SQUHADRON

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University of Pavia, INFN, Jefferson Lab

Theory and phenomenology of TMDs: progress and open questions

Correlations in Partonic and Hadronic Interactions

March 7, 2022

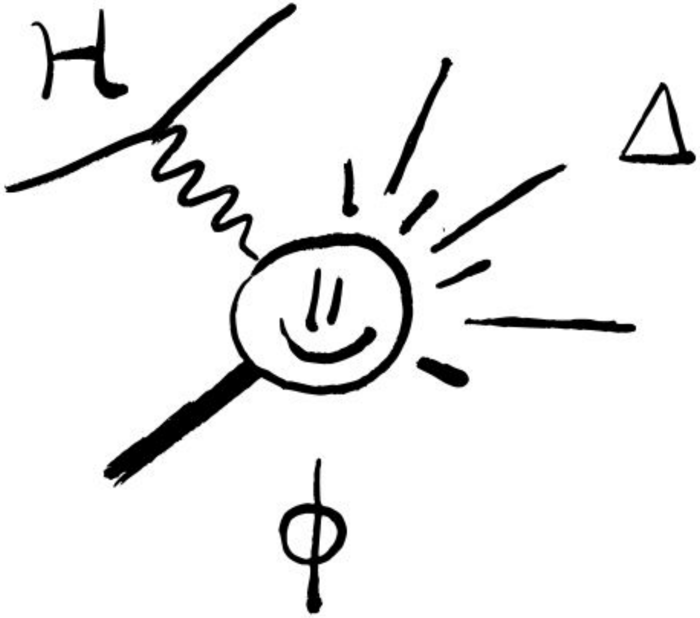
Outline

1. **Theory**
2. **Phenomenology**
3. **(Computational tools)**

A beautiful landscape

$$\begin{tikzcd}
 & & & & & & \overset{\text{parton correlation function}}{H(k,P,\Delta)} \arrow[dllllll, "\Delta=0"'] \arrow[dd, "\int dk^-"] \\
 f(k,P)\ \text{parton correlation function} \arrow[dddd, "\int dk^-"'] & & & & & & \\
 & & & & & H(x,\boldsymbol{k},\xi,\boldsymbol{b}) \arrow[r, leftrightarrow, "\mathrm{FT}"] \arrow[dlll, "\xi=0"'] & H(x,\boldsymbol{k},\xi,\boldsymbol{\Delta})\ \text{GTMD} \arrow[dd, "\int d^2\boldsymbol{k}"] \\
 & & W(x,\boldsymbol{k},\boldsymbol{b})\ \text{Wigner distribution} \arrow[dl, "\int d^2\boldsymbol{b}"] \arrow[dr, "\int d^2\boldsymbol{k}"] & & & & \\
 & & & & & H(x,\xi,\boldsymbol{b}) \arrow[r, leftrightarrow, "\mathrm{FT}"] \arrow[dll, "\xi=0"'] & H(x,\xi,\Delta^2)\ \text{GPD} \arrow[d, "\int dx\, x^{n-1}"] \\
 f(x,\boldsymbol{z}) \arrow[r, leftrightarrow, "\mathrm{FT}"] & \underset{\text{TMD}}{f(x,\boldsymbol{k})} \arrow[ddr, "\int d^2\boldsymbol{k}"] & & f(x,\boldsymbol{b})\ \begin{array}{r}\text{impact parameter}\\ \text{distribution}\end{array} \arrow[ddl, "\int d^2\boldsymbol{b}"] \arrow[ddr, "\int dx\, x^{n-1}"] & & & \underset{\text{GFFs}}{\sum_{k=0}^{n} A_{nk}(\Delta^2)\,(2\xi)^k} \arrow[dl, "\xi=0"'] \\
 & & & & & & \\
 & & \underset{\text{PDF}}{f(x)} & & F_n(\boldsymbol{b}) \arrow[r, leftrightarrow] & \underset{\text{form factor}}{F_n(\Delta^2)} &
\end{tikzcd}$$

Credit picture: M. Diehl - [arXiv 1512.01328]



Theory

Quarks

Drell-Yan / Z / W production (hh)

Semi-Inclusive DIS (eh)

2h-inclusive
e+e- annihilation

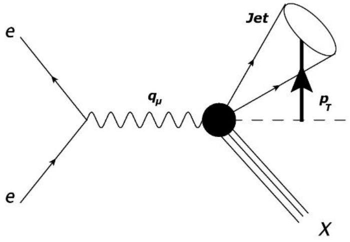
Gluons

Higgs production
in hadronic collisions

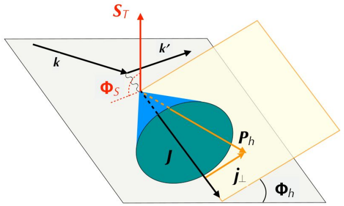
Quarkonium production (e.g. $\eta_{b,c}$
in hadronic collisions)

TMD factorization

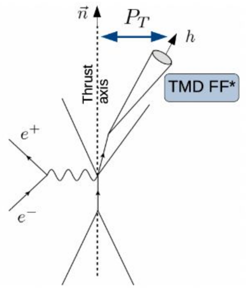
Jets:
(e.g. jet SIDIS,
di-jet SIDIS)



Hadron **“in jet”**:
(eh, hh, e+e-)



Y **“+ jet”**:
(e.g. Y = γ, h)



Quarks

Drell-Yan / Z / W production (hh)

Semi-Inclusive DIS (eh)

2h-inclusive
e+e- annihilation

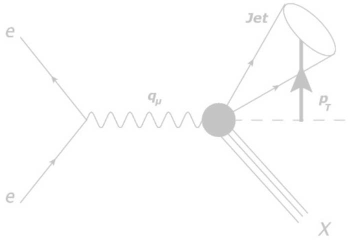
Gluons

Higgs production
in hadronic collisions

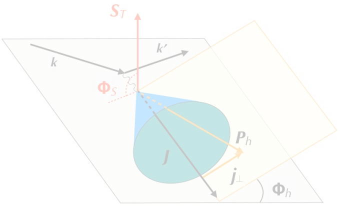
Quarkonium production ($\eta_{b,c}$)
in hadronic collisions

Global fits (unpolarized)

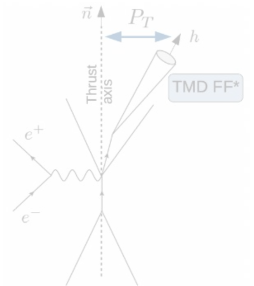
Jets:
(e.g. jet SIDIS,
di-jet SIDIS)



Hadron **"in jet"**:
(eh, hh, e+e-)



Y **"+ jet"**:
(e.g. Y = γ, h)



**No data yet
(or not enough)
for the other
processes!**

TMD factorization

$\boxed{q_T \ll Q}$

$pp \longrightarrow \gamma^{\cdot} \,/\, Z \longrightarrow l\,\bar{l} + X$

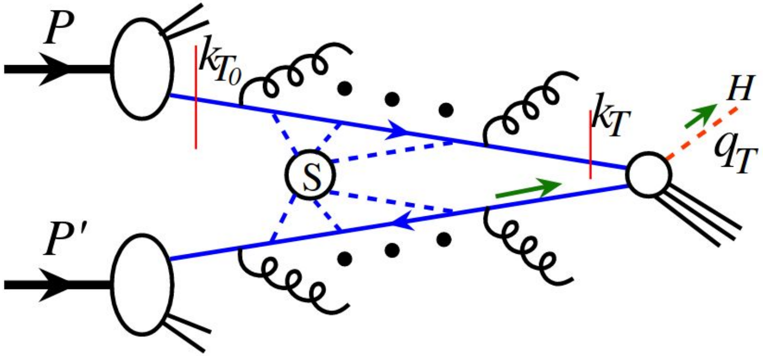
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}(q_T - k_{T\,a} - k_{T\,b}) \; + \mathcal{O}(q_T/Q) \, + \, \mathcal{O}(\Lambda/Q)$$

- TMDs & partonic cross section:
  same **IR poles** = same non-perturbative physics

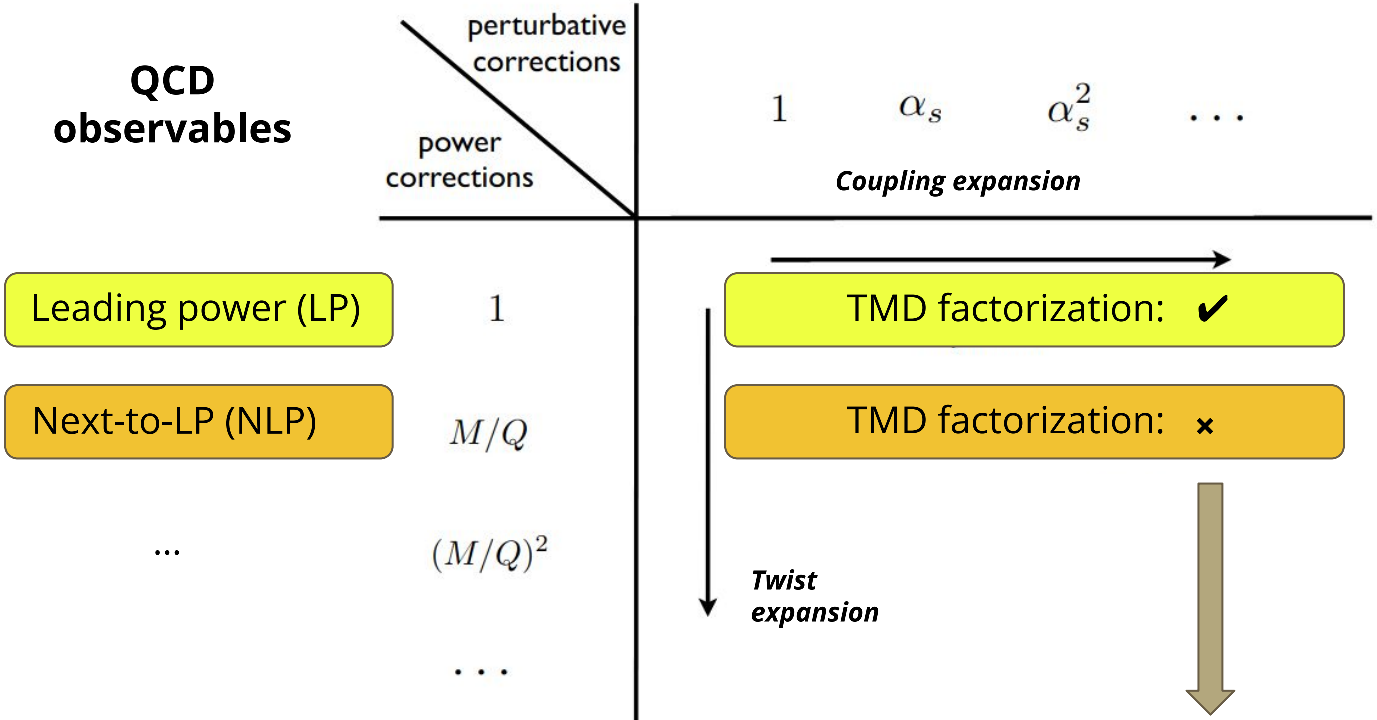
- **observed transverse momentum** :
  transverse momenta of **quarks**

- quark transverse momentum :
  radiative (perturbative) and **intrinsic** (non-perturbative) components

- Renormalization = **evolution** equations tell us how to distinguish between the two



Sub-leading power (twist)



QCD observables

perturbative corrections

power corrections

1 α_s α_s^2 $\dots$

Coupling expansion

Leading power (LP) 1 TMD factorization: ✔ (⟶previous slide)

Next-to-LP (NLP) M/Q TMD factorization: ×

… $(M/Q)^2$

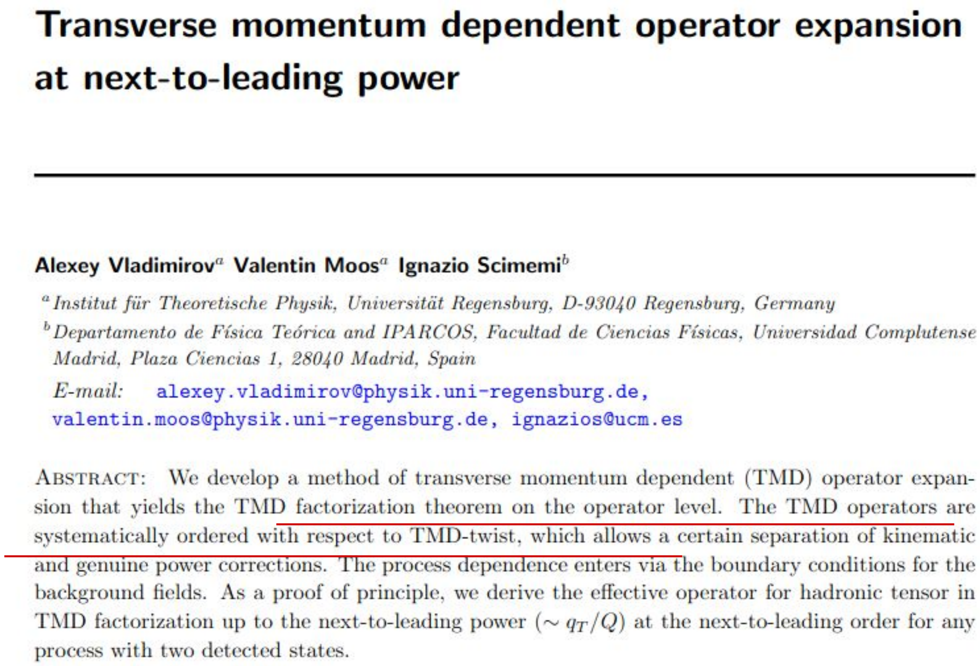
$\dots$

Twist expansion

How can we interpret the higher twist effects in QCD?

Sub-leading power (twist)

https://inspirehep.net/literature/1925147



Transverse momentum dependent operator expansion at next-to-leading power

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[b] *Departamento de Física Teórica and IPARCOS, Facultad de Ciencias Físicas, Universidad Complutense Madrid, Plaza Ciencias 1, 28040 Madrid, Spain*
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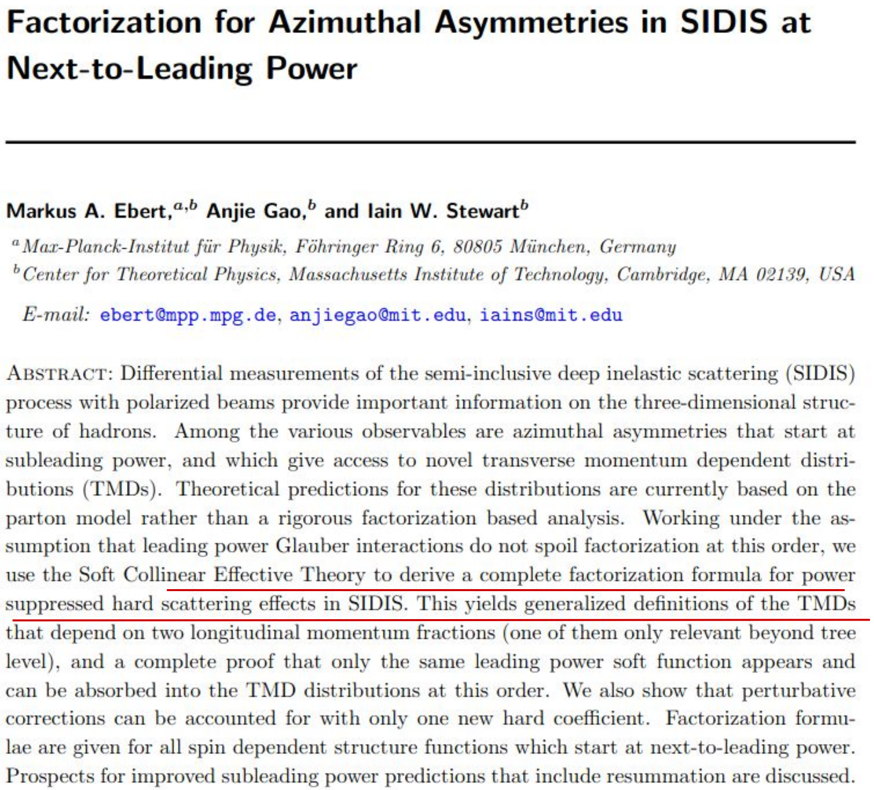
ABSTRACT: We develop a method of transverse momentum dependent (TMD) operator expansion that yields the TMD factorization theorem on the operator level. The TMD operators are systematically ordered with respect to TMD-twist, which allows a certain separation of kinematic and genuine power corrections. The process dependence enters via the boundary conditions for the background fields. As a proof of principle, we derive the effective operator for hadronic tensor in TMD factorization up to the next-to-leading power ($\sim q_T/Q$) at the next-to-leading order for any process with two detected states.

