Determination of Collins-Soper kernel

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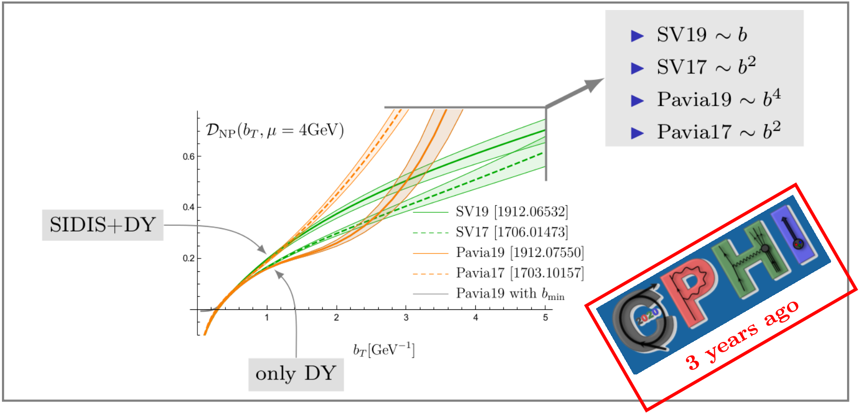
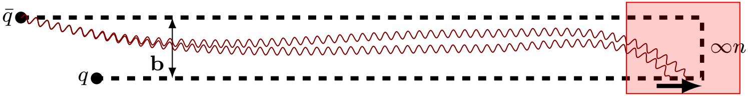


Where do we stand nowadays?

Outline

- Appearance and definition of CS kernel
- Theory determination
- Extractions of CS kernel

$$O_{\rm TMD} = \bar{q}(\lambda n + b)[\lambda n + b, -\infty n + b]...[-\infty n, 0]q(0)$$



Rapidity divergence

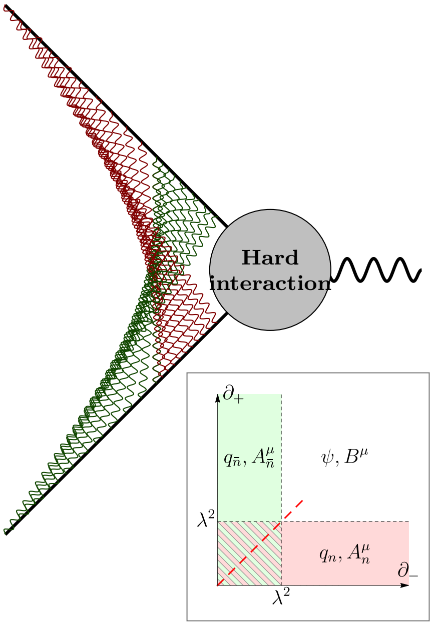
- Non-Local (depends on b)
- Not regularized by dim.reg.
- Multiplicatively renormalizable

$$O_{\rm TMD} = R\left(b^2, \zeta\right) O_{\rm TMD}(\zeta)$$

- Rapidity anomalous dimension (=**CS kernel**)

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x, b; \mu, \zeta) = -\mathcal{D}^f(b, \mu) F_{f\leftarrow h}(x, b; \mu, \zeta)$$

$$\mathcal{D} = -\frac{1}{2}K = \frac{1}{2}F_{q\bar{q}} = -\frac{1}{2}\gamma_\nu^{f_\perp} = -\frac{1}{2}\gamma_\zeta$$



The counter-part of rap. div. is the soft-gluon exchanges between in/out-going partons.
They are absorbed into a soft-factor, which subtract the overlap of modes

$$d\sigma \sim \int F_1 \times \frac{1}{S} \times F_2$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$d\sigma \sim \int F_1(\zeta){\color{red}R} \times \frac{1}{{\color{red}R}\Sigma_0{\color{red}R}} \times {\color{red}R}F_2(\bar\zeta)$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$d\sigma \sim \int F_1^{\text{phys}}(\zeta) \times F_2^{\text{phys}}(\bar\zeta)$$

