EICUG UG Early Career Workshop 2022

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# An overview of gluon TMD PDFs and polarization


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ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

FONDAZIONE BRUNO KESSLER

FUTURE BUILT ON KNOWLEDGE

TIFPA

Trento Institute for Fundamental Physics and Applications

HAS QCD

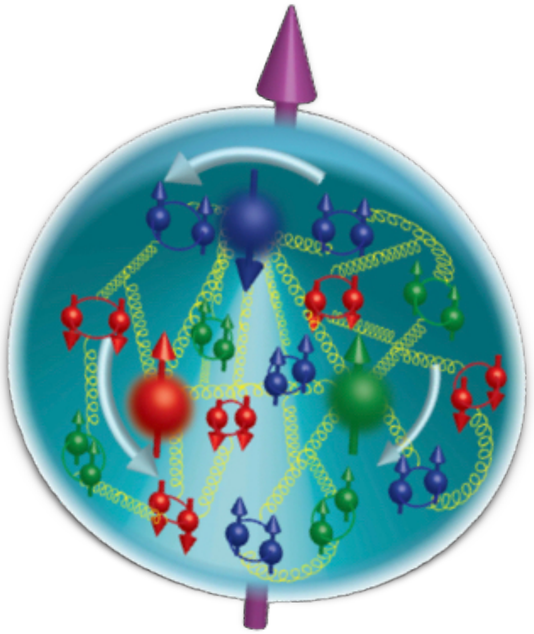
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

# Parton densities: hors d'œuvre

**Parton densities** → relevant for the search of **New Physics**…

→ …crucial role in the understanding and exploration of **QCD**

- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- **Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*



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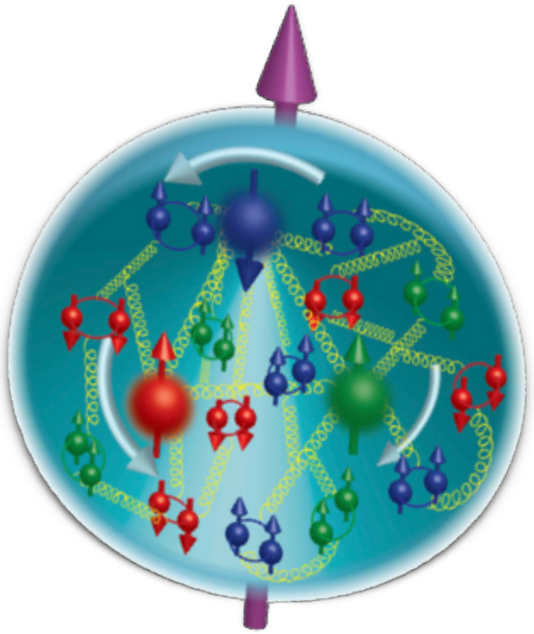
Several types of distributions (1D collinear, **3D TMD**, …)

- Respect different **factorization theorems**
- Exhibit peculiar **universality properties**
- Obey distinct **evolution equations**

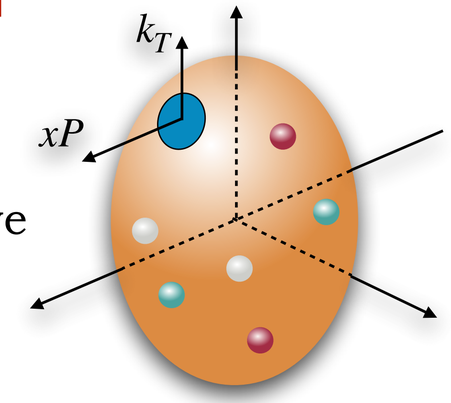


# Parton densities: an incomplete family tree

**3D**

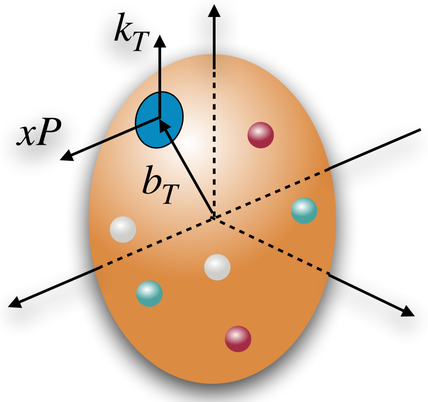
TMDs

(semi-)inclusive




**5D**

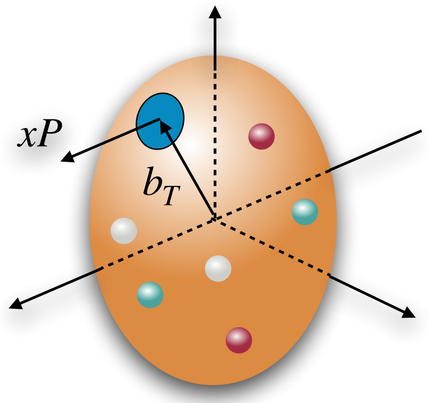
Wigner distributions




**3D**

FT of GPDs

exclusive




PDFs

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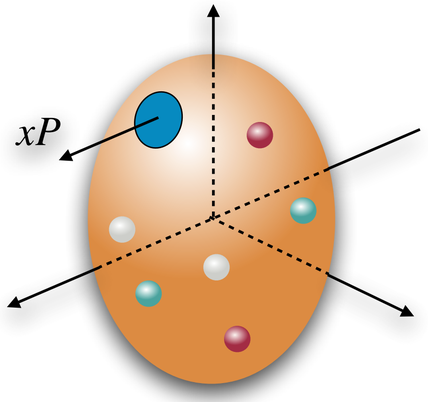
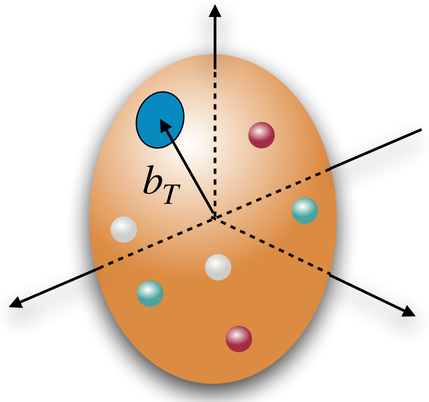



FT of Form Factors




**1D**

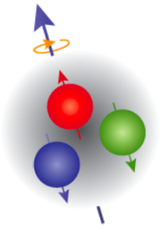
**2D**

$\vec{b}_\perp$ dependence

$\vec{k}_\perp$ dependence

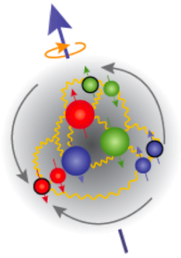
these two variables are NOT Fourier conjugate

# The proton spin crisis



**quarks helicity**

~ 30 %



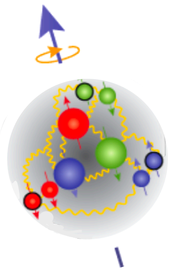
**orbital angular momentum**

unknown!

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

**gluons helicity**

~ 20% – 50 % **?**



**Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description…**

🔗 (proton spin crisis) [EMC Collaboration, CERN (1987)]

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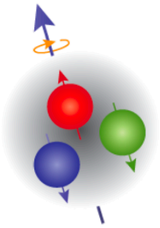

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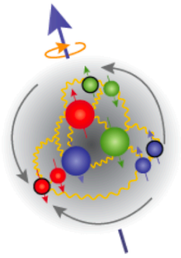

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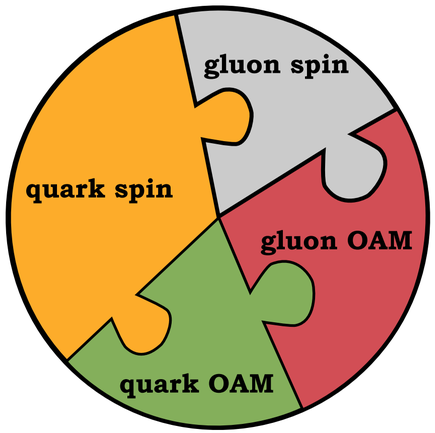


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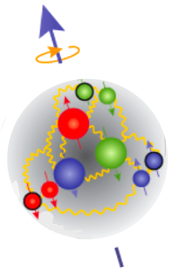

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**Hors d'œuvre**

slide adapted from C. Bissolotti

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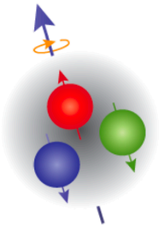

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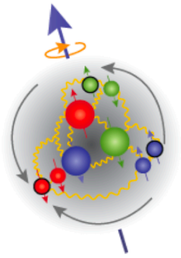

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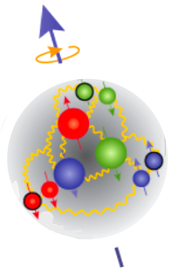

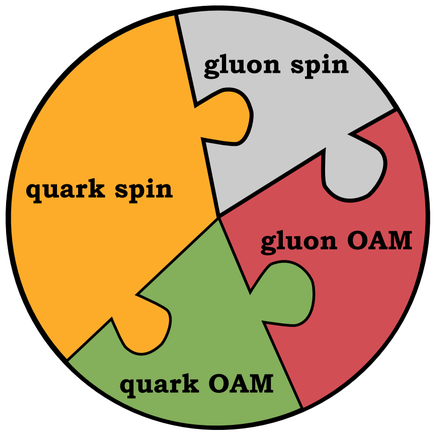


**Hors d'œuvre**

**…many other effects in hadronic interactions cannot be understood in the purely collinear approach**

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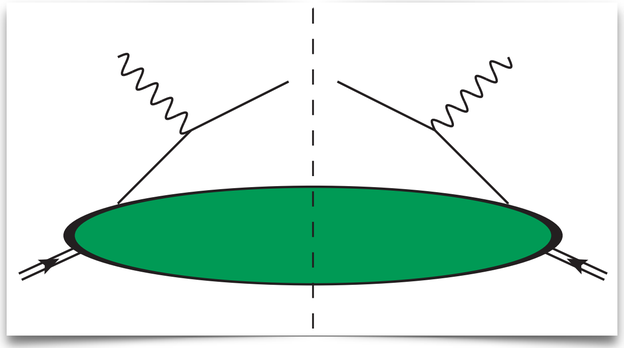
# Gauge links and process dependence



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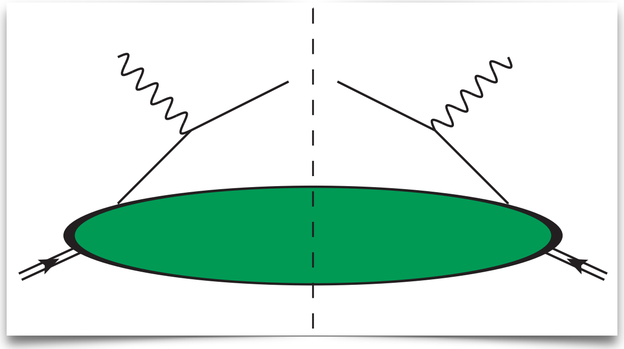
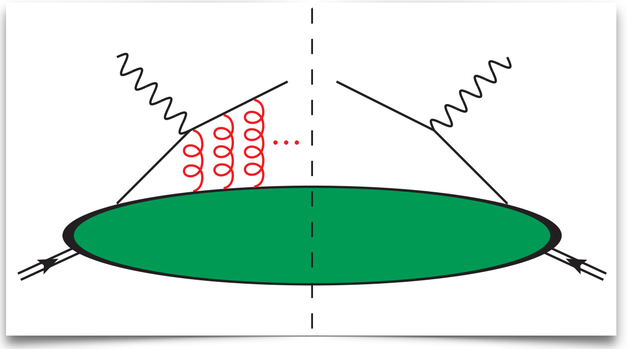


$$\bar{\psi}_j(0)\; U(0,\xi)\; \psi_i(\xi)$$

**gauge link**

**resummation** of (calculable)
**infinite gluon emissions**



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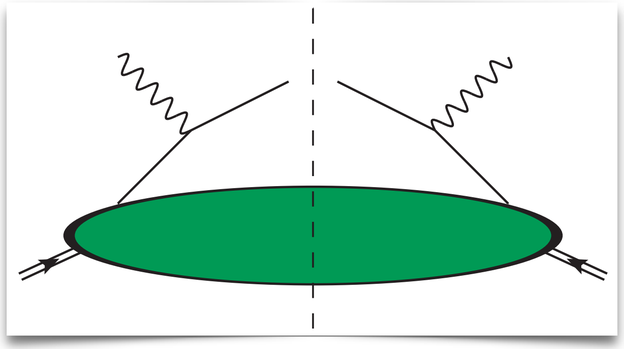

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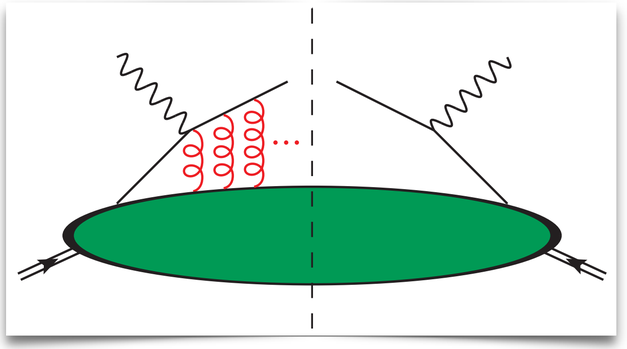
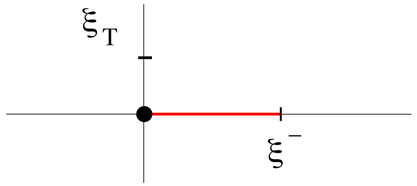


## Collinear PDFs

$$\Phi_{ij}(x) \doteq \int d^2\boldsymbol{p}_T \Phi_{ij}(x,\boldsymbol{p}_T) = \int \frac{d\xi^-}{2\pi} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\psi_i(\xi)|P\rangle_{|\xi^+=0,\xi_T=0}$$




- Light-cone: $\xi^+ = 0$, $\boldsymbol{\xi} = 0$
- **Straight** gauge link (unique!)
- $(A^+ = 0)$ light-cone: WL $= \hat{1}$

☑ **Universality warranted**

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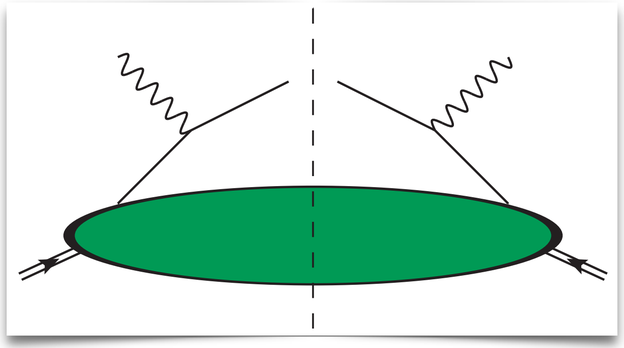

