EIC User Group Early Career Workshop 2022

24-25 July 2022
CFNS Stony Brook University

Electron-jet production at EIC

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Zhong-Bo Kang, Kyle Lee and Ding Yu Shao and **FZ**, arXiv:2106.15624

Roli Esha, Zhong-Bo Kang, Kyle Lee, Ding Yu Shao and **FZ**, arXiv:2207.XXXX

25 Jul. 2022

Overview

- Jet anisotropy
  - $ep \to e + \text{jet} + X$
- Spin Asymmetries
  - $ep \to e + \text{jet} + X$
  - $ep \to e + \text{jet}(h) + X$
- Summary & Outlook

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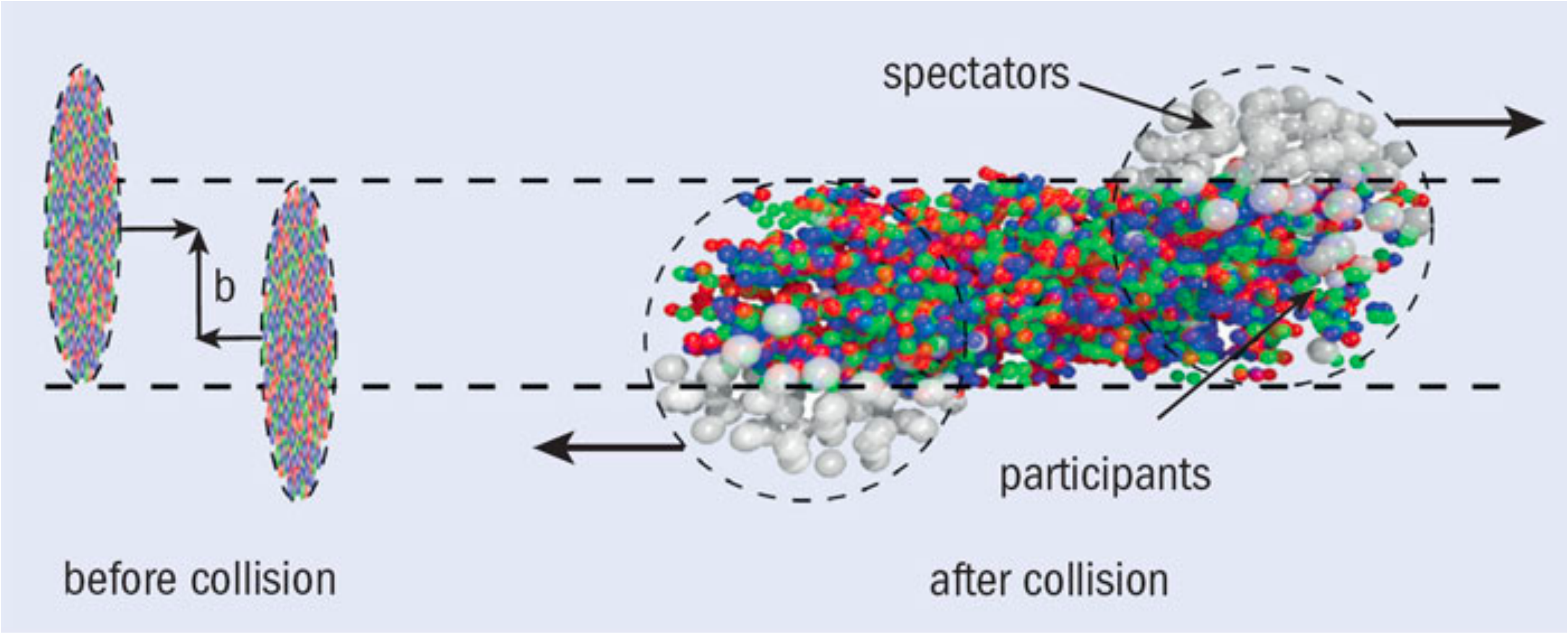
- Summary & Outlook

- In heavy-ion collisions,



[1]

$$E\frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n\left(\varphi - \Psi_{RP}\right)\right]\right)$$

$$v_n\left(p_t, y\right) = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$$

v_1 : directed flow

v_2 : elliptic flow

v_3 : triangular flow

v_4 : quadrapole flow

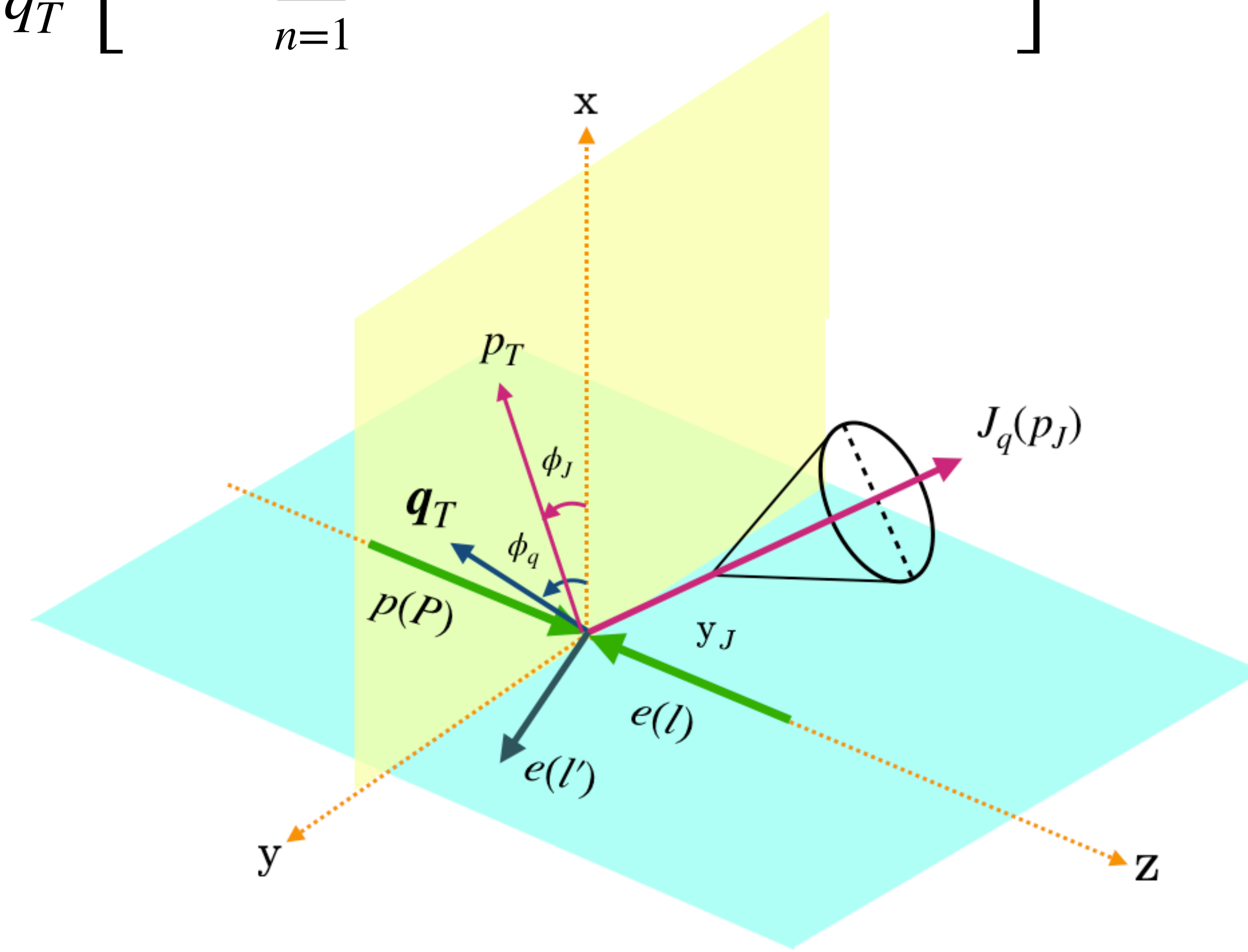
...

[1] Snellings, Raimond. "Elliptic flow: a brief review." *New Journal of Physics* 13.5 (2011): 055008.

- Back-to-back electron-jet production from *ep* collision,

$$e(l) + p(P) \to e(l') + J_q(p_J) + X$$

$$\frac{d\sigma}{d^2\boldsymbol{p}_T dy_J d\phi_J d^2\boldsymbol{q}_T} = \frac{d\sigma}{2\pi d^2\boldsymbol{p}_T dy_J q_T dq_T}\left[1 + 2\sum_{n=1}^{\infty} v_n(p_T, y_T)\cos(n(\phi_q - \phi_J))\right]$$



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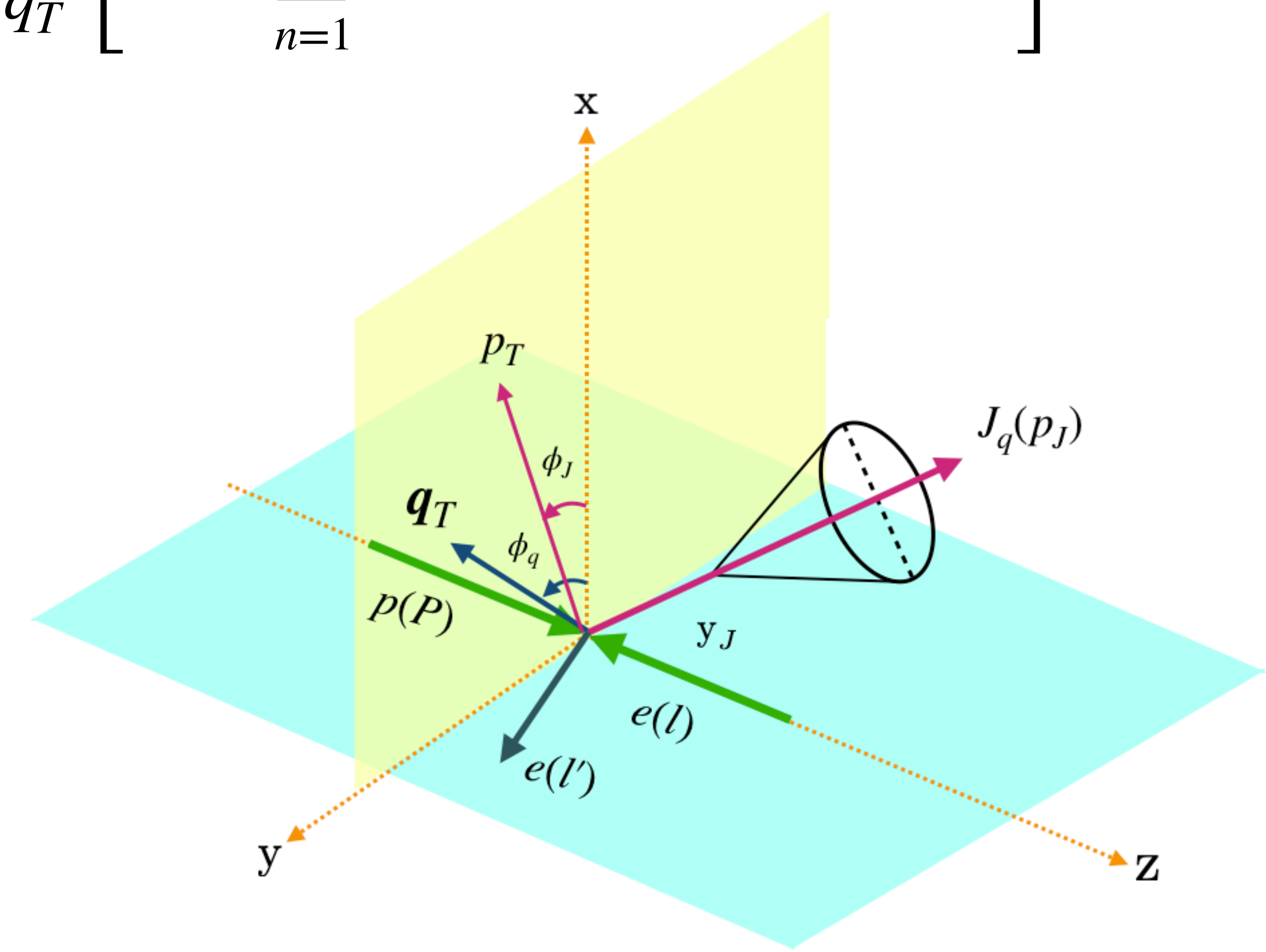
q_T : transverse momentum imbalance

$$\boldsymbol{q}_T = \boldsymbol{l}'_T + \boldsymbol{p}_{JT}$$

p_T : jet transverse momentum

y_J : jet rapidity



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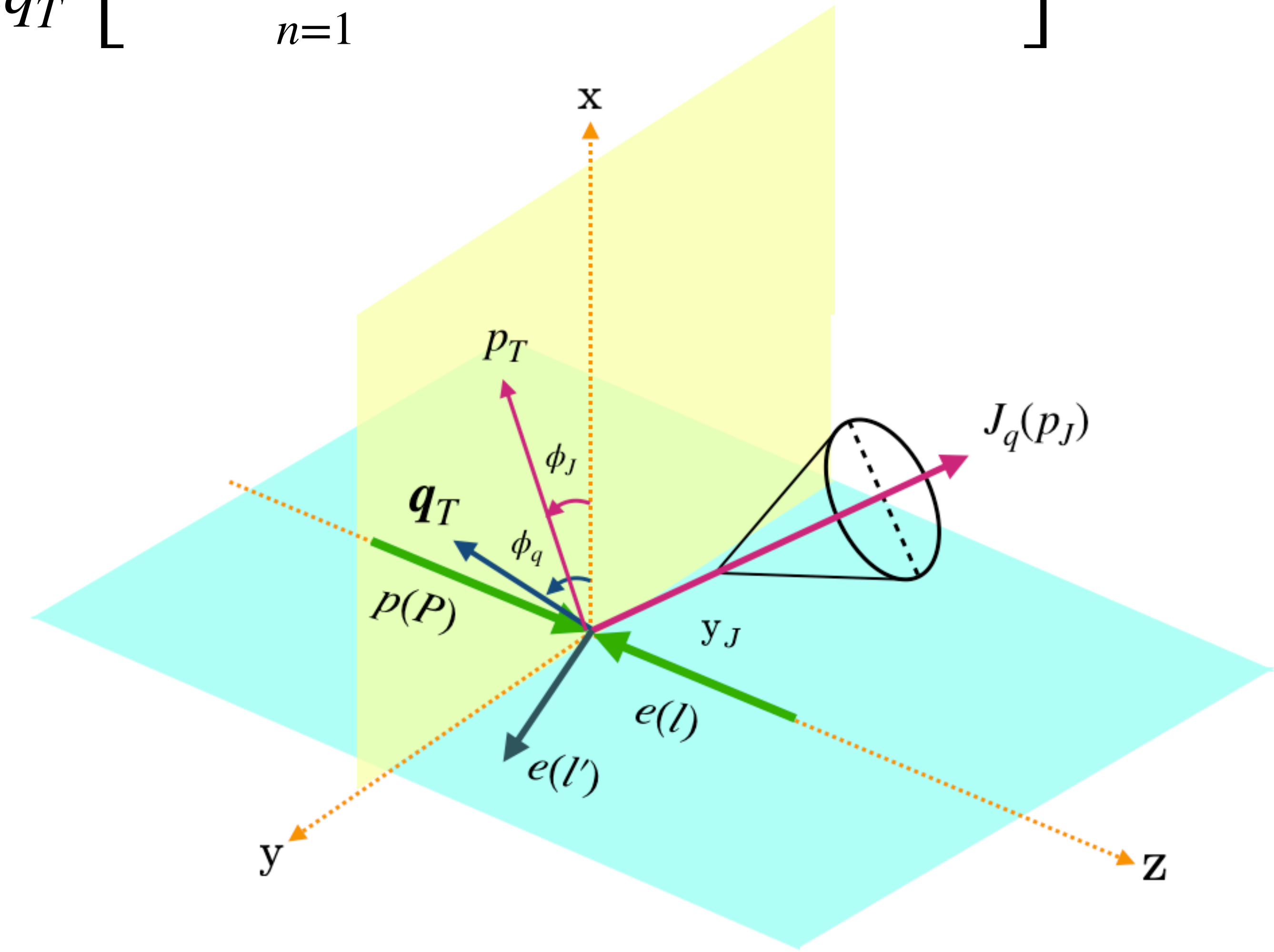
ϕ_q : azimuthal angle of transverse momentum imbalance

