Study of TMDs in DIS single-jet production

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EIC User Group Early Career Workshop 2022

Istituto Nazionale di Fisica Nucleare

Outline

Jet-Sidis Process

Jet-TMDFF

Phenomenological Estimates

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Jet-TMDFF

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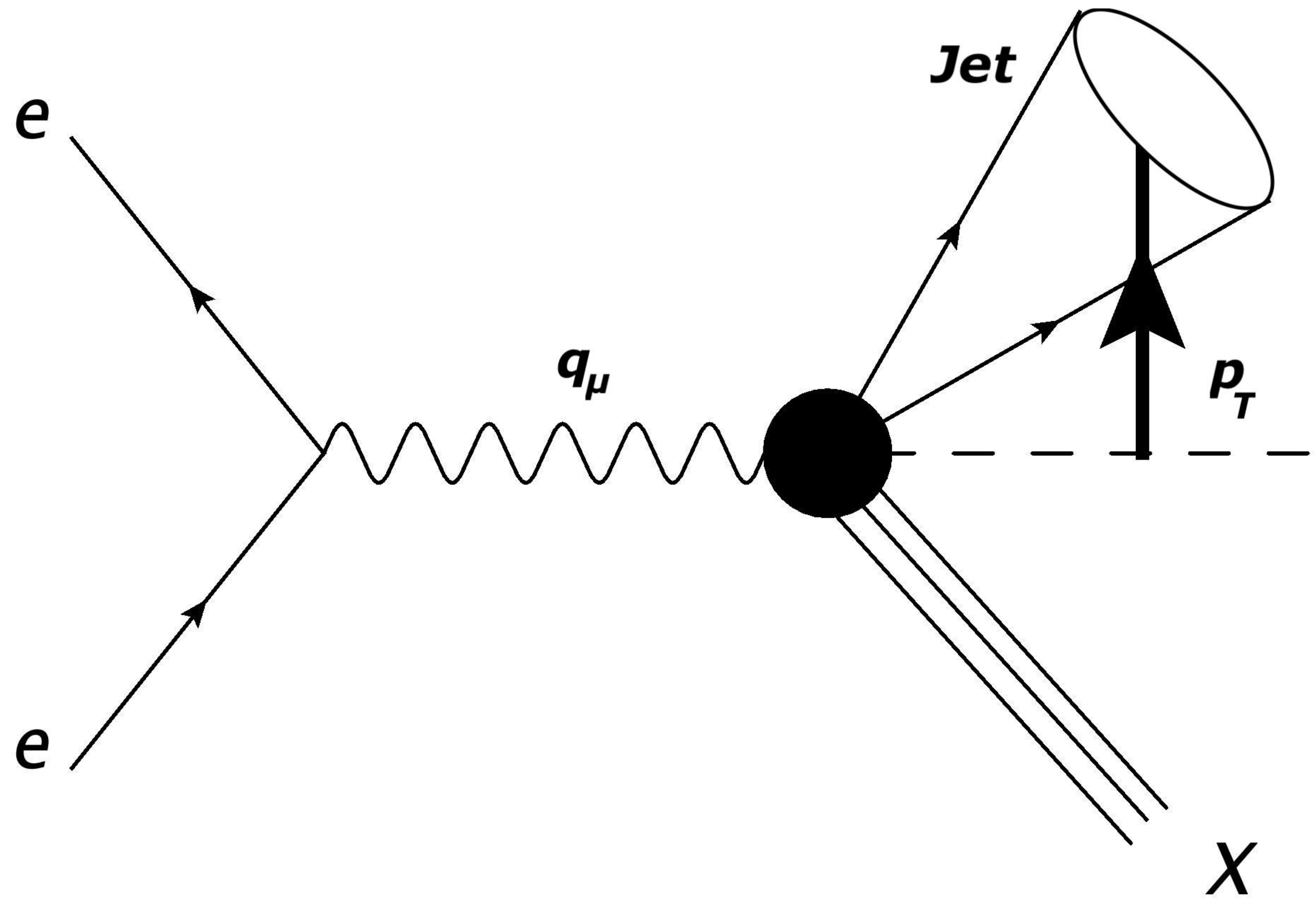
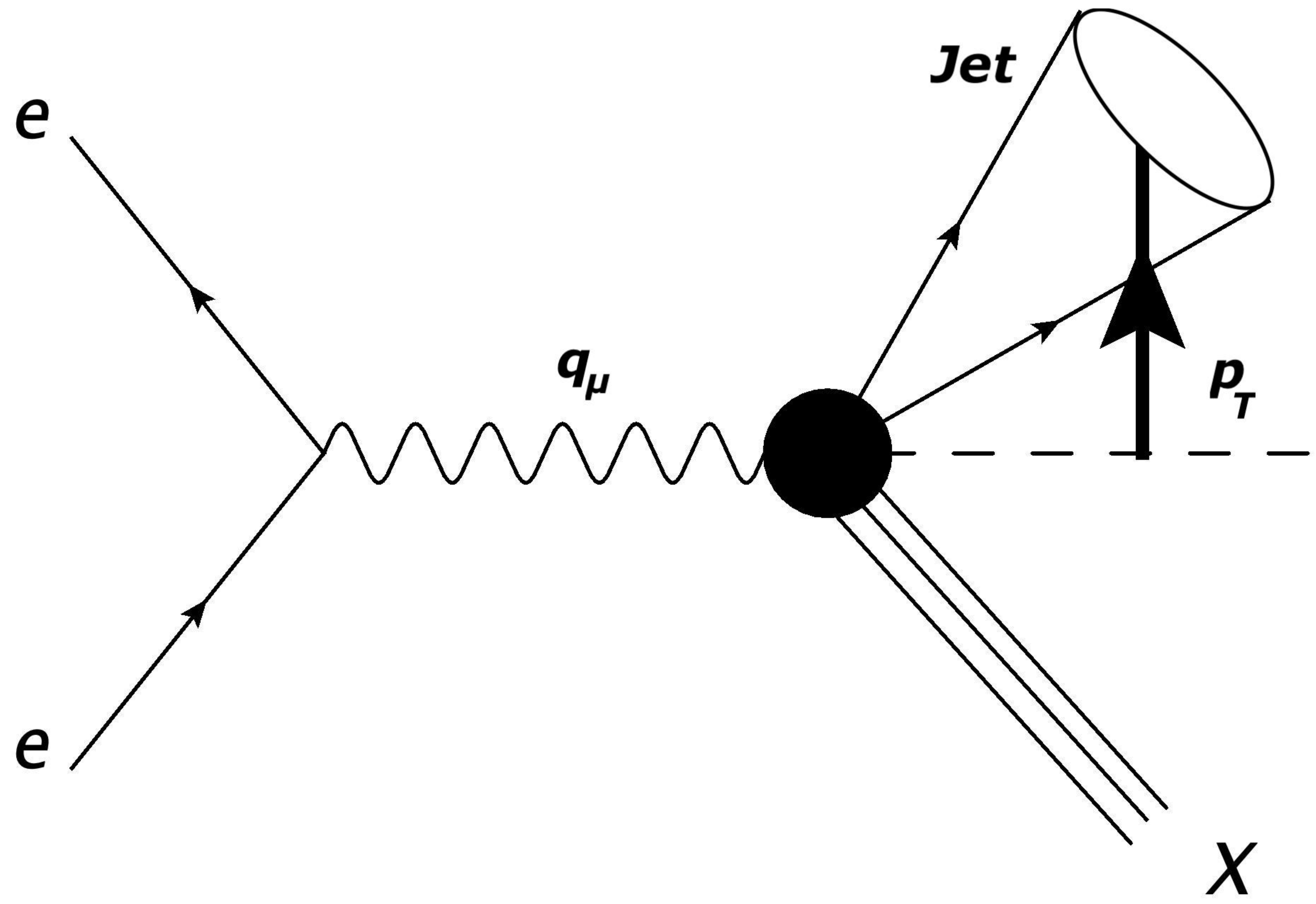
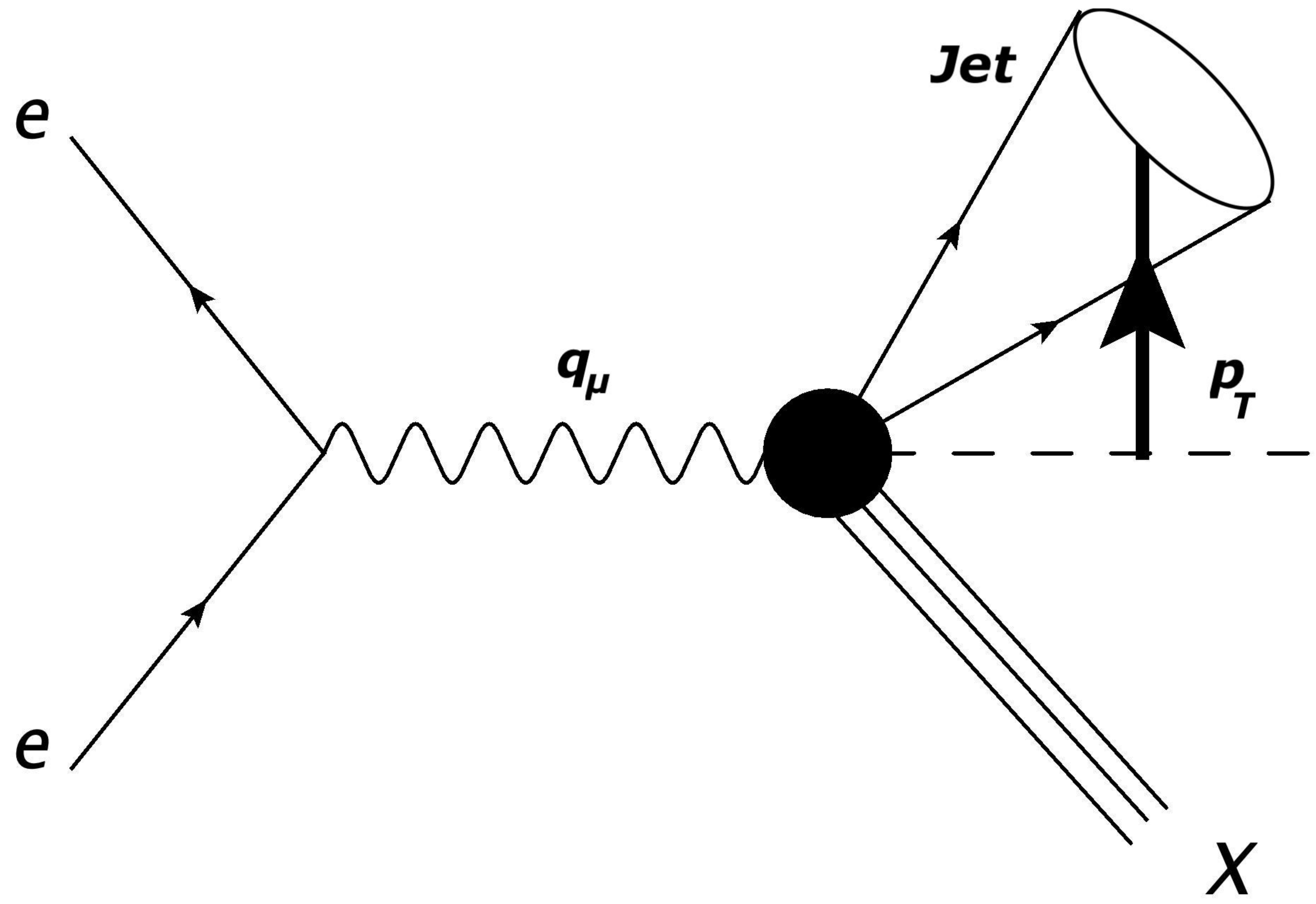