"Derivation of hadronic tensor in TMD factorization at NLP and NLO"

(two out of the most recent papers, but not the only ones available)

Sub-leading power (twist)

https://inspirehep.net/literature/1991138



Factorization for Azimuthal Asymmetries in SIDIS at Next-to-Leading Power

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^{b}*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

E-mail: `ebert@mpp.mpg.de`, `anjiegao@mit.edu`, `iains@mit.edu`

ABSTRACT: Differential measurements of the semi-inclusive deep inelastic scattering (SIDIS) process with polarized beams provide important information on the three-dimensional structure of hadrons. Among the various observables are azimuthal asymmetries that start at subleading power, and which give access to novel transverse momentum dependent distributions (TMDs). Theoretical predictions for these distributions are currently based on the parton model rather than a rigorous factorization based analysis. Working under the assumption that leading power Glauber interactions do not spoil factorization at this order, we use the Soft Collinear Effective Theory to derive a complete factorization formula for power suppressed hard scattering effects in SIDIS. This yields generalized definitions of the TMDs that depend on two longitudinal momentum fractions (one of them only relevant beyond tree level), and a complete proof that only the same leading power soft function appears and can be absorbed into the TMD distributions at this order. We also show that perturbative corrections can be accounted for with only one new hard coefficient. Factorization formulae are given for all spin dependent structure functions which start at next-to-leading power. Prospects for improved subleading power predictions that include resummation are discussed.

Factorization for SIDIS at NLP,
with one strong assumption

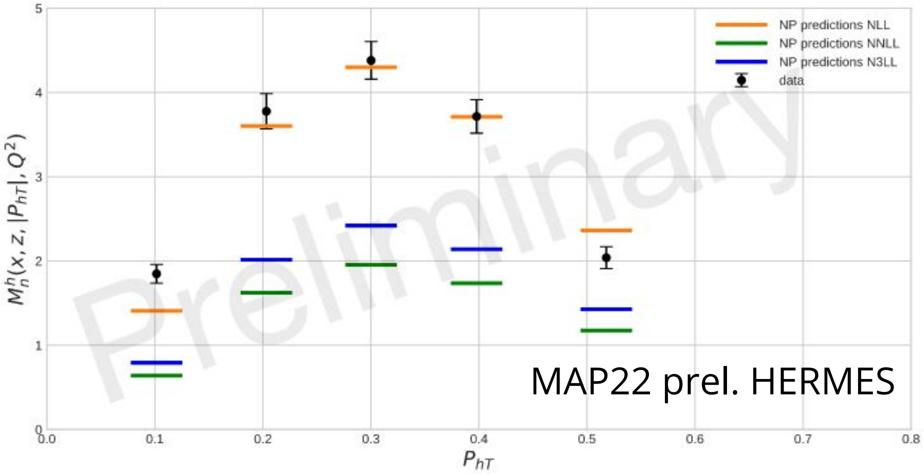
Only the same LP soft function appears

Only one extra hard function at NLP

(two out of the most recent papers, but not the only ones available)

Normalization issues

***Small** transverse momentum*



MAP22 prel. HERMES

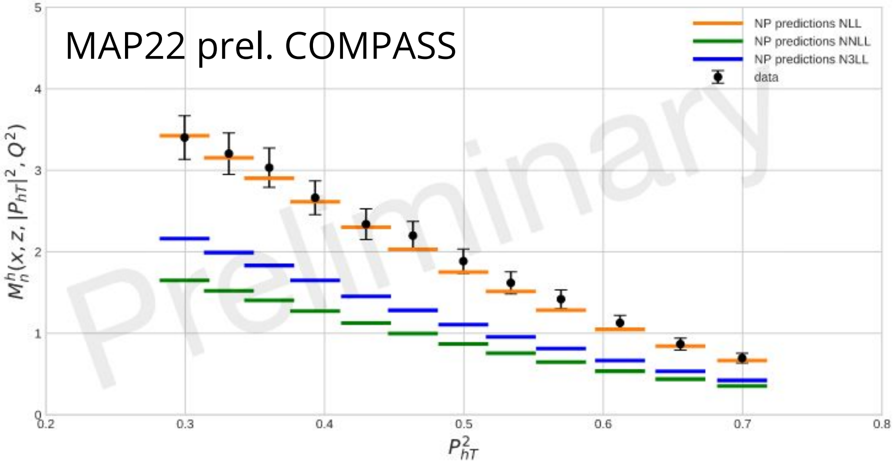
But **no consensus** in literature,
even about the problem

- **SV 19** : *not seen;* power corrections from the start?
- **MAP 22** : power corrections from pre-computed normalization coefficients

Beyond the NLL, the **theoretical** prediction is **way too low**

Who to blame:

- hard function (large coeffs.)
- low Q



MAP22 prel. COMPASS

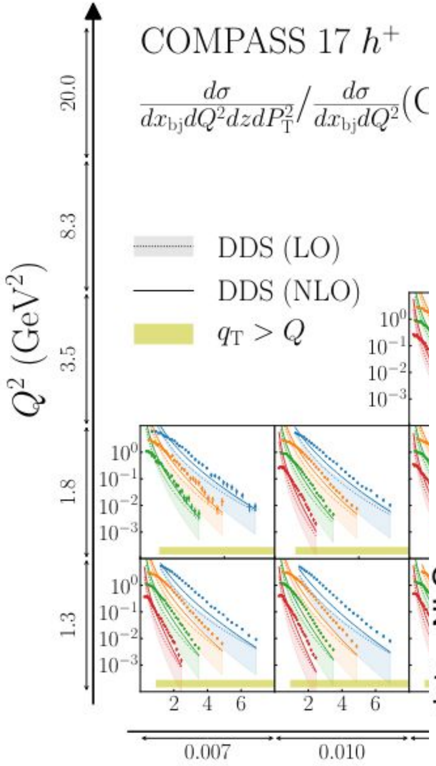
Normalization issues

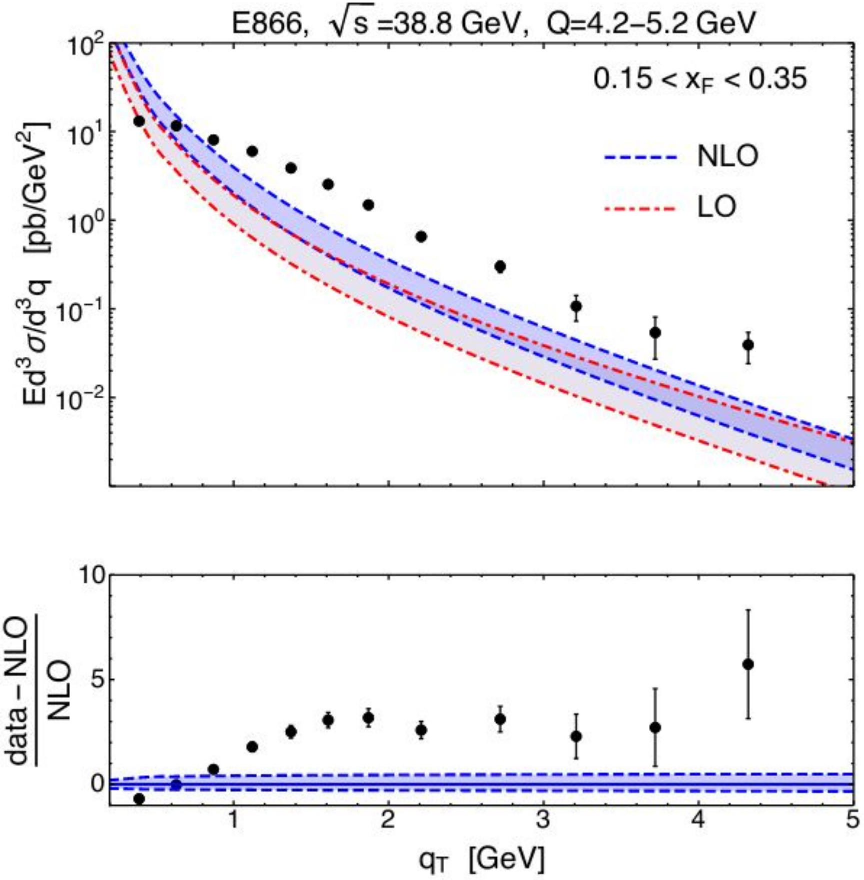
https://inspirehep.net/literature/1723777

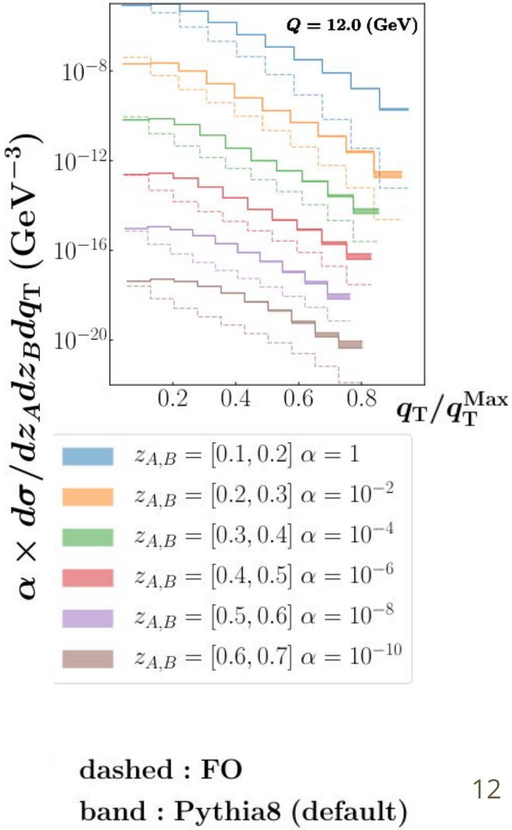
https://inspirehep.net/literature/1716140

https://inspirehep.net/literature/1752934

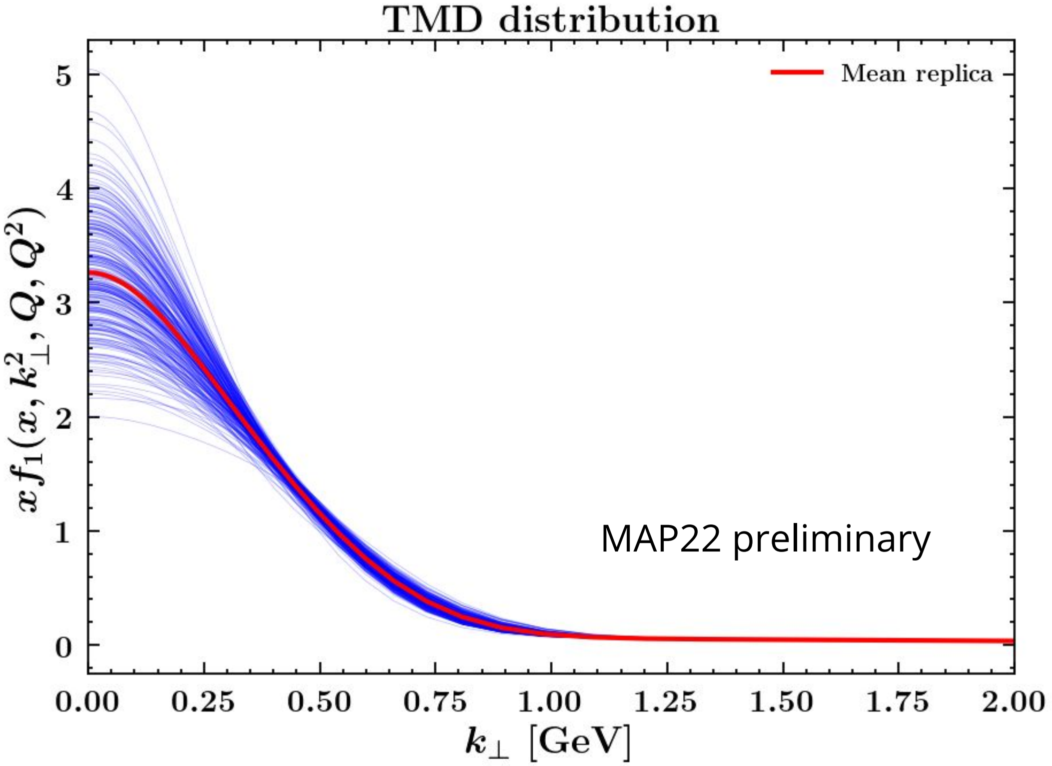
***Large** transverse momentum*







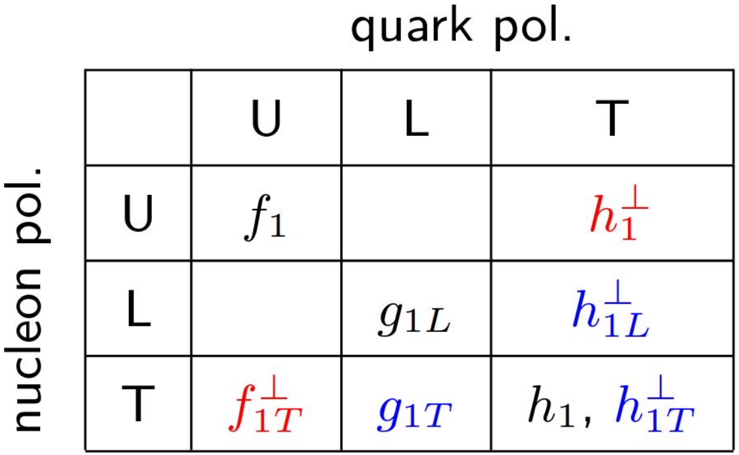
TMDs & recent phenomenology



MAP22 preliminary

TMD PDF of the u at $Q = 2$ GeV and $x = 0.001$

TMD PDFs for quarks in nucleon



quark pol.

nucleon pol.		U	L	T
	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

$$\Phi_{ij}(k,P) \;=\; \mathrm{F.T.}\,\langle P\big|\,\overline{\psi}_j(0)\,U\,\psi_i(\xi)\,\big|P\rangle$$