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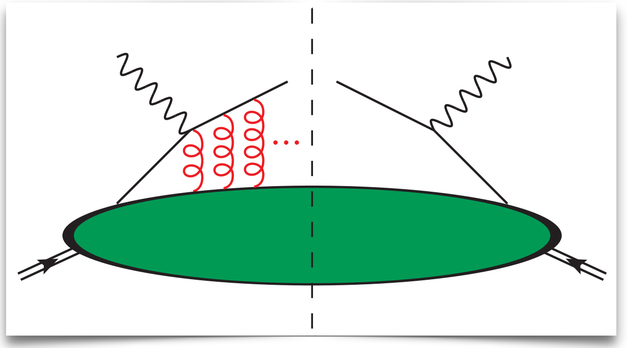

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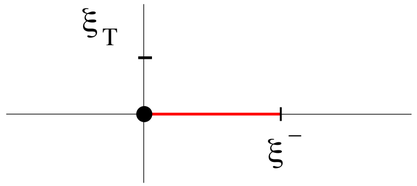


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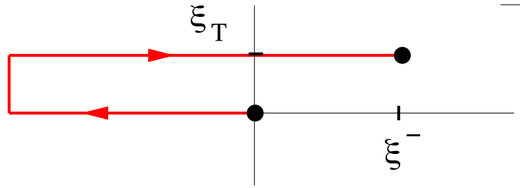
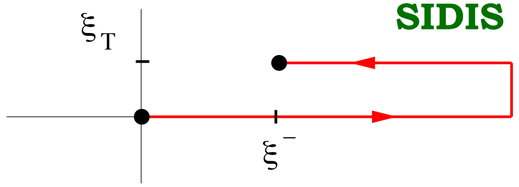
## TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)
- ¡! **Process dependence**




**Drell-Yan**




**SIDIS**

# An overview on gluon TMDs

# Gluon TMDs: gauge links and modified universality

* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
* **Color flow** → integration paths of gauge links calculable
* Gluon TMDs → more complicated structure with respect to quark **staple links**
* **Factorization-preserving** processes → two main kinds of **modified universality**
* Different classes of processes → distinct gluon TMDs, **not related** to each other

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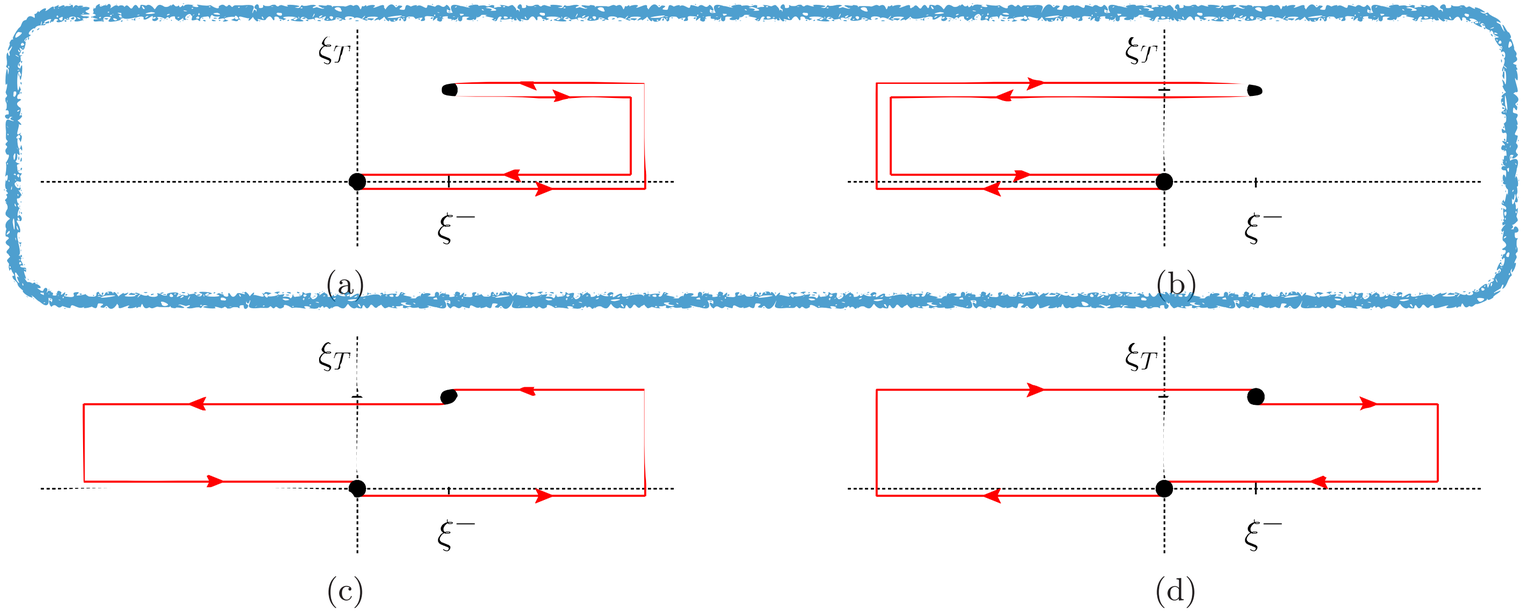
(a) [ + , + ] or (b) [ − , − ]



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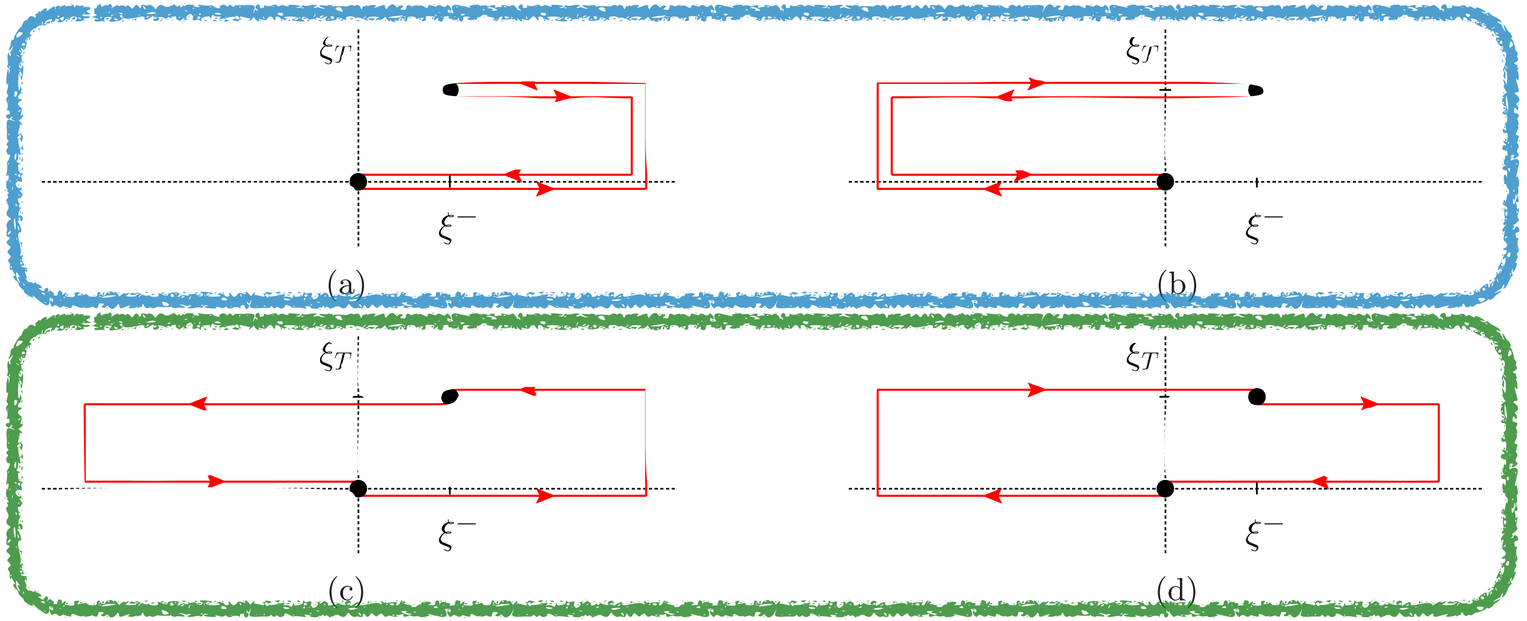
(a) [ + , + ] or (b) [ − , − ]

## *d*-type (dipole)

(c) [ + , − ] or (d) [ − , + ]



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

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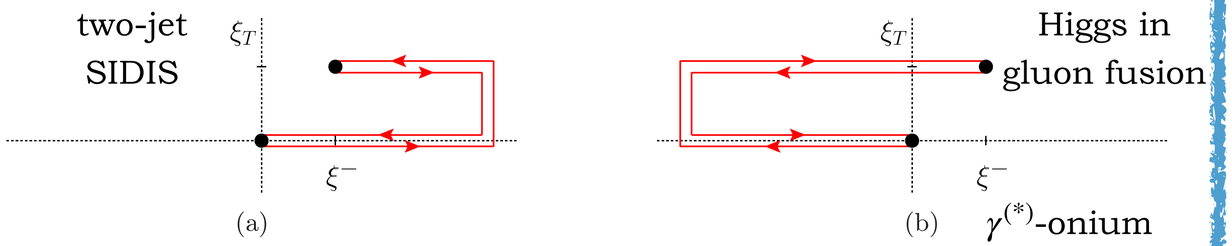


Higgs in gluon fusion

$\gamma^{(*)}$-onium hadroproduction

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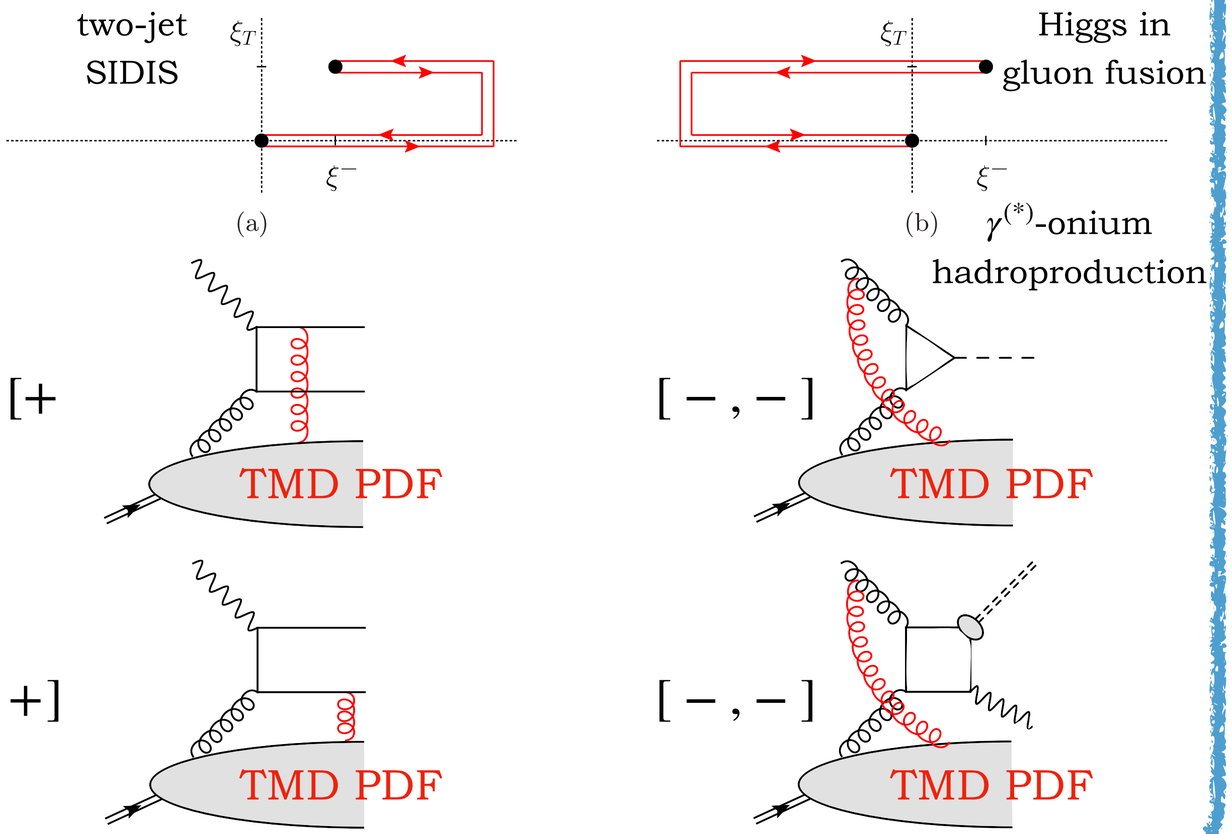

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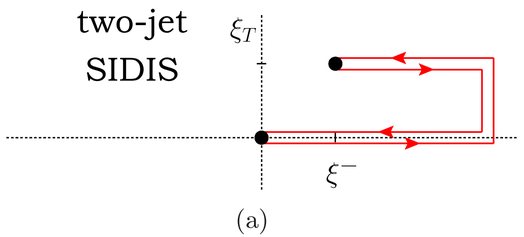
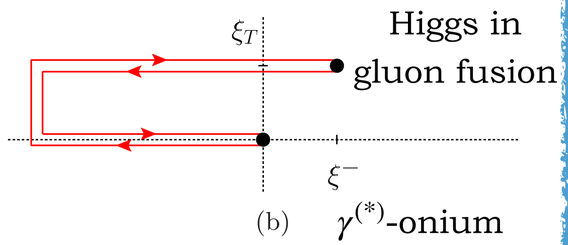
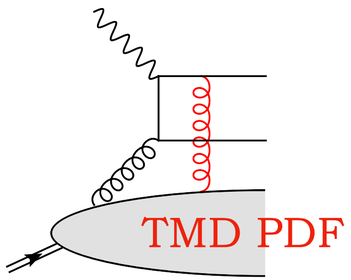


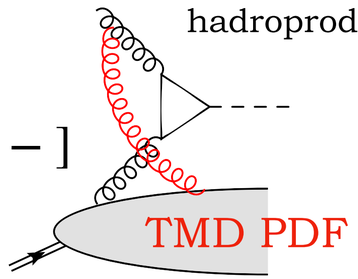




$\gamma^{(*)}$-onium hadroproduction

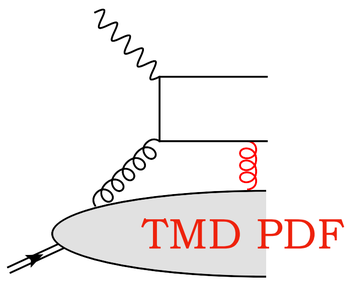
$[+$




$[-,-]$




$+]$

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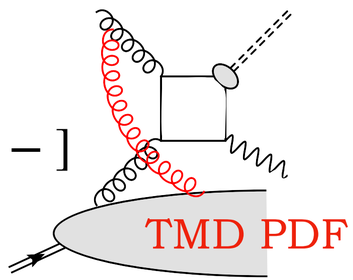


