Correlations between v_2^2 and multiplicity in CGC particle production.

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The Ridge in p-Pb.

The motivation originally comes from attempts to understand the observation of "Ridge" correlations in p-p and p-Pb at LHC.

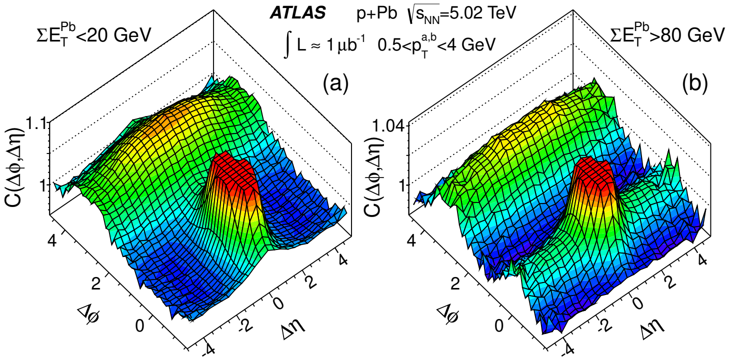


Figure: Ridge in p-Pb at ATLAS, $\sim 10^{-2}$ events

Currently exploring the relevance of these effects for EIC as well.

How do we calculate particle production in CGC?

Basics: eikonal scattering of the projectile gluons on the target fields.

Projectile color charge density $\rho^a(x)$ produces soft gluons via the Weiszacker-Williams field.
Gluons "decohere" due to eikonal interaction with the target and are scattered into final state.

$$a^{\dagger a}(x) \to U^{ab}(x_\perp) a^{\dagger b}(x)$$

$U^{ab}(x_\perp)$ - eikonal scattering matrix.
Work within the dilute-dense CGC approach: "p-A at mid rapidity".
No hadronization is accounted for: final state gluons = final state hadrons.
Final state interactions not considered until recently - perturbative classical field dynamics after collision may be taken into account.

The CGC hadron wave function.

High energy factorisation: the fast partons are dressed by the soft gluon cloud.

Fast partons: color charge density in the transverse plane $\rho^a(x_\perp)$.

Soft gluons: the Weiszacker-Williams cloud.

CGC wave function of the projectile (approximate!):

$$\Psi[A] = e^{i\int_{x_\perp} b_i[\rho]A_i(x_\perp)}|0\rangle_{soft}|v\rangle$$

Solution of classical Yang-Mills equation:

$$\partial_i b_i^a(x_\perp) = g\rho^a(x_\perp); \qquad b_i^a(k) = g\frac{ik_i}{k^2}\rho^a(k)$$

ρ has to be averaged over with some weight functional. Need to know $|v\rangle$.

Don't know - therefore model e.g. simplest Gaussian:

McLerran-Venugopalan model.

The flow harmonics

The azimuthal flow harmonics v_n^2 are defined as

$$v_n^2(k_1, k_2) \equiv \frac{\int d\phi_1 d\phi_2 e^{in(\phi_1-\phi_2)} \frac{d^2N}{d^2k_1 d^2k_2}}{\int d\phi_1 d\phi_2 \frac{d^2N}{d^2k_1 d^2k_2}} ,$$

where ϕ_1, ϕ_2 are the azimuthal angles of the corresponding transverse momenta.

For $n = 2$ this is equally sensitive to correlations at $\Delta\phi = 0$ - "forward", and $\Delta\phi = \pi$ - "back to back" correlations in the transverse plane.

So both "forward" and "backward" ridge contribute to v_2 equally.

Single inclusive gluon production

The master formula:

$$\frac{dN}{d^2p\,d\eta} = \langle \int_{z,\bar{z}} e^{ik(z-\bar{z})} \int_{x_1,x_2,\bar{x}_1,\bar{x}_2} \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)$$

$$\mathrm{Tr}\ \tilde{\rho}(x_1)[U^\dagger(x_1)-U^\dagger(z)][U(\bar{x}_1)-U(\bar{z})]\ \tilde{\rho}(\bar{x}_1)\rangle_{P,T}$$

$$f_i(x-y) = \frac{(x-y)_i}{(x-y)^2}, \qquad \tilde{\rho} \equiv -iT^a\rho^a$$

Have to average over the projectile (distribution of ρ) and target (distribution of U)

Double inclusive production.

The master formula:

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = \langle\, \sigma(k)\,\sigma(p) + O(\rho^2) + O(\rho^3)\,\rangle_{P,T}$$

with

$$\sigma(k) = \int_{z,\bar z} e^{ik(z-\bar z)} \int_{x_1,x_2,\bar x_1,\bar x_2} \vec f(\bar z - \bar x_1)\cdot \vec f(x_1 - z)$$

$$\mathrm{Tr}\,\tilde\rho(x_1)[U^\dagger(x_1) - U^\dagger(z)][U(\bar x_1) - U(\bar z)]\;\tilde\rho(\bar x_1)$$

$O(\rho^2)$ - production of both gluons from a single "Pomeron". Contributes to back-to-back correlations subtracted. Not frequently discussed (but see Haowu Duan's talk).