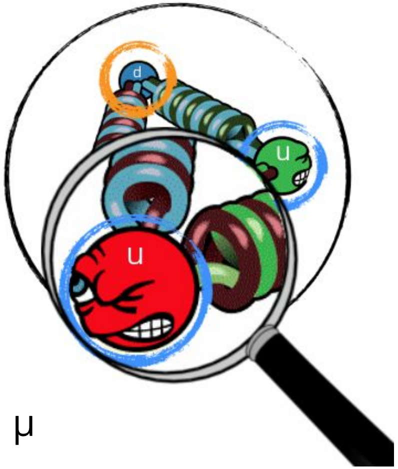
At leading twist: 8 TMD PDFs

(similar classification for gluons and for FFs)

- **Black**: time-reversal even AND collinear
- **Blue**: time-reversal even
- **Red**: time-reversal odd (*process dependence*)

The **symmetries of QCD** play a crucial role in this classification

QCD evolution of a TMD PDF



$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

→ TMD distribution
at initial scales

x $$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \boxed{\gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)}\right]$$

→ evolution in μ

Calculable in pQCD

x $$\left(\frac{\zeta}{\zeta_0}\right)^{-\boxed{D(b_T\mu_0, \alpha_s(\mu_0))}\boxed{+g_K(b_T;\lambda)}}$$

→ evolution in ζ

Non-pert. corrections
(large bT)

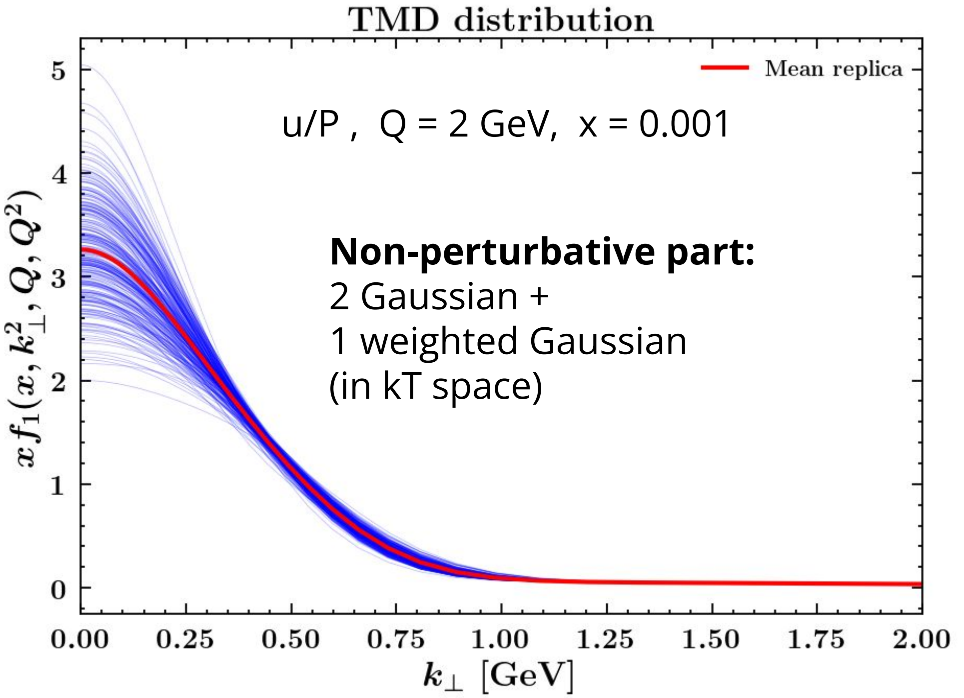
$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b \boxed{C_{a/b}(x, b_T^2, \mu_0, \zeta_0)} \otimes \underline{f_b(x, \mu_0)} \boxed{F_{NP}(b_T; \lambda)}$$

Prior knowledge
assumed (?)

See e.g. https://inspirehep.net/literature/1785810 (but also JCC book and many other references)

MAP22 : TMDs

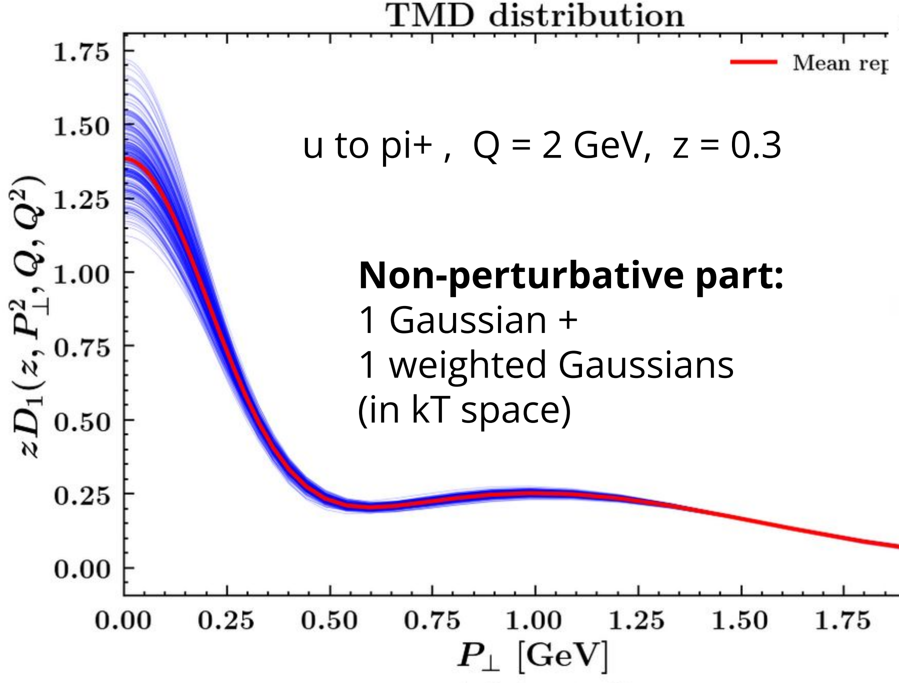
In preparation



u/P , Q = 2 GeV, x = 0.001

Non-perturbative part:
2 Gaussian +
1 weighted Gaussian
(in kT space)

TMD PDF of the u at Q = 2 GeV and x = 0.001



u to pi+ , Q = 2 GeV, z = 0.3

Non-perturbative part:
1 Gaussian +
1 weighted Gaussians
(in kT space)

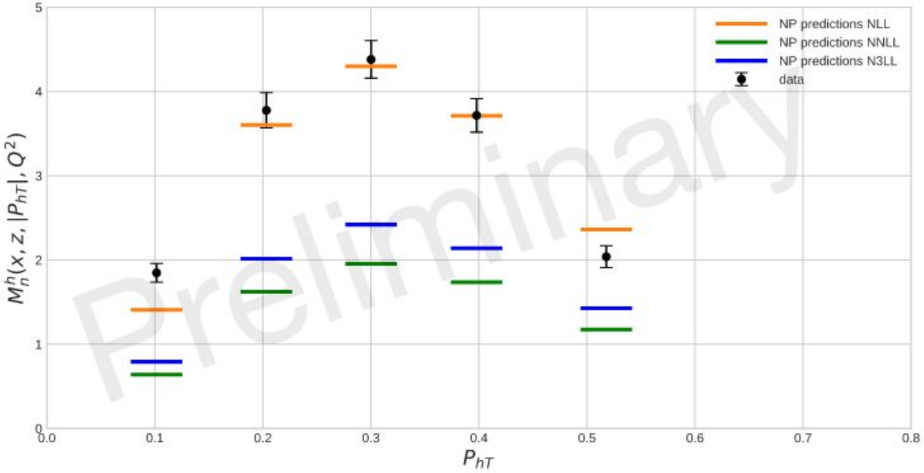
TMD FF of u -> π^+ at Q = 2 GeV and z = 0.3

No TMD flavor dependence yet

(computationally much
more demanding)

MAP22 : SIDIS normalization at low q_T

In preparation



$$\boxed{w(x,z,Q) = \frac{\frac{d\sigma^h}{dx\,dQ^2\,dz}}{\int \mathrm{W} d^2\boldsymbol{q}_T}}$$

Beyond NLL:

- The integral over qT of **SIDIS TMD** cross section **does not** yield the **collinear** cross section

- The hard part **heavily suppresses** the TMD cross section

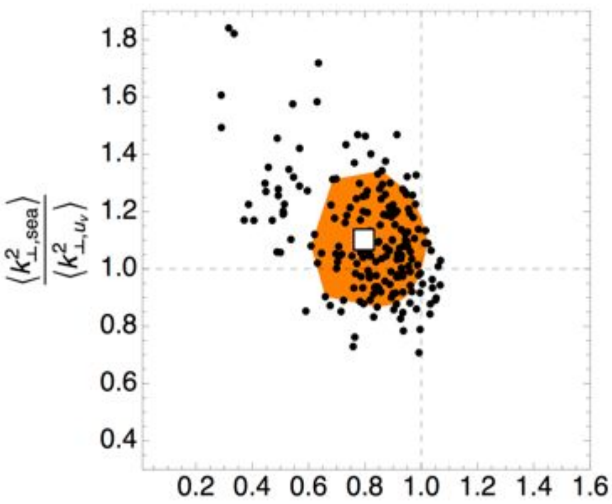
We **enhance** the predictions with this factor that **restores** the correct normalization

Effectively a ~1/Q correction & *no dependence on fit parameters*

Flavor structure of TMDs (PV13)

Imaging from **SIDIS** data from Hermes experiment

Ratio of width of sea / width of up valence

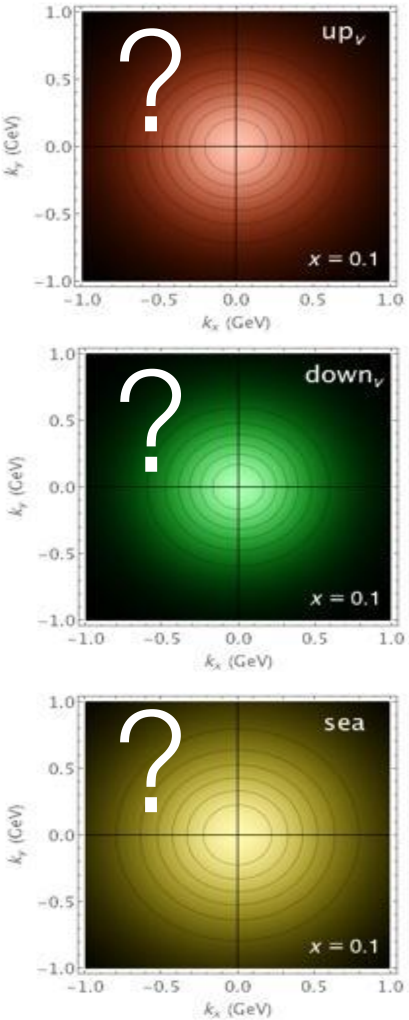


Ratio width of down valence/ width of up valence $\frac{\langle k^2_{\perp,d_v} \rangle}{\langle k^2_{\perp,u_v} \rangle}$

A lot of room
for flavor-dependent distributions

NEWS:
Recent analysis, Interplay with collinear PDFs

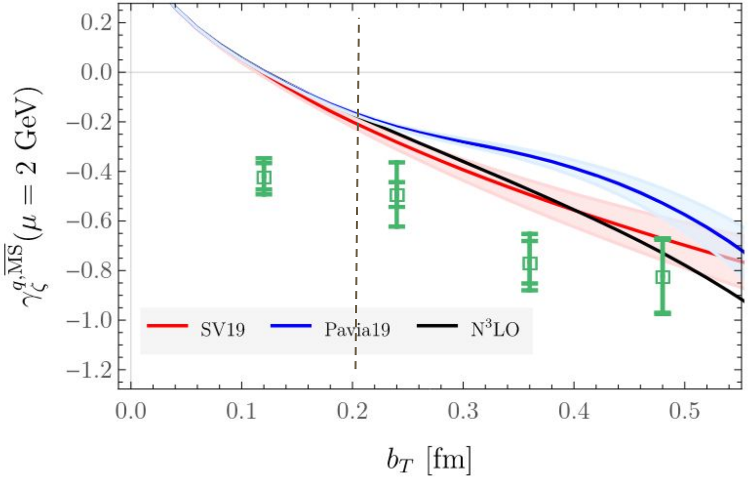
https://inspirehep.net/literature/2012944



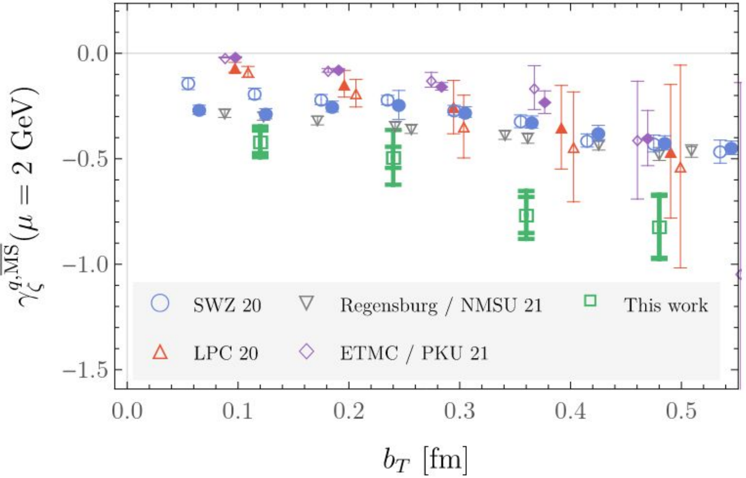
arXiv 1309.3507

Lattice input: non-perturbative evolution

Lattice QCD can also calculate some of the quantities that we are trying to extract from experimental data: e.g. **gK**



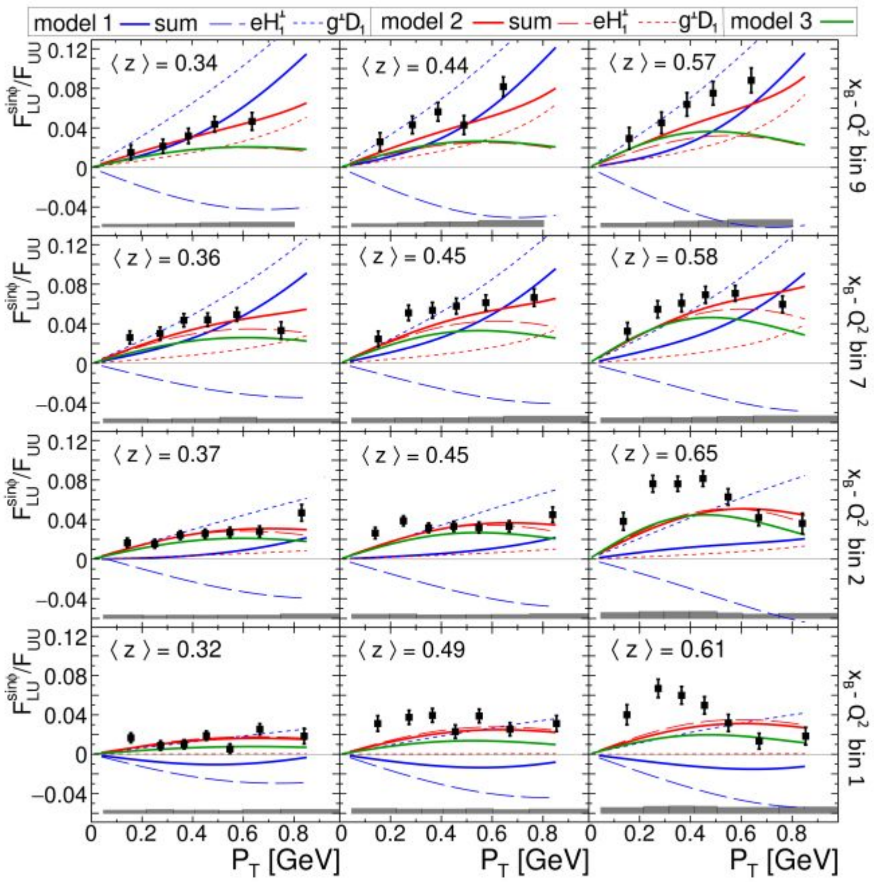
(a) Comparison with the SV19 [4] and Pavia19 [5] phenomenological parameterizations and the next-to-next-to-next-to-leading order (N^{3}LO) perturbative result [42, 43].



