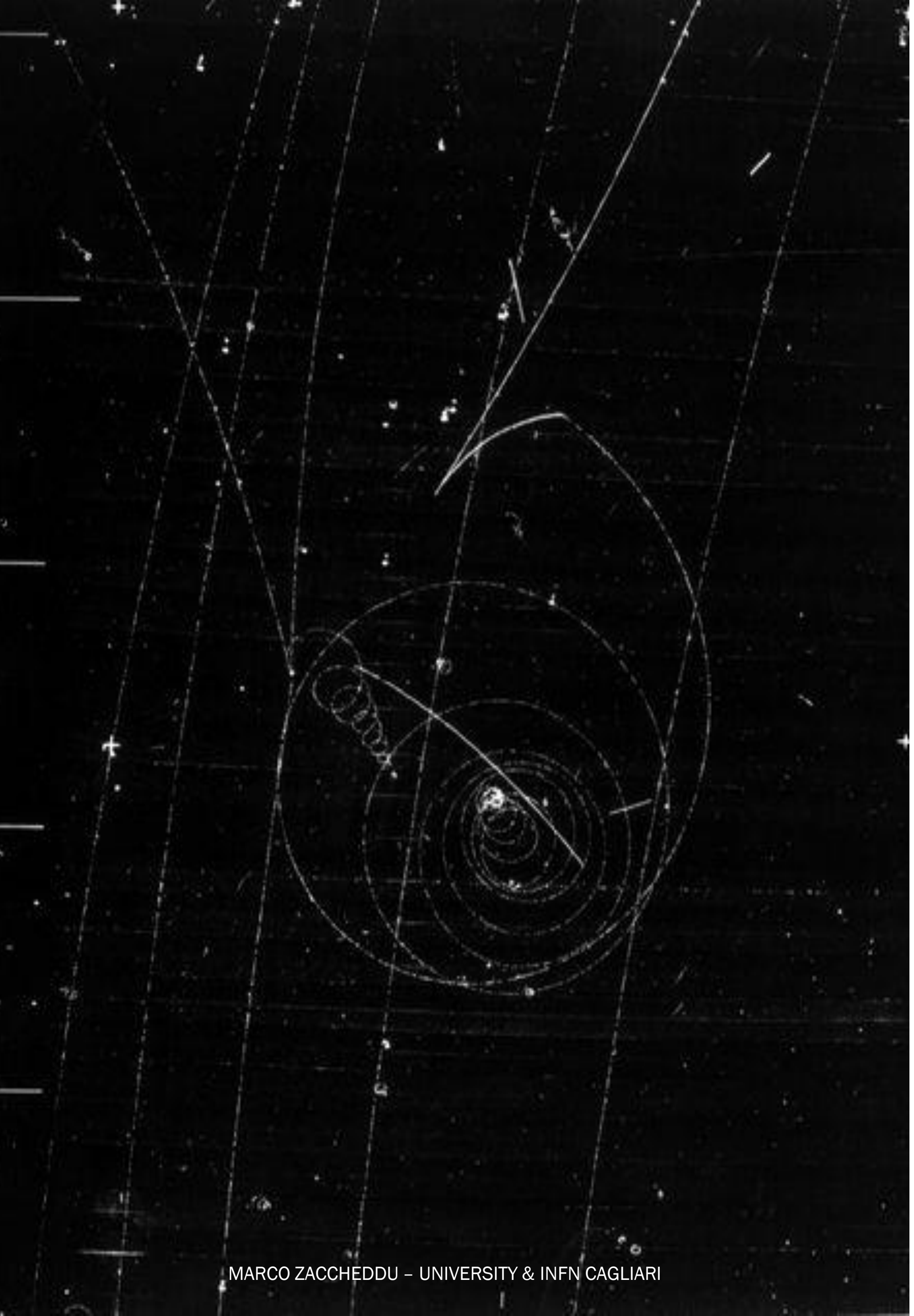

The transverse Λ polarization in e^+e^- collisions and SIDIS

Marco Zaccheddu – University & INFN Cagliari

In collaboration with:

Umberto D’Alesio, Leonard Gamberg and Francesco Murgia

EIC User Group Early Career Workshop 2022

Motivations and Contents

Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

- 2 data set @$\sqrt{s} = 10.58$ GeV

[Y. Guan et al., Phys. Rev. Lett. 122. 042001 (2019)]

Double hadron production:

- $e^+e^- \rightarrow \Lambda\pi/K + X$: 128 points - bins of the energy fractions z_Λ - $z_{\pi,K}$

Single-inclusive hadron production:

- $e^+e^- \rightarrow \Lambda(jet) + X$: 32 points - $\Lambda(jet)$, in bins of z_Λ - $p_\perp$

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First extraction of the Λ pFF
[D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)]

- $e^+e^- \to h^{\uparrow}{}_1 h_2 X \;\; \to \;\; \mathrm{X}^2_{dof} = 1.26$
- $e^+e^- \to h^{\uparrow}{}_1 h_2 X$ and $e^+e^- \to \mathrm{h}_1^{\uparrow}(\mathrm{jet})X \to \;\; \mathrm{X}^2_{dof} = 1.94$

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- No evolution equations;
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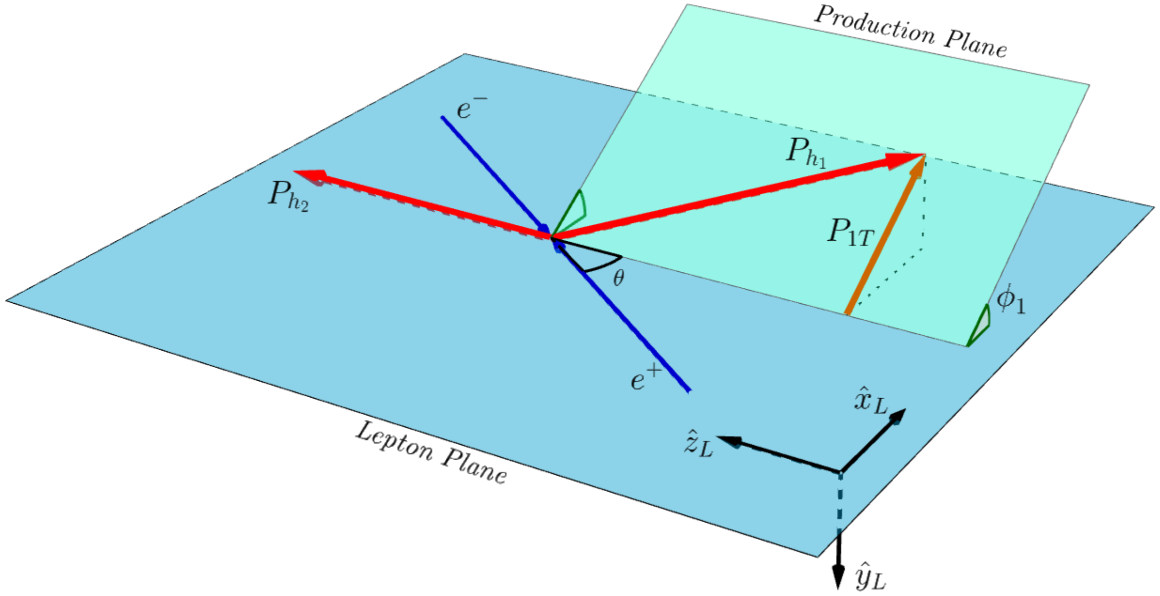
- Fit results: 2-h and (2-h +1-h)
- Opal data
- Conclusions

Convolutions: Double-hadron production

Convolutions for the transverse polarization in b_T- space

$$F_{UU} = \mathcal{F}[D_1 \bar{D}_1] = \mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right]$$
$$= \sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T J_0(b_T\, q_T)\widetilde{D}_1(z_1, b_T)\widetilde{\bar{D}}_1(z_2, b_T)$$

$$F_{TU}^{\sin(\phi_1-\phi_{S_1})} = \mathcal{F}\left[\frac{\hat{h}\cdot \mathbf{k}_T}{M_{h_1}} D_{1T}^{\perp}\bar{D}_1\right] = M_{h_1}\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}_1\right]$$
$$= M_{h_1}\sum_q e_q^2 \int \frac{db_T}{2\pi}\, b_T^2 J_1(q_T\, b_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, b_T)\widetilde{\bar{D}}_1(z_2, b_T)$$



Polarizing FF first moment

$$\Delta^N D_{h^\uparrow/q}(z, p_\perp) = \frac{p_\perp}{z M_h} D_{1T}^{\perp}(z, p_\perp)$$

$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2 z M_h}\Delta^N D_{h^\uparrow/q}(z, p_\perp)$$

Transverse Polarization:

$$P_T^h(z_1, z_2) = \frac{\int d^2\mathbf{q}_T\, F_{TU}^{\sin(\phi_1-\phi_S)}}{\int d^2\mathbf{q}_T\, F_{UU}} = \frac{M_{h_1}\int d^2\mathbf{q}_T\, \mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}\right]}{\int d^2\mathbf{q}_T\, \mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right]}$$

Polarization 2-h: Double-hadron Production

Solving the CSS evolution equations we obtain the full form of convolutions

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\,\widetilde{\bar{D}}_1\right] = \frac{\mathcal{H}^{(e^+e^-)}(Q)}{z_1^2 z_2^2}\sum_q e_q^2 \int \frac{db_T}{(2\pi)}\, b_T^2 J_1(b_T\, q_T)\widetilde{D}_{1T}^{\perp(1)}(z_1;\bar{\mu}_b)\, d_{\bar{q}/h_2}(z_2;\bar{\mu}_b)$$

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$$\times\exp\left\{\widetilde{K}(b_*;\bar{\mu}_b)\ln\frac{Q^2}{\bar{\mu}_b^2}+\int_{\bar{\mu}_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(g(\mu');1)-\gamma_K(g(\mu'))\ln\frac{Q^2}{\mu'^2}\right]\right\},$$

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$$\times \exp\left\{\widetilde{K}(b_*;\bar{\mu}_b)\ln\frac{Q^2}{\bar{\mu}_b^2} + \int_{\bar{\mu}_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(g(\mu');1) - \gamma_K(g(\mu'))\ln\frac{Q^2}{\mu'^2}\right]\right\},$$

Polaring FF first moment

$$\widetilde{D}_{1T,\Lambda/q}^{\perp(1)}(z;\mu_b) = \mathcal{N}_q^p(z) d_{q/\Lambda}(z;\mu_b)$$

$$\mathcal{N}_q^p(z) = N_q z^{a_q}(1-z)^{b_q}\frac{(a_q+b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}}$$

- Gaussian model
- Power Law model

DSS FF for π/K
AKK FF for Λ

Different models

Different $g_K(b_T)$ functions

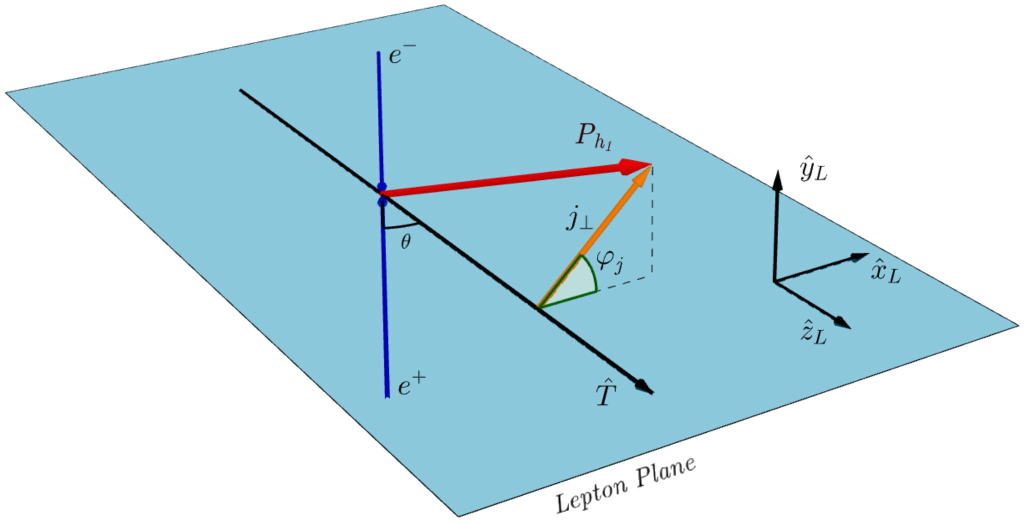
Perturbative Sudakov factor

Polarization: Single hadron with thrust

Z.-B. Kang, D.Y. Shao, F. Zhao, J. High Energy Phys. 12 (2020) 127

L. Gamberg et al., Phys.Lett.B 818 (2021) 136371

$$\mathcal{P}(z_1, p_\perp) = \frac{d\Delta\sigma/dz_1 d^2 p_\perp}{d\sigma/dz_1 d^2 p_\perp}$$



$$\frac{d\Delta\sigma}{dz_1 d^2\mathbf{j}_\perp} = \frac{\sigma_0}{z_1^2}\sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T\, q_T)\widetilde{D}_{1T}^{\perp(1)}(z_1, \bar{\mu}_b)\, U_{NG}(\bar{\mu}_b, Q)$$
$$\times M_D^\perp(b_c(b_T); b_{\max})\, \exp\left\{ - g_K(b_c(b_T); b_{\max}) \ln\left(\frac{Qz_1}{M_{h_1}}\right)\right\}$$
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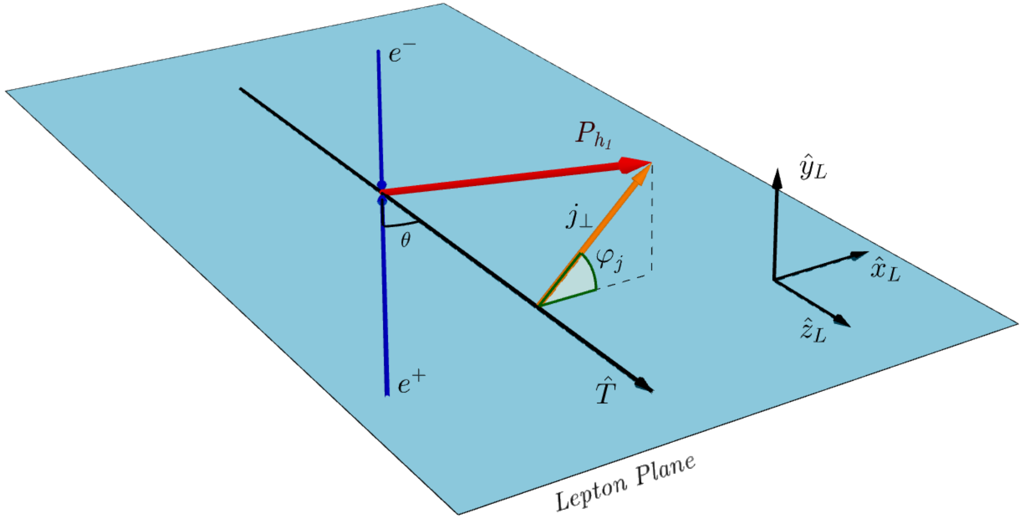