- $F^{\text{phys}}(\zeta) = F(\zeta)/\sqrt{\Sigma_0}$
- $\zeta\bar\zeta = Q^4$

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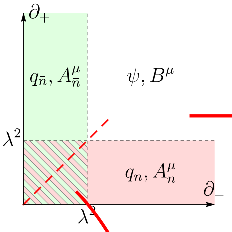
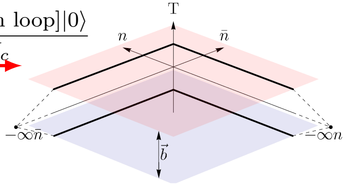
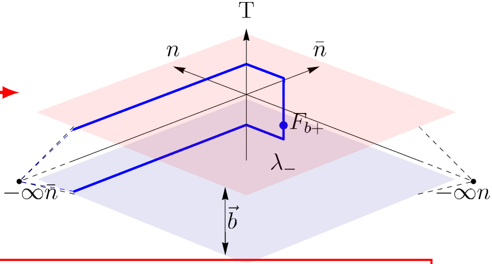


Soft factor

$$= \frac{\langle 0|[\text{Wilson loop}]|0\rangle}{N_c}$$



Variation
of overlap size
gives CS kernel



[AV, PRL 125 (2020) 19]

$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\int_0^1 d\beta \langle 0|F_{b+}(-\lambda_- n + b\beta) W_{C'}|0\rangle}{\langle 0|W_{C'}|0\rangle} + Z_{\mathcal{D}}(\mu)$$

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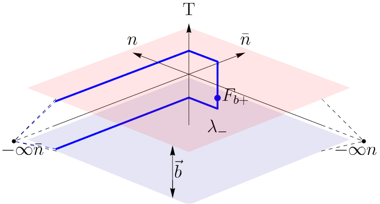
Process independent self-contained definition of CS-kernel



$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \int_0^1 d\beta \frac{\langle 0|F_{b+}(-\lambda_- n + b\beta) W_{C'}|0\rangle}{\langle 0|W_{C'}|0\rangle} + Z_{\mathcal{D}}(\mu)$$

- λ_- independent (any finite)
- Renormalization group equation (CS-equation)

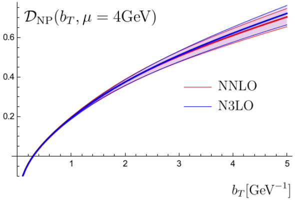
$$\mu \frac{d}{d\mu}\mathcal{D}(b,\mu) = \Gamma_{\text{cusp}}(\mu)$$

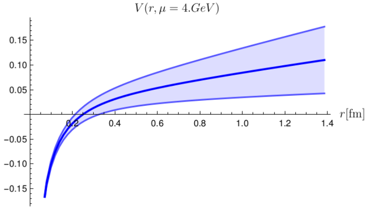
CS kernel is not "just a part of TMD factorization" but a self-contained nonperturbative function
It is as important and interesting as TMDs or PDFs

How can we interpret CS kernel?

Not clear so far...

- CS-kernel "knows" only about QCD vacuum
- Similar to inter-quak potential matrix element but light-like (Wilson criterium)
- In models can be computed, e.g. in SVM [Brambilla,Vairo,hep-ph/9606344]





$$V(r)=r\frac{\pi}{4}\mathcal{D}''(0)+\frac{\mathcal{D}'(0)}{2}+\frac{r^2}{2}\int_r^\infty\frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}}+...$$

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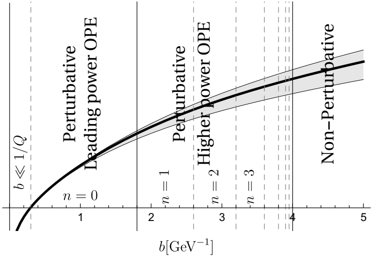
The field-theoretical definition allows first-principle computation
Systematic small-b expansion

$$\mathcal{D}(b) = \mathcal{D}_0(\ln(b)) + \mathbf{b}^2\mathcal{D}_2(\ln(b)) + \mathbf{b}^4\mathcal{D}_4(\ln(b)) + ...$$