Jet-Sidis Process



★ SCET approach

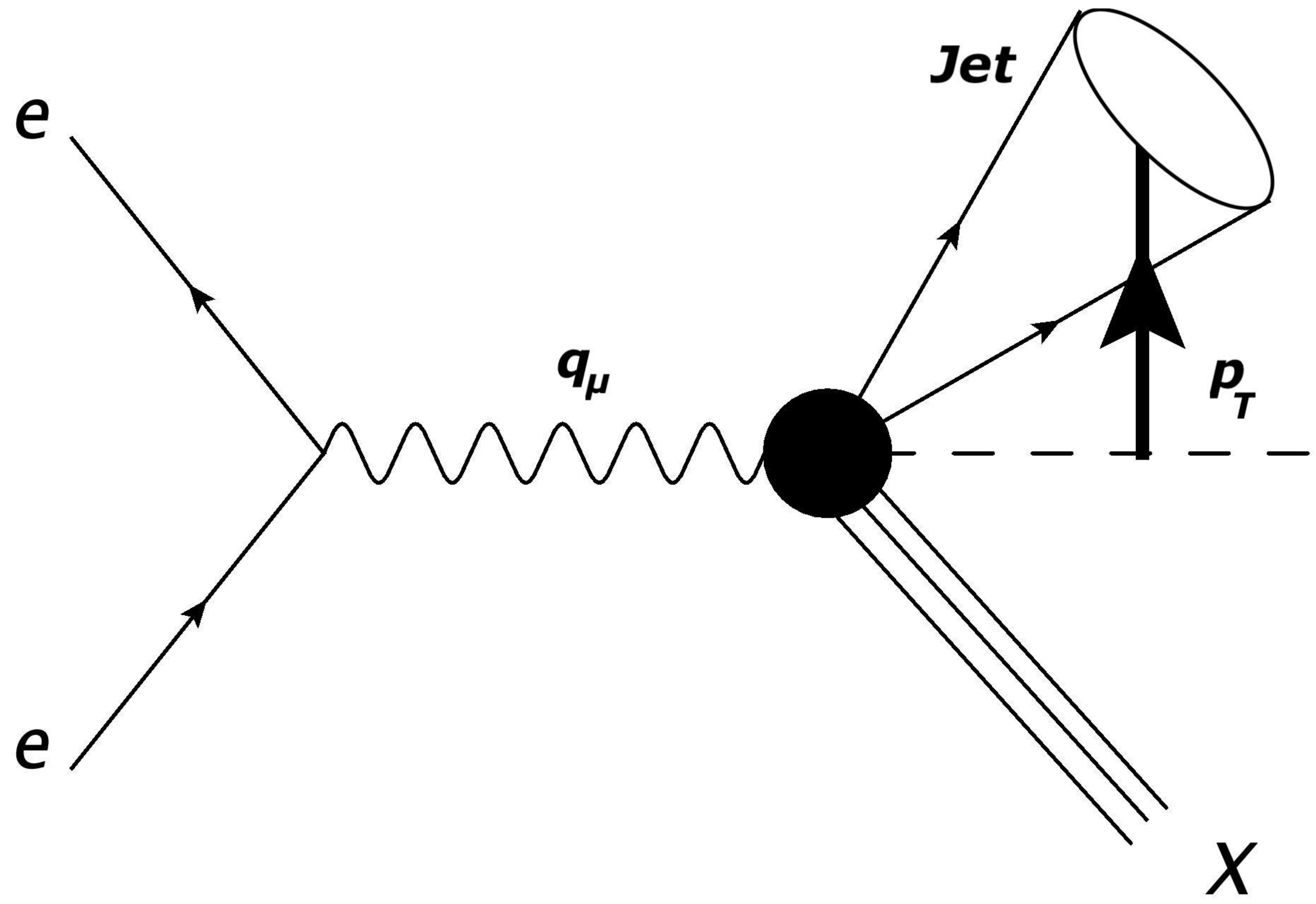
Jet-Sidis Process



★ SCET approach

★ $p_T \ll Q^2 = -q^2$

Jet-Sidis Process



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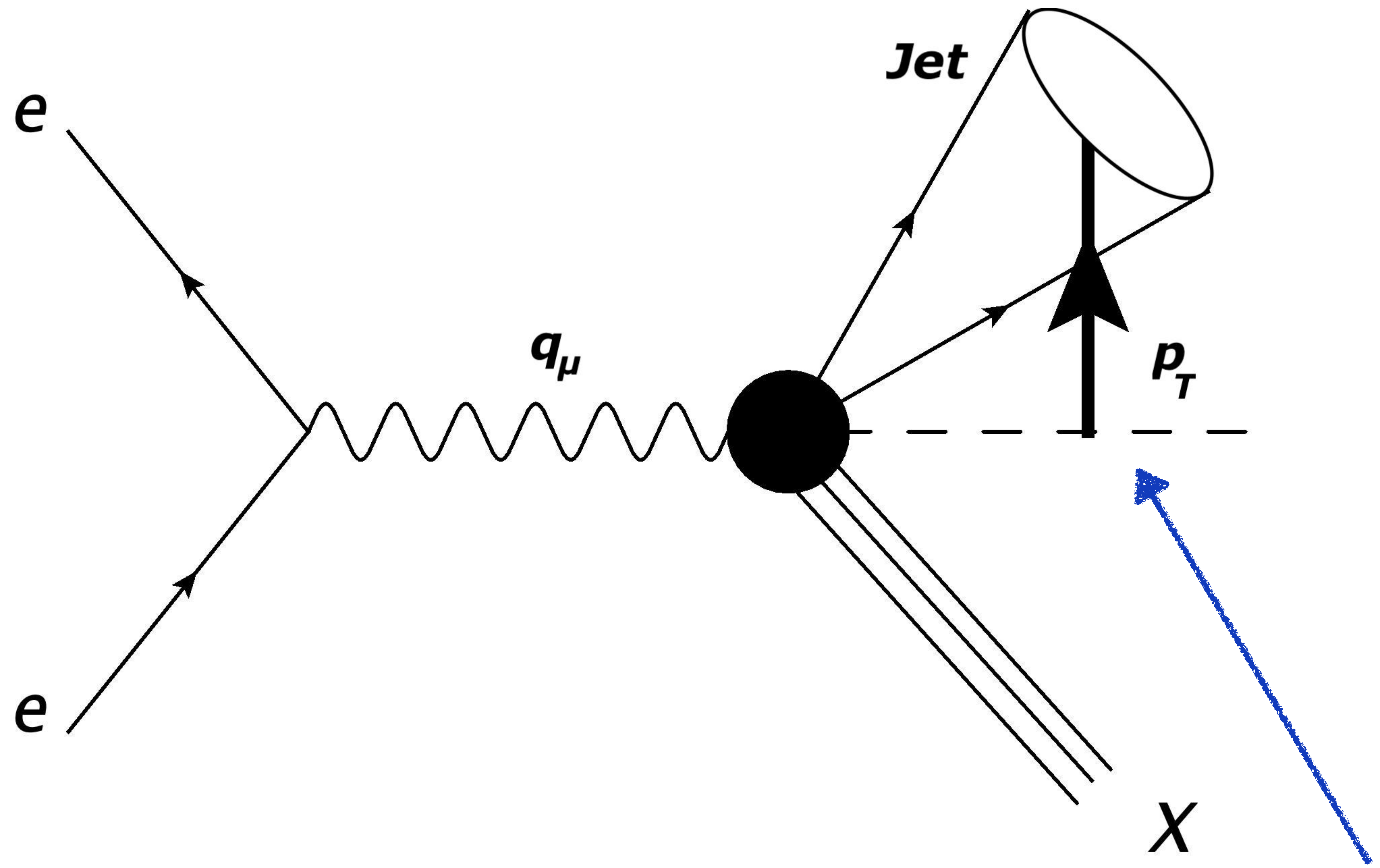
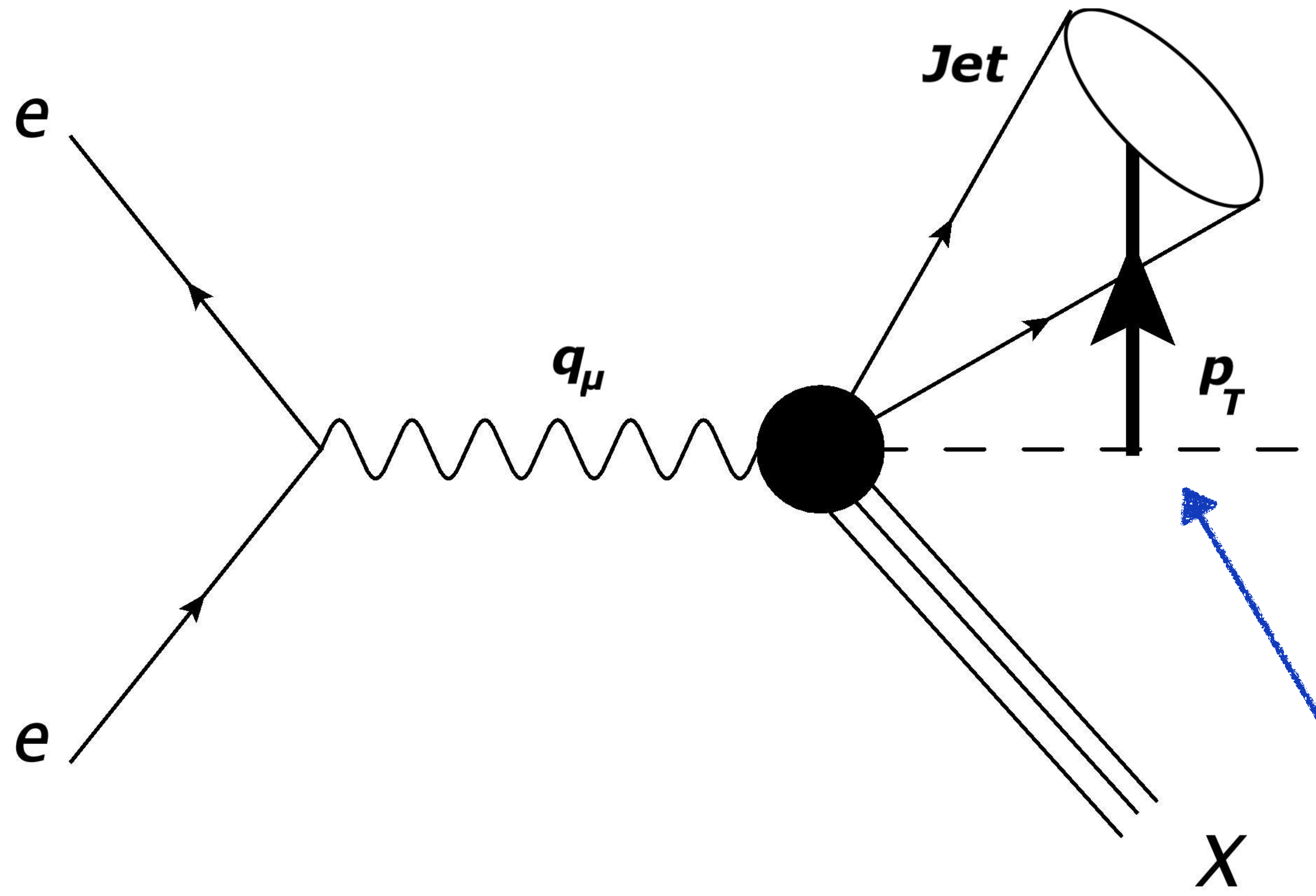
★ Large Radius

Jet-Sidis Process



- ★ SCET approach
- ★ $p_T \ll Q^2 = -q^2$
- ★ Large Radius
- ★ Photon axis

Jet-Sidis Process



★ SCET approach

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Usual TMDPDFs and TMDFFs

Jet-Sidis Process

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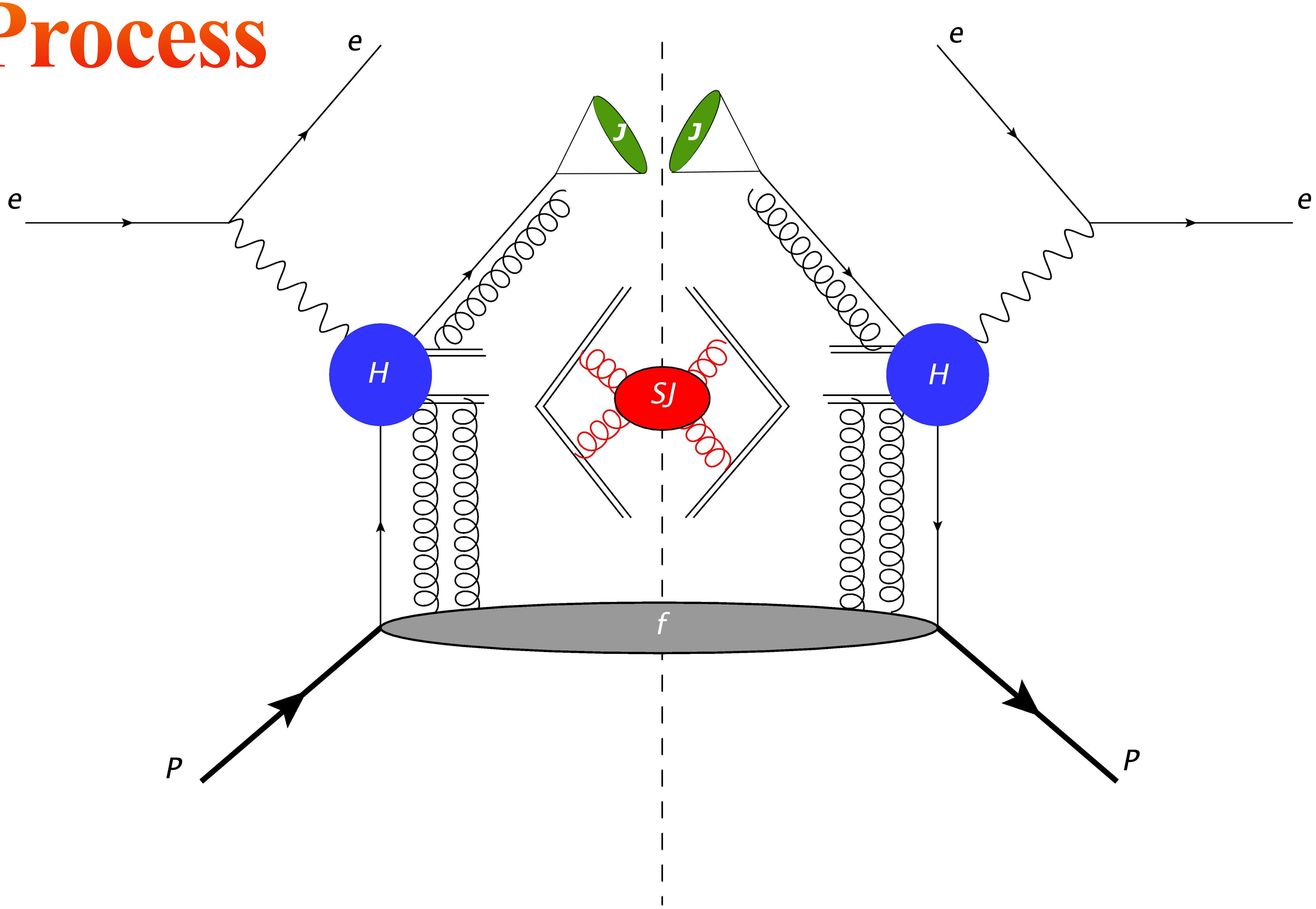

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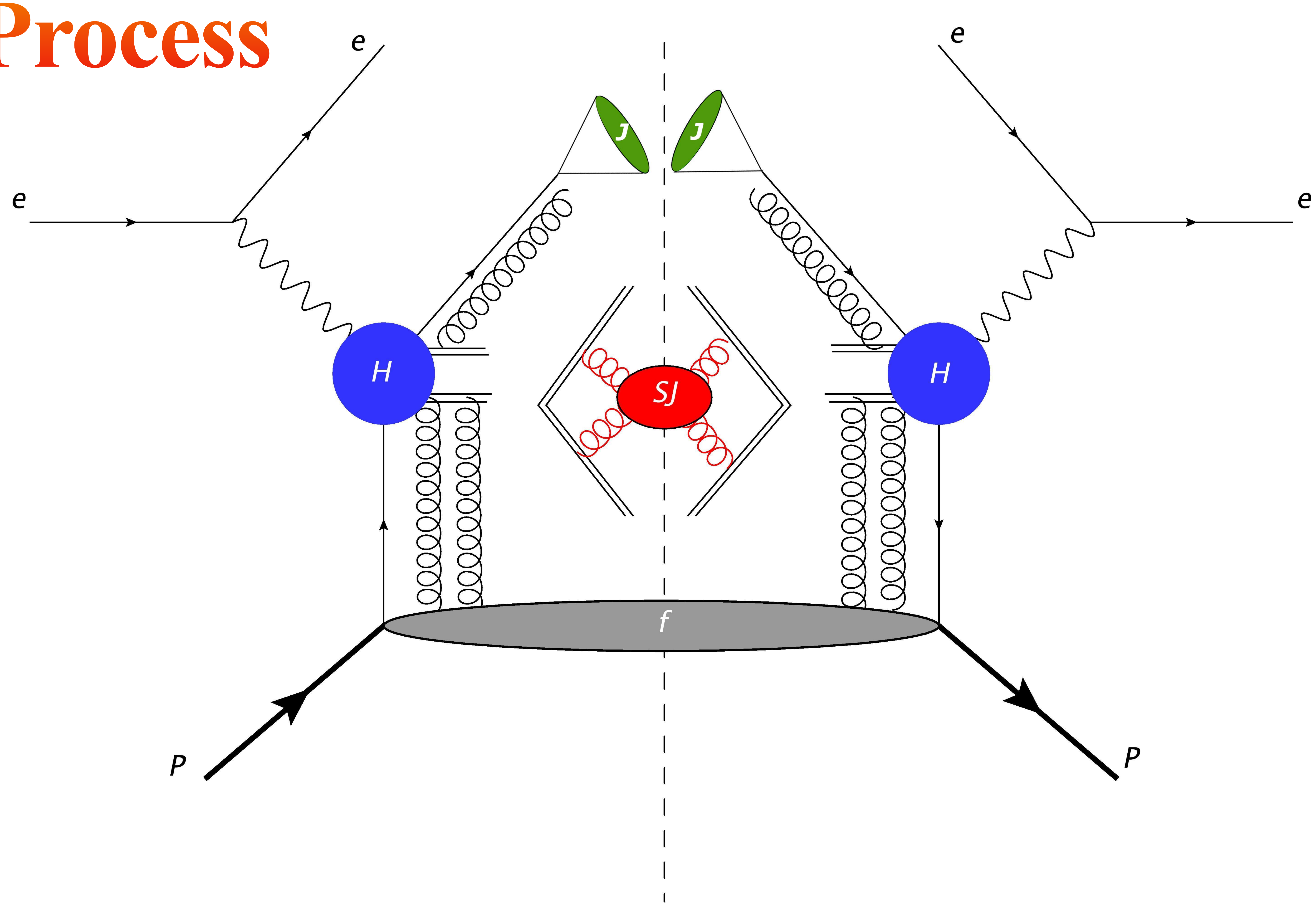
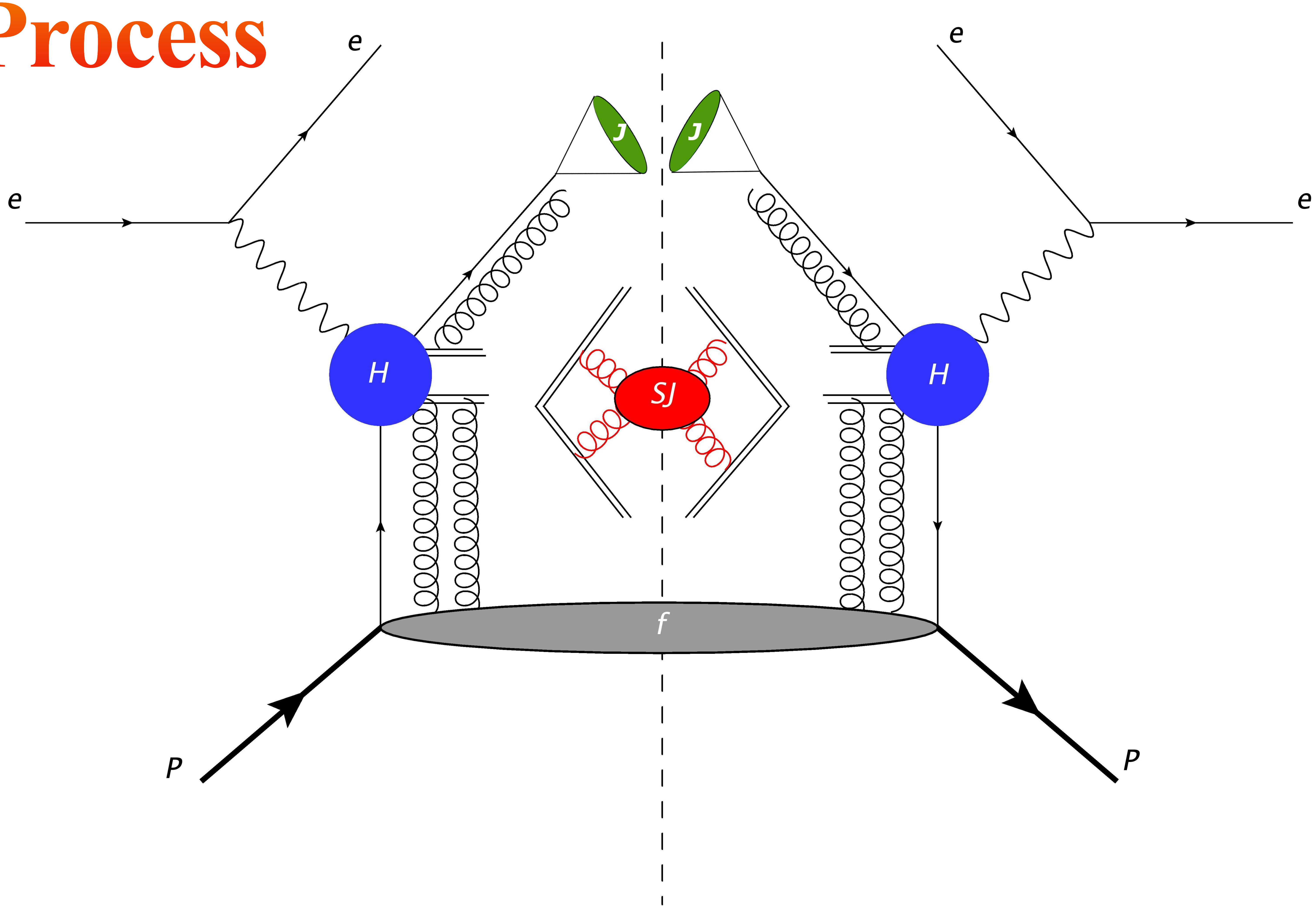


$$F_{UU,T}^{\text{jet}} = \sum_{ij} H_{ij}(Q;\mu_Q)\,\mathscr{B}_0\Big[$$

Jet-Sidis Process



$$F_{UU,T}^{\text{jet}} = \sum_{ij} H_{ij}(Q;\mu_Q)\,\mathscr{B}_0\Big[\, f_{1j}^{\text{uns.}}(x, b_T, Q; \mu, \zeta_2)$$