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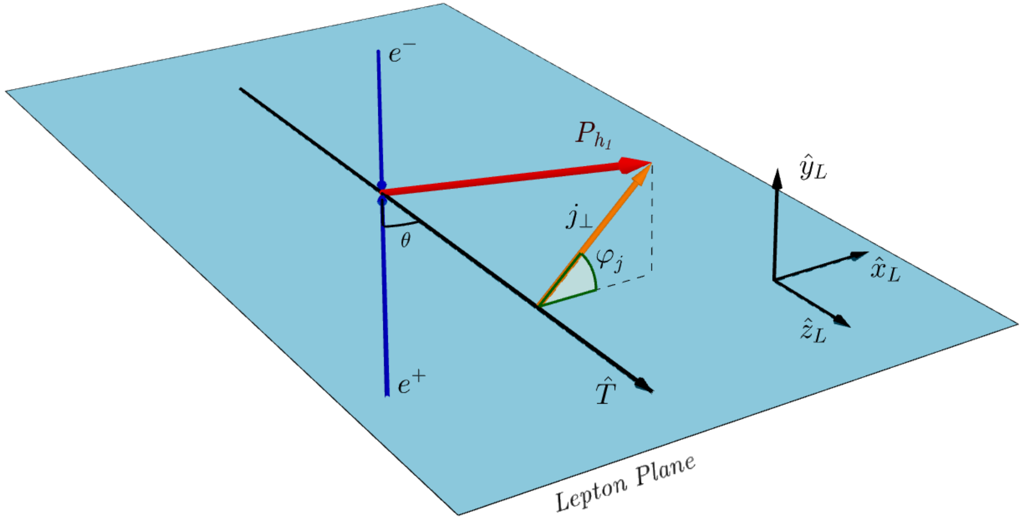

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$$U_{NG}(\mu_{b_*}, Q) = \exp\left[- C_A C_F \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu^c)}\right]$$

M. Dasgupta, G.P. Salam, Phys. Lett. B 512 (2001) 323

$$u = \frac{1}{\beta_0} \ln\left[\frac{\alpha_s(\mu_b)}{\alpha_s(Q)}\right]$$

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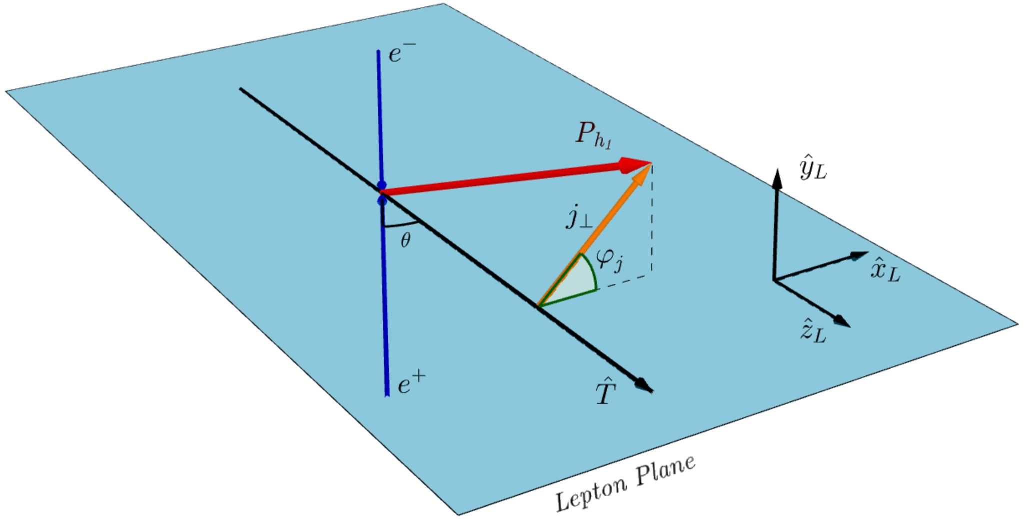

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M. Dasgupta, G.P. Salam, Phys. Lett. B 512 (2001) 323 $\quad u = \frac{1}{\beta_0}\ln\left[\frac{\alpha_s(\mu_b)}{\alpha_s(Q)}\right]$

In general the FF in 2-h and 1-h could not be the same, but at NLO the FFs in 2-h and 1-h are equal.

$g_K(b_T)$ Non-perturbative Function

- It cannot be computed from first principles, but it has to be extracted from Fit;
- Universal Function

Functions employed:

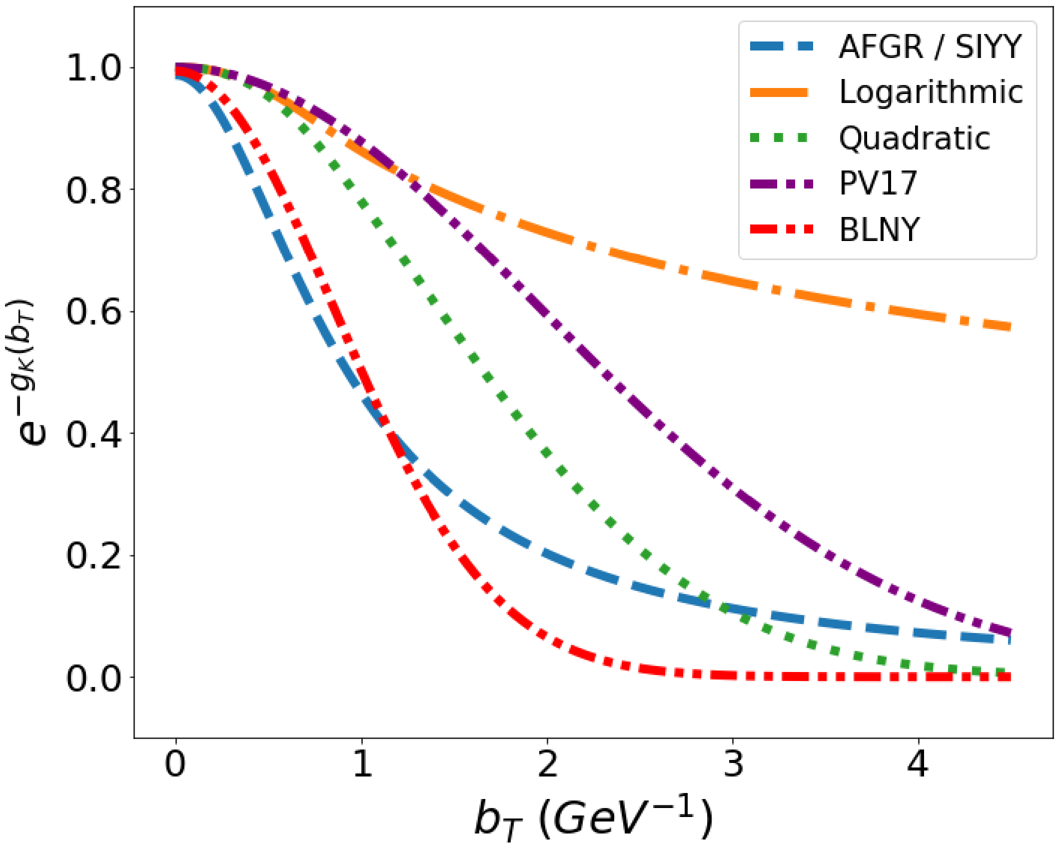
$$g_K(b_T; b_{\max}) = \frac{g_2 b_T^2}{2}; \quad g_2 = 0.68\,\mathrm{GeV}^2$$ BLNY [3]

$$g_K(b_T; b_{\max}) = \frac{C_F}{\pi}\frac{b_T^2}{b_{\max}^2}\alpha_s(\mu_{b_*})$$ Quadratic [2]

$$g_K(b_T; b_{\max}) = \frac{\alpha_s(C_1/b_*)C_F}{\pi}\ln(1 + b_T^2/b_{\max}^2)$$ [2] Logarithmic

$$g_K(b_T; b_{\max}) = g_2 \ln\left(\frac{b_T}{b_*}\right) \quad g_2 = 0.84$$ AFGR / SIYY [1]

$$g_K(b_T; b_{\max}) = -\frac{g_2 b_T^2}{2}; \quad g_2 = 0.13\,\mathrm{GeV}^2$$ PV17 [4]

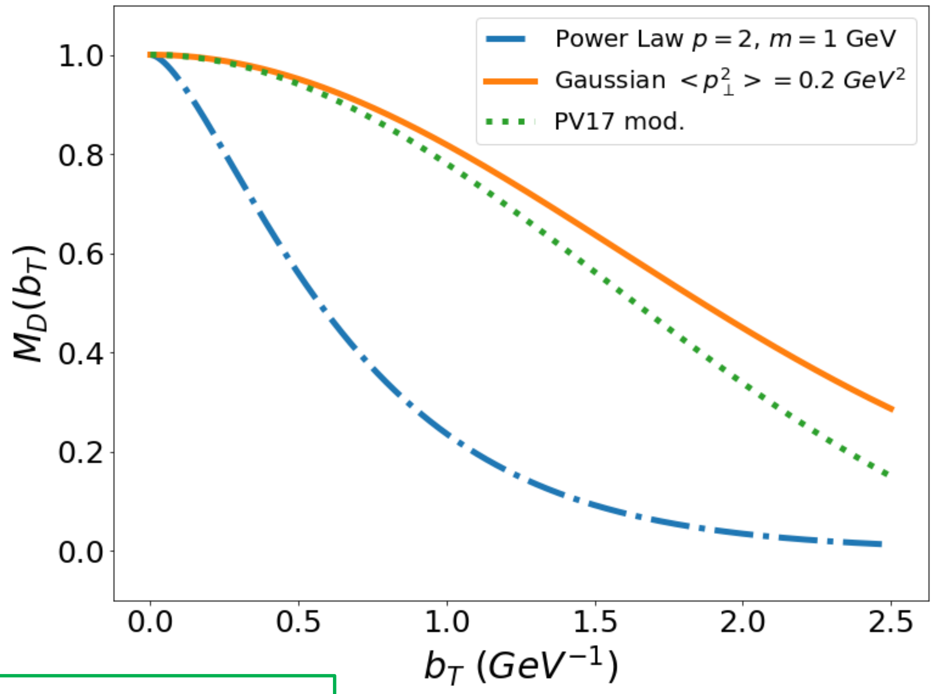


[1] C.A. Aidala, B. Field, L.P. Gamberg, T.C. Rogers, Phys. Rev. D 89 (2014) 094002
P. Sun, J. Isaacson, C.P. Yuan, F. Yuan, Int. J. Mod. Phys. A 33 (2018) 1841006

[2] J. Collins, T. Rogers, Phys.Rev.D 91 (2015) 7, 074020
[3] F. Landry et al., *Phys.Rev.D* 67 (2003)
[4] Bacchetta et al., *JHEP* 06 (2017) 081

$M_D(b_T)$ Hadronic Models Parameterizations

- They cannot be computed from first principles, but they have to be extracted from Fit;
- Universal Function (for same hadron)



Unpolarized π/K

Gaussian Model

$$M_D(b_T) = \exp\left(-\frac{\langle p_\perp^2\rangle b_T^2}{4z_p^2}\right)$$

Unpolarized Λ

PV17 hadron model

$$M_D(b_T) = \frac{g_3\, e^{-b_T^2\frac{g_3}{4z^2}} + \frac{\lambda_F}{z^2} g_4^2 (1 - g_4 \frac{g_4}{4z^2}) e^{-b_T^2\frac{g_4}{4z^2}}}{g_3 + \frac{\lambda_F}{z^2} g_4^2}$$

Polarized Λ

Power Law Model

$$M_D(b_T, p, m) = \frac{2^{2-p}}{\Gamma(p-1)} (b_T m/z_p)^{p-1} K_{p-1}(b_T m/z_p)$$

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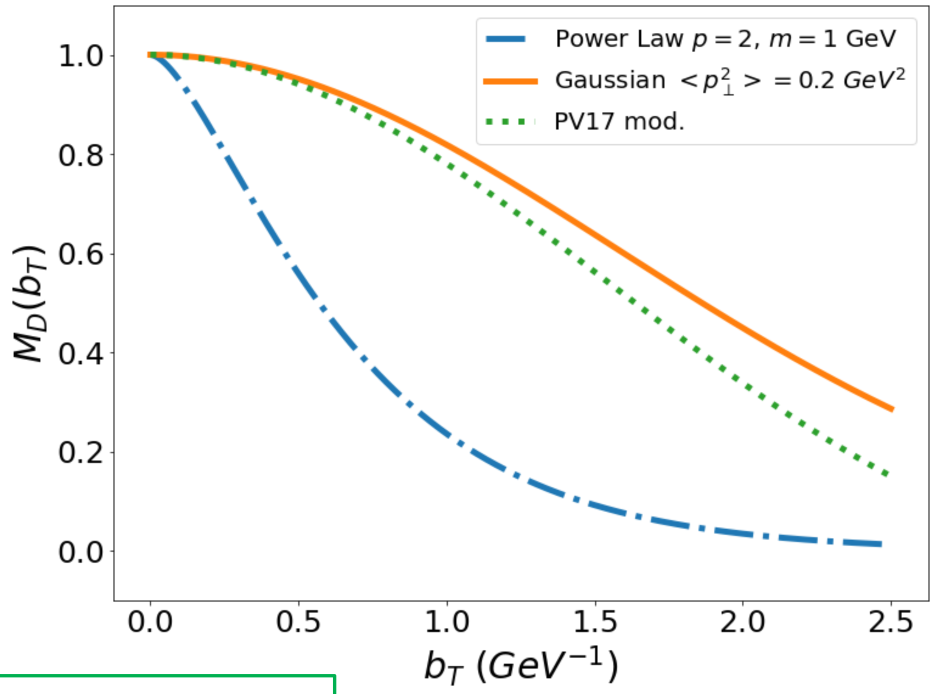

