EIC User Group Early Career Workshop 2022

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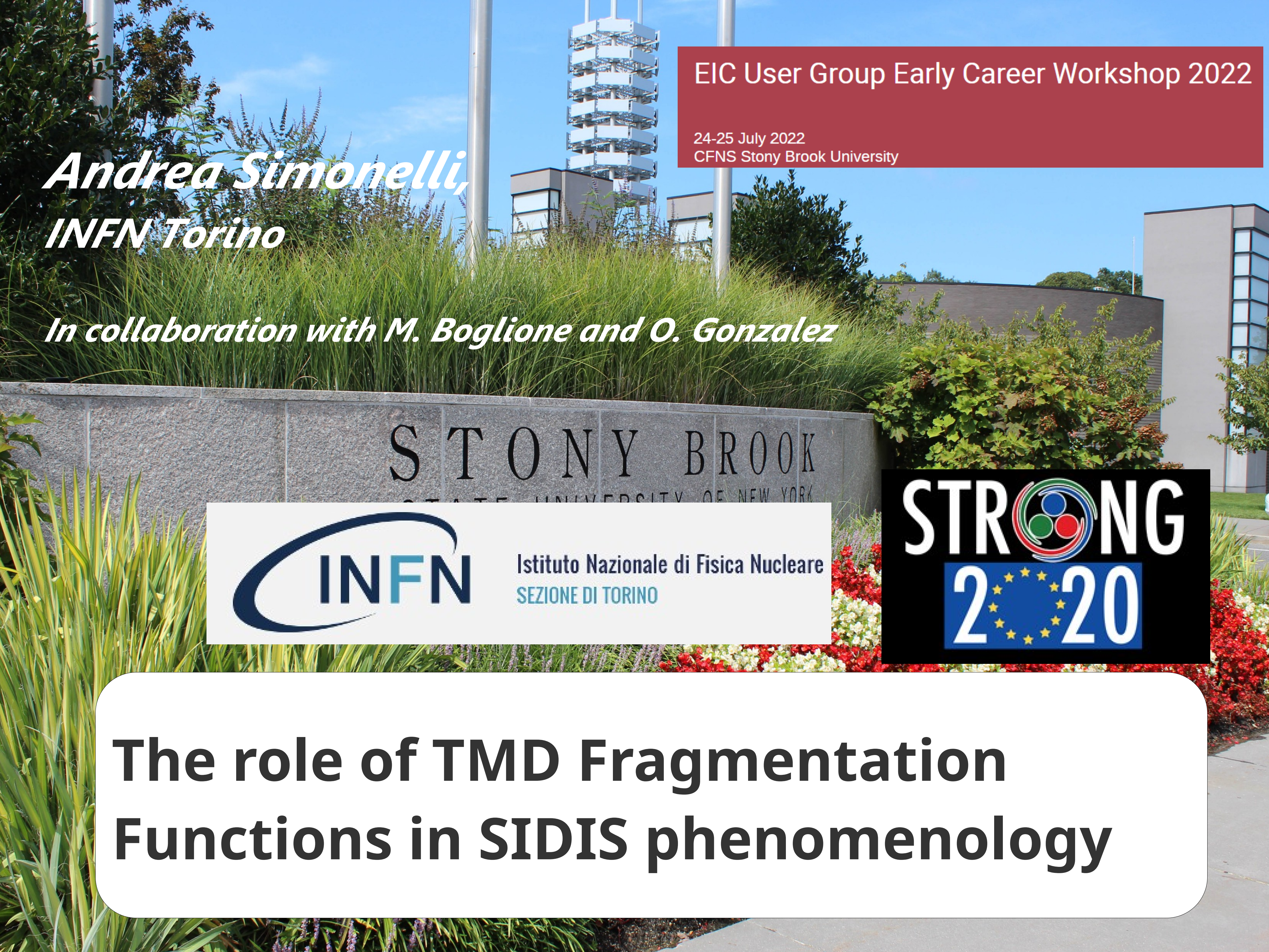
CFNS Stony Brook University

Andrea Simonelli,
INFN Torino

In collaboration with M. Boglione and O. Gonzalez



Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

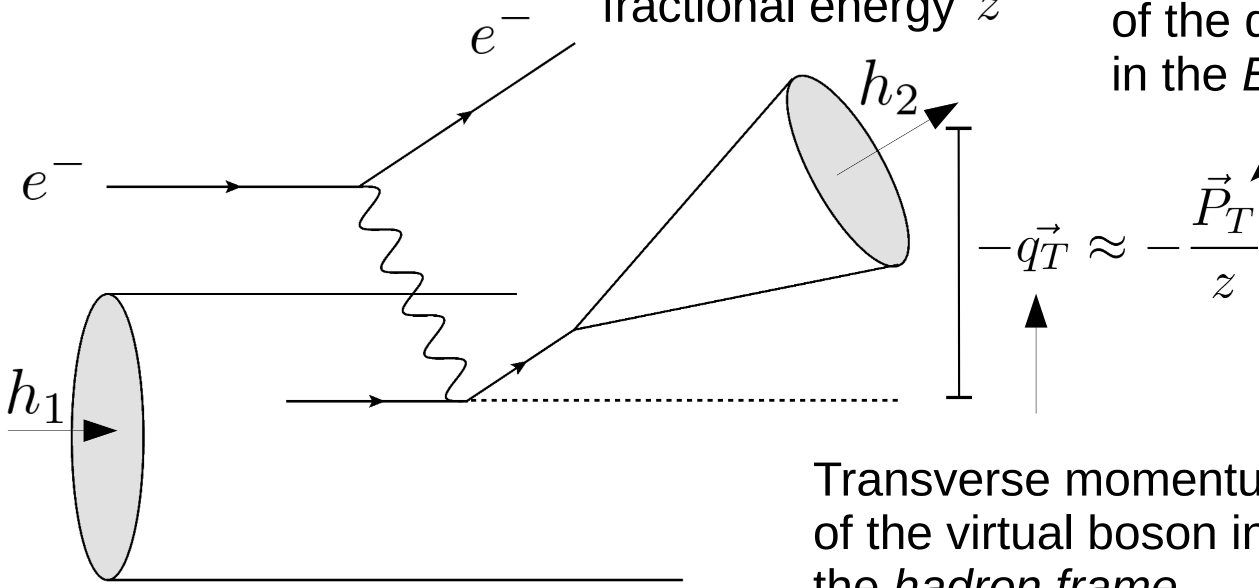
The role of TMD Fragmentation Functions in SIDIS phenomenology

SIDIS process

Detected hadron with fractional energy z

Transverse momentum of the detected hadron in the *Breit frame*

Target hadron (proton, light ion…). A fraction x of its total momentum is carried by the struck quark



Transverse momentum of the virtual boson in the *hadron frame*

The *factorization properties* are well-known:

2 different kinematic regions ⟷ 2 different factorization theorems

TMD factorization

$$q_T \ll Q$$

$$d\sigma \sim H \int \frac{d^2\vec{b}_T}{2\pi^2} e^{-i\vec{q}_T \cdot \vec{b}_T} F(x,\, b_T)\, D(z,\, b_T)$$

collinear factorization

$$q_T \gtrsim Q$$

$$d\sigma \sim d\widehat{\sigma}(q_T) \otimes f \otimes d[x,\, z]$$

By using **evolution equations** and the **b* prescription**, the TMDs can be written as:

$$b_T^\star = b_T \left(1 + \frac{b_T^2}{b_{MAX}^2}\right)^{-1/2}$$

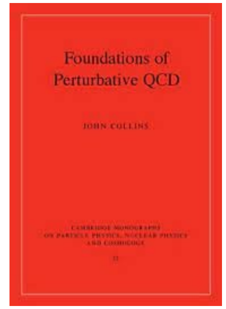
$$\log\left(\frac{\mu\, b_T}{2e^{-\gamma_E}}\right)$$

$$\widetilde{C}_{h,j}(z,\, a_S(\mu), L_b, \log\frac{\sqrt{\zeta}}{\mu};\, b_T) = \sum_k \int_z^1 \frac{d\rho}{\rho}\left[\rho^2\, \widetilde{\mathcal{C}}_{k/j}(\rho, a_S(\mu_b^\star))\right] \boldsymbol{c_{h/k}(z/\rho, \mu_b^\star)}$$
$$\times\ \exp\left\{\widetilde{K}_\star\left(a_S(\mu_b^\star)\right)\, \log\frac{\sqrt{\zeta}}{\mu_b^\star} + \int_{\mu_b^\star}^{\mu} \frac{d\mu'}{\mu'}\, \gamma_C\left(a_S(\mu'),\, \log\left(\frac{\sqrt{\zeta}}{\mu'}\right)\right)\right\}$$
$$\times\ \boldsymbol{M_C(z, b_T;\, j, h)}\, \exp\left\{-\boldsymbol{g_K(b_T)}\, \log\frac{\sqrt{\zeta}}{M_h}\right\}.$$

Collinear analogues (generally not extracted in TMD analyses and get from other extractions – e.g. LHAPDF database)

Extracting a TMD means to extract these two **NP** functions, which (re)organize the b_T dependence outside the logs

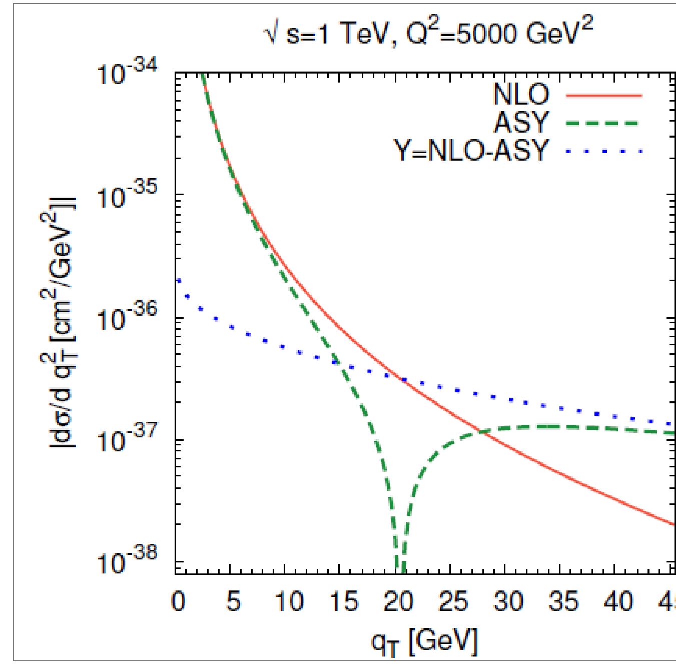
The theory is well understood….



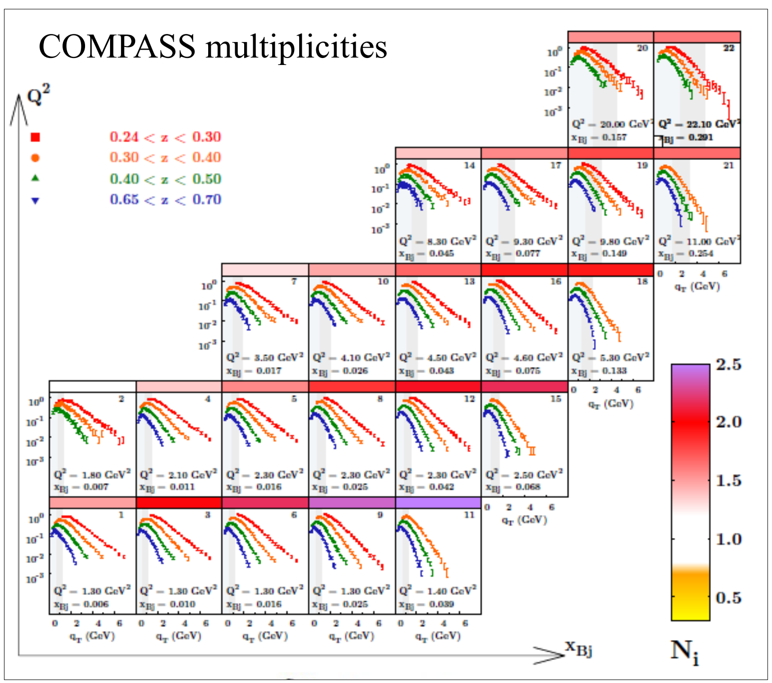
…but the phenomenology is very complicated!

- Matching between small and large q_T regions

- Normalization issue



Boglione, Gonzalez, Prokudin, JHEP 02 (2015) 095



SIDIS (TMD) phenomenology is intrinsically difficult also because it requires a **simultaneous** extraction of a TMD PDF and a TMD FF

From Osvaldo Gonzalez's talk @ Sar Wors 2019

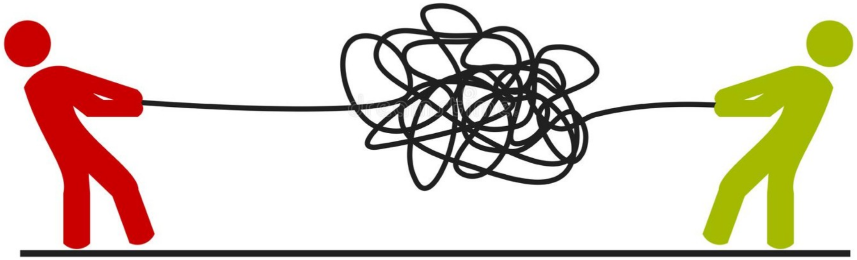
More generally, phenomenological analyses involving the **three benchmark processes** require the simultaneous extraction of two TMDs:

DY	SIDIS	DIA
$h_1\, h_2 \to e^+\, e^- + X$	$e\, h_1 \to e\, h_2 + X$	$e^+\, e^- \to h_1\, h_2 + X$
Allows extraction of distribution functions	Allows extraction of distribution and fragmentation functions	Allows extraction of fragmentation functions

$$\frac{d\sigma}{dq_T\, d\xi_1\, d\xi_2} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, C_1(\xi_1,\, b_T)\, C_2(\xi_2,\, b_T)$$

First Hadron

Second Hadron



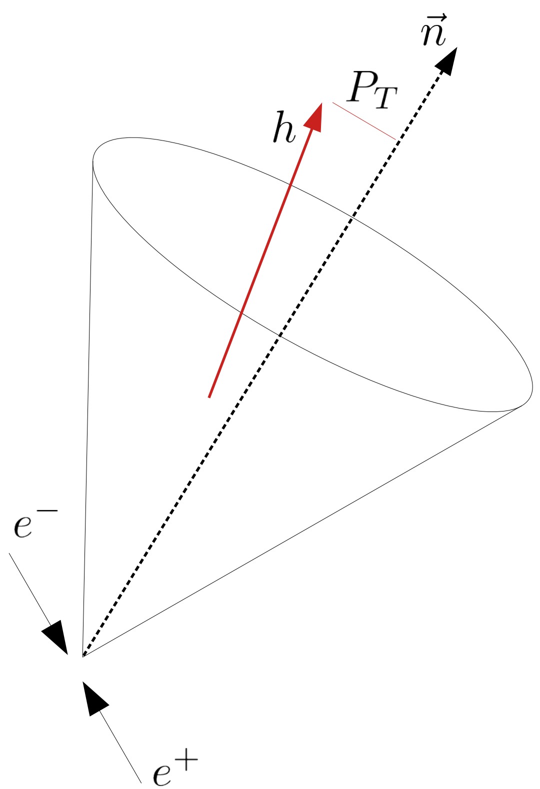
It would be much easier if we could exploit a process in which a **single TMD** appears.

