, $C_f^{(0)}(Q)$

- Plug these currents into $J_{\mathcal{P}}^{(1)\dagger\mu}J^{(0)\nu} + J^{(0)\dagger\mu}J_{\mathcal{P}}^{(1)\nu}$

$$\hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f \int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)\,\mathcal{S}(b_T)$$
$$\times\left\{\mathrm{Tr}\left[\hat{B}_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\,\gamma^\nu\right] + \mathrm{Tr}\left[\hat{B}_f(x,\vec{b}_T)\,\gamma^\mu\,\hat{\mathcal{G}}_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\right]\right\}.$$

$$\hat{B}_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T)$$
$$\equiv \frac{1}{2Q}\theta(\omega_a)\left\{\left\langle N\middle|\bar{\chi}_n^\beta(b_\perp^\mu)\,\left[\not{\mathcal{P}}_\perp\not{\bar{n}}\,\chi_{n,\omega_a}(0)\right]^{\beta'}\middle|N\right\rangle + \left\langle N\middle|\left[\bar{\chi}_n(b_\perp^\mu)\,\not{\bar{n}}\,\not{\mathcal{P}}_\perp^\dagger\right]^\beta\,\chi_{n,\omega_a}^{\beta'}(0)\middle|N\right\rangle\right\}$$
$$= \mathrm{i}\,\frac{1}{2Q}\,\frac{\partial}{\partial b_\perp^\rho}\left[\gamma_\perp^\rho\,\not{\bar{n}}\,,\,\hat{B}_f(x,\vec{b}_T)\right]^{\beta'\beta},$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

- Define $B_{\mathcal{P}f}$, $\mathcal{G}_{\mathcal{P}f}$ and $W_{\mathcal{P}}^{(1)\mu\nu}$

$$B_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T) \equiv \mathrm{i}\frac{1}{2Q}\frac{\partial}{\partial b_\perp^\rho}\left[\gamma_\perp^\rho\,\bar{n}\!\!\!/\,,\, B_f(x,\vec{b}_T)\right]^{\beta'\beta}\,,$$ where $B_f^{\beta'\beta}(x,\vec{b}_T) = \hat{B}_f^{\beta'\beta}(x,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c}\sum_f \int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)$$
$$\times\left\{\mathrm{Tr}\left[B_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu\right] + \mathrm{Tr}\left[B_f(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\right]\right\}.$$

- Equivalent to $\hat{W}_{\mathcal{P}}^{(1)\mu\nu}$ (noticing that $(n_\mu - \bar{n}_\mu)P_i^{\mu\nu} = \mathcal{O}(P_{hT}/Q)$)

$$W_{\mathcal{P}}^{(1)\mu\nu} - \hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)\,\frac{\mathrm{i}}{Q}\left(\frac{\partial}{\partial b_\perp^\rho}\sqrt{\mathcal{S}(b_T)}\right)\sqrt{\mathcal{S}(b_T)}$$
$$\times\left\{(\bar{n}^\nu - n^\nu)\,\mathrm{Tr}\left[\gamma_\perp^\rho\,\hat{B}_f(x,\vec{b}_T)\,\gamma^\mu\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\right] + (n^\mu - \bar{n}^\mu)\,\mathrm{Tr}\left[\hat{B}_f(x,\vec{b}_T)\,\gamma_\perp^\rho\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\,\gamma^\nu\right]\right\}$$

- Same leading power functions appear, in momentum space

$$B_{\mathcal{P}f}(x,\vec{p}_T) = \frac{1}{2Q}\left[p\!\!\!/_\perp\bar{n}\!\!\!/\,,\,B_f\right] = \frac{1}{2Q}\left\{f_1 p\!\!\!/_\perp - ih_1^\perp\frac{p_T^2\,[n\!\!\!/,\bar{n}\!\!\!/]}{2M_N}\right\} + \dots$$