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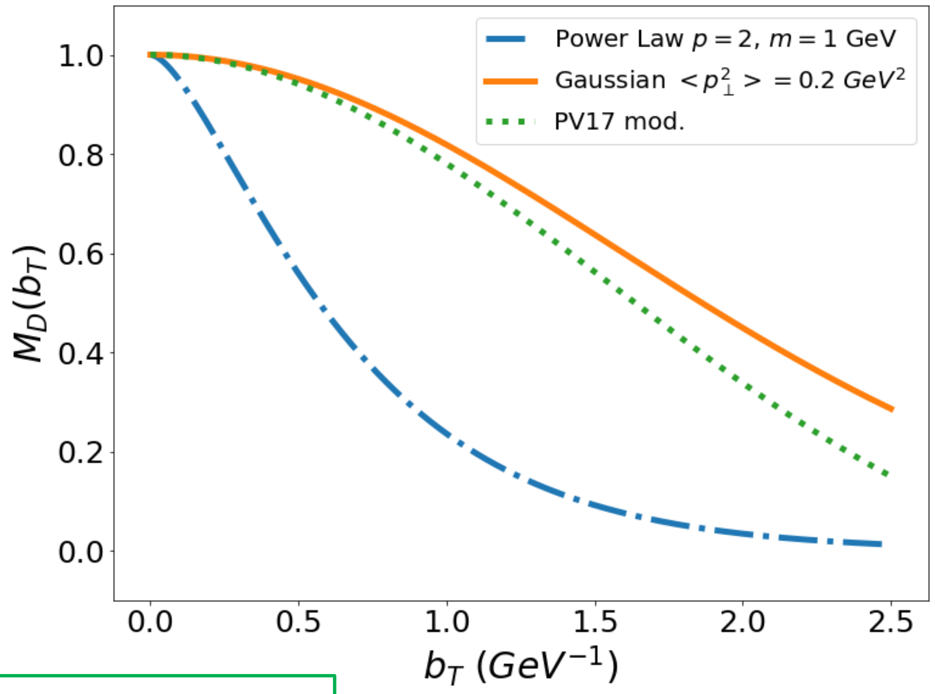

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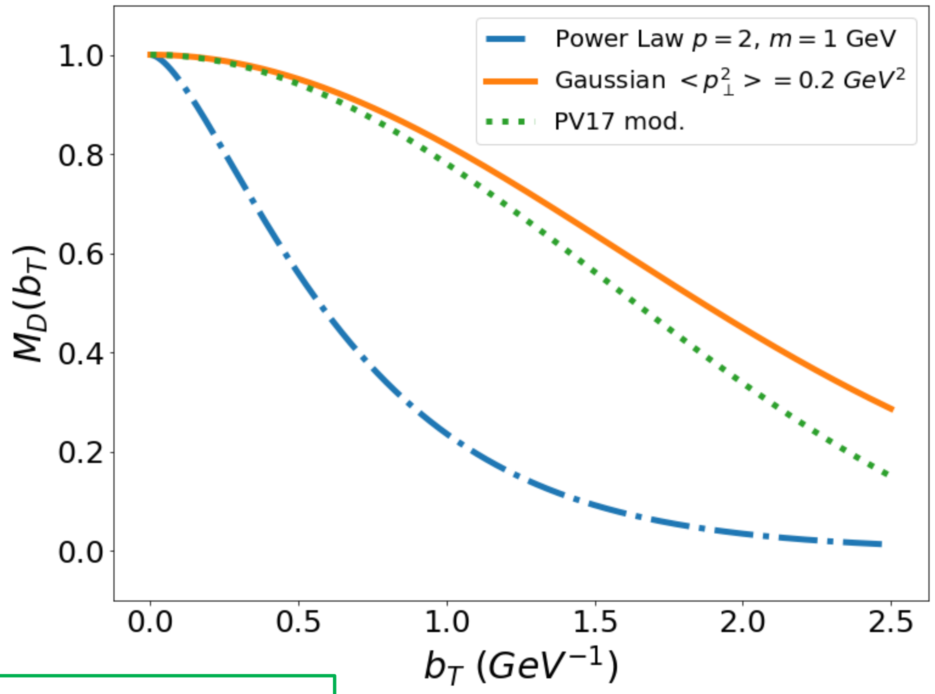

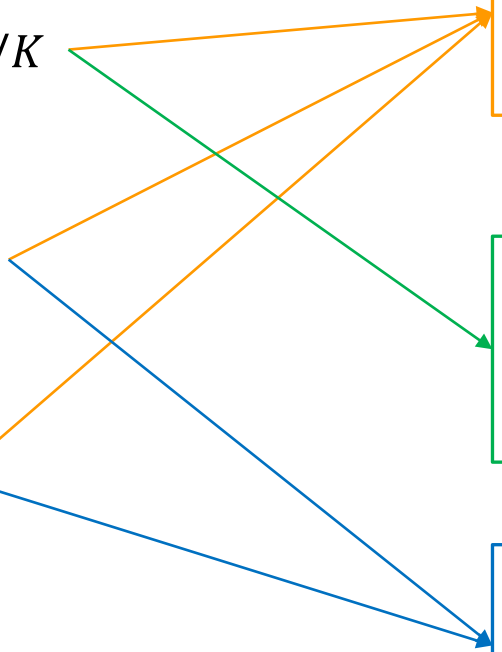

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Double-hadron production (2-h) data Fit

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5\ - 0.9]$ bin excluded → 96 data points

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Around 40 different combinations and Fit

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Fitted 8-9 parameters

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N_u
N_d
N_s
N_{sea}
a_s
b_u
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$\langle p_\perp^2 \rangle_p$
p
m

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pFF first moment parameters extracted

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I parameterization
- u
- d
- s
- $sea = \bar{u}, \bar{d}, \bar{s}$

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N_d
N_s
N_{sea}
a_s
b_u
b_{sea}
$\langle p_\perp^2 \rangle_p$
p
m

$\langle p_\perp^2 \rangle_p$ → Gaussian width

pFF first moment parameters extracted

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I parameterization

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- d
- s
- $sea = \bar{u}, \bar{d}, \bar{s}$

$\langle p_\perp^2 \rangle_p$ → Gaussian width

p, m → Power-Law parameters

Fitted 8-9 parameters

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$q_{T_{max}}/Q$ = 0,25

Double-hadron production (2-h) data Fit

Best Results

Polarizing	Unpolarized	g_K	$M_D^{h_2}$	$\chi^2_{\rm dof}$ (2-h)
Gaussian	Power-Law	Logarithmic	Gaussian	1.192
Power-Law	Power-Law	Logarithmic	Gaussian	1.21
Gaussian	Power-Law	PV17	PV17	1.198

- $\chi^2_{dof} \simeq 1.2$
- First moment parameters are consistent;
- Up pFF is positive
- Up and Down: opposite contribution
- The $M_D^{\perp}$ models are compatible

Double-hadron production (2-h) data Fit

Best Results

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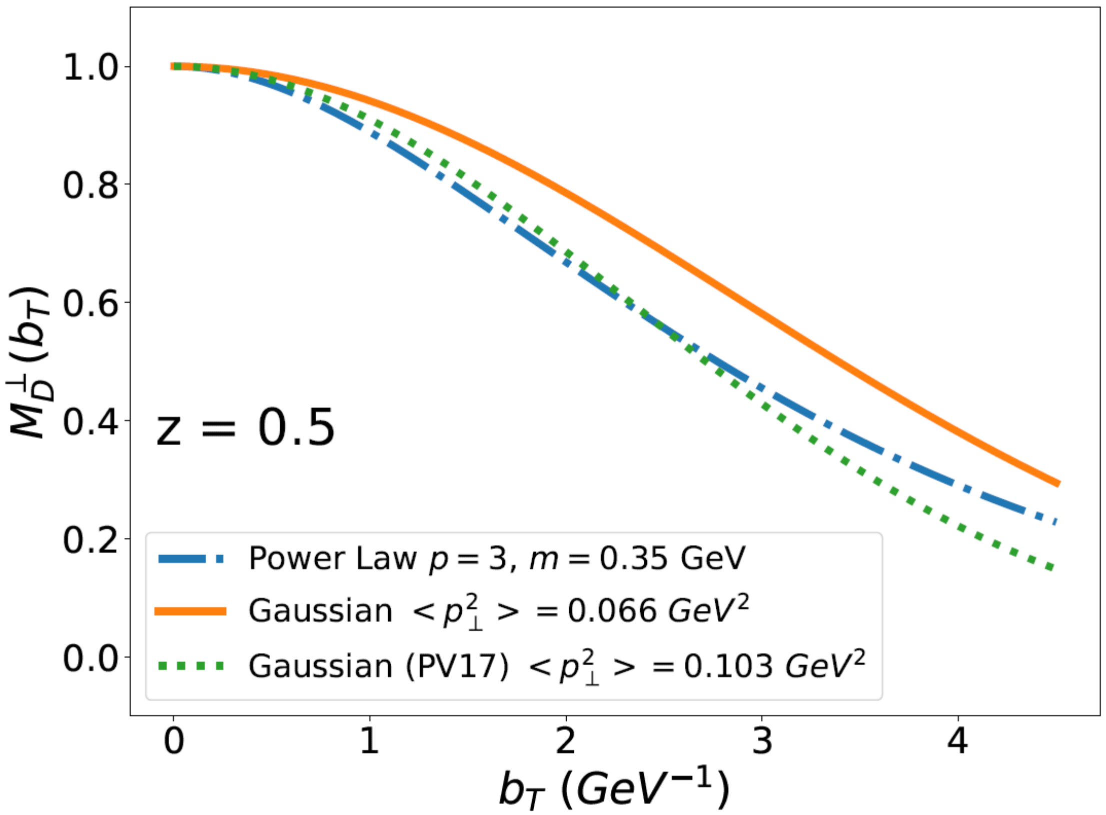

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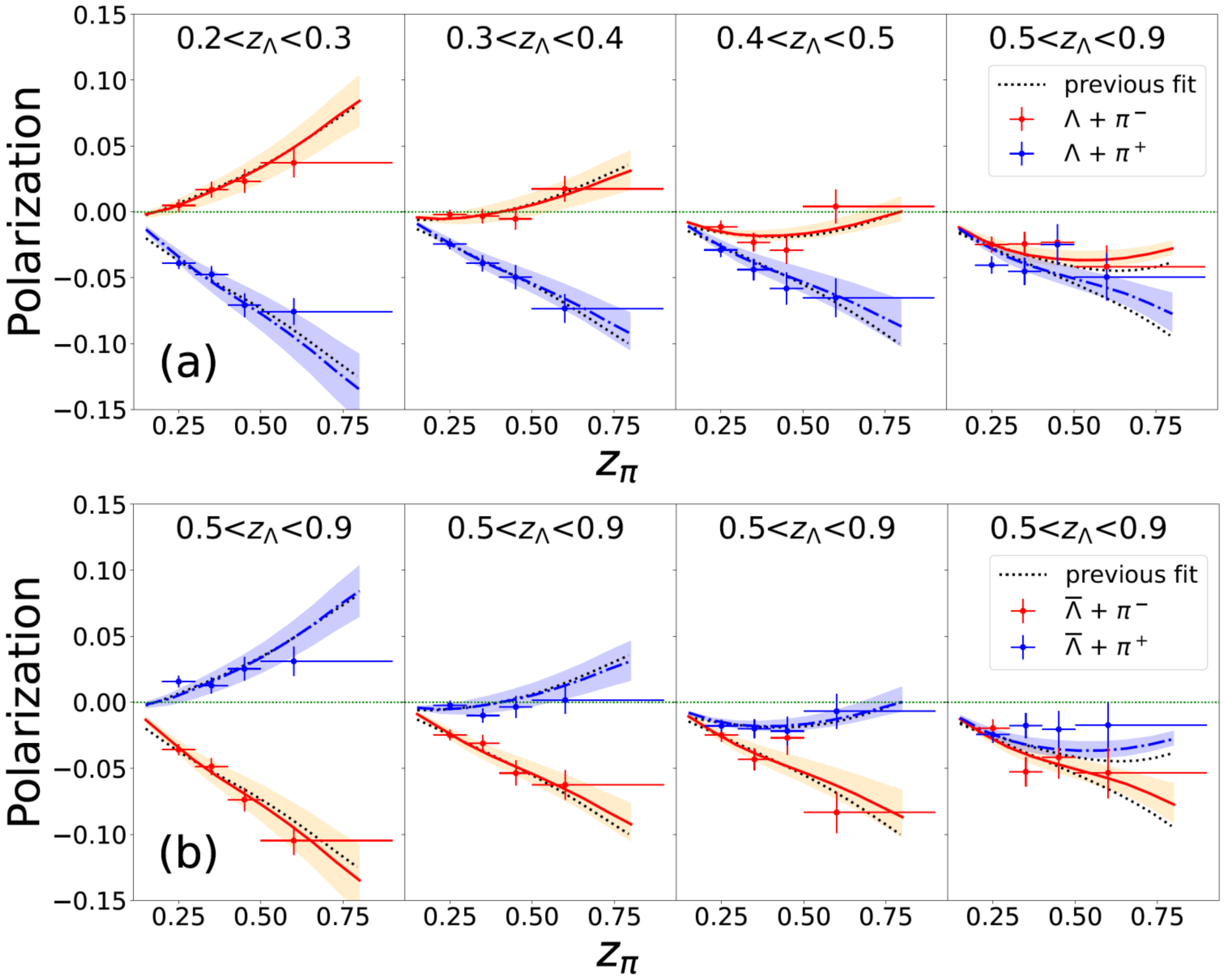
- The $M_D^\perp$ models are compatible