ϕ_J : azimuthal angle of jet transverse momentum

v_n : anisotropic Fourier coefficients

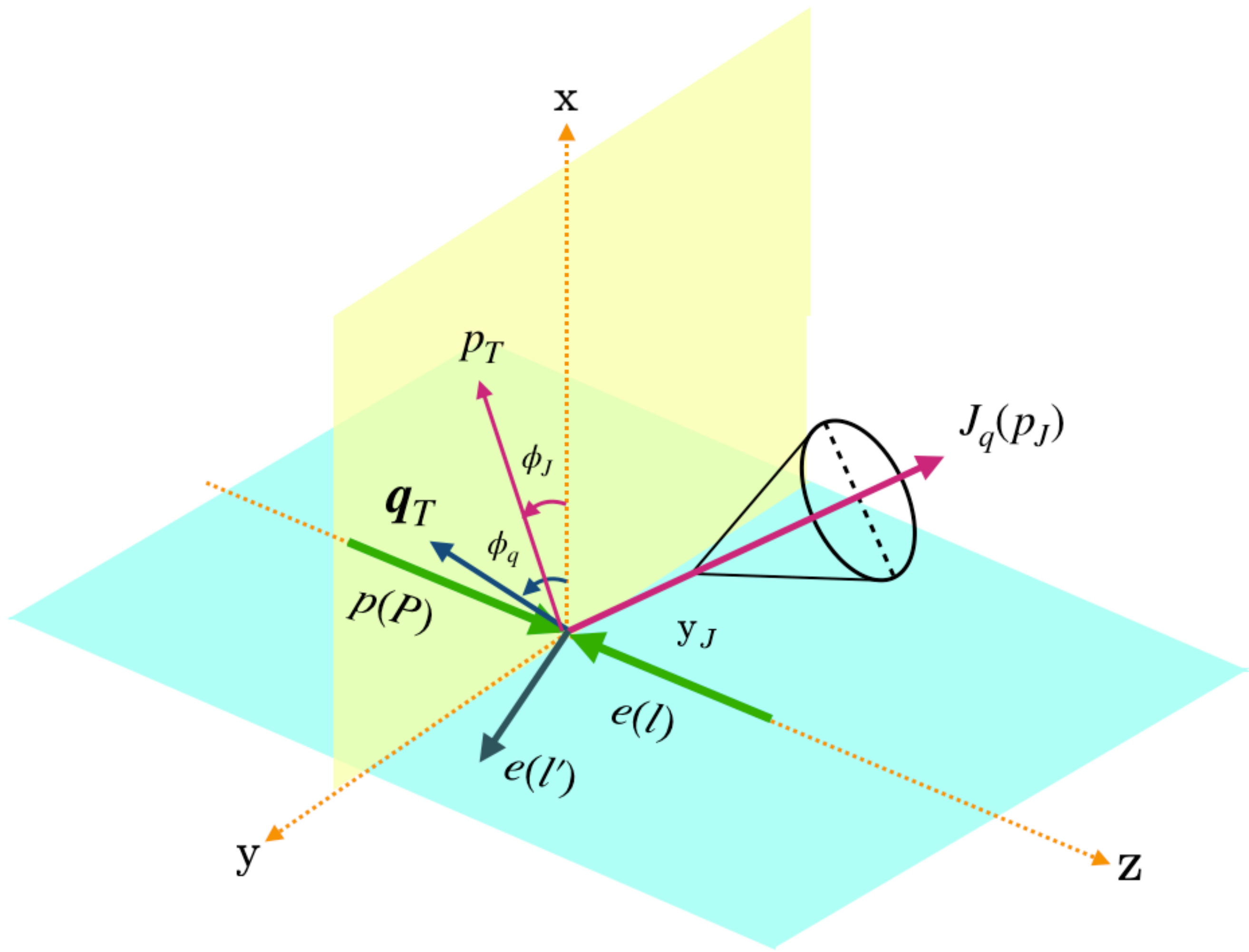
$$\sim \langle \cos(n(\phi_q - \phi_J)) \rangle$$



- At small $|\boldsymbol{q}_T|$ limit, TMD factorization gives [2]

$$\frac{d\sigma^{e+p\to e+\text{jet}+X}}{d^2p_T dy_J d\boldsymbol{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(-i\boldsymbol{b}\cdot\boldsymbol{q}_T)\, x\tilde{f}_1(x,b,\mu)$$

$$\times S_q(\boldsymbol{b},R,\mu) J_q(p_T R,\mu)$$



[2] Arratia, Miguel, et al. "Jet-based measurements of Sivers and Collins asymmetries at the future electron-ion collider." *Physical Review D* 102.7 (2020): 074015.

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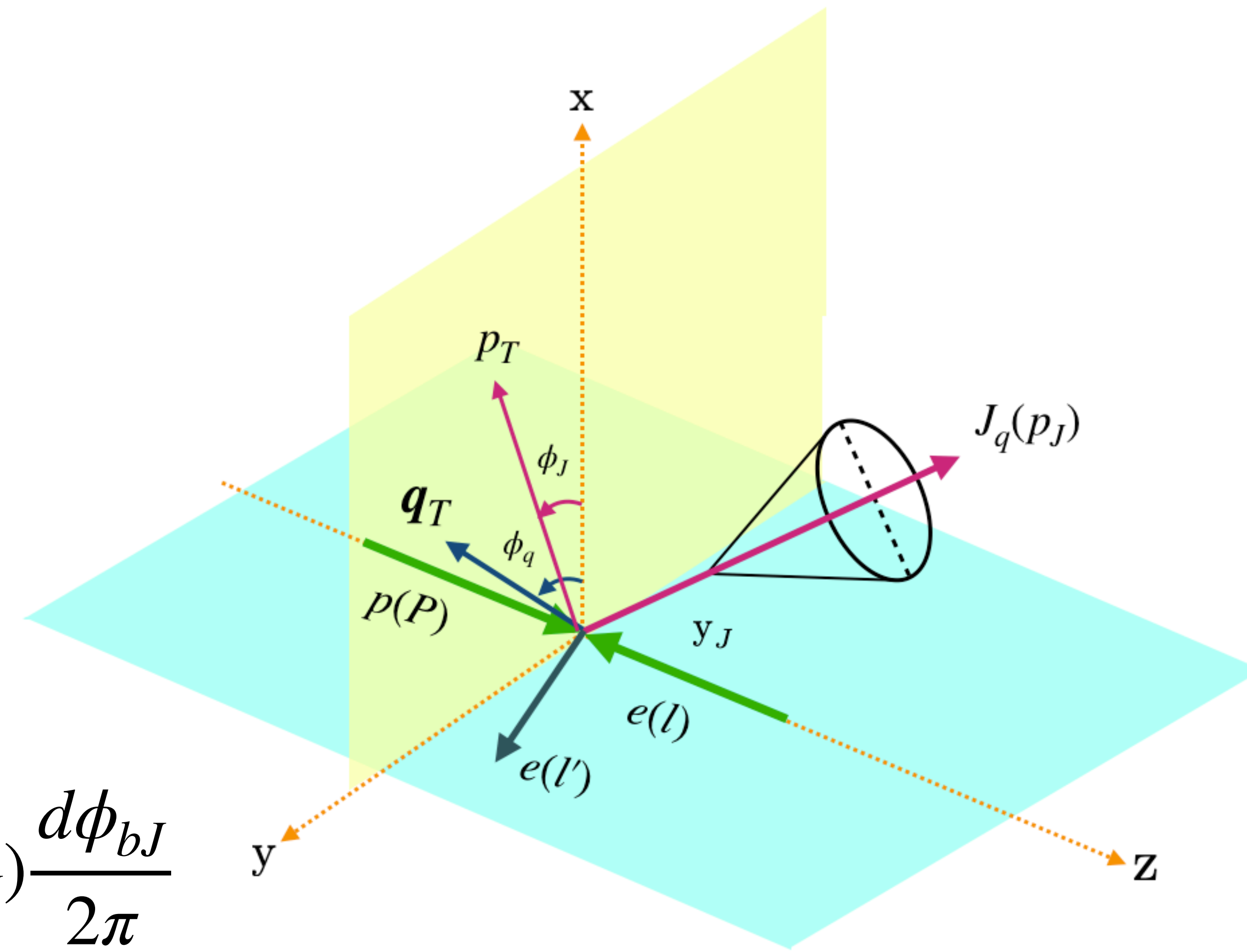
$$\times S_q(\boldsymbol{b},R,\mu) J_q(p_T R,\mu)$$

Soft function: $S_q(\boldsymbol{b},R,\mu)$

- Depends on the magnitude b and azimuthal angle ϕ_b of the vector $\boldsymbol{b}$

$$S_q^{\langle\cos(n\phi_{qJ})\rangle}(b,R,\mu) = \int S_q(\boldsymbol{b},R,\mu)\cos(n\phi_{bJ})\frac{d\phi_{bJ}}{2\pi}$$



$$n = 0 \;\Rightarrow\; \bar{S}_q(b,R,\mu)$$

[2] Arratia, Miguel, et al. "Jet-based measurements of Sivers and Collins asymmetries at the future electron-ion collider." *Physical Review D* 102.7 (2020): 074015.

- Azimuthal anisotropy of particle spectrum

$$v_n \sim \frac{\tilde{f}_1^{\mathrm{TMD}}(x,b,\mu) \otimes J_n(bq_T)(-i)^n S_q^{\langle\cos(n\phi_{qJ})\rangle}(\boldsymbol{b},R,\mu)}{\tilde{f}_1^{\mathrm{TMD}}(x,b,\mu) \otimes J_0(bq_T)\bar{S}_q(\boldsymbol{b},R,\mu)}$$

⇒ Can be measured in experiments as the expectation of the n-th order harmonics

$$v_n = \langle\cos(n\phi_{qJ})\rangle$$

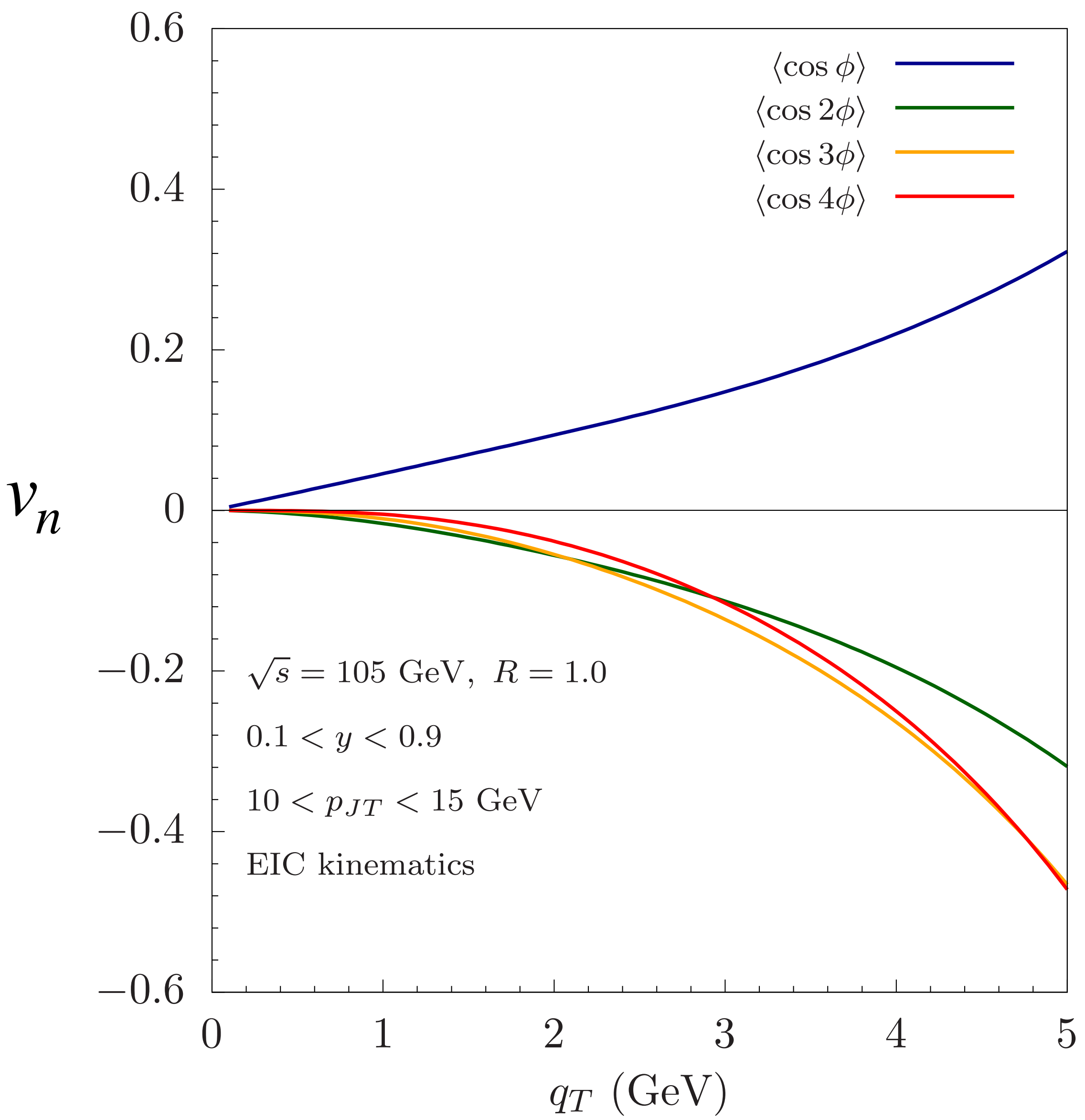
$\langle\cos(\phi_{qJ})\rangle$	v_1 :	directed flow
$\langle\cos(2\phi_{qJ})\rangle$	v_2 :	elliptic flow
$\langle\cos(3\phi_{qJ})\rangle$	v_3 :	triangular flow
$\langle\cos(4\phi_{qJ})\rangle$	v_4 :	quadrapole flow

$$\frac{d\sigma^{e+p\to e+\mathrm{jet}+X}}{d^2p_T dy_J d\boldsymbol{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu) \int \frac{bdb}{2\pi}\; x\tilde{f}_1(x,b,\mu) J_q(p_T R,\mu)$$