(b) Comparison with quenched results of Ref. [19] (SWZ), as well as results from the LPC [20], Regensburg/NMSU [21], and ETMC/PKU [22] collaborations. Different sets of points with the same color show different sets of results from the same collaboration.

https://inspirehep.net/literature/1892223

Higher twist: beam-spin asymmetry @ CLAS12



Among the first CLAS12 publications

https://inspirehep.net/literature/1840207

Models can reproduce at least the size of the signal (not always the sign)

Higher twist: beam-spin asymmetry

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(x\underline{e}\,\underline{H_1^\perp} + \frac{M_h}{M}\,\underline{f_1}\,\frac{\underline{\tilde{G}^\perp}}{z}\Big)$$

$$+\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(x\underline{g^\perp}\,\underline{D_1} + \frac{M_h}{M}\,\underline{h_1^\perp}\,\frac{\underline{\tilde{E}}}{z}\Big)\Big].$$

Leading twist parts:

- f1, D1 : known with good precision from global anaylises
- Collins and Boer-Mulders : accessible and partially known

We can tackle the **higher twist** parts

TMD factorization at **NLP** (see the theory section)

Tools for TMD physics

Different frameworks, same observable

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \quad \begin{aligned} &\overset{q_T-\text{res.}}{\underset{\text{PB}}{\propto}} \quad e^{2S}\left[f_1 \otimes \mathcal{H} \otimes f_2\right] \\ &\overset{\text{TMD}}{\propto} \quad H \times F_1 \times F_2 \qquad + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right] \\ &\overset{\text{SCET}}{\propto} \quad H \times B_1 \times B_2 \times S \end{aligned}$$

$$\mathcal{H} = H C_1 C_2$$

$$F_i = e^S C_i \otimes f_i$$

$$F_i = \sqrt{S} \times B_i$$

Dictionary to compare different factorization frameworks

“equivalent” to the extent of describing TMD physics

G. Bozzi at SarWors 2021 - https://agenda.infn.it/event/27742/

Codes

SCETlib
[https://confluence.desy.de/display/scetlib]

CuTe
[https://cute.hepforge.org/]

SCET

DYRes/DYTurbo, DYqT, etc.
[https://gitlab.cern.ch/DYdevel/DYTURBO]

ReSolve
[https://github.com/fkhorad/reSolve]

ResBos
[https://resbos.hepforge.org/]

qT resummation

TMD factorization

arTeMiDe
[https://teorica.fis.ucm.es/artemide/]

Nanga Parbat
[https://github.com/MapCollaboration/NangaParbat]

Parton branching

RadISH
[https://arxiv.org/pdf/1705.09127.pdf]

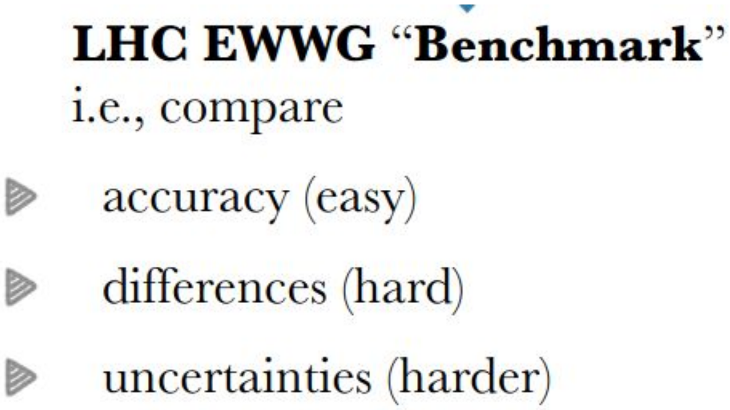
PB-TMDs
[https://arxiv.org/pdf/1906.00919.pdf]

G. Bozzi at SarWors 2021 - https://agenda.infn.it/event/27742/

Codes

Excellent accuracy **BUT** *only unpolarized and leading twist*!

Basic ingredients **common** to all codes



LHC EWWG "Benchmark"
i.e., compare

- accuracy (easy)
- differences (hard)
- uncertainties (harder)

Main **differences**:

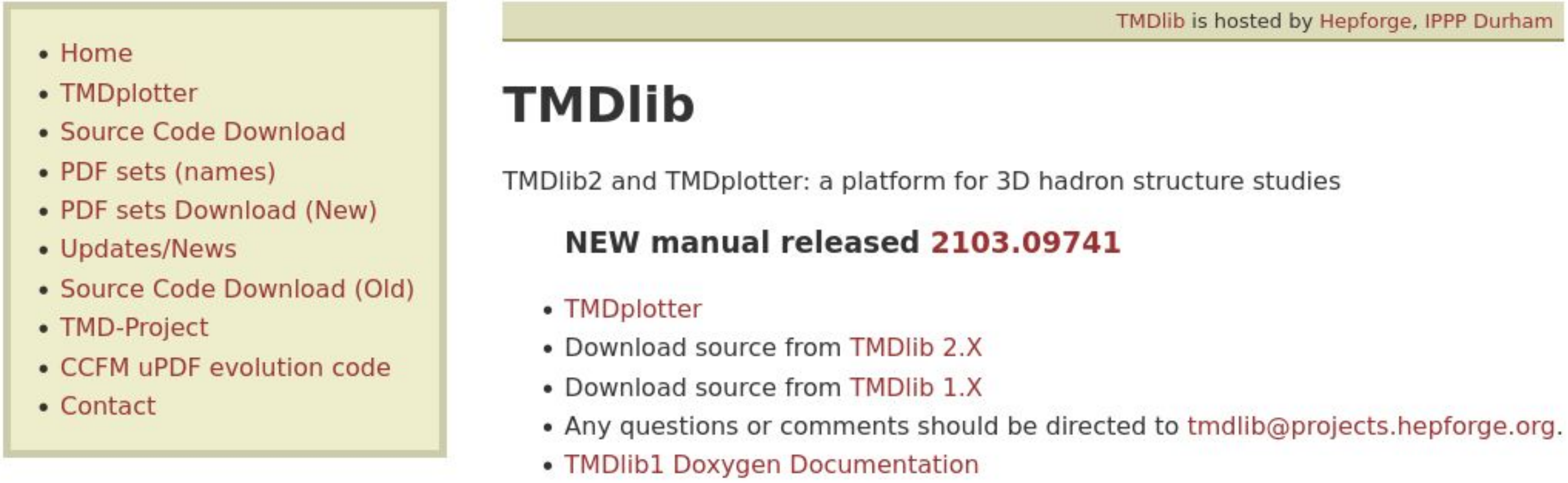
- "Space": position vs momentum space
- Perturbative QCD: PDF evolution, scale variation, matching with fixed-order
- Non-perturbative QCD: treatment of Landau pole, intrinsic-kT

G. Bozzi at SarWors 2021 - https://agenda.infn.it/event/27742/

TMDlib

https://inspirehep.net/literature/1852038

https://tmdlib.hepforge.org/



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- Source Code Download
- PDF sets (names)
- PDF sets Download (New)
- Updates/News
- Source Code Download (Old)
- TMD-Project
- CCFM uPDF evolution code
- Contact

TMDlib is hosted by Hepforge, IPPP Durham

TMDlib

TMDlib2 and TMDplotter: a platform for 3D hadron structure studies

NEW manual released 2103.09741

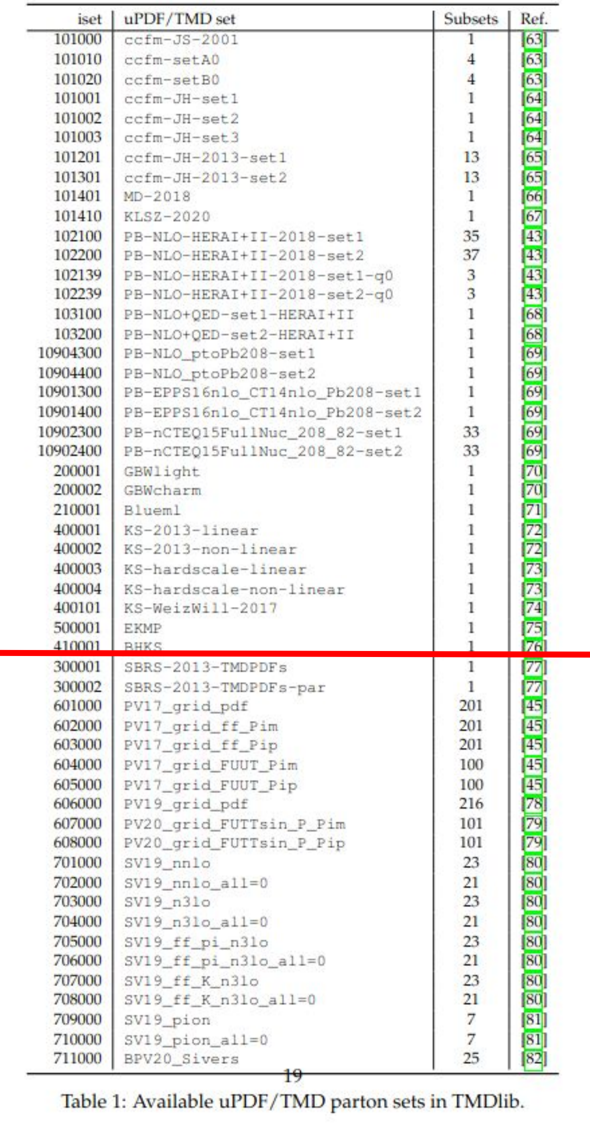
- TMDplotter
- Download source from TMDlib 2.X
- Download source from TMDlib 1.X
- Any questions or comments should be directed to tmdlib@projects.hepforge.org.
- TMDlib1 Doxygen Documentation

TMDlib

https://inspirehep.net/literature/1852038

https://tmdlib.hepforge.org/

PB TMDs, etc



iset	uPDF/TMD set	Subsets	Ref.
101000	ccfm-JS-2001	1	[63]
101010	ccfm-setA0	4	[63]
101020	ccfm-setB0	4	[63]
101001	ccfm-JH-set1	1	[64]
101002	ccfm-JH-set2	1	[64]
101003	ccfm-JH-set3	1	[64]
101201	ccfm-JH-2013-set1	13	[65]
101301	ccfm-JH-2013-set2	13	[65]
101401	MD-2018	1	[66]
101410	KLSZ-2020	1	[67]
102100	PB-NLO-HERAI+II-2018-set1	35	[43]
102200	PB-NLO-HERAI+II-2018-set2	37	[43]
102139	PB-NLO-HERAI+II-2018-set1-q0	3	[43]
102239	PB-NLO-HERAI+II-2018-set2-q0	3	[43]
103100	PB-NLO+QED-set1-HERAI+II	1	[68]
103200	PB-NLO+QED-set2-HERAI+II	1	[68]
10904300	PB-NLO_ptoPb208-set1	1	[69]
10904400	PB-NLO_ptoPb208-set2	1	[69]
10901300	PB-EPPS16nlo_CT14nlo_Pb208-set1	1	[69]
10901400	PB-EPPS16nlo_CT14nlo_Pb208-set2	1	[69]
10902300	PB-nCTEQ15FullNuc_208_82-set1	33	[69]
10902400	PB-nCTEQ15FullNuc_208_82-set2	33	[69]
200001	GBWlight	1	[70]
200002	GBWcharm	1	[70]
210001	Blueml	1	[71]
400001	KS-2013-linear	1	[72]
400002	KS-2013-non-linear	1	[72]
400003	KS-hardscale-linear	1	[73]
400004	KS-hardscale-non-linear	1	[73]
400101	KS-WeizWill-2017	1	[74]
500001	EKMP	1	[75]
410001	BHKS	1	[76]
300001	SBRS-2013-TMDPDFs	1	[77]
300002	SBRS-2013-TMDPDFs-par	1	[77]
601000	PV17_grid_pdf	201	[45]
602000	PV17_grid_ff_Pim	201	[45]
603000	PV17_grid_ff_Pip	201	[45]
604000	PV17_grid_FUUT_Pim	100	[45]
605000	PV17_grid_FUUT_Pip	100	[45]
606000	PV19_grid_pdf	216	[78]
607000	PV20_grid_FUTTsin_P_Pim	101	[79]
608000	PV20_grid_FUTTsin_P_Pip	101	[79]
701000	SV19_nnlo	23	[80]
702000	SV19_nnlo_all=0	21	[80]
703000	SV19_n3lo	23	[80]
704000	SV19_n3lo_all=0	21	[80]
705000	SV19_ff_pi_n3lo	23	[80]
706000	SV19_ff_pi_n3lo_all=0	21	[80]
707000	SV19_ff_K_n3lo	23	[80]
708000	SV19_ff_K_n3lo_all=0	21	[80]
709000	SV19_pion	7	[81]
710000	SV19_pion_all=0	7	[81]
711000	BPV20_Sivers	25	[82]

19

Table 1: Available uPDF/TMD parton sets in TMDlib.

TMD factorization:

Unpolarized TMDs (PV13, PV17, SV19)
Sivers TMD PDF (PV20, BPV20)

SIDIS structure functions (PV17, PV20)

Event generators

Based on TMDs:

- Cascade (PB TMDs)
  [https://cascade.hepforge.org/]

- gmctrans/TMDgen
  - parton model level TMDs
  - includes polarization and higher twist, but no evolution: too primitive for EIC?
  - semi-inclusive
  [https://wiki.bnl.gov/eic/index.php/Gmc_trans
  Hermes collaboration + independent work]

Exclusive generators with transverse momentum effects

- Pythia [https://pythia.org/]
- Herwig [https://herwig.hepforge.org/]
- Geneva [https://stash.desy.de/projects/GENEVA]
- ...

Enjoy the workshop!