Subleading Operators: with $\mathcal{B}_\perp$ Insertion

- Fields and currents of definite helicity [Moult et al '15]

$$\mathcal{B}^a_{n\pm} = -\varepsilon_{\mp\mu}(n,\bar{n})\,\mathcal{B}^{a\mu}_{n\perp,\omega_c}\,,\ \chi^\alpha_{n\pm} = \frac{1\pm\gamma_5}{2}\chi^\alpha_{n,\omega_a}\,,\ J^{\bar{\alpha}\beta}_{\bar{n}n\pm} = \mp\sqrt{\frac{2}{\omega_a\,\omega_b}}\,\frac{\varepsilon^\mu_\mp(\bar{n},n)}{\langle n\mp|\bar{n}\pm\rangle}\,\bar{\chi}^{\bar{\alpha}}_{\bar{n}\pm}\,\gamma_\mu\chi^\beta_{n\pm}$$

$$\varepsilon^\mu_+(p,r) = \frac{\langle p+|\gamma^\mu|r+\rangle}{\sqrt{2}\langle rp\rangle}\,, \qquad \varepsilon^\mu_-(p,r) = -\frac{\langle p-|\gamma^\mu|r-\rangle}{\sqrt{2}[rp]}\,,$$

- The complete set of operators in the helicity basis [Feige et al '17]

$$O^{(1)a\,\bar{\alpha}\beta}_{1+-} = \mathcal{B}^a_{n+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,, \qquad O^{(1)a\,\bar{\alpha}\beta}_{1-+} = \mathcal{B}^a_{n-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,,$$
$$O^{(1)a\,\bar{\alpha}\beta}_{2--} = \mathcal{B}^a_{\bar{n}-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,, \qquad O^{(1)a\,\bar{\alpha}\beta}_{2++} = \mathcal{B}^a_{\bar{n}+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,.$$

- Parity and charge conjugation invariance $\Rightarrow C^{(1)}_{\lambda_3\lambda_{12}} = C^{(1)}_{-\lambda_3-\lambda_{12}}$

$\Rightarrow$ Combination of helicity operators appear as

$$\mathcal{B}_{n+}J_{\bar{n}n-} + \mathcal{B}_{n-}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\,[S^\dagger_{\bar{n}}S_n]\,\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
$$\mathcal{B}_{\bar{n}-}J_{\bar{n}n-} + \mathcal{B}_{\bar{n}+}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\not{\mathcal{B}}_{\perp\bar{n},\omega_c}\,[S^\dagger_{\bar{n}}S_n]\,\chi_{n,\omega_a}$$

- Same soft Wilson lines as LP since fields always appears as $S_n\chi_n$, $S_n\mathcal{B}_nS^\dagger_n$

Subleading Current: with $\mathcal{B}_\perp$ Insertion

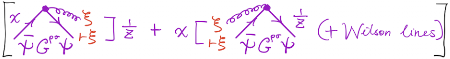
$$\frac{1}{\sqrt{\omega_a\omega_b}}\,\bar\chi_{\bar n,\omega_b}[S^\dagger_{\bar n}S_n]\,\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}\,, \qquad \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar\chi_{\bar n,\omega_b}\not{\mathcal{B}}_{\perp \bar n,\omega_c}\,[S^\dagger_{\bar n}S_n]\,\chi_{n,\omega_a}$$

- Hermiticity + $n \leftrightarrow \bar n$ symmetry: only one Wilson coefficient $C_f^{(1)}$
- Summing over helicities gives

$$\sum_{\lambda_e,\lambda_{12},\lambda_3} C_f^{(1)}\,\mathcal{B}_{\lambda_3}\,J_{\bar n n\lambda_{12}}J_{\lambda_e} \sim J_{\mathcal{B}}^{(1)\mu}\,J_{e\mu}$$

where $J_{e\mu} = \bar e\gamma_\mu e$ and (denoting $\xi = \omega_c/Q$),

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar n^\mu)\int \mathrm{d}\omega_a\mathrm{d}\omega_b\mathrm{d}\omega_c\, C_f^{(1)}(Q,\xi)$$
$$\times\Big[\delta(\omega_a+\omega_c-Q)\,\delta(\omega_b-Q)\,\bar\chi_{\bar n,\omega_b}[S^\dagger_{\bar n}S_n]\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
$$+\,\delta(\omega_a-Q)\,\delta(\omega_b+\omega_c-Q)\,\bar\chi_{\bar n,\omega_b}\not{\mathcal{B}}_{\perp \bar n,\omega_c}[S^\dagger_{\bar n}S_n]\chi_{n,\omega_a}\Big]$$

