The Drell-Yan q_T Spectrum and Its Uncertainty at $\mathrm{N^3LL'}$

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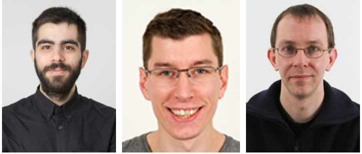
[to appear soon]

in collaboration with

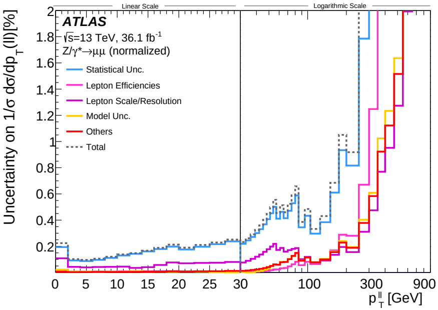
G. Billis, M. Ebert, F. Tackmann

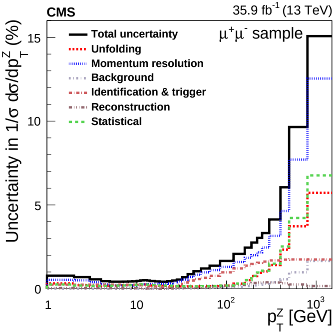


Advantages:

✓ Sub-percent uncertainties, in particular for normalized spectra in $q_T \equiv p_T^Z$

- ▶ Backbone of TMD global fits at N^{3}LL, in particular for x dependence
  [Scimemi, Vladimirov '19; Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro '19]





Introduction: Why LHC Z data for TMD physics?

Advantages:

✓ Sub-percent uncertainties, in particular for normalized spectra in $q_T \equiv p_T^Z$

- Backbone of TMD global fits at N^{3}LL, in particular for x dependence
  [Scimemi, Vladimirov '19; Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro '19]

✓ Large $Q = m_Z$ suppresses power corrections q_T^2/Q^2 to TMD factorization

✓ Synergies with LHC precision program: m_W at LHC needs TMD flavor structure
[Bacchetta, Bozzi, Radici, Ritzmann, Signori '18]

Downsides:

- Only unpolarized TMD PDF accessible
- Spectrum peaks at $q_T = 4 - 5\,\mathrm{GeV}$
  - Nonperturbative effects already suppressed by $(\Lambda_{\mathrm{QCD}}/q_T)^2$
  - Turn this into a virtue and parametrize leading (quadratic) corrections in order to make model-independent statements
    [See talk by Z. Sun for more on this!]

Goal

Percent-level perturbative description of $pp \to Z/\gamma^* \to \ell^+\ell^-$ at the LHC to be able to make definitive statements about nonperturbative TMD dynamics.

Perturbative ingredients: Factorized TMD cross section at N^{3}LL$'$

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\mathrm{res}}_{\mathrm{fact}}}{dq_T} + \Big[\frac{d\sigma^{\mathrm{FO}}_{\mathrm{full}}}{dq_T} - \frac{d\sigma^{\mathrm{FO}}_{\mathrm{fact}}}{dq_T}\Big] \equiv \frac{d\sigma^{\mathrm{res}}_{\mathrm{fact}}}{dq_T} + \frac{d\sigma^{\mathrm{nons}}_{\mathrm{fact}}}{dq_T}$$

$$\frac{d\sigma_{\mathrm{fact}}}{dQ\,dY\,dq_T} = \sum_q H_{q\bar q}(Q,\mu)\;q_T \int_0^\infty db_T\, b_T\, J_0(q_T b_T) \times f_q^{\mathrm{TMD}}(x_a, b_T, \mu, \zeta)\, f_{\bar q}^{\mathrm{TMD}}(x_b, b_T, \mu, \zeta) + (q \leftrightarrow \bar q)$$

Implemented in SCETlib C++ numerical library [Ebert, JM, Tackmann]:

- Three-loop hard function [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10; Czakon et al. '21]
- Three-loop matching of TMD PDFs onto collinear PDFs [Li, Zhu, '16; Luo, Yang, Zhu, Zhu '19; Ebert, Mistlberger, Vita '20]
  - Prediction includes complete three-loop RG boundary conditions (N^{3}LL$'$)
  - Integral of spectrum is N^{3}LO-accurate
- Four-loop cusp, three-loop noncusp anomalous dimensions [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06]
- Three-loop Collins-Soper kernel [Li, Zhu, '16; Vladimirov '16]

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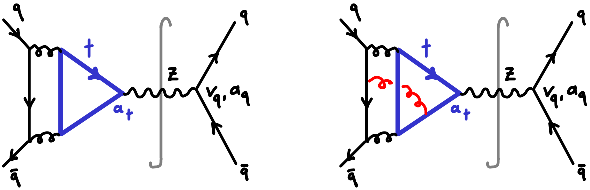

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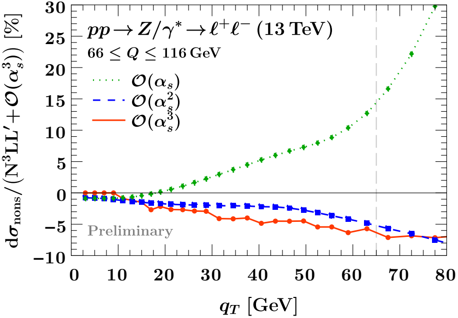
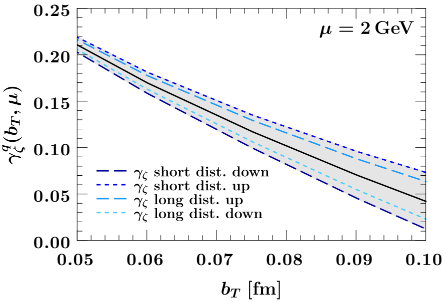
- Three-loop Collins-Soper kernel [Li, Zhu, ’16; Vladimirov ’16]

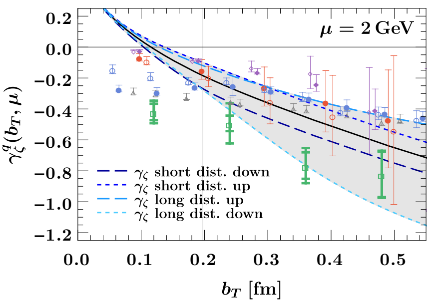
$$\frac{d\sigma_{\rm nons}}{dq_T} = \frac{d\sigma^{\rm FO}_{\rm full}}{dq_T} - \frac{d\sigma^{\rm FO}_{\rm fact}}{dq_T} = \frac{1}{q_T}\mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big)$$



- In-house analytic implementation of all helicity structure functions at $\mathcal{O}(\alpha_s)$
- Fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^2)$ from MCFM
  [Campbell, Ellis, et al. '99, '15]
- Very recently: Precise fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^3)$ from NNLOjet
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]

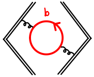
NOTE: Extraction of $\sigma_{\rm nons}$ (and derived results) at α_s^3 is still preliminary.





$$\gamma^q_{\zeta\,\mathrm{NP}}(b_T) = c^i_\zeta \tanh\Big(\frac{\omega^2_{\zeta,i}}{|c_\zeta|} b_T^2\Big) = \mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i} b_T^2 + \mathcal{O}(b_T^4)$$

- Pick central value of $\mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i}$ to serve as bias correction for known leading (NNLL) bottom quark mass effect in γ^q_ζ:



$$\Delta\gamma^q_\zeta(b_T, m_b, \mu) = \frac{\alpha_s^2}{\pi^2} C_F T_F (m_b b_T)^2 \Big(\ln\frac{b_T^2 m_b^2}{4e^{-2\gamma_E}} - 1\Big) \approx -(0.25\,\mathrm{GeV})^2$$

- Vary either ω_ζ ("short distance") or c_ζ ("long distance")

[Collection of lattice data reproduced from Shanahan, Wagman, Zhao, 2107.11930]