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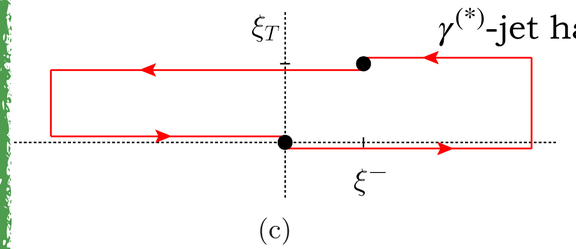
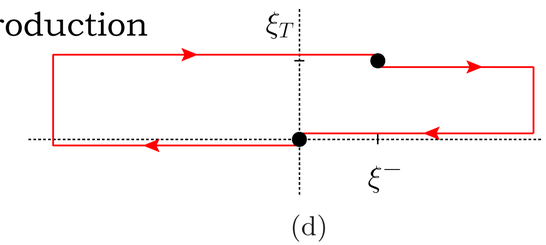
(c) $[+,-]$ or (d) $[-,+]$



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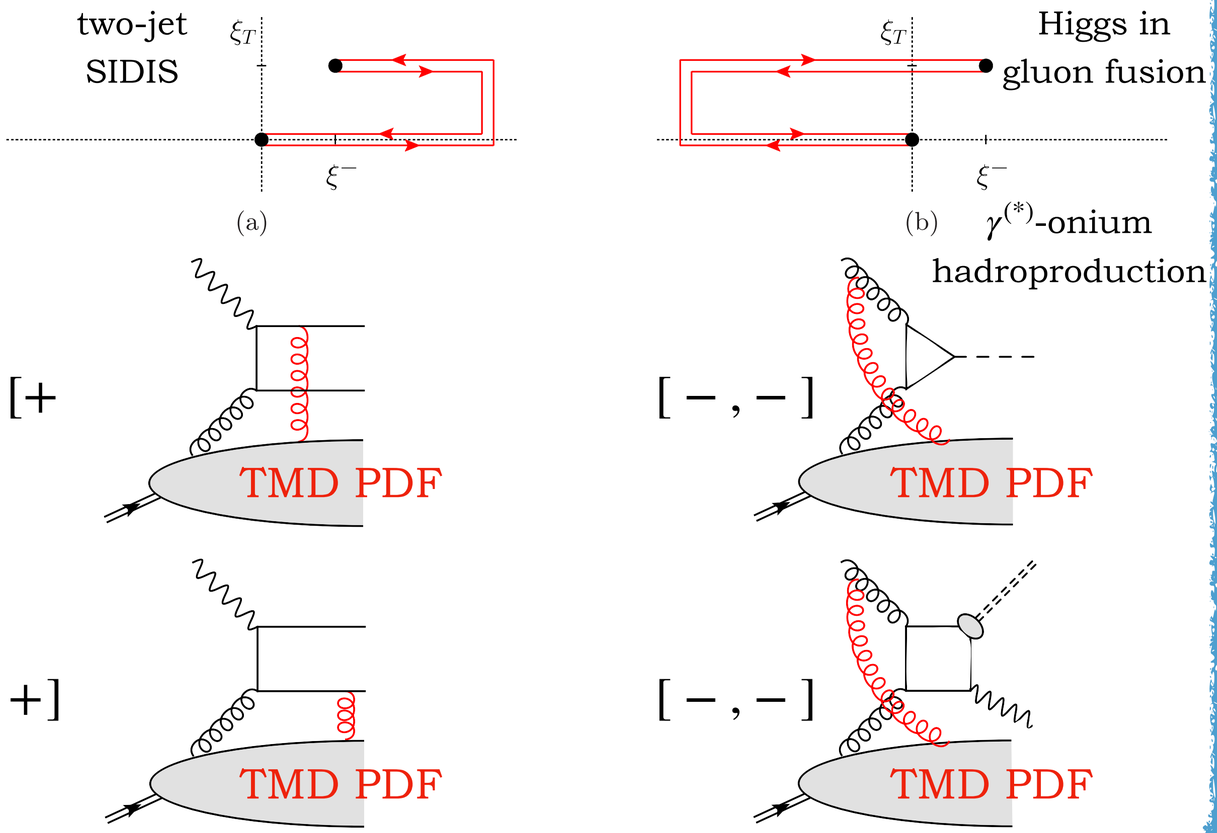


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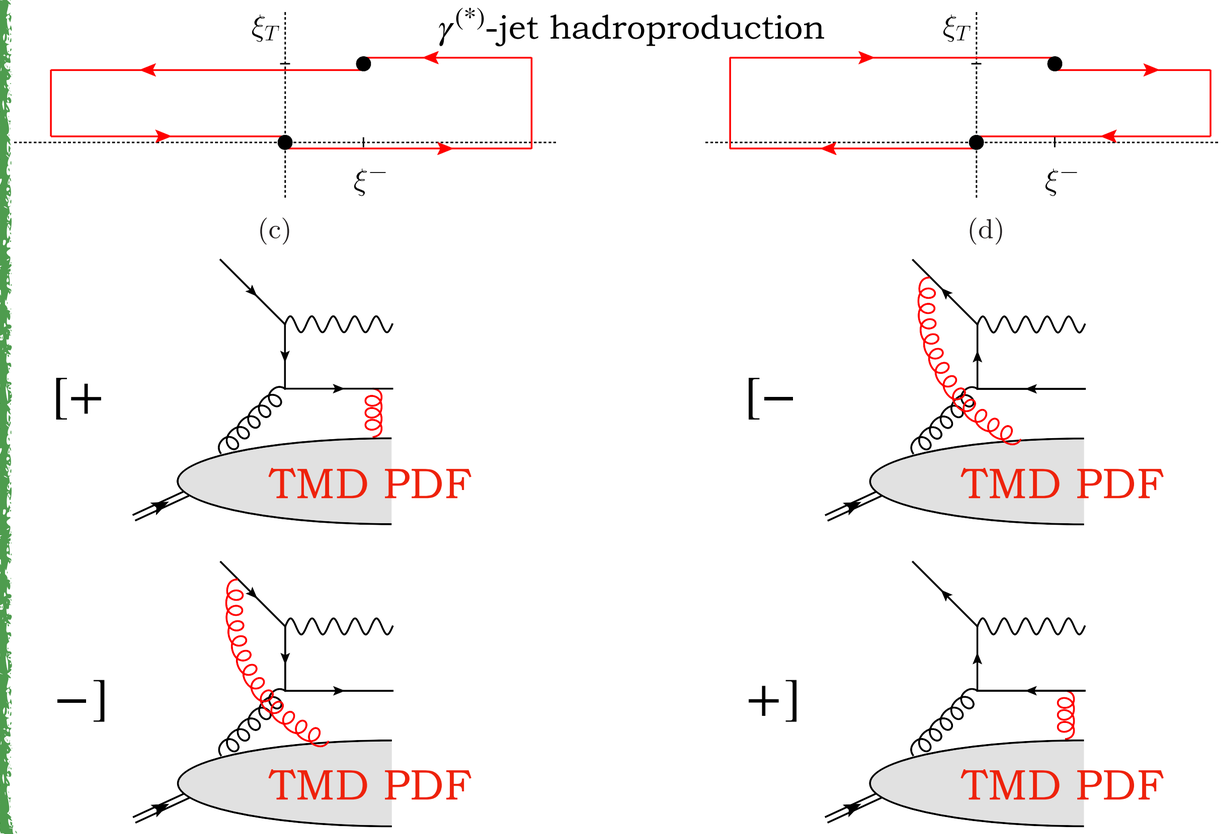

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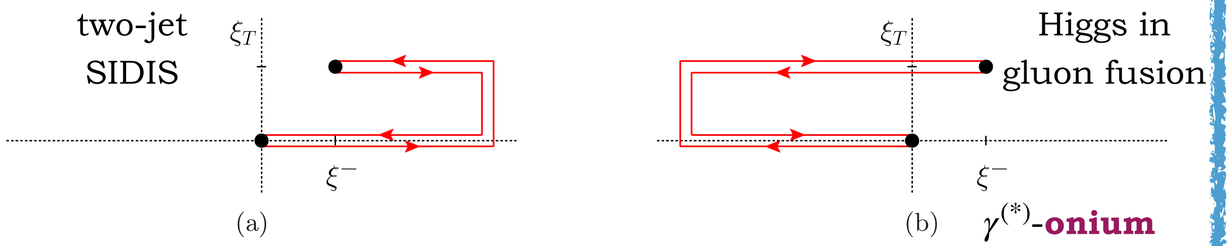





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$\gamma^{(*)}$-**onium** hadroproduction

* Color flow annihilated within final/initial state
* $f$-type gluon TMDs $\rightarrow$ $f^{abc}$ color structure
* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

* Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in $pp$, two-jet SIDIS, heavy-quark pair SIDIS

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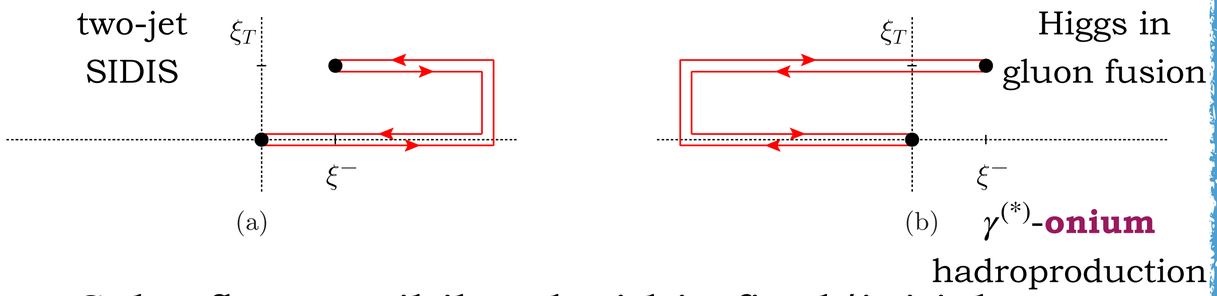
## Weiszäcker-Williams (WW)

(a) $[+,+]$ or (b) $[-,-]$



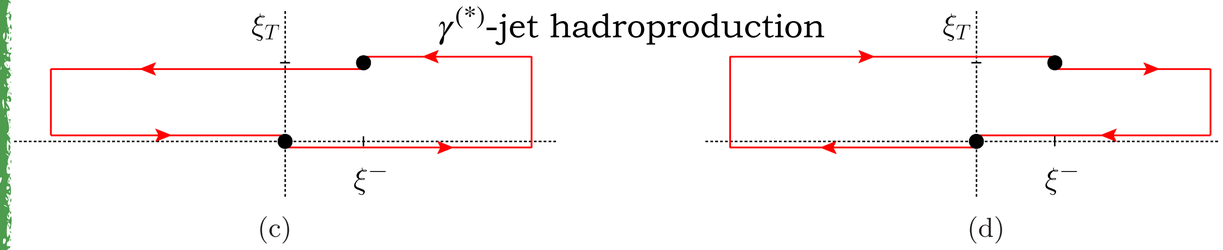

- Color flow annihilated within final/initial state
- $f$-type gluon TMDs $\rightarrow$ $f^{abc}$ color structure
- Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

- Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in $pp$, two-jet SIDIS, heavy-quark pair SIDIS

## Dipole (DP)

(c) $[+,-]$ or (d) $[-,+]$




- Color flow involving both initial and final states
- $d$-type gluon TMDs $\rightarrow$ $d^{abc}$ color structure
- Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]},$$
$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

- Phenomenology: single hadron or $\gamma^{(*)}$-jet hadroproduction, SIDIS or Drell-Yan (subleading)

Gauge link $\rightarrow$ two main independent sets of TMDs, **not related** to each other

# Modeling gluon TMDs

# T-even and T-odd gluon TMD PDFs at leading-twist

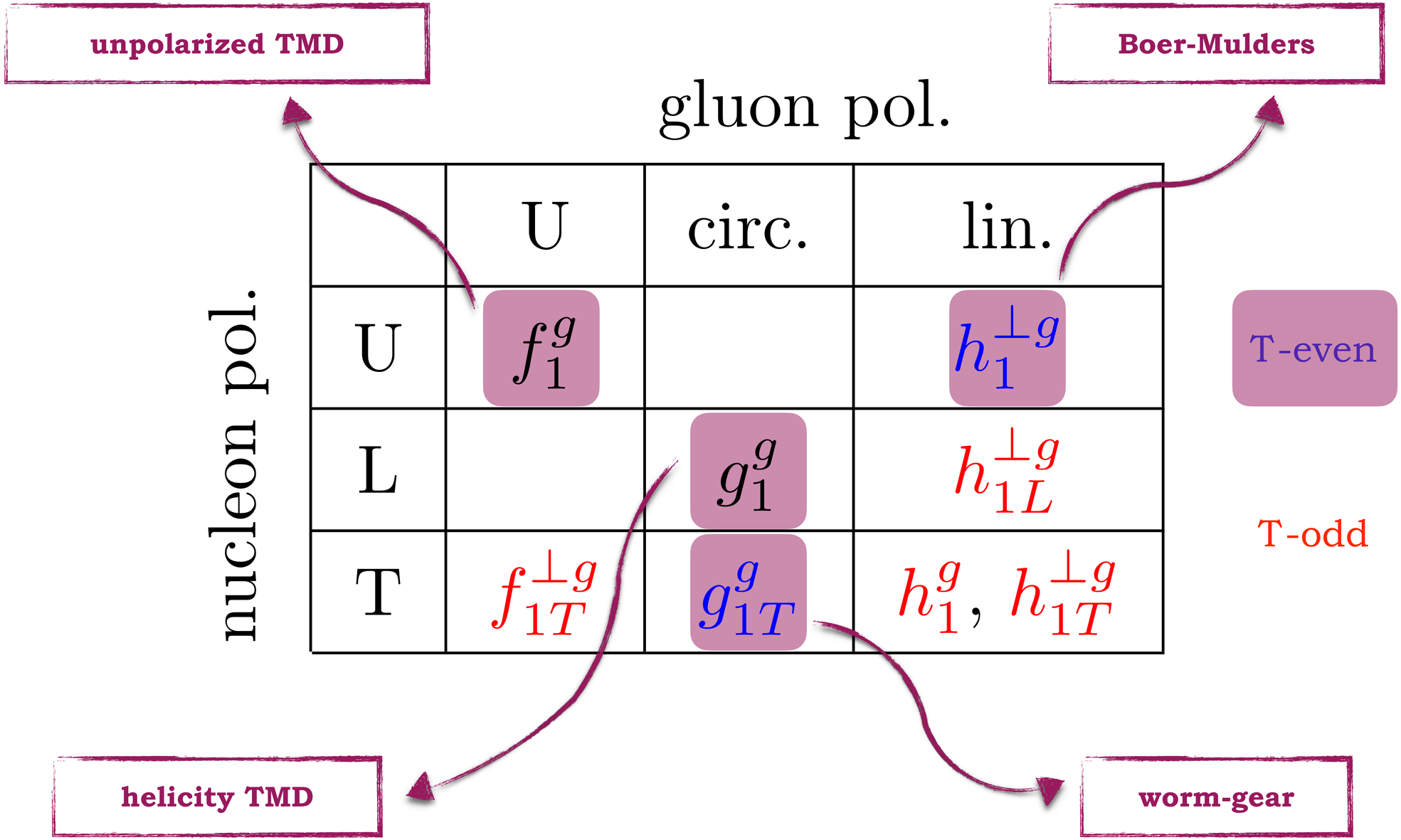
gluon pol.