$\sim C_f^{(1)}(Q,\xi)$ 

Subleading Current: with $\mathcal{B}_\perp$ Insertion

Denoting $\xi = \omega_c/Q$, define the q-g-q correlators as

$$\hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) \equiv Q\, \langle N|\, \big[\bar{\chi}^{\beta}_{n,\omega_a}\, \mathcal{B}^{\rho}_{\perp n,-\omega_c}\big](b^\mu_\perp)\, \chi^{\beta'}_{n}(0)\,|N\rangle\ ,$$

$$\hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) \equiv \frac{Q}{2z} \sum\!\!\!\!\!\!\!\int_X \langle 0|\, \big[\bar{\chi}^{\beta}_{\bar{n},\omega_b}\, \mathcal{B}^{\rho}_{\perp\bar{n},\omega_c}\big](b^\mu_\perp)\,|h,X\rangle\,\langle h,X|\,\chi^{\beta'}_{\bar{n}}(0)\,|0\rangle$$

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) = \hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)},$$
$$\tilde{\mathcal{G}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) = \hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W^{(1)\mu\nu}_{\mathcal{B}} = \frac{2z}{Q}\sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\int_0^1 \mathrm{d}\xi\, \mathcal{H}^{(1)}(Q,\xi)(n^\mu+\bar{n}^\mu)$$
$$\times\,\mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\,\gamma_\rho\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu_\perp + B_f(x,\vec{b}_T)\,\gamma^\nu_\perp\,\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}\,f}(z,\xi,\vec{b}_T)\,\gamma_\rho\Big] + \text{h.c.}\,.$$

$$\mathcal{H}^{(1)}(Q,\xi) = C^{(1)}_f(Q,\xi)\,C^{(0)}_f(Q)$$

In momentum space, $\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}$ can be decomposed as [Boer, Mulders, Pijlman '03] [Bacchetta, Mulders, Pijlman '04]

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}(x,\xi,\vec{p}_T) = \frac{xM_N}{2}\,\bigg\{\Big[\big(\tilde{f}^\perp - \mathrm{i}\tilde{g}^\perp\big)\frac{p_{\perp\sigma}}{M_N}\big(g^{\rho\sigma}_\perp - \mathrm{i}\epsilon^{\rho\sigma}_\perp\gamma_5\big) + \mathrm{i}\big(\tilde{h}+\mathrm{i}\,\tilde{e}\big)\gamma^\rho_\perp\Big]\frac{\not{n}}{2}\bigg\}^{\beta'\beta} + \ldots$$

Vanishing Soft Contributions

- Subleading soft contributions exist in general, and are important in other processes
- In subleading SIDIS, we showed that they all vanish:

(1) Operators involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ yield

$\hat{\mathcal{S}}_1^\rho(b_\perp) \sim \mathrm{tr} \left\langle 0 \middle| \left[S_n^\dagger(b_\perp)\, S_{\bar{n}}(b_\perp)\right] \left[S_{\bar{n}}^\dagger(0)\, S_n(0) g \mathcal{B}_{s\perp}^{(n)\rho}(0)\right] \middle| 0 \right\rangle$

Vanish due to charge conjugation & parity invariance & Lorentz invariance & translation invariance of the vacuum

(2) Operators involving a $n \cdot \partial_s$, $n \cdot \mathcal{B}_s^{(\bar{n})}, \ldots$ give $\frac{\partial}{\partial b_s^{\mp}} \mathcal{S}(b_T, b_s^+ b_s^-)\Big|_{b_s^{\pm} \to 0}$, which scales linear in $\bar{n}$ or n under RPI-III ($n \to e^\alpha n$, $\bar{n} \to e^{-\alpha}\bar{n}$) of SCET , thus vanishes

Vanishing Soft Contributions

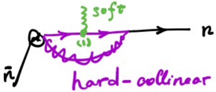
(3) SCET$_{\text{II}}$ Subleading Lagrangian insertions

$$W_{\mathcal{L}}^{(1)\mu\nu} \sim \langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\mathrm{d}^4y\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1/2)}(x)\mathcal{L}^{(1/2)}(y)\big]|N\rangle$$
$$+\,\langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1)}(x)\big]|N\rangle + \dots$$