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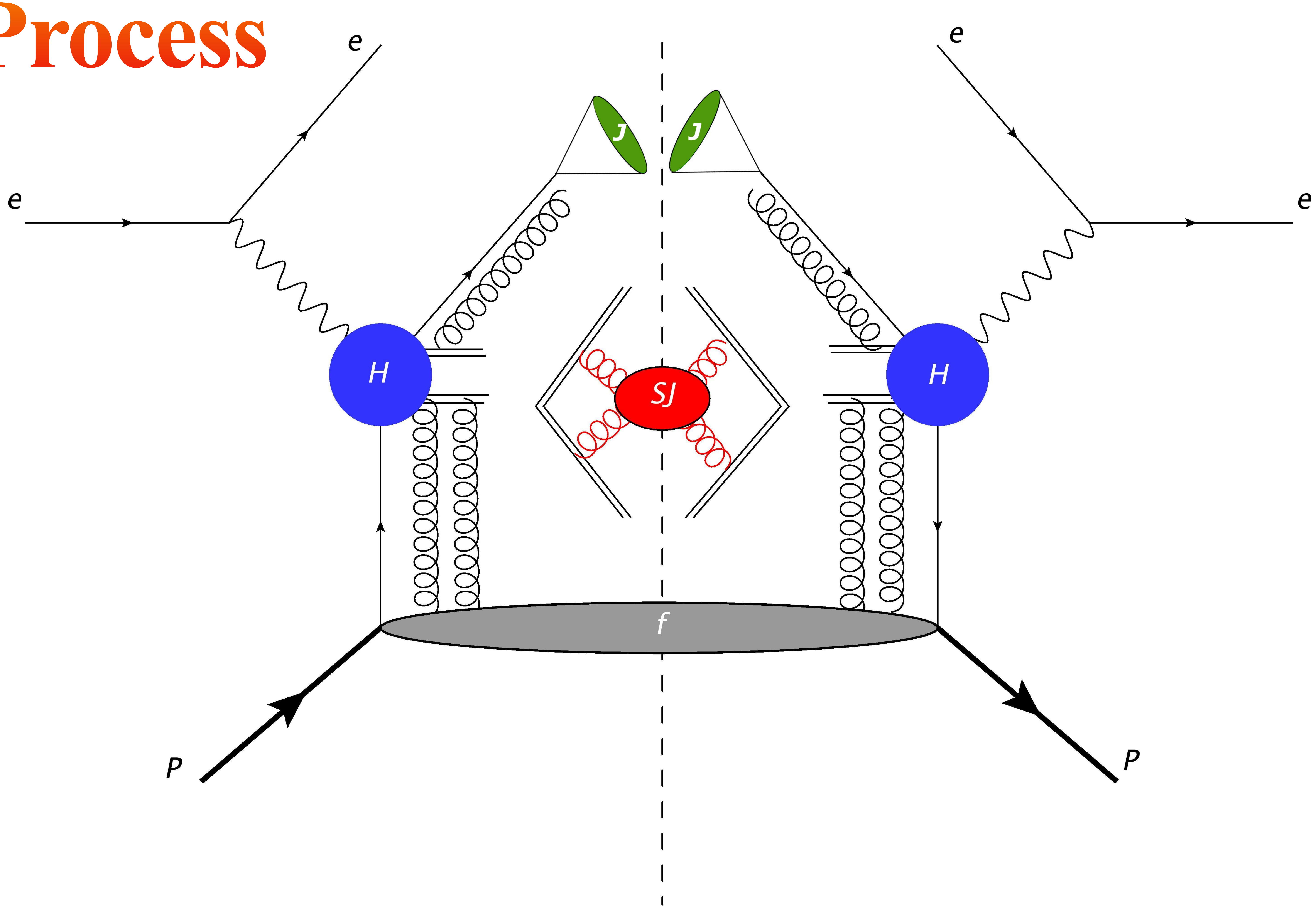

$$F_{UU,T}^{\text{jet}} = \sum_{ij} H_{ij}(Q;\mu_Q)\,\mathscr{B}_0\Big[f_{1j}^{\text{uns.}}(x, b_T, Q; \mu, \zeta_2)\, J_{i\to\text{jet}}(Q, R; \mu_j)$$

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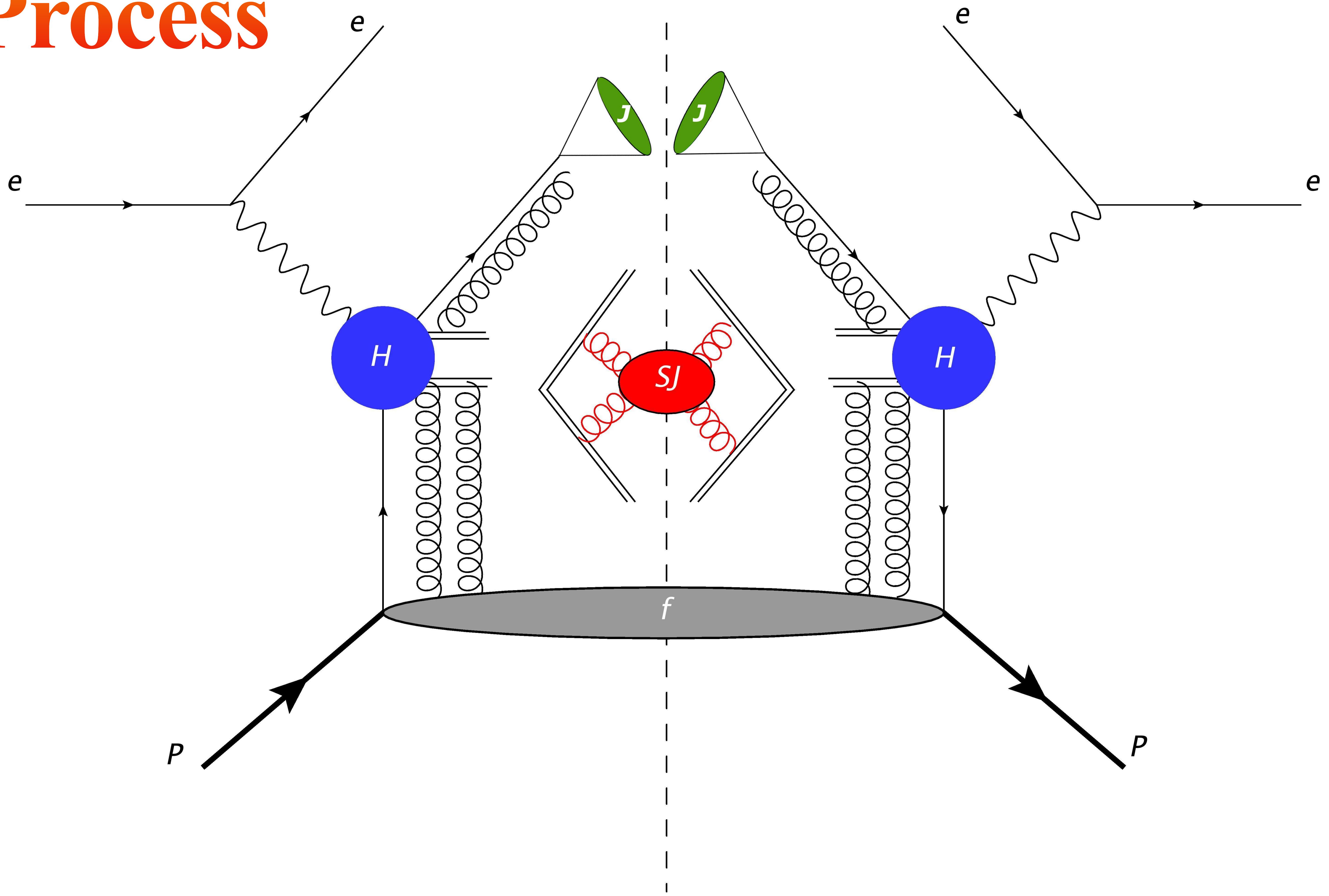
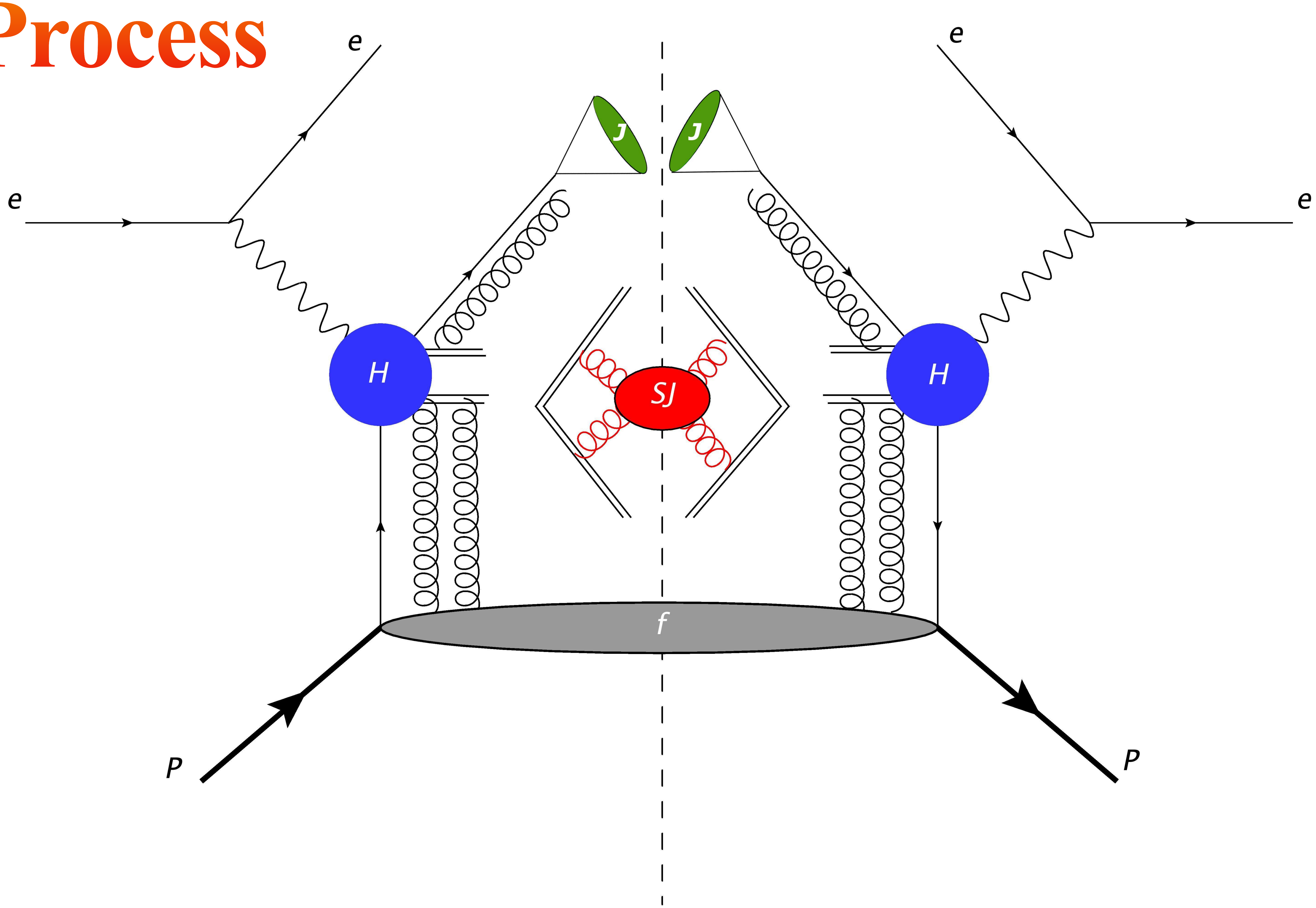


$$F_{UU,T}^{\text{jet}} = \sum_{ij} H_{ij}(Q;\mu_Q)\mathscr{B}_0\left[f_{1j}^{\text{uns.}}(x,b_T,Q;\mu,\zeta_2)J_{i\to\text{jet}}(Q,R;\mu_j)\mathcal{S}_J(b_T,R;\mu_{Sj},\zeta_R)\right],$$

Jet-Sidis Process



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where $$\mathscr{B}_0[f\,D] = \frac{x}{2\pi}\int_0^\infty db_T\, b_T J_0(b_T p_T) f\big(x, b_T; Q^2\big)\, D\big(z, b_T^2; Q^2\big)\,.$$

Parallelism to SIDIS

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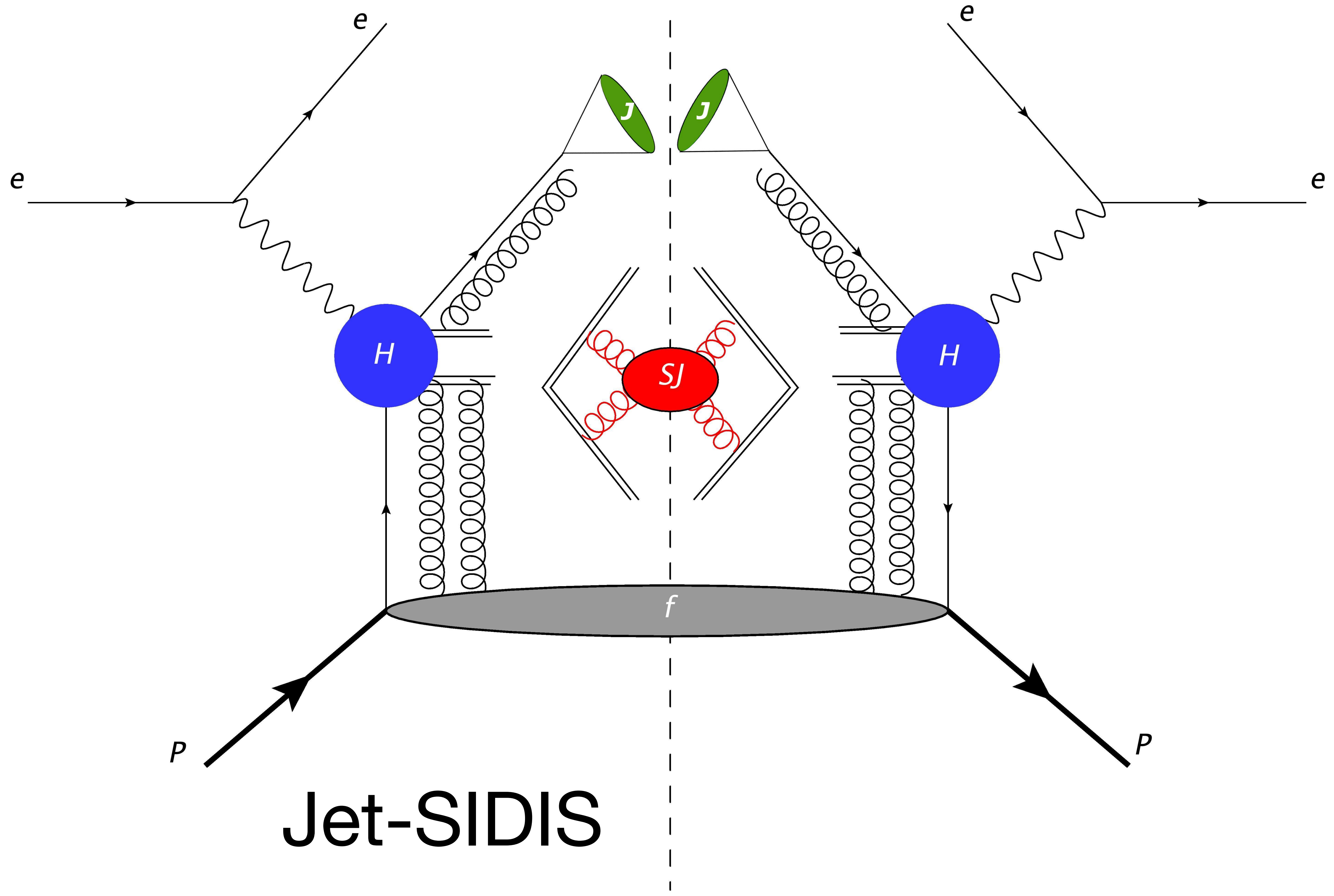

