Double inclusive gluon production in ultra-peripheral collisions

Haowu Duan

North Carolina State University

In preparation, with Alex Kovner and Vladi Skokov

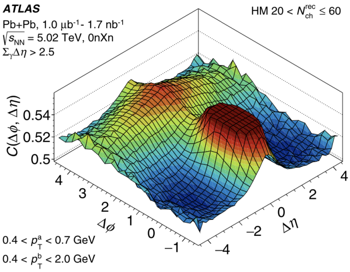
Correlations in Partonic and Hadronic Interactions, 2022

Supported by DOE

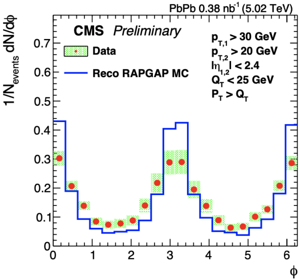
NC STATE UNIVERSITY

Motivation

Two particle angular correlation observed in UPC measurement at LHC



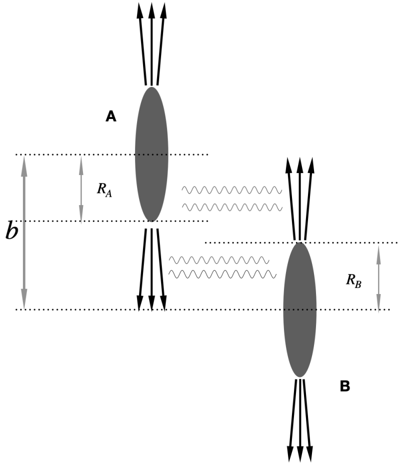
(a) PHYSICAL REVIEW C 104, 014903 (2021), ATLAS



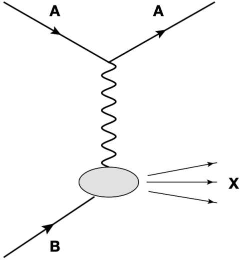
(b) CMS PAS HIN-18-011

NC STATE
UNIVERSITY

Ultra-peripheral collisions



- $b > R_A + R_B$
- equivalent photon approximation
- photon-nuclear interaction



NC STATE UNIVERSITY

Existing calculation

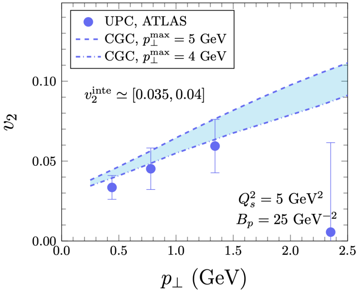
- Parton Wigner distribution to model real photon

$$W(x, \boldsymbol{b}, \boldsymbol{k}) = f_{\gamma/p}(x) \frac{1}{\pi^2} \exp\{-\boldsymbol{b}^2/B_p - \boldsymbol{k}^2/\Delta^2\}$$

where $f_{\gamma/p}(x)$ is the collinear parton distribution of photon

- Forward (photon direction) rapidity kinematics

- Ignored color and momentum correlation in the projectile.

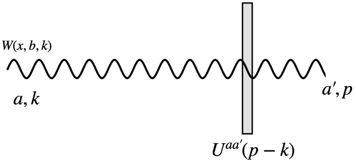


NC STATE UNIVERSITY

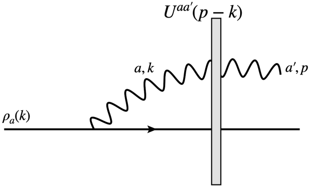
Phys.Rev.D 103 (2021) 5, 054017, Yu Shi et al.