Clearly, it cannot be one of the benchmark processes!

The factorization is not granted

BELLE collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

At the beginning of 2019, BELLE collaboration have provided data for SIA$^{\mathrm{thr}}$, i.e. $e^+e^- \to h\,X$, sensitive to:



$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i} \cdot \widehat{n}|}{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

Naively, it is the **cleanest way to access a TMD** (FF). However, relating it with those encountered in standard TMD factorization is subtle and non-trivial.

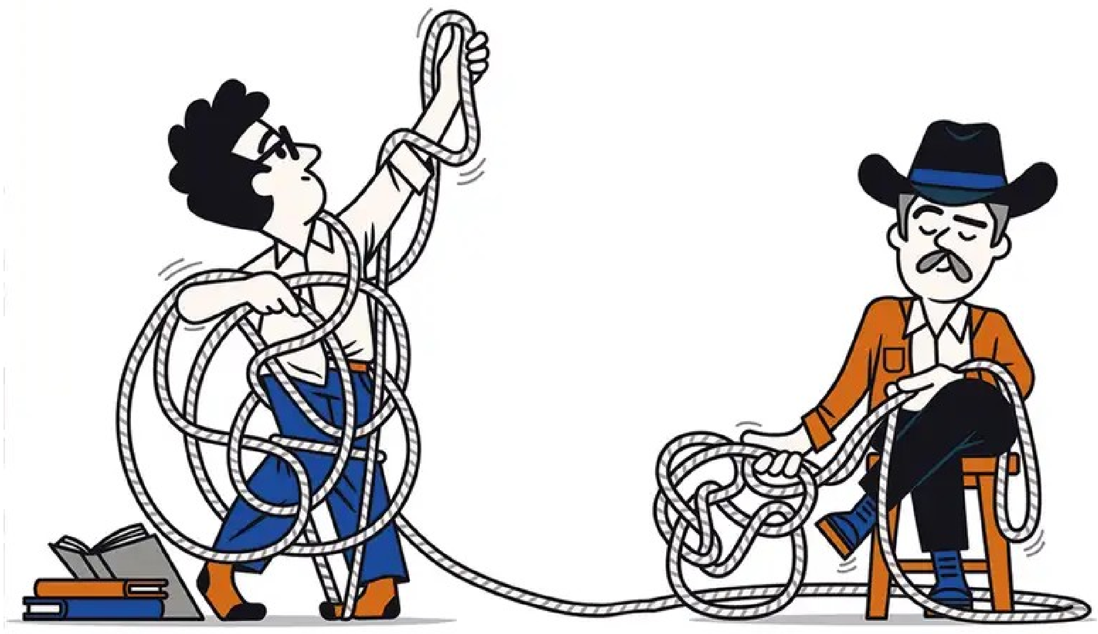
A **strategy** begins to take shape:

1. Extract/constraint the TMD FFs from SIA^{thr} ← Crucial role of TMD FFs in TMD phenomenology

2. Use the result in SIDIS and extract the TMD PDFs

In this way, we have to deal with a single TMD unknown at a time…

...and trying to untangling the TMD convolution!



A **strategy** begins to take shape:

NAIVE picture

1. Extract/constraint the TM… …rom SIA^{thr}

Crucial role of TMD FFs in TMD phenomenology

2. Use the result in SI… …d extract the TMD PDFs

In this way, we h… …deal with a single TMD unknown at a time…

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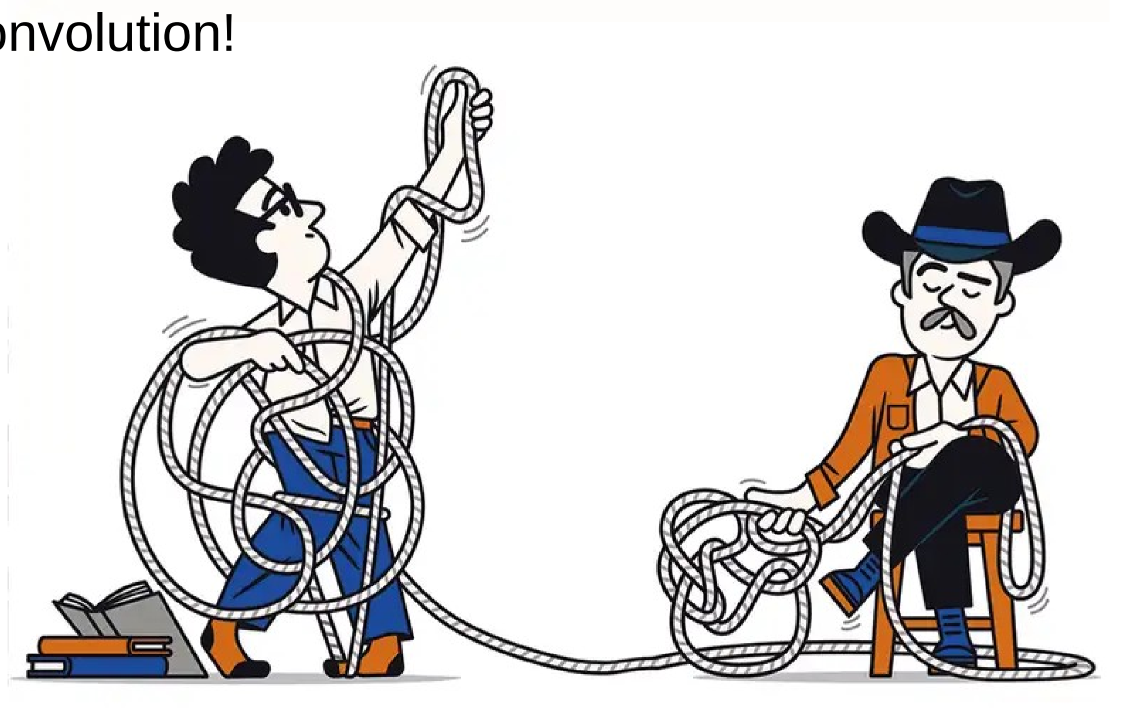


- We have to prove factorization for SIA^{thr}.
- We have to clarify the relation between the TMD FF of standard TMD factorization with its eventual analogue in SIA^{thr}.
- We have to take into account different energies:
  - SIA^{thr} @ $Q = 10.58$ GeV
  - COMPASS $1 \text{ GeV} \leq Q \leq 9 \text{ GeV}$
  - HERMES $1 \text{ GeV} \leq Q \leq 3.87 \text{ GeV}$
  - EIC $20 \text{ GeV} \leq Q \leq 140 \text{ GeV}$

SIDIS

Let's compare SIA^{thr} and the benchmark processes...where do they differ?

We have to think in terms of hard, **soft** and collinear radiation contributions

Associated with long-distance correlation *between* the TMDs…

...but in SIA^{thr} we expect to have only one TMD!

So the differences are expected to be associated with the soft gluons. However, it seems that there is no track of soft radiation in the SIDIS cross section...why?

$$d\sigma \sim H \int \frac{d^2\vec{b}_T}{2\pi^2} e^{-i\vec{q}_T\cdot\vec{b}_T} F(x,\, b_T)\, D(z,\, b_T)$$

This is not the result of the *sole* factorization procedure, but it has been properly manipulated in order *to absorb the contribution of the soft gluons inside the TMDs*

The sole factorization procedure gives instead:

$$d\sigma \sim H \int \frac{d^2\vec{b}_T}{2\pi^2} e^{-i\vec{q}_T\cdot\vec{b}_T} F^{\text{fact}}(x,\, b_T)\, \boxed{S(b_T)}\, D^{\text{fact}}(z,\, b_T)$$

No direct experimental probe

SOFT FACTOR ISSUE

Soft radiation contributions cannot be accessed *directly:*

- They “escape” from the detector
- They strongly correlate all the collinear parts

This issue makes phenomenological analyses problematic and consequently undermines the power of the factorization theorem

sqrt.

$$d\sigma \sim H \int \frac{d^2\vec{b}_T}{2\pi^2} e^{-i\vec{q}_T\cdot\vec{b}_T} F^{\text{fact}}(x,\, b_T)\, \cancel{S(b_T)}\, D^{\text{fact}}(z,\, b_T)$$

sqrt.

Recasting terms

$$= H \int \frac{d^2\vec{b}_T}{2\pi^2} e^{-i\vec{q}_T\cdot\vec{b}_T} F(x,\, b_T)\, D(z,\, b_T)$$

- The usual TMDs are not associated to pure collinear contributions, as they are "contaminated" by soft radiation effects.

- The usual TMD definition has been devised specifically for the benchmark processes and hence it can be hardly extended beyond them.

- The factorization definition of the TMDs implies that the soft factors are treated on the same footage of the TMDs.
  By exploiting the evolution equations and the b* prescription, the 2-h soft factor of the benchmark processes can be written as:

Same g_K appearing in TMDs

$$\mathbb{S}_{\text{2-h}}(b_T;\,\mu,\,y_1-y_2) = e^{\frac{y_1-y_2}{2}\widetilde{K}(b_T^\star;\,\mu)}\, M_S(b_T)\, e^{-\frac{y_1-y_2}{2}\,g_K(b_T)} + \mathcal{O}\left(e^{-(y_1-y_2)}\right)$$

SOFT MODEL
(Soft counterpart of MC)

M. Boglione, A. Simonelli, Eur.Phys.J.C 81 (2021) 1, 96

The usual definition of the TMDs (also called **square-root definition**) is then recovered as:

$$\boxed{\widetilde{D}\Big(z,\, b_T;\, \mu,\, y_P - y_1\Big) = \widetilde{D}^{\text{fact}}\Big(z,\, b_T;\, \mu,\, y_P - y_1\Big) \times \sqrt{M_S(b_T)}}$$

In practice, this means:

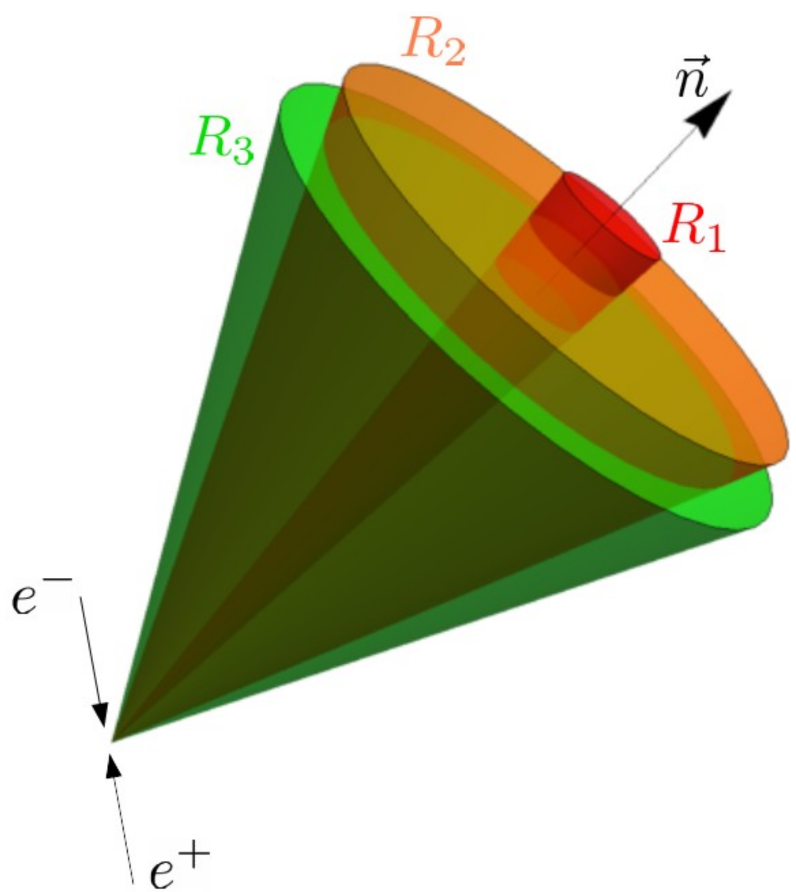
$$\begin{cases} M_D = M_D^{\text{fact}} \times \sqrt{M_S} \\ g_K = \dfrac{1}{2}\, g_K^{\text{fact}} \end{cases}$$

The two definitions differ only in the non-perturbative region (large distances) and exactly by a square root of the soft model.