| nucleon pol. | U | circ. | lin. |
|---|---|---|---|
| U | $f_1^g$ | | $h_1^{\perp g}$ |
| L | | $g_1^g$ | $h_{1L}^{\perp g}$ |
| T | $f_{1T}^{\perp g}$ | $g_{1T}^g$ | $h_1^g,\ h_{1T}^{\perp g}$ |

T-even

T-odd

# T-even and T-odd gluon TMD PDFs at leading-twist



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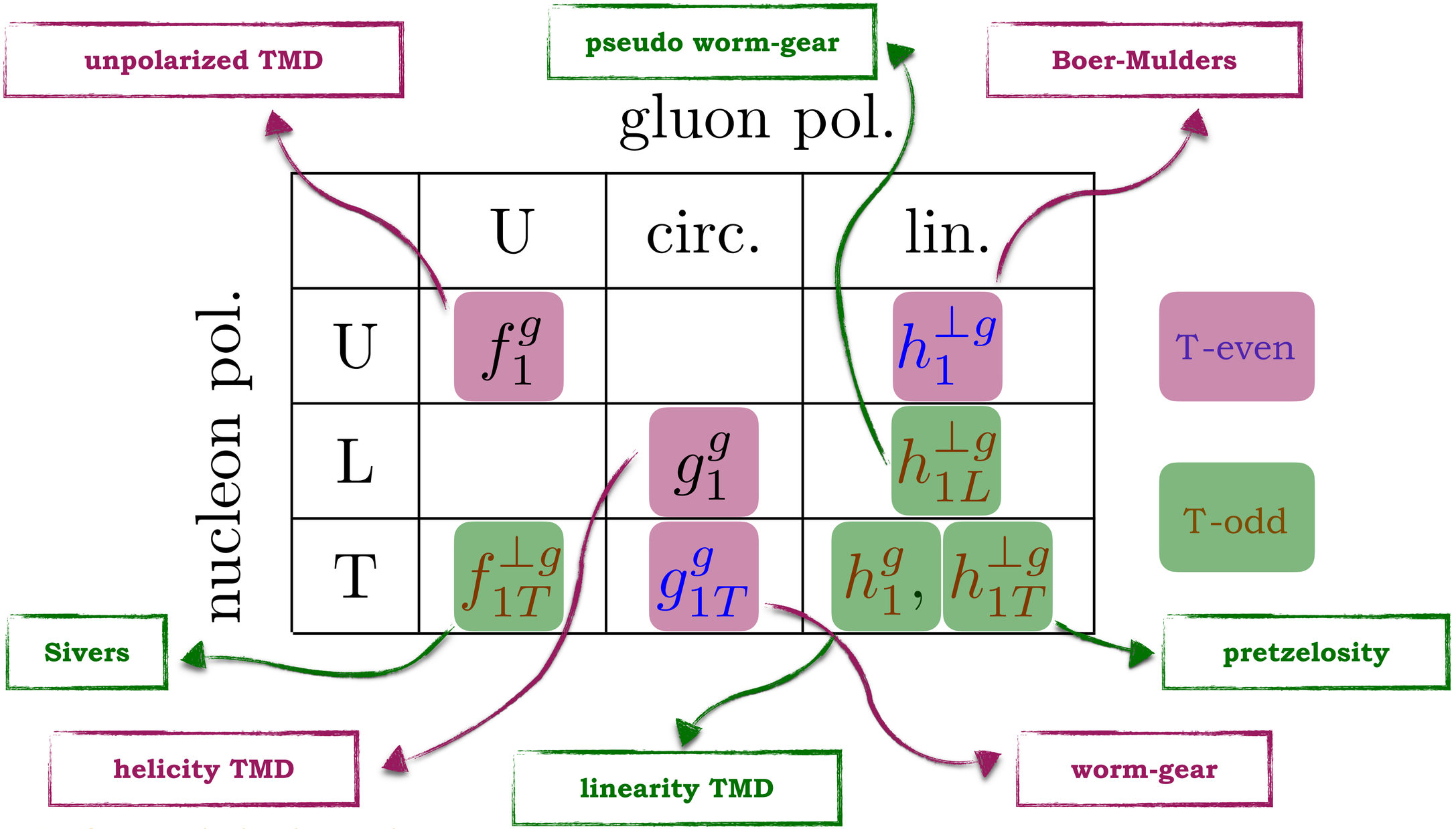
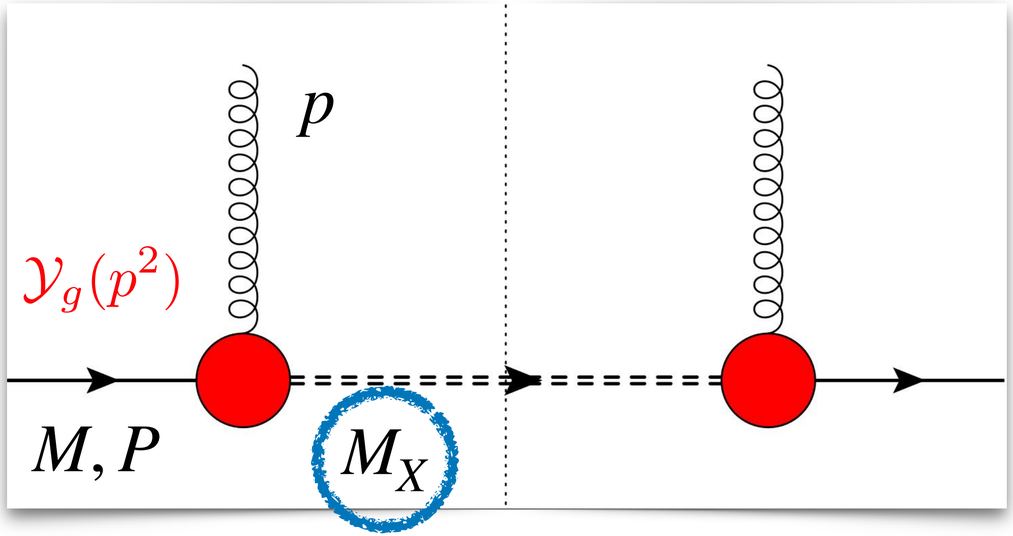


# Assumptions of the model

**Spin-1/2 spectator**

Lowest Fock state:
**tri-quark** spectator
on-shell and
with mass $M_X$



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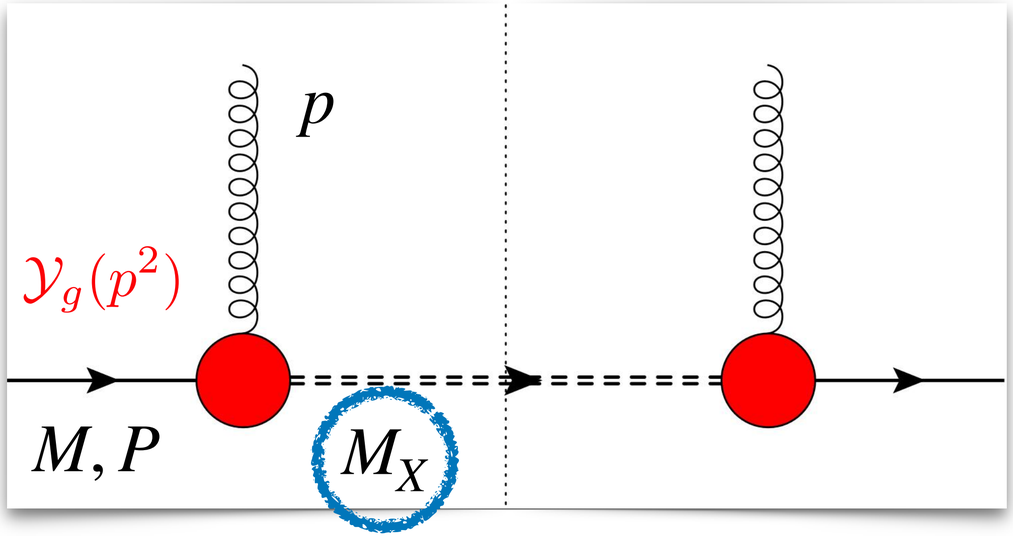


## Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not{P}+M)\frac{1+\gamma^5\not{S}}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}_g^{\rho *}\mathcal{Y}_{g\sigma}(\not{P}-\not{p}+M)\right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M}\sigma^{\mu\nu}p_\nu$$

mimics proton form factors
(conserved EM current
of a free nucleon)

# Assumptions of the model

**Link with collinear factorization**

1. $p_T$-integrated TMDs **have to** reproduce PDFs at the lowest scale ($Q_0$) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

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## Link with collinear factorization

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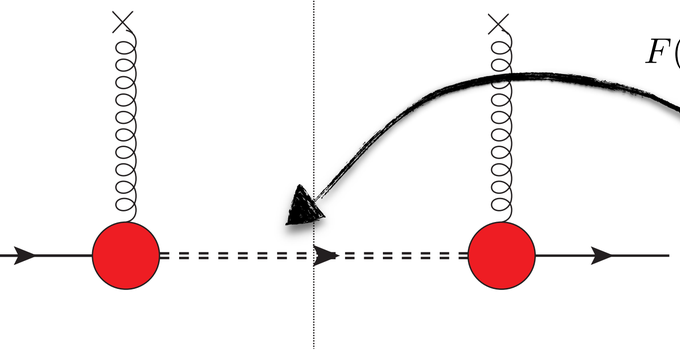
## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high $p_T$
3. Compensates log divergences arising from $p_T$-integration
4. Adds three more parameters: $\kappa_{1,2}$ and $\Lambda_X$

# Our model at a glance

## Spectator-system spectral-mass function



$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

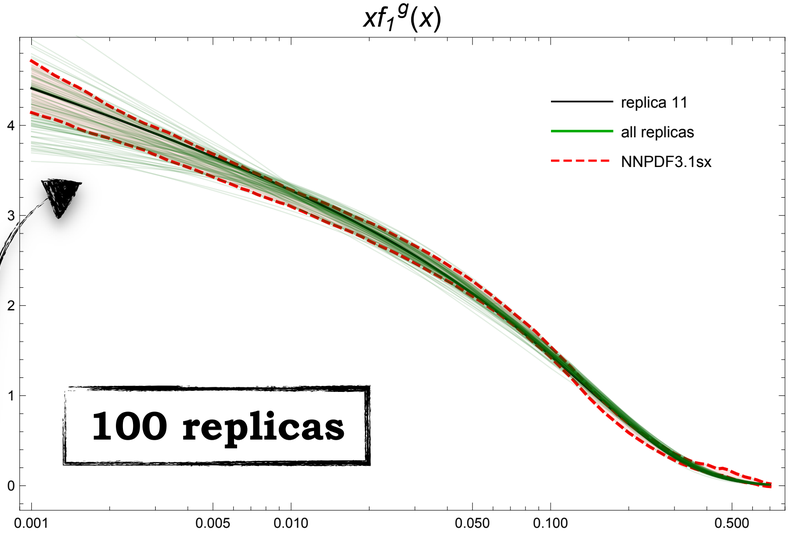
Instead of a single on-shell spectator, a continuum of spectators

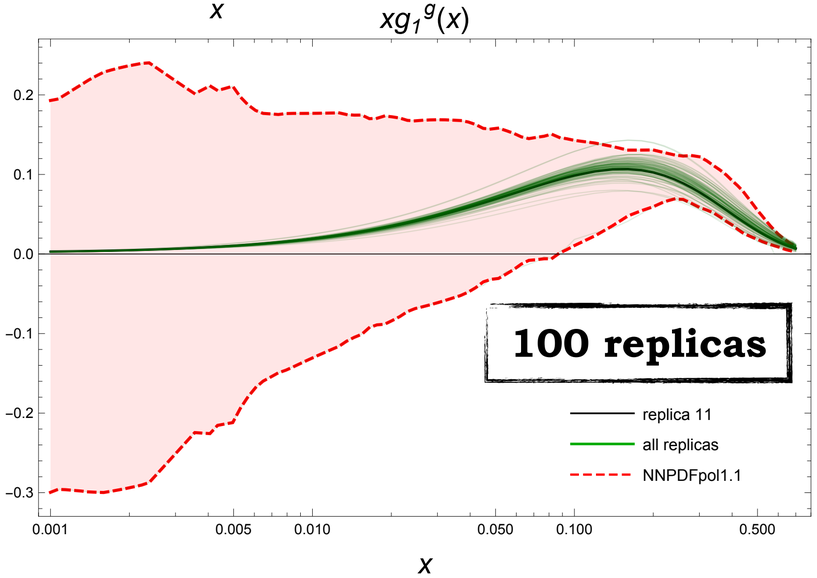
Spectral function **learns** small- and moderate-$x$ info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)

- ☑ **Simultaneous fit** of $f_1$ and $g_1$ PDFs
- ☑ Inclusion of small-$x$ resummation effects (**BFKL**)
- ☑ Calculation of all leading-twist T-even gluon TMDs

## Link with collinear factorization

$p_T$-integrated TMDs **have to** reproduce PDFs at the lowest scale ($Q_0$) *before* evolution