Difference between forward and central rapidity kinematics

- Experimental data was taken at *mid* rapidity
- *Forward* scattering at photon going direction



(a) Forward



(b) Mid-rapidity

- Mid-rapidity gluon production contains Lipatov vertex which prevents zero momentum transfer $(p = k)$.

$$L^i(\boldsymbol{p},\boldsymbol{k}) = \frac{\boldsymbol{p}^i}{\boldsymbol{p}^2} - \frac{\boldsymbol{k}^i}{\boldsymbol{k}^2}$$

NC STATE UNIVERSITY

Compute cross section

Main ingredients of the approach:

- Wave function of the projectile
- Gluon production in the CGC formalism from the valance degrees of freedom
- Eikonal scattering
- Ensemble average over projectile and the target

Angular correlation from the cross section

From the cross section of the two gluon production

$$\Sigma = \frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2}$$

one can extract the angular correlation function

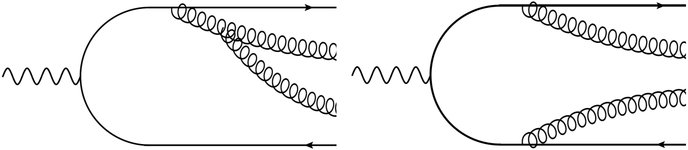
$$C(q,\theta) = \frac{\Sigma(q,\theta)}{\frac{1}{2\pi}\int_0^{2\pi}\Sigma(q,\theta)d\theta}$$

set $|q_1| = |q_2| = q$, and θ is the angle between the two particles

NC STATE
UNIVERSITY

Wave function of the projectile

Dipole model

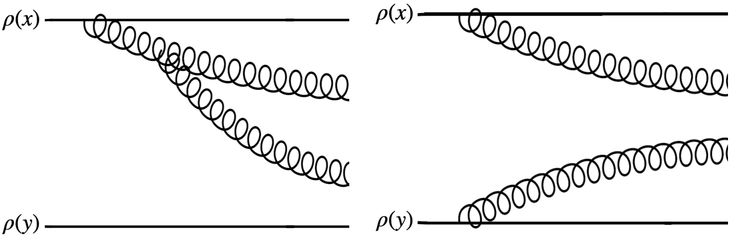


- Dipole model to approximate the photon
  Small Q^2 suppresses the longitudinal polarization

$$\Psi_\lambda^T(z, \boldsymbol{r}, s_1) = -i\frac{2ee_f}{2\pi}\delta_{s_1,-s_2}(2z - 1 + 2\lambda s_1)\sqrt{z(1-z)}\frac{\boldsymbol{r}\cdot\boldsymbol{\epsilon_\lambda}}{|r|}\varepsilon_f K_1(\varepsilon_f|r|)$$

NC STATE UNIVERSITY

MV model



- Inspired by Vector Meson Dominance Model
- Due to the existence of the high energy fixed point, ρ-meson w.f. at asymptotically high energy $\equiv$ nucleus w. f.
- Valence degrees of freedom $\rho_a(\boldsymbol{x})$ follow the distribution defined by McLerran-Venugopalan (MV) model

$$W(\rho_a) = \exp\left\{-\int_{\boldsymbol{x}} \frac{\rho_a(\boldsymbol{x})\rho_a(\boldsymbol{x})}{2\mu^2}\right\}$$

NC STATE UNIVERSITY

Gluon production

Create gluons within initial states

One account for the emission of the gluons using coherent operators

$$C = \mathcal{P} e^{i\sqrt{2}\int d^2x d\xi \, \hat{b}^i_a(\xi,\boldsymbol{x})\left[a^\dagger_{i,a}(\xi,\boldsymbol{x}) + a_{i,a}(\xi,\boldsymbol{x})\right]}$$

with the background field

$$\hat{b}^i_a(\xi,\boldsymbol{x}) = \frac{g}{2\pi}\int d^2y \frac{(\boldsymbol{x}-\boldsymbol{y})^i}{|\boldsymbol{x}-\boldsymbol{y}|^2}\hat{\rho}^a_{\mathrm{P}}(\xi,\boldsymbol{y})$$