As a general criterium, it should be possible to define consistently the TMDs through their usual <u>sqrt definition whenever the soft gluons generate significant non-perturbative TMD effects.</u>

Kinematic regions of SIA^{thr}

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems



The hadron is detected very close to the axis of the jet:
- Extremely small P_T
- Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

TMD FF + non-pert. SOFT contribution

The hadron is detected in the central region of the jet:
- Most common scenario
- Majority of experimental data fall into this case

TMD FF

The hadron is detected near the boundary of the jet:
- Moderately small P_T
- The hadron P_T causes the spread of the jet affecting the topology of the final state (i.e. the value of thrust)

Generalized FJF

M. Boglione, A. Simonelli, JHEP 02 (2022) 013

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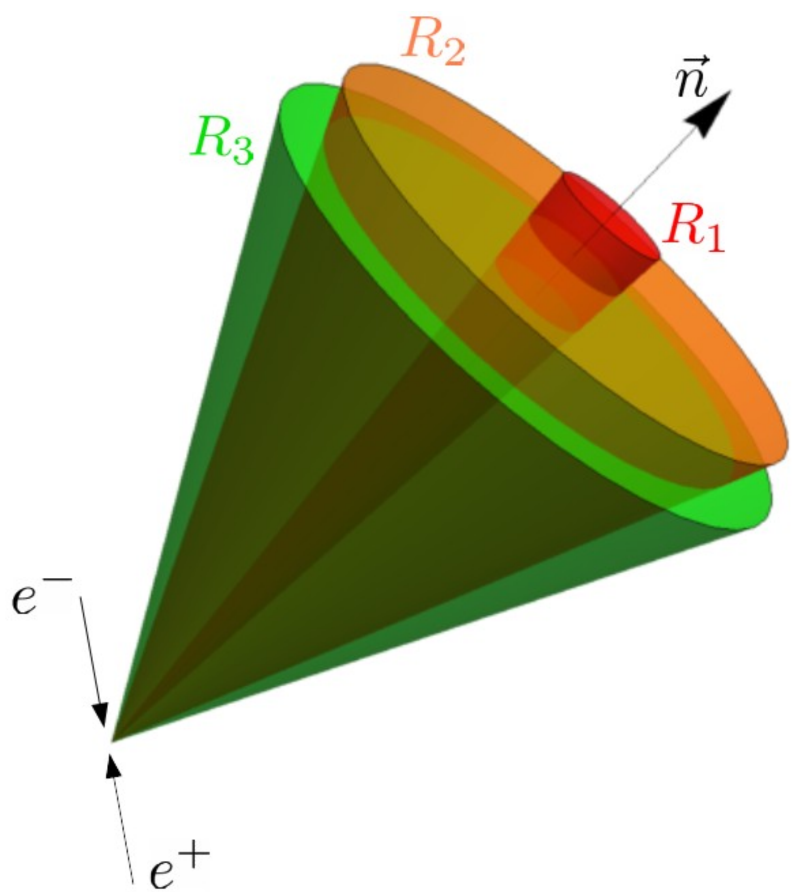

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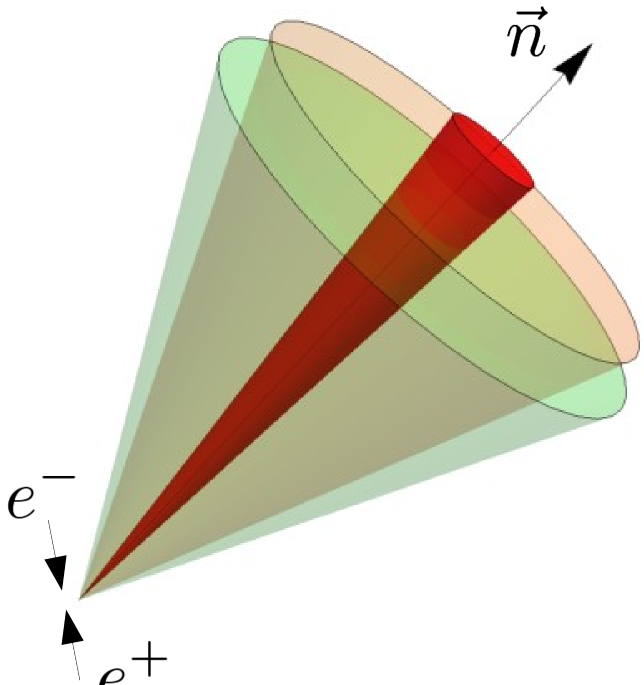
Generalized FJF

Region 1: TMD Factorization



Role of soft gluons:

In this case there is a **generalized soft factor,** $\widetilde{\Sigma}\,(u, b_T,\, y_1, y_2)$ defined as the usual soft factor of benchmark processes in the forward hemisphere and as the usual soft thrust function in the backward hemisphere.

It is associated with Non-Global Logs.

Its combination with a TMD$^{\text{(fact)}}$ can be rearranged to give a TMD$^{\text{(sqrt)}}$, by performing the same manipulations used in the benchmark processes:

$$\widetilde{\Sigma}(u,\, b_T,\, y_1, y_2)\; \widetilde{D}^{\text{fact}}_{j \to h}\,(z,\, b_T,\, y_1) \propto$$

$$\sqrt{M_S(b_T)}\, M_D^{\text{fact}}(b_T, \dots) = M_D(b_T, \dots)$$

$$\boxed{d\sigma_{R_1} \sim H \cdot J(u) \cdot \frac{\widetilde{\Sigma}(u,\, b_T,\, y_1, y_2)}{\mathcal{Y}_L(u,\, y_2)} \cdot \widetilde{D}^{\text{fact}}_{j \to h}(z,\, b_T,\, y_1)}$$

Region 1: TMD Factorization

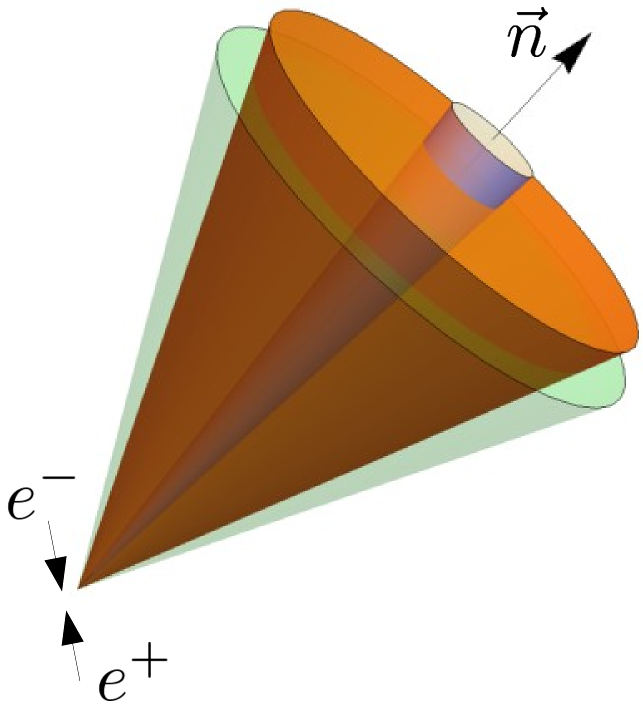
Despite the Region 1 may seem the perfect framework where to develop our phenomenological strategy, it actually presents **hidden pitfalls** to watch out for. These are essentially:

- Presence of NGLs

- Covering of a kinematic region which is basically at the boundary of the phase space

Therefore, even if a TMD FF defined as in SIDIS appears in the factorization theorem of Region 1, this is not the most appealing starting point for a strategy aiming at global fits with the benchmark processes.

Region 2: a new kind of Factorization



Role of soft gluons:

In this case there is a **generalized soft thrust function,** $\mathcal{S}(u,\, y_1, y_2)$ defined as the usual soft thrust function but with the Wilson lines tilted off the light-cone. The two functions coincide after subtractions:

$$S(u) = \frac{\mathcal{S}(u,\, y_1, y_2)}{\mathcal{Y}_L(u,\, y_2)\mathcal{Y}_R(u,\, y_1)}$$

The soft gluons do not generate any non-perturbative TMD effect and the TMD FF is a TMD$^{(\text{fact})}$.

$$\boxed{d\sigma_{R_2} \sim H \cdot J(u) \cdot \frac{\mathcal{S}(u,\, y_1, y_2)}{\mathcal{Y}_L(u,\, y_2)} \cdot \widetilde{D}^{\text{fact}}_{j\to h}(z,\, b_T,\, y_1)}$$

Region 2: a new kind of Factorization

The hadronization process in Region 2 is naturally described by a TMD FF defined by the factorization definition.

Moreover, this kinematic region corresponds to the bulk of the phase space and hence it is particularly suitable to be used as a starting point for our strategy.

The result obtained for Region 2 is NEW (never encountered before in literature) and its most peculiarity regards the role of the rapidity cut-off y_1 and its intertwining with the thrust and the transverse momentum.

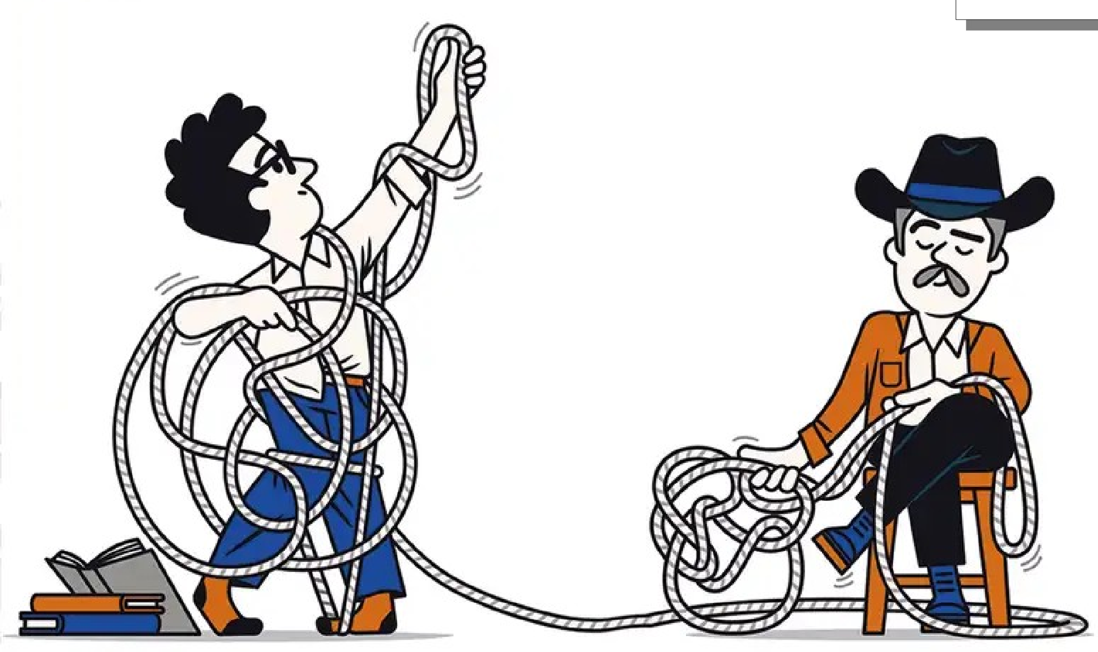
This is due to the fact that in this case part of the rapidity divergences are regulated (naturally) by the thrust, while the others are regulated (artificially) by the rapidity cut-off. The overlap of the two regulators causes the rapidity cut-off to be directly related to the measured variables:

$$y_1 = L_u - L_b^\star \left(1 + \frac{1 - \exp\left\{ -\frac{2\beta_0}{\gamma_K^{[1]}} \left(g_K(b_T) - \widetilde{K}_\star(a_S(\mu_b^\star)) \right) \right\}}{2 a_S(\mu_b^\star)\, \beta_0} \right)$$

A promising strategy for TMD phenomenology

How to deal with **just one non-perturbative unknown at a time** and perform GLOBAL phenomenological analyses

1. TMD FF (fact. def.) extraction from **Region 2** SIA$^{\text{thr}}$ → $M_D^{\text{fact}};\ g_K^{\text{fact}}$
2. TMD FF (sqrt def.) extraction from DIA /**Region 1** SIA$^{\text{thr}}$ → M_S
3. TMD PDF (fact. def.) extraction from SIDIS → M_F
4. Global fit including DY



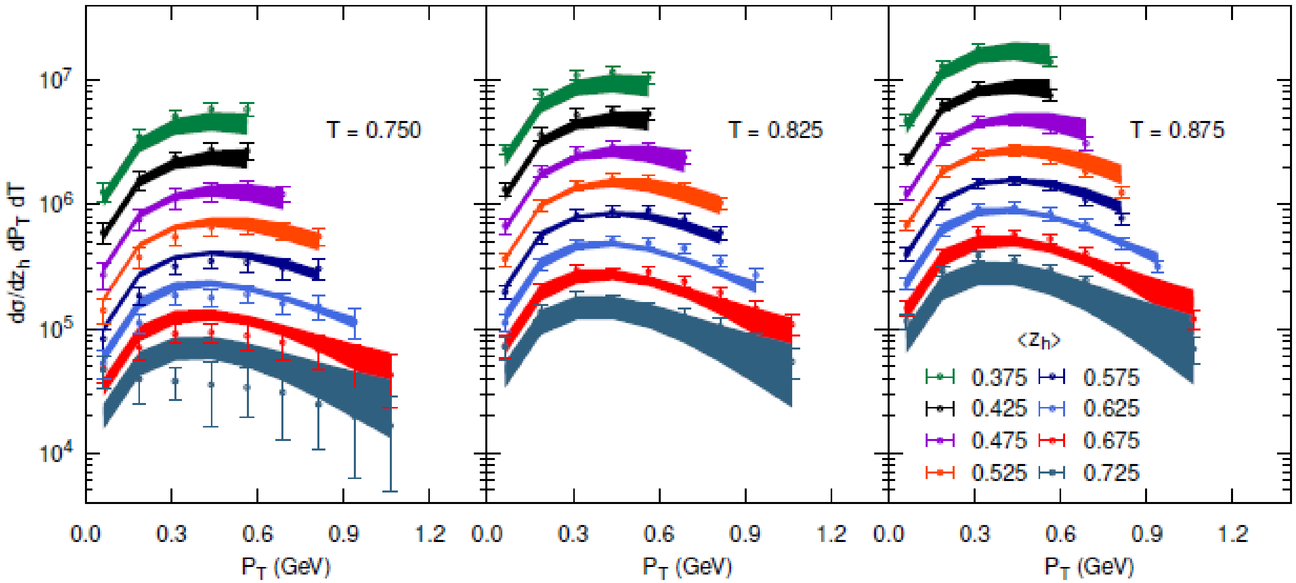
A promising strategy for TMD phenomenology

Where we are: **step 1.**

We recently presented some preliminary result obtained in a strongly simplified version of the factorization theorem of Region 2.