- Most general structure of leading NP correction $b_T^2 \Lambda_i^{(2)}(x)$ is complicated
- However, can show that for a given process and fiducial volume, only *a single average coefficient* $\overline{\Lambda}$ remains after the integral over hard phase space Φ_B:
  [Ebert, JM, Stewart, Sun '22]

$$\tilde{\sigma}(b_T) = \tilde{\sigma}^{(0)}(b_T)\Big\{1 + b_T^2\Big(2\overline{\Lambda}^{(2)} + \gamma_{\zeta,q}^{(2)} L_{Q^2}\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\mathrm{QCD}} b_T)^4\big]$$

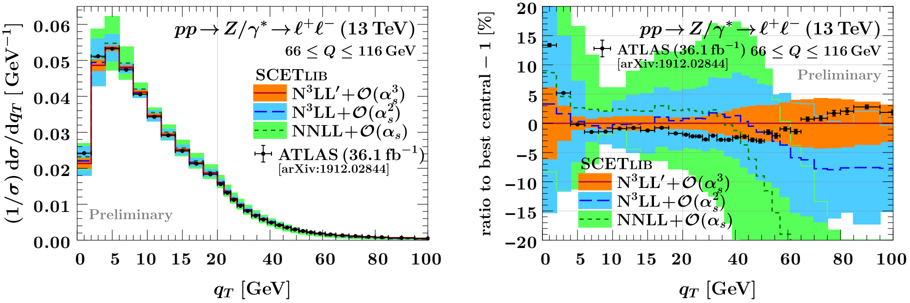
$$\overline{\Lambda}^{(2)} = \frac{\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a,\mu_0)\, f_j^{(0)}(x_b,\mu_0)\big[\Lambda_i^{(2)}(x_a) + \Lambda_j^{(2)}(x_b)\big]}{2\int \mathrm{d}\Phi_B\, A(\Phi_B)\, \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a,\mu_0)\, f_j^{(0)}(x_b,\mu_0)}$$

- Idea: Promote $\overline{\Lambda}^{(2)}$ to a single-parameter Gaussian model

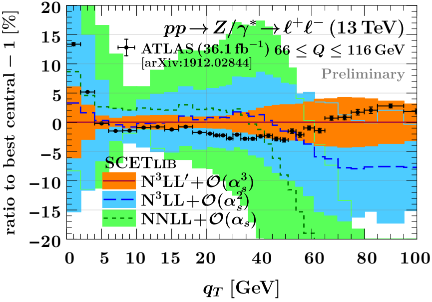
$$f_i^{\mathrm{NP}}(x, b_T) = \exp(-\Omega^2 b_T^2) \quad \text{with} \quad \overline{\Lambda}^{(2)} = -\Omega^2$$

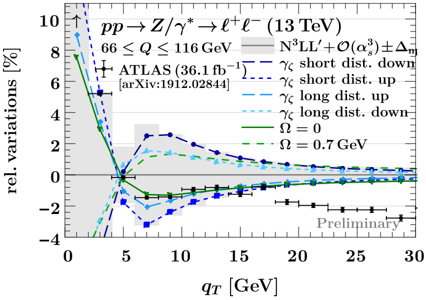
- Take central $\Omega = 0.5\,\mathrm{GeV}$ and vary it as $\Omega = \{0, 0.7\}\,\mathrm{GeV}$
- For $q_T \gg \Lambda_{\mathrm{QCD}}$, this captures the most general form that the leading NP correction to the rapidity-integrated q_T spectrum can take
  [Consistent with observations in Pavia fit, 1912.07550]

Results



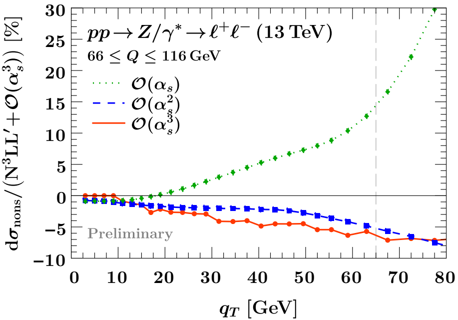
- Perturbative uncertainties estimated by varying all RG boundary scales and parameters governing resummation turn-off
- Excellent perturbative convergence towards three-loop result
- Higher orders are covered by uncertainty estimate at lower orders