- MV model source ρ_a
- $\hat{\rho}^a_D(\boldsymbol{x}) = b^\dagger_{\alpha\sigma}(\boldsymbol{x_1}) t^a_{\alpha\beta} b_{\beta\sigma}(\boldsymbol{x_1})\delta^{(2)}(\boldsymbol{x}-\boldsymbol{x_1}) - d^\dagger_{\alpha\sigma}(\boldsymbol{x_2}) t^a_{\beta\alpha} d_{\beta\sigma}(\boldsymbol{x_2})\delta^{(2)}(\boldsymbol{x}-\boldsymbol{x_2})$
- $\hat{\rho}^a_g(\zeta,\boldsymbol{x}) = a^{i\dagger}_b(\eta,\boldsymbol{x}) T^a_{bc} a_c(\eta,\boldsymbol{x})$

NC STATE
UNIVERSITY

The cross section

$$\frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2} = \frac{1}{(2\pi)^4}\int d^2u_1 d^2u_2 d^2\bar{u}_1 d^2\bar{u}_2 e^{-i\boldsymbol{q}_1(\boldsymbol{u}_1-\bar{\boldsymbol{u}}_1)} e^{-i\boldsymbol{q}_2(\boldsymbol{u}_2-\bar{\boldsymbol{u}}_2)}$$
$$\times \langle\gamma^*|C^\dagger\hat{S}^\dagger C a^\dagger_{i,a}(\eta,\boldsymbol{u}_1) a^\dagger_{j,b}(\xi,\boldsymbol{u}_2) a_{i,a}(\eta,\bar{\boldsymbol{u}}_1) a_{j,b}(\xi,\bar{\boldsymbol{u}}_2) C^\dagger \hat{S} C|\gamma^*\rangle$$

where $C = C_\xi C_\eta$, and $\eta \gg \xi$,

$$C_\eta \simeq 1 + i\sqrt{2}\int d^2v_1 \hat{b}^i_{Da}(\boldsymbol{v}_1)\left[a^{i\dagger}_a(\eta,\boldsymbol{v}_1) + a^i_a(\eta,\boldsymbol{v}_1)\right]$$
$$C_\xi \simeq 1 + i\sqrt{2}\int d^2v_2 \left(\hat{b}^j_{Db}(\boldsymbol{v}_2) + \delta\hat{b}^j_b(\eta,\boldsymbol{v}_2)\right)\left[a^{j\dagger}_b(\xi,\boldsymbol{v}_2) + a^j_b(\xi,\boldsymbol{v}_2)\right]$$

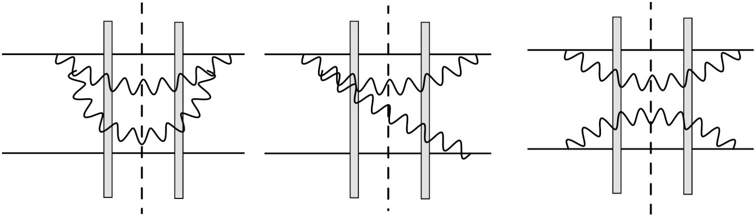
- $C|\gamma^*\rangle$ Initial state
- $\hat{S}$ S-matrix
- $Ca_{j,b}(\xi,\bar{\boldsymbol{u}}_2)C^\dagger$ dressed gluons in the final state

NC STATE UNIVERSITY

Organize the cross section

Organize the cross section Σ according to the order of ρ

$$\Sigma = \Sigma_2 + \Sigma_3 + \Sigma_4$$



(a) $\Sigma_2(\rho^2)$ (b) $\Sigma_3(\rho^3)$ (c) $\Sigma_4(\rho^4)$

NC STATE UNIVERSITY

Eikonal scattering

Eikonal scattering

- Eikonal scattering

$$V(\boldsymbol{x}) = \mathcal{P}\exp\left\{ig\int_{-\infty}^{\infty} dx^+ t^a A_a^-(x^+,\boldsymbol{x})\right\}$$
$$U(\boldsymbol{x}) = \mathcal{P}\exp\left\{ig\int_{-\infty}^{\infty} dx^+ T^a A_a^-(x^+,\boldsymbol{x})\right\}$$