Jet-SIDIS

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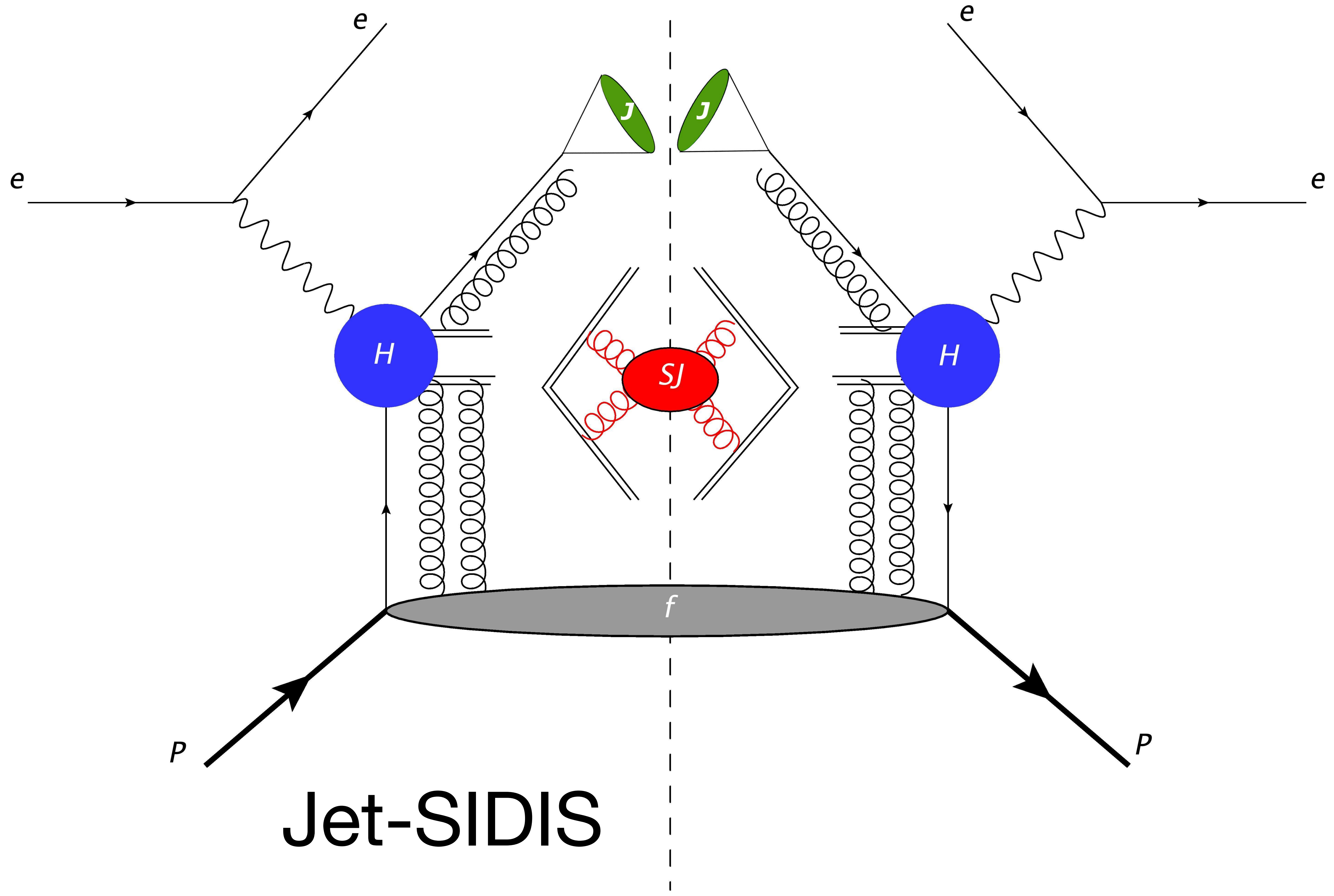

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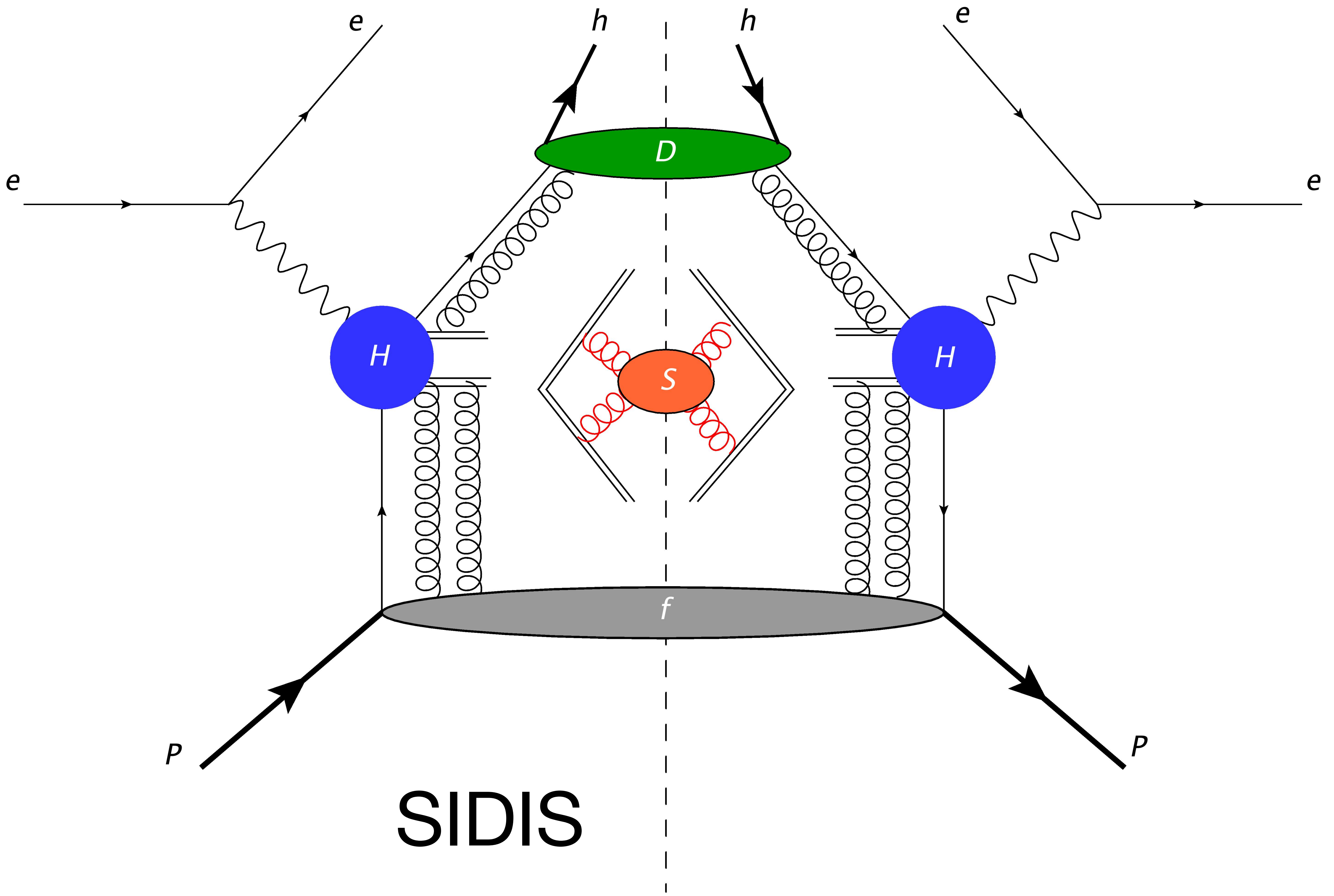

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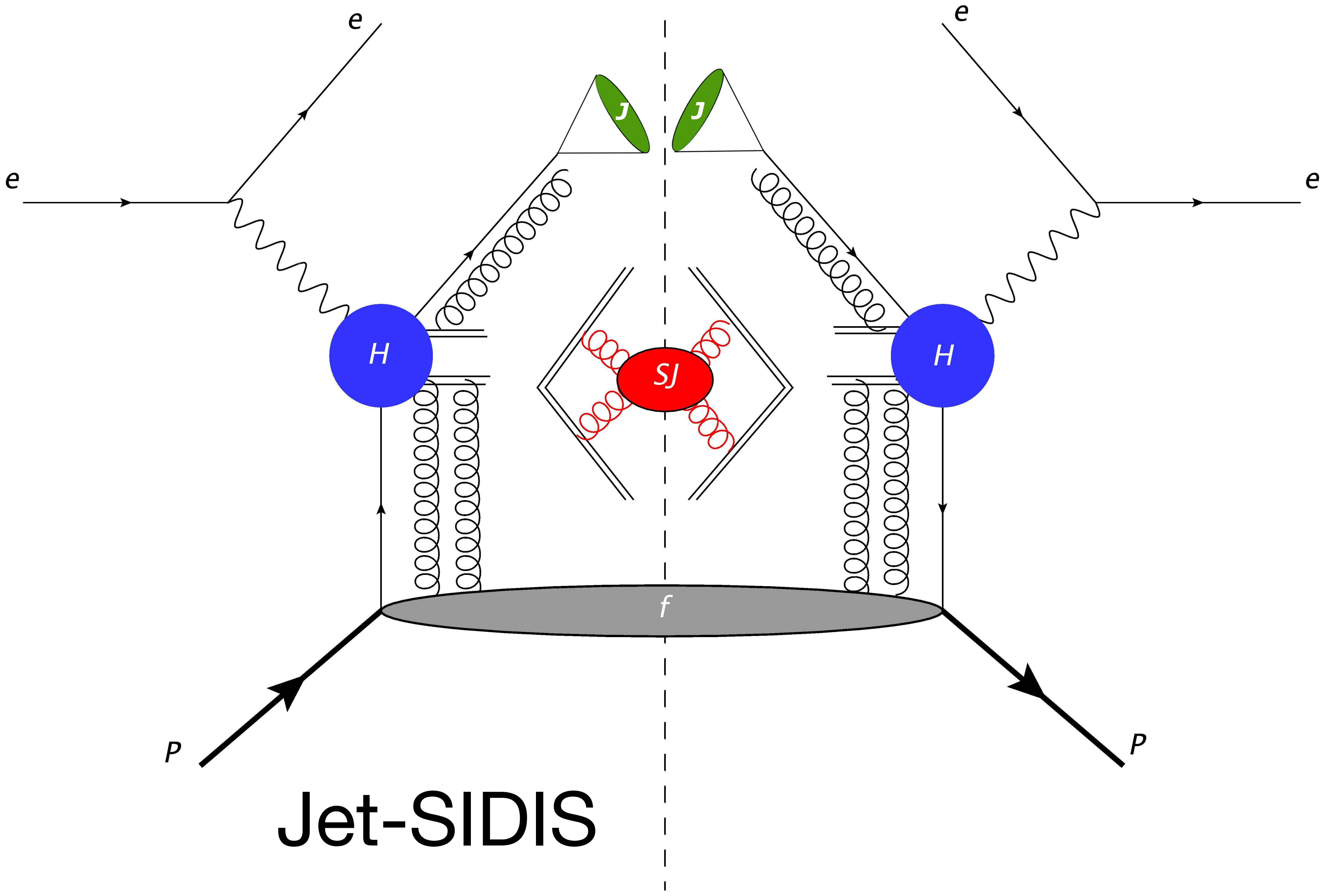

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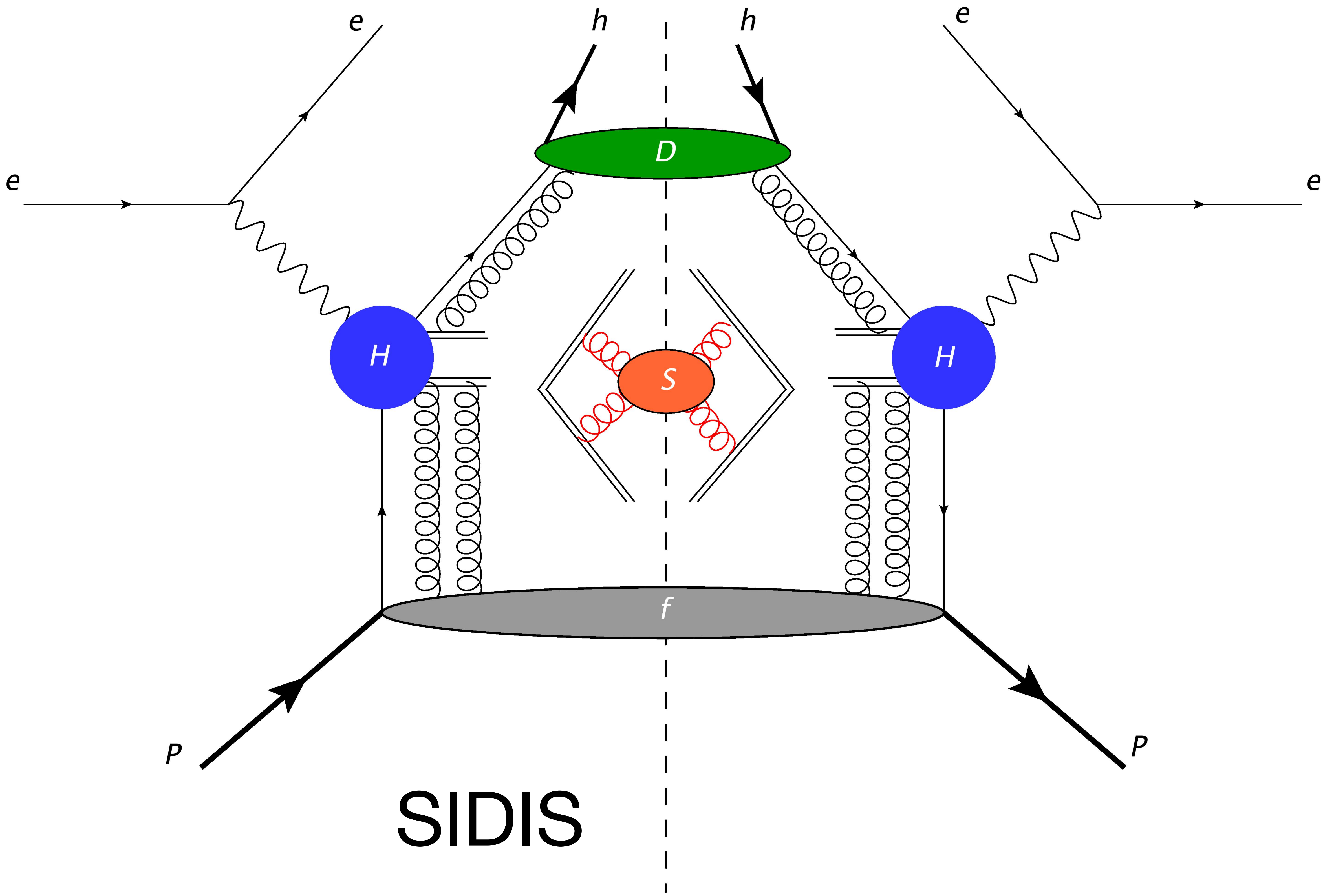