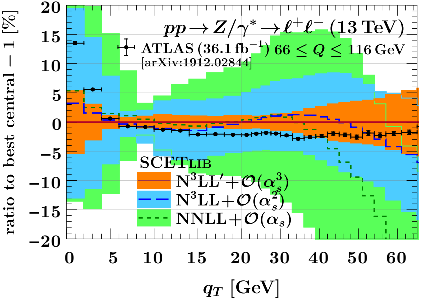


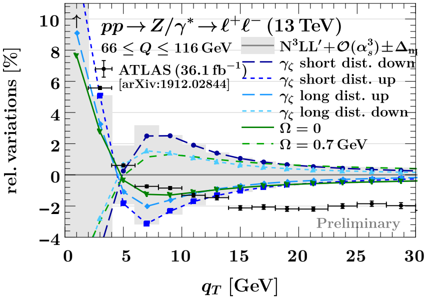
- Taken at face value, the lowest bins seem to prefer *weaker* NP effects
- N^{3}LL closer to data for $q_T \le 15\,\mathrm{GeV}$ with our default NP parameters, suggesting that three-loop and NP corrections can be traded off for one another
- Short-distance variations of γ_ζ dominate in all bins
- Overshoot data at $q_T = 20 - 30\,\mathrm{GeV}$, way outside NP effect strength

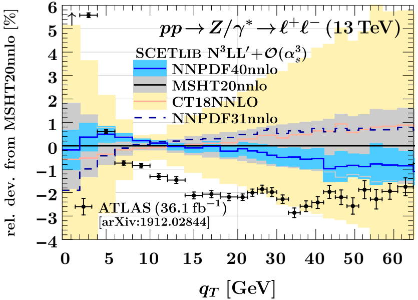
- Would like to estimate parametric uncertainties from collinear PDF and α_s
- Challenge: MC data for α_s^2 and α_s^3 nonsingular cross sections are expensive



- Idea: Restrict to $q_T \leq 65\,\mathrm{GeV}$ where nonsingular is still a small correction
- Perform parametric variations in fast & flexible analytic resummed calculation
- Consistently self-normalize to integral over $q_T \leq 65\,\mathrm{GeV}$



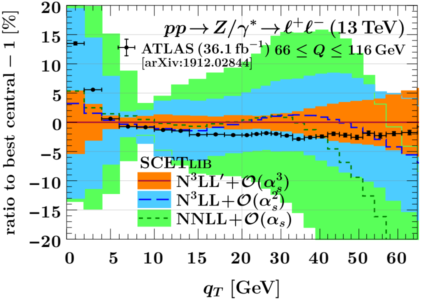


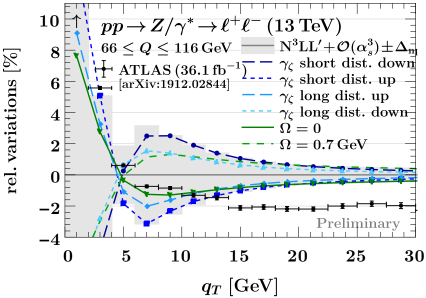


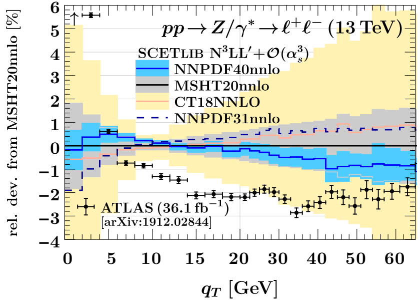
- Conclusions so far are the same for $[0, 65]$ GeV
- Small PDF uncertainty on normalized Y-integrated spectrum with recent PDF sets

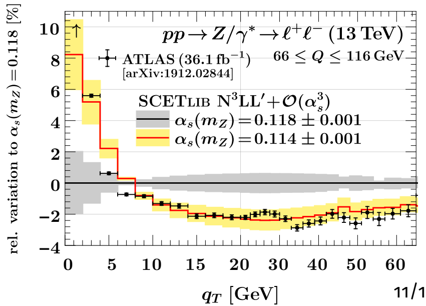
  [But see Ignazio's talk for more differential.]