- Rotation in the color space

$$\hat{S}^\dagger b_\alpha(\boldsymbol{x})\hat{S} = V_{\alpha\beta}(\boldsymbol{x}) b_\beta(\boldsymbol{x})$$
$$\hat{S}^\dagger d_\alpha(\boldsymbol{x})\hat{S} = V^\dagger_{\beta\alpha}(\boldsymbol{x}) d_\beta(\boldsymbol{x})$$
$$\hat{S}^\dagger a^i_a(\zeta,\bar{\boldsymbol{u}})\hat{S} = U^{aa'}(\bar{\boldsymbol{u}}) a^i_{a'}(\zeta,\bar{\boldsymbol{u}})$$
$$\hat{S}^\dagger \hat{\rho}_{D,a}(\bar{\boldsymbol{u}})\hat{S} = U^{aa'}(\bar{\boldsymbol{u}}) \hat{\rho}_{D,a'}(\bar{\boldsymbol{u}})$$
$$\hat{S}^\dagger \hat{\rho}_a(\zeta,\bar{\boldsymbol{u}})\hat{S} = a^{i\dagger}_b(\zeta,\bar{\boldsymbol{u}}) a_c(\zeta,\bar{\boldsymbol{u}}) \left[U^\dagger(\bar{\boldsymbol{u}}) T^a U(\bar{\boldsymbol{u}})\right]_{bc} .$$

NC STATE UNIVERSITY

Continue the calculation of Σ

Use Σ_2 as example, in coordinate space,

$$\Sigma_2 = 4\int d^2\boldsymbol{x}\int d^2\bar{\boldsymbol{x}} f^i(\bar{u}_1-\boldsymbol{x}) f^i(u_1-\bar{\boldsymbol{x}}) f^j(\bar{u}_2-\bar{u}_1) f^j(u_2-u_1)\langle\rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})\rangle_P$$
$$\left\langle\Big[[U^\dagger(u_1)T^aU(u_1)][U^\dagger(u_2)-U^\dagger(u_1)][U(\bar{u}_2)-U(\bar{u}_1)][U^\dagger(\bar{u}_1)T^aU(\bar{u}_1)]\Big]_{d'd}\right\rangle_T$$

where $f^i(\boldsymbol{x}) = \frac{g}{(2\pi)^2}\frac{x_i}{x^2}$.

- Kinematic factors (Eikonal emission vertices)
- Projectile (photon)
- Target (nucleus)

NC STATE
UNIVERSITY

Expectation values for projectile and target

NC STATE
UNIVERSITY

Dipole expectation values

- Expectation values for $q\bar{q}$

$$\langle q\bar{q}|\hat{\rho}_{d'}(\bar{\boldsymbol{x}})\hat{\rho}_d(\boldsymbol{x})|q\bar{q}\rangle = \frac{\delta^{dd'}}{2}\left(\delta^2(\bar{\boldsymbol{x}}-z_1)-\delta^2(\bar{\boldsymbol{x}}-z_2)\right)\left(\delta^2(\boldsymbol{x}-z_1)-\delta^2(\boldsymbol{x}-z_2)\right)$$

$$\begin{aligned}&\langle q\bar{q}|\hat{\rho}^a(\boldsymbol{x_1})\hat{\rho}^b(\boldsymbol{x_2})\hat{\rho}^c(\boldsymbol{x_3})|q\bar{q}\rangle\\ =&\frac{if_{abc}}{4}\left(\delta^{(2)}(\boldsymbol{x_2}-\boldsymbol{z_1})+\delta^{(2)}(\boldsymbol{x_2}-\boldsymbol{z_2})\right)\prod_{i=1,3}\left(\delta^{(2)}(\boldsymbol{x_i}-\boldsymbol{z_1})-\delta^{(2)}(\boldsymbol{x_i}-\boldsymbol{z_2})\right)\end{aligned}$$

$\boldsymbol{z}_1, \boldsymbol{z}_2$ are the transverse coordinates of quark and anti-quark.