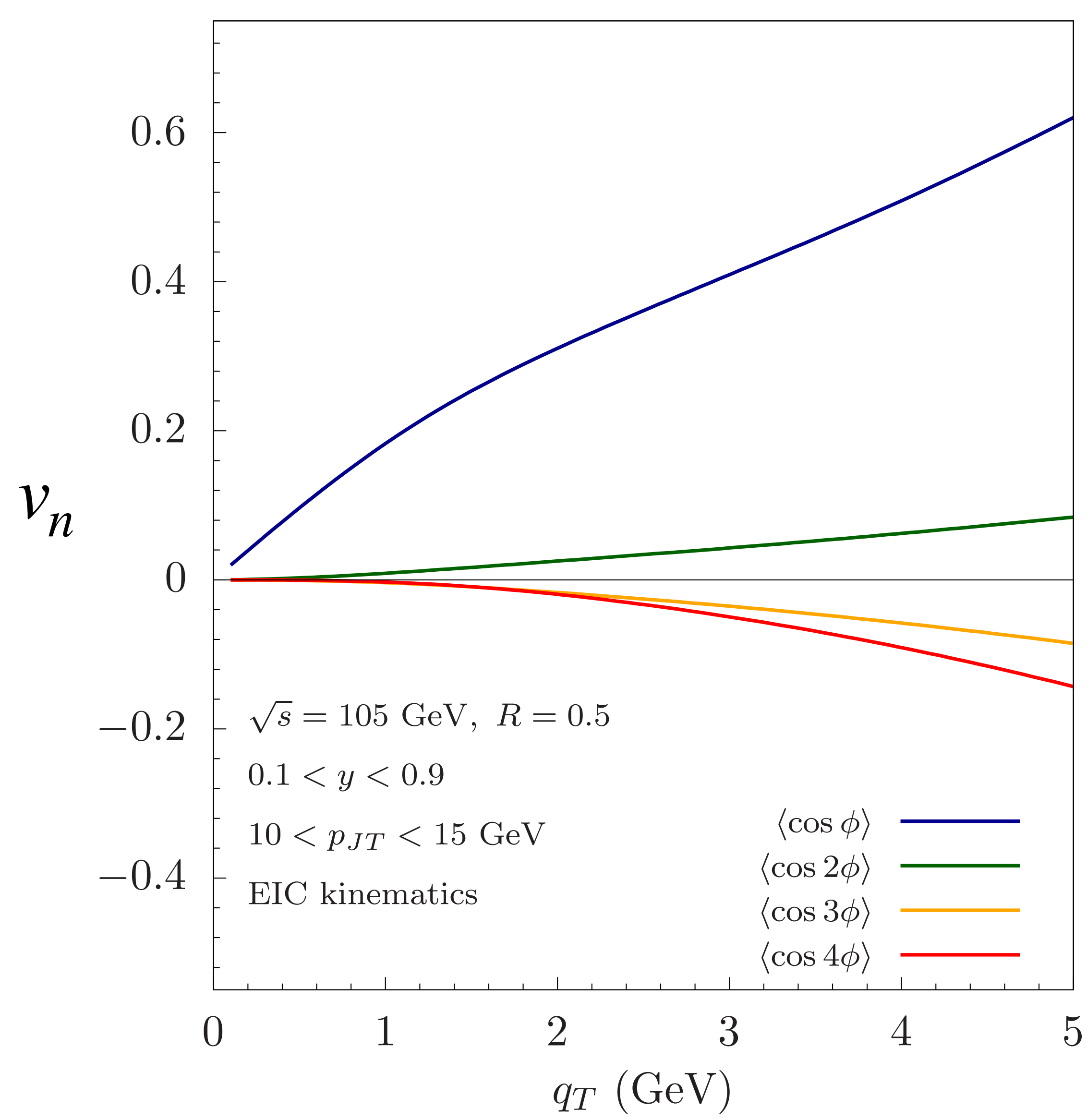
$$\times \left\{ J_0(bq_T)\bar{S}_q(\boldsymbol{b},R,\mu) + 2\sum_{n=1} J_n(bq_T)\left[\cos(n\phi_{qJ})(-i)^n S_q^{\langle\cos(n\phi_{qJ})\rangle}(\boldsymbol{b},R,\mu)\right]\right\}$$

EIC kinematics

R=1



R=0.5



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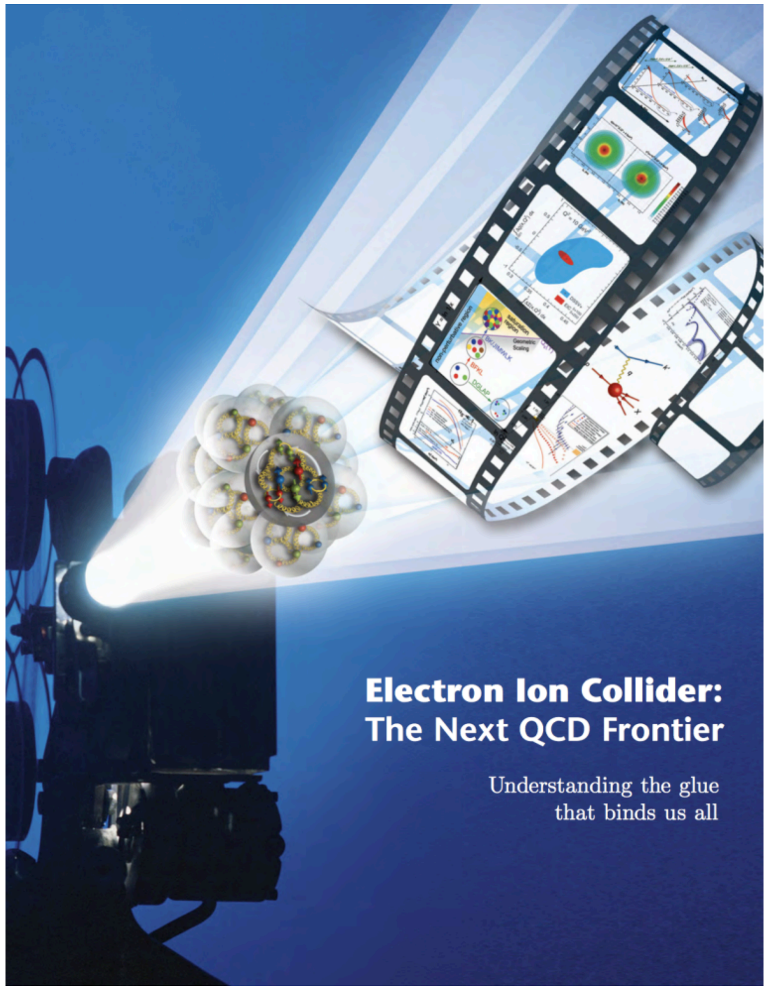
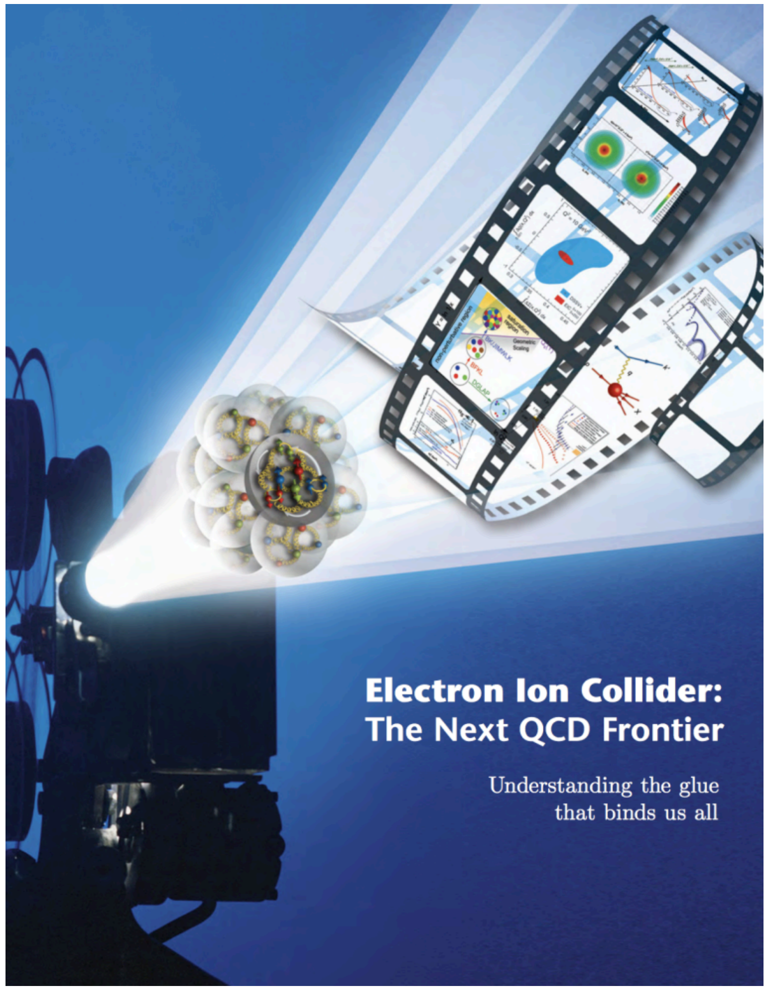
- Summary & Outlook

- Studies of jets have been used as an important probe to test the fundamental properties of hadrons.

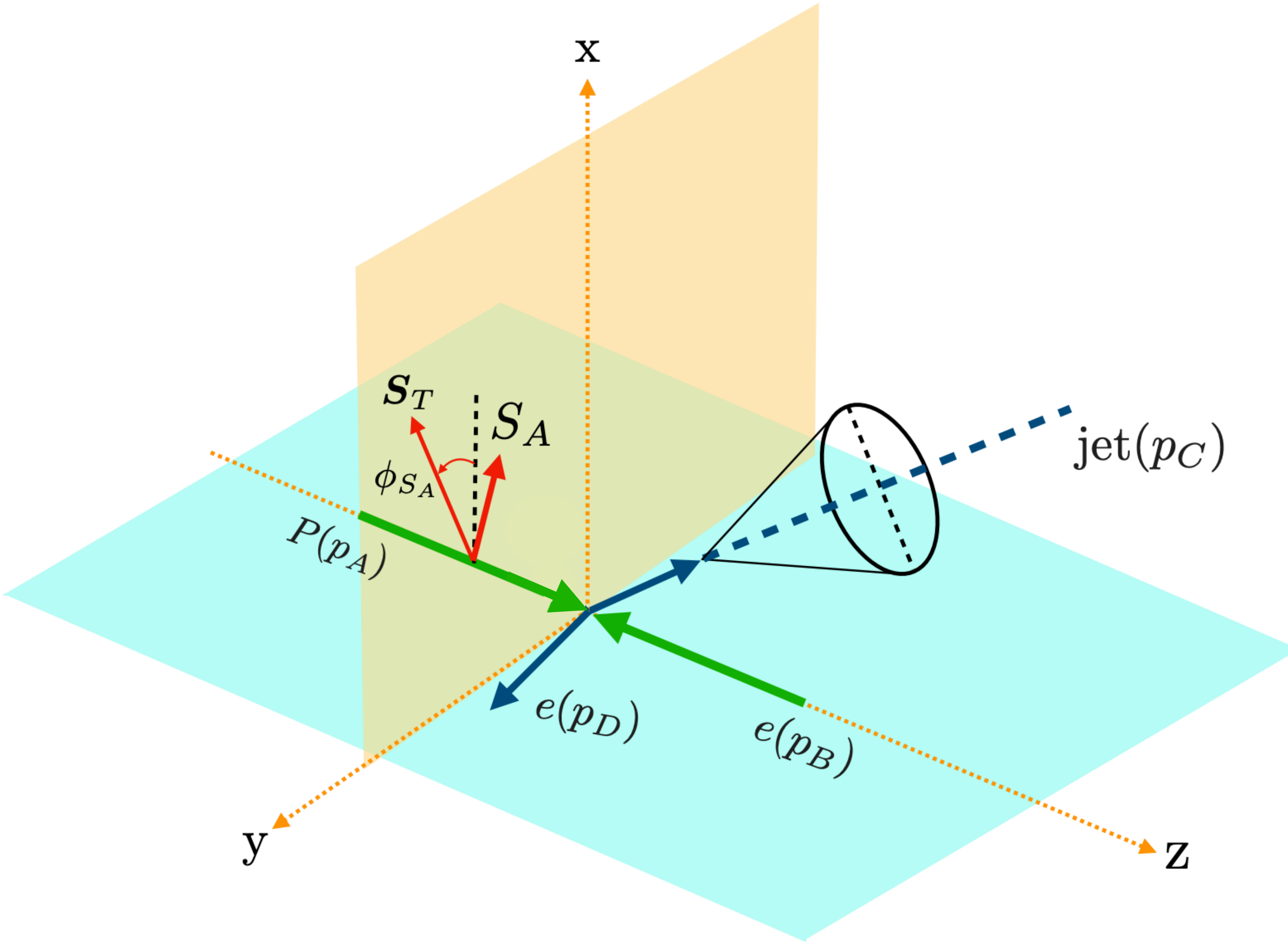


- Studies of jets have been used as an important probe to test the fundamental properties of hadrons.