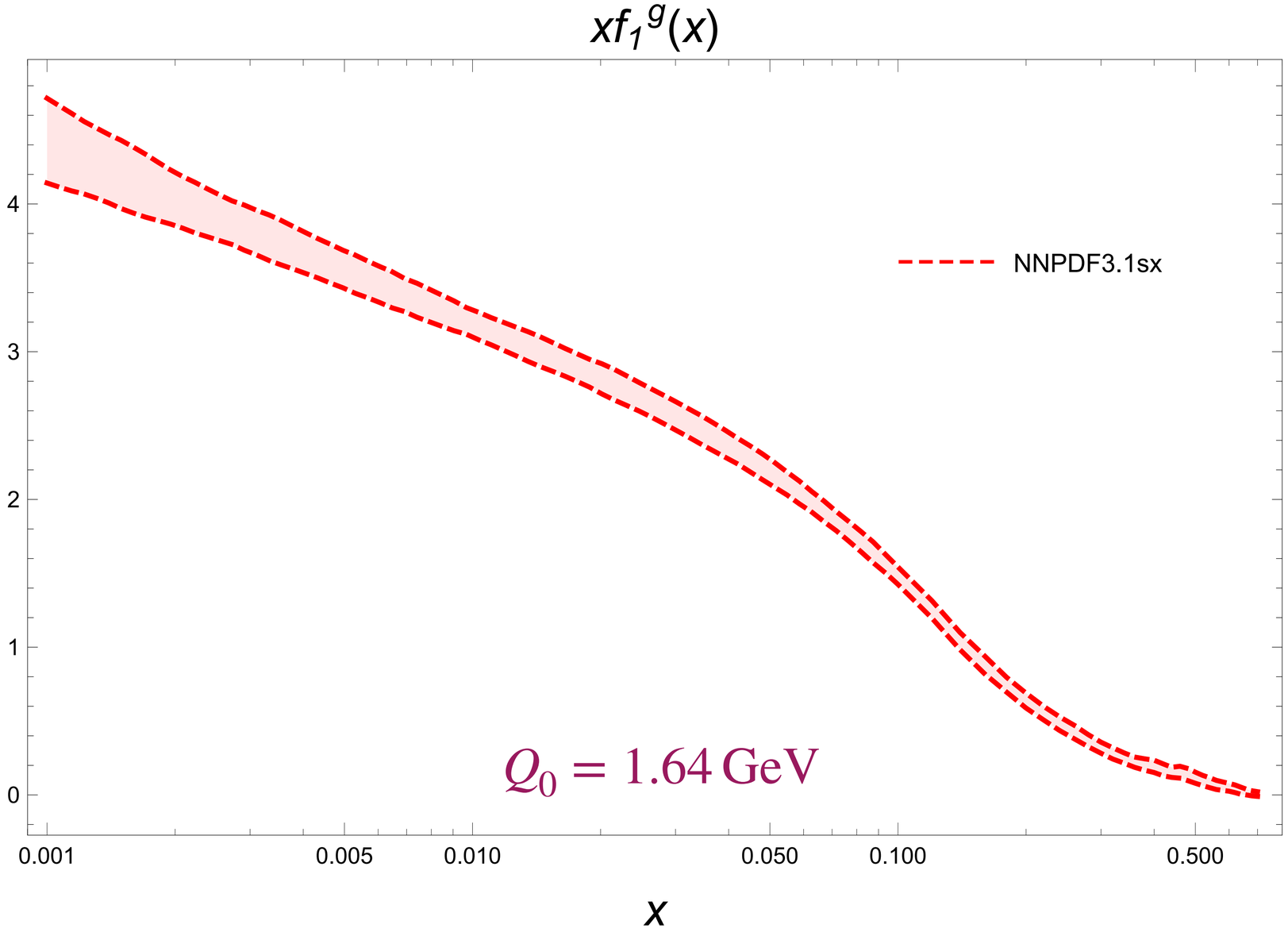



**2.3 Modeling gluon TMDs** 🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

# Unpolarized gluon PDF

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



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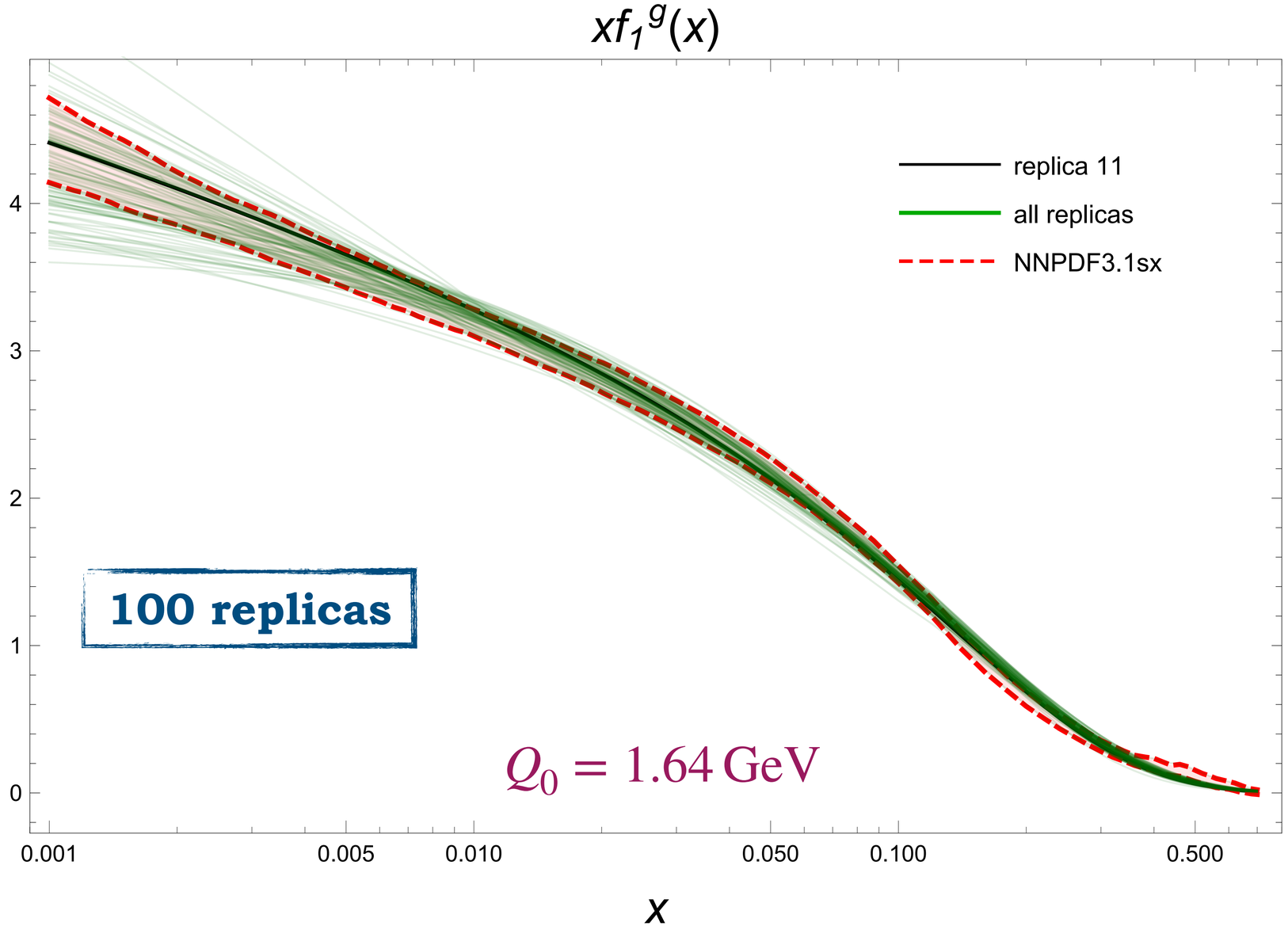
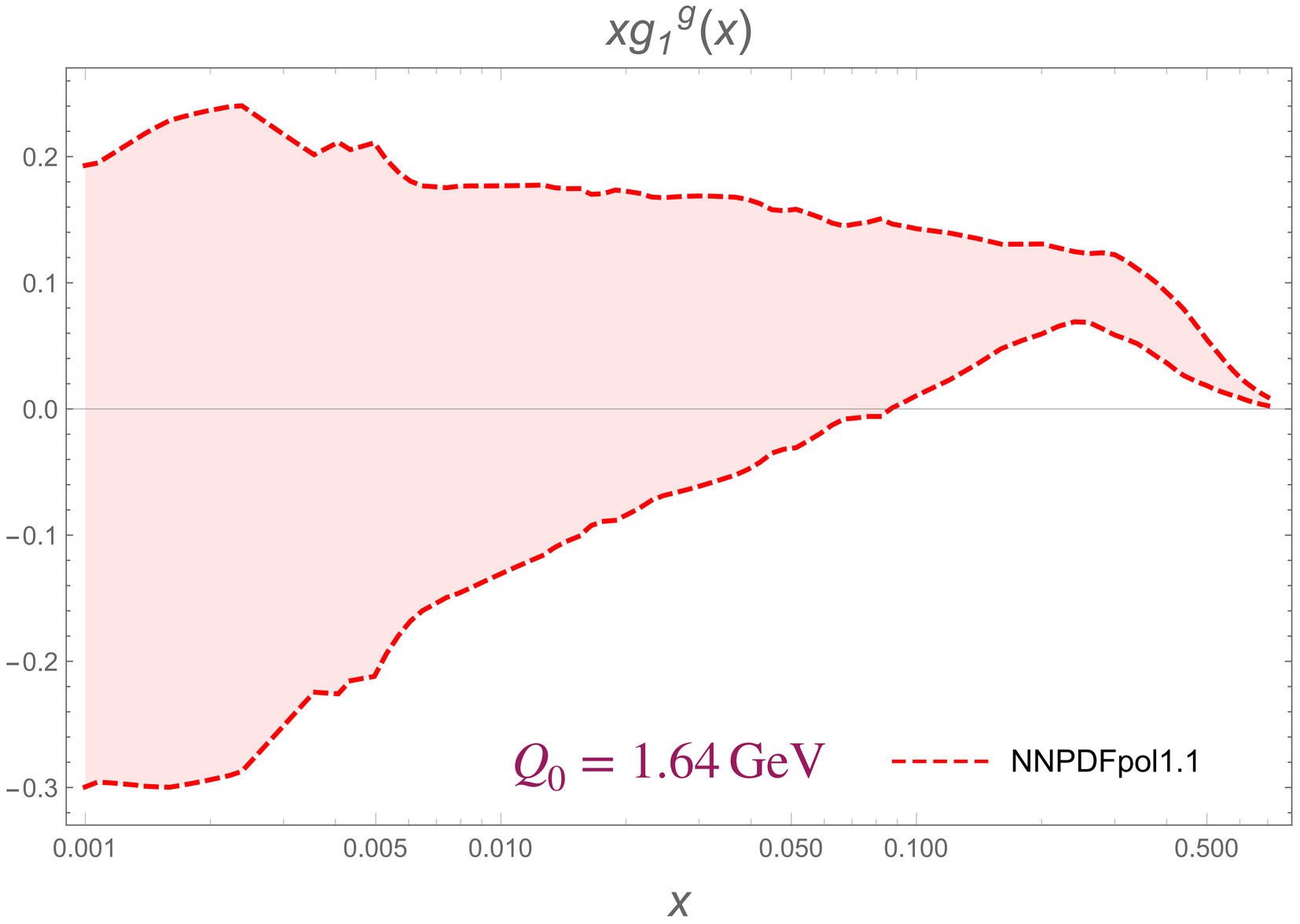