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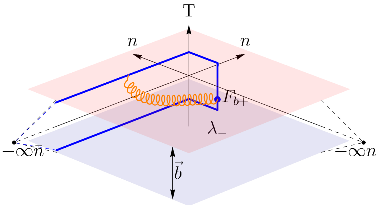
- $\mathcal{D}_0$ is known up to NNLO (α_s^3)

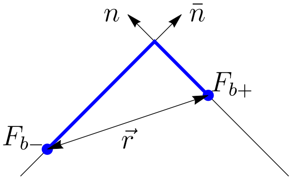


$$\begin{aligned}\mathcal{D}_0 \quad = \quad & 2a_s C_F \mathbf{L}_b \\ & +2a_s^2 C_F\left\{\beta_0\mathbf{L}_b^2 + 2\left[\left(\frac{67}{2} - 2\zeta_2\right)C_A - \frac{10}{9}N_f\right]\right. \\ & \qquad \left. +C_A\left(\frac{404}{27} - 14\zeta_3\right) - \frac{56}{27}N_f\right\} \\ & +a_s^3\,(\text{known}) + ...\end{aligned}$$

The field-theoretical definition allows first-principle computation
Systematic small-b expansion

$$\mathcal{D}(b) = \mathcal{D}_0(\ln(b)) + \mathbf{b}^2\mathcal{D}_2(\ln(b)) + \mathbf{b}^4\mathcal{D}_4(\ln(b)) + ...$$

- $\mathcal{D}_0$ is known up to NNLO (α_s^3)
- $\mathcal{D}_2$ is known at LO (α_s^0)



$$\mathcal{D}_2 = \frac{1}{2}\int_0^\infty d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2)}{\mathbf{r}^2} + \mathcal{O}(a_s^2)$$

- Value is unknown
- But can be estimated

$$\mathcal{D}_2 \sim \frac{\pi^2}{72}\frac{G_2}{\Lambda^2_{QCD}} \simeq (1.-5.)\times 10^{-2}\text{GeV}^{-2}$$

	Pavia17	SV19	SV17	Pavia19	BLNY(03/14)
$\mathcal{D}_2 \times 10^2$	2.8 ± 0.5	2.9 ± 0.6	$0.7^{+1.2}_{-0.7}$	0.9 ± 0.2	$20 - 35$
	DY+SIDIS		DY only		

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How it behaves at $b \to \infty$?

So far, only models...

In Stochastic vacuum model

Linear asymptotic

$$\lim_{\mathbf{b}^2 \to \infty} \mathcal{D}(b) = \sqrt{\mathbf{b}^2} \int_0^\infty d\mathbf{y}^2 2\sqrt{\mathbf{y}^2}\Delta(\mathbf{y}^2) = \sqrt{\mathbf{b}^2} c_\infty$$

Lattice computation of c_∞ [Bali,Brambilla,Vairo,97; Meggiolaro,98]

$c_\infty \simeq 0.01 - 0.4\text{GeV}$ compare to $c_\infty^{\text{SV19}} \simeq 0.06 \pm 0.01\text{GeV}$

Non-abelian Stockes theorem (in leading approximation)

Slower than linear asymptotic

$$\lim_{\mathbf{b}^2 \to \infty} \mathcal{D}(b) \sim (\mathbf{b}^2)^{1/2-\delta}, \qquad \delta > 0$$

Determining
Collins-Soper kernel
from measurements

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CS kernel within TMD factorization

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq)_T} H(Q) F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2)$$

Evolving to a reference scale

$$\mu^2 \frac{d}{d\mu^2} F(\mu,\zeta) = \gamma_F(\mu,\zeta) F(\mu,\zeta)$$
$$\zeta \frac{d}{d\zeta} F(\mu,\zeta) = -\mathcal{D}(\mu) F(\mu,\zeta)$$