Boglione, Simonelli, JHEP 02 (2021) 076

Boglione, Gonzalez-Hernandez, Simonelli, 2206.08876 [hep-ph]



This sounds really promising, although very difficult to pursue in practice.
One of the most problematic issues regards the *crucial second step* (soft model extraction), which involves a comparison in b_T space.

Notice than an important cross-check would be to *verify the z-independence of* M_S.

The **EIC** will be instrumental for the success of this work plan, since it will give access to SIDIS over a wide range of energy scales.


THANK YOU

BACK-UP SLIDES

Preliminary results from “simplified” R2 treatment:

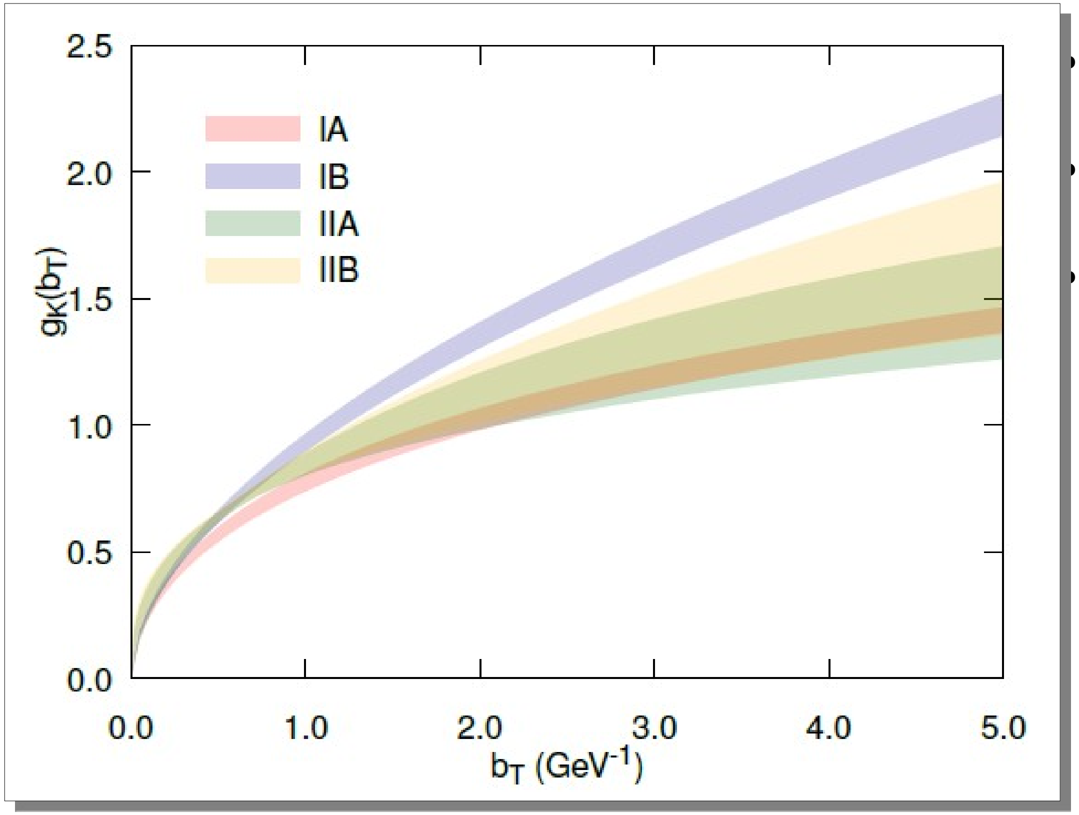
Boglione, Simonelli, JHEP 02 (2021) 076

Boglione, Gonzalez-Hernandez, Simonelli, 2206.08876 [hep-ph]

- Rapidity cut-off associated directly (and naively) to thrust.
- Any b_T-dependence outside the TMD FF is integrated out.
- Formal resummation of thrust cannot be performed

➢ Extraction of g_K

Vladimirov, Phys.Rev.Lett. 125 (2020) 19, 192002
Collins, Rogers, Phys.Rev.D 91 (2015) 7, 074020



Theory constraints:

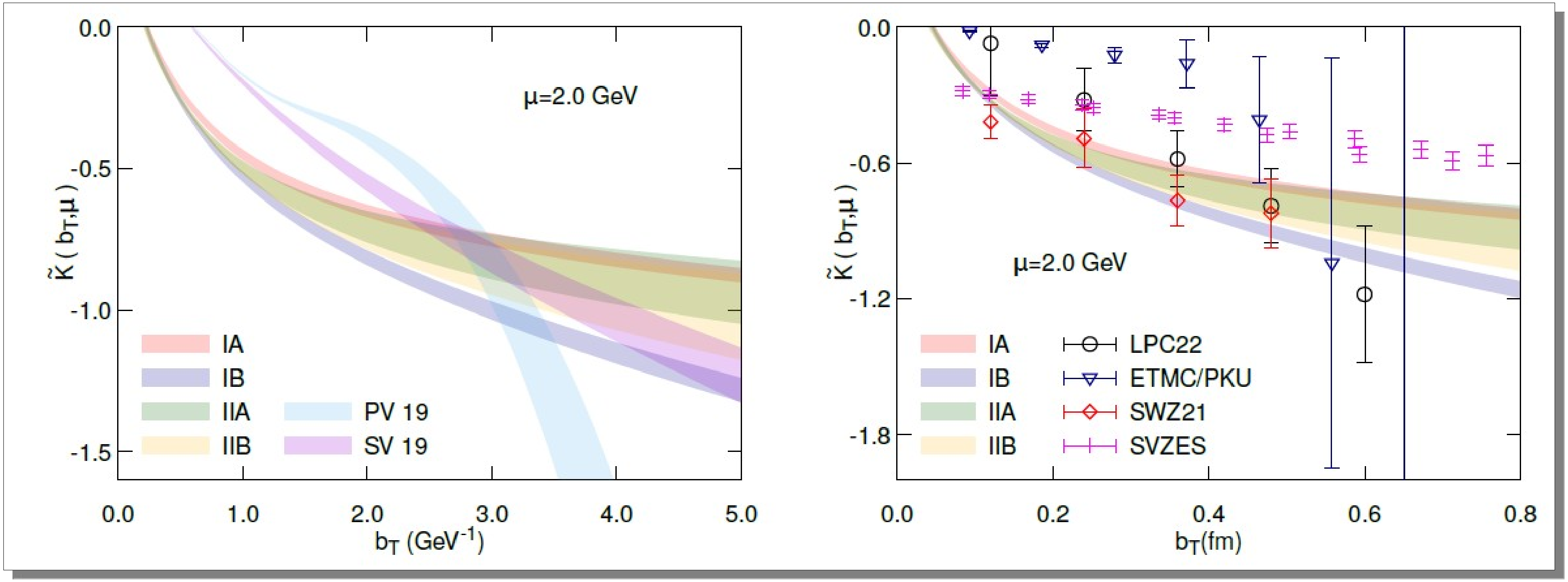
- $g_K \to 0; \quad b_T \to 0$ ✓
- $g_K = g_2\, b_T^2 + g_4\, b_T^4 + \ldots; \quad b_T \to 0$ ✗
- g_K sub-linear; $\quad b_T \to \infty$ ✓

A) $g_K = \log\left(1 + M_k\, b_T^{p_k}\right)$

B) $g_K = M_k\, b_T^{1-2\,p_k}$

BELLE data sensitive to large b_T (larger than b_{MAX}). ⟷

- Low energy $Q = 10.58\ \mathrm{GeV}$
- Thrust T close to 1



Boglione, Gonzalez-Hernandez, Simonelli, 2206.08876 [hep-ph]

Extraction of M_D

Scimemi, Tarason, Vladimirov, JHEP 05 (2019) 125
Collins, Rogers, Phys.Rev.D 91 (2015) 7, 074020

Theory constraints:

- $\log M_D \to 0; \quad b_T \to 0$ ✔
- $\log M_D = -c_2\, b_T^2 + c_4\, b_T^4 + \ldots; \quad b_T \to 0$ ✔
- $\log M_D$ linear (plus logs) ; $\quad b_T \to \infty$ ✔

■ Model I:

$$\left(\frac{1 + \log\left(1 + (M_z\, b_T)^2\right)}{1 + (M_z\, b_T)^2} \right)^q$$

$$M_D(z_h,\, b_T) = \boxed{\frac{2^{2-p}}{\Gamma(p-1)}\, (b_T\, M)^{p-1}\, K_{p-1}(b_T\, M)} \times \boxed{F(z_h,\, b_T)}$$

$$\boxed{\text{F.T.} \left\{ \frac{\Gamma(p)}{\pi\, \Gamma(p-1)} M^{2\,(p-1)} \left(M^2 + \frac{P_T^2}{z_h^2} \right)^{-p} \right\}}$$

$$M_z = -M_1 \log z$$
$$p = 1.51; \quad q = 8$$

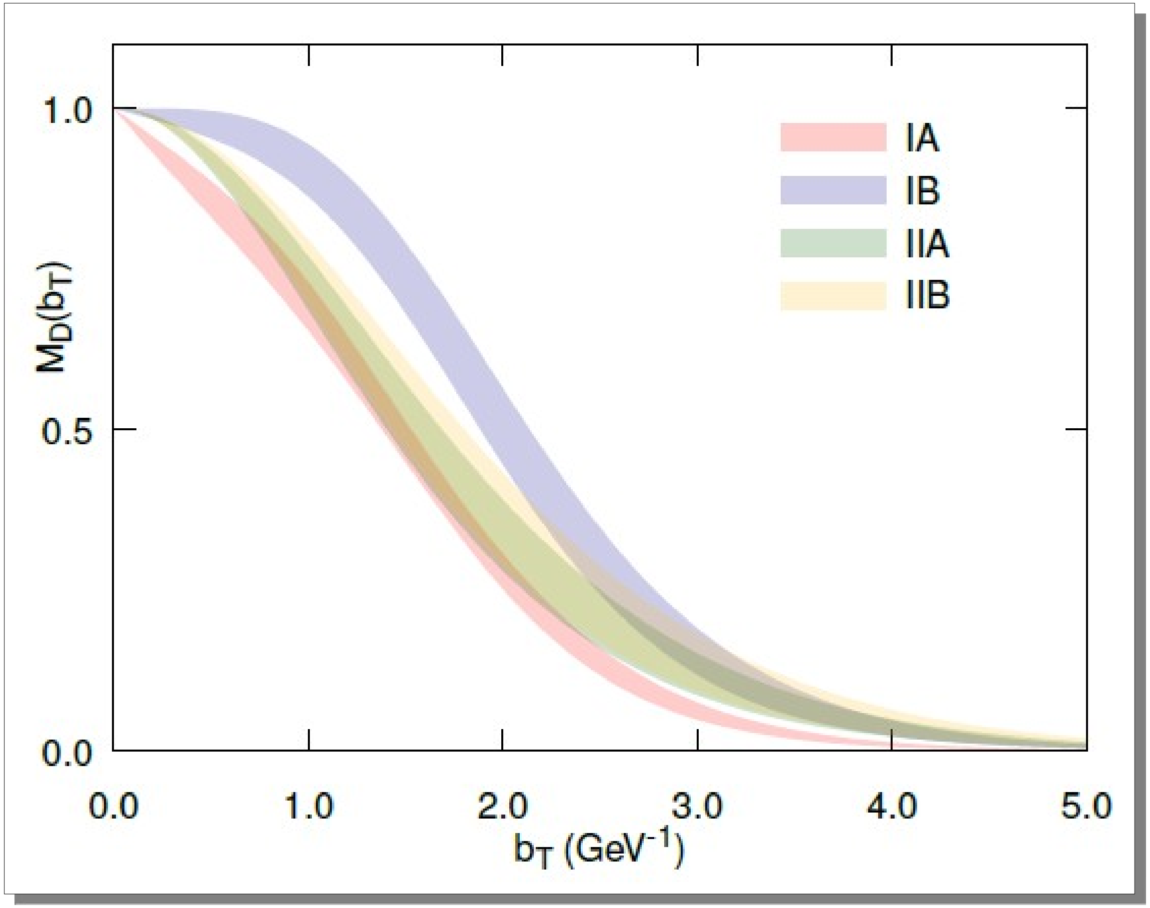
- Model II:

$$M_D(z_h,\, b_T) = \frac{2^{2-p}}{\Gamma(p-1)}\, (b_T\, M)^{p-1}\, K_{p-1}(b_T\, M)$$