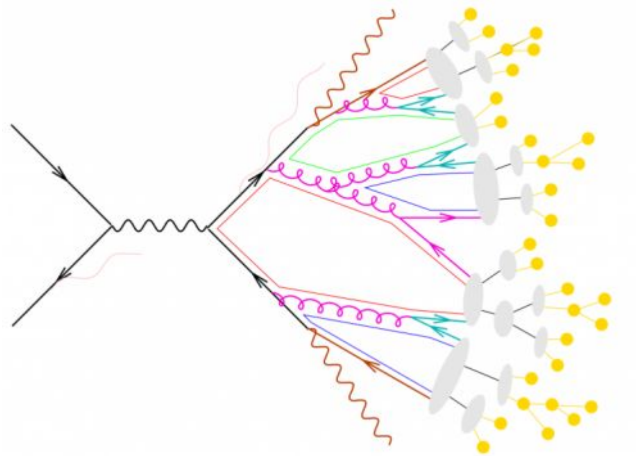
U. of Pavia, 23-27 May 2022

https://agenda.infn.it/event/19219/

Backup

Hadronization and fragmentation functions (FFs)

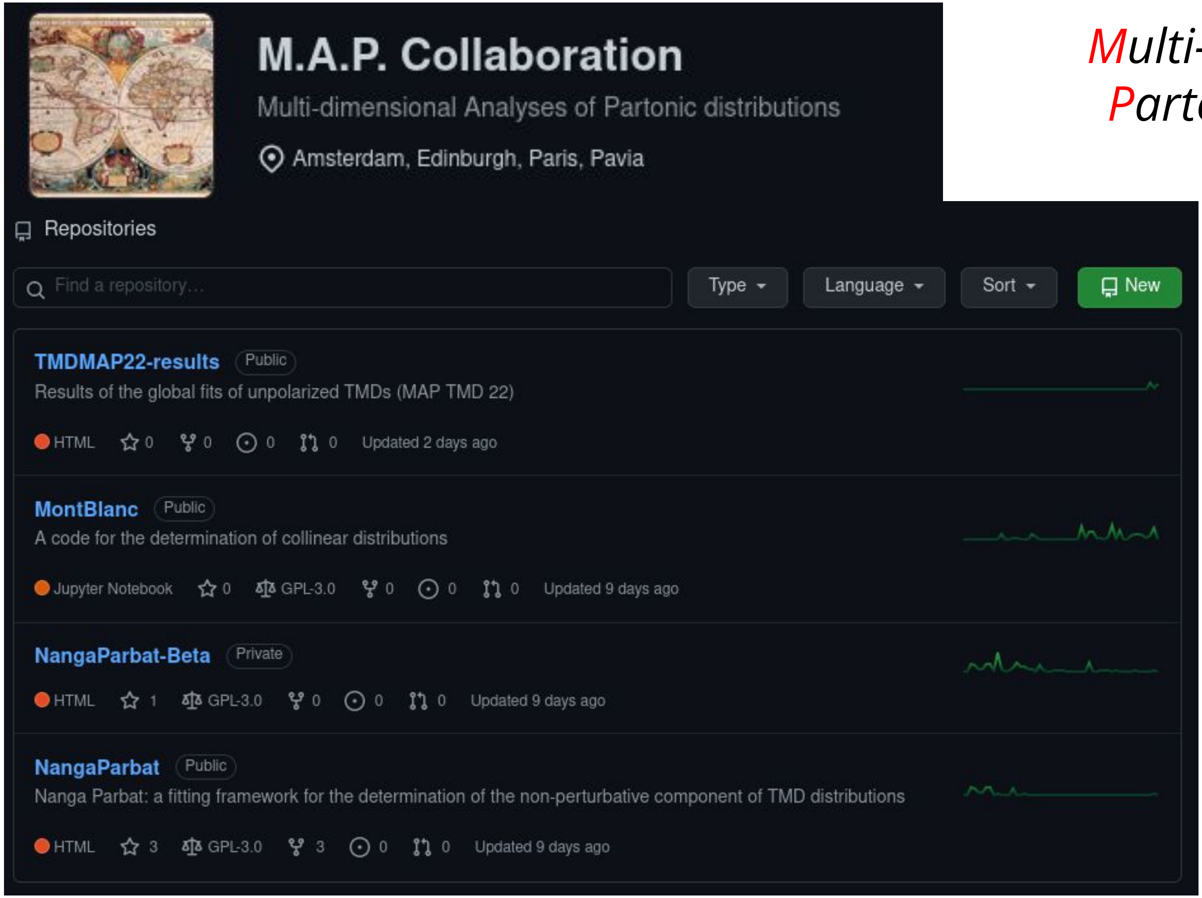
“Maps” of hadron formation in momentum space



$D_1^h(z)$	single-hadron collinear FF
$D_1^h(z, P_T^2)$	single-hadron TMD FF
$D_1^{h_1\,h_2}(z, \zeta)$	di-hadron FF
$J(s)$	inclusive jet FF
$\mathcal{G}^h(s, z)$	in-jet FF

MAP collaboration

https://github.com/MapCollaboration



M.A.P. Collaboration

Multi-dimensional Analyses of Partonic distributions

Amsterdam, Edinburgh, Paris, Pavia

TMDMAP22-results Public
Results of the global fits of unpolarized TMDs (MAP TMD 22)
HTML ☆ 0 · 0 forks · 0 issues · 0 pull requests · Updated 2 days ago

MontBlanc Public
A code for the determination of collinear distributions
Jupyter Notebook ☆ 0 · GPL-3.0 · 0 forks · 0 issues · 0 pull requests · Updated 9 days ago

NangaParbat-Beta Private
HTML ☆ 1 · GPL-3.0 · 0 forks · 0 issues · 0 pull requests · Updated 9 days ago

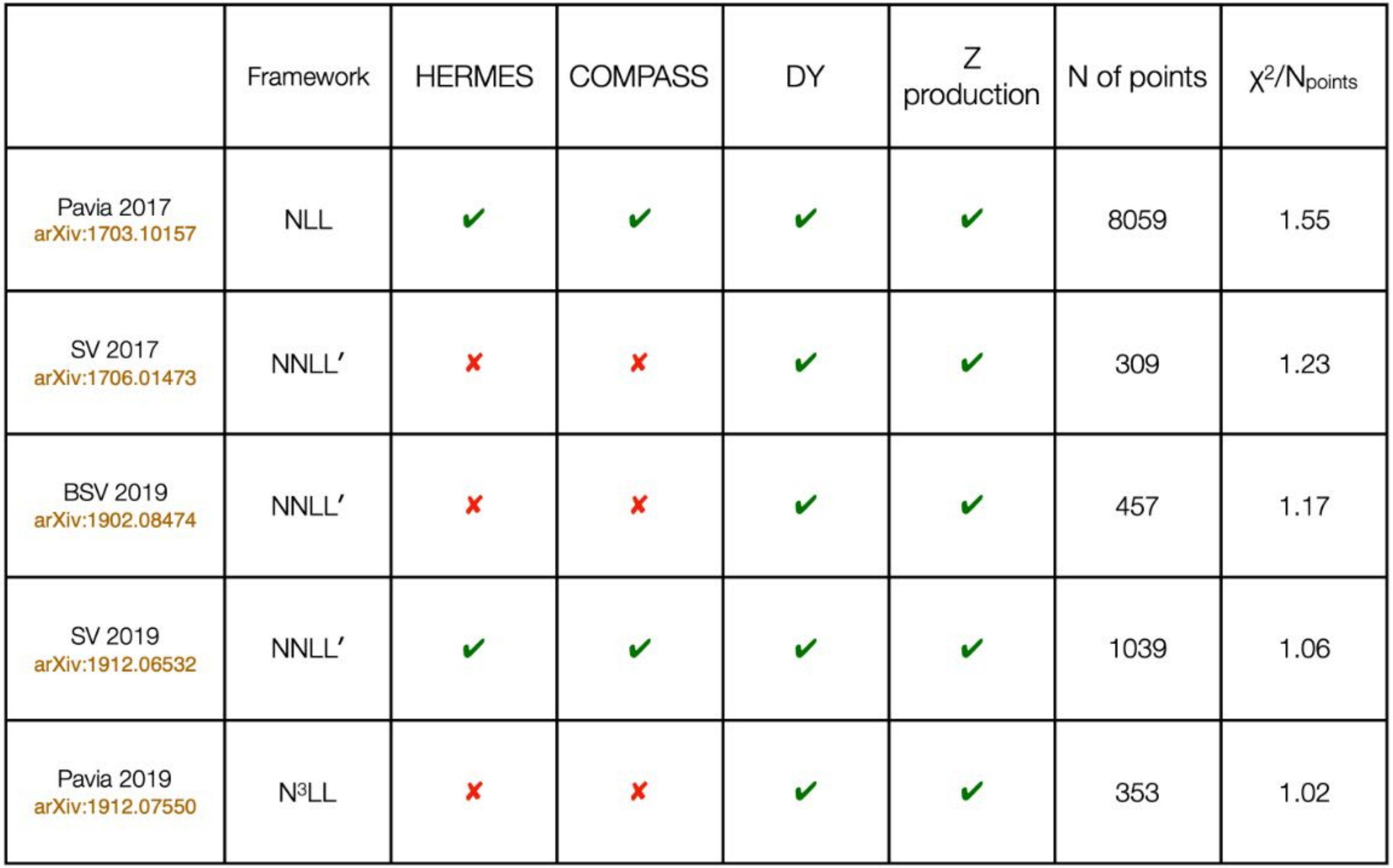
NangaParbat Public
Nanga Parbat: a fitting framework for the determination of the non-perturbative component of TMD distributions
HTML ☆ 3 · GPL-3.0 · 3 forks · 0 issues · 0 pull requests · Updated 9 days ago

Multi-dimensional Analyses of Partonic distributions (MAP)

At the moment:
- Edinburgh
- JLab
- Paris
- Pavia

“A framework for discussion and for analyses of partonic distributions”

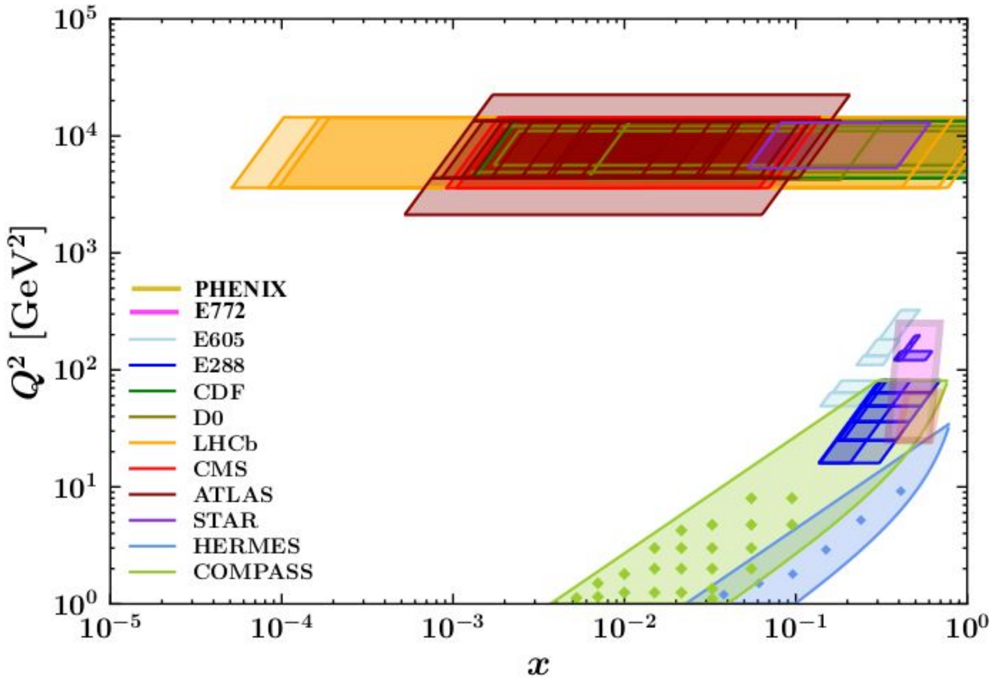
A selection of recent fits



	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

“MAP22” fit : kinematic coverage

In preparation



“***Global***” fit of ***unpolarized TMDs***
at ***N3LL*** accuracy

Drell-Yan/Z and **SIDIS** data

2031 data
21 parameters

Global chi2: 1.00

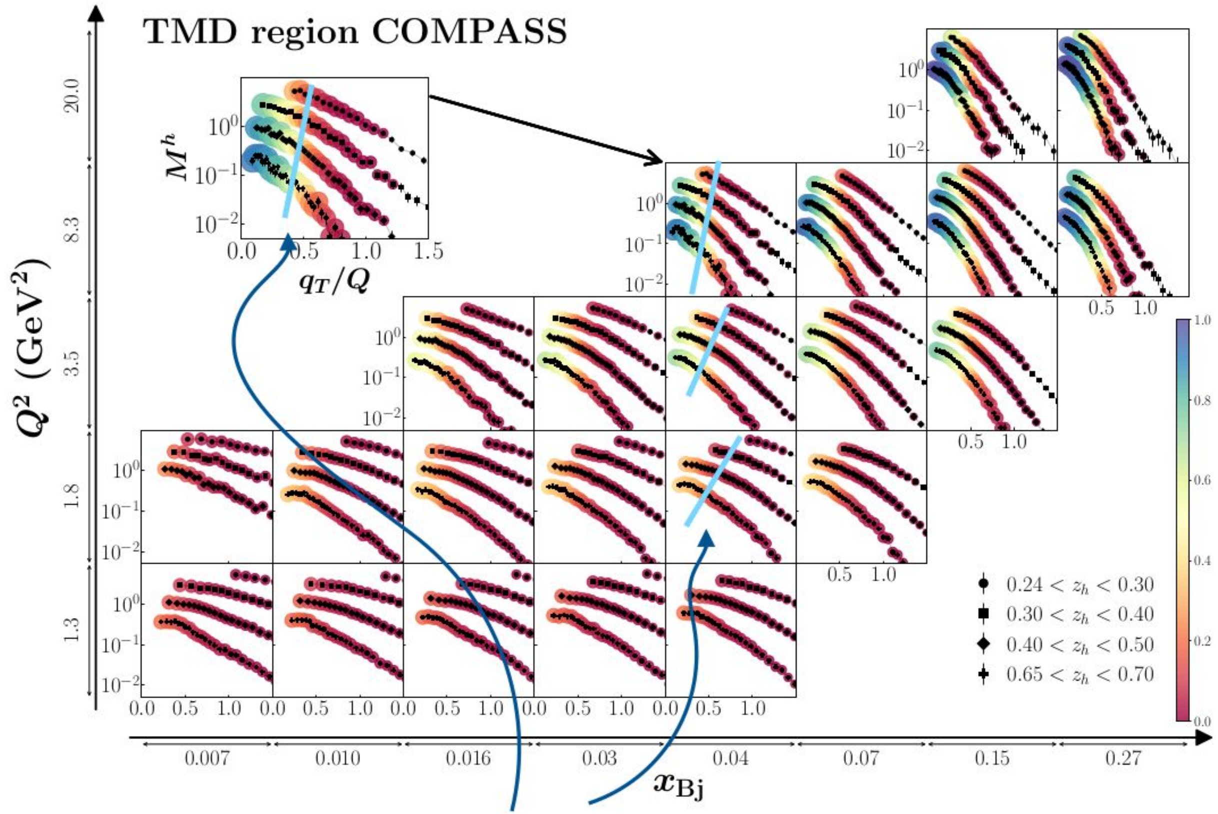
MAP22 : TMD region

https://inspirehep.net/literature/2021571

see A. Bacchetta, recent
“CLAS collaboration meeting”

MAP22 implementation
of TMD region for SIDIS:

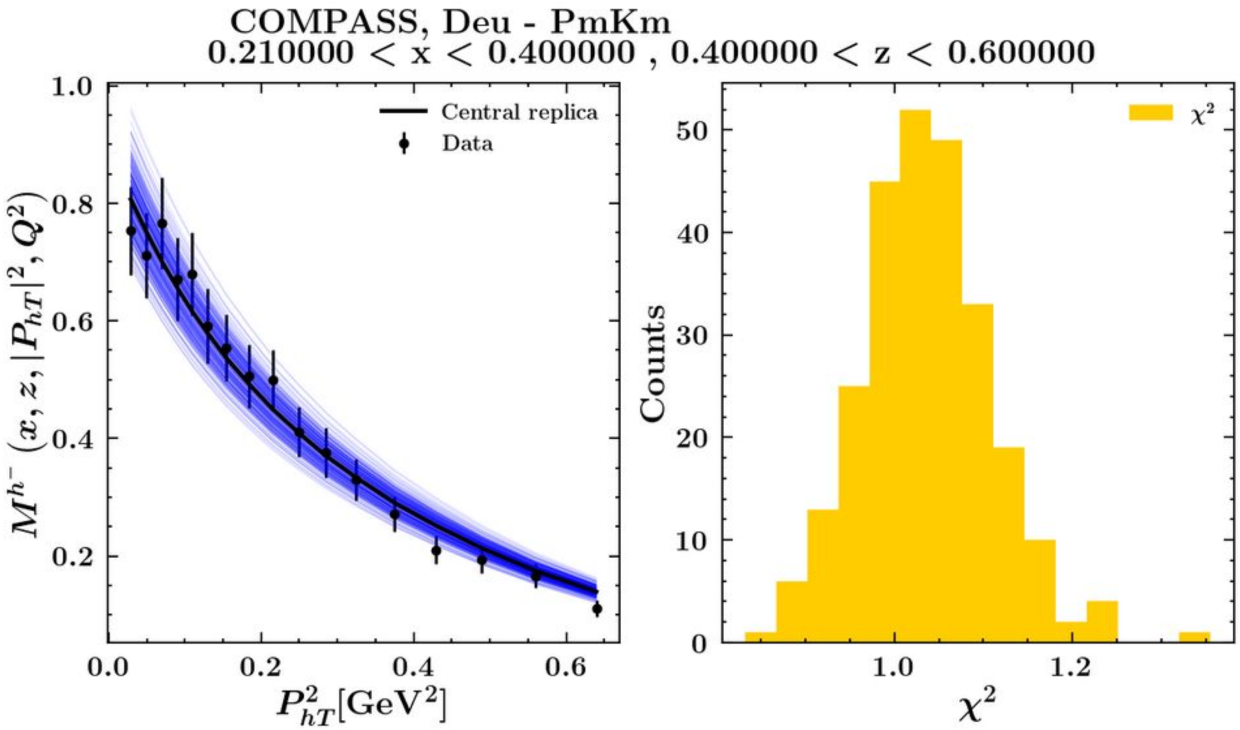
qT < Q at most



Approximate region included in MAP22 fit

MAP22 : comparison with data

In preparation



300 Monte Carlo replicas (bootstrap)

SIDIS data:
overall satisfactory

Drell-Yan data:
major problems with ATLAS data

The normalization coefficients play a crucial role

***BUT** they do **not** depend on q_T and the fit parameters*

Conclusions and outlook

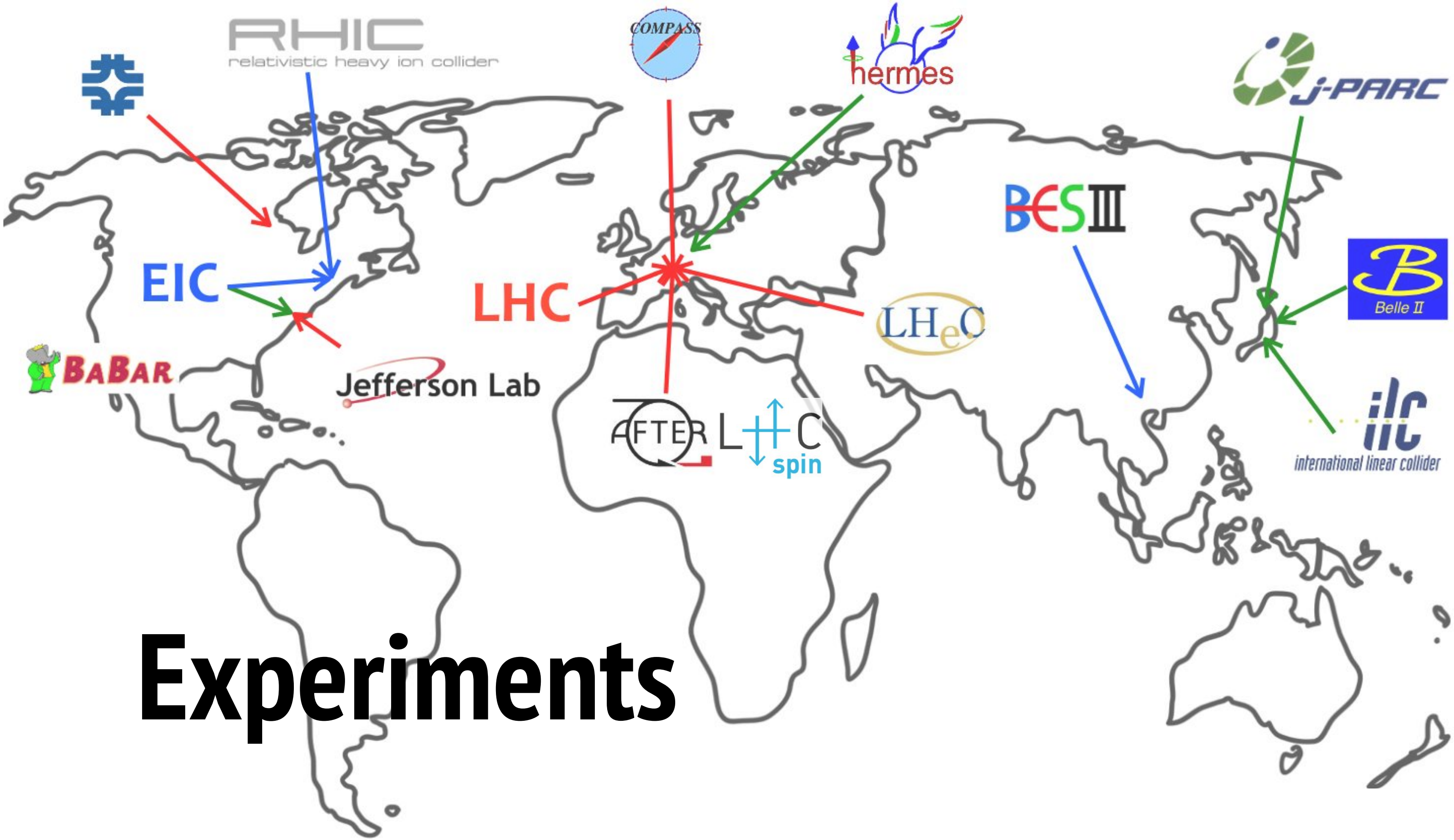
1. We are working hard to build “**maps**” of hadron structure and formation: **parton distribution and fragmentation functions** and the like, connected to fundamental properties of QCD

2. **Crucial input** is provided by **experiments**.
   The **Electron-Ion Collider** is the next experimental frontier of QCD and will provide us with a wealth of information: **we have to be ready for that!**

3. Which **tools for TMD physics** are most needed?
   How can **theorists** and **experimentalists work together** to develop these tools?

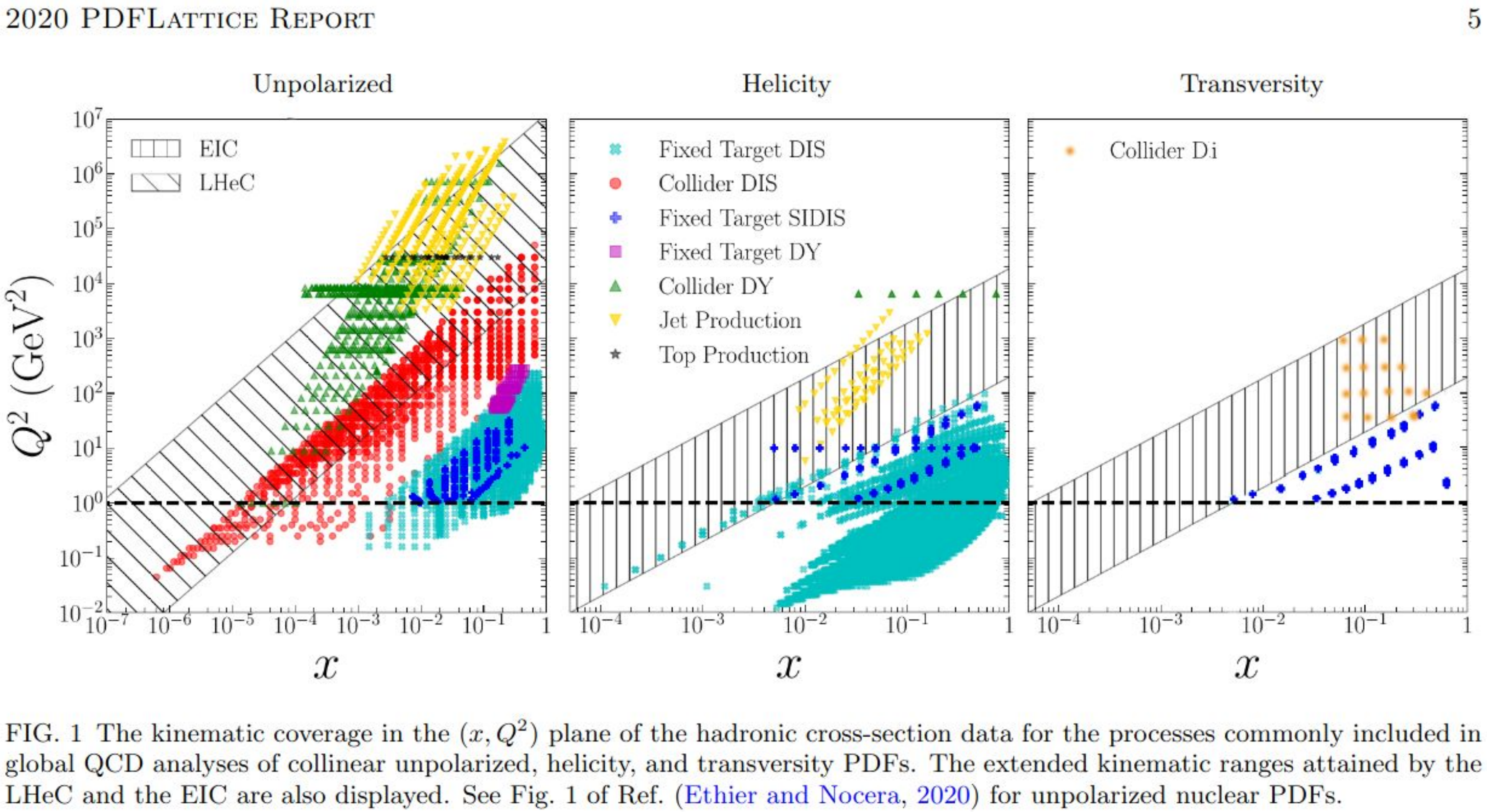
4. Can we define **“best practices”** for these tools? Standard formats, availability, etc.?

Experiments



Enough data ..?

See https://inspirehep.net/literature/1801417



2020 PDFLattice Report 5

FIG. 1 The kinematic coverage in the (x, Q^2) plane of the hadronic cross-section data for the processes commonly included in global QCD analyses of collinear unpolarized, helicity, and transversity PDFs. The extended kinematic ranges attained by the LHeC and the EIC are also displayed. See Fig. 1 of Ref. (Ethier and Nocera, 2020) for unpolarized nuclear PDFs.

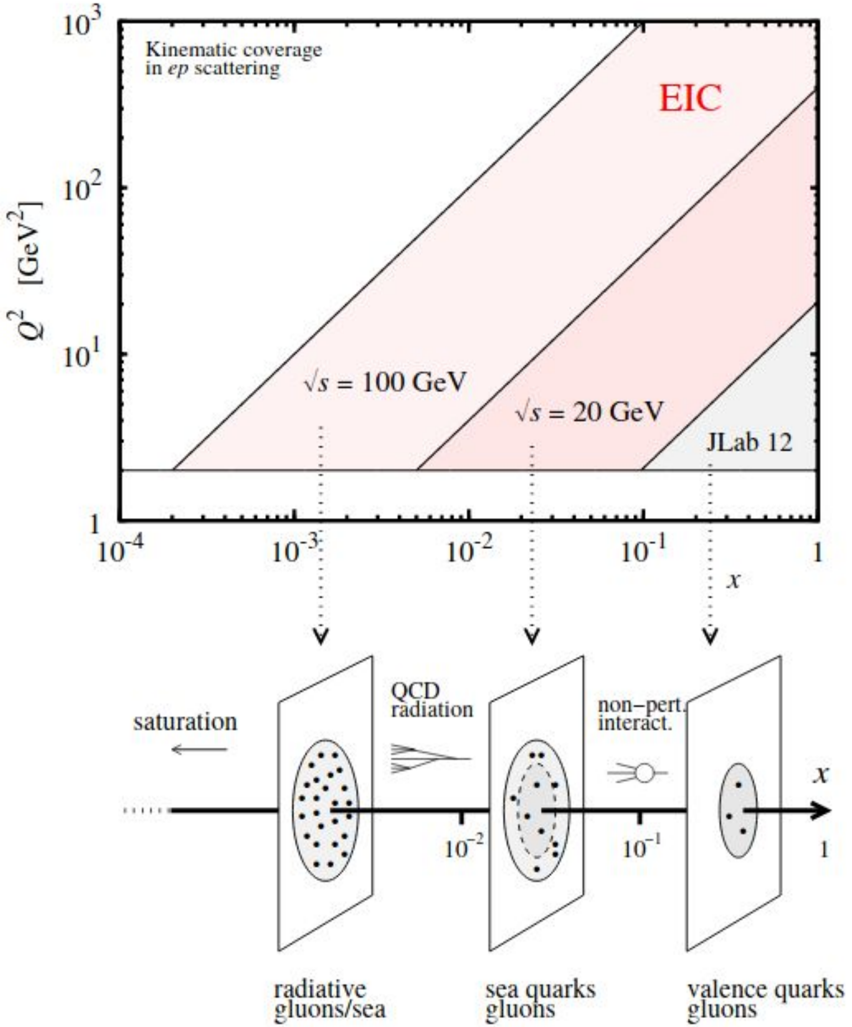
SIDIS coverage

Importance of complementary experiments

from JLab 12 GeV, Hermes, Compass
to the EIC

zooming into hadron structure

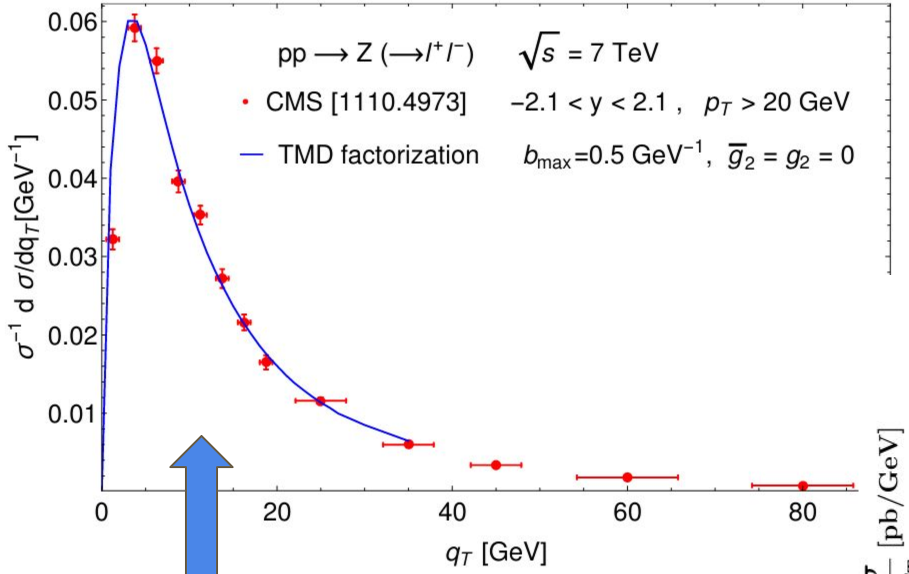
Credit picture: C. Weiss



TMD region: low transverse momentum

$q_T \ll Q$

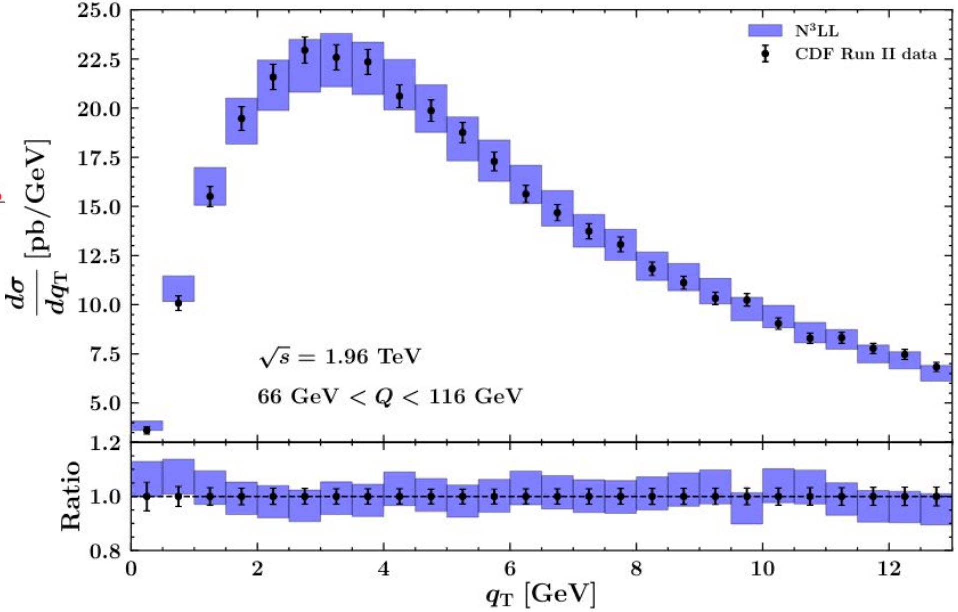
https://inspirehep.net/literature/1785810