- The advent of the Electron-Ion Collider (EIC) with polarized beams unlock the full potential of jets for probing 3D structure of the nucleon and nuclei (encoded in **TMDPDFs**).



- All the possible spin asymmetries in back-to-back electron-jet production, $ep \to e + \text{jet} + X$, at the EIC



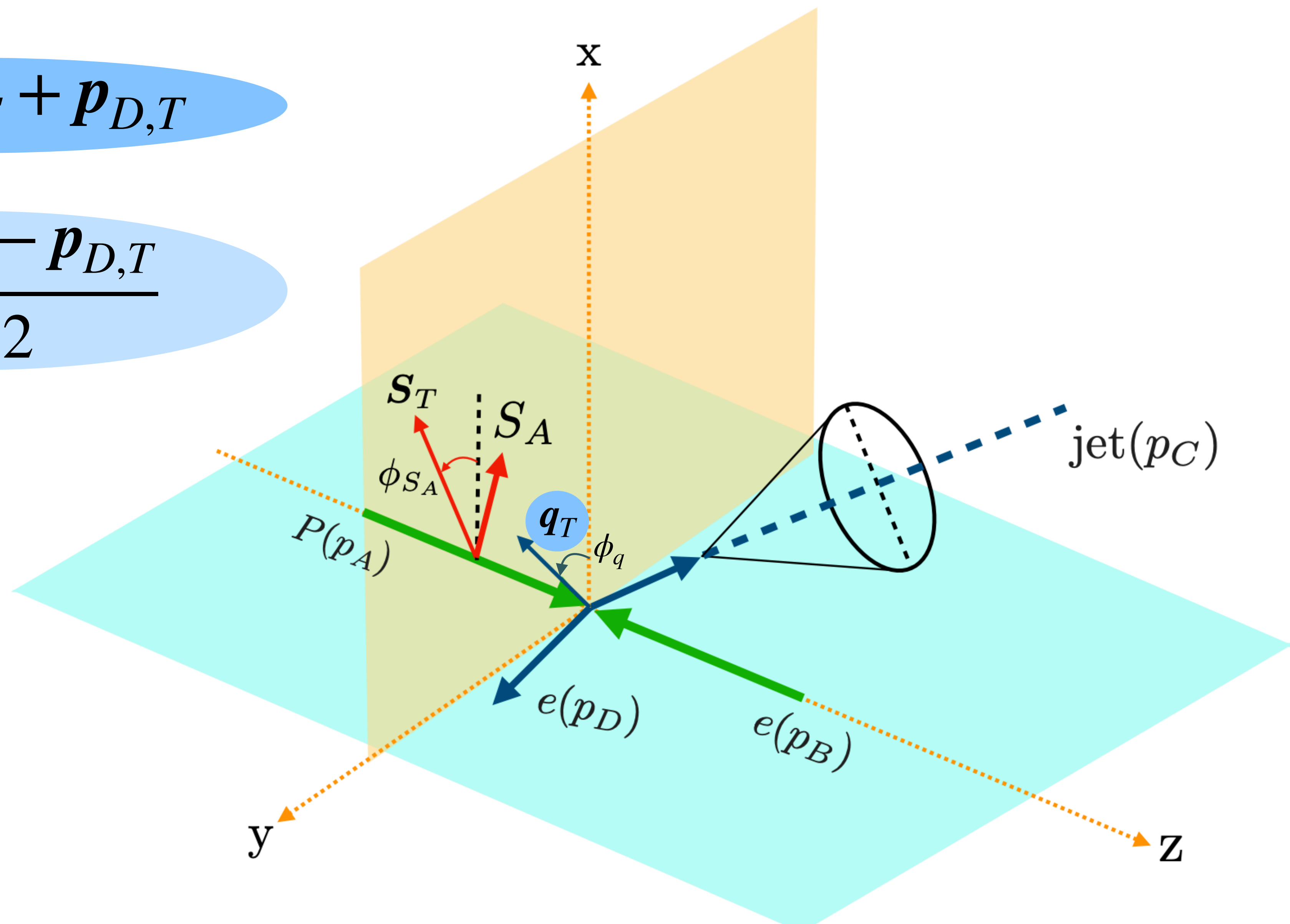
$$ep \to e + \text{jet} + X$$

Spin asymmetry

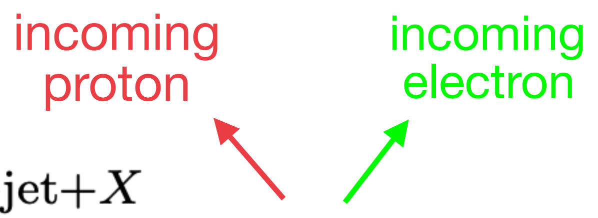
- All the possible spin asymmetries in back-to-back electron-jet production, $ep \to e + \text{jet} + X$, at the EIC

$$\boldsymbol{q}_T = \boldsymbol{p}_{C,T} + \boldsymbol{p}_{D,T}$$

$$\boldsymbol{p}_T = \frac{\boldsymbol{p}_{C,T} - \boldsymbol{p}_{D,T}}{2}$$



$$ep \to e + \text{jet} + X$$



$$\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+\text{jet}+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T} = F_{UU} + \lambda_p\lambda_e F_{LL}$$

$$+ S_T\left\{\sin(\phi_q - \phi_{S_A})F_{TU}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e\cos(\phi_q - \phi_{S_A})F_{TL}^{\cos(\phi_q-\phi_{S_A})}\right\}$$

y_J : the rapidity of the jet

$$\boldsymbol{q}_T = \boldsymbol{p}_{C,T} + \boldsymbol{p}_{D,T}$$

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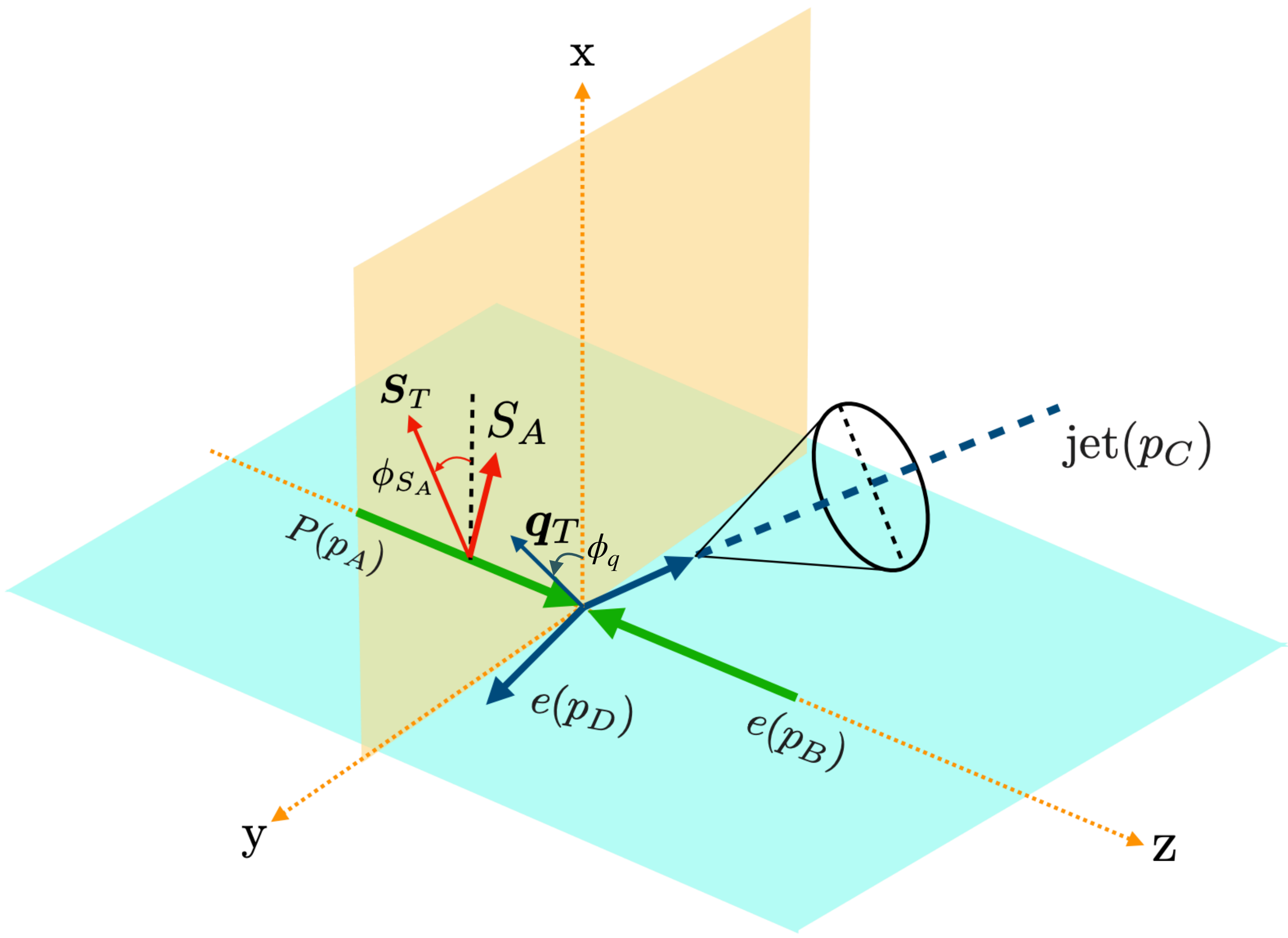

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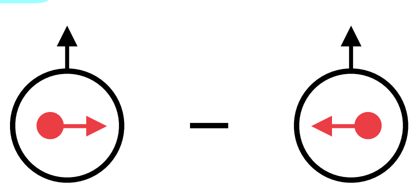
$$+ S_T \left\{ \sin(\phi_q - \phi_{S_A}) F_{TU}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL}^{\cos(\phi_q-\phi_{S_A})} \right\}$$

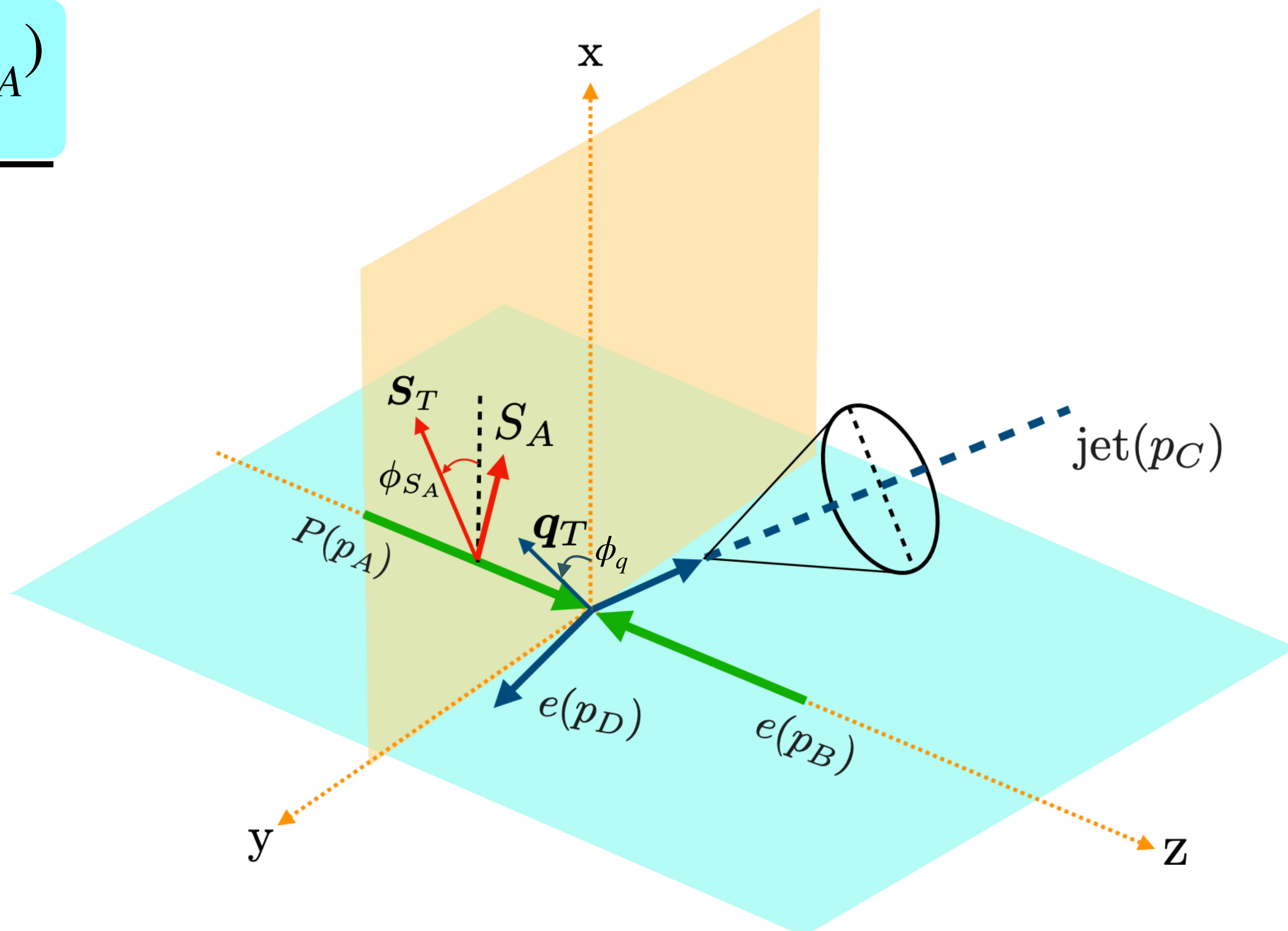
$$A_{TL}^{\cos(\phi_q-\phi_{S_A})} = \frac{F_{TL}^{\cos(\phi_q-\phi_{S_A})}}{F_{UU}}$$