Parametric PDF and α_s uncertainties

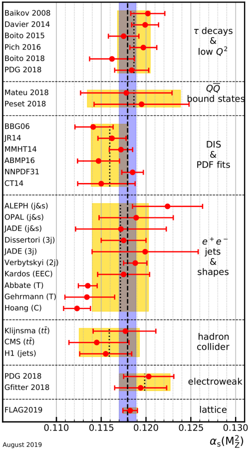








This is not unprecedented …



- Lower values of $\alpha_s(m_Z)$ have previously been reported in fits to e^+e^- event shapes (thrust and C parameter)
- Like TMD observables, these are strongly sensitive to all-order resummation …

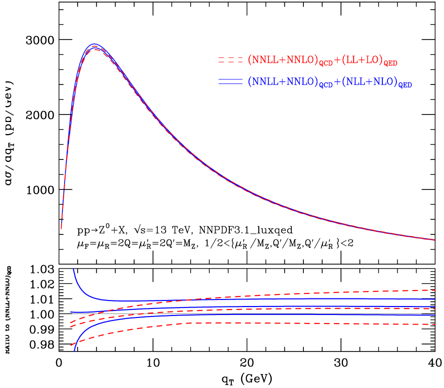
T. Rex Might Have Had Close Cousins

New York Times, March 1, 2022

"That's not the kind of thing you should be doing based on femur robusticity and the presence or absence of a tooth," Dr. Hone added. "If you're going to shoot for the king, don't miss."

Outlook: Systematics at the theory frontier:

- QED effects for on-shell Z well understood
  [Bacchetta, Echevarria '18; Cieri, Ferrera, Sborlini '18; Billis, Tackmann, Talbert '19]
  - Expected to be $\sim 1\%$, but would bring the tail up *more*



[Cieri, Ferrera, Sborlini 1805.11948]

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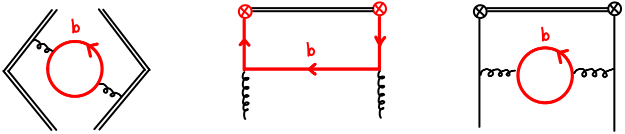
  - Expected to be $\sim 1\%$, but would bring the tail up *more*
- QED radiative corrections to full process and interplay with TMDs are *challenging*
  [But see session on Wed for some recent progress in SIDIS!]
- Subleading power resummation of *nonsingular* cross section
  [Progress towards doing this at least for $\mathcal{O}(q_T/Q)$ azimuthal correlations!]
  [Moos, Scimemi, Vladimirov '21; Ebert, Gao, Stewart '21 $\longrightarrow$ see talk by A. Gao]
- Probably most important: Full treatment of mass effects/flavor thresholds
  - Expect impact on spectrum (and cumulative cross section)
    to be suppressed by $\# \, m_b^2/q_T^2$?



Summary

The Drell-Yan q_T Spectrum at N^{3}LL$'$ and Its Uncertainty:

- Percent-level perturbative predictions at the LHC are possible, and are required to fully leverage LHC data for TMD determinations
- Intriguing hints that the data may prefer a lower value of α_s ...
- Even small changes in α_s strongly impact the peak shape, in ways similar to nonperturbative effects
- Important to consistently match to fixed-order perturbation theory beyond $q_T \sim 20\,\mathrm{GeV}$ to isolate the two effects

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Thank you for your attention!