$O(\rho^3)$ - production from "Odderon". Ditto as above

$\langle\sigma\sigma\rangle$ term is the main source of "forward" correlations in CGC calculations.

Nature of correlations in CGC

Leading order CGC calculations: correlations are flat in rapidity; forward and back-to-back are of equal strength.

The correlations come in two varieties: "classical" and "quantum".

Classical: local anisotropy (AKA color field domains); local density variations.

Quantum: Bose enhancement of gluons in the initial projectile wave function; Hanbury-Brown, Twiss correlations of emitted gluons.

Here - only the correlations of Quantum origin. Leading when the area of the projectile ($\sim 1/\Lambda^2_{QCD}$) is much larger than the (square of the) color correlation length in the target ($1/Q_s^2$)

Bose enhancement

Bose enhancement in the incoming wave function a.k.a. "Glasma graphs"

The CGC soft gluon state is "classical" - coherent state.

But when averaged over the valence color charges, the density matrix is not classical!

$$\hat{\rho} = \mathcal{N} \int D[\rho]\ W[\rho] e^{i \int_q b^i_b(q)[a^i_b(-q) + a^{\dagger i}_b(q)]} |0\rangle\langle 0|\ e^{-i \int_p b^j_c(p)[a^j_c(-p) + a^{\dagger j}_c(p)]}$$

Explicit calculation with McLerran-Venugopalan model

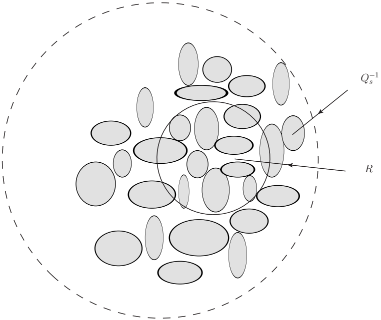
$$W[\rho] = e^{-\int_k \frac{1}{2\mu^2} \rho_a(k)\rho_a(-k)}$$

yields Bose enhancement effect: enhanced probability to find two gluons with the same transverse momentum.

More gluons come in with $k_\perp = p_\perp \to$ more gluons are produced with $k_\perp \approx p_\perp$.

Hanbury-Brown, Twiss

Gluonic Hanbury-Brown - Twiss effect



Scattering randomizes color phases in the projectile on transverse scale Q_s^{-1} - the projectile after scattering turns into a bunch of sources of incoherent emission. Typical HBT situation.

BE vs HBT 1

Concentrate on target averages.

E.g.:

$$\int_{z,\bar{z}} F(z,\bar{z})\langle [U(z_1)U^\dagger(\bar{z}_1)]^{ab}[U(z_2)U^\dagger(\bar{z}_2)]^{cd}\rangle_T$$

Roughly: The further apart the points are, the larger the contribution due to integration (no area suppression.)

But has to be color invariant!

Color neutralization in the target on distance scales $r \sim 1/Q_s$.

So points should be pairwise close to each other - otherwise the average vanishes.

Any product of U's factorizes *a la* Wick with basic "contraction" (also Haowu Duan's talk)

$$\left\langle U^{ab}(x)U^{cd}(y)\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{N_c^2-1}d(x,y),$$

BE vs HBT 2

$$\langle Q(z_1, \bar{z}_1, z_2, \bar{z}_2)\rangle_T = d(z_1, \bar{z}_1)d(z_2.\bar{z}_2) + d(z_1, \bar{z}_2)d(z_2, \bar{z}_1) + \frac{1}{N_c^2 - 1} d(z_1, z_2)d(\bar{z}_1, \bar{z}_2),$$

First term: two gluons scatter independently. But arise with larger probability from the wave function. Bose enhancement.
Width of the correlation $\Delta k \sim Q_s^T$

Second term: correlates directly momenta of produced gluons. Hanbury Brown - Twiss effect.
Width of the correlation $\Delta k \sim 1/R$ - size of the projectile.

Both peaked around $|k| = |p|$, but HBT peak is much narrower and much higher. One can tune in and out of HBT peak by varying the width of the momentum bin.

$\langle v_2 N \rangle$

Experimentally - strength of correlations grows with multiplicity.

Can we probe this correlation in the CGC calculation?

Ideally we should average over a sub ensemble (projectile and target) to concentrate on high multiplicity events. But this is difficult.