- Average over different dipole size $\boldsymbol{r} = \boldsymbol{z}_1 - \boldsymbol{z}_2$

$$\langle \rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})\rangle_P \approx \sum_{s_1}\int_z \int d^2\boldsymbol{r}\,\Psi_\lambda^{T*}(z,r,s_1)\Psi_\lambda^T(z,r,s_1)\langle q\bar{q}|\rho_{d'}(\bar{\boldsymbol{x}})\rho_d(\boldsymbol{x})|q\bar{q}\rangle$$

NC STATE UNIVERSITY

MV model projectile average

- MV model describes the distribution of classical color source not quantum operators.

$$W(\rho_a) = \exp\left\{ -\int_{\boldsymbol{x}} \frac{\rho_a(\boldsymbol{x})\rho_a(\boldsymbol{x})}{2\mu^2} \right\}$$

- Two and three point correlators

$$\langle \hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y})\rangle_{\rm MV} = \langle \rho_a(\boldsymbol{x})\rho_b(\boldsymbol{y})\rangle_{\rm MV} = \mu^2 \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta_{ab}$$

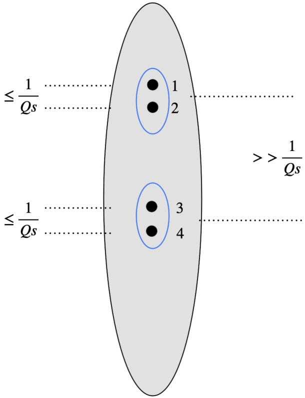
$$\langle \hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y})\hat{\rho}_c(\boldsymbol{z})\rangle_{\rm MV} = -\frac{1}{2}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta^{(2)}(\boldsymbol{y}-\boldsymbol{z})T^a_{bc}\,\mu^2$$

- Symmetrization of $\hat{\rho}s$

$$\begin{aligned}\hat{\rho}_a(x)\hat{\rho}_b(y) &= \{\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})\} + [\hat{\rho}_a(\boldsymbol{x}), \hat{\rho}_b(\boldsymbol{y})] \\ &= \rho_a(\boldsymbol{x})\rho_b(\boldsymbol{y}) - \frac{1}{2}\delta^{(2)}(x-y)T^c_{ab}\rho_c(\boldsymbol{x})\end{aligned}$$

NC STATE
UNIVERSITY

Target average(I)



- "Dipole" approximation

  Phys. Rev. D 96, 074018, Kovner, Rezaeian

- Dense target $\rightarrow$ Saturated

- $\frac{1}{Q_s}$ serves the role of correlation length in transverse plane

- For the example configuration

  $\mathrm{Tr}\left[U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)\right]$

  $\approx$

  $\frac{1}{N_c^2-1}\,\mathrm{Tr}\left[U(x_1)U^\dagger(x_2)\right]\mathrm{Tr}\left[U(x_3)U^\dagger(x_4)\right]+$

  $\ldots$

see also talk by A. Kovner

NC STATE
UNIVERSITY

Target average (II)

We only have one type of Wilson line correlator in momentum space

$$
\begin{aligned}
&\left\langle \mathrm{Tr}\left[U(k_1)T^a U^\dagger(k_2)U(k_3)T^a U^\dagger(k_4)\right]\right\rangle_T \\
=&T^a_{bc}T^a_{de}\left\langle\left[U^{fb}(k_1)U^{\dagger cg}(k_2)U^{gd}(k_3)U^{\dagger ef}(k_4)\right]\right\rangle_T \\
\approx&T^a_{bc}T^a_{de}(\frac{(2\pi)^2}{N_c^2-1})^2\Big\{(N_c^2-1)\delta^{bc}\delta^{de}\delta^{(2)}(k_1-k_2)D(k_1)\delta^{(2)}(k_3-k_4)D(k_3) \\
&+(N_c^2-1)\delta^{bd}\delta^{ce}\delta^{(2)}(k_1+k_3)D(k_1)\delta^{(2)}(k_2+k_4)D(-k_2) \\
&+(N_c^2-1)^2\delta^{be}\delta^{cd}\delta^{(2)}(k_1-k_4)D(k_1)\delta^{(2)}(k_2-k_3)D(-k_2)\Big\}
\end{aligned}
$$

here the dipole $D(p)$ is defined as

$$D(p)=\frac{1}{N_c^2-1}\int dx^2 e^{ipx}\langle\mathrm{Tr}\Big(U^\dagger(x)U(0)\Big)\rangle_T$$