Lambda – pion: fit comparison



Bin excluded
$z_\pi = [0.5 - 0.9]$

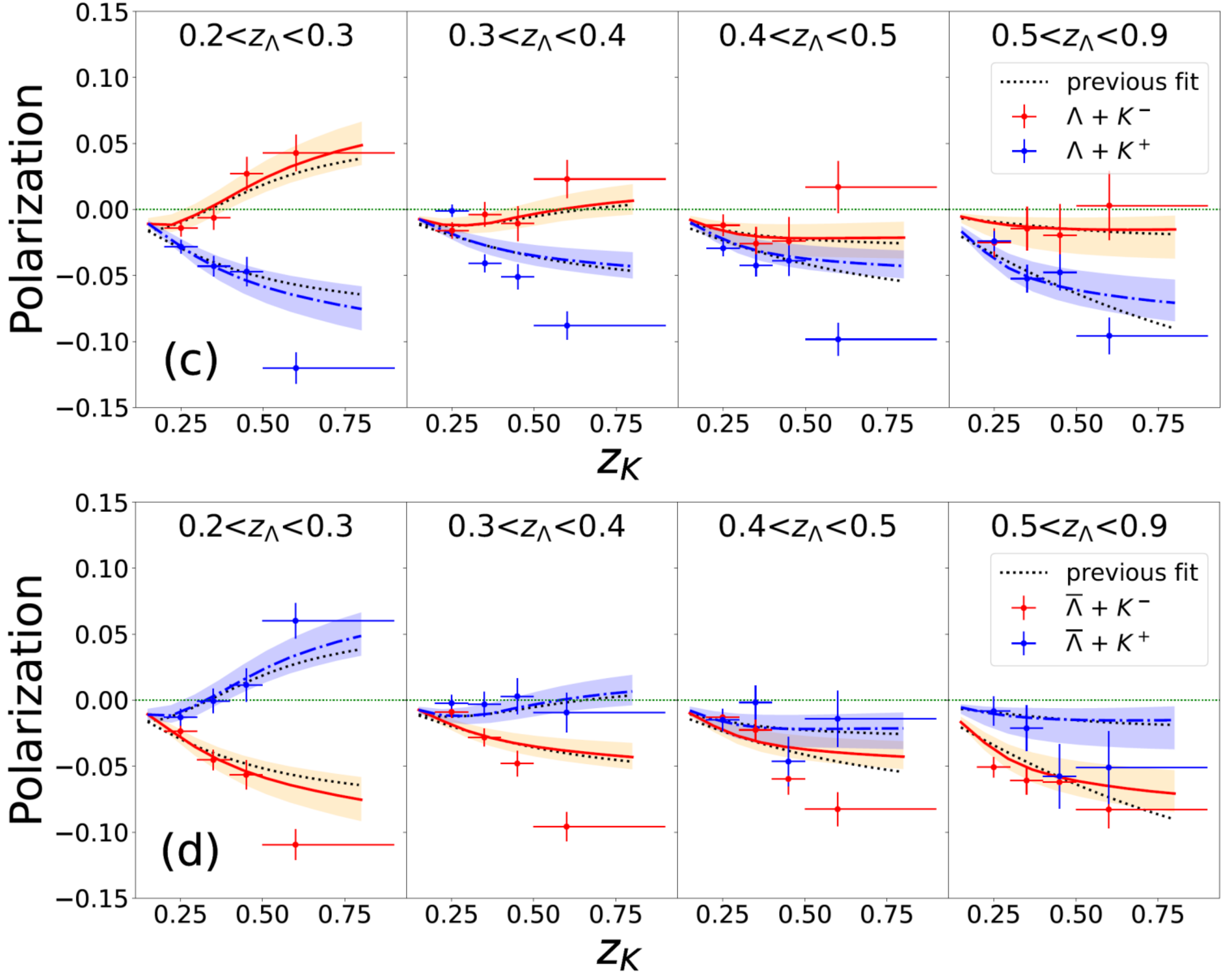
Gaussian Model

$$\chi^2_{dof} = 1.192$$

- Reference Fit [4]

[4] D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)

Lambda – kaon: fit comparison



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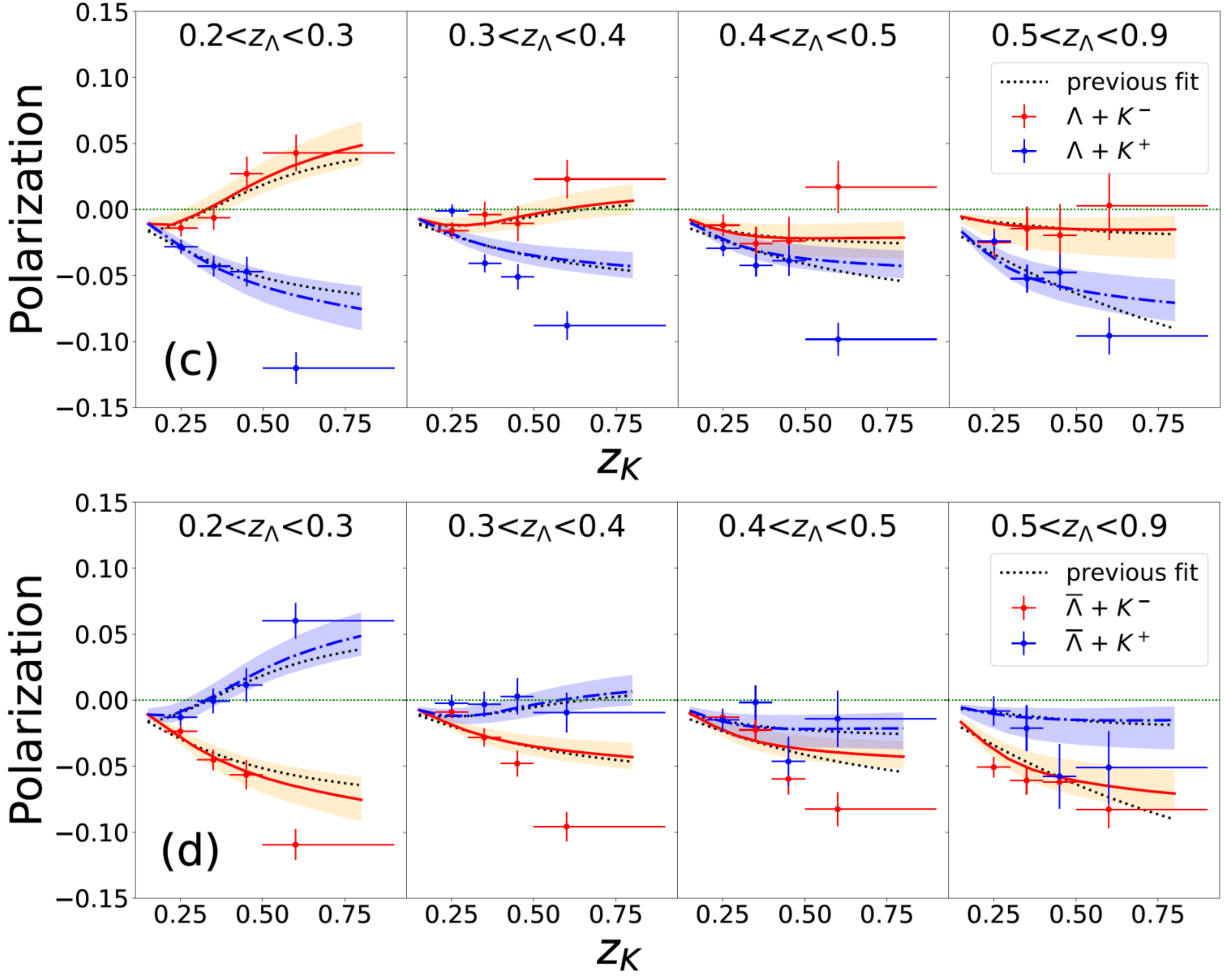

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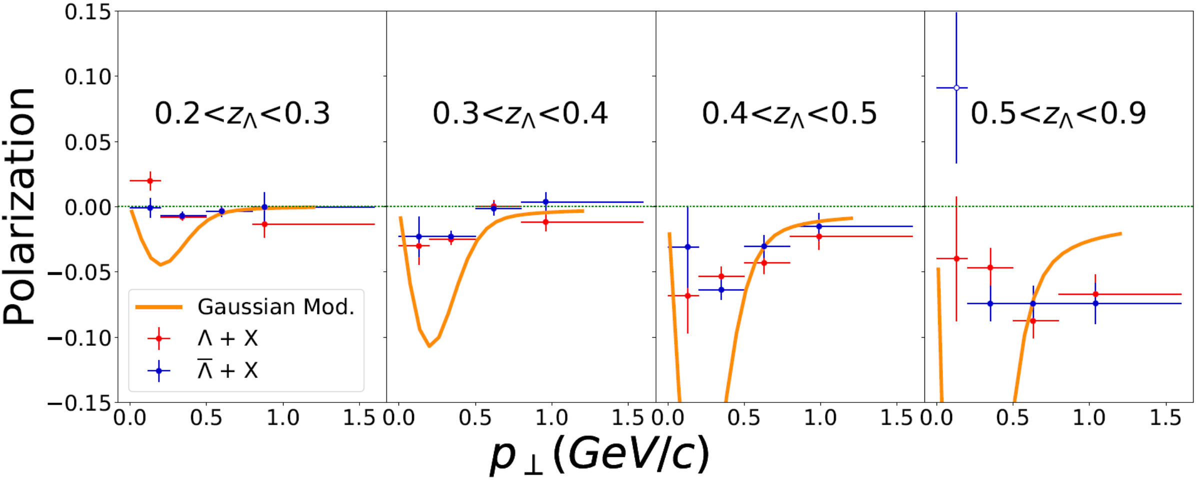
- Both (old and new) fits present problems in describing $\Lambda K^+ \ - \bar{\Lambda} K^-$;
- TMD evolution does not help;
- To be further investigated: heavier quark flavors contribution?

Single-inclusive Polarization

The parameters extracted in 2-h Fit cannot reproduce the 1-h data



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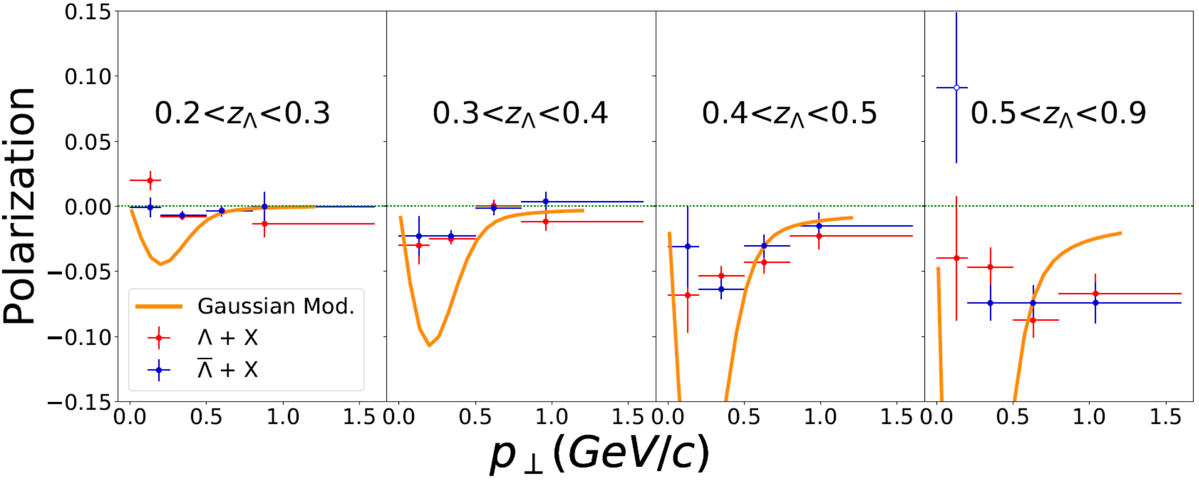

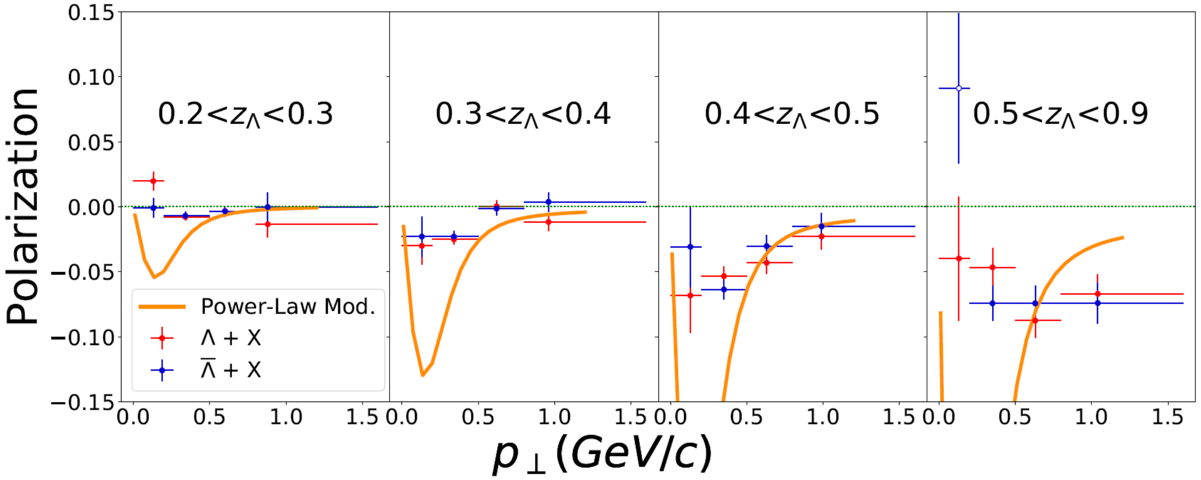

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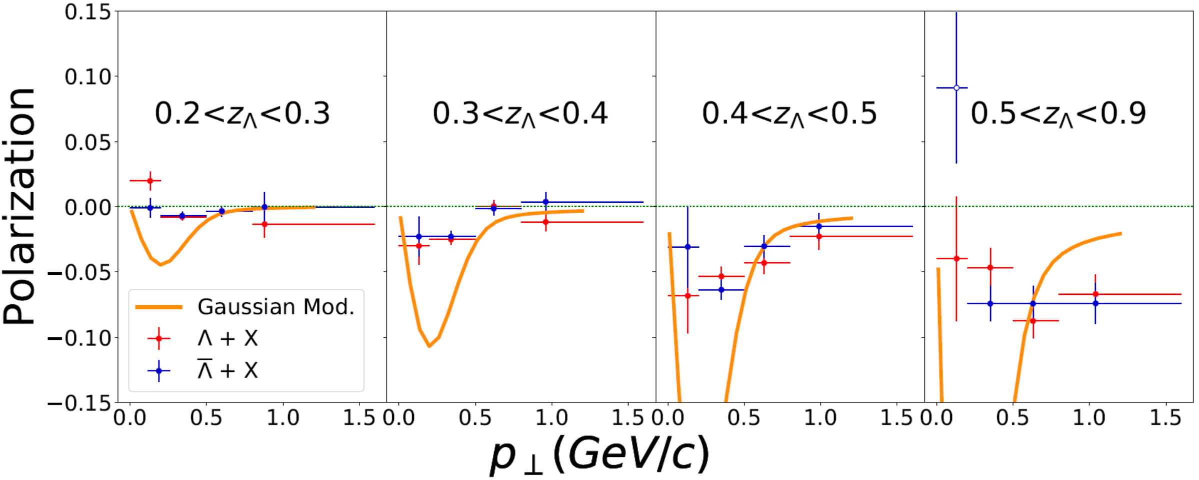