incoming proton (T) — incoming electron (L)

$$F_{UU} \sim f_1^q \otimes J_q$$

$$F_{TL}^{\cos(\phi_q-\phi_{S_A})} \sim g_{1T}^q \otimes J_q$$





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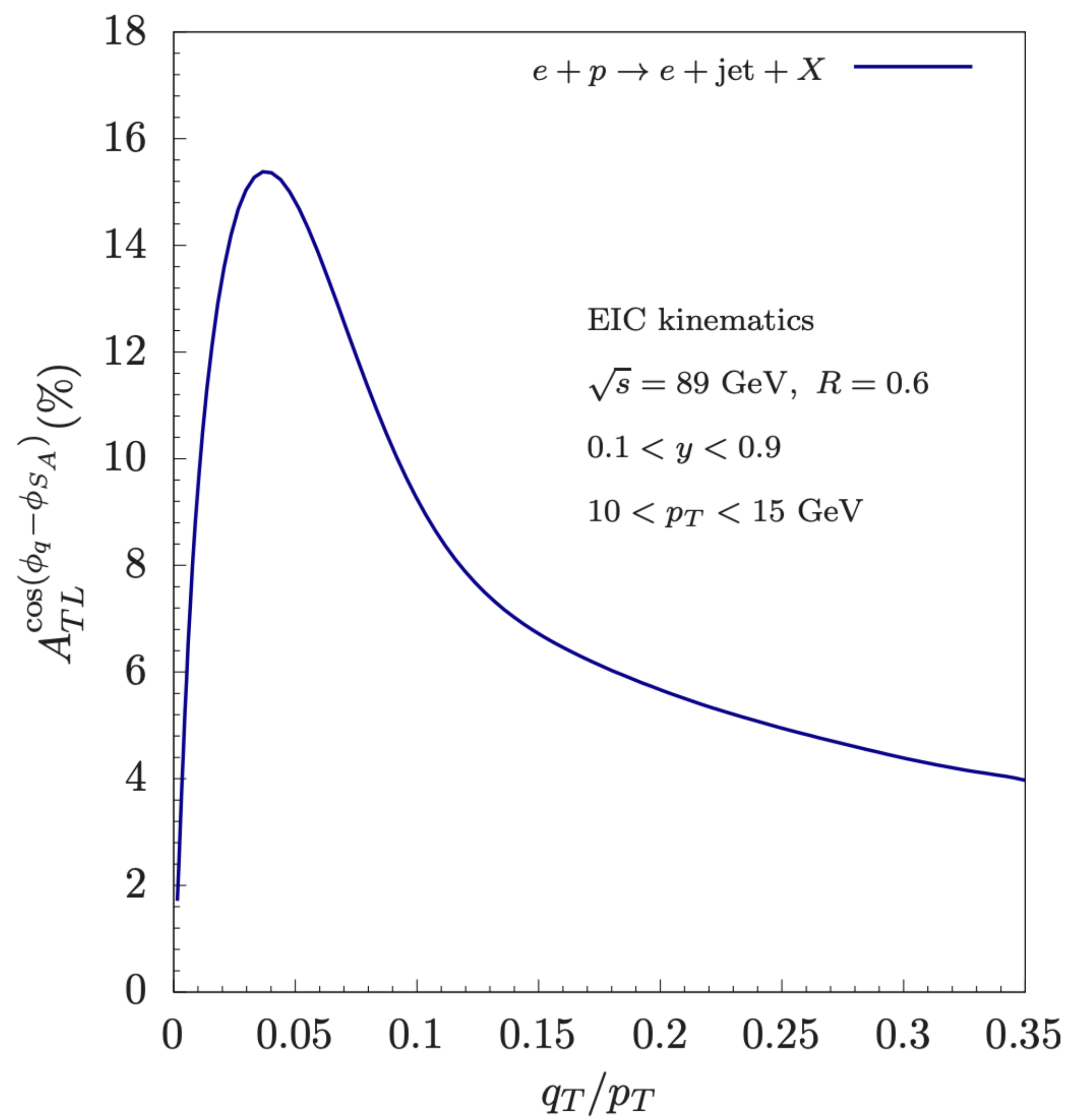

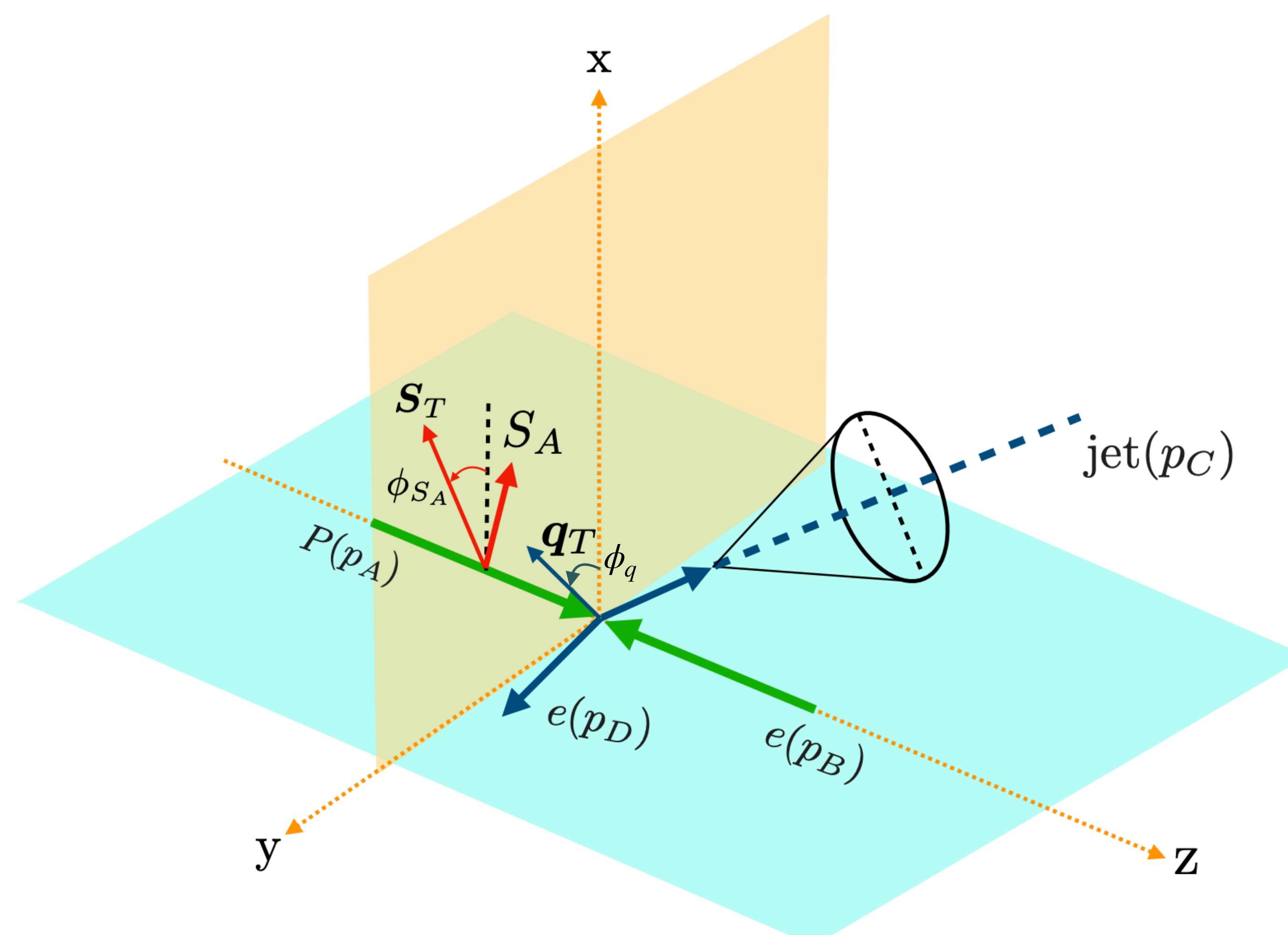

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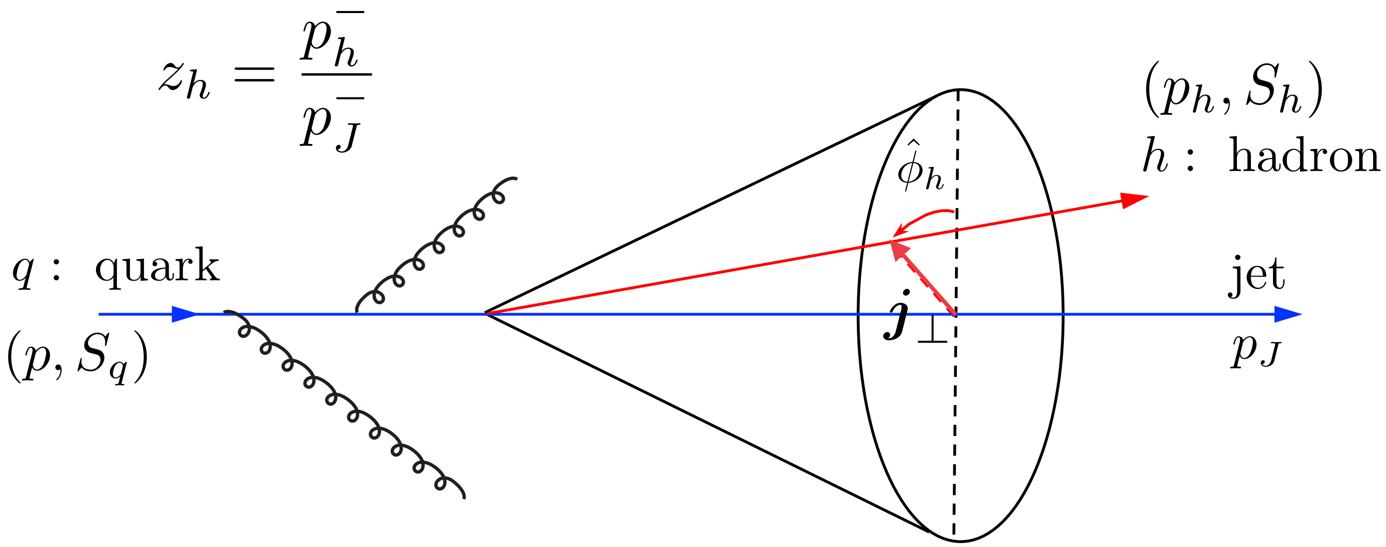
TMDJFFs

$$p(p_A, S_A) + e(p_B) \to (\text{jet}(\eta_J, p_{JT}, R)\ h(z_h, j_\perp, S_h)) + e(p_D) + X$$



Leading TMDFFs

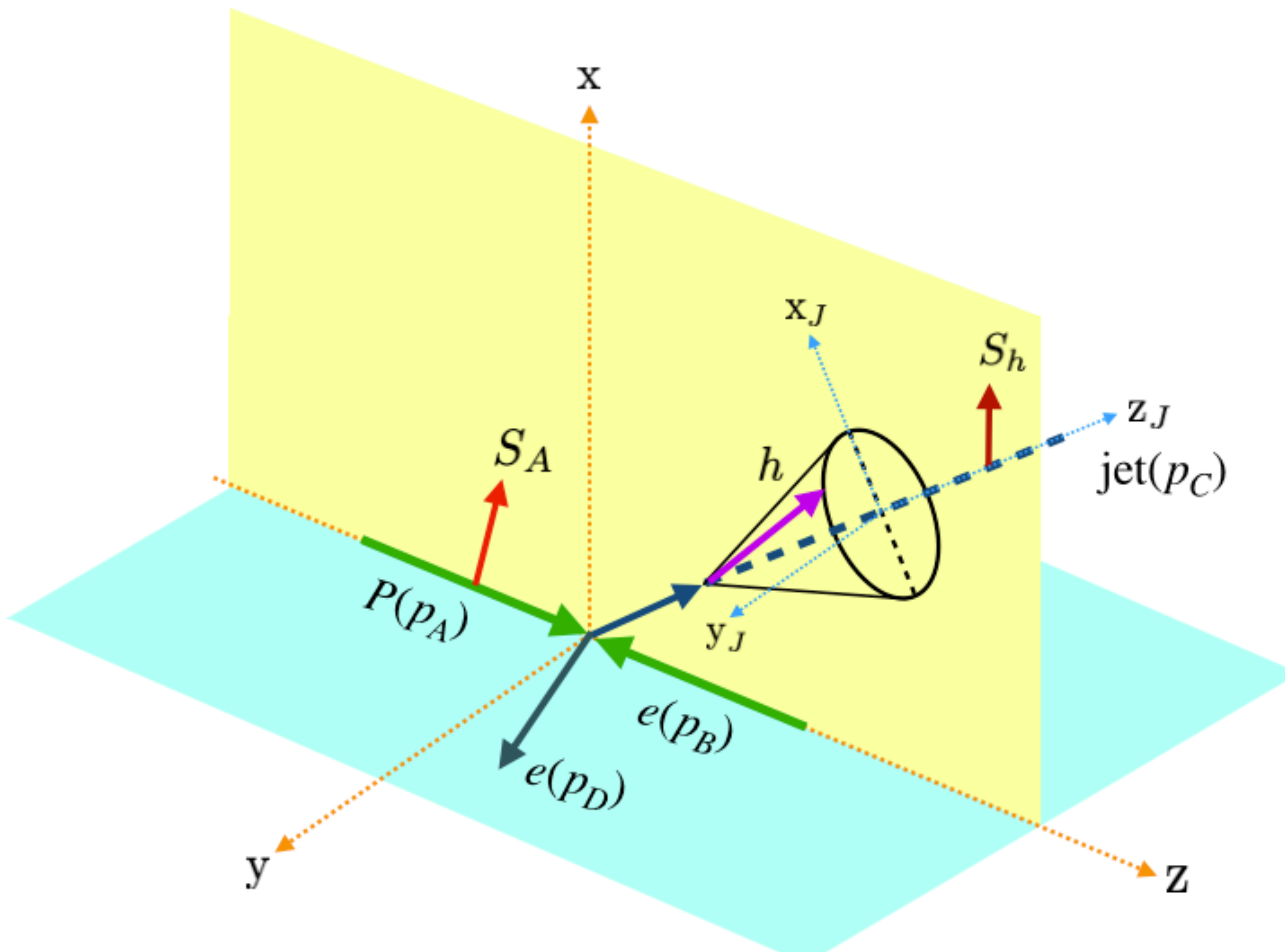
h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1\mathrm{L}}^\perp$
T	$D_{1\mathrm{T}}^\perp$	$G_{1\mathrm{T}}$	$H_1, H_{1\mathrm{T}}^\perp$