NC STATE UNIVERSITY

The structure of analytic results(Σ_2)

- Dipole model

$$\frac{dN^{(2)}}{d\eta dq_1^2 d\xi dq_2^2} \propto \int_{k,\bar{k}} S_\perp \mathcal{I}(\varepsilon_f, |q_1+q_2+k+\bar{k}|) D(-\bar{k}) D(-k) \frac{(N_c^2-1)\Gamma(q_2, q_2+k, q_2+k) - \Gamma(q_2, k+q_2, \bar{k}+q_2)}{(q_1+q_2+k+\bar{k})^2}$$

  - $S_\perp$-Transverse area
  - $\mathcal{I}(\varepsilon_f, |\boldsymbol{P}|) = \int drr\varepsilon_f^2 K_1^2(\varepsilon_f r)(1 - J_0(|r||\boldsymbol{P}|))$

- MV model

$$\frac{dN_2}{dq_1^2 dq_2^2 d\eta d\xi} \propto \int d^2x \mu^2(x) \int_{k_1,k_2} D(k_1+q_1) D(-k_2+q_2) \Big\{ (N_c^2-1)\Big(\frac{(k_2-k_1)\cdot L(q_2,k_2)}{(k_2-k_1)^2}\Big)^2 - \frac{(k_2-k_1)\cdot L(q_2,k_2)}{(k_2-k_1)^2} \frac{(k_2-k_1)\cdot L(q_2, q_2-k_2-k_1)}{(k_2-k_1)^2} \Big\}$$

  - $L^i(\boldsymbol{p},\boldsymbol{q}) = \frac{\boldsymbol{p}^i}{\boldsymbol{p}^2} - \frac{\boldsymbol{q}^i}{\boldsymbol{q}^2}$ (Lipatov vertex)
  - $\Gamma(\boldsymbol{p},\boldsymbol{k},\boldsymbol{q}) = L(\boldsymbol{p},\boldsymbol{k}) \cdot L(\boldsymbol{p},\boldsymbol{q})$

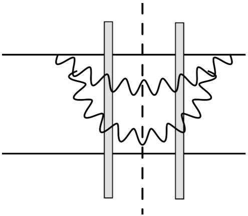
NC STATE UNIVERSITY

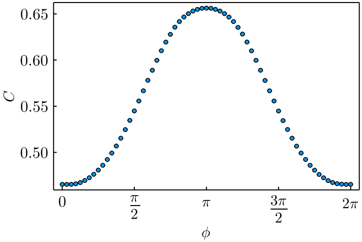
Numerical results

We now consider different contribution to the total cross section.
We normalize them to show their relative strength.

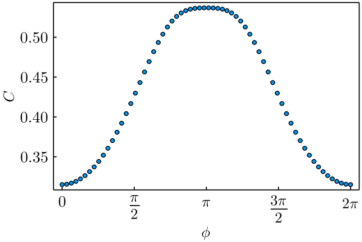
NC STATE
UNIVERSITY

$\Sigma_2,\ q = Q_s$





(a) Dipole

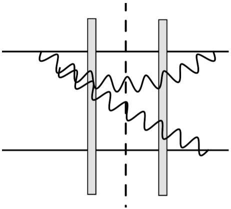


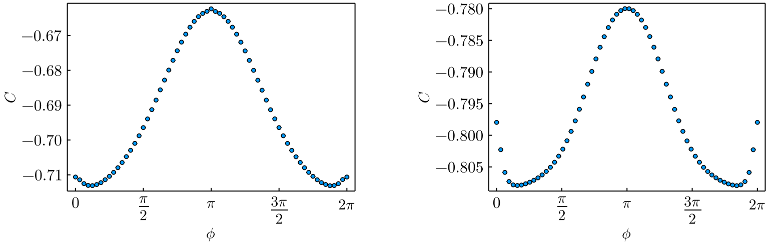
(b) MV

NC STATE UNIVERSITY

As expected, a strong back-to-back correlation.