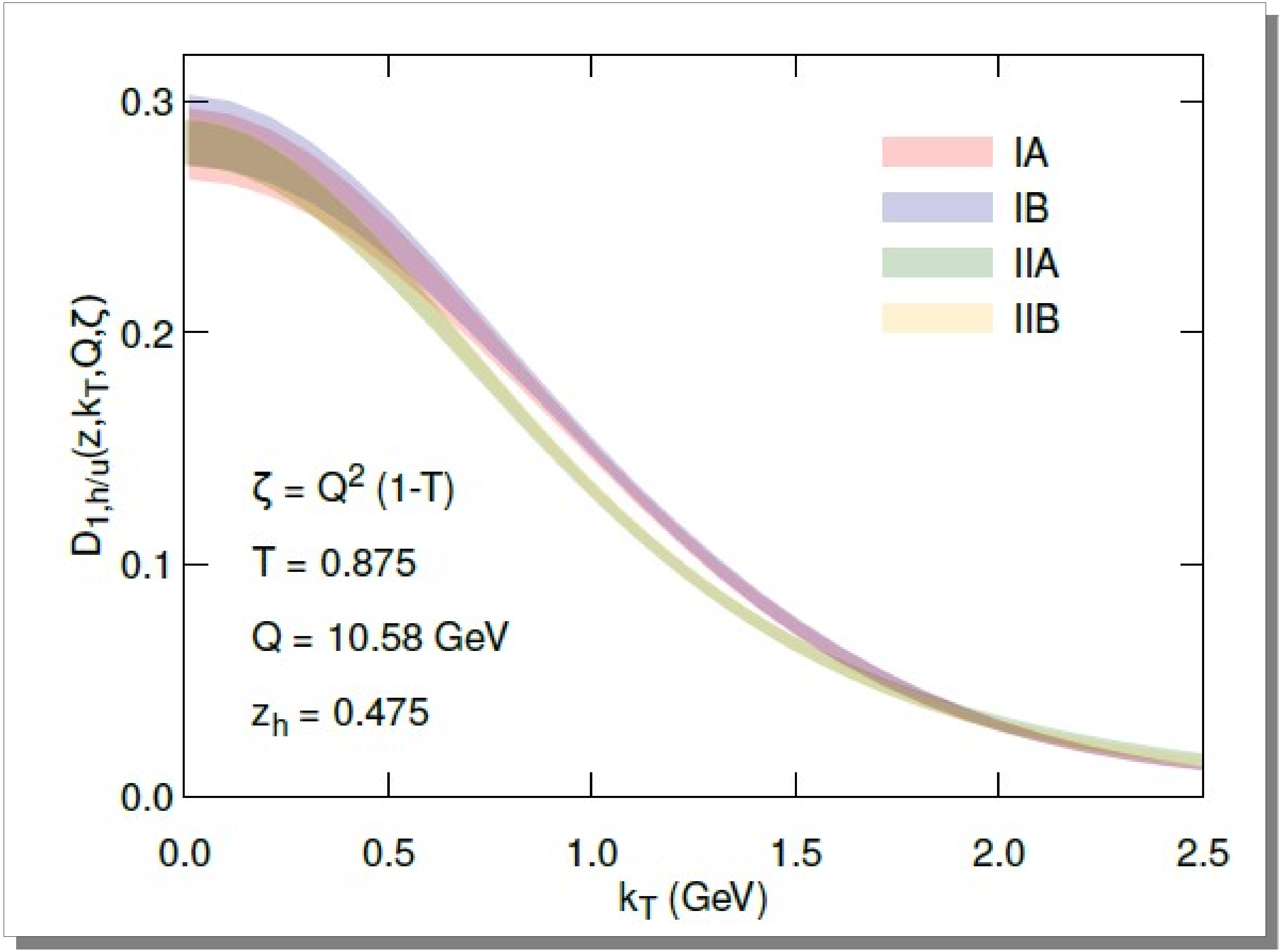
$$\begin{cases} p = \dfrac{1}{2}\left(\dfrac{3}{1-R} - 1\right) \\ M = \dfrac{W}{z_h}\sqrt{\dfrac{3}{1-R}} \end{cases}$$

With:

$$\begin{cases} R = 1 - (1-z)^{z_0/1-z_0}, \\ W = \dfrac{M_h}{R^2} \end{cases}$$



Extraction of the **TMD FF** from Region 2 (preliminary)

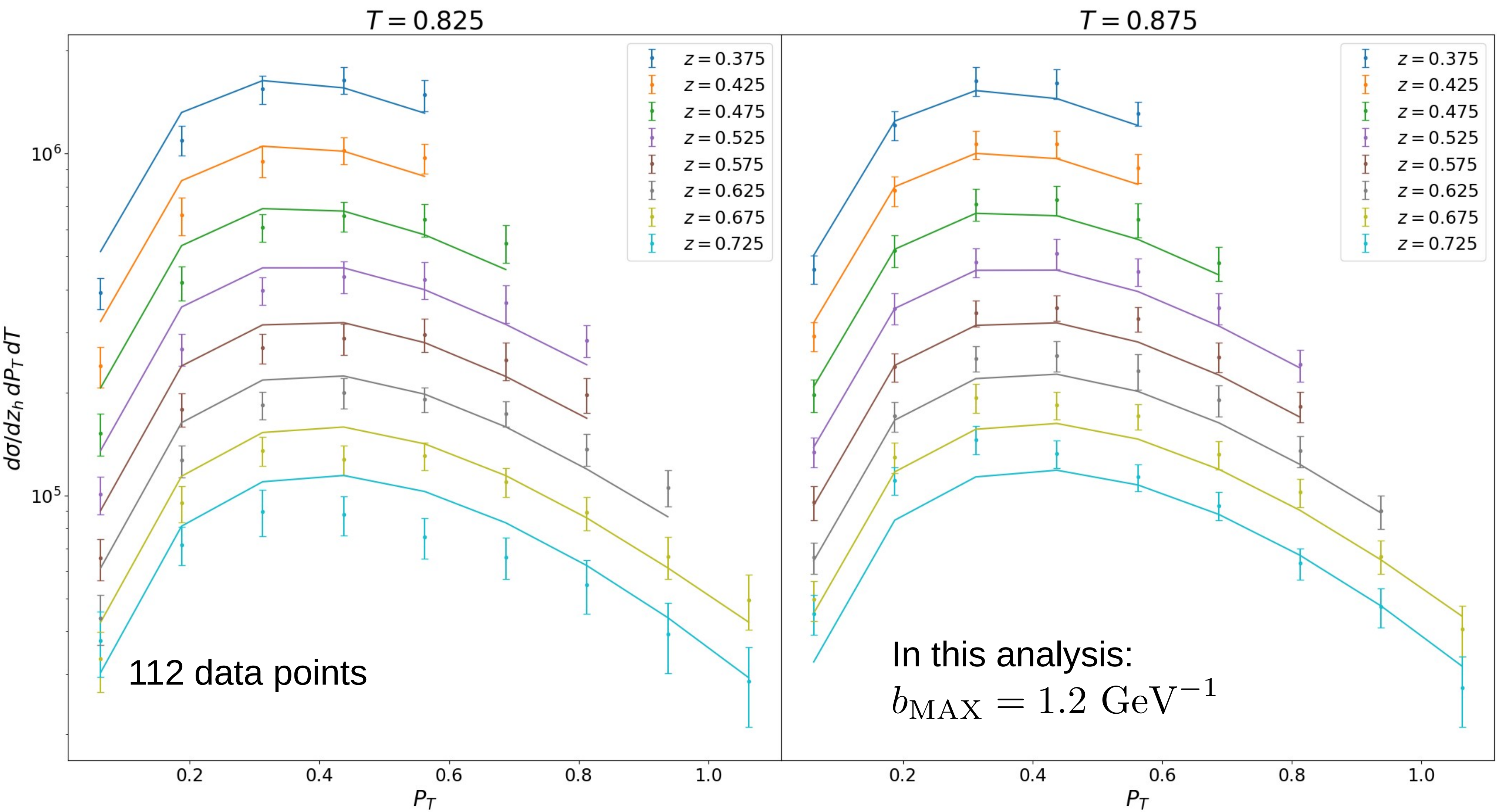


Phenomenology: preliminary FIT of BELLE data @ N²LL*

$\chi^2 = 1.25$

$\mathrm{M_D}:$ $z_0 = 0.729 \pm 0.011$
$\alpha = 0.175 \pm 0.004$

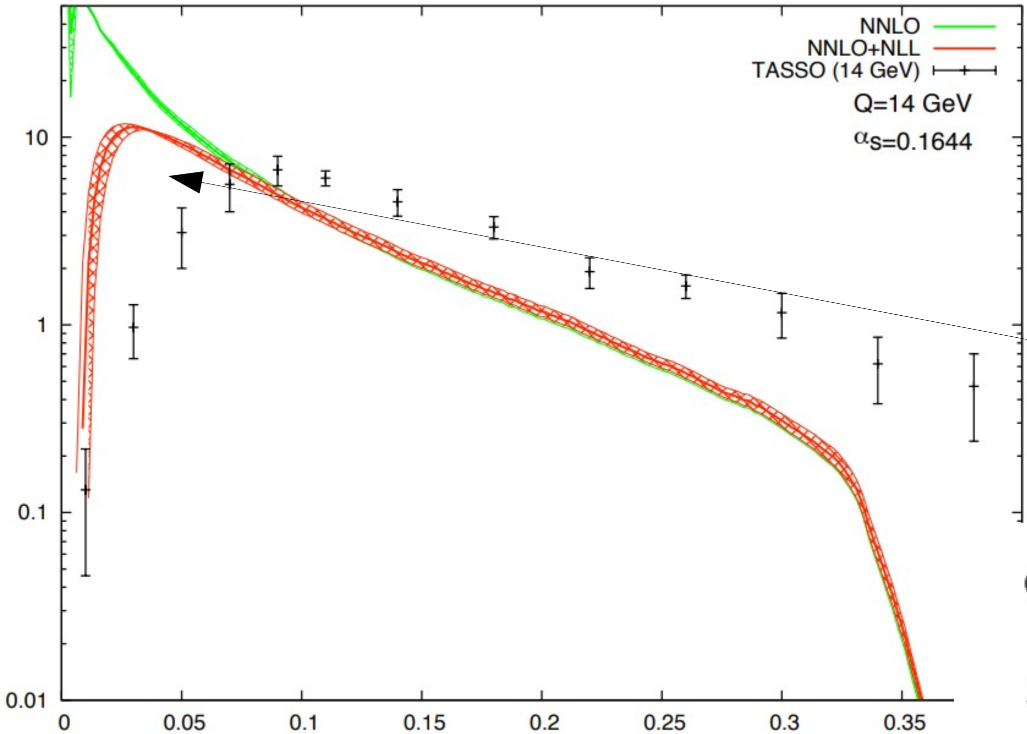
$\mathrm{g_K}:$ $g_a = 0.445 \pm 0.026\ \mathrm{GeV}^{-1}$
$g_b = 3.690 \pm 0.026$



112 data points

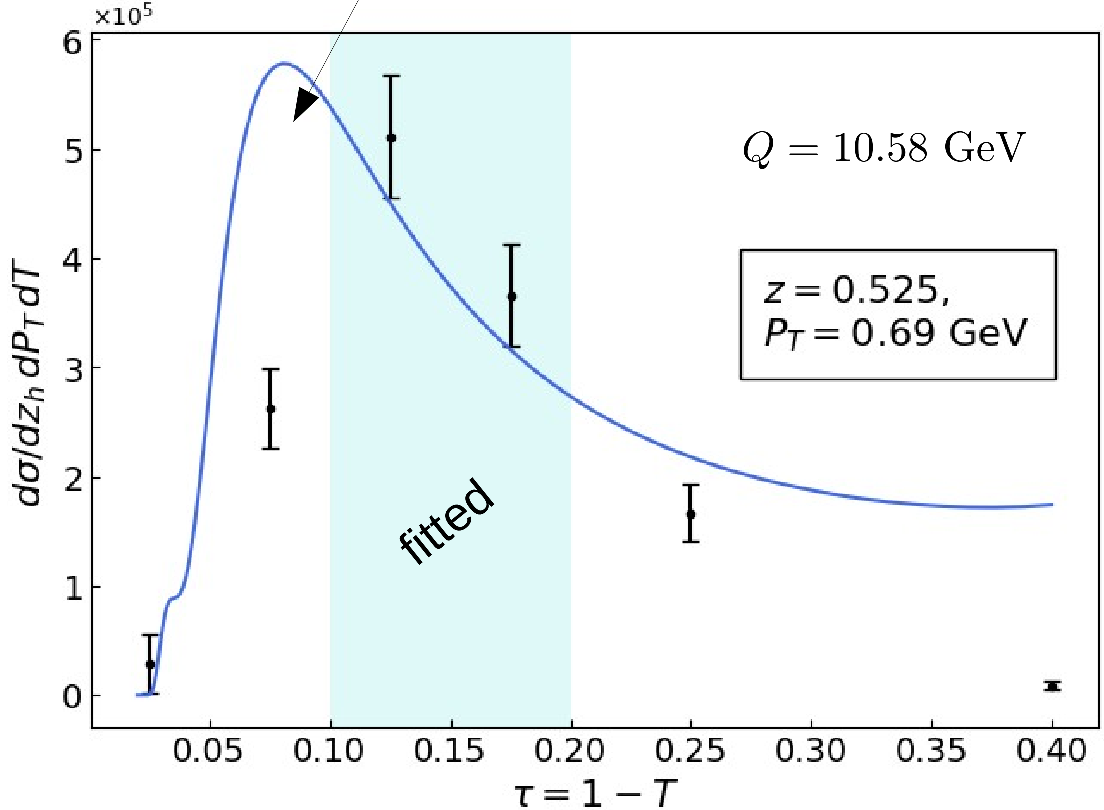
In this analysis:
$b_{\mathrm{MAX}} = 1.2\ \mathrm{GeV}^{-1}$

Peak shift and Thrust dependence



R.A. Davison and B.R. Webber, *Non-Perturbative Contribution to the Thrust Distribution in e^+e^- Annihilation*, Eur. Phys. J. C 59 (2009) 13 [0809.3326].

Same NON-PERTURBATIVE effect, *genuinely* due to thrust dependence.



Behaviour of g_K

- At small-b_T :

$$g_K = g_2\, b_T^2 + \dots$$

with:

$$g2 = 68.879 \pm 8.016\ \mathrm{GeV}^2$$

Two orders of magnitude larger than in previous extractions!