$q_T \,/\, Q \;<\; 0.3$

Hadronic collisions

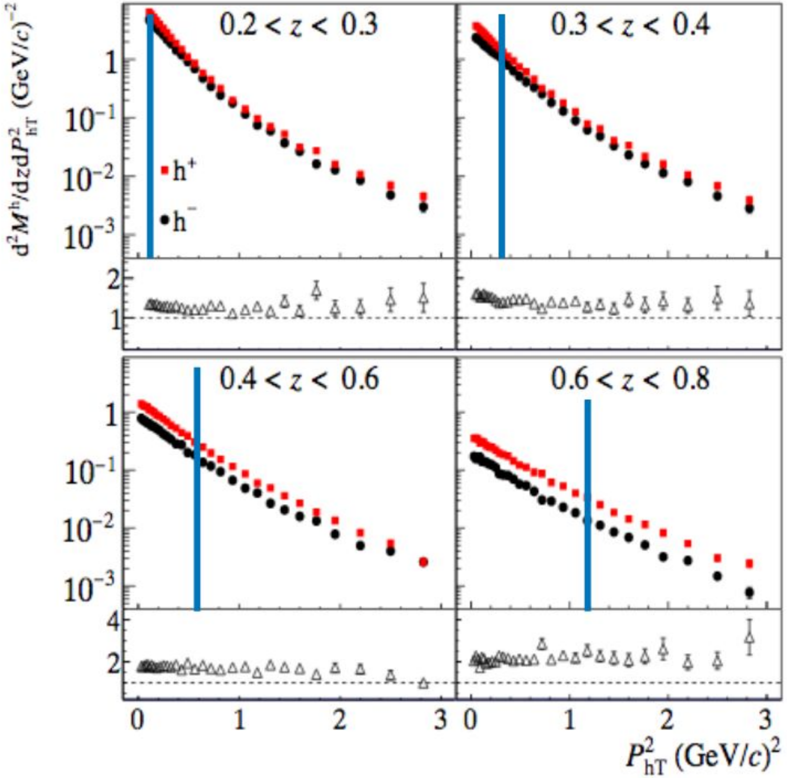
https://inspirehep.net/literature/1771006



$q_T \,/\, Q \;<\; 0.2$

TMD region: low transverse momentum

$$\boxed{q_T \ll Q}$$



COMPASS unpolarized SIDIS multiplicities - arxiv 1709.07374

SIDIS - TMD region

$$\boxed{P_{hT}^2/z^2 \ll Q^2}$$

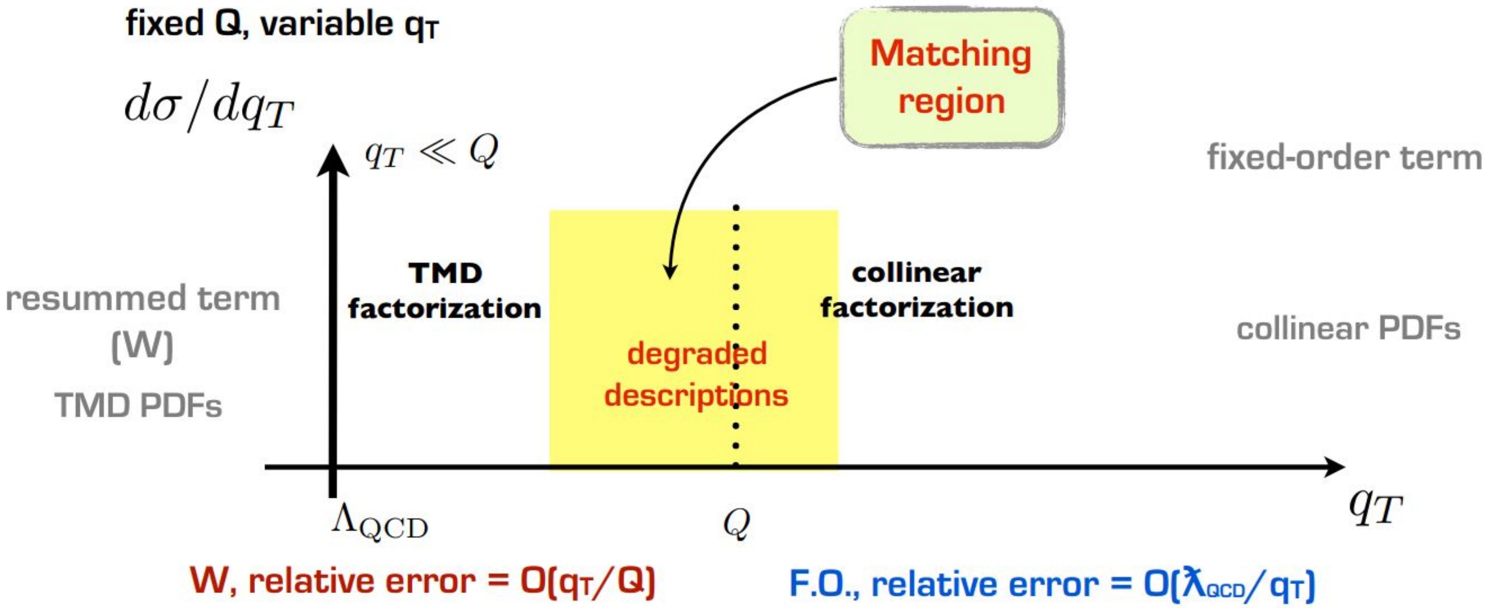
Let's highlight

$$P_{hT}^2/z^2 \sim 0.25\ Q^2$$

One of the bins with highest Q:

$$\langle Q^2 \rangle = 9.78\ \mathrm{GeV}^2$$
$$\langle x \rangle = 0.149$$

Matching TMD and collinear factorization



fixed Q, variable q_T

$d\sigma/dq_T$

$q_T \ll Q$

Matching region

fixed-order term

resummed term (W)

TMD PDFs

TMD factorization

degraded descriptions

collinear factorization

collinear PDFs

Λ_{QCD}

Q

q_T

W, relative error = $O(q_T/Q)$

F.O., relative error = $O(\Lambda_{QCD}/q_T)$

Matching schemes

- **“Subtraction” schemes :**

  cross section = **W + (FO - ASY)** = **W + Y**

  cross section = W * FO / ASY

At low Q (e.g. SIDIS) these cancellations do not work well as expected

- **“Average” scheme :**

  AS et al. https://inspirehep.net/literature/1646273

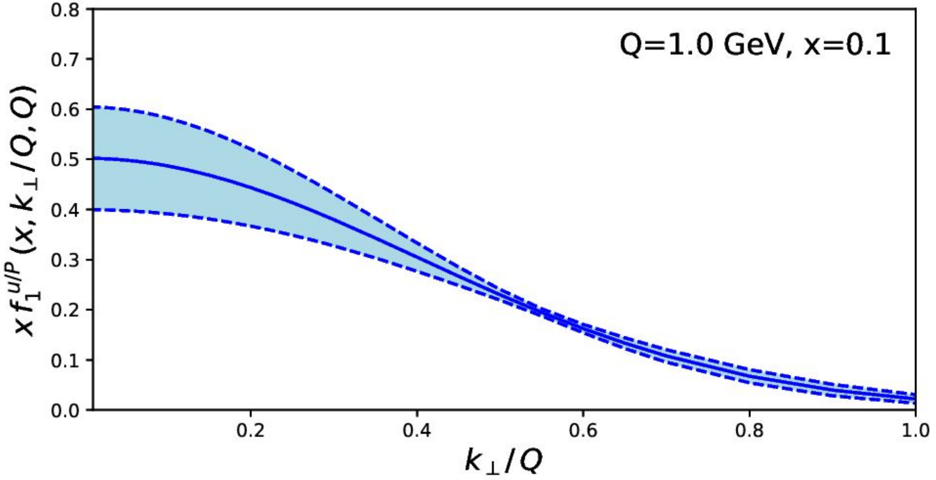
  Cross section = **a W + b FO**
  a, b : weights related to *power corrections to factorization theorems*

  (weighted average scheme: model-dependence better under control)

Unpolarized TMDs: PV17

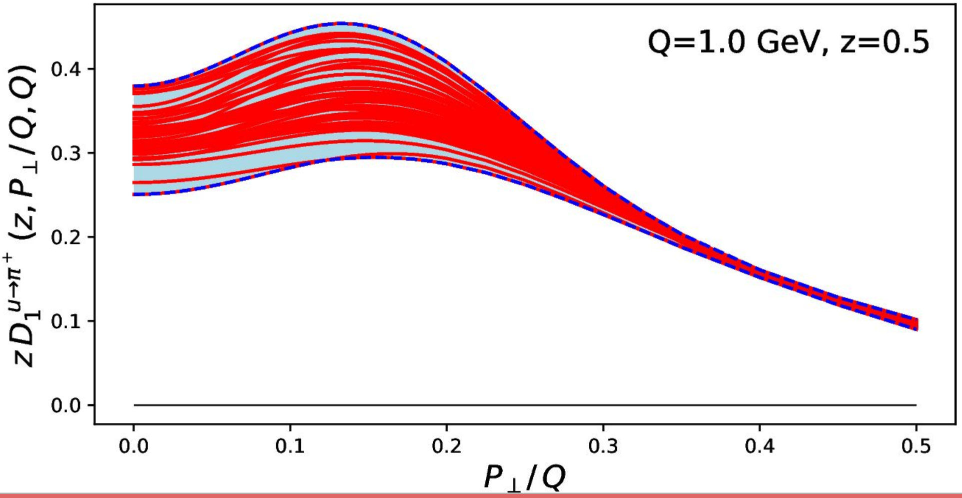
see https://inspirehep.net/literature/1520011

Imaging from **SIDIS** data (Hermes and Compass)
and **Drell-Yan** data (fixed-target & Z production @ Fermilab)



Unpolarized TMD PDF

Unpolarized TMD FF



Combining SIDIS and Drell-Yan:
Possibility to disentangle
hadron structure and formation

See https://inspirehep.net/literature/1520011

TMD impact studies: PV17

200 replicas are compared
with pseudodata

$$\chi^2_k = \chi^2_{k,\mathrm{EIC}} + \chi^2_{k,\mathrm{PV17}}$$

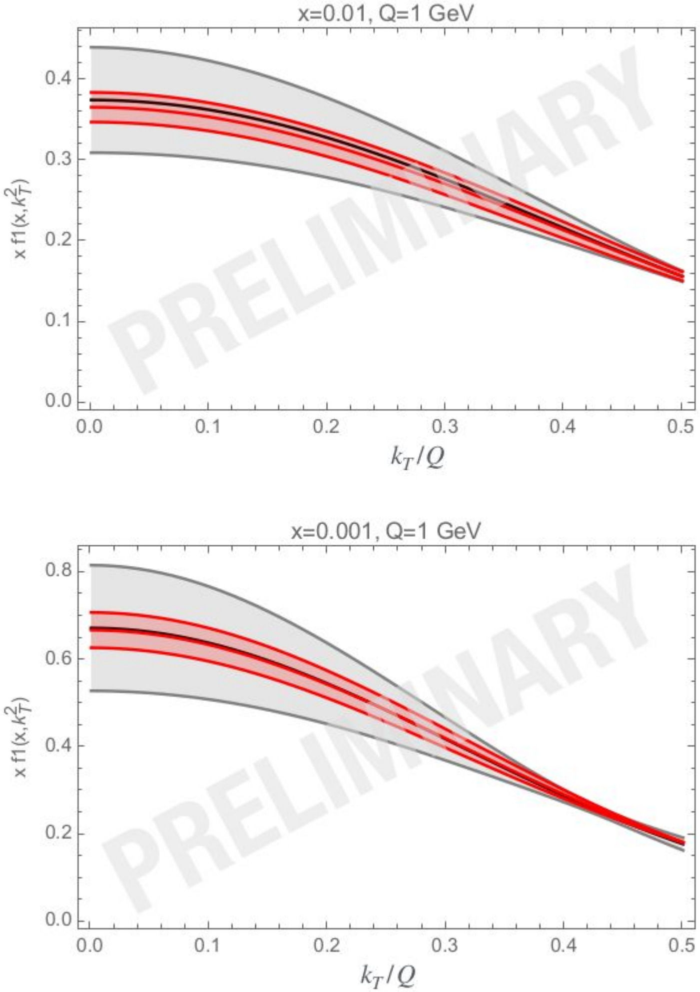
'original' χ^2
with respect to PV17 data

weights

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

Reweighting technique (no fit of EIC pseudo-data)

(see C. Bissolotti's talk at DIS 2021)