# Helicity gluon PDF

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



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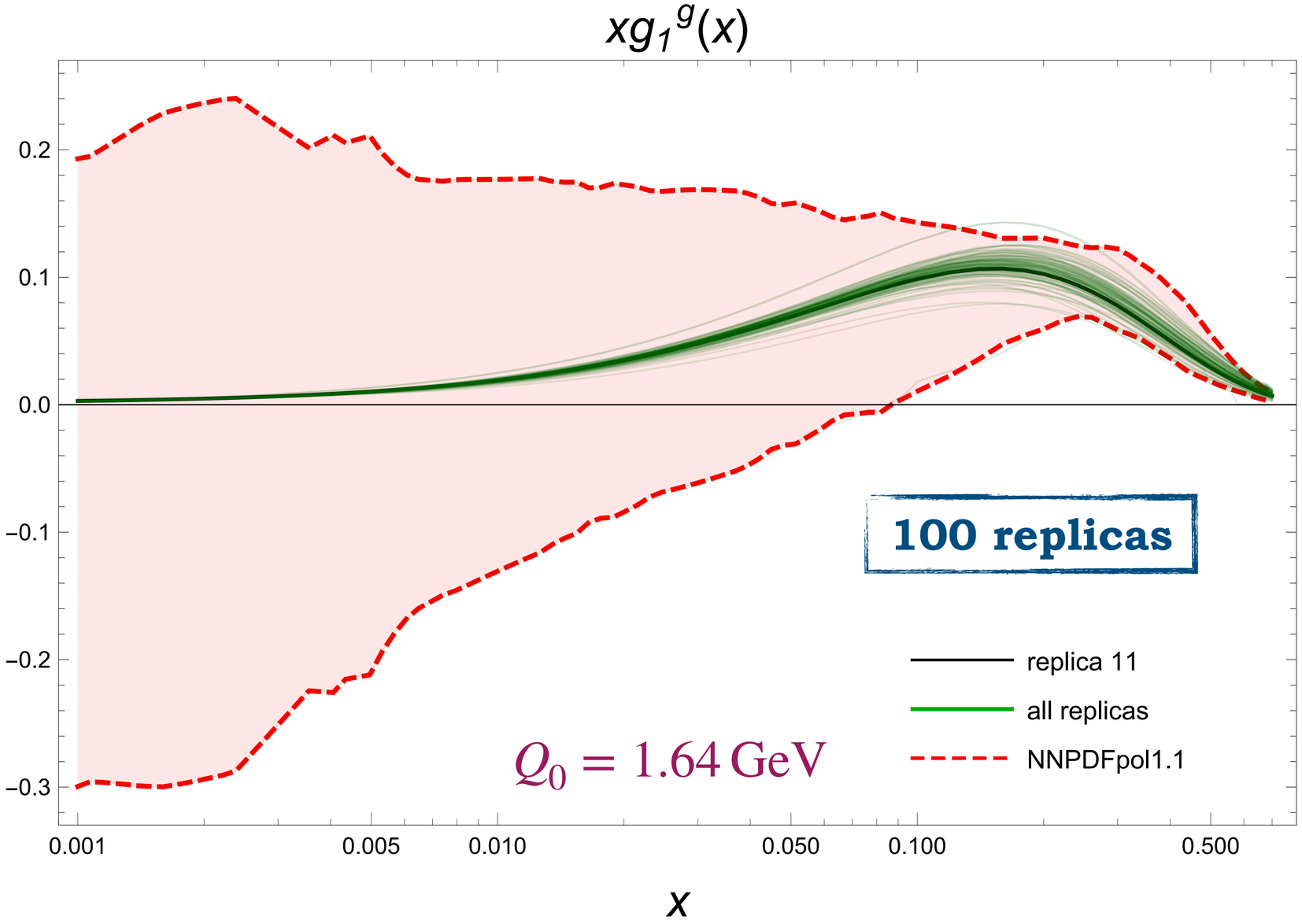
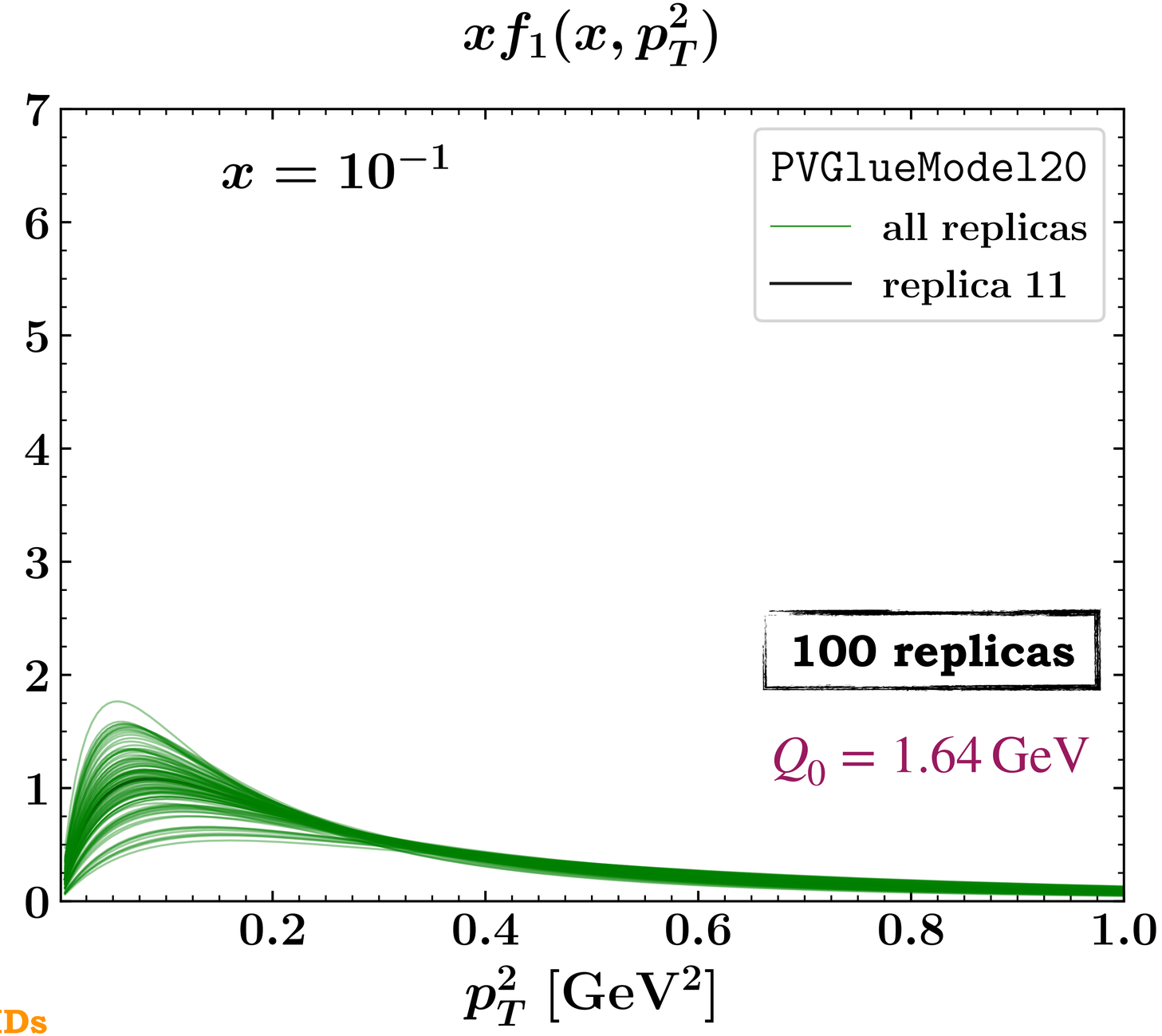


# Unpolarized gluon TMD

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



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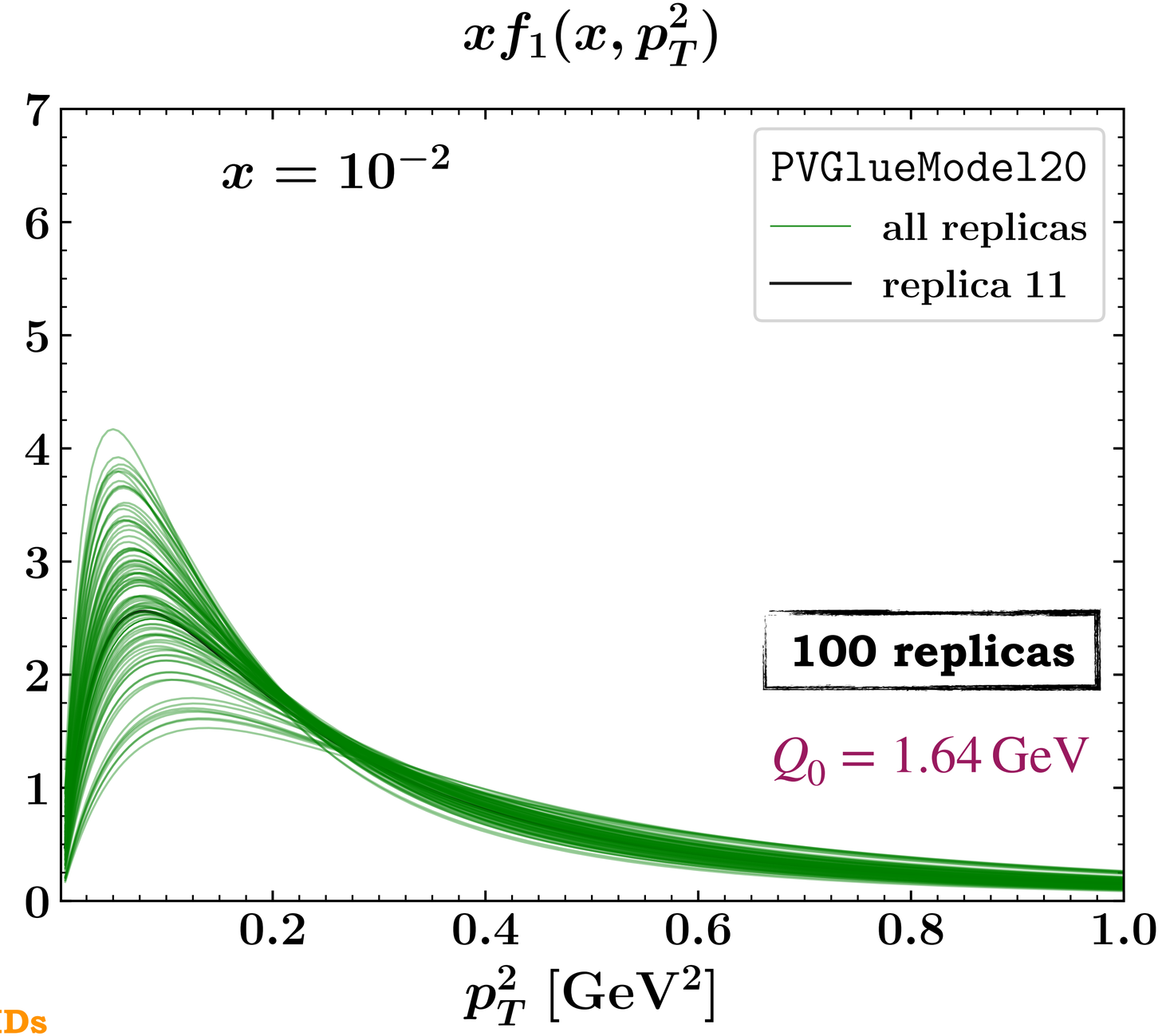

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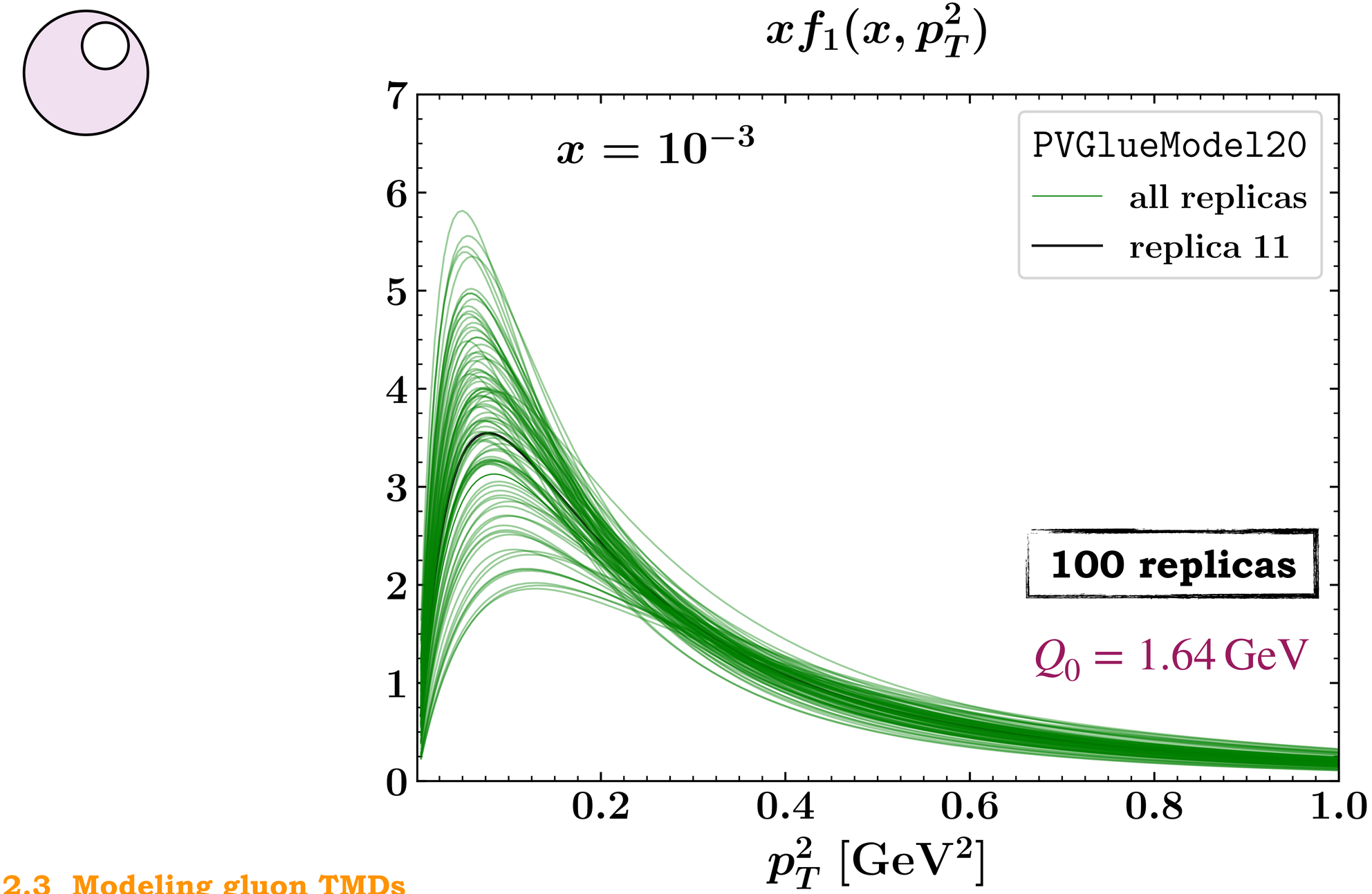
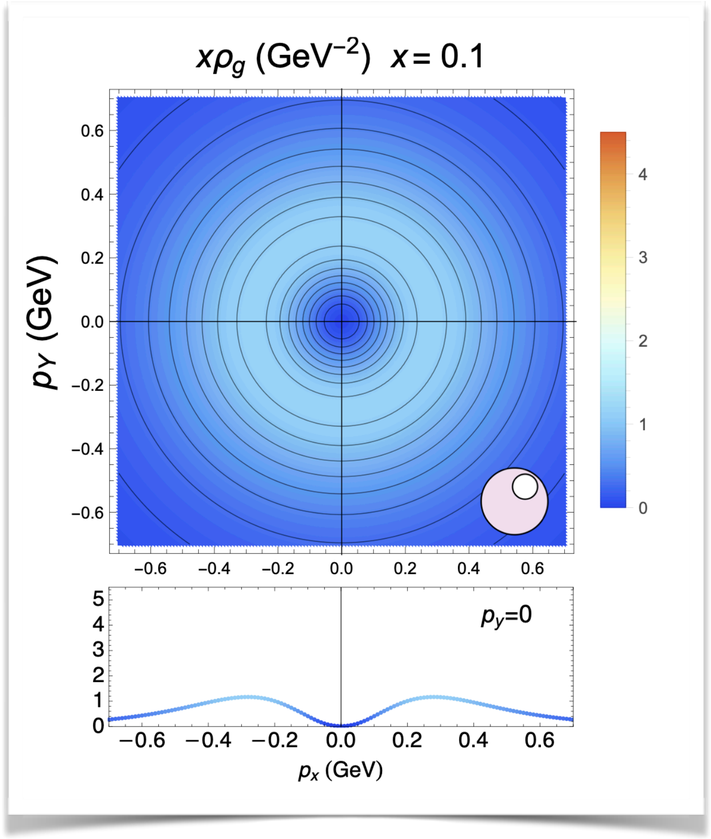


# 3D imaging of the proton

# 3D tomography: the gluon content in the proton

**unpolarized TMD**




🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels, *Eur. Phys. J. C* **80** (2020) no.8 [arXiv:2005.02288]]