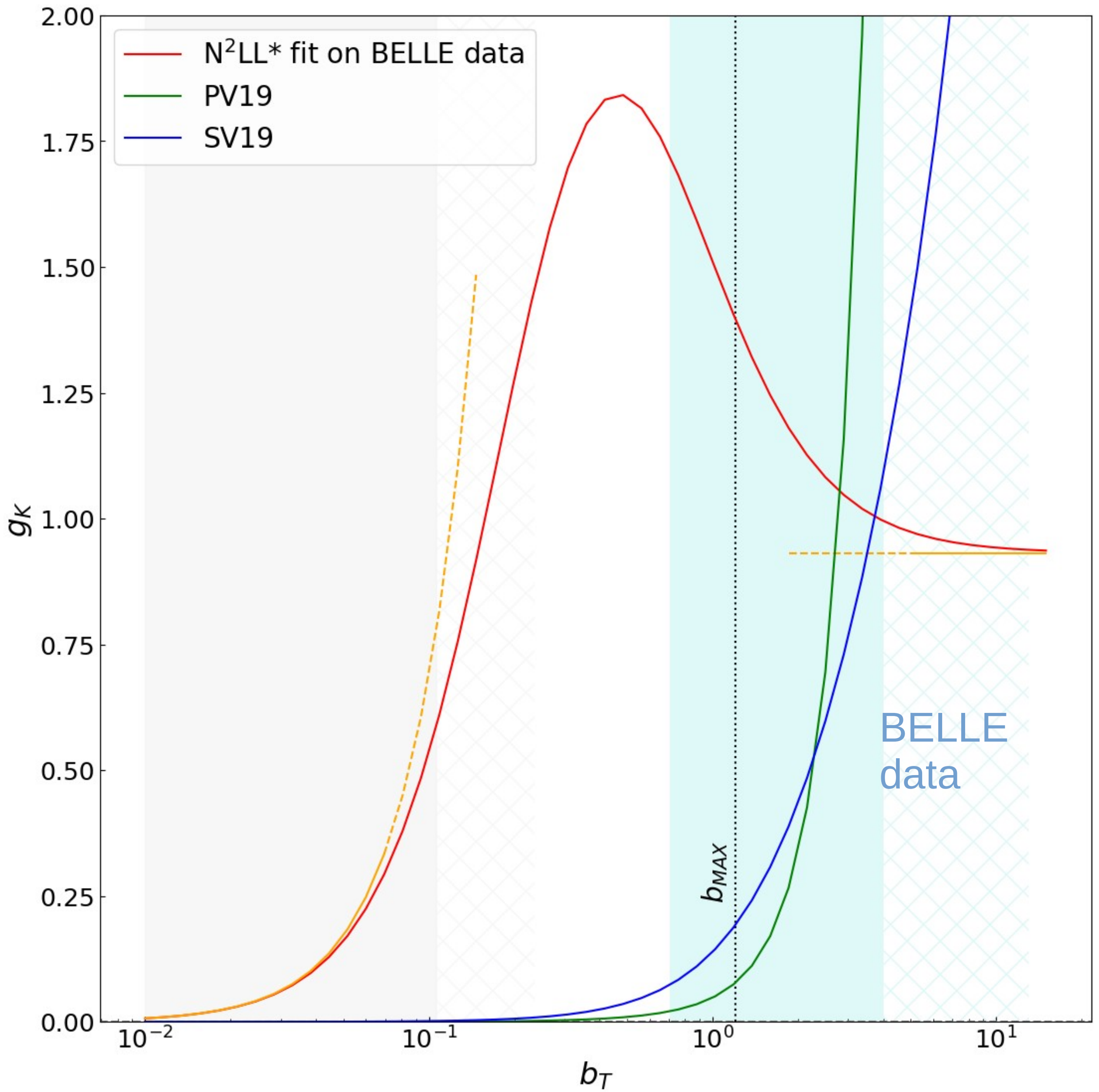
- At large-b_T :

$$g_K \sim g_0$$

with:

$$g0 = 0.932 \pm 0.001$$

Asymptotic constant behaviour



Collins, Rogers, Phys.Rev.D 91 (2015) 7, 074020

Behaviour of g_K

- At small-b_T :

$$g_K = g_2 \, b_T^2 + \ldots$$

with:

$$g2 = 68.879 \pm 8.016 \ \mathrm{GeV}^2$$

Two orders of magnitude larger than in previous extractions!

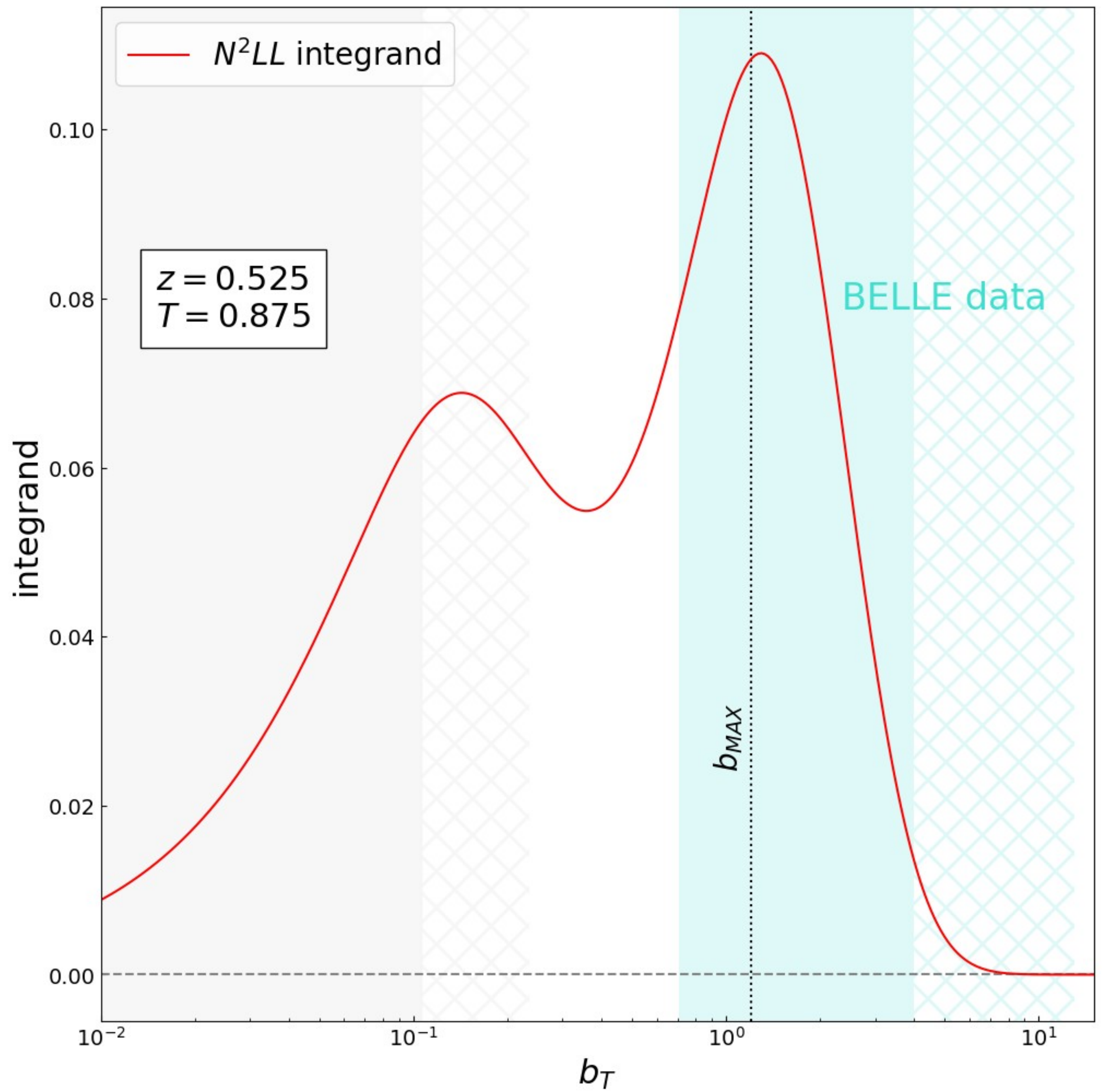
- At large-b_T :

$$g_K \sim g_0$$

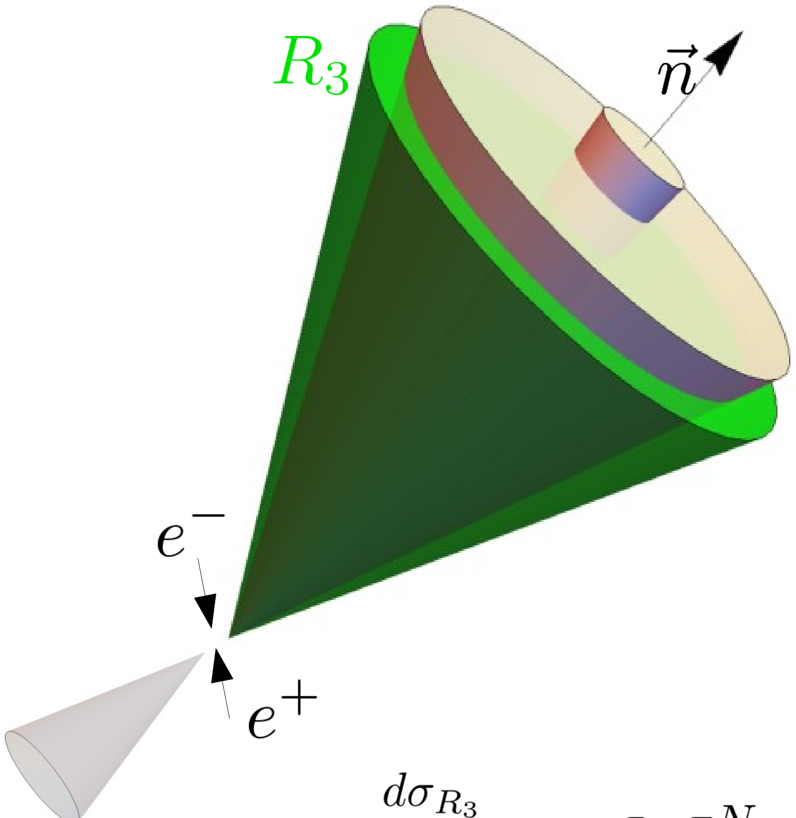
with:

$$g0 = 0.932 \pm 0.001$$

Asymptotic constant behaviour



Region 3 for a 2-jet topology



The hadron is detected near the boundary of the jet:

- Moderately small P_T
- The hadron P_T causes the spread of the jet affecting the topology of the final state (i.e. the value of thrust)

Generazlized FJF

- (forward) soft radiation and soft-collinear radiation are TMD-irrelevant
- Only collinear radiation is TMD-relevant

$$\frac{d\sigma_{R_3}}{dz_h\,dP_T^2\,dT} = \sigma_B\,\pi N_C \int d\tau_S\,d\tau_A\,d\tau_B\,\delta(\tau-\tau_S-\tau_A-\tau_B)\sum_f e_f^2 \times$$

$$\times \underset{\text{Virtual}}{V}\,\underbrace{J(\tau_B)}_{\text{Backward}}\,\underbrace{S(\tau_S)\,\Gamma_{h/f}\left(z_h,\frac{P_T}{z_h},\tau_A\right)}_{\text{Forward}}$$

Region 3: **Generalized Collinear Factorization**

$$\frac{d\sigma_{R_3}}{dz_h\, dP_T^2\, dT} = \sigma_B\, \pi N_C \int d\tau_S\, d\tau_A\, d\tau_B\ \delta(\tau - \tau_S - \tau_A - \tau_B) \sum_f e_f^2 \times$$
$$\times\, V\, J(\tau_B)\, S(\tau_S)\, \Gamma_{h/f}\left(z_h,\, \frac{P_T}{z_h},\, \tau_A\right)$$

In literature:

Joint thrust and TMD resummation in electron-positron and electron-proton collisions, Y. Makris, F. Ringer, W.J. Waalewijn **JHEP 02 (2021) 070**

Generalized Fragmenting Jet Function
- A lot in common with collinear FFs (e.g. DGLAP) but carries TMD information
- Its non-perturbative part depends EXPLICITLY on the thrust (invariant mass of the jet)

A generalized collinear factorization theorem:

- None of the functions in the cross section depend on a rapidity cut-off
- “Clean” way to access GFJFs

GFJFs could play a central role at EIC ($e\,p \to h_{\text{jet}} + X$)

How to distinguish between the regions

- Algorithm based on the comparison of ratios that describe the kinematics of each region. Such ratios are inspired by 1-loop explicit computation.