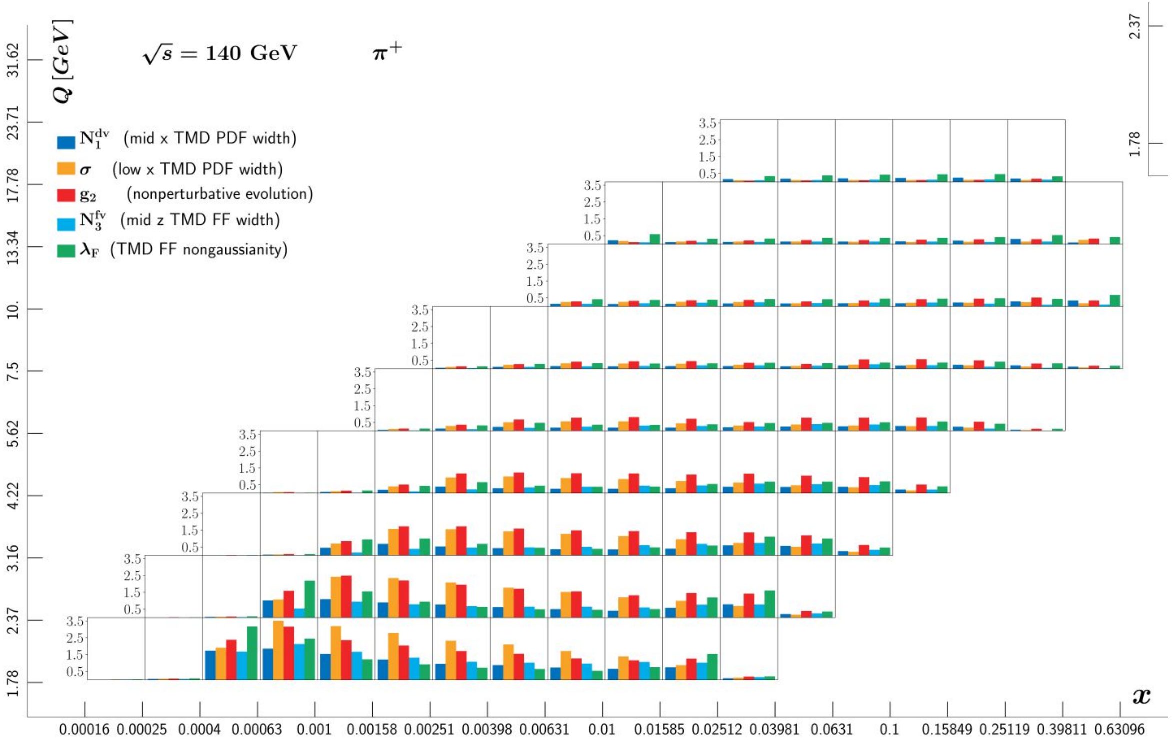


TMD impact studies: PV17

(see C. Bissolotti’s talk at DIS 2021)

$$S[f_i,\mathcal{O}] = \frac{\langle \mathcal{O}\cdot f_i\rangle - \langle \mathcal{O}\rangle\langle f_i\rangle}{\delta\mathcal{O}\Delta f_i}$$

O: e.g. a SIDIS structure function
fi : the non-perturbative TMD parameters



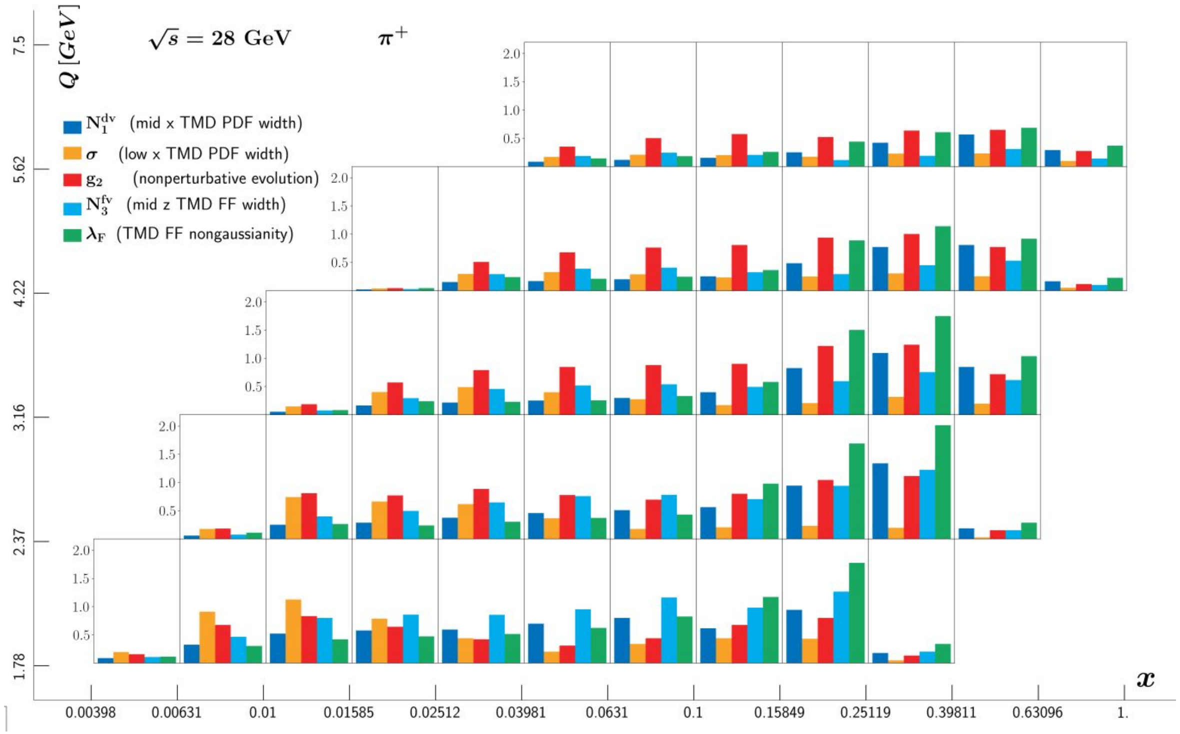
$$\sqrt{s} = 140 \text{ GeV}$$

TMD impact studies: PV17

(see C. Bissolotti’s talk at DIS 2021)

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\delta \mathcal{O} \Delta f_i}$$

O: e.g. a SIDIS structure function
fi : the non-perturbative TMD parameters



$$\sqrt{s} = 28 \text{ GeV}$$

Stronger effect at
lower energies

Unpolarized TMDs: SV19

see https://inspirehep.net/literature/1770788

Extraction from **SIDIS** (Hermes, Compass)
and **Drell-Yan** data (Phenix, fixed-target at Fermilab, CDF, DO, ATLAS, CMS, LHCb)

No problems with normalization in SIDIS - several source of power corrections

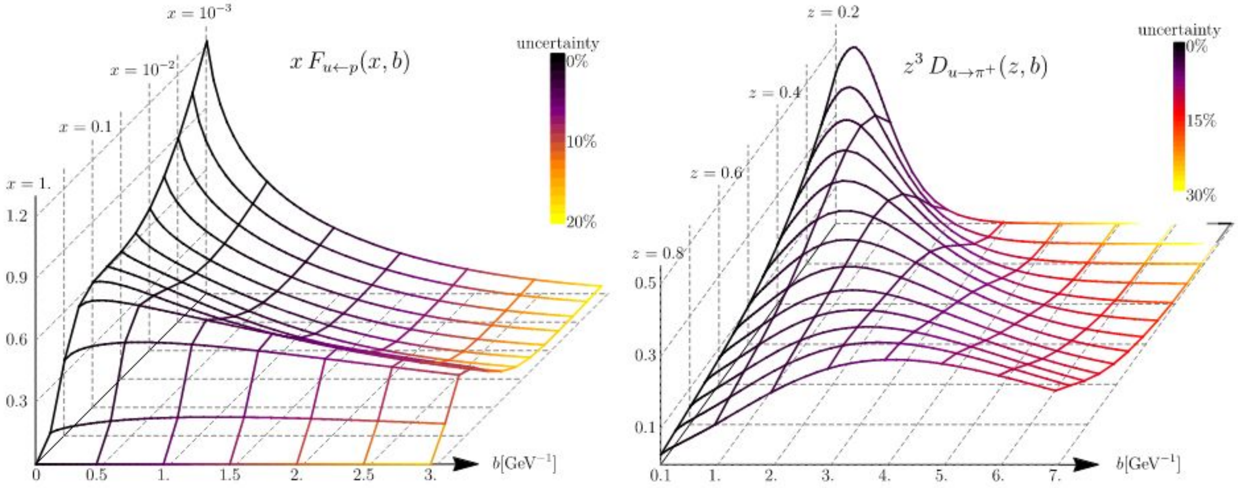


Figure 24. Example of extracted (optimal) unpolarized TMD distributions. The color indicates the relative size of the uncertainty band

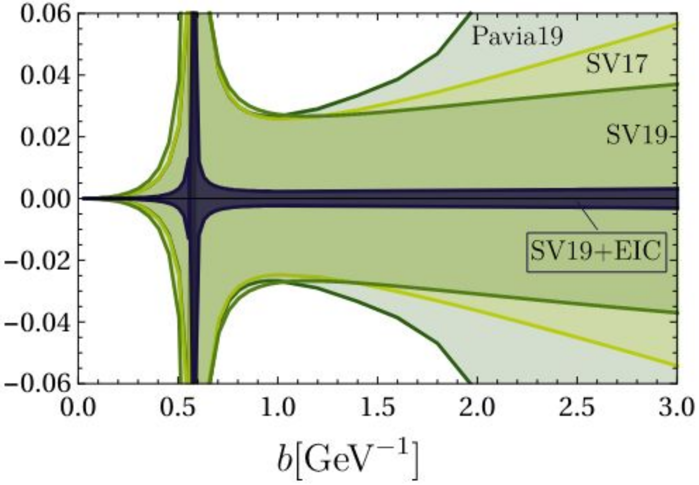
TMD impact studies: SV19

See https://inspirehep.net/literature/1851258

$$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T\mu_0,\alpha_s(\mu_0))\boxed{+g_K(b_T;\lambda)}}$$

→ evolution in ζ

Non-pert. corrections (large bT)



Typically a function of bT^2 with one or two parameters (with variations of course)

Huge impact of EIC SIDIS program on **non-perturbative TMD evolution**

TMD impact studies: SV19

See https://inspirehep.net/literature/1851258

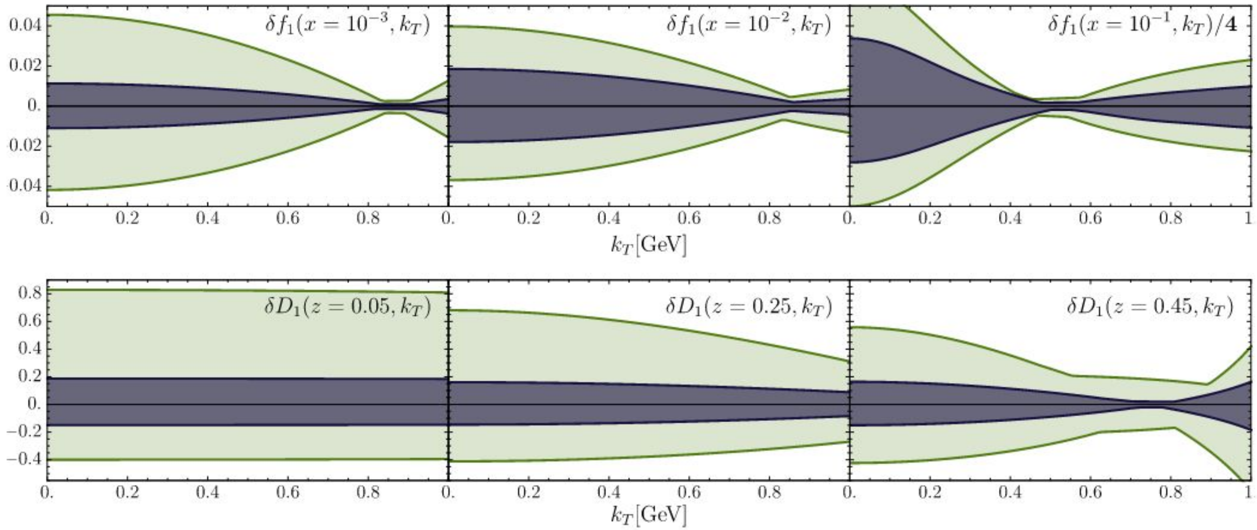


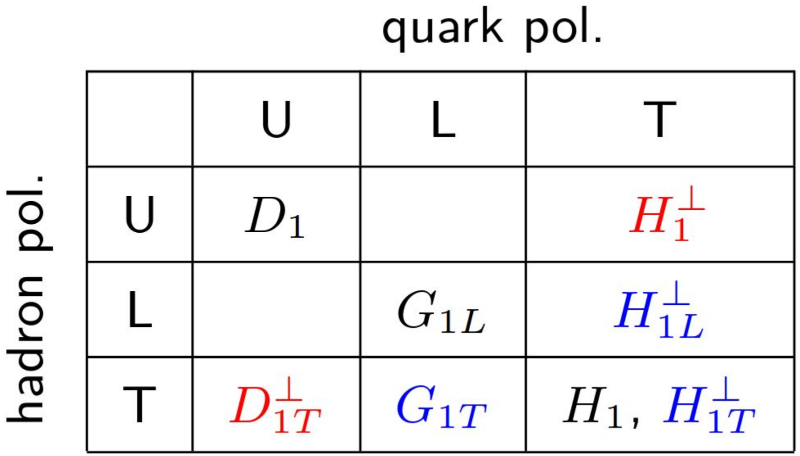
Figure 7.52: Comparison of relative uncertainty bands (i.e. uncertainties normalized by central value) for up-quark unpolarized TMD PDFs (upper panel) and $u \to \pi^+$ pion TMD FFs (lower panel), at different values of x and z as a function of k_T, for $\mu = 2$ GeV. Lighter band is the SV19 extraction, darker is SV19 with EIC pseudodata.

Up in proton
TMD PDF

Up to pion+
TMD FF

Fit with EIC
pseudo-data

Collinear and TMD single-hadron FFs



quark pol.

hadron pol.

	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

At leading twist:
8 TMD FFs and
3 collinear FFs (diagonal)

The **symmetries of QCD** play
a crucial role in this classification

Universality..!

Separating small and large bT

One needs to “separate” the small (perturbative) bT region from the large (non-perturbative) bT region:

$$\alpha_s(\mu = \mu_b \sim 1/b_T) \longrightarrow b_T < b_{max}$$

Avoid the Landau pole of QCD

$$\int_{\mu_b \sim 1/b}^{Q} \gamma_F \;, \quad \mu_b < Q \longrightarrow b_T > b_{\min}$$

Otherwise gluon “absorption” instead of “emission”

For more details see https://inspirehep.net/literature/1520011

Separating small and large bT

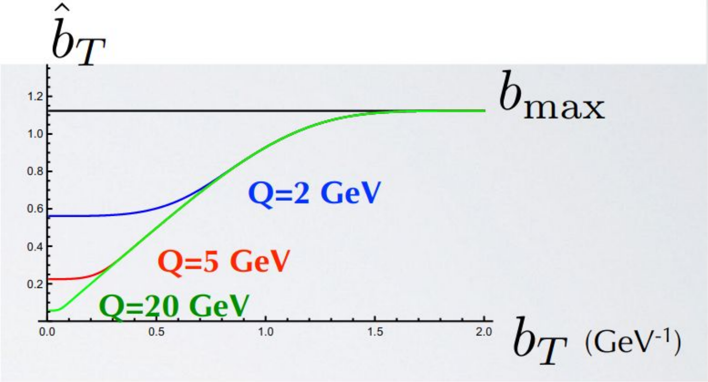
One needs to “separate” the small (perturbative) bT region from the large (non-perturbative) bT region:

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right) \begin{array}{l} \nearrow\; b_{\max}\;, \quad b_T \to +\infty \\[2ex] \searrow\; b_{\min}\;, \quad b_T \to 0 \end{array}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

These choices guarantee that for Q=1 GeV the TMD coincides with the NP model



For more details see https://inspirehep.net/literature/1520011

Some open questions

A non-exhaustive *personal* list of open questions:

- deepen our understanding of **sea** quarks
- **flavor structure** of TMDs
- experimental confirmation of **sign change** relation
- **gluon** observables and **spin-1** effects
- what can **hadronization** teach us about **confinement**?
- interplay between **nuclear/hadron** and **high-energy** physics
- ..