Instead we calculate the correlations between v_2 and multiplicity.

$$\langle v_2^2 N \rangle = \frac{1}{\langle v_2^2 \rangle \langle N \rangle} \int d^2k_1 d\phi_2 d\phi_3 e^{i2(\phi_2 - \phi_3)} \langle \frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3} \rangle =$$

$$\frac{1}{\langle v_2^2 \rangle \langle N \rangle} \int d^2k_1 d\phi_2 d\phi_3 e^{i2(\phi_2 - \phi_3)} \left\langle \left\langle \frac{dN}{d^2k_1 dy_1} \frac{dN}{d^2k_2 dy_2} \frac{dN}{d^2k_3 dy_3} \right\rangle_p \right\rangle_t$$

We integrate $v_2^2(k_1, k_2)$ over bins in transverse momentum and vary the central position of the bin..

Inputs

For projectile averages use MV model

$$W_p(\rho_p) = \exp\left(-\int \frac{d^2q}{(2\pi)^2}\rho_p^a(-q)\frac{1}{2\mu^2(q)}\rho_p^a(q)\right) .$$

For target averages we use the saturation approximation, which factorizes the correlators of U into pairs

$$\langle U_{ab}(p)U_{cd}(q)\rangle_{\rm t} = \frac{(2\pi)^2}{N_c^2-1}\delta_{ac}\delta_{bd}\delta^2(p+q)d(p)\,,$$

with the "adjoint dipole" amplitude

$$d(p) = \frac{1}{N_c^2-1}\int d^2x e^{ix\cdot p}\langle {\rm tr}\left[U^\dagger(x)U(0)\right]\rangle_{\rm t}\,.$$

Adopt the Golec-Biernat - Wusthoff (GBW) model for the dipole

$$d(p) = \frac{4\pi}{Q_s^2}\, e^{-p^2/Q_s^2}\,.$$

Results 1

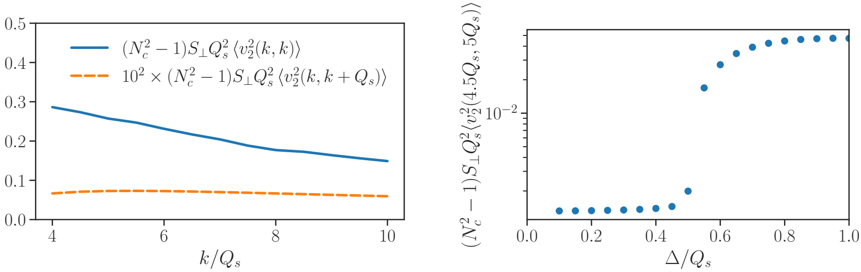


Figure: Left panel: The second flow harmonic, v_2^2 as a function of the momentum. The calculation of v_2^2 is performed for two cases: a) the same momentum of the pair, b) the momentum of the pair is offset by the saturation momentum of the target in order to avoid the gluon HBT effect. The bin width in both cases is $\Delta = Q_s/2$.
Right panel: The second flow harmonic, v_2^2 as a function of the bin width. The centers of the two bins are chosen at $k = 4.5Q_s$, $k' = 5Q_s$.

Results 2

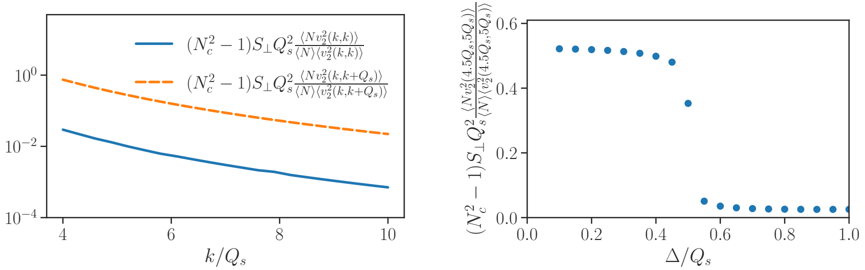


Figure: Left panel: The three particle correlation function $\langle v_2^2 N\rangle$ defined by the normalized correlations between v_2^2 and the total multiplicity of produced particles. The calculation of v_2^2 is performed for two cases: a) the same momentum of the pair, b) the momentum of the pair is offset by the saturation momentum of the target in order to avoid the gluon HBT effect. The bin width in both cases is $\Delta = Q_s/2$.
Right panel: The three particle correlation function $\langle v_2^2 N\rangle$ as a function of the bin width.

Discussion

There is a clear positive correlation between v_2^2 and multiplicity.

For BE the correlation is rather significant - the normalized correlator is of order $\sim .1$.

HBT is much weaker correlated with multiplicity. This is understandable, since the multiplicity is mostly driven by soft gluons which are strongly BE correlated.

The difference is very striking when we vary the bin width at fixed offset between momenta of the two measured gluons. v_2^2 drops sharply when the bin width becomes smaller than the offset, while the correlation sharply increases at the same point.