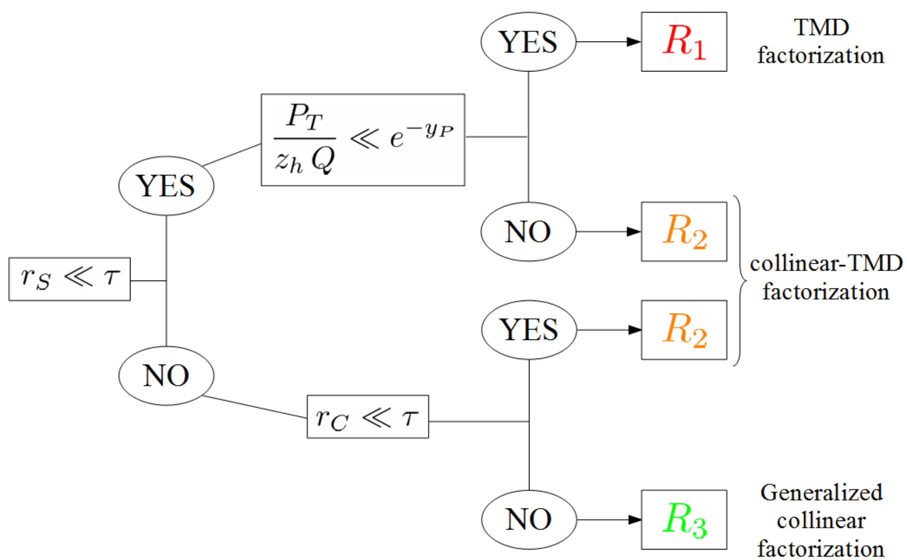
SOFT RATIO

$$r_S = \frac{P_T}{z_h\, Q} e^{-y_P}$$

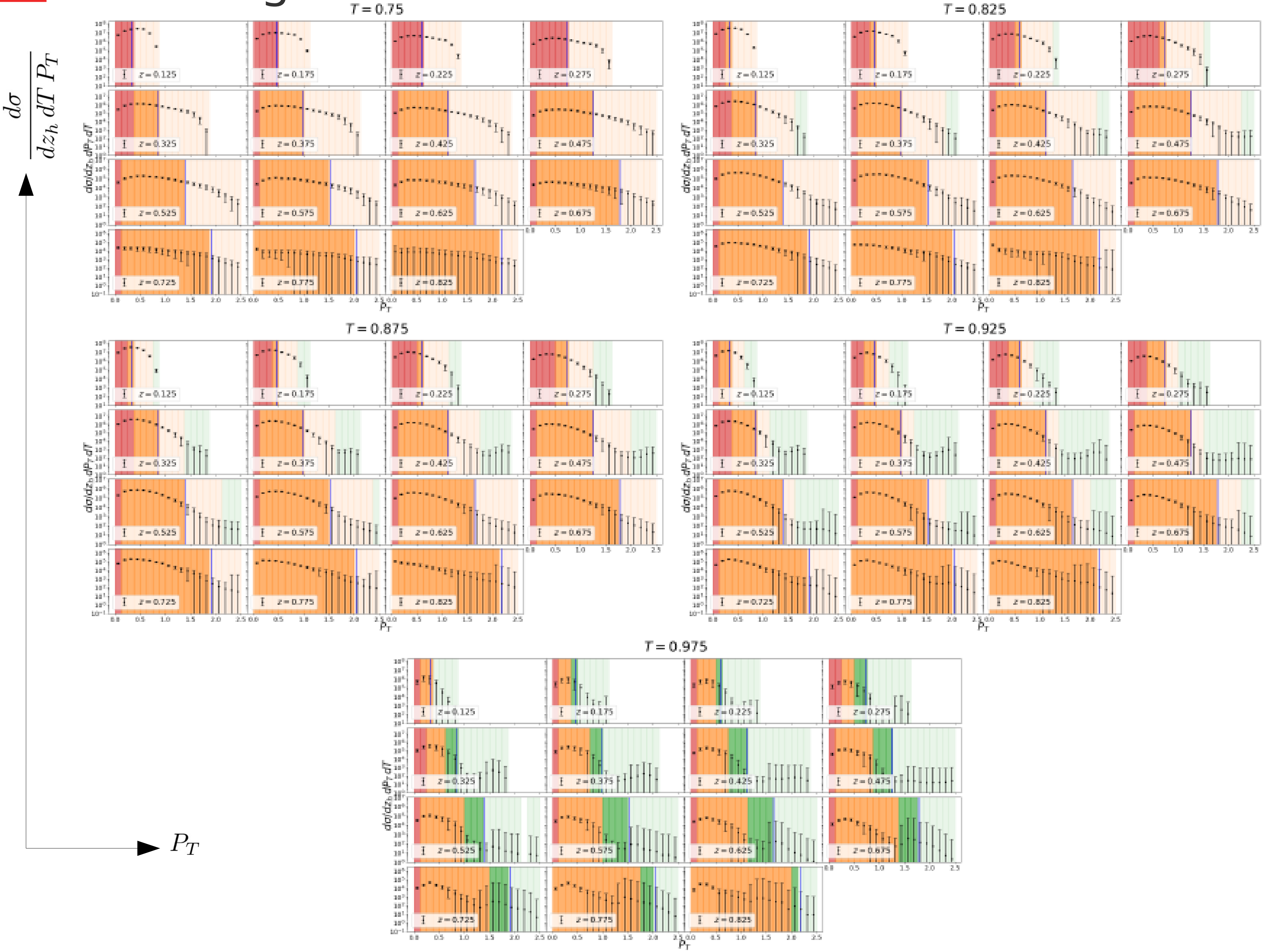
COLLINEAR RATIO

$$r_C = z_h\,(1 - z_h)\, e^{-2\, y_P}$$

- The rapidity of the hadron is taken explicitly into account
- Region 2 is the widest region as expected
- The problem of the matching is not urgent, as there are many **monocromatic** bins