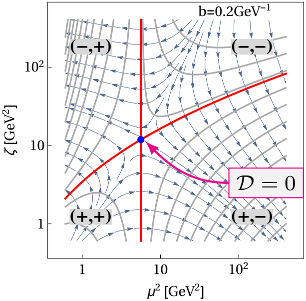
$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq)_T} H(Q) R^2[Q \to (\mu_0, \zeta_0), b] F_1(x_1, b; \mu_0, \zeta_0) F_2(x_2, b; \mu_0, \zeta_0)$$

$$R[Q \to (\mu_0, \zeta_0), b] = \exp\left[\int_P \left(\gamma_F \frac{d\mu}{\mu} - \mathcal{D}\frac{d\zeta}{\zeta}\right)\right]$$

Any path connecting initial and final points in (μ, ζ)-plane

- There are **three** functions to extract $\to \{F_1, F_2, \mathcal{D}\}$
- TMD distributions **internally depend** on CS-kernel $F = F[\mathcal{D}]$

The only way to decorrelate CS kernel and TMD is use select the reference scale with constant $\mathcal{D}$



$$(\mu,\zeta)[b,\mathcal{D}]$$

- The position of reference point depends on nonperturbative $\mathcal{D}$ and b
  - I.e. it must be determined together with the determination of CS kernel at each value of b
- Solution is not unique (equipotential lines)
- **The best option** is the saddle point $\mathcal{D}=0$.
  - Optimal TMD distribution [Scimemi,AV,17]

$$F(b,Q)=\left(\frac{\zeta}{\zeta_Q[\mathcal{D}]}\right)^{-\mathcal{D}}F(b)$$

The floating reference point is build-in into ζ-prescription
If the reference point is fixed than CS and TMDs cannot be disentangled
e.g. CSS formalism

CS kernel within CSS formulation

- CS kernel can be determined in this case

$$Q^2 \frac{dF(x,b;Q,Q^2)}{dQ^2} = (\gamma_F(Q,Q^2) - \mathcal{D}(b,Q))F(x,b;Q,Q^2)$$

- But the value of TMDs are not "universal" they depend on CS kernel

[Collins,Rogers,1705.07167]

$$\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\rm DY}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}$$
$$\times e^{-g_{j/A}(x_A,b_T;b_{\max})} \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}^{\rm PDF}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times e^{-g_{\bar{j}/B}(x_B,b_T;b_{\max})} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\rm PDF}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times \exp\left\{-g_K(b_T;b_{\max})\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$
$$+ \text{suppressed corrections.} \tag{11}$$

g_K is a part of TMD definition

$$\mathcal{D}(b,\mu) = \frac{-1}{2}\tilde{K}(b_*;\mu_{b^*}) - \int_{\mu_{b^*}}^{\mu}\frac{d\mu'}{\mu'}\gamma_K + \frac{1}{2}g_K(b,b_{\max})$$

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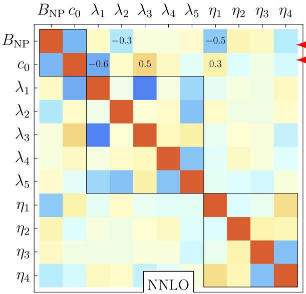
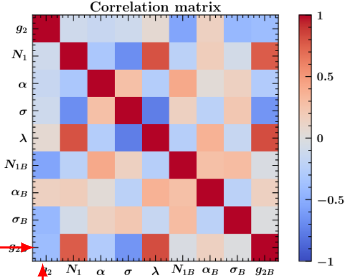
SV19

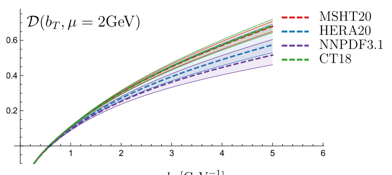
[Scimemi,AV, 1912.06532]



Pavia 19

[Bacchetta, et al, 1912.07550]





Bury, et al,2201.07114

The correlation is still very significant

- Dependent on PDF

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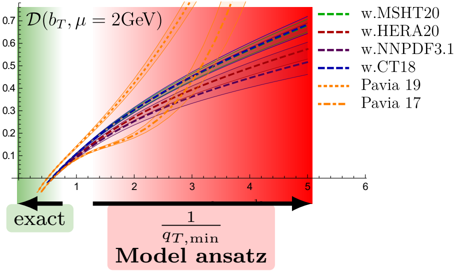
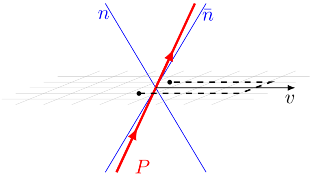
Data allows us to extract in a narrow region

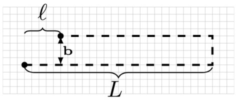


- **Total dependence on fitting ansatz**
- Adding SIDIS helps a lot!