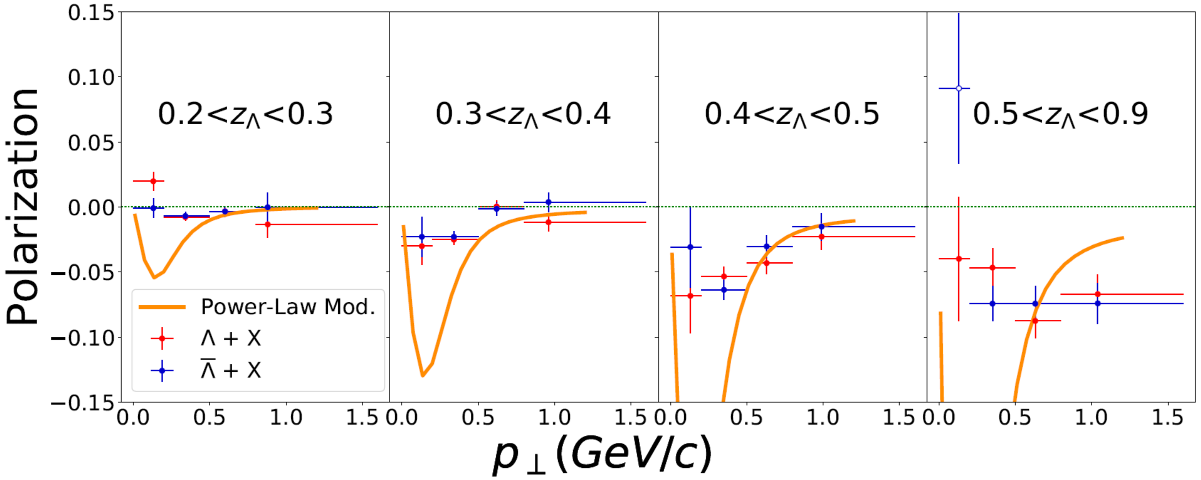

If we include 1-h data

Polarizing	Unpolarized	g_K	$M_D^{h_2}$	$\chi^2_{\rm dof}$ (2-h)	$\chi^2_{\rm dof}$ (2-h + 1-h)
Gaussian	Power-Law	Logarithmic	Gaussian	1.192	2.813
Power-Law	Power-Law	Logarithmic	Gaussian	1.21	2.39
Gaussian	Power-Law	PV17	PV17	1.198	3.159

Different combinations of NP functions fits give χ^2_{dof} = [2.4 – 5.4]

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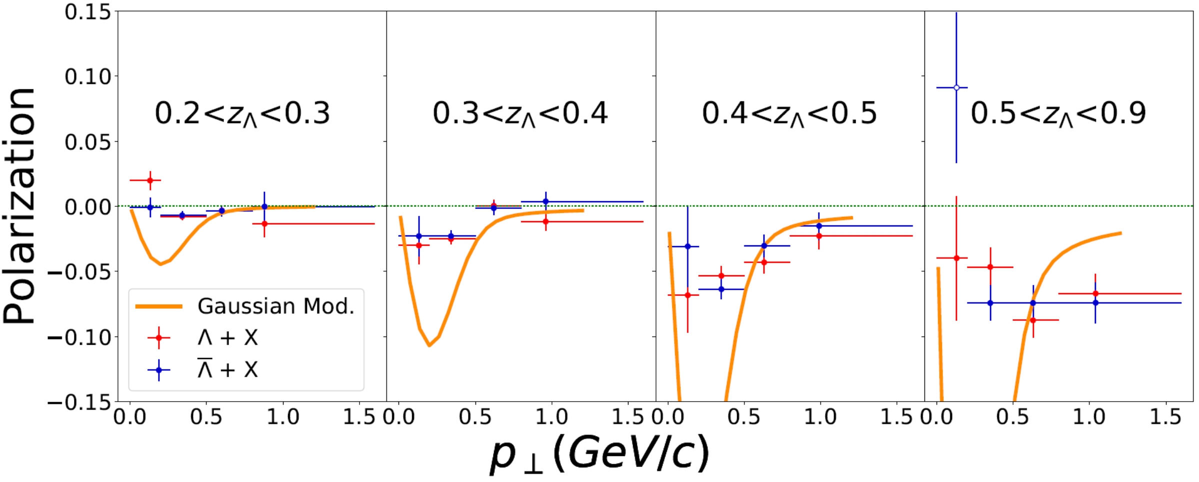

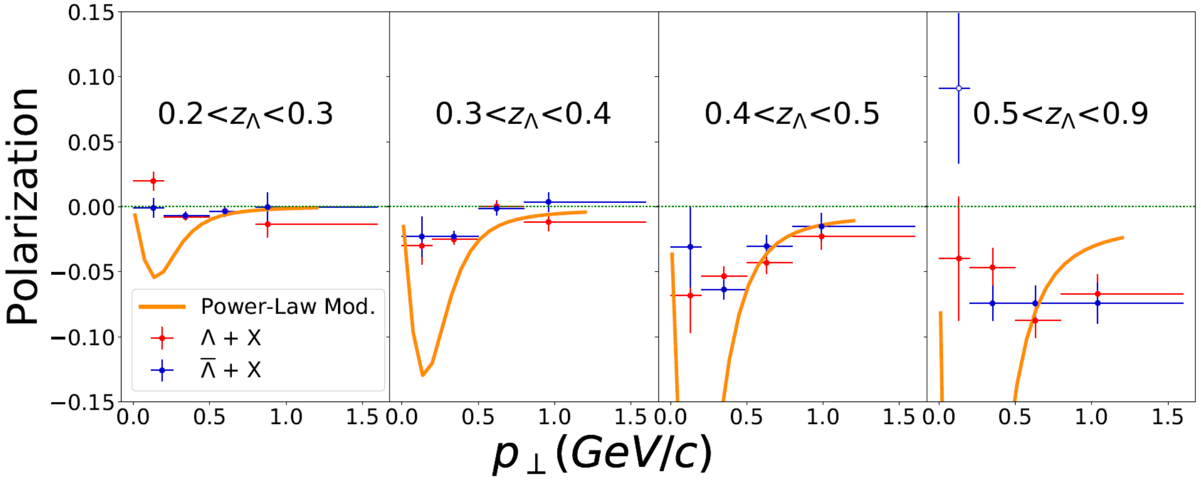

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Different factorization or different hadronic model?

Different combinations of NP functions fits give χ^2_{dof} = [2.4 – 5.4]

Combined Fit: Double Model

- Same parametrization for $D_{1T}^{\perp(1)}(b_T)$
- Two set of parameters for hadron models

Gaussian mod.
$\langle p_\perp^2 \rangle$
1-h 2-h

Power-Law mod.
(p, m)
1-h 2-h

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Gaussian mod. $\langle p_\perp^2 \rangle$ → 1-h, 2-h

Power-Law mod. (p, m) → 1-h, 2-h

Gaussian	Power-Law
$\chi^2_{dof} = 1.801$	$\chi^2_{dof} = 1.565$

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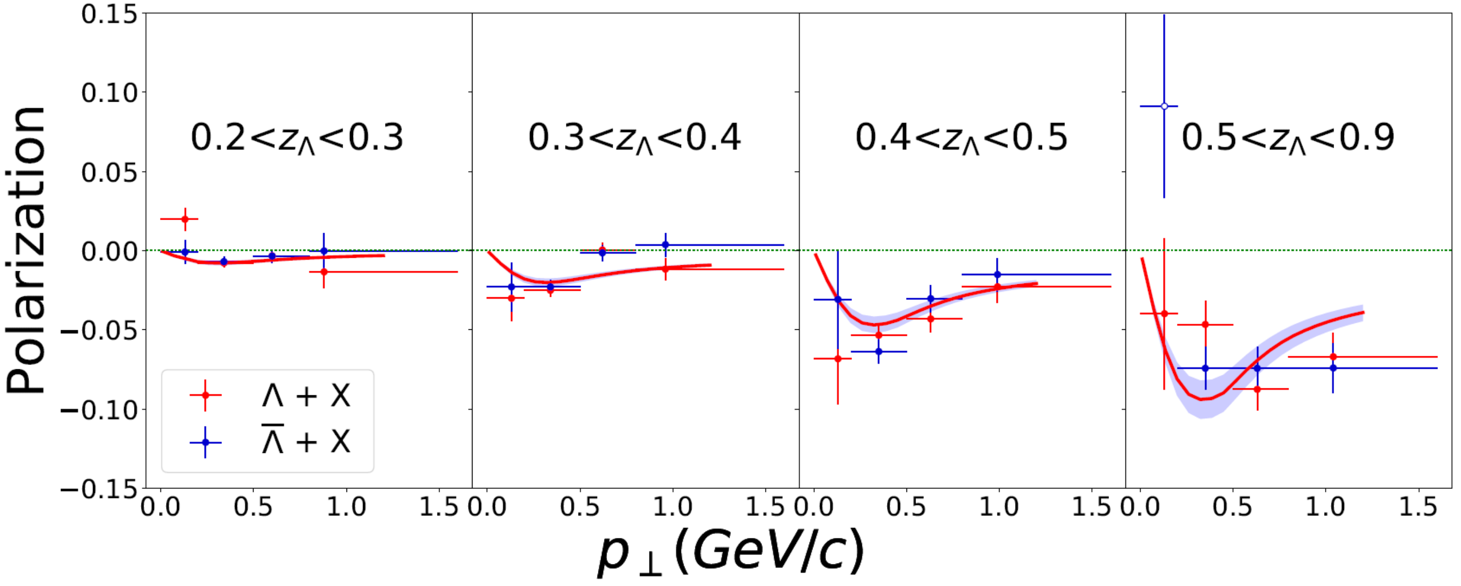
With Power-Law model



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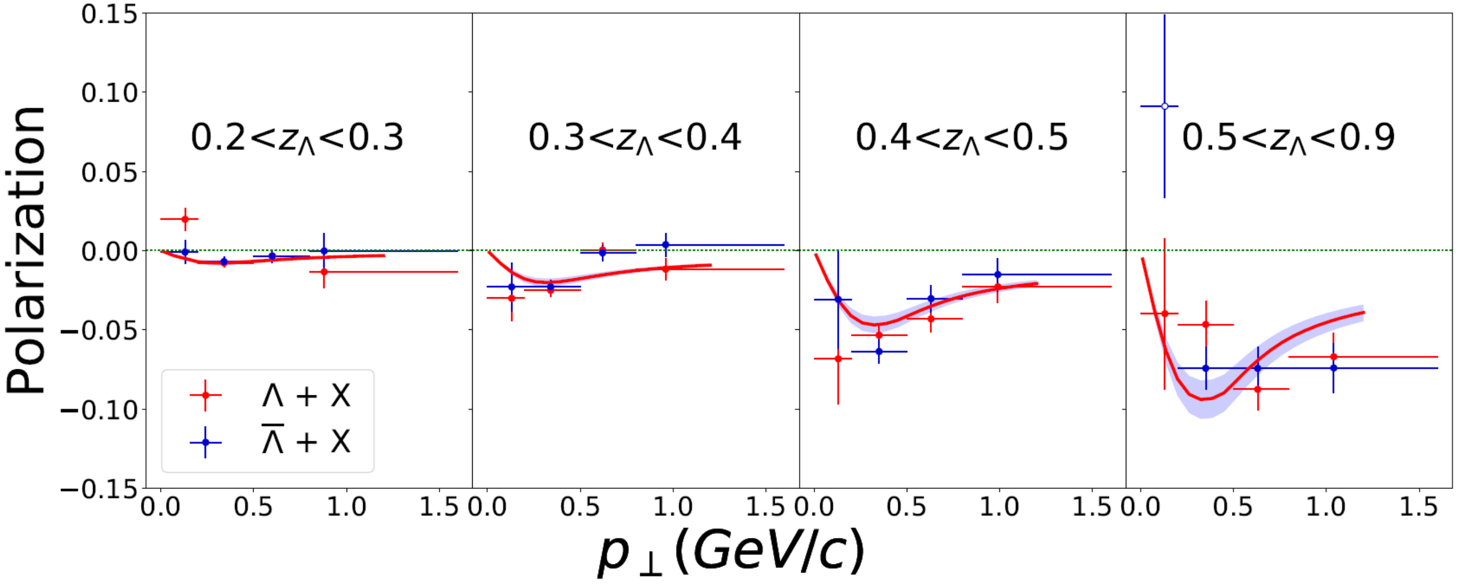
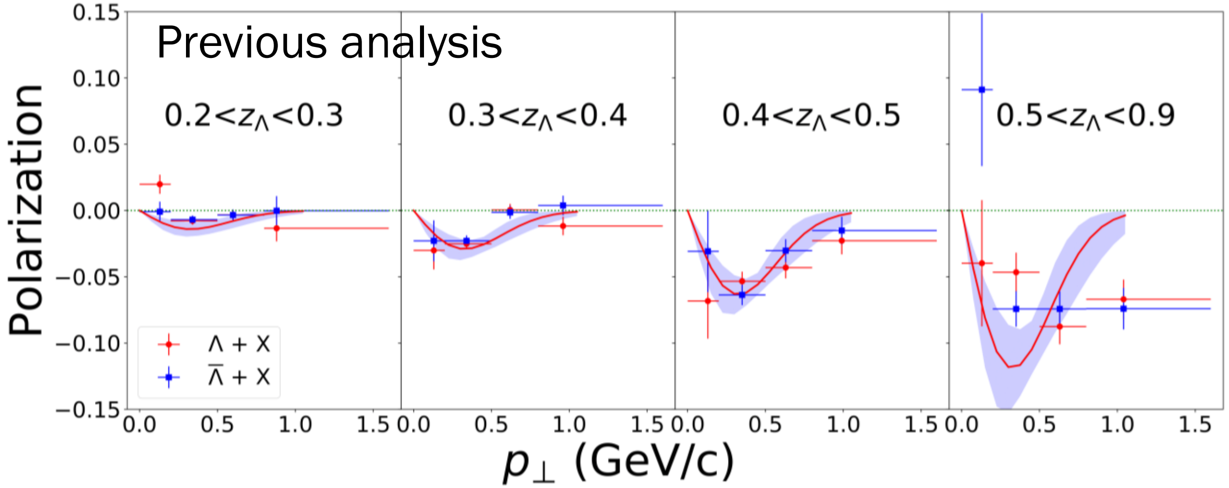
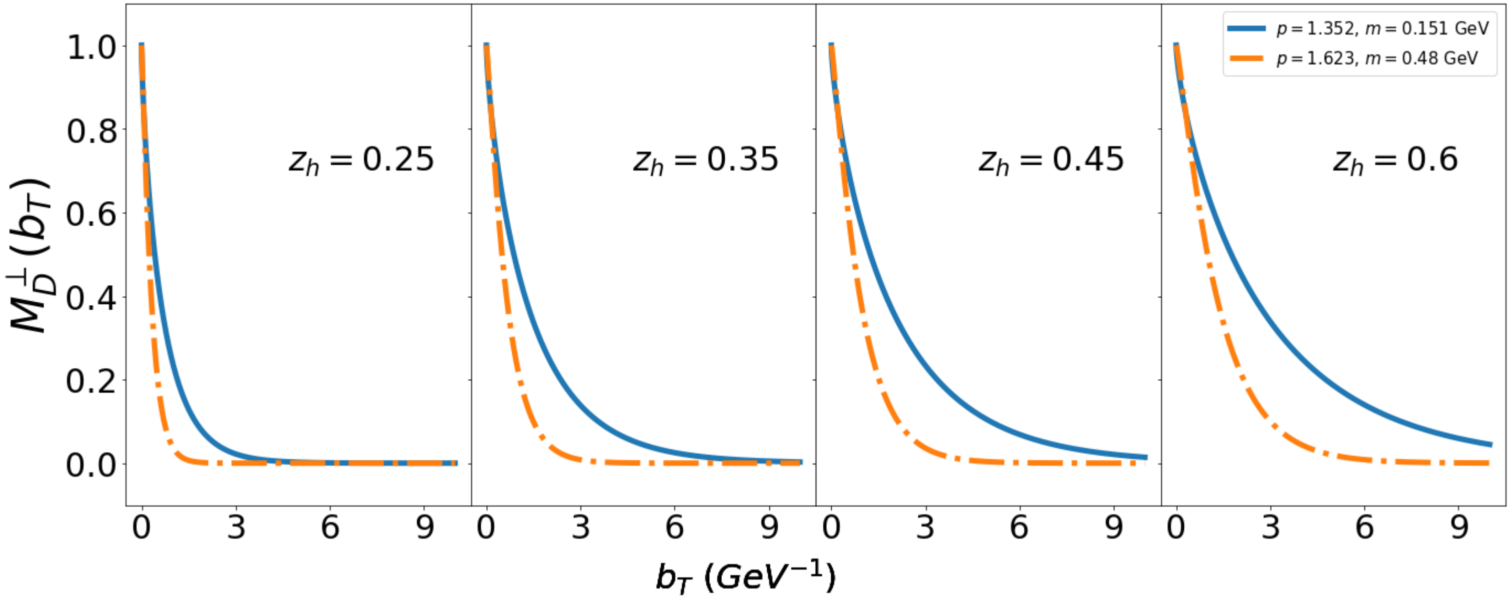
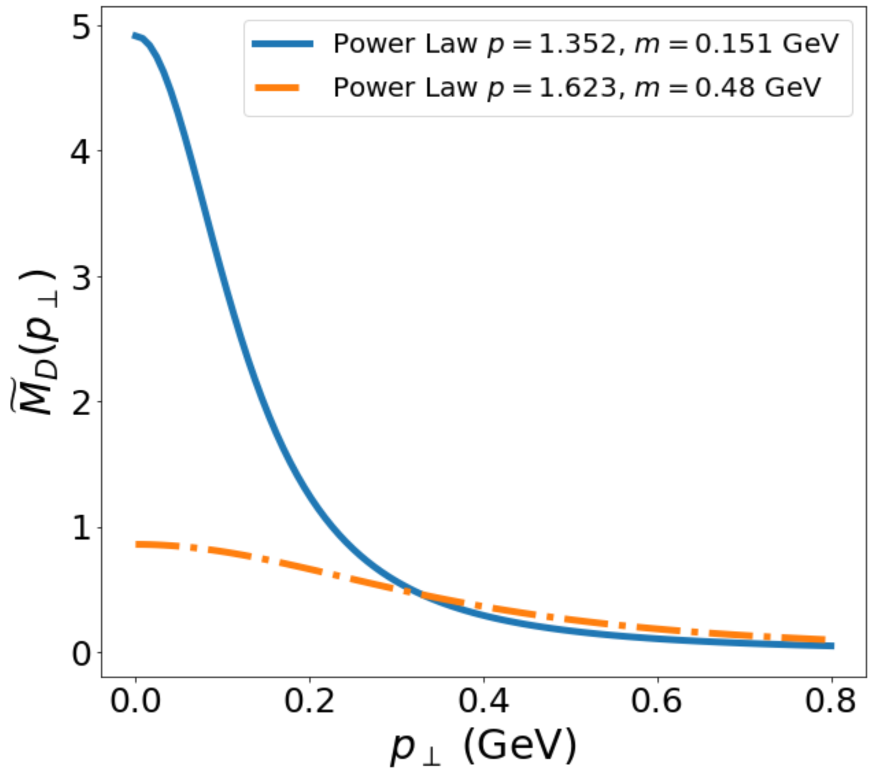