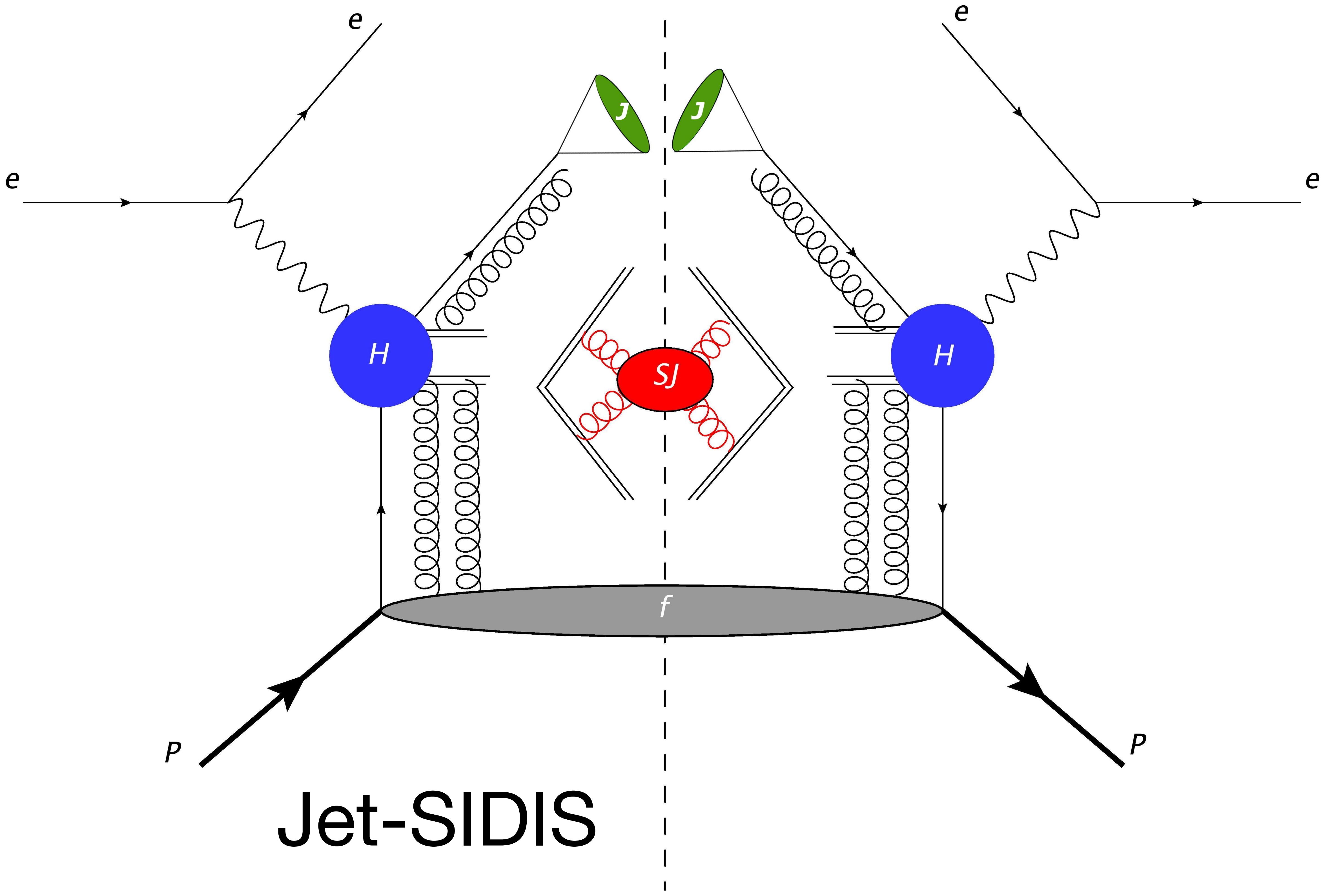

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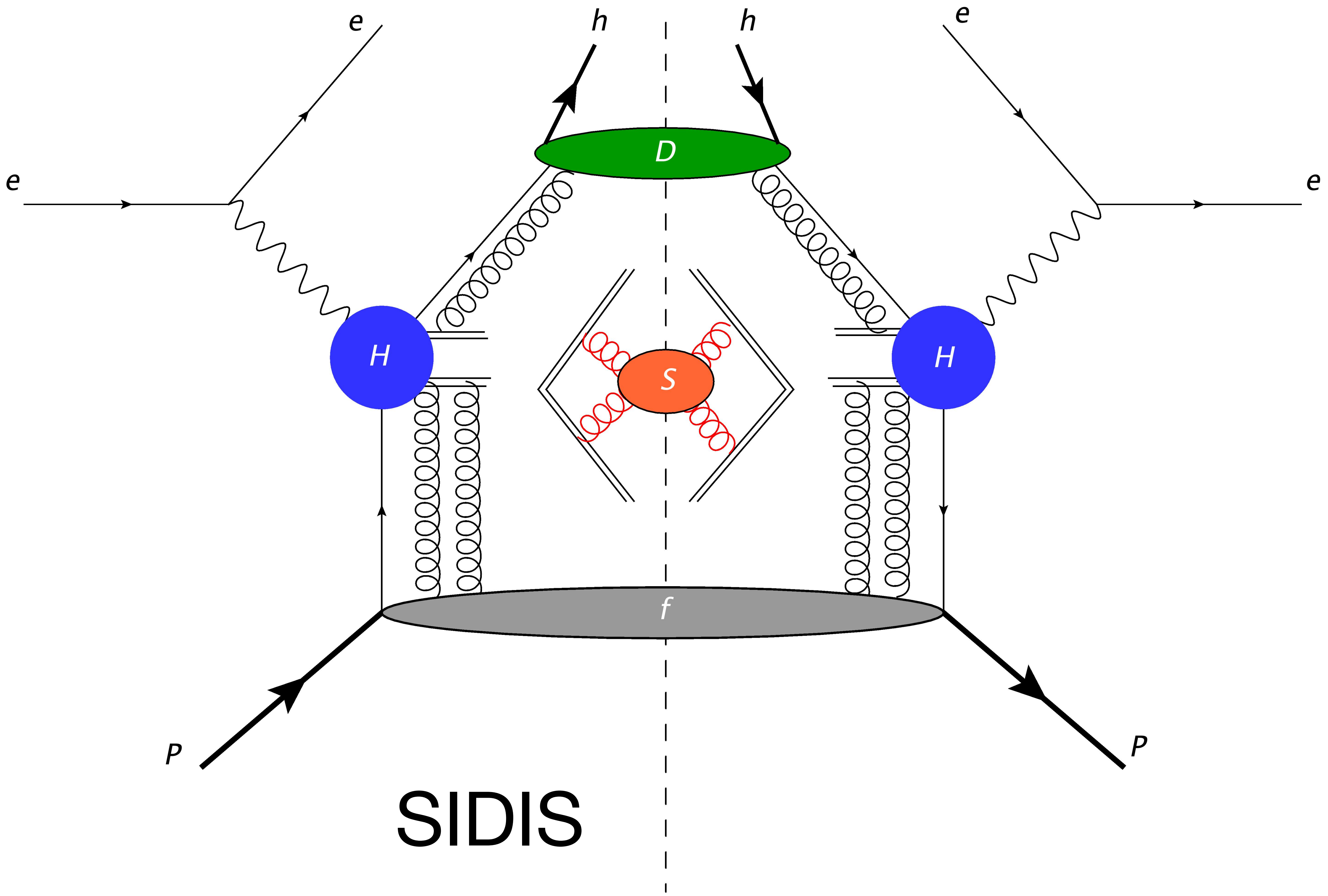

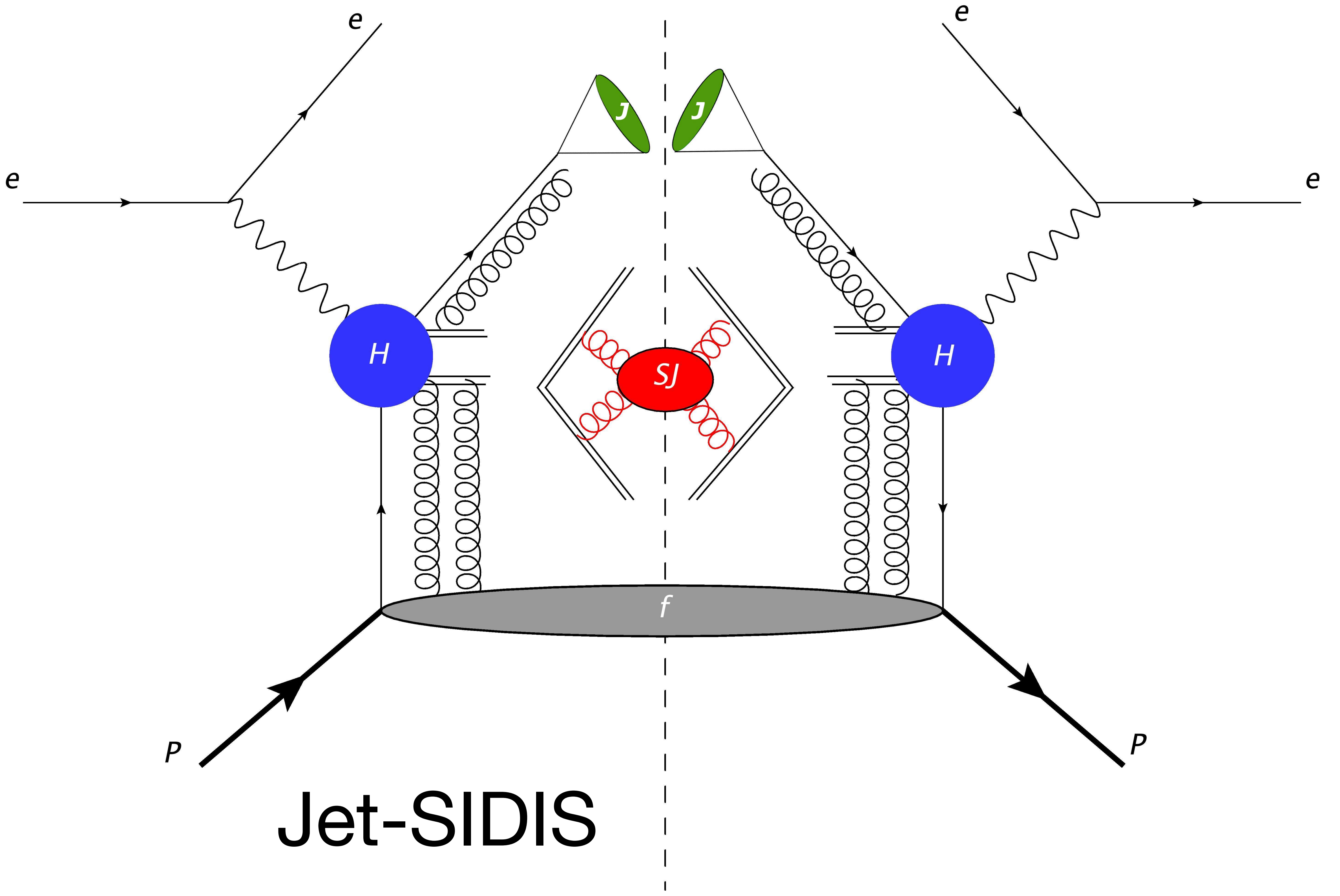

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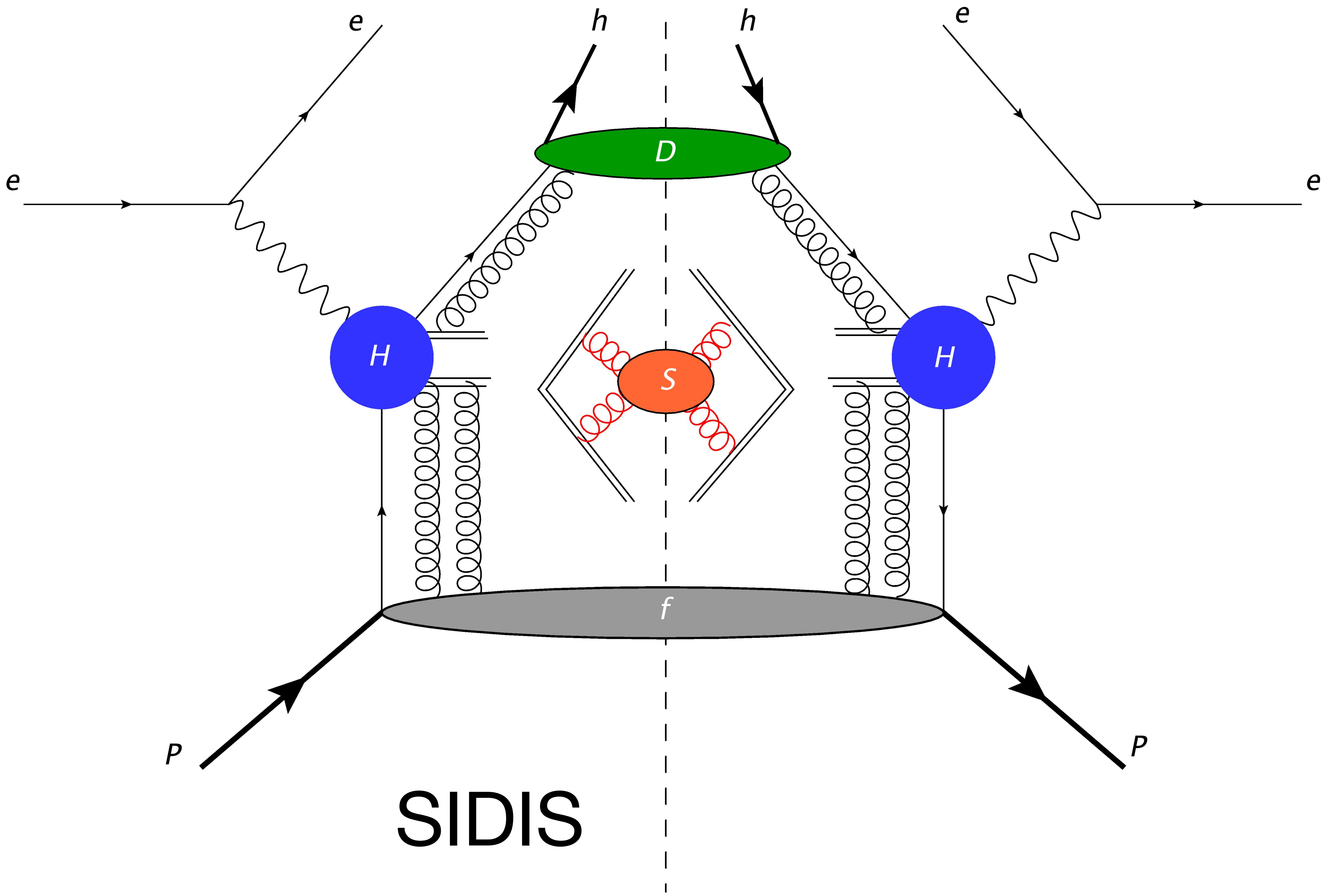
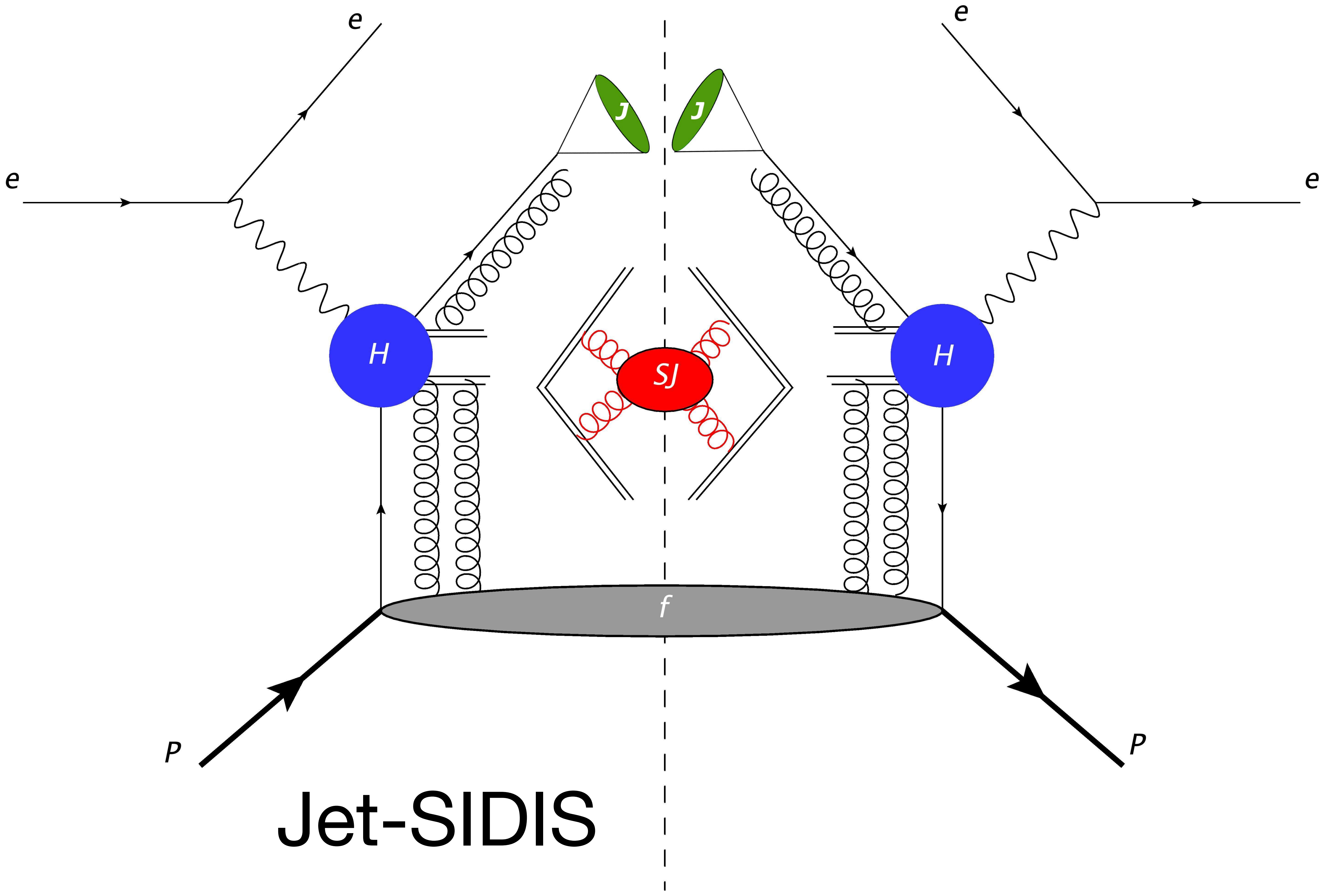


SIDIS



Jet-SIDIS

$$F_{UU,T}^{\mathrm{SIDIS}} = \sum_{ij} H_{ij}(Q;\mu_Q)\mathscr{B}_0\Big[f_{1j}^{\mathrm{uns.}}(x, b_T, Q;\mu,\zeta_2)\,D_{i\to h}^{\mathrm{uns.}}(z, b_T;\mu,\zeta)\Big)$$