> Before EIC we should look for alternative sources of information

Lattice simulations is an alternative access to CS kernel





- Factorization theorem for quasi-TMDs
$$W = \int dx e^{i\ell x p^+} H\Big(\frac{|x|p_+}{\mu}\Big) F(x, b; \mu, \zeta)\Psi(b, \mu, \bar{\zeta})$$
$$\zeta\bar{\zeta} = 2(xp^+)^2\mu^2$$
- Just alike ordinary TMD factorization but already in position space!
- $L \to \infty$ (power corrections $\sim \{\frac{b}{L}, \frac{\ell}{L}\}$)
- Factorization scale is (xp^+)
  - Cannot be large at lattice ($p_+ \sim$ 2GeV, max)
  - Power corrections at small b, $\sim \dfrac{1}{|b|p^+}$

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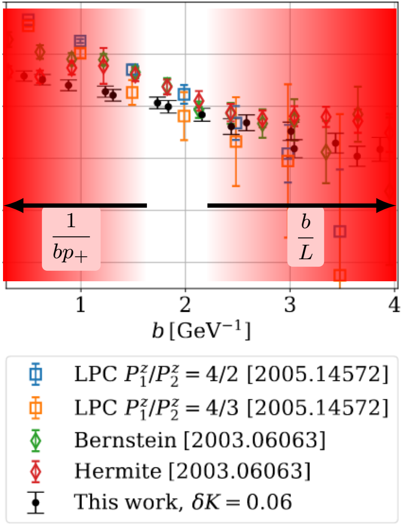

CS kernel is extracted from the ratio

$$\frac{W(P_1)}{W(P_2)} \sim \left(\frac{P_1^+}{P_2^+}\right)^{-2\mathcal{D}(b)} \mathbf{r}(..)$$

- Problems at large and small b
- Shown error bars are statistical only
- + Lattice systematics
  - Can be huge!

Yet, lattice extractions are in very early stage.

see Thuesday session

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Same idea can be used with any source of data

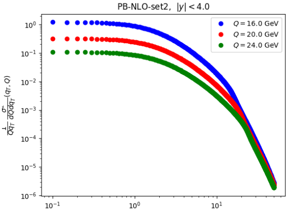
Extracting CS kernel from Monte-Carlo even generator(s)

[A.Bermudez Martinez, AV,in progress]

$$\frac{d\sigma}{\Delta P.S.} = \int dy \int dQ \int dq_T \int d^2b e^{-i(qb)_T} \sigma_0 H\Big(\frac{Q}{\mu}\Big) F_1\Big(\sqrt{\frac{Q^2}{s}}e^{y}, b; \mu, \zeta\Big) F_2\Big(\sqrt{\frac{Q^2}{s}}e^{-y}, b; \mu, \zeta\Big)$$

Extraction process

- Generate data in a two narrow bins of $Q = \{Q_1, Q_2\}$
  - with $\{s, y\}$ such that ranges of x's **exactly coincides**
  - with very fine binning in q_T



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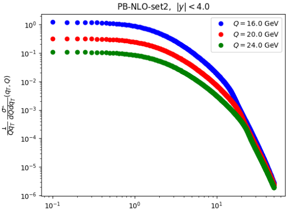
Extraction process