Leading TMDJFFs

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\ h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{h/q}$
T	$\mathcal{D}_{1T}^{\perp\ h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}, \mathcal{H}_{1T}^{\perp\ h/q}$

Transverse momentum dependent FFs/JFFs for quarks. Here U, L, and T represent unpolarized, longitudinally, and transversely polarized state

- All the possible spin asymmetries in back-to-back electron-jet production with jet fragmentation process, $ep \to e + \text{jet}(h) + X$, at the future electron ion collider (EIC)



$$ep \to e + \text{jet}(h) + X$$

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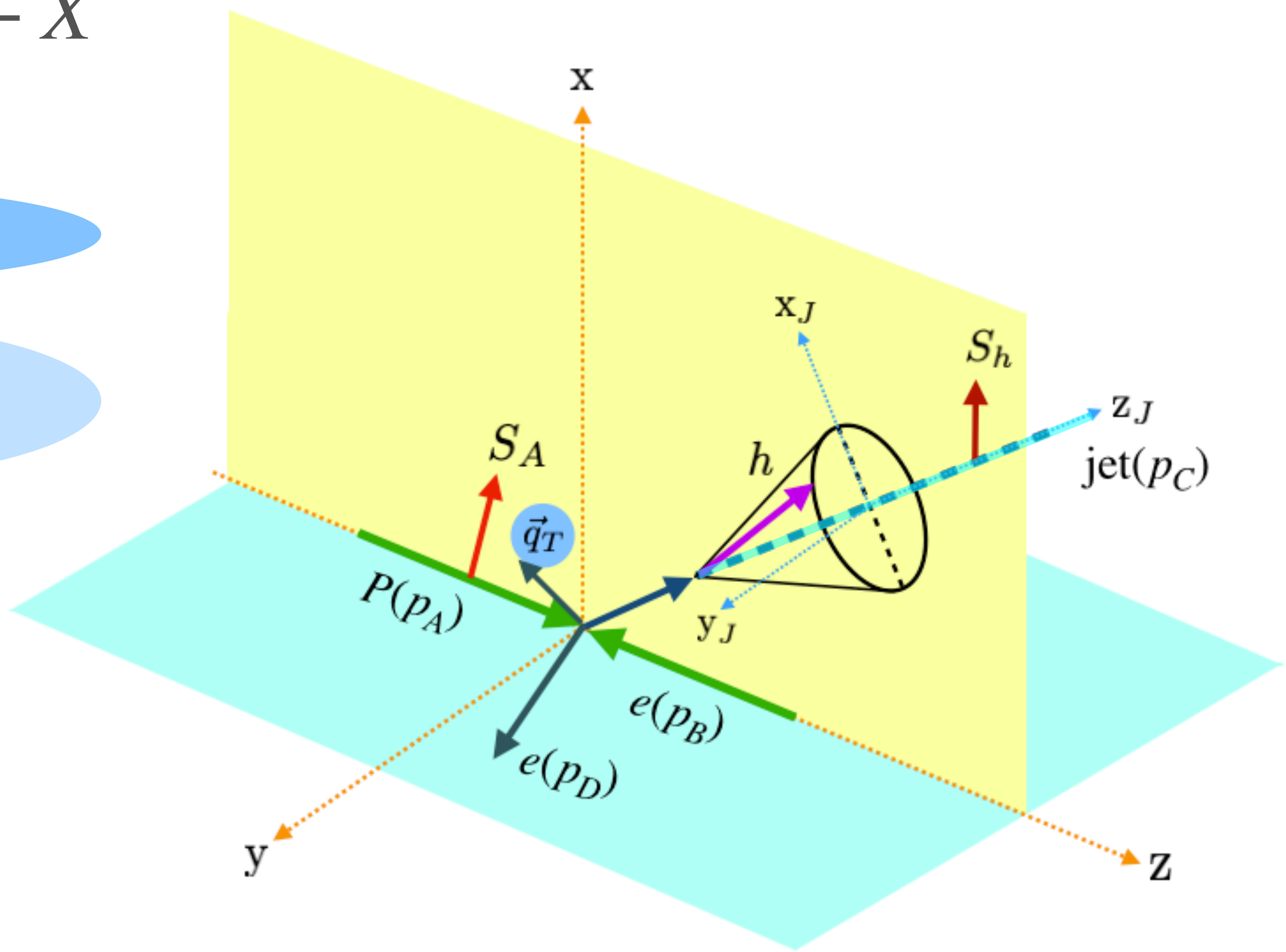
$$\boldsymbol{p}_T = \frac{\boldsymbol{p}_{C,T} - \boldsymbol{p}_{D,T}}{2}$$

$$z_h = \frac{p_h^-}{p_J^-}$$

$$\boldsymbol{j}_\perp$$

$$\boldsymbol{S}_{h\perp}$$

$$\boldsymbol{S}_T$$



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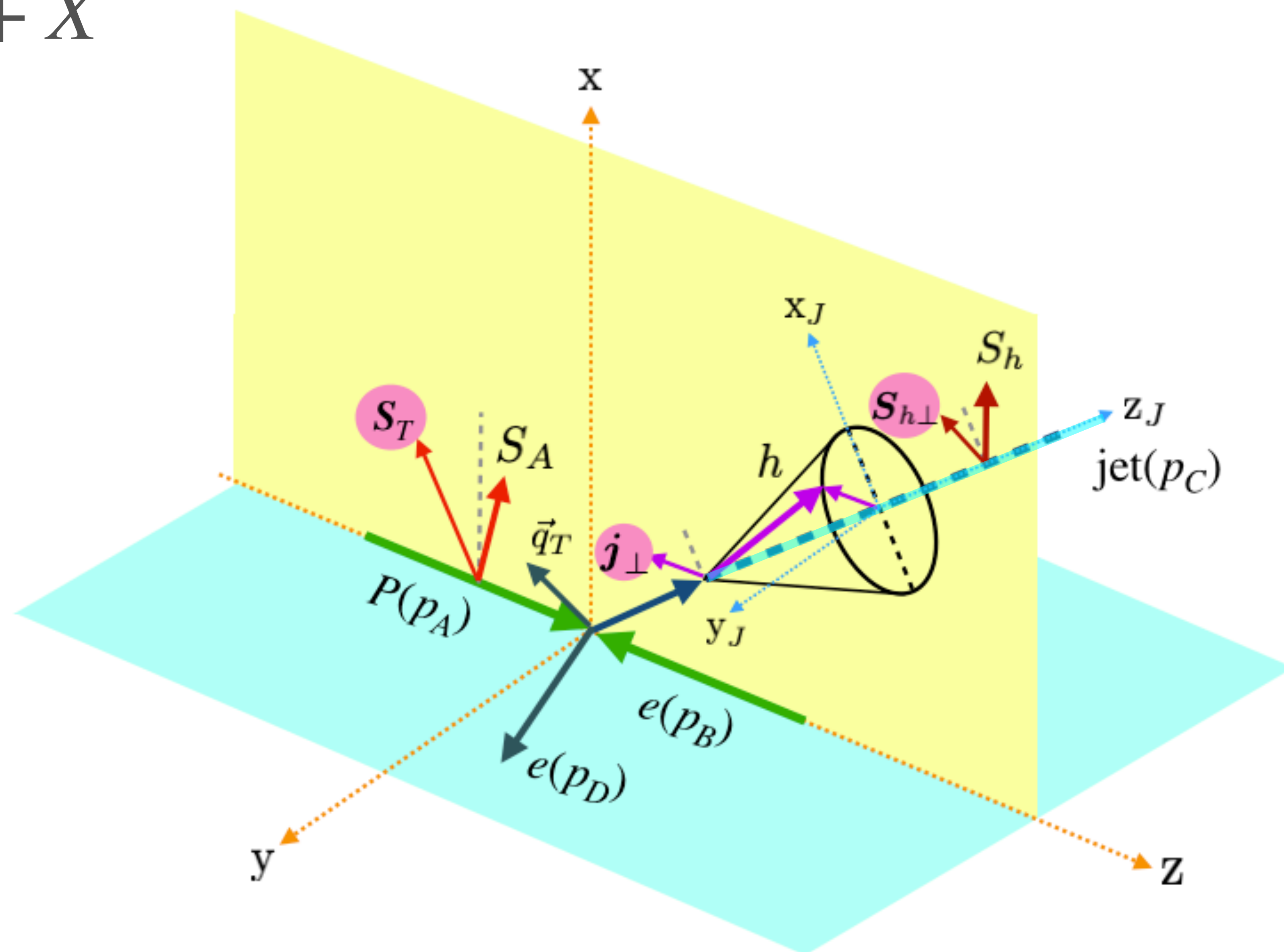

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ϕ_q $\quad \boldsymbol{q}_T = \boldsymbol{p}_{C,T} + \boldsymbol{p}_{D,T}$

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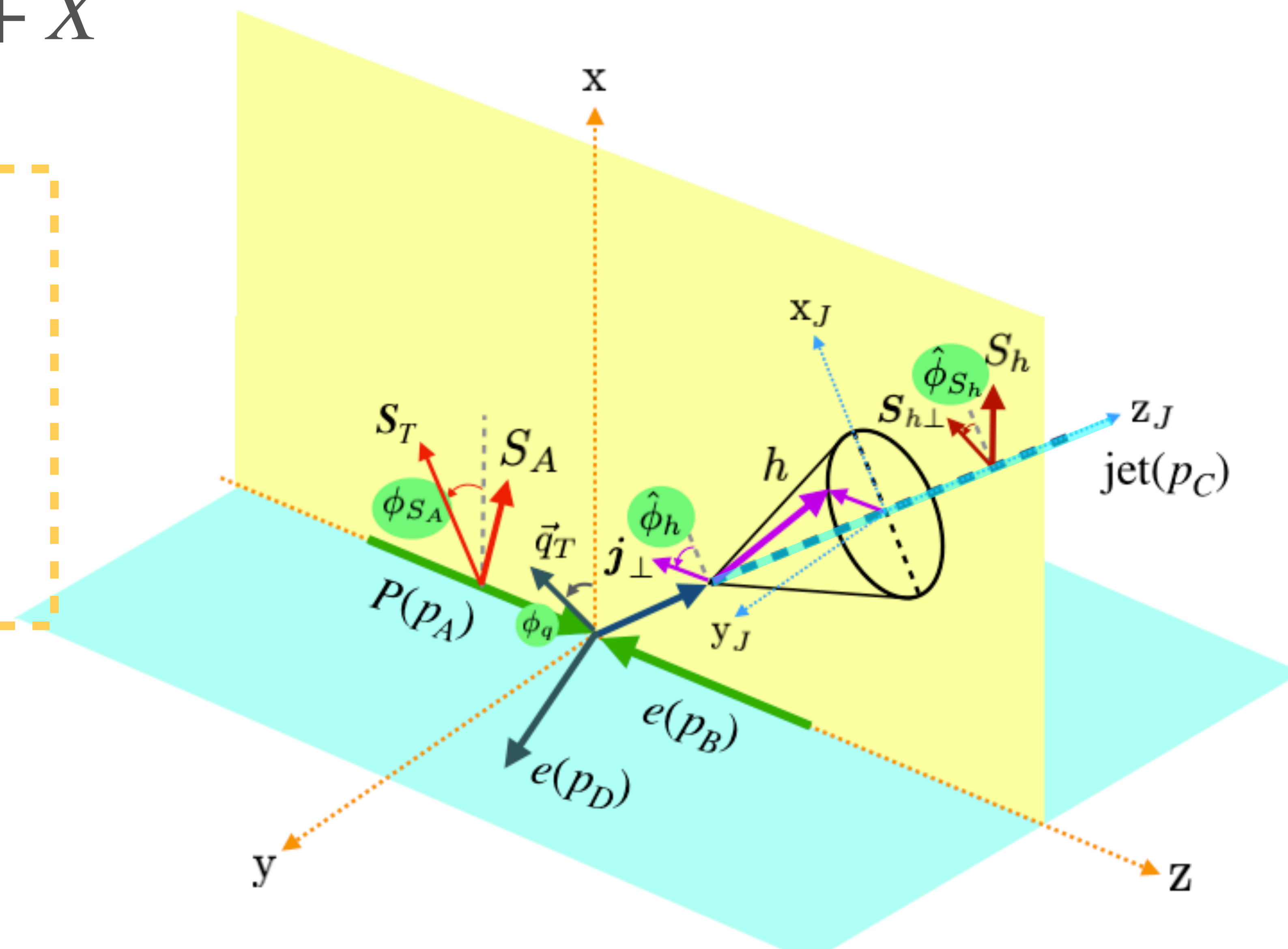
ϕ_{S_A} $\quad \boldsymbol{S}_T$

$$z_h = \frac{p_h^-}{p_J^-}$$

$\hat{\phi}_h$ $\quad \boldsymbol{j}_\perp$

$\hat{\phi}_{S_h}$ $\quad \boldsymbol{S}_{h\perp}$



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ϕ_{S_A} $$\boldsymbol{S}_T$$