Combined Fit: Double Model

- 2-h Power-Law model
- 1-h Power-Law model





- Both models have same value at small b_T → collinear limit
- In $p_\perp$- space: same value at large $p_\perp$

- 2-h wider than 1-h: different behaviour at large b_T
- In $p_\perp$- space: different value at small $p_\perp$
- Possible different contribution from Soft gluons

OPAL data: Predictions

- 5 points: bins of $p_\perp$
- $\sqrt{s} = M_Z$
- Integrated over energy fractions $z_h = \ [0.15 - 1]$
- Good data set to check the TMD evolution

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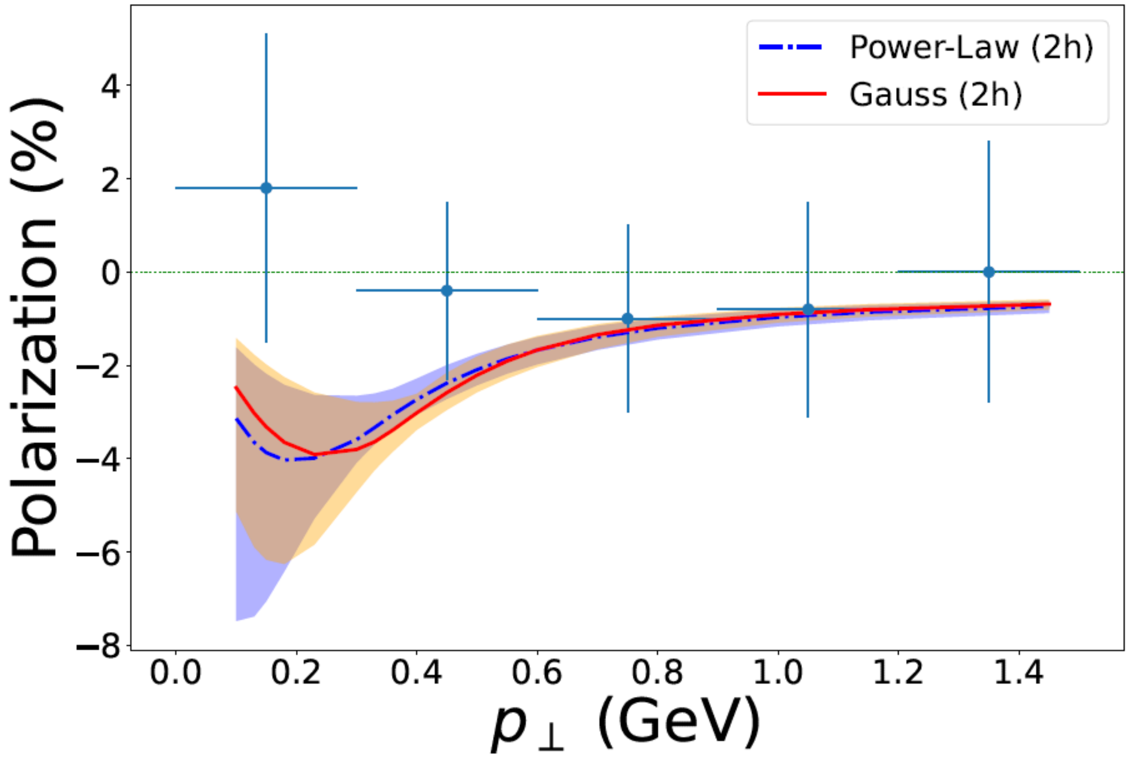

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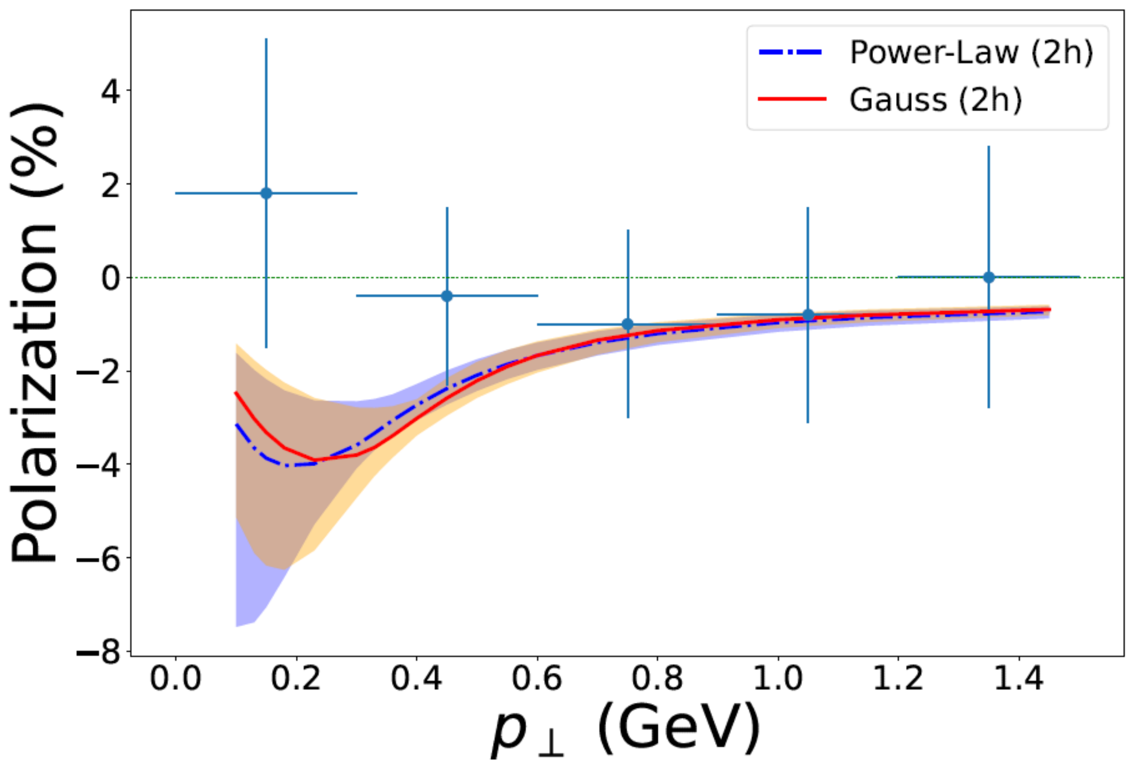
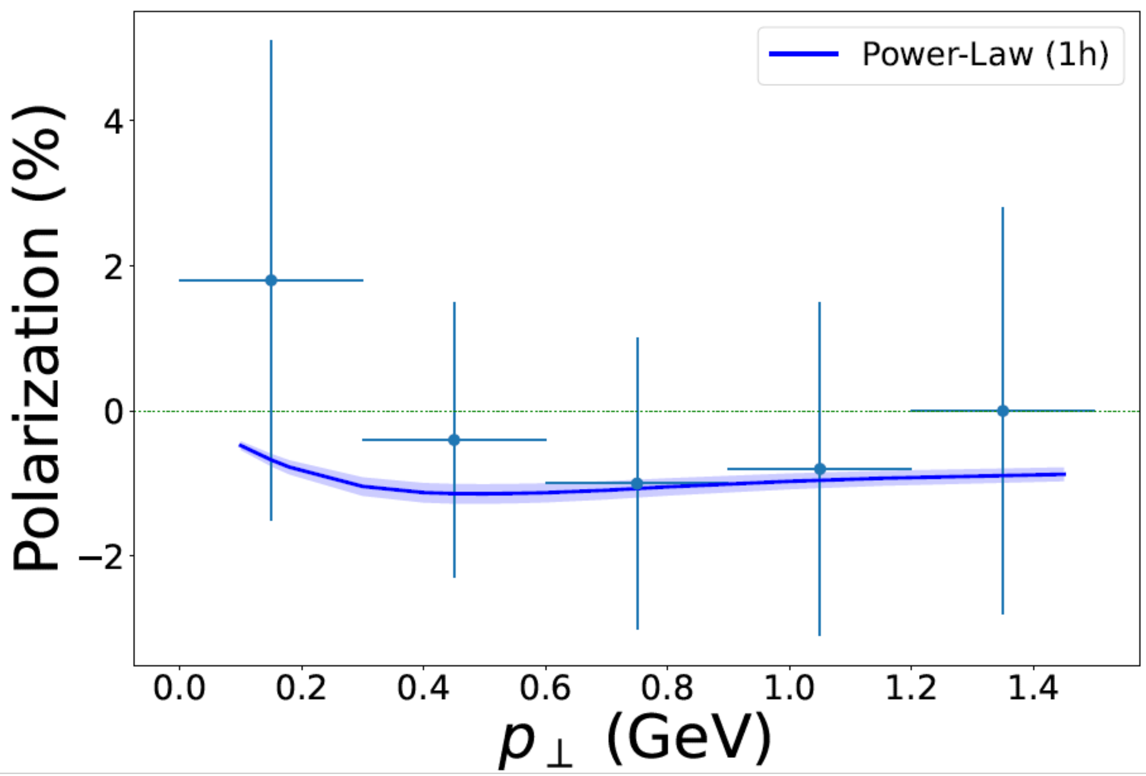
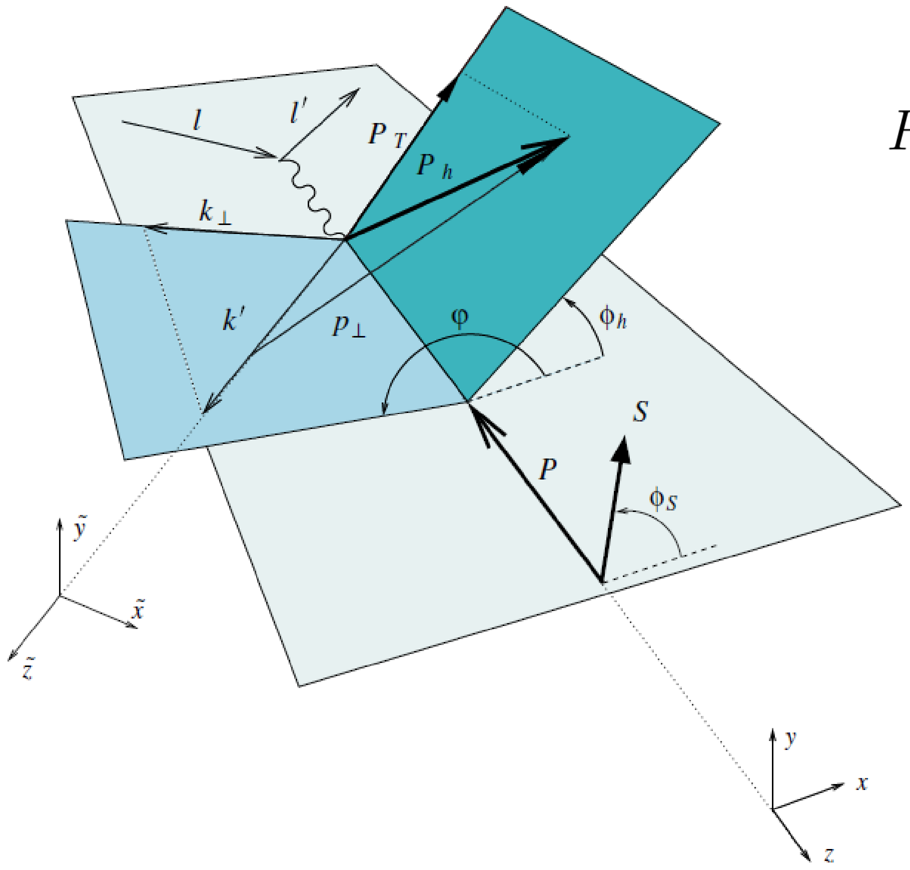