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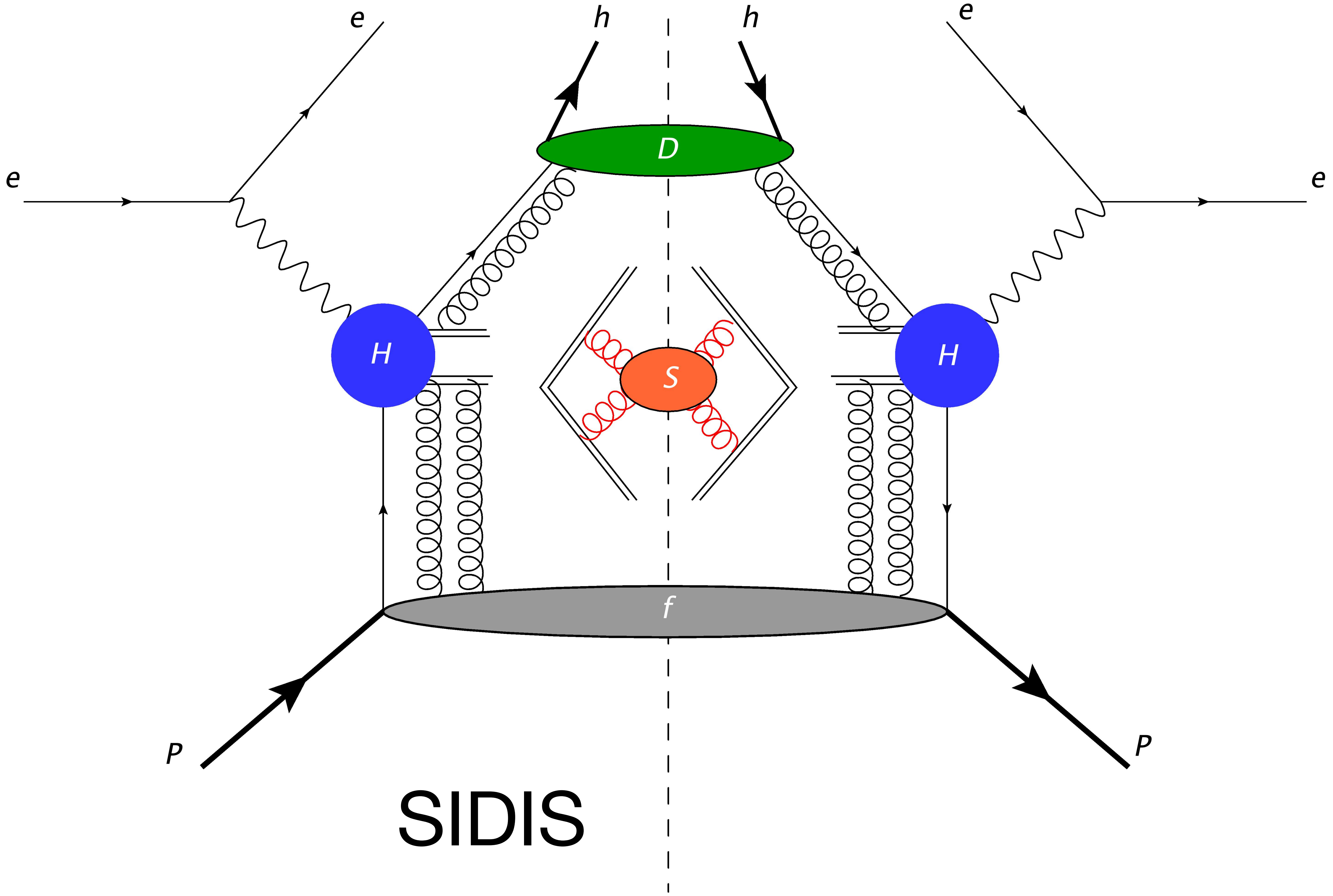
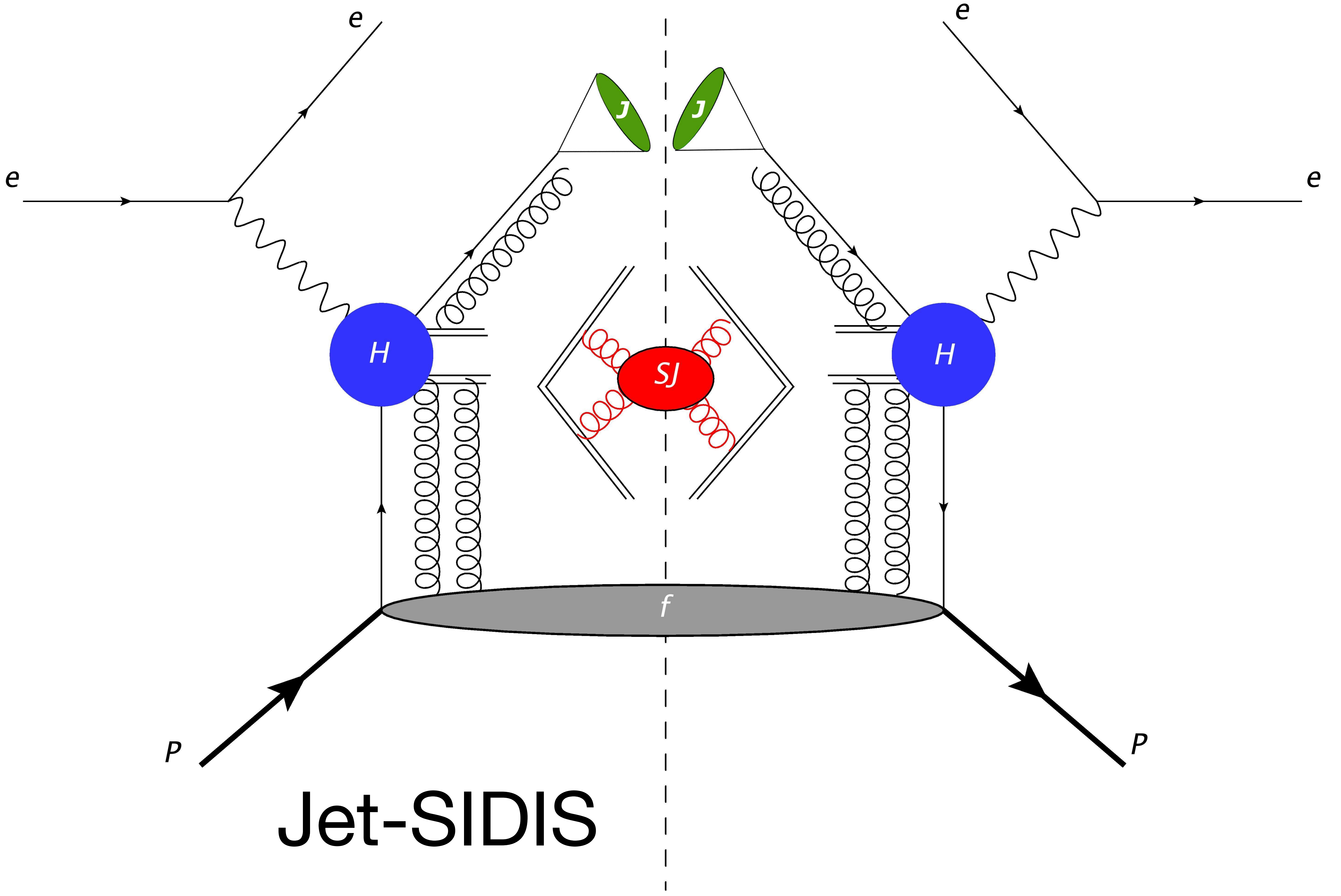
Parallelism to SIDIS





$$F^{\mathrm{SIDIS}}_{UU,T} = \sum_{ij} H_{ij}(Q;\mu_Q)\,\mathscr{B}_0\left[f_{1j}^{\mathrm{uns.}}(x, b_T, Q; \mu, \zeta_2)\, D^{\mathrm{uns.}}_{i\to h}(z, b_T; \mu, \zeta)\, S(b_T; \mu_S, \zeta_S) \right],$$

$$F^{\mathrm{jet}}_{UU,T} = \sum_{ij} H_{ij}(Q;\mu_Q)\,\mathscr{B}_0\left[f_{1j}^{\mathrm{uns.}}(x, b_T, Q; \mu, \zeta_2)\, J_{i\to \mathrm{jet}}(Q, R; \mu_j)\ \mathcal{S}_J(b_T, R; \mu_{Sj}, \zeta_R) \right]$$

Outline

Jet-Sidis Process

Jet-TMDFF

Phenomenological Estimates

Jet-TMDFF

Jet-TMDFF

It's useful to introduce a new function called Jet-TMDFF:

$$D_{i\to\mathrm{jet}}(b_T, Q, R; \mu, \zeta_1) = \frac{J_{i\to\mathrm{jet}}(Q, R; \mu_J)\mathscr{S}_J(b_T, R; \mu_R, \zeta_R)}{\sqrt{S(b; \mu_S, \zeta_S)}}$$

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$$\Longrightarrow \qquad F_{UU,T}^{\mathrm{jet}} = \sum_{ij} H_{ij}(Q; \mu_Q)\mathscr{B}_0\left[f_{1j}^{\mathrm{uns.}}(b_T, x, Q; \mu, \zeta_2)\sqrt{S(b; \mu_S, \zeta_S)}D_{i\to\mathrm{jet}}(b_T, Q, R; \mu, \zeta_1)\right]$$

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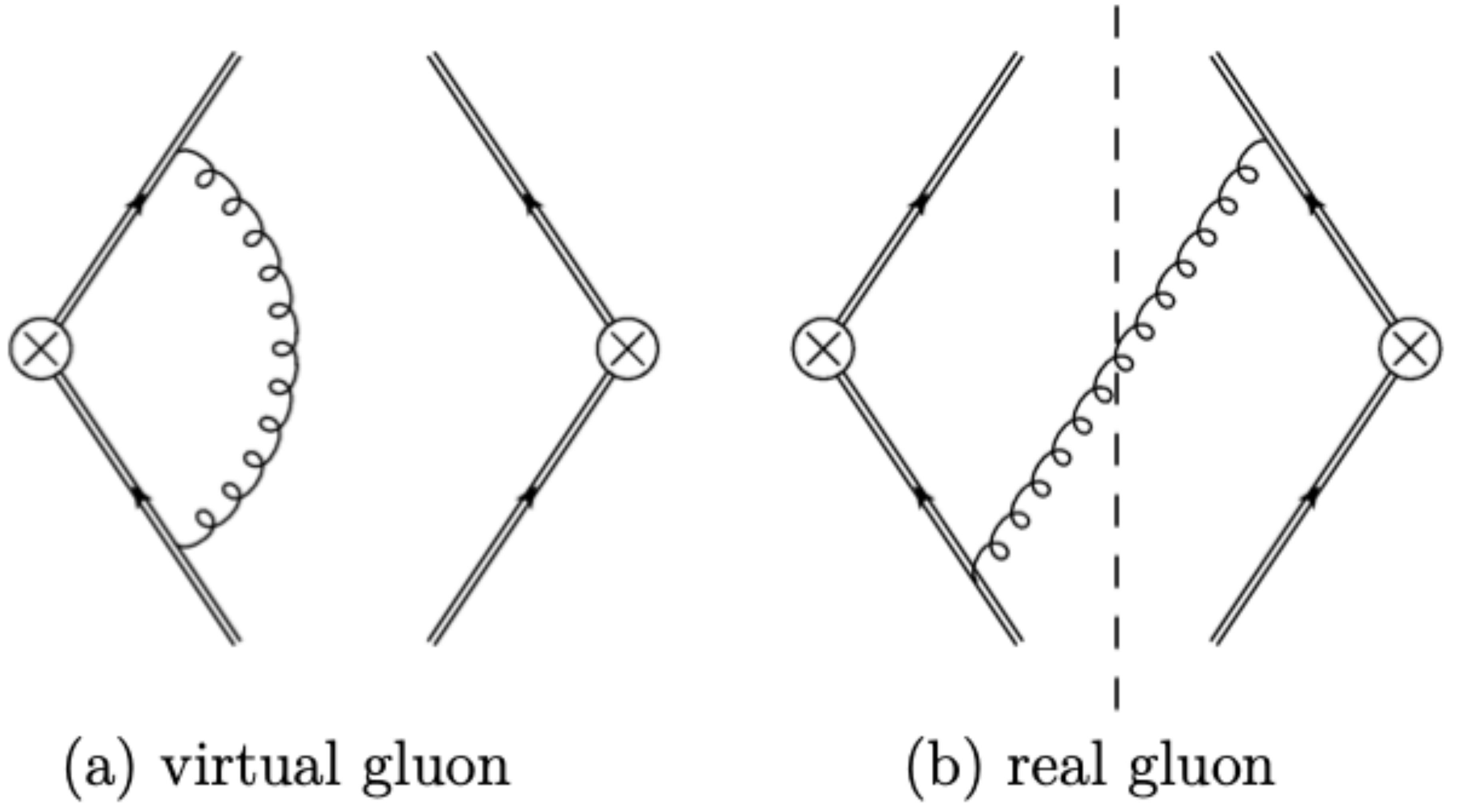
$$\Longrightarrow \qquad \boxed{F_{UU,T}^{\text{jet}} = \sum_{ij} H_{ij}(Q; \mu_Q)\mathcal{B}_0\left[f_{1j}^{\text{sub}}(b_T, x, Q; \mu, \zeta_2)D_{i\to\text{jet}}(b_T, Q, R; \mu, \zeta_1)\right]}$$

Jet Soft Function

Definition:

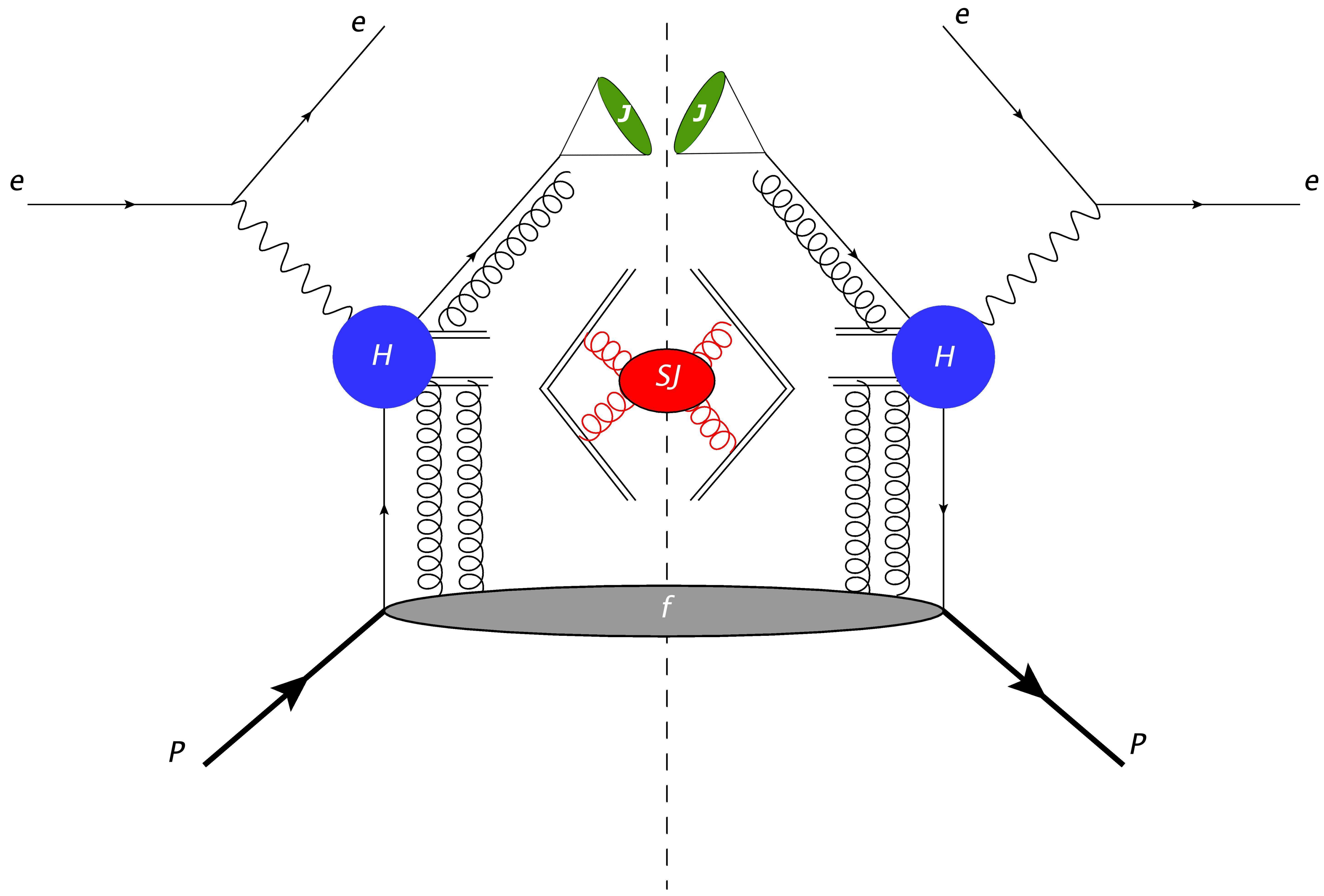
$$\mathcal{S}_J(b) = \frac{1}{N_c} \sum_{X_s} \langle 0 | T[S_n(b_\perp) S^\dagger_{\bar{n}}(b_\perp)] | X_s \rangle \langle X_s | \bar{T}[S_{\bar{n}}(0) S^\dagger_n(0)] | 0 \rangle$$



Diagrams contributing at next-to-leading order in the perturbative calculation of the jet Soft Function.
There are other eight mirror diagrams, which are not shown in this figure.