Backup

RG evolution, profile scales, and Landau pole prescription

- Use exact analytic solutions of CS equation and μ evolution of TMD PDFs, combined with fast numerically exact solution of β function [Ebert '21]
  - Eliminates source of truncation error at fraction of cost of full Runge-Kutta

- Choose RG boundary scales as *hybrid profile scales*, e.g. for $\mu_i(b_T, q_T)$:
  [Lustermans, JM, Tackmann, Waalewijn '19]

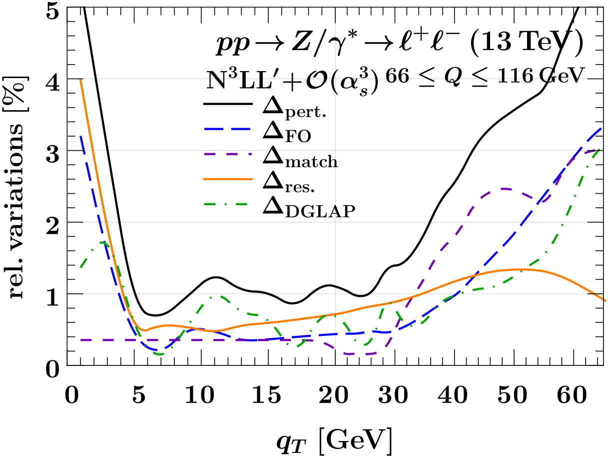
$$\mu_i(b_T, q_T \ll Q) = \frac{2e^{-\gamma_E}}{b_T} \quad \text{but} \quad \mu_i(b_T, q_T \to Q) \to \mu_f = Q$$

  - Turns off resummation in the tail, ensures FO prediction is recovered

- Apply "local" b^* prescription starting at b_T^4 to virtuality scales:
  [Absence of quadratic correction is similar to prescription used in Pavia fit.]

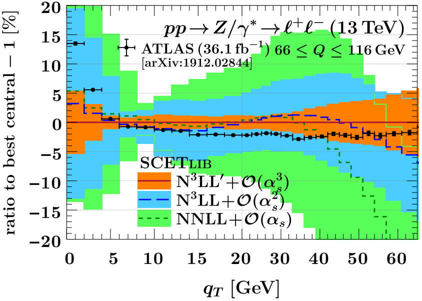
$$\mu_i \to \mu_i^* = \Big[(\mu_i^{\min})^4 + \Big(\frac{2e^{-\gamma_E}}{b_T}\Big)^4\Big]^{1/4} = \frac{2e^{-\gamma_E}}{b_T}\Big\{1 + \mathcal{O}\big[(\mu_i^{\min} b_T)^4\big]\Big\}$$

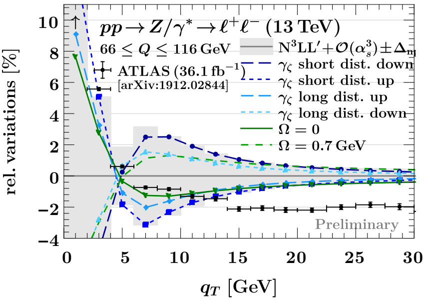
  - Avoids "contaminating" nonperturbative corrections at quadratic order
    [Scimemi, Vladimirov '18; Ebert, JM, Stewart, Sun '22; see also talk by Z. Sun]

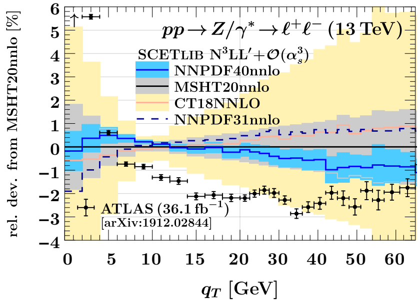


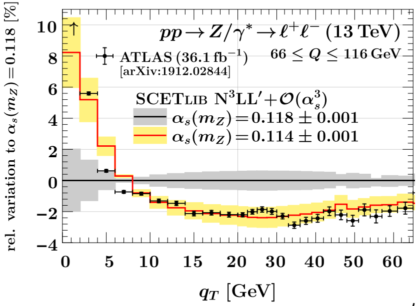
$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$ $66 \leq Q \leq 116$ GeV
$\Delta_{\mathrm{pert.}}$
Δ_{FO}
Δ_{match}
$\Delta_{\mathrm{res.}}$
Δ_{DGLAP}
rel. variations [%]
0
1
2
3
4
5
0
5
10
15
20
30
40
50
60
q_T [GeV]