$\Sigma_3,\ q = Q_s$



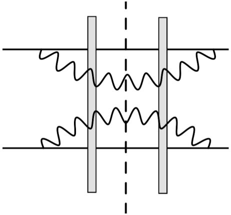


(a) Dipole (b) MV

NC STATE UNIVERSITY

Back-to-back correlation

Symmetrization of Σ_4



- MV model

$$\langle \hat{\rho}_a(\boldsymbol{x})\hat{\rho}_b(\boldsymbol{y})\hat{\rho}_c(\boldsymbol{z})\hat{\rho}_d(\boldsymbol{u})\rangle_{\mathrm{MV}}$$
$$= \mu^4\delta^{ab}\delta^{cd}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta^{(2)}(\boldsymbol{z}-\boldsymbol{u}) + \mu^4\delta^{ac}\delta^{bd}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{z})\delta^{(2)}(\boldsymbol{y}-\boldsymbol{u})$$
$$+ \mu^4\delta^{ad}\delta^{bc}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{u})\delta^{(2)}(\boldsymbol{z}-\boldsymbol{u}) + \mathcal{O}(\mu^2)(\text{not fully symmetric})$$

- Dipole model

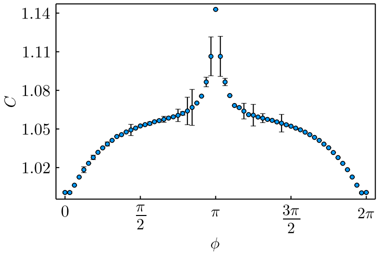
$$\mathrm{Tr}\left(t^a t^b t^c t^d\right) = \frac{1}{12N_c}(1+\frac{1}{6N_c})(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}) + (\text{not fully symmetric})$$

- $\Sigma_4 = \Sigma_4^{sym} + \Sigma_4^{nsym}$

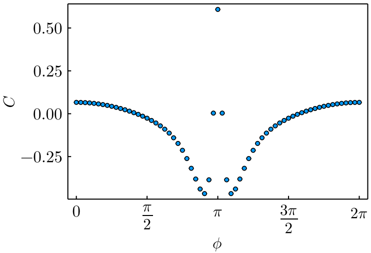
- Σ_4^{sym} is the signal.

NC STATE UNIVERSITY

Σ_4^{nsym}, non-symmetric part, $q = Q_s$



(a) Dipole

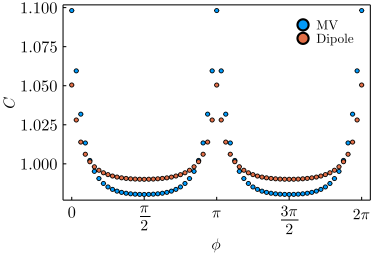


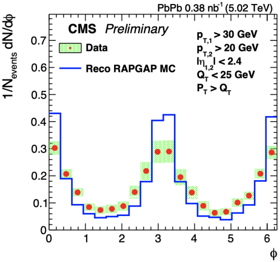
(b) MV

Also gives us back-to-back correlation.

NC STATE UNIVERSITY

Σ_4^{sym}, symmetric part, $q = Q_s$





As what was done in experimental analysis, we subtract backgrounds and normalize the signal.The preliminary results shows similar correlations in CGC calculation.

Stay tuned for the comparision with (ATLAS) experimental data!

NC STATE
UNIVERSITY

Summary and outlook

- We analytically derived inclusive two gluon production in UPC at mid-rapidity.
- To estimate systematic uncertainty originated from the poor knowledge of the real photon wave function, we studied two limiting cases.
- Both models result in qualitatively similar correlation. Quantitatively, the amplitude of azimuthal anisotropy for MV model is about two times the dipole model.
- Our results show similar correlation as experimental data.
- Further development
  - To compare with experimental data
  - To incorporate rapidity difference
  - To extend to EIC physics

NC STATE UNIVERSITY