With model 1-h in ''double-model'' fit



SiDIS – Polarized Lambda Production

$e^{-}P \rightarrow e^{-}\Lambda$



$$F_{UU} = \sum_q e_q^2 \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp \delta^{(2)}(\mathbf{P}_T - \xi_p \mathbf{k}_\perp - \mathbf{p}_\perp) f_{q/p}(x, k_\perp) D_{h/q}(z, p_\perp)$$

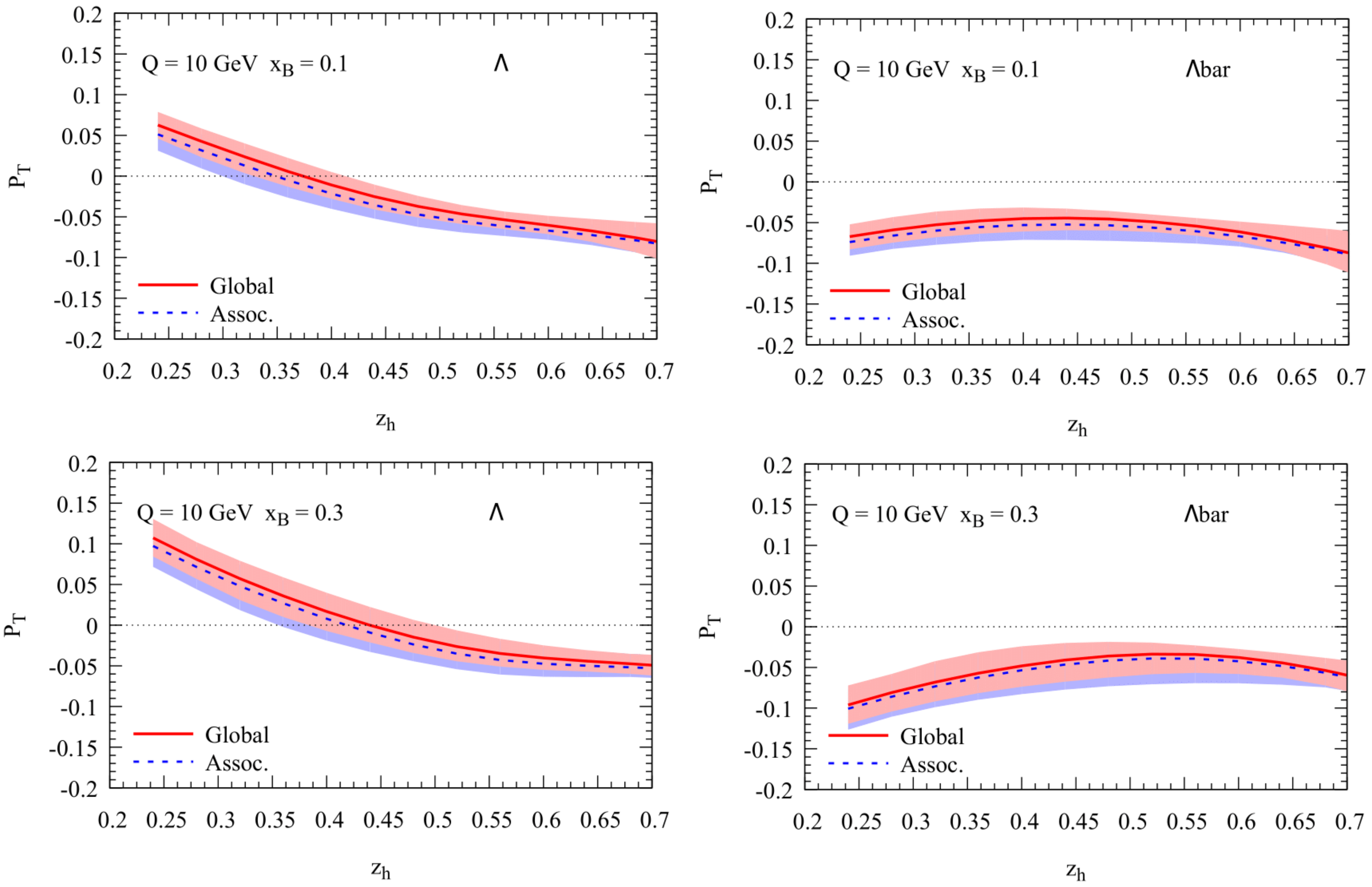
$$F_{TU}^{\sin(\phi_1 - \phi_{S_h}^L)} = \sum_q e_q^2 \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp \delta^{(2)}(\mathbf{P}_T - \xi_p \mathbf{k}_\perp - \mathbf{p}_\perp) \times \left[\xi_p \frac{\mathbf{k}_\perp \cdot \hat{\mathbf{P}}_T}{p_\perp} - \frac{P_T}{p_\perp}\right] f_{q/p}(x, k_\perp) \Delta^N D_{h^\uparrow/q}(z, p_\perp)$$

$$P_n^{h_1}(x, z) = \frac{\int d^2\mathbf{P}_T F_{TU}^{\sin(\phi_1 - \phi_{S_h}^L)}}{\int d^2\mathbf{P}_T F_{UU}}$$

x_B Bjorken-x
z_h energy fraction

$$\xi_p = z_h \left(1 - \frac{m_h^2}{z_h^2 Q^2} \frac{x_B}{1 - x_B}\right)$$

SiDIS – Polarized Lambda Production



Prediction for the Λ polarization:

- x_B = 0.1
- x_B = 0.3

Polarization ≃10%

Conclusions

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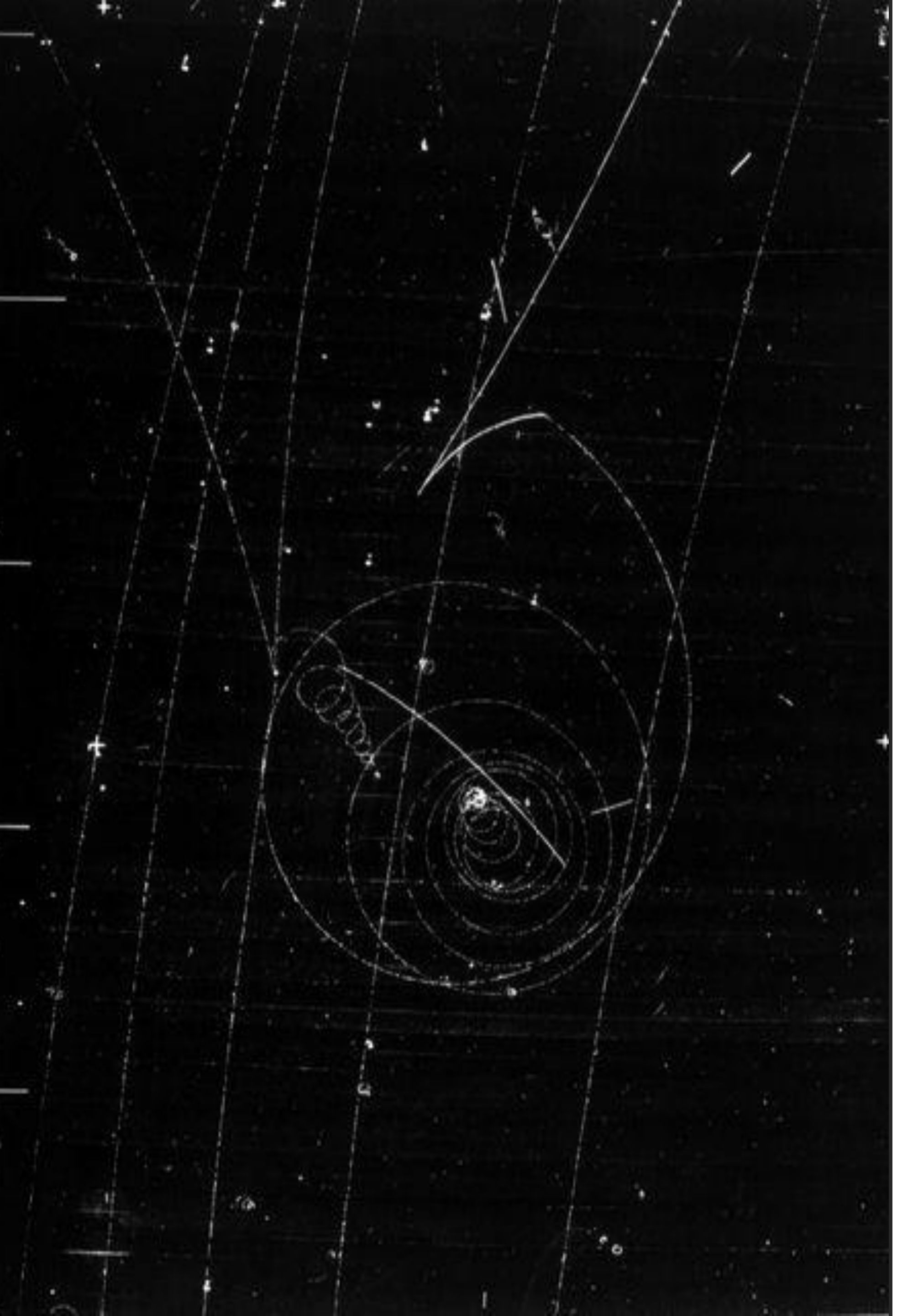
- pFF (2-h) $\neq$ pFF(1-h)
- Good description with different models

- Predictions for the transverse Lambda polarization in SIDIS at energies compatible w/ future EIC.



Thanks for your attention!

Backup Slides

Polarization 2-h: Double-hadron Production

$$\mathcal{B}_1\left[\widetilde{D}_{1T}^{\perp(1)}\widetilde{\bar{D}}_1\right]=\frac{\mathcal{H}^{(e^+e^-)}(Q)}{z_1^2 z_2^2}\sum_q e_q^2\int\frac{db_T}{(2\pi)}\,b_T^2 J_1(b_T\,q_T)\widetilde{D}_{1T}^{\perp(1)}(z_1;\bar{\mu}_b)\,d_{\bar{q}/h_2}(z_2;\bar{\mu}_b)$$
$$\times M_D^{\perp}(b_c(b_T);b_{\max})\,M_{D_2}(b_c(b_T);b_{\max})\exp\left\{-g_K(b_c(b_T);b_{\max})\ln\left(\frac{Q^2 z_1 z_2}{M_{h_1}M_{h_2}}\right)\right\}$$
$$\times\exp\left\{\widetilde{K}(b_*;\bar{\mu}_b)\ln\frac{Q^2}{\bar{\mu}_b^2}+\int_{\bar{\mu}_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(g(\mu');1)-\gamma_K(g(\mu'))\ln\frac{Q^2}{\mu'^2}\right]\right\},$$

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$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2b_T}\{J_1(b_T\, q_{T_{max}})H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}})H_1(b_T\, q_{T_{max}})\}$$

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$$\mathcal{B}_0\left[\widetilde{D}\widetilde{\bar{D}}\right] = \frac{\mathcal{H}^{(e^+e^-)}(Q)}{z_1^2 z_2^2}\sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T q_T)\, d_{q/h_1}(z_1;\bar{\mu}_b) d_{\bar{q}/h_2}(z_2;\bar{\mu}_b)$$
$$\times M_{D_1}(b_c(b_T);b_{\max})\, M_{D_2}(b_c(b_T);b_{\max}) \exp\left\{-g_K(b_c(b_T);b_{\max}) \ln\left(\frac{Q^2 z_1 z_2}{M_{h_1}M_{h_2}}\right)\right\}$$
$$\times \exp\left\{\widetilde{K}(b_*;\bar{\mu}_b)\ln\frac{Q^2}{\bar{\mu}_b^2} + \int_{\bar{\mu}_b}^{Q}\frac{d\mu'}{\mu'}\left[2\gamma_D(g(\mu');1) - \gamma_K(g(\mu'))\ln\frac{Q^2}{\mu'^2}\right]\right\},$$

$J_{0,1}(b_T q_T)$ Bessel Function

$H_{0,1}(b_T q_T)$ Struve Function

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_1(b_T\, q_T) = \frac{\pi q_{T_{max}}}{2 b_T}\{J_1(b_T\, q_{T_{max}}) H_0(b_T\, q_{T_{max}}) - J_0(b_T\, q_{T_{max}}) H_1(b_T\, q_{T_{max}})\}$$

$$\int_0^{q_{T_{max}}} dq_T\, q_T J_0(b_T\, q_T) = \frac{q_{T_{max}}}{b_T} J_1(b_T\, q_{T_{max}})$$

$$q_{Tmax} = Q * \eta$$

η = [0,15 – 0,3]

Region of
TMD factorization

Double-hadron production (2-h) data Fit

Best Results

Polarizing	Unpolarized	g_K	$M_D^{h_2}$	χ^2_{dof} (2-h)
Gaussian	Power-Law	Logarithmic	Gaussian	1.192
Power-Law	Power-Law	Logarithmic	Gaussian	1.21
Gaussian	Power-Law	PV17	PV17	1.198

Parameters	Gaussian	Power-Law	Gaussian
N_u	0.093	0.100	0.168
N_d	-0.100	-0.107	-0.138
N_s	-0.117	-0.115	-0.161
N_{sea}	-0.055	-0.058	-0.104
a_s	2.19	2.12	2.19
b_u	3.5	3.5	4.02
b_{sea}	2.3	2.3	2.91
$\langle p_\perp^2 \rangle_p$	0.066		0.103
p		3.0	
m		0.35	