Since μ, ν are transverse $\left(J^{(0)\mu} \sim (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\,\bar\chi^\alpha_{\bar n,\omega_b}[S^\dagger_{\bar n} S_n]\,\chi^\beta_{n,\omega_a}\right)$,
when contracting with $P_1^{\mu\nu} = -(\tilde t^\mu \tilde x^\nu + \tilde x^\mu \tilde t^\nu)$, $P_2^{\mu\nu} = \mathrm{i}\big(\tilde t^\mu \tilde x^\nu - \tilde x^\mu \tilde t^\nu\big)$, ...
such contributions vanish

(4) $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(1)}\big]$, $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(2)}\big]$, $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(1)}\mathcal{L}_{\text{I}}^{(1)}\big]$ in SCET$_{\text{I}}$

$\rightarrow$ hard scattering operators in SCET$_{\text{II}}$
Vanish since μ, ν in $J^{(0)}$ are (again) transverse



Results

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$

(Kinematic corrections)

$$-\,\mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q}\,f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h}\,h_1^\perp H_1^\perp\Big]$$

(From the $\mathcal{P}_\perp$ operators)

$$+\,\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h}k_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N}p_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

$$W_2 = \mathcal{F}\Big\{\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{g}^\perp D_1 + \frac{M_N}{M_h}k_{Tx}\,\tilde{e}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1\tilde{G}^\perp + \frac{M_h}{M_N}p_{Tx}\,h_1^\perp\tilde{E}\Big)\Big]\Big\}$$

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f\int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T+\vec{p}_T-\vec{k}_T\big)\,\omega(\vec{p}_T,\vec{k}_T) \times\int_0^1 \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),p_T)\,D_f(z,(\xi),k_T)$$

New in our results

- Vanishing of the subleading soft contributions
- Soft function, same as leading power (as conjectured in [Bacchetta et al '19])
- Appearance of two hard functions, $\mathcal{H}^{(0)}(Q)$ and $\mathcal{H}^{(1)}(Q,\xi)$
- Dependence on ξ in $\mathcal{H}^{(1)}(Q,\xi)$ and the functions $\tilde{f}^\perp$, $\tilde{D}^\perp$, ...

Structure Functions with Full Spin Dependence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\psi\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\alpha^2}{2Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L\cdot W)_{UU} + S_L(L\cdot W)_{UL} + \lambda_\ell(L\cdot W)_{LU} + \lambda_\ell S_L(L\cdot W)_{LL} + S_T(L\cdot W)_{UT} + \lambda_\ell S_T(L\cdot W)_{LT}\Big],$$

$$(L\cdot W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h)W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h)W_{UU}^{\cos(2\phi_h)},$$

$$(L\cdot W)_{UL} = \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_h)W_{UL}^{\sin(\phi_h)} + \epsilon\sin(2\phi_h)W_{UL}^{\sin(2\phi_h)},$$

$$(L\cdot W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h)W_{LU}^{\sin(\phi_h)},$$

$$(L\cdot W)_{LL} = \sqrt{1-\epsilon^2}W_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_h)W_{LL}^{\cos(\phi_h)},$$

$$(L\cdot W)_{UT} = \sin(\phi_h-\phi_S)\left[W_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon W_{UT,L}^{\sin(\phi_h-\phi_S)}\right] + \epsilon\left[\sin(\phi_h+\phi_S)W_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)W_{UT}^{\sin(3\phi_h-\phi_S)}\right] + \sqrt{2\epsilon(1+\epsilon)}\left[\sin(\phi_S)W_{UT}^{\sin(\phi_S)} + \sin(2\phi_h-\phi_S)W_{UT}^{\sin(2\phi_h-\phi_S)}\right],$$

$$(L\cdot W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S)W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h-\phi_S)W_{LT}^{\cos(2\phi_h-\phi_S)}\right].$$

- We also have results for spin-dependent $\mathcal{O}(P_{hT}/Q)$ structure functions