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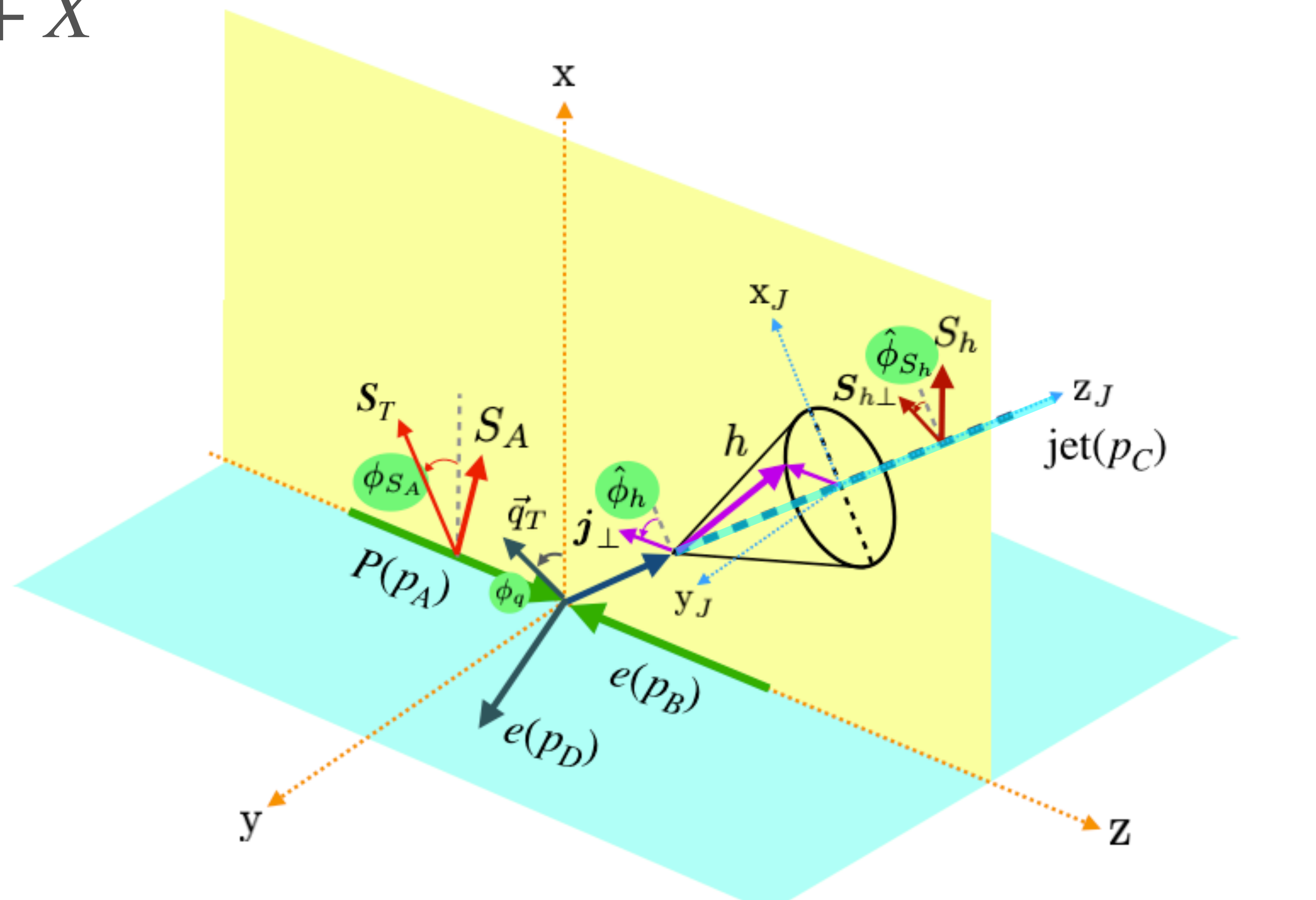
$\hat{\phi}_h$ $$\boldsymbol{j}_\perp$$

$\hat{\phi}_{S_h}$ $$\boldsymbol{S}_{h\perp}$$



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$$ep \to e + \text{jet}(h) + X$$

TMDPDFs

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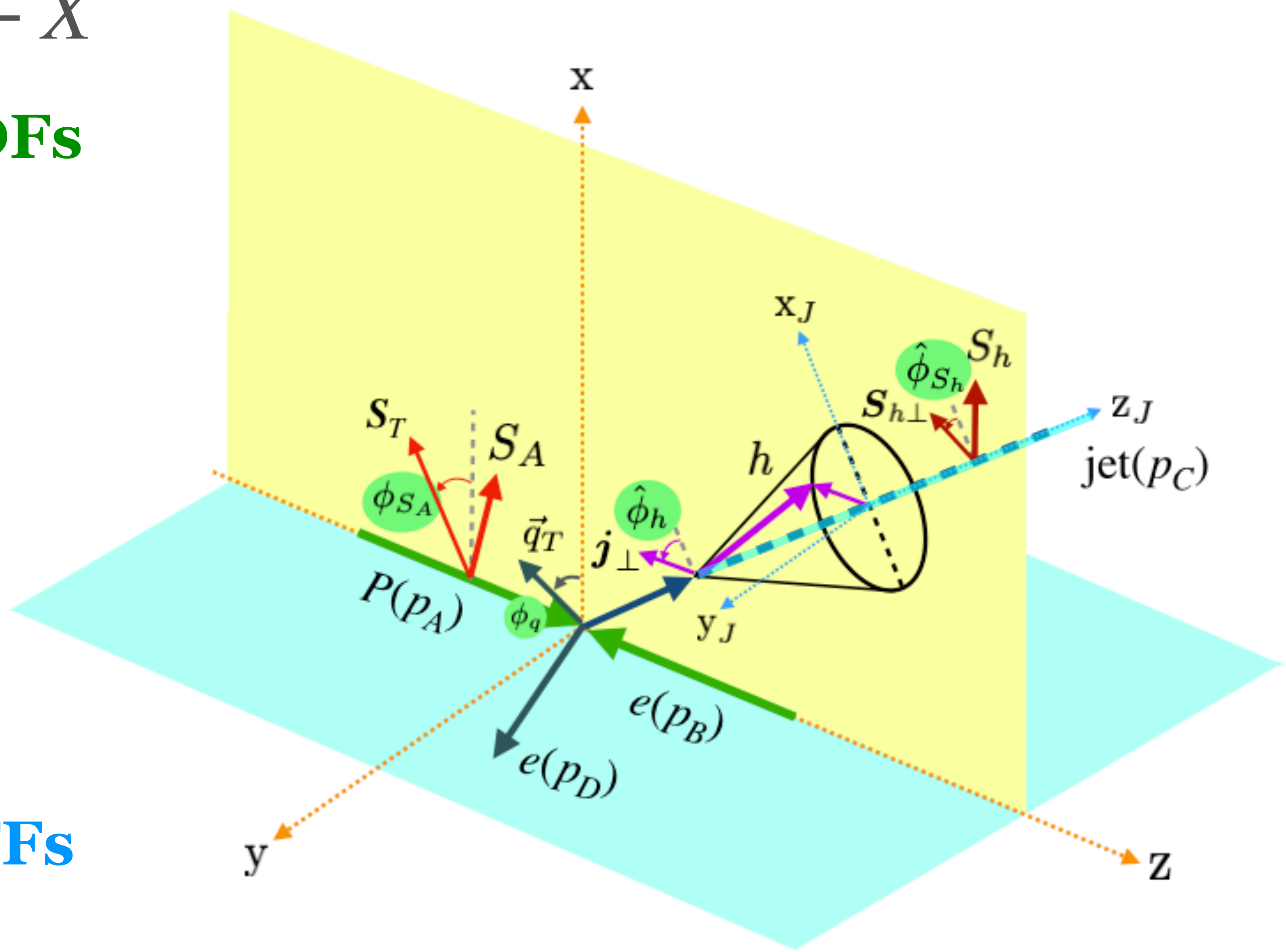
ϕ_{S_A} $$\boldsymbol{S}_T$$

$$z_h = \frac{p_h^-}{p_J^-}$$

$\hat{\phi}_h$ $\boldsymbol{j}_\perp$ **TMDJFFs**

$\hat{\phi}_{S_h}$ $$\boldsymbol{S}_{h\perp}$$



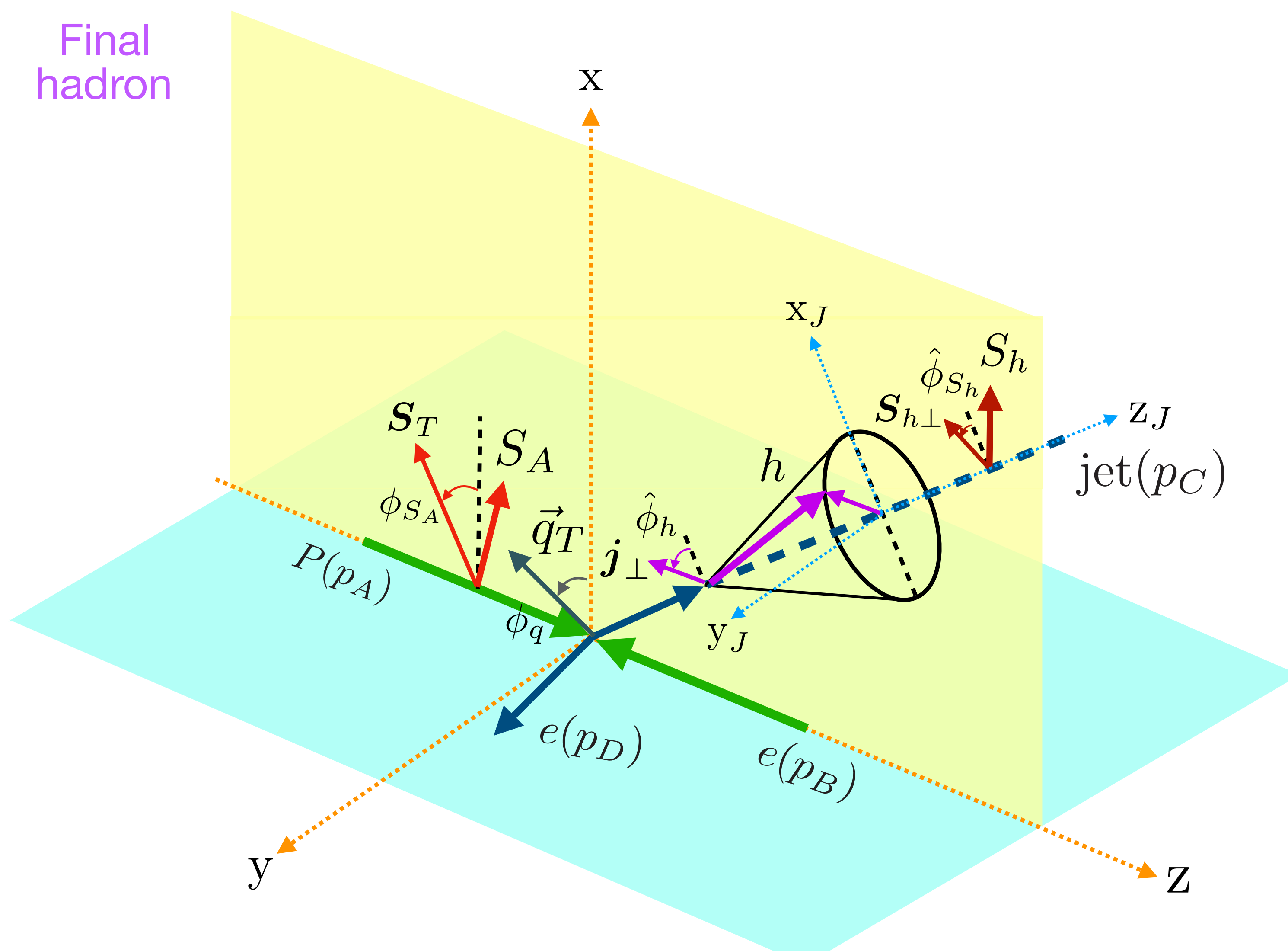
Spin asymmetry

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{\sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots\right.$$

$$A_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}$$

$$ep \to e + \text{jet}(h^\uparrow) + X$$

incoming proton | incoming electron | Final hadron



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$$A_{UU,T}^{\sin(\hat\phi_h - \hat\phi_{S_h})} = \frac{F_{UU,T}^{\sin(\hat\phi_h - \hat\phi_{S_h})}}{F_{UU,U}}$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

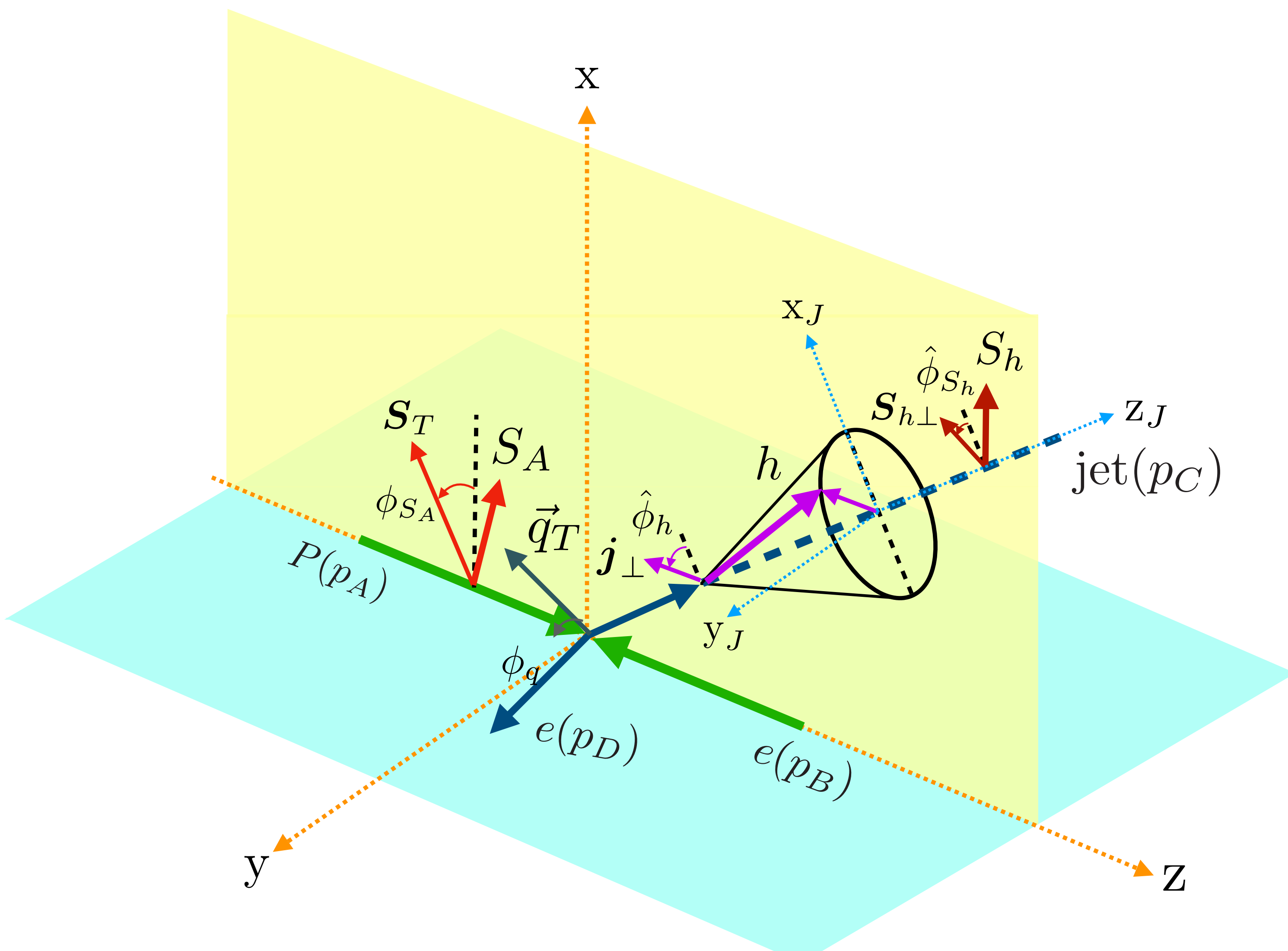
$$F_{UU,T}^{\sin(\hat\phi_h - \hat\phi_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\, h/q}$$

Unpolarized TMDPDF

match to

Polarizing TMDFF

arXiv: [2003.04828]



$$ep \to e + \text{jet}(\Lambda^\uparrow) + X$$

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\, h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{\perp\, h/q}$
T	$\mathcal{D}_{1T}^{\perp\, h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q},\ \mathcal{H}_{1T}^{\perp\, h/q}$

TMDJFFs for quarks.