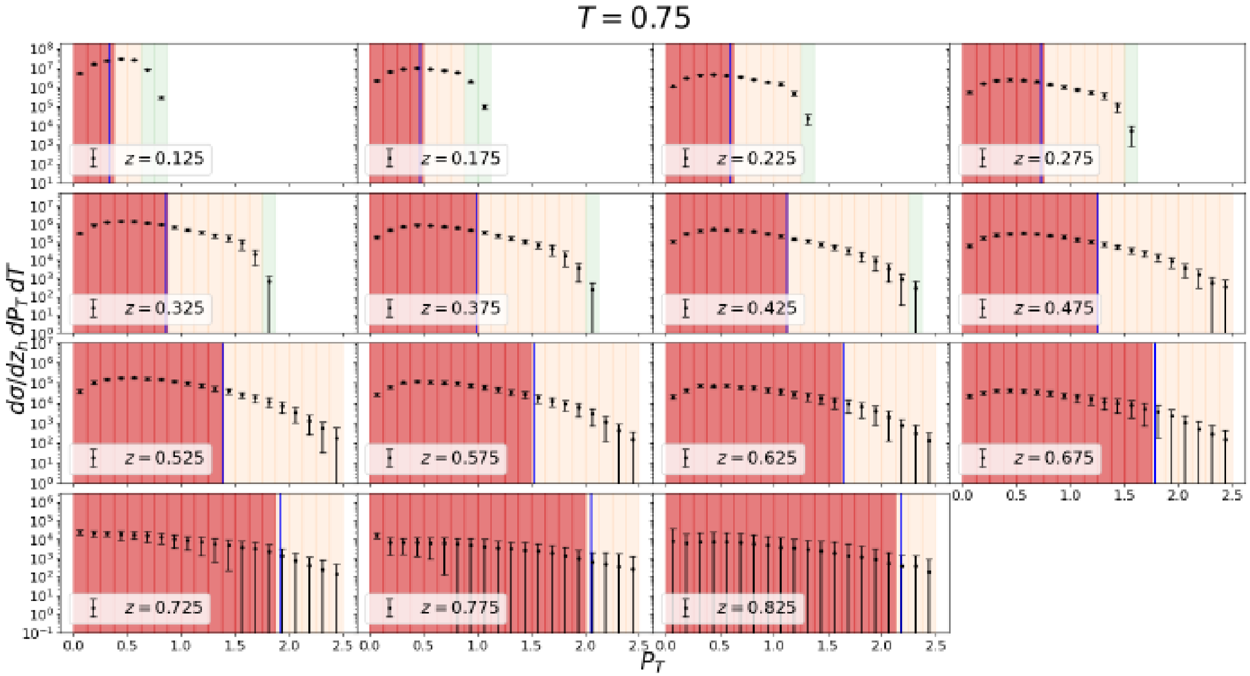


Ratios algorithm on BELLE data

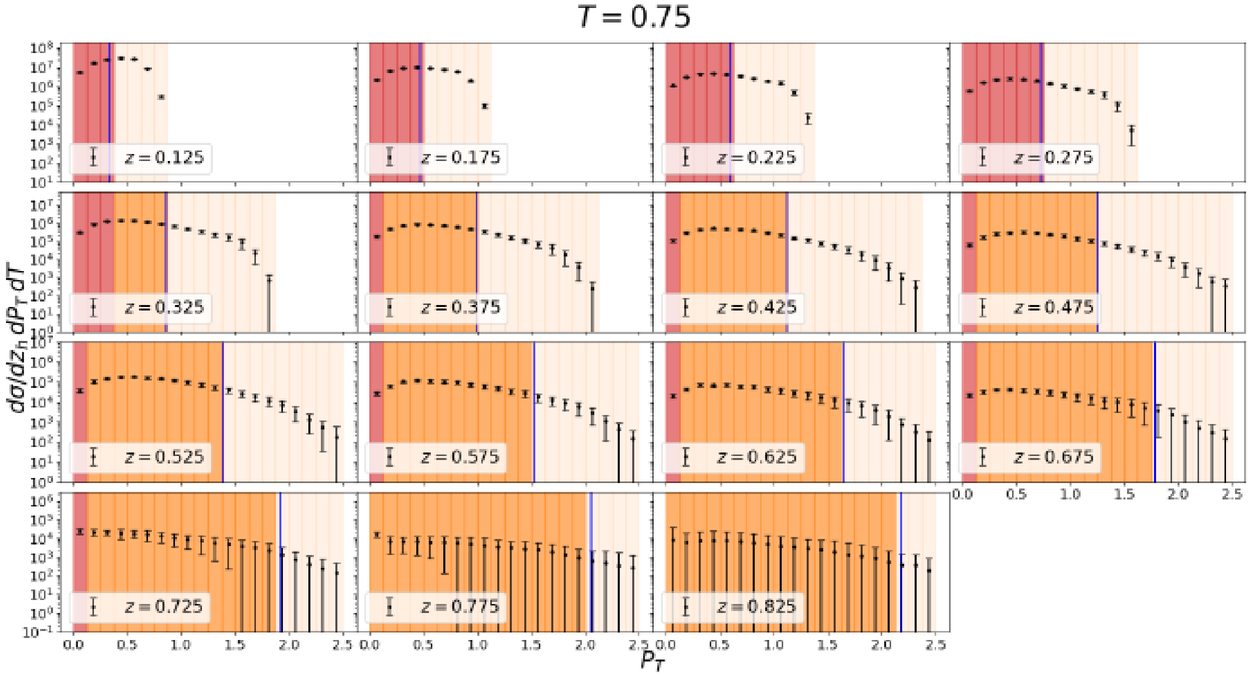


Comparison between data selection criteria

Power-counting criterium

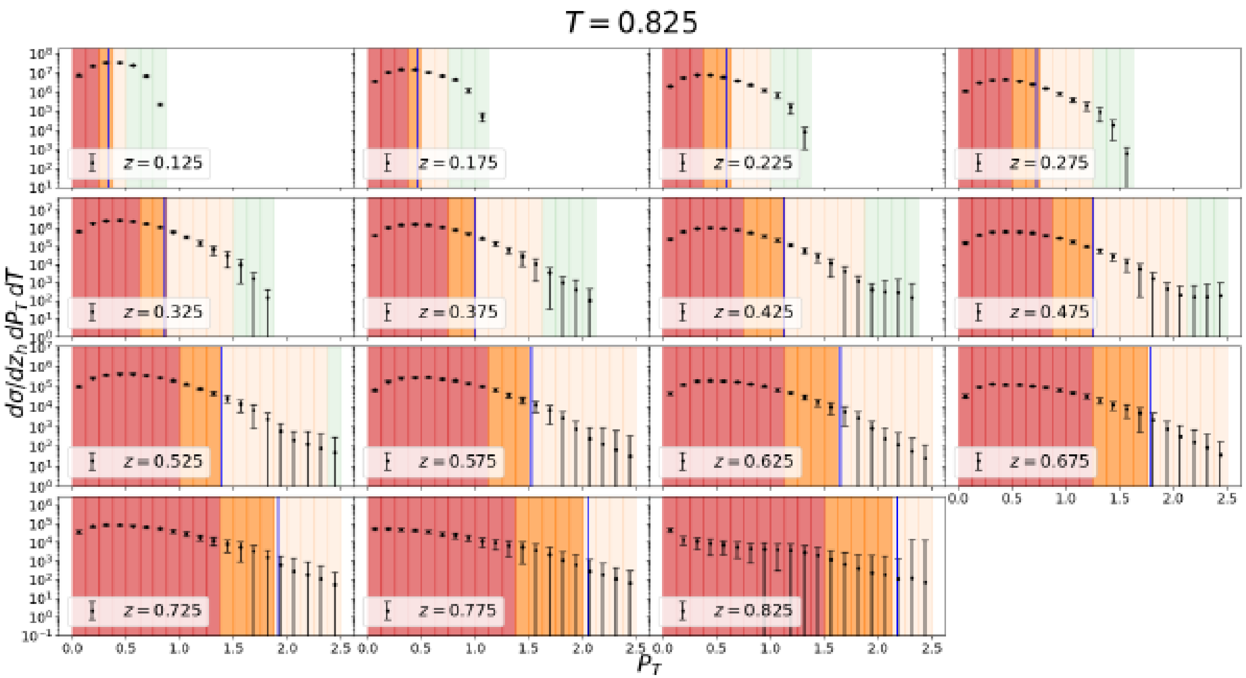


Ratios algorithm

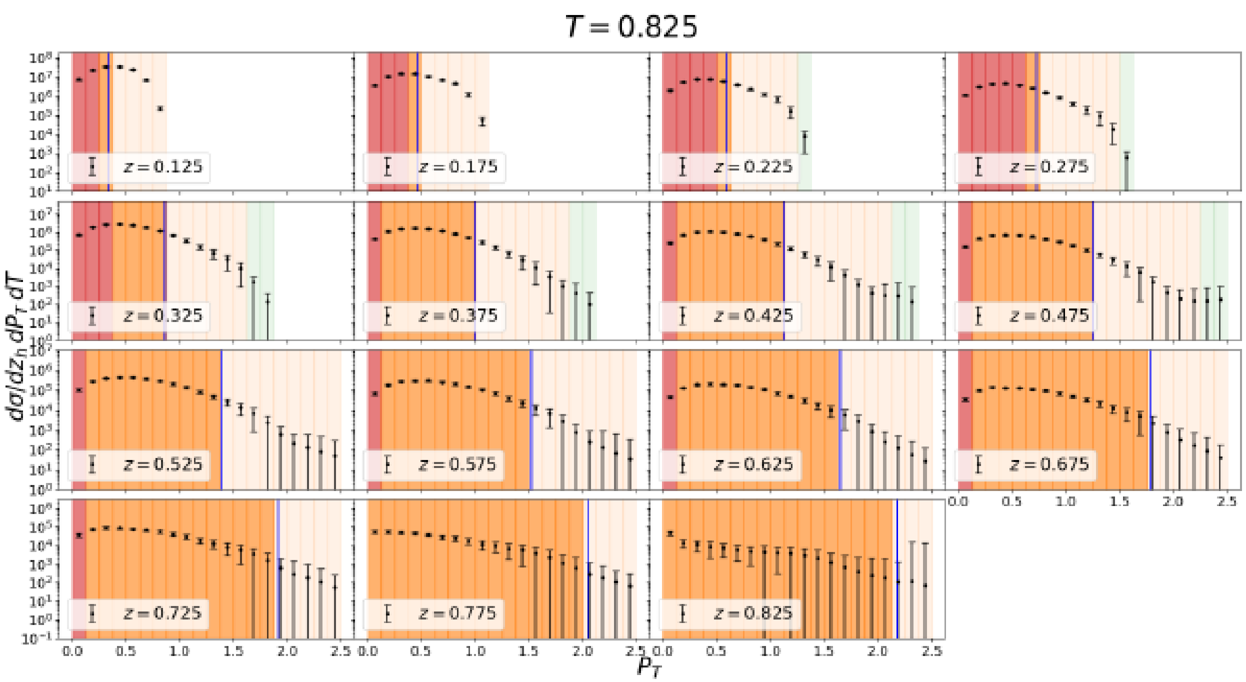


Comparison between data selection criteria

Power-counting criterium



Ratios algorithm



Comparison between data selection criteria

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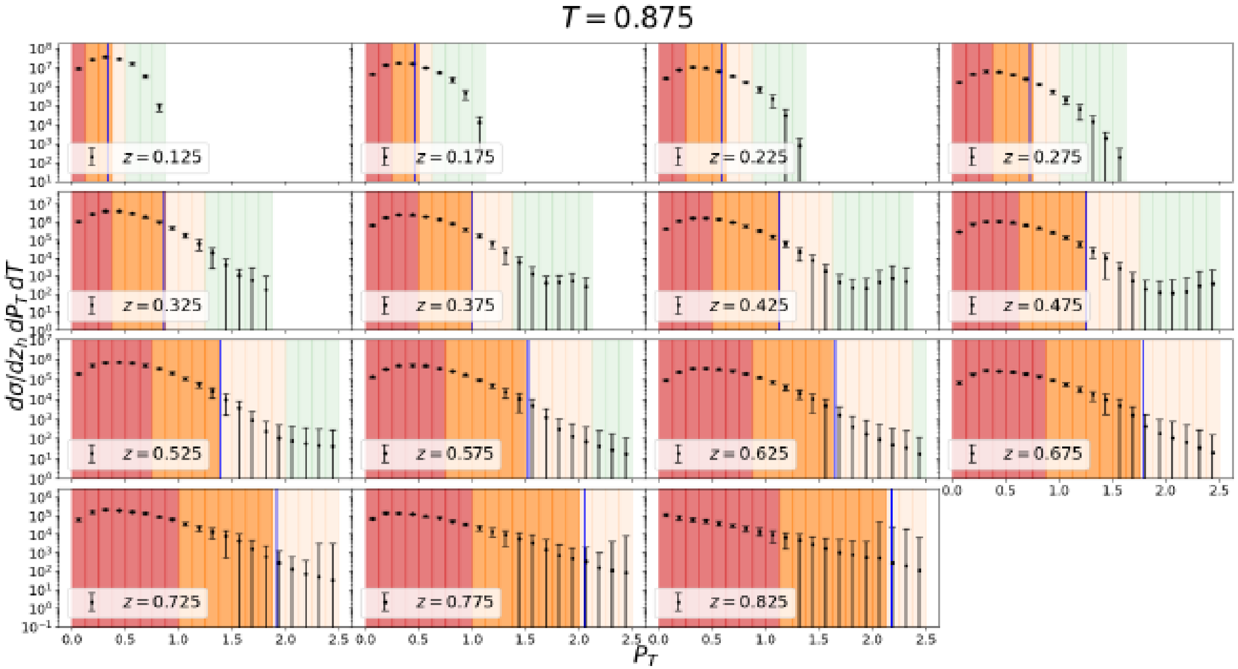

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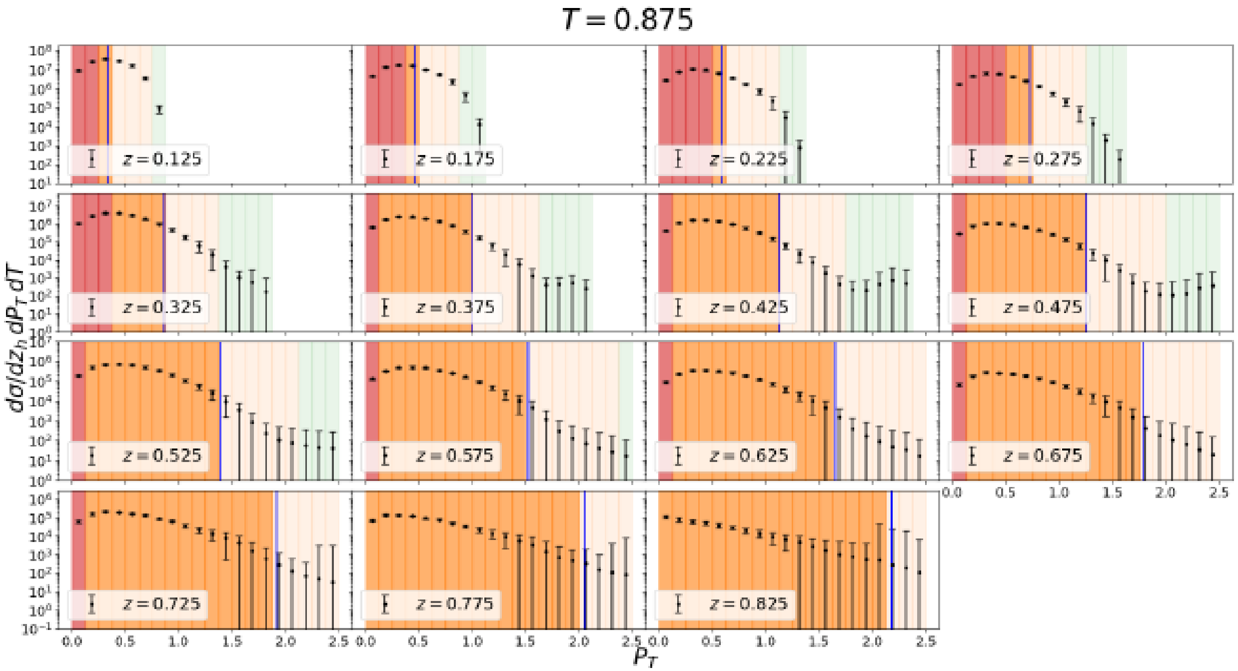

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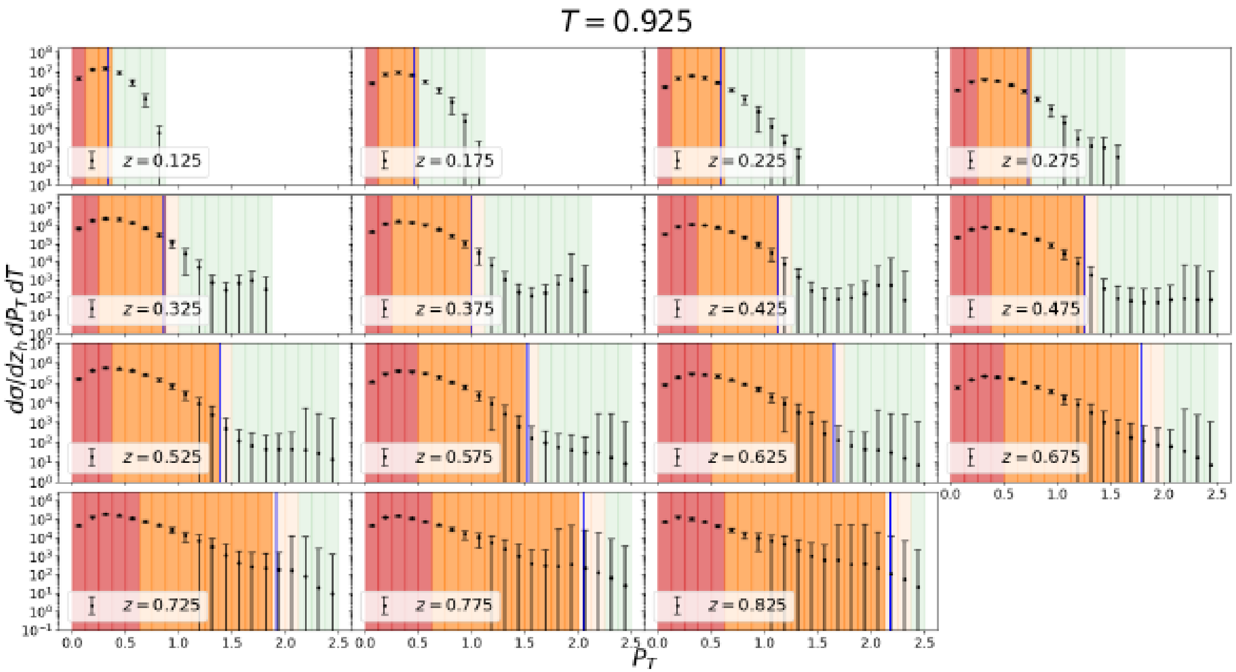
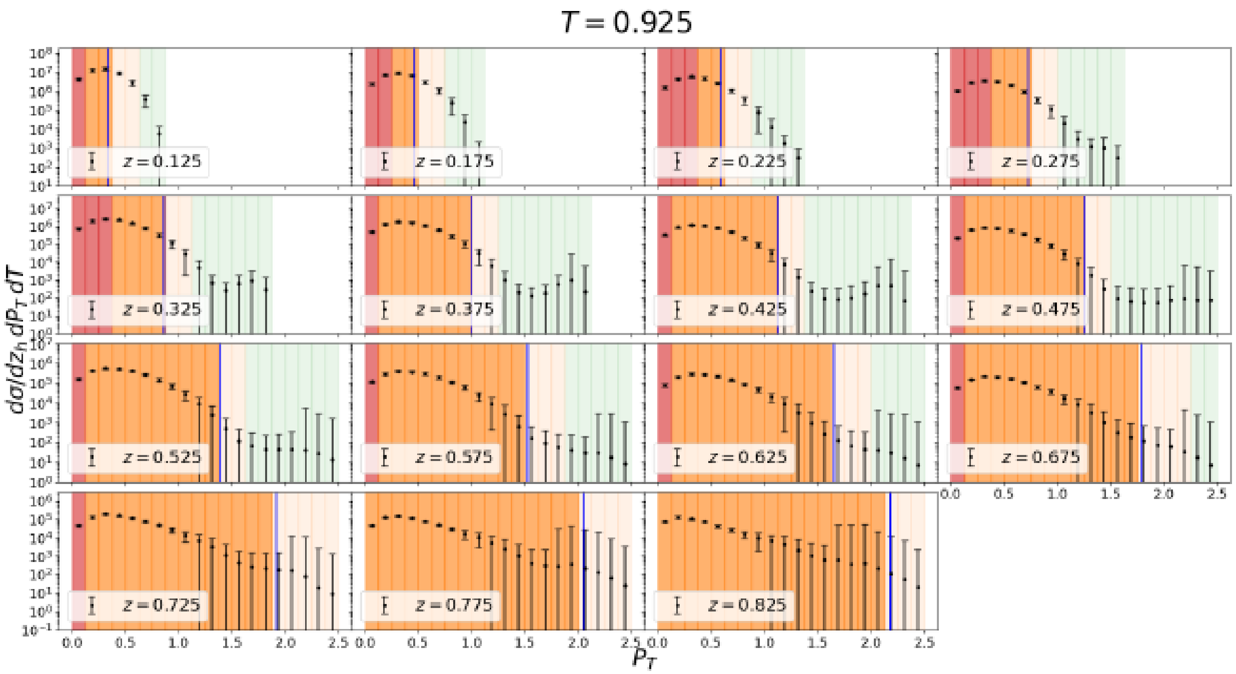


Ratios algorithm



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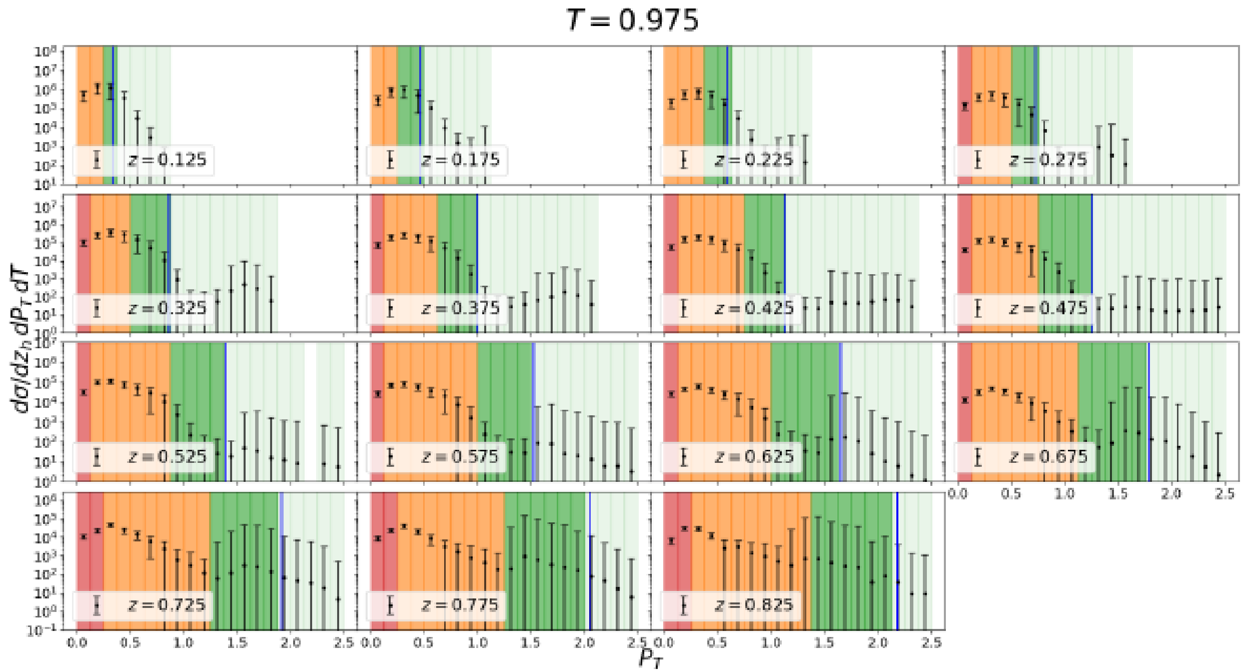

Ratios algorithm