Jet Soft Function

Jet Soft Function



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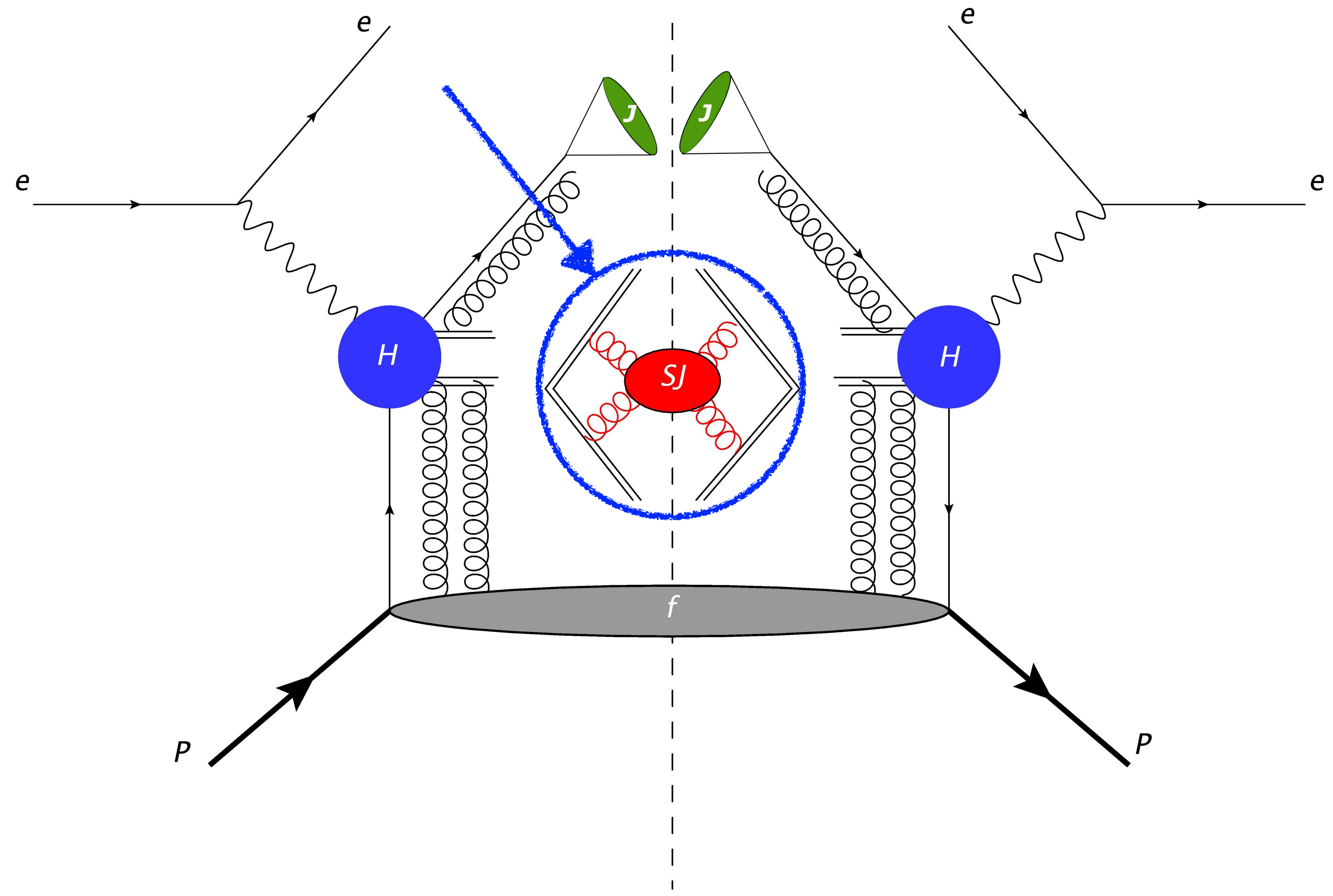

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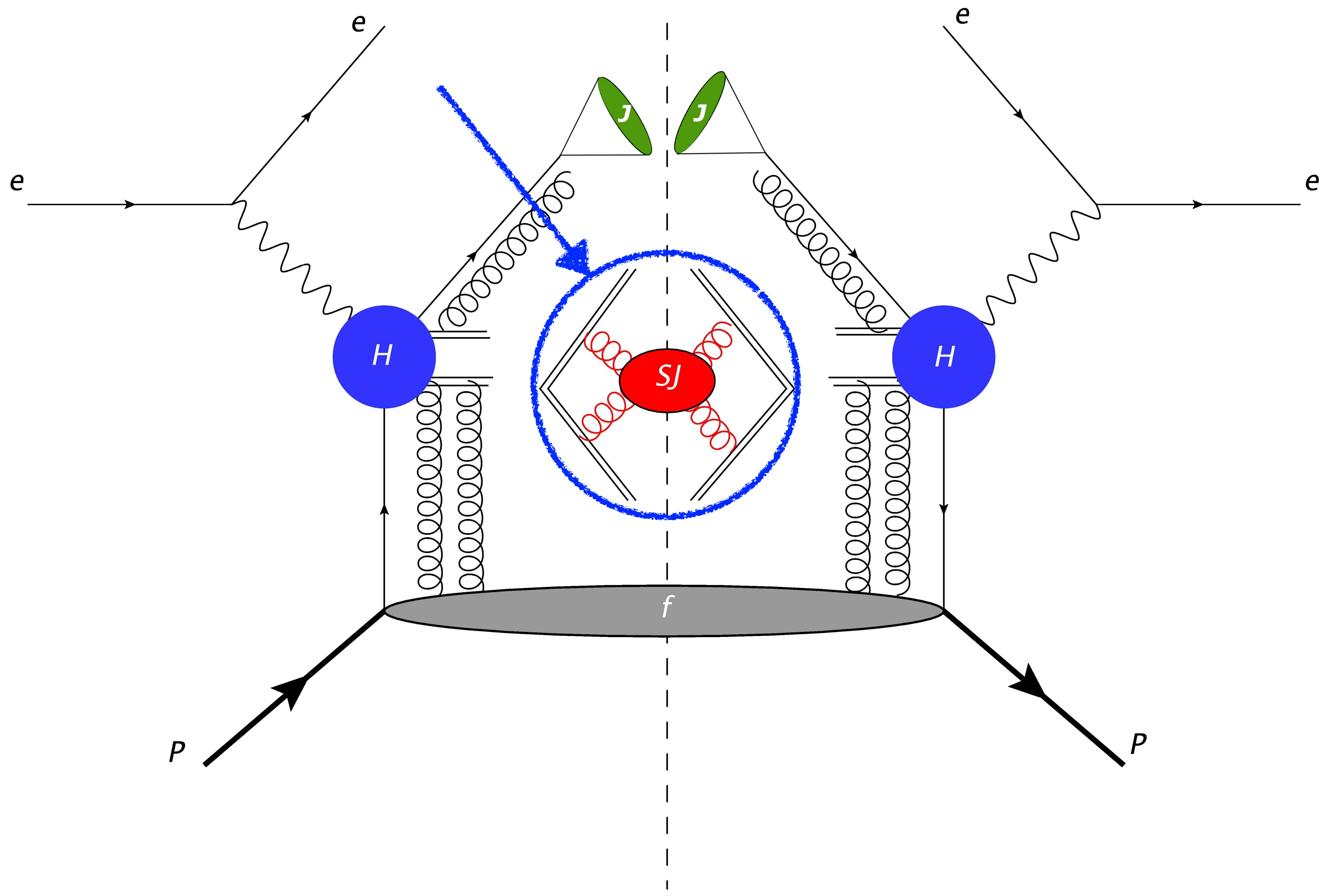
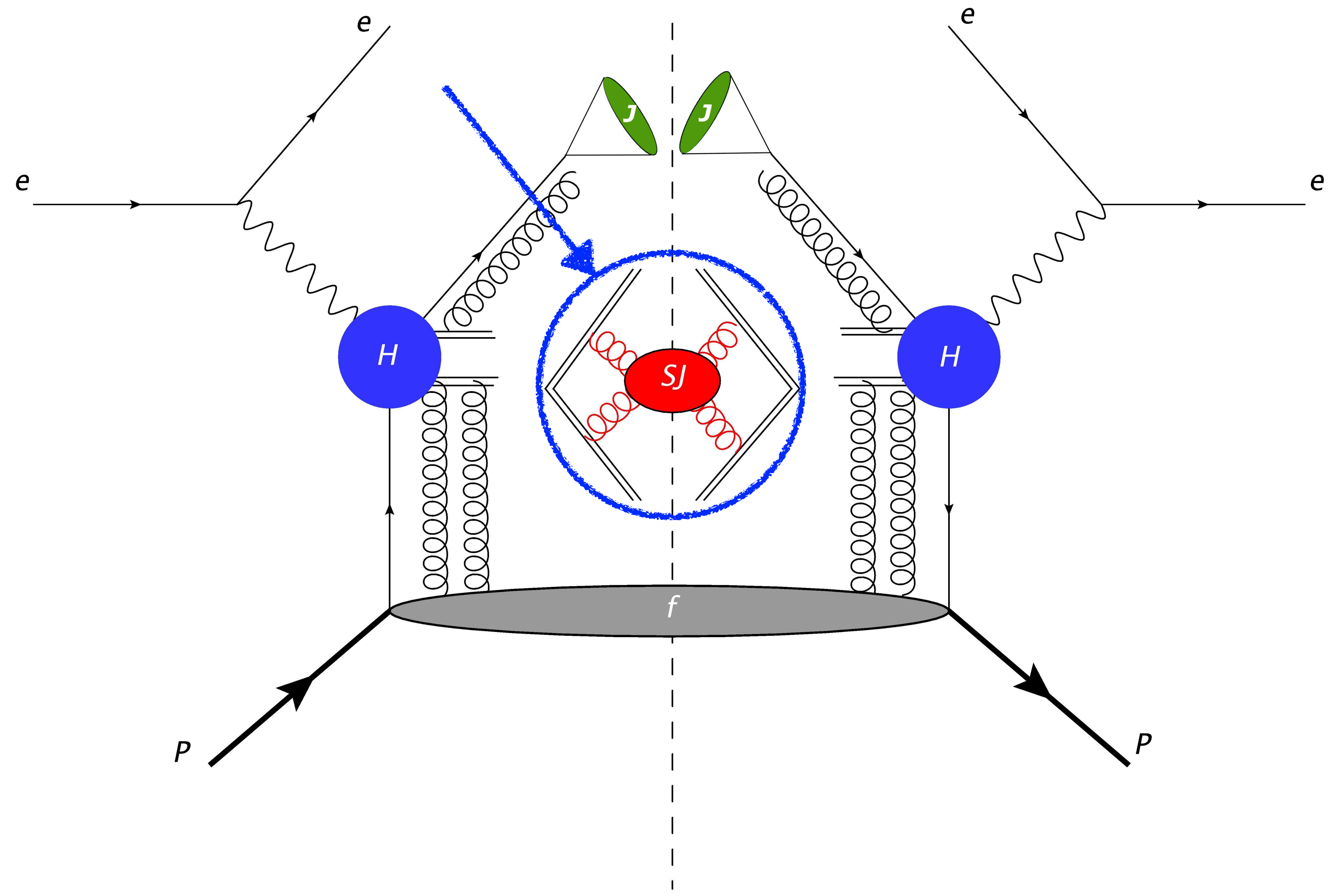


NLO expression

Jet Soft Function

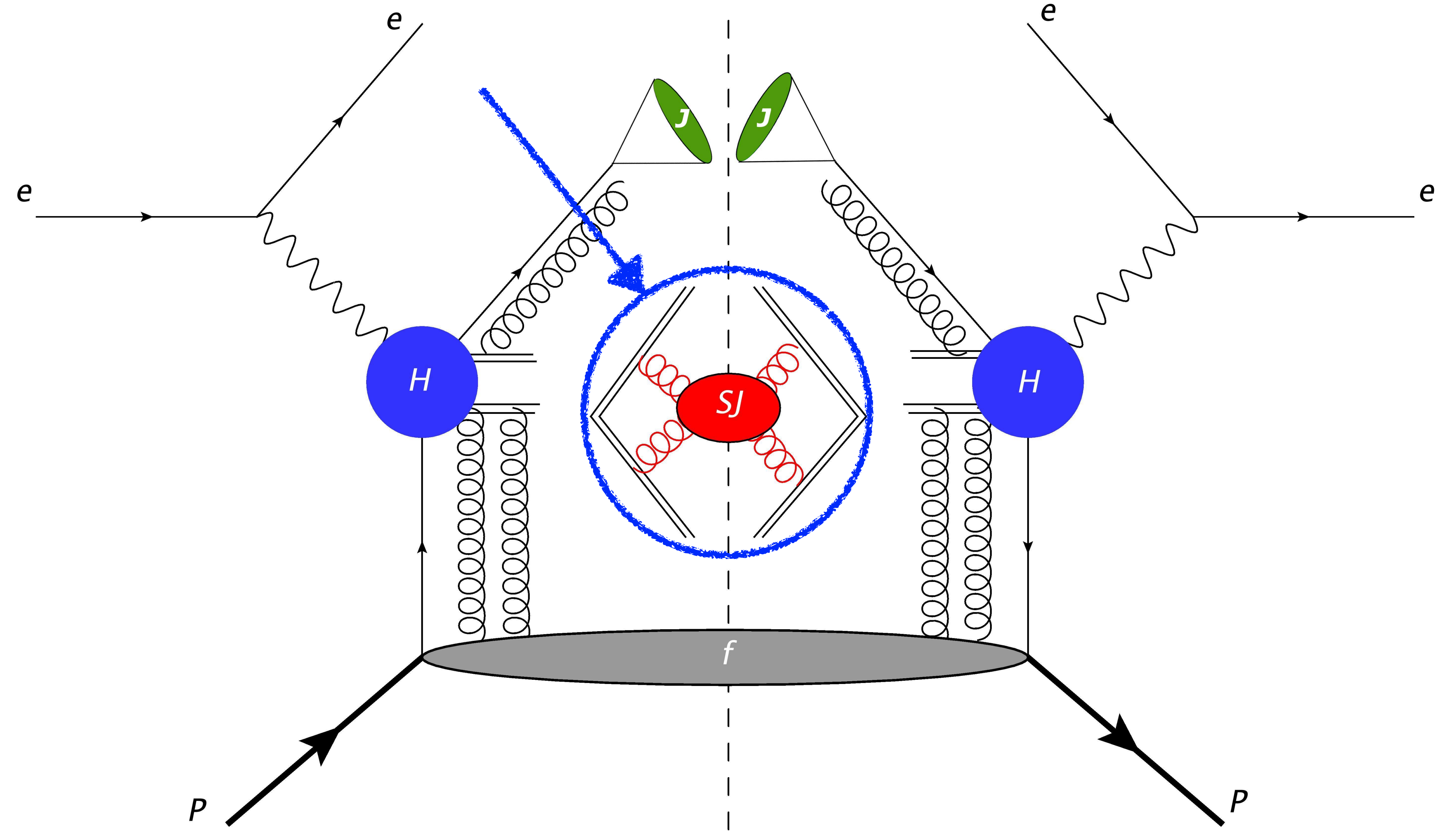


NLO expression

$$\mathcal{S}_J(b) = 1 + \frac{\alpha_s C_F}{\pi}\Bigg[\frac{1}{2\epsilon^2} - \frac{1}{\epsilon\alpha} - \frac{1}{\epsilon}\ln\frac{\nu}{\mu\tan(\frac{R}{2})} - \frac{1}{\alpha}\left(\ln\frac{\mu^2}{\mu_b{}^2}\right) - \frac{\pi^2}{24} - \frac{1}{4}\ln^2\left(\frac{\mu^2}{\mu_b{}^2}\right) + $$

$$-\ln\left(\frac{\mu^2}{\mu_b{}^2}\right)\ln\left(\frac{\nu}{\mu\tan(\frac{R}{2})}\right)\Bigg] + \mathcal{O}(\alpha^2) + \mathcal{O}(\epsilon^2)\,.$$

Jet Soft Function



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$$-\ln\Big(\frac{\mu^2}{\mu_b^2}\Big)\ln\Big(\frac{\nu}{\mu\tan(\frac{R}{2})}\Big)\Bigg] + \mathcal{O}(\alpha^2) + \mathcal{O}(\epsilon^2)\,.$$

Jet-TMDFF

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Let's remind the definition of the Jet-TMDFF:

$$D_{i\to\text{jet}}(b_T, Q, R; \mu, \zeta_1) = \frac{J_{i\to\text{jet}}(Q, R; \mu_J)\mathcal{S}_J(b_T, R; \mu_R, \zeta_R)}{\sqrt{S(b; \mu_S, \zeta_S)}}$$

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Next-to-Leading Order Expression:

$$D_{i\to\mathrm{jet}} = 1 + \frac{\alpha_s(\mu_J)}{2\pi}\Big[\frac{3C_F}{2}\ln\Big(\frac{\mu_J^2}{Q^2\tan^2\frac{R}{2}}\Big) + \frac{C_F}{2}\ln^2\Big(\frac{\mu_J^2}{Q^2\tan^2\frac{R}{2}}\Big) + d_J^{q,\mathrm{alg}}\Big] + \frac{\alpha_s(\mu_R)C_F}{\pi}\Big) +$$
$$-\frac{\pi^2}{6} - \ln^2\Big(\frac{\mu_R^2}{\mu_b{}^2}\Big) - 4\ln\Big(\frac{\mu_R^2}{\mu_b{}^2}\Big)\ln\Big(\frac{\nu_R}{\mu_R\tan\frac{R}{2}}\Big)\Big] - \frac{\alpha_s(\mu_S)C_F}{4\pi}\Big[-\ln^2\Big(\frac{\mu_S^2}{\mu_b^2}\Big) +$$
$$+4\ln\Big(\frac{\mu_S^2}{\mu_b^2}\Big)\ln\Big(\frac{\mu_0}{\nu_S}\Big) - \frac{\pi^2}{6}\Big].$$

Jet-TMDFF

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Choice of initial scales:

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Jet-TMDFF

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$$\boxed{\begin{aligned} D_{i\to\text{jet}} = 1 + \frac{\alpha_s(\mu_J)}{2\pi}\left[\frac{3C_F}{2}\ln\left(\frac{\mu_J^2}{Q^2\tan^2(R/2)}\right) + \frac{C_F}{2}\ln^2\left(\frac{\mu_J^2}{Q^2\tan^2(R/2)}\right) + d_J^{q,alg}\right] + \\ -4\frac{\alpha_s(\mu_0)C_F}{\pi}\ln\left(\frac{\mu_0^2}{\mu_b{}^2}\right)\ln\left(\frac{\nu_R}{\nu_S\tan(R/2)}\right) \end{aligned}}$$

Jet-TMDFF

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$$d\sigma_{\text{jet}} \sim H_{ij}(Q;\mu_H) \otimes D_{i\to\text{jet}}(b,Q,R;\mu_1,\zeta_1) \times f^{sub.}_{j\leftarrow P}(x,b,Q;\mu_2,\zeta_2)$$

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$$\gamma_\mu^{D_{i\to\text{jet}}} + \gamma_\mu^H + \gamma_\mu^f = 0 \qquad \gamma_\nu^{D_{i\to\text{jet}}} + \gamma_\nu^f = 0$$

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$$d\sigma_{\text{sidis}} \sim H_{ij}(Q;\mu_H) \otimes D^{sub.}_{i\to h}(b,Q,z;\mu_1,\zeta_1) \times f^{sub.}_{j\leftarrow P}(x,b,Q;\mu_2,\zeta_2)$$

Jet-TMDFF

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Jet-TMDFF

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The Jet-TMDFF has the same evolution of TMD-FF.