Spin asymmetry

$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots \right.$$

$$A_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

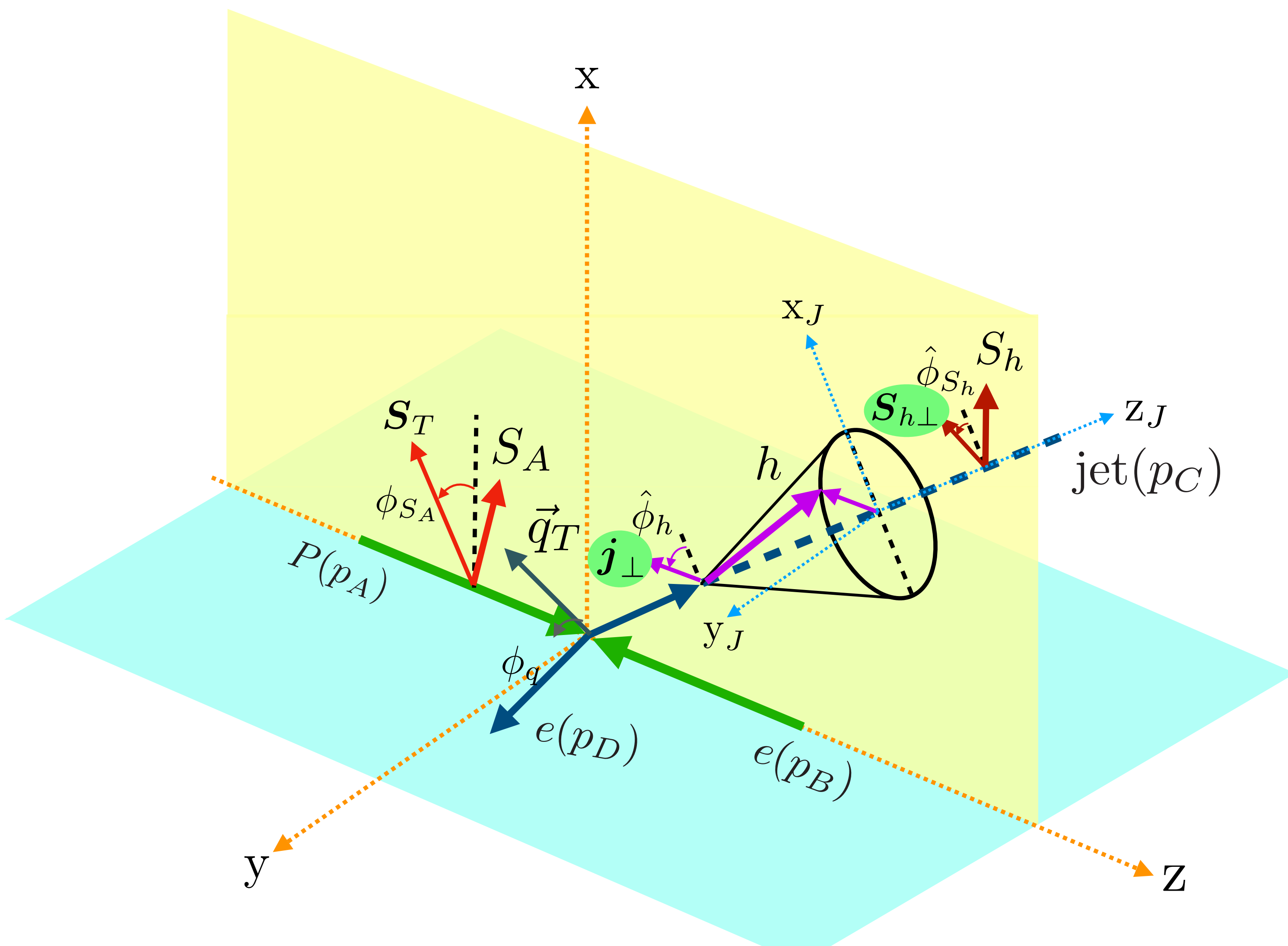
$$F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\, h/q}$$

Unpolarized TMDPDF

match to

Polarizing TMDFF

arXiv: [2003.04828]



$$ep \to e + \text{jet}(\Lambda^\uparrow) + X$$

h\ q	U	L	T
U	$\mathcal{D}_1^{h/q}$		$\mathcal{H}_1^{\perp\, h/q}$
L		$\mathcal{G}_{1L}^{h/q}$	$\mathcal{H}_{1L}^{\perp\, h/q}$
T	$\mathcal{D}_{1T}^{\perp\, h/q}$	$\mathcal{G}_{1T}^{h/q}$	$\mathcal{H}_1^{h/q}$, $\mathcal{H}_{1T}^{\perp\, h/q}$

TMDJFFs for quarks.

Spin asymmetry

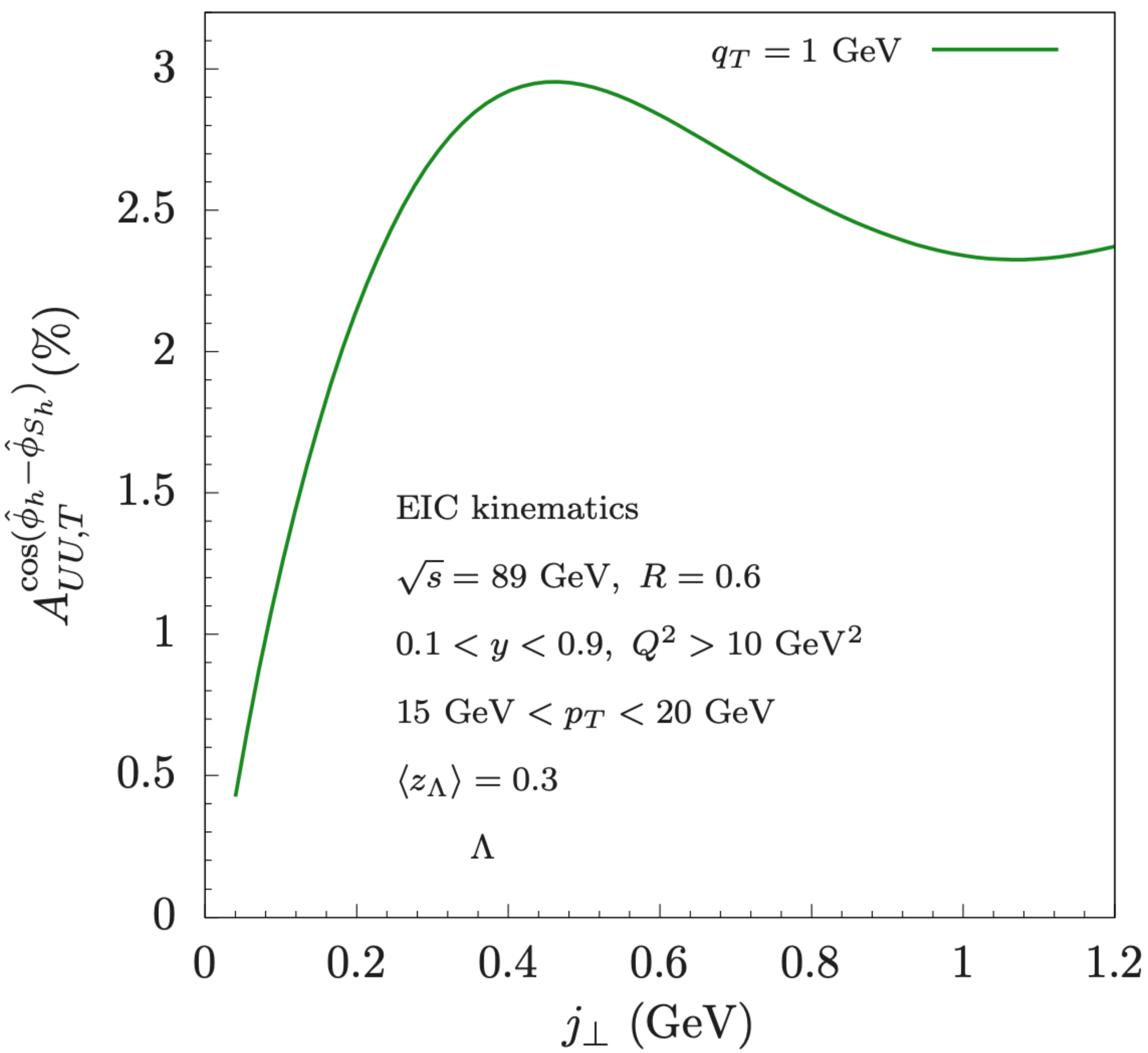
$$\frac{d\sigma}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cdots + S_{h\perp}\left\{\sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} + \cdots\right.$$

$$F_{UU,U} \sim f_1 \otimes \mathcal{D}_1^{h/q}$$

$$F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} \sim f_1 \otimes \mathcal{D}_{1T}^{\perp\, h/q}$$

q_T $\quad j_\perp$

$$A_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_h - \hat{\phi}_{S_h})}}{F_{UU,U}}$$

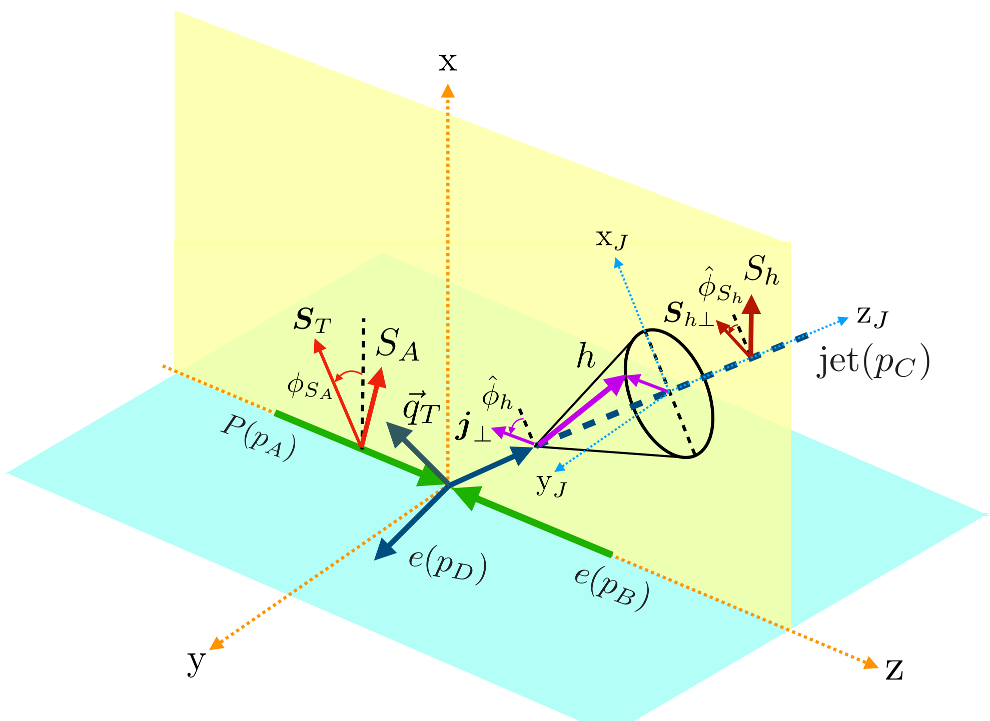


Summary & Outlook

- In summary, we study the azimuthal anisotropy for the azimuthal angle difference $\phi_{qJ} = \phi_q - \phi_J$ for EIC in the back-to-back lepton-jet production in *ep* collisions.

- Also we have developed the theoretical framework for all spin asymmetries in back-to-back e + jet and e + jet(h) productions. Sizable asymmetry can be measured with EIC kinematics.

- These observables can open new and exciting opportunities for studying lepton-jet correlations and spin-dependent hadron structures in future experiments.



Thank you

Thank you

$$
\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}
$$

$$
+ \Lambda_p \Big\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \Big\}
$$

$$
+ S_T \Big\{ \sin(\phi_q - \phi_{S_A}) F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL,U}^{\cos(\phi_q-\phi_{S_A})}
$$

$$
+ \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \Big\}
$$

$$
+ \Lambda_h \Big\{ \lambda_e F_{UL,L} + \sin(\hat{\phi}_h - \phi_q) F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \Lambda_p \Big[F_{LU,L} + \cos(\hat{\phi}_h - \phi_q) F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)} \Big]
$$

$$
+ S_T \Big[\cos(\phi_q - \phi_{S_A}) F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e \sin(\phi_q - \phi_{S_A}) F_{TL,L}^{\sin(\phi_q-\phi_{S_A})}
$$

$$
+ \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \Big] \Big\}
$$

$$
+ S_{h\perp} \Big\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})}
$$

$$
+ \sin(\hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)}
$$

$$
+ \Lambda_p \Big[\cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})}
$$

$$
+ \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \Big]
$$

$$
+ S_T \Big[\cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})}
$$

$$
+ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_q - \phi_{S_A}) F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_q-\phi_{S_A})}
$$

$$
+ \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_q - \phi_{S_A})) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_q-\phi_{S_A})}
$$

$$
+ \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})}
$$

$$
+ \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})}
$$

$$
+ \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)}
$$

$$
+ \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \Big] \Big\}
$$