Structure Function with Full Spin Dependence

- For example

$$W_{UT}^{\sin\phi_S} = \mathcal{F}\Bigg\{ -\frac{q_T}{2Q}\,\mathcal{H}^{(0)}\left(\frac{k_{Tx}}{M_N} f_{1T}^{\perp} D_1 - \frac{2p_{Tx}}{M_h} h_1 H_1^{\perp}\right) \text{(Kinematic corrections)}$$

$$+\,\mathcal{H}^{(0)}\left(-\frac{k_T^2 + \vec{k}_T\cdot\vec{p}_T}{2M_N Q} f_{1T}^{\perp} D_1 + \frac{p_T^2 + \vec{k}_T\cdot\vec{p}_T}{M_h Q} h_1 H_1^{\perp}\right) \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+\mathcal{H}^{(1)}\Bigg[\frac{xM_N}{Q}\left(2\tilde{f}_T D_1 - \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(\tilde{h}_T - \tilde{h}_T^{\perp}\right) H_1^{\perp}\right)$$

$$-\frac{M_h}{zQ}\left(2h_1\tilde{H} + \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(g_{1T}\tilde{G}^{\perp} + f_{1T}^{\perp}\tilde{D}^{\perp}\right)\right)\Bigg]\Bigg\} \text{(From the } \mathcal{B}_{n\perp} \text{ operators)}$$

Discussion

Anomalous dimensions

- Rapidity anomalous dimension is the same as at leading power
$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\zeta) = \hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\nu^2/\zeta)\sqrt{\mathcal{S}(b_T,\mu,\nu)}$,
$$\Rightarrow \frac{\mathrm{d}\log \tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}}{\mathrm{d}\log\zeta} = \frac{1}{4}\frac{\mathrm{d}\log\mathcal{S}}{\mathrm{d}\log\nu} = \frac{1}{4}\gamma_\nu(\mu,b_T)$$
- Anomalous dimension of $C_f^{(1)}$ have been calculated to one loop, with single log dependence on ξ [Beneke et al, '17 '18]
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_f^{(1)}(Q,\xi,\mu) = \int\frac{\mathrm{d}\xi'}{\xi'}\gamma^{(1)}_{f\,f'}(\xi,\xi',Q,\mu)C^{(1)}_{f'}(\xi',Q,\mu)$$

Discussion

- At leading order, $C_f^{(1)}$ is independent on ξ from tree level matching,
- ξ can be integrated in q-g-q correlators. $W_1, W_2, \cdots$ then fully agree with [Bacchetta et al '06] at leading order (after inclusion of the soft function, as conjectured in [Bacchetta et al '19])
- Anomalous dimension results for $C_f^{(1)}(Q, \xi, \mu)$ confirms the nontrivial ξ dependence

$\Rightarrow$ Disproves the simpler factorization theorem in [Bacchetta et al '19]

- Our results includes radiative corrections: fixed order + resumed logs
- LO $\stackrel{\text{Fact.}}{\Longrightarrow}$ NLO + LL + NLL + ...!
- Recent overlapping work on operator basis, perturbative corrections and anomalous dimensions [Vladimirov, Moos, Scimemi '21]
  - ▷ Calculated $C^{(1)}$ at $\mathcal{O}(\alpha_s)$

Summary & Outlook

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$

(Kinematic corrections)

$$-\,\mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big]$$

(From the $\mathcal{P}_\perp$ operators)

$$+\,\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

- Derived factorization of $W^{\mu\nu}$ at subleading power, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_\perp$
- Obtained factorization formulae for subleading structure functions $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, $W_{LU}^{\sin(\phi_h)}$, $W_{LL}^{\cos(\phi_h)}$, $W_{UT}^{\sin(\phi_S)}$, $W_{UT}^{\sin(2\phi_h-\phi_S)}$, $W_{LT}^{\cos(\phi_S)}$, $W_{LT}^{\cos(2\phi_h-\phi_S)}$, including contributions from the kinematic correction and the subleading operators
- Future Directions: phenomenology including perturbative and resummation effects for $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, ...

Summary & Outlook

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp \Big] \qquad \text{(Kinematic corrections)}$$

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp \Big] \qquad \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp \Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\,\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\,\tilde{H} \Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

- Derived factorization of $W^{\mu\nu}$ at subleading power, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_\perp$
- Obtained factorization formulae for subleading structure functions $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, $W_{LU}^{\sin(\phi_h)}$, $W_{LL}^{\cos(\phi_h)}$, $W_{UT}^{\sin(\phi_S)}$, $W_{UT}^{\sin(2\phi_h-\phi_S)}$, $W_{LT}^{\cos(\phi_S)}$, $W_{LT}^{\cos(2\phi_h-\phi_S)}$, including contributions from the kinematic correction and the subleading operators
- Future Directions: phenomenology including perturbative and resummation effects for $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, ...

Thanks for your attention!