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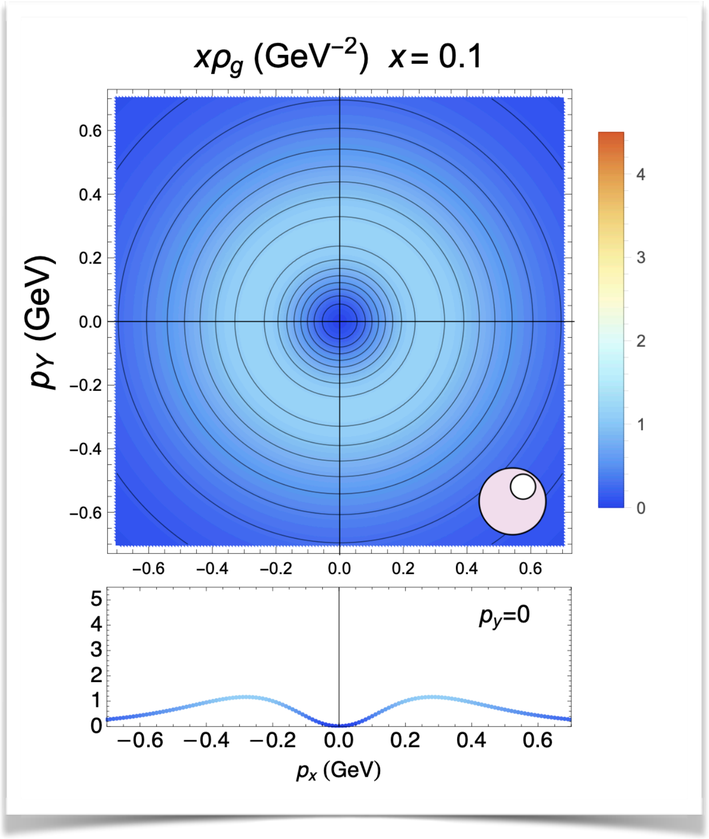
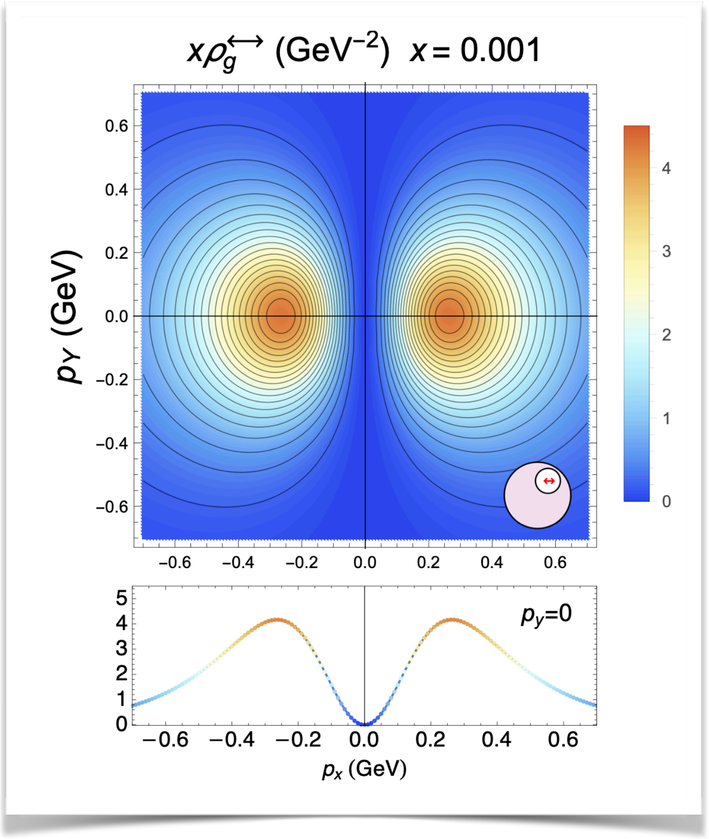



**Boer-Mulders**

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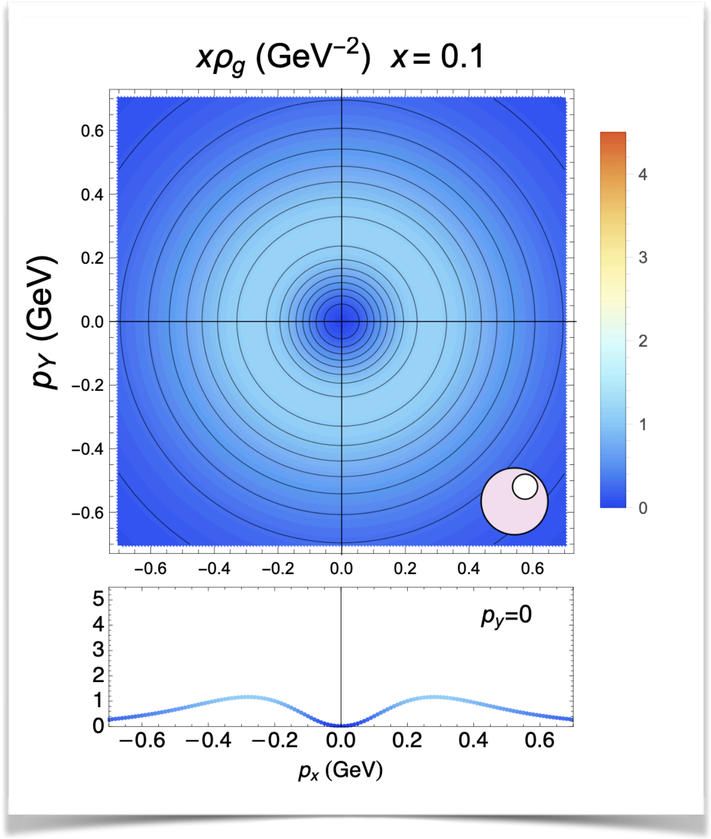


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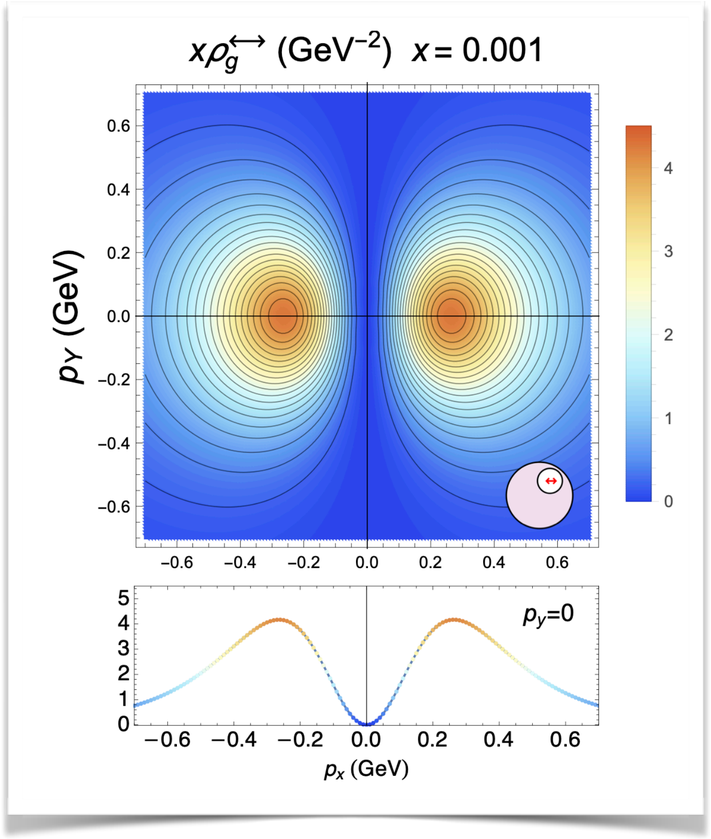
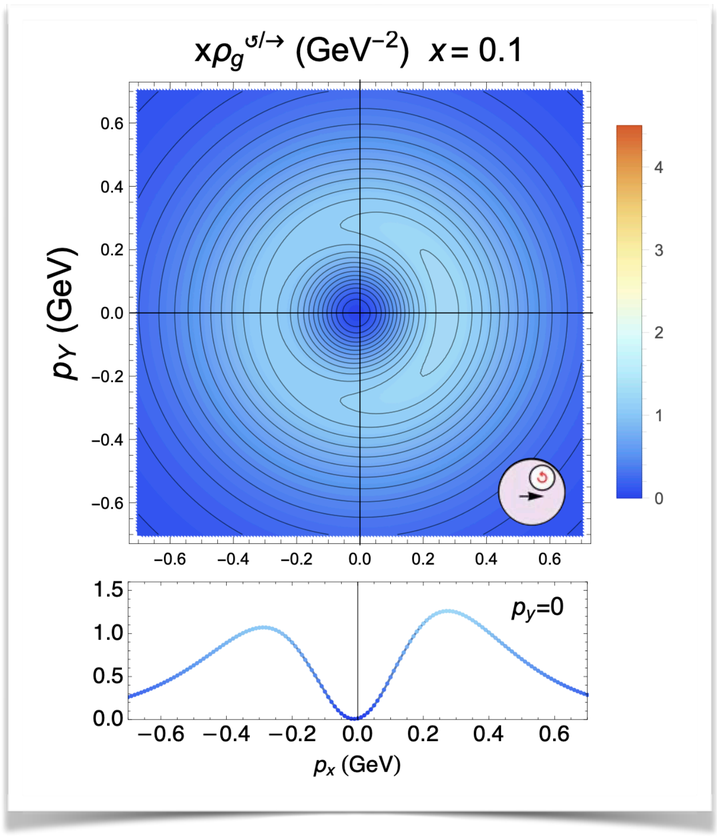



**worm-gear**

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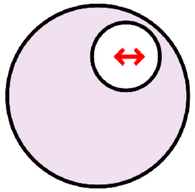
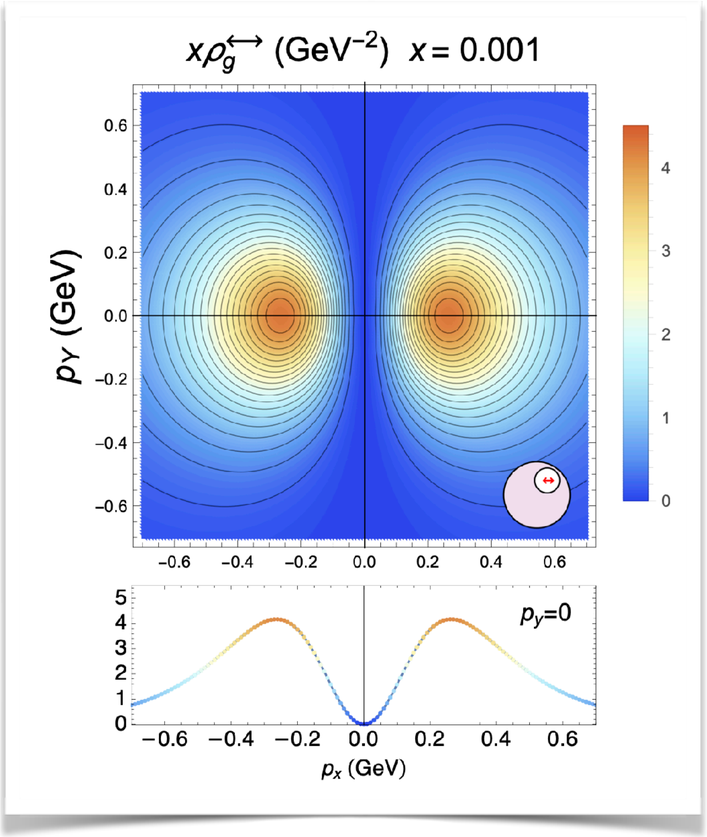
# Boer-Mulders effect in unpolarized *pp* collisions



$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x,p_x,p_y)$$

unpol. Boer-Mulders

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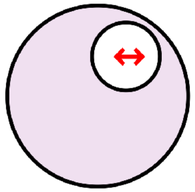

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unpol. Boer-Mulders

**(Pseudo)scalar $p_T$-distribution: Higgs, $\eta_{b,c}$**

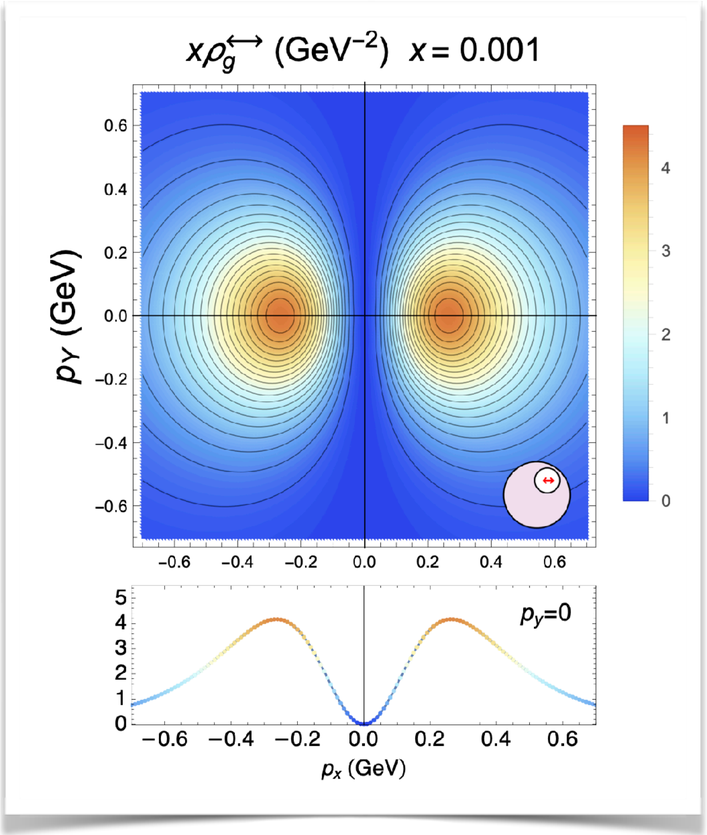
$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \, \Phi_B^U \, |\mathcal{M}|^2 \qquad \text{at low transverse momentum for (pseudo)scalar state}$$

$$\sim \mathcal{C}[\, f_1^{g/A} \; f_1^{g/B} \,] \pm \mathcal{C}[\, h_1^{\perp g/A} \; h_1^{\perp g/B} \,]$$

$\mathcal{C}[\, f_1^{g/A} \; f_1^{g/B} \,]$: **unpolarized** gluons

$\mathcal{C}[\, h_1^{\perp g/A} \; h_1^{\perp g/B} \,]$: **lin. polarized** gluons

🔗 [D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) 🔗 [D. Boer, C. Pisano (2015)]

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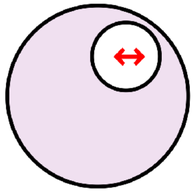
# Boer-Mulders effect in unpolarized *pp* collisions



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unpol. Boer-Mulders

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**unpolarized** gluons — **lin. polarized** gluons

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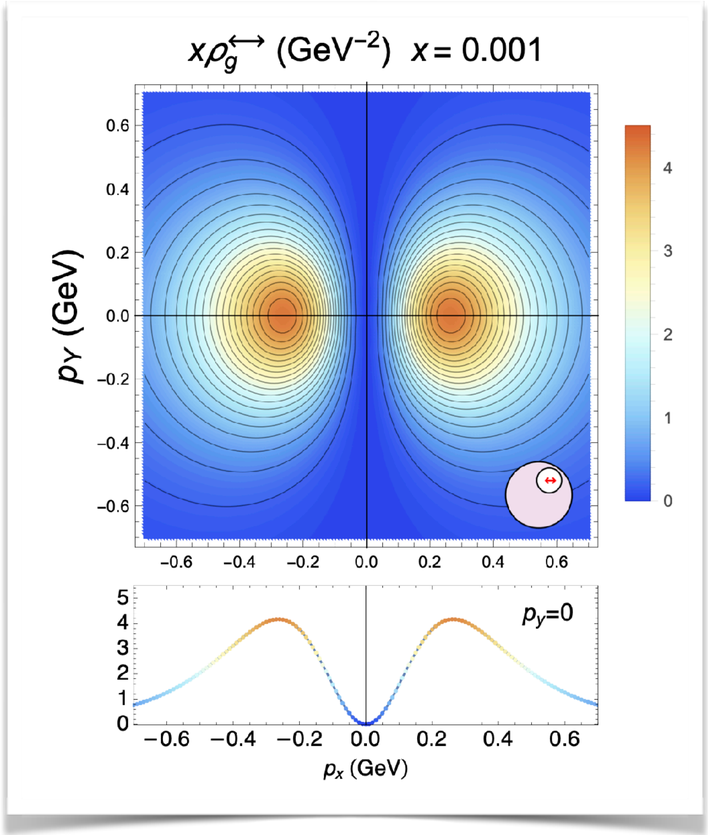