- $\chi^2_{dof} \simeq 1.2$
- First moment parameters are consistent;
- Up pFF is positive
- Up and Down: opposite contribution
- The $M_D^\perp$ models are compatible

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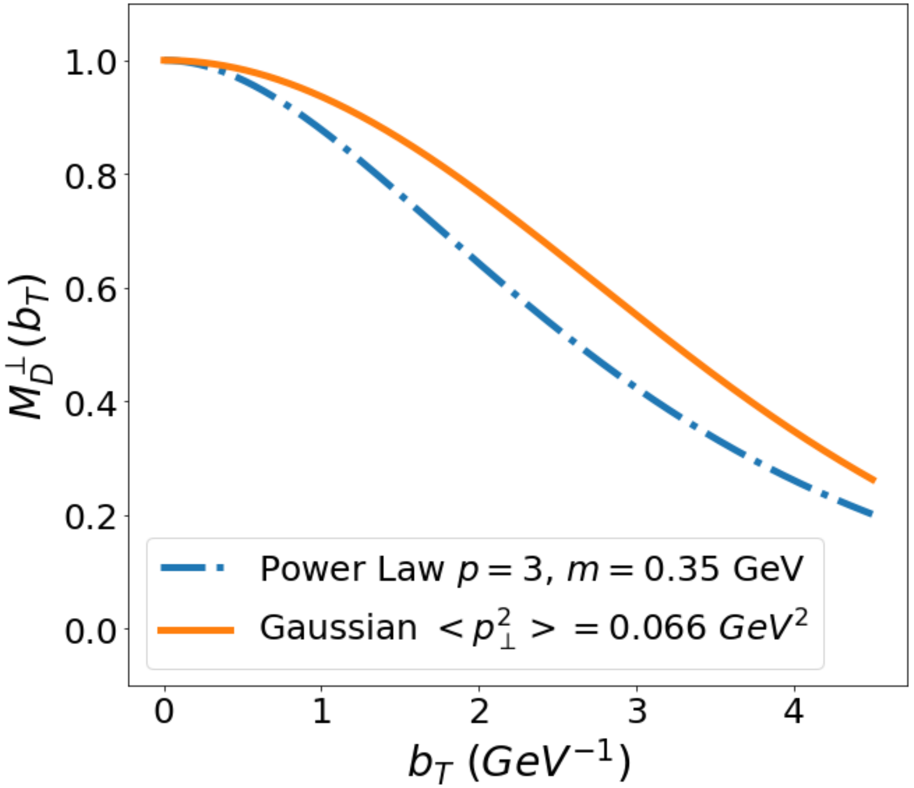
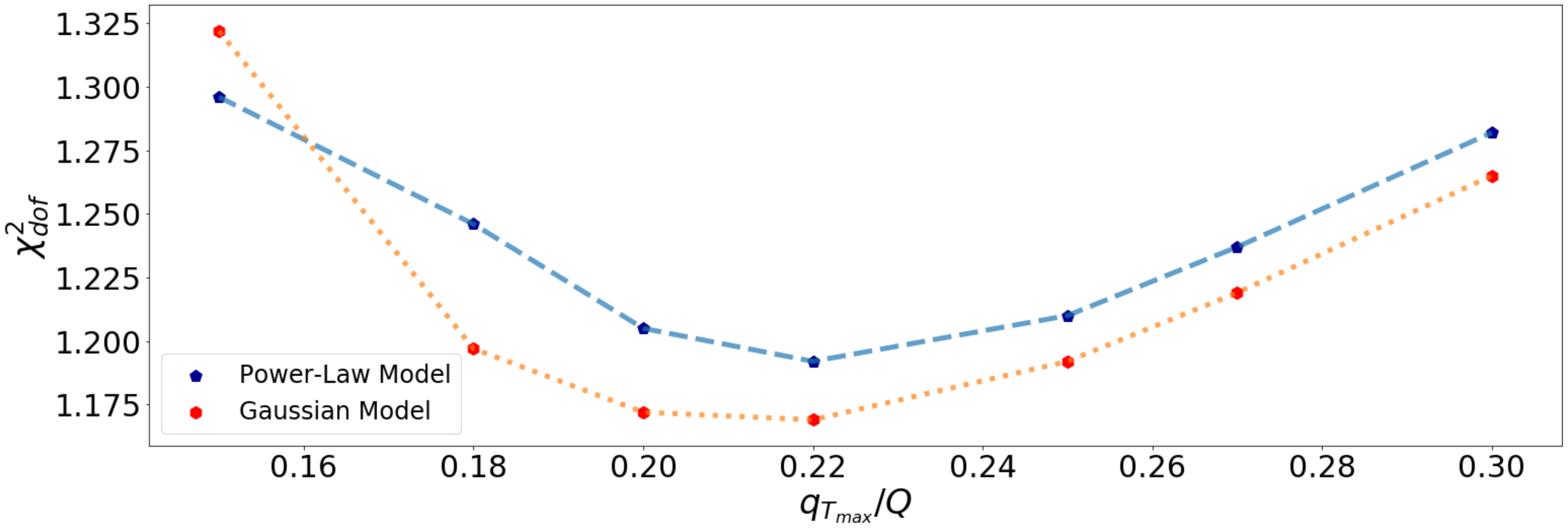


Double-hadron production (2-h) data Fit

Impact of different $q_{T_{max}}/Q$ values on the quality of the fit:



- Gaussian model gives a smaller χ^2_{dof} than Power-Law model
- Both models reach their minimum at $\frac{q_{T_{max}}}{Q} = 0.22$

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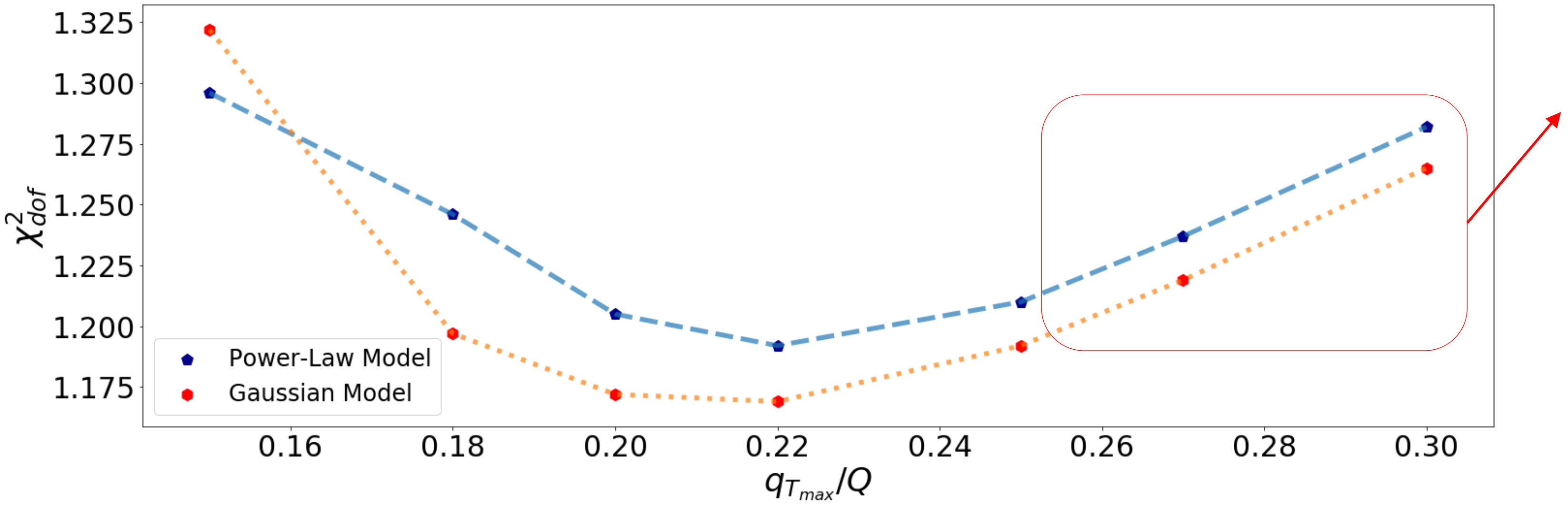

Gradually going out of the validity region of the TMD Factorization $q_{T_{max}} \ll Q$

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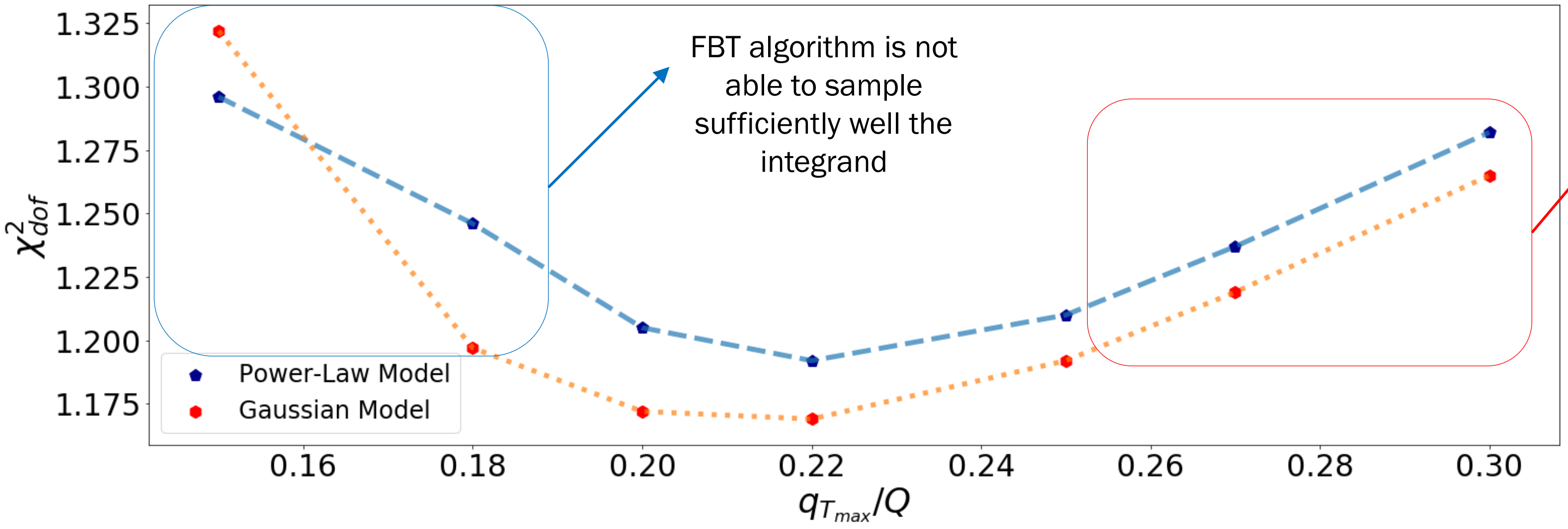


Gradually going out of the validity region of the TMD Factorization $q_{T_{max}} \ll Q$

- The Fast Bessel Transform (FBT) algorithm is based on the density nodes of Bessel functions.
- The smaller is the value of $q_{T_{max}}/Q$, the larger is the distance between the nodes

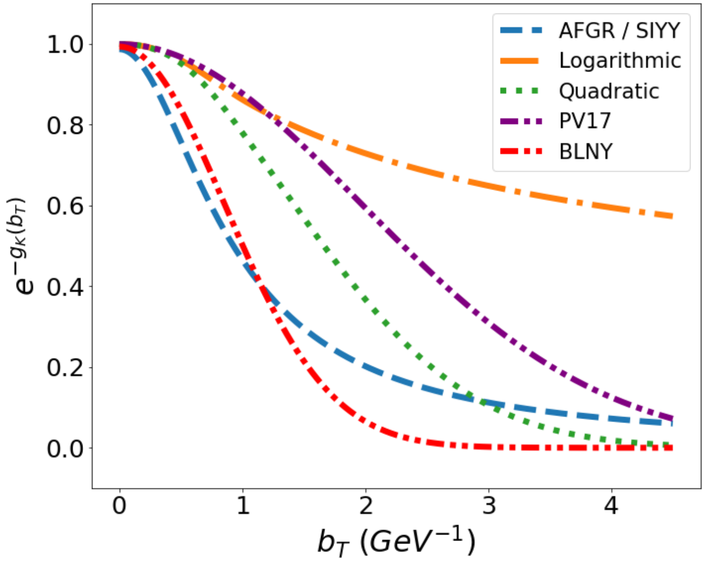
- Gaussian model gives a smaller χ^2_{dof} than Power-Law model
- Both models reach their minimum at $\frac{q_{T_{max}}}{Q} = 0.22$

Double-hadron production (2-h) data Fit

g_K	χ^2_{dof} range
Logarithmic	1.192 - 1.287
Quadratic	1.4 - 1.472
AFGR	1.474 - 1.514
BLNY	1.67 - 1.783

- Extractions are stable when employing the same $g_K(b_T)$
- Best fits with Logarithmic function $\chi^2_{dof} < 1.3$
- Quadratic/AFGR $\chi^2_{dof} = [1.4 - 1.5]$
- Worst fits with BLNY $\chi^2_{dof} > 1.7$

$g_K(b_T)$ better if it goes like a constant for large b_T



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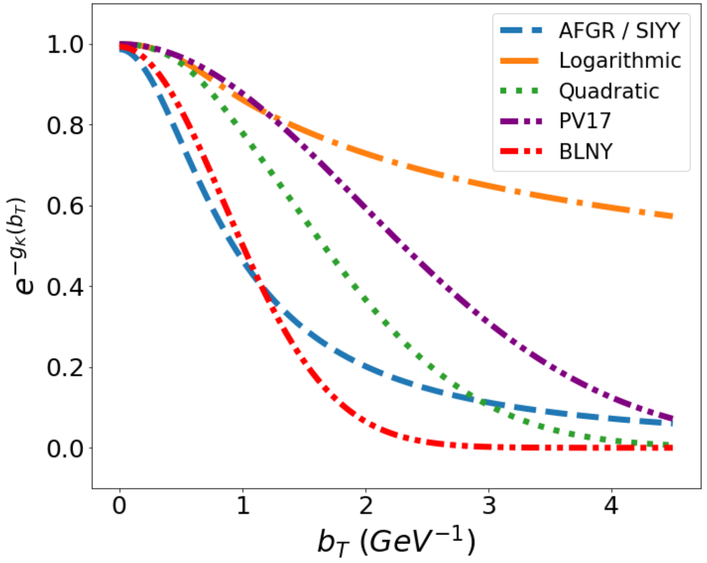

We have 36 different fits that can describe 2-h data well enough.

Can they also describe the 1-h data set?