- Generate data in a two narrow bins of $Q = \{Q_1, Q_2\}$
  - with $\{s, y\}$ such that ranges of x's **exactly coincides**
  - with very fine binning in q_T
- Make an inverse (discrete) Fourier transform

$$\Sigma(b) = \int_0^\infty dq_T q_T J_0(q_T b) \frac{d\sigma}{\Delta P.S.}$$



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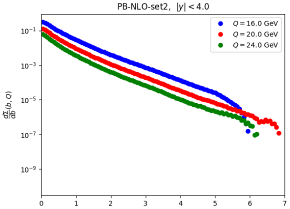


Same idea can be used with any source of data

Extracting CS kernel from Monte-Carlo even generator(s)

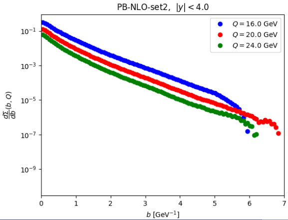
[A.Bermudez Martinez, AV,in progress]

Extraction process

- Make a ratio
  - All TMDs **exactly** cancel
  - Only CS kernel and perturbative terms are left
- Invert the formula

$$\mathcal{D}(b,\mu_0) = \frac{\ln(\Sigma_1/\Sigma_2) - \ln Z(Q_1,Q_2) - 2\Delta_R(Q_1,Q_2,\mu_0)}{4\ln(Q_2/Q_1)} - 1$$

  - $Z \sim H(Q_1)/H(Q_2)$
  - $\Delta_R \sim \int \frac{d\mu}{\mu}\Gamma$ evolution of $\mathcal{D}$ to same μ_0.



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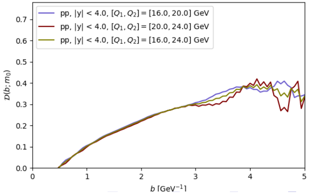
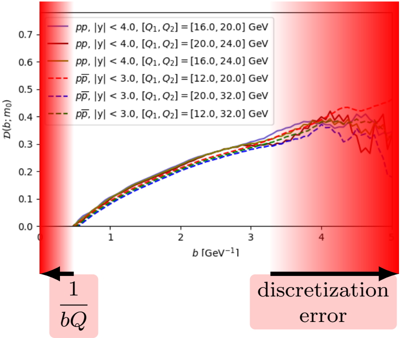


MC generator build having in mind NLO evolution ala Altarelli-Parisi

A complicated model that describes data

here CASCADE



- **Ideally,** different Q's must coincide
- **Ideally,** different processes must coincide

Testing factorization statement with MC generators

Passed!

MC generators knowns about vacuum...

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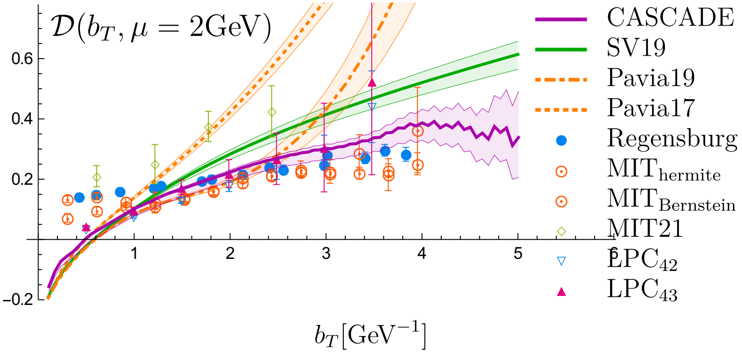
All extractions in a single plot



- MC extraction agrees with perturbation theory
- There is an agreement between MC and lattice in $b \in [1.5, 2.5]$
- There is a general agreement in shapes between MC, Lattice, SV19 and "linear-to-constant" model

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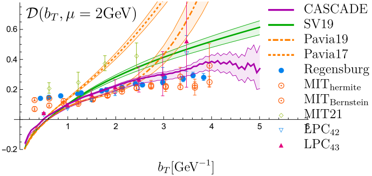
Conclusion



There was a huge progress in last 3 years

- Theory
  - Self-contained definition
  - Ideas of interpretation
  - Some estimations and models
- Phenomenology
  - PDF bias also affect CS kernel
  - First lattice results
  - Extractions of CS kernel from event generators

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