Jet-TMDFF

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Outline

Jet-Sidis Process

Jet-TMDFF

Phenomenological Estimates

Non-Perturbative Parts Jet-TMDFF

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Collins-Soper kernel

Fixed by the MAP22[1] proton replicas.

[1] To parametrize the non-perturbative effects in the TMD-PDFs, we use 200 sets of values of the free parameters obtained in the so-called MAP22 extraction. arXiv:2206.07598

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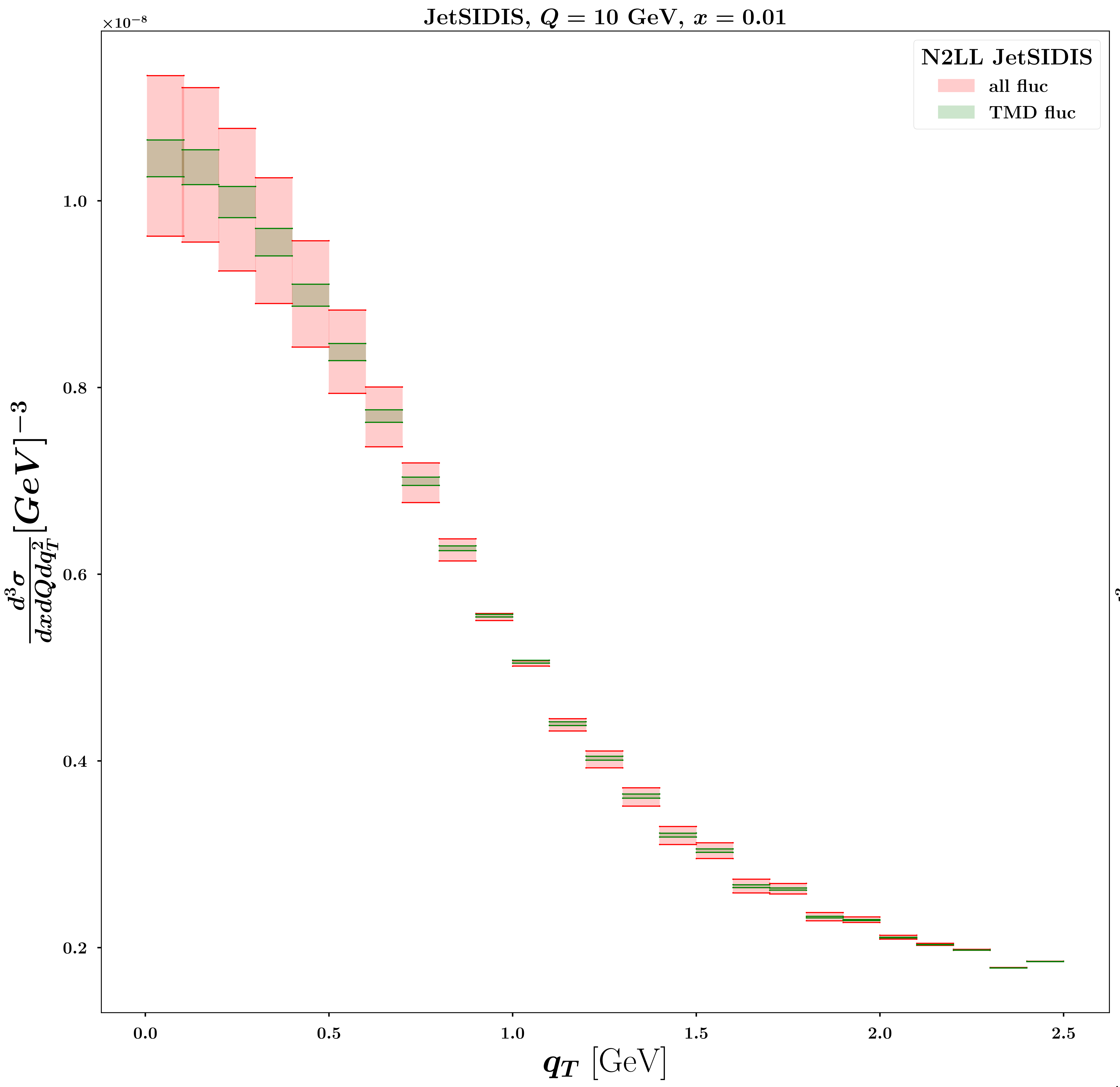
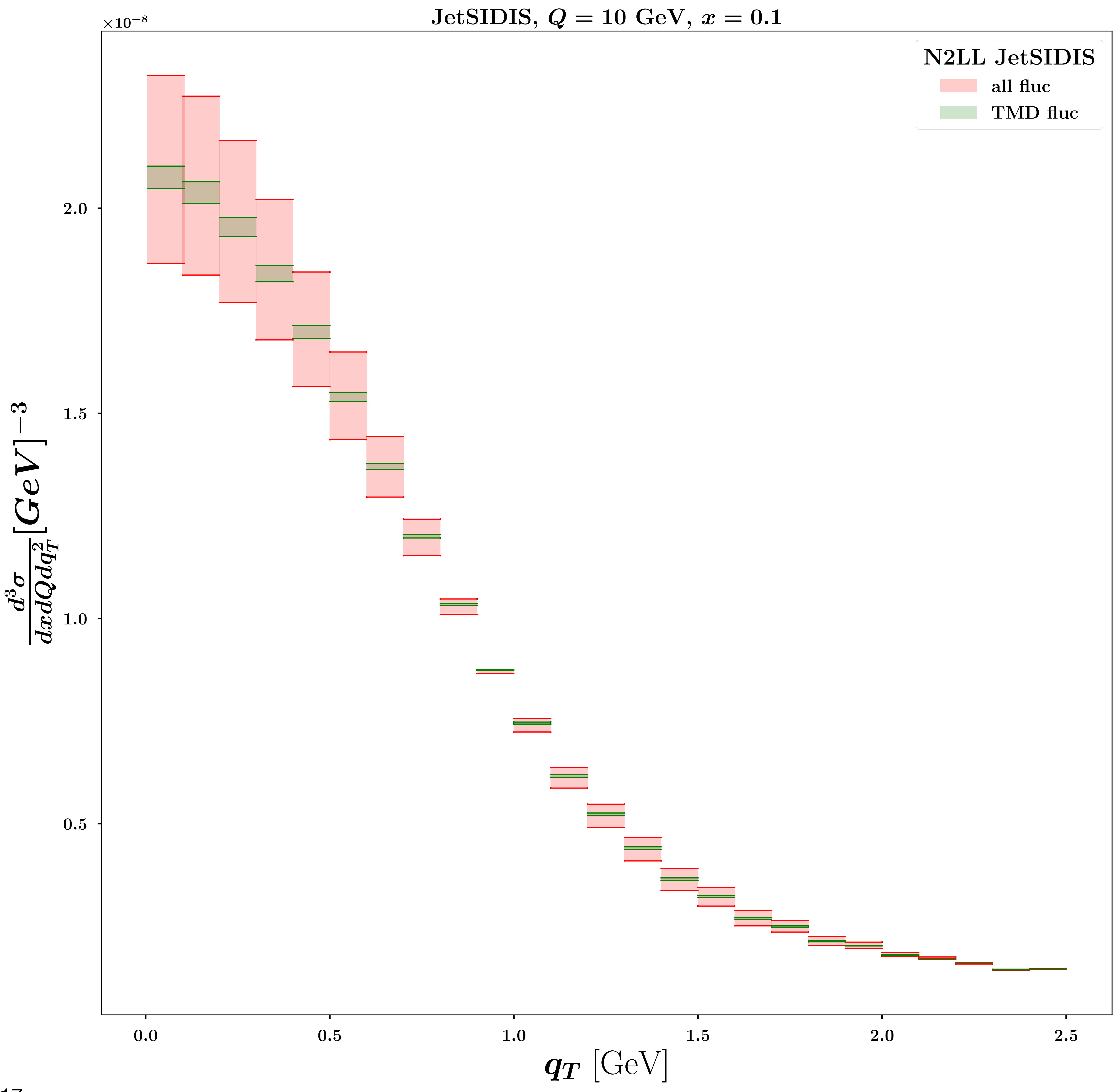
We expect NP effect relatively high!

1

Phenomenological Estimates





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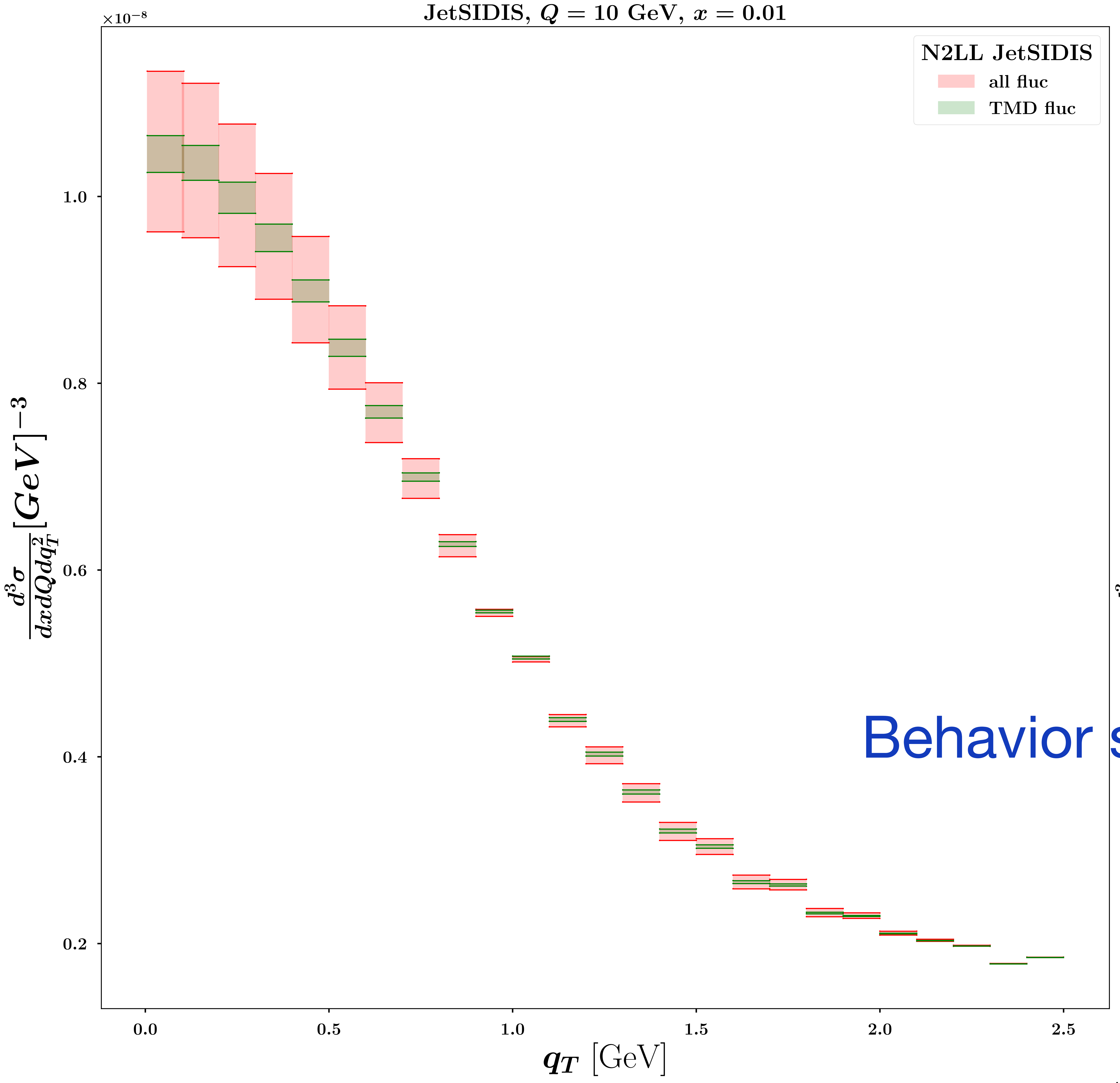

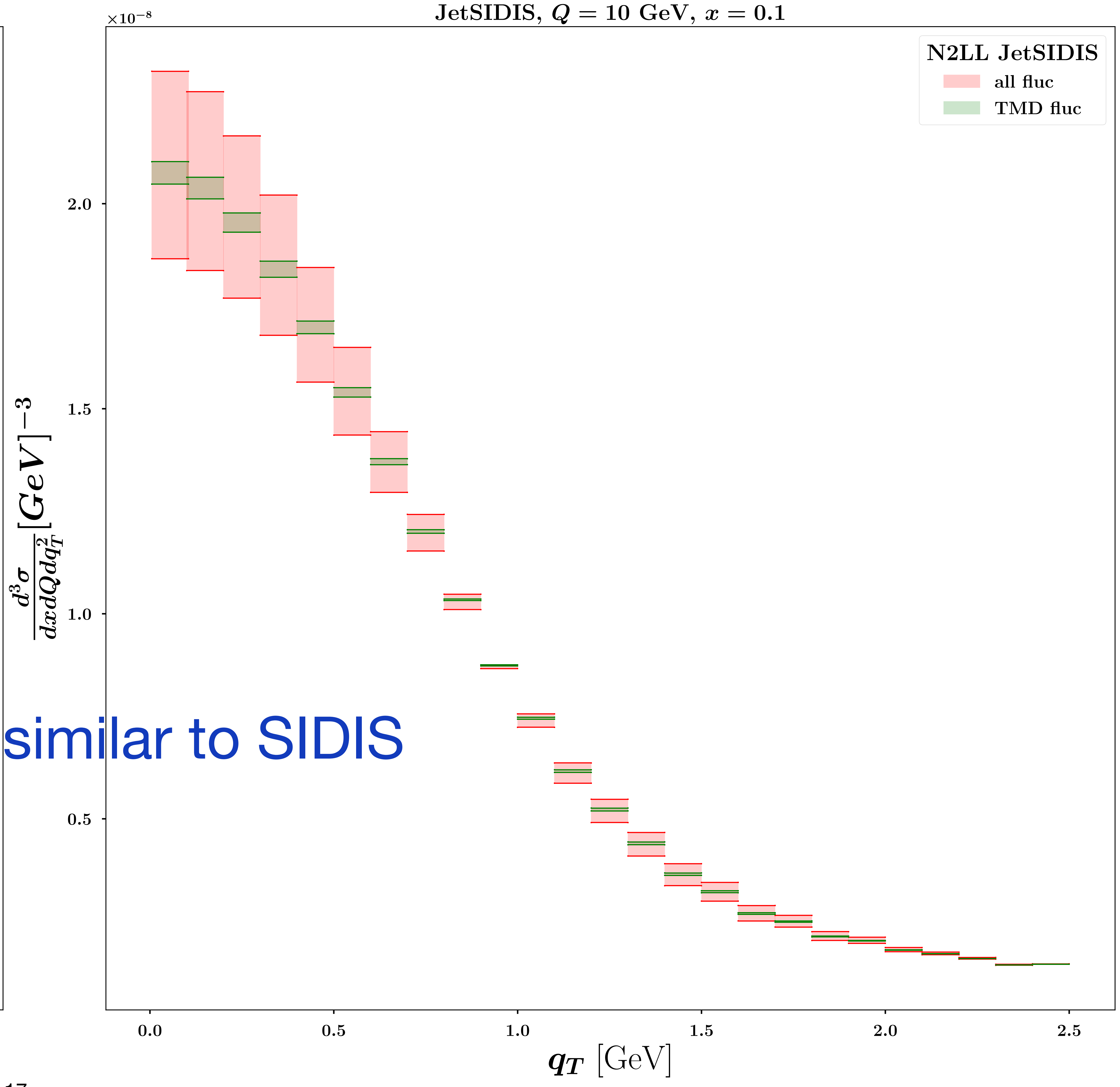

Behavior similar to SIDIS

Phenomenological Estimates

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Summary & Outlooks

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Obtained Results:

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Possible outlooks:

- Extension to N^3LL accuracy
- Predictions for the Sivers polarized Structure Function $F_{UT,T}^{\sin(\phi_h-\phi_s)}$
- Reanalyzing of data from HERA