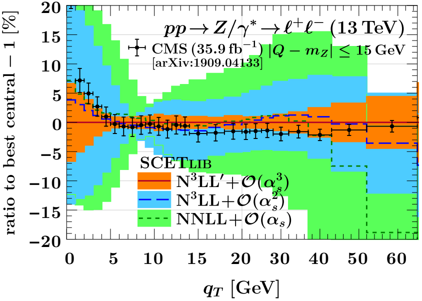
ATLAS normalized spectrum

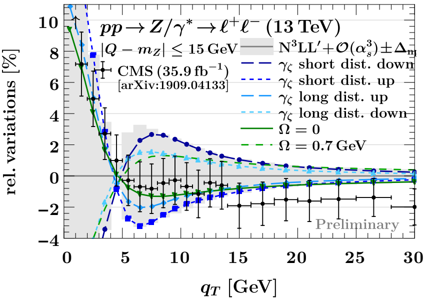









$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
CMS (35.9 fb^{-1}) $|Q - m_Z| \leq 15$ GeV
[arXiv:1909.04133]
SCETlib
N^{3}LL$'$+$\mathcal{O}(\alpha_s^3)$
N^{3}LL+$\mathcal{O}(\alpha_s^2)$
NNLL+$\mathcal{O}(\alpha_s)$
ratio to best central − 1 [%]
q_T [GeV]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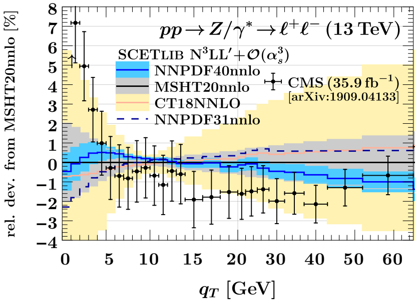
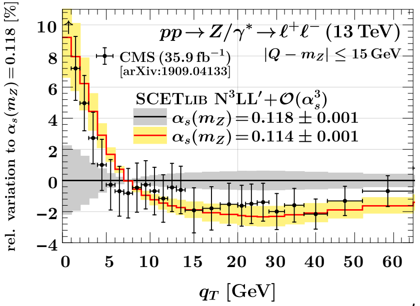
$|Q - m_Z| \leq 15$ GeV
CMS (35.9 fb^{-1})
[arXiv:1909.04133]
N^{3}LL$'$+$\mathcal{O}(\alpha_s^3)\pm\Delta_{\rm m}$
γ_ζ short dist. down
γ_ζ short dist. up
γ_ζ long dist. up
γ_ζ long dist. down
$\Omega = 0$
$\Omega = 0.7$ GeV
Preliminary
rel. variations [%]
q_T [GeV]


$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
SCETlib N^{3}LL$'$+$\mathcal{O}(\alpha_s^3)$
NNPDF40nnlo
MSHT20nnlo
CT18NNLO
NNPDF31nnlo
CMS (35.9 fb^{-1})
[arXiv:1909.04133]
rel. dev. from MSHT20nnlo [%]
q_T [GeV]


$pp \to Z/\gamma^* \to \ell^+\ell^-$ (13 TeV)
$|Q - m_Z| \leq 15$ GeV
CMS (35.9 fb^{-1})
[arXiv:1909.04133]
SCETlib N^{3}LL$'$+$\mathcal{O}(\alpha_s^3)$
$\alpha_s(m_Z) = 0.118 \pm 0.001$
$\alpha_s(m_Z) = 0.114 \pm 0.001$
rel. variation to $\alpha_s(m_Z) = 0.118$ [%]
q_T [GeV]