[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

**Model prediction at low-$x$**

$$\frac{f_1^g(x,p_T^2)}{h_1^{\perp g}(x,p_T^2)} \underset{x\to 0^+}{\sim} \text{constant}$$

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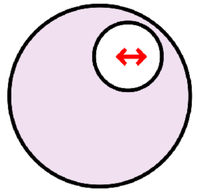

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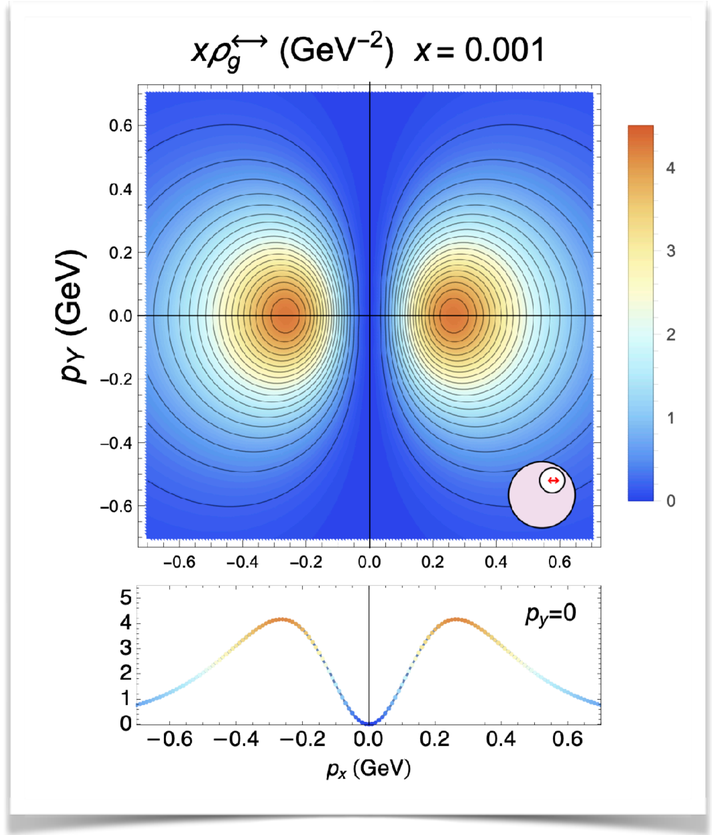


[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

**Model prediction at low-$x$**

$$\frac{f_1^g(x,p_T^2)}{h_1^{\perp g}(x,p_T^2)} \underset{x\to 0^+}{\sim} \text{constant}$$

$$f_1^g(x,p_T^2) = h_1^{\perp g}(x,p_T^2) + \text{higher twist}$$

**HEF regime (linear low-$x$ evolution)**

# Checkpoints and further steps

- ☑ Systematic calculation of all twist-2 T-even gluon TMDs
- ☑ Spectral mass to catch small- and large-$x$ effects
- ☑ **Simultaneous fit** of $f_1$ and $g_1$ PDFs via **replica method**
- ☑ Inclusion of standard CSS evolution (investigation on *collinear matching*)

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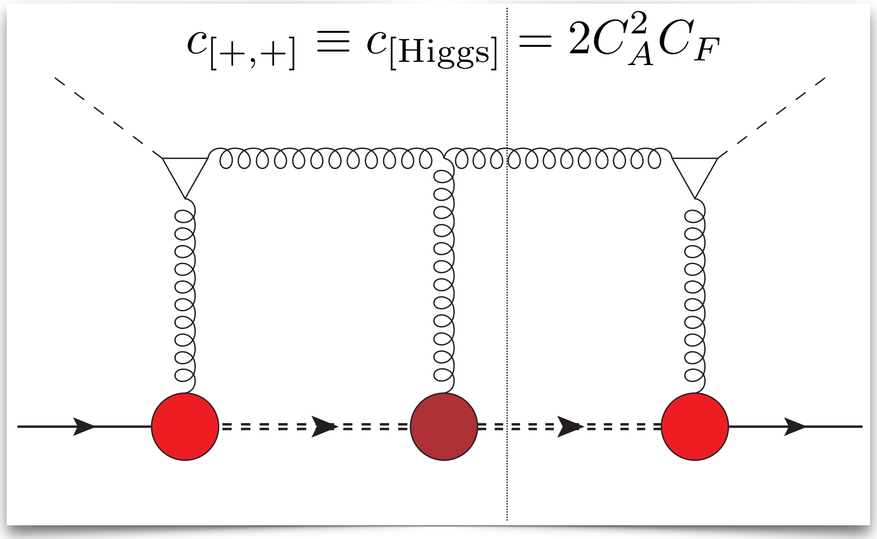
- ☑ Inclusion of standard CSS evolution (investigation on *collinear matching*)
- ☐ Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- ☐ Twist-2 T-odd gluon TMDs (**Sivers**, etc.) almost done!
- ☐ Explorative studies on gauge-link sensitivity and factorization

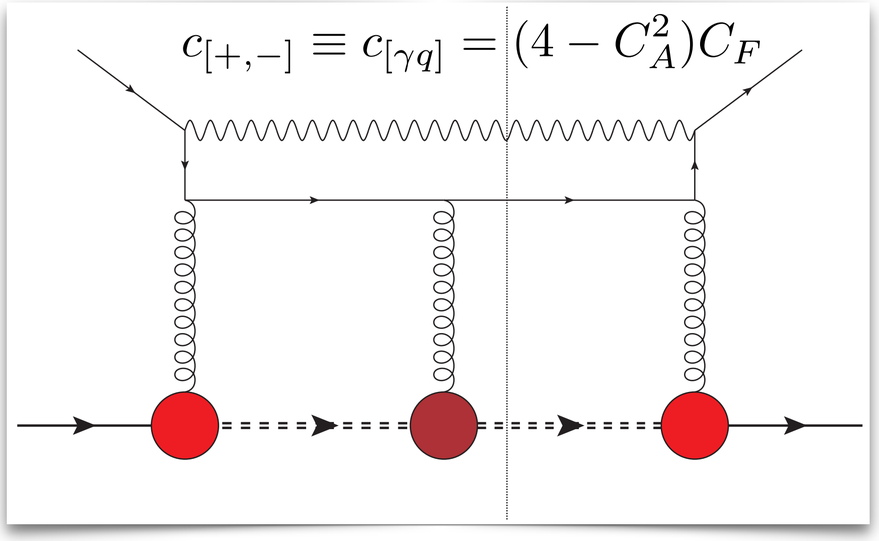
# Backup slides

# Gauge-link dependence of T-odd gluon TMDs

Higgs-gluon fusion $\Rightarrow$ $f$-type [ + , + ]




Photon-jet emission $\Rightarrow$ $d$-type [ + , − ]




Backup

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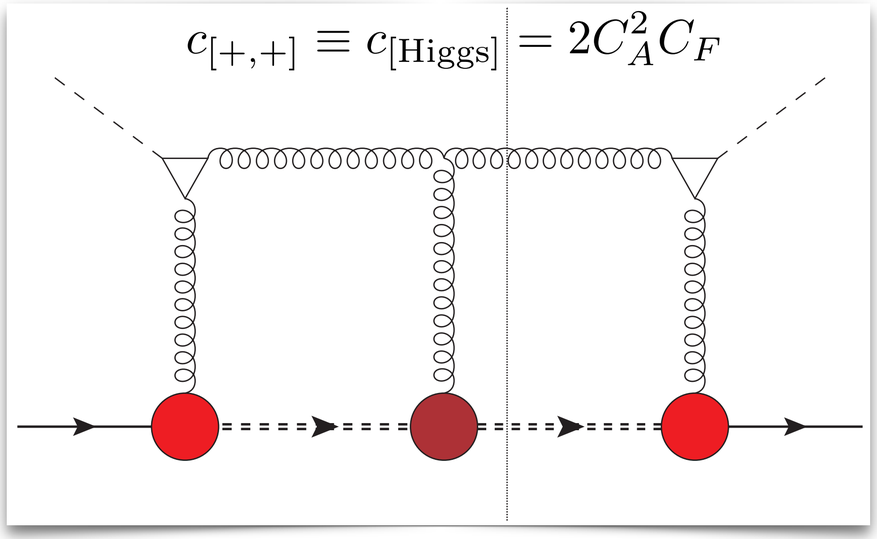
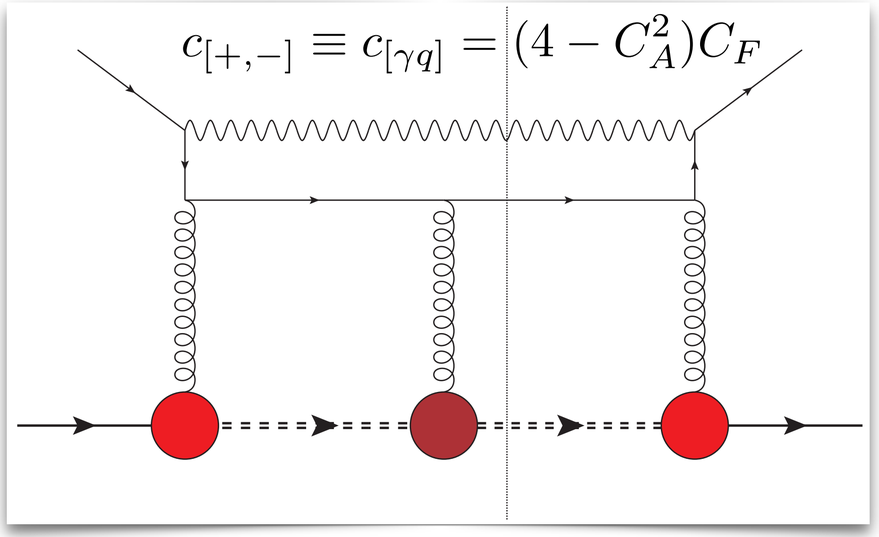
Higgs-gluon fusion $\Rightarrow$ $f$-type $[+,+]$




Photon-jet emission $\Rightarrow$ $d$-type $[+,-]$



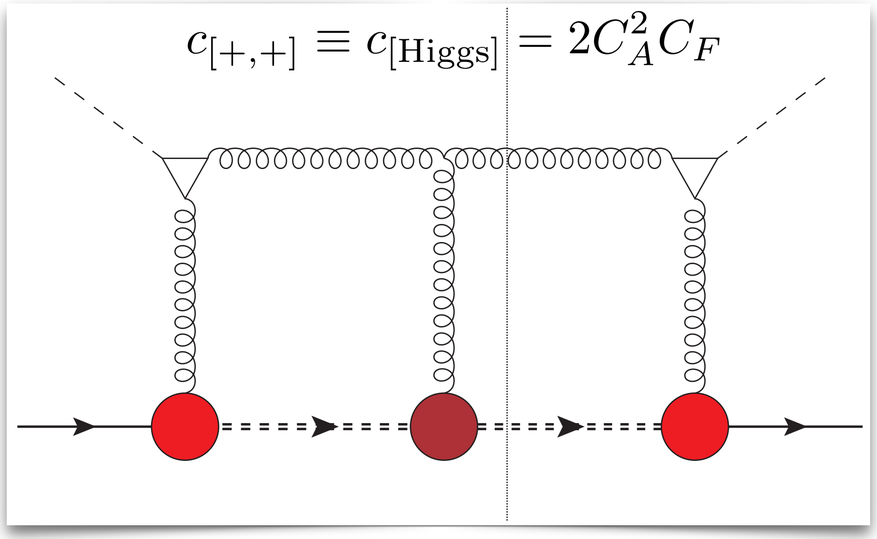

nucleon-gluon-spectator $\quad \mathcal{Y}^{\mu}_{bc}(p^2) = \delta_{bc}\left[g_1(p^2)\,\gamma_\mu + g_2(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$

spectator-gluon-spectator $\quad \mathcal{X}^{\mu}_{abc}(p^2) = f^{abc}\left[g_1^f(p^2)\,\gamma^\mu + g_2^f(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right] - i\,d^{abc}\left[g_1^d(p^2)\,\gamma^\mu + g_2^d(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$

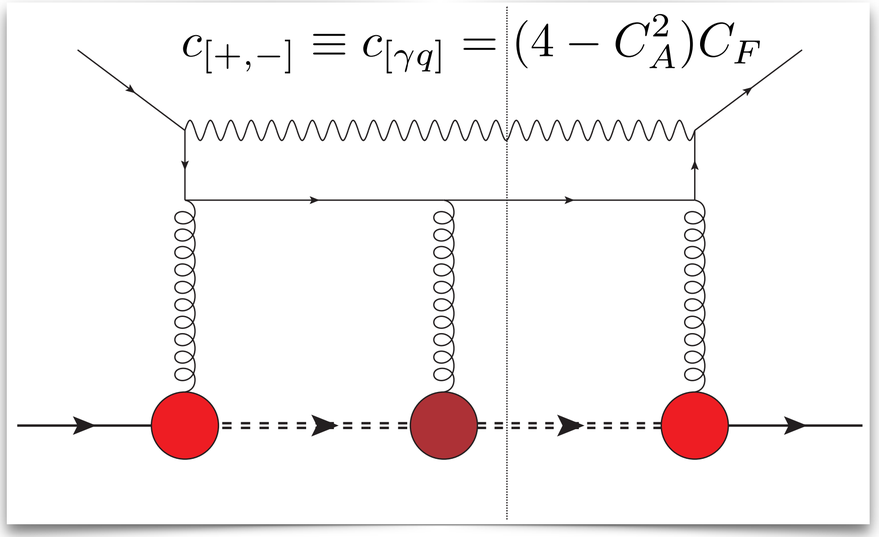
Backup

# Gauge-link dependence of T-odd gluon TMDs

Higgs-gluon fusion $\Rightarrow$ $f$-type $[+,+]$




Photon-jet emission $\Rightarrow$ $d$-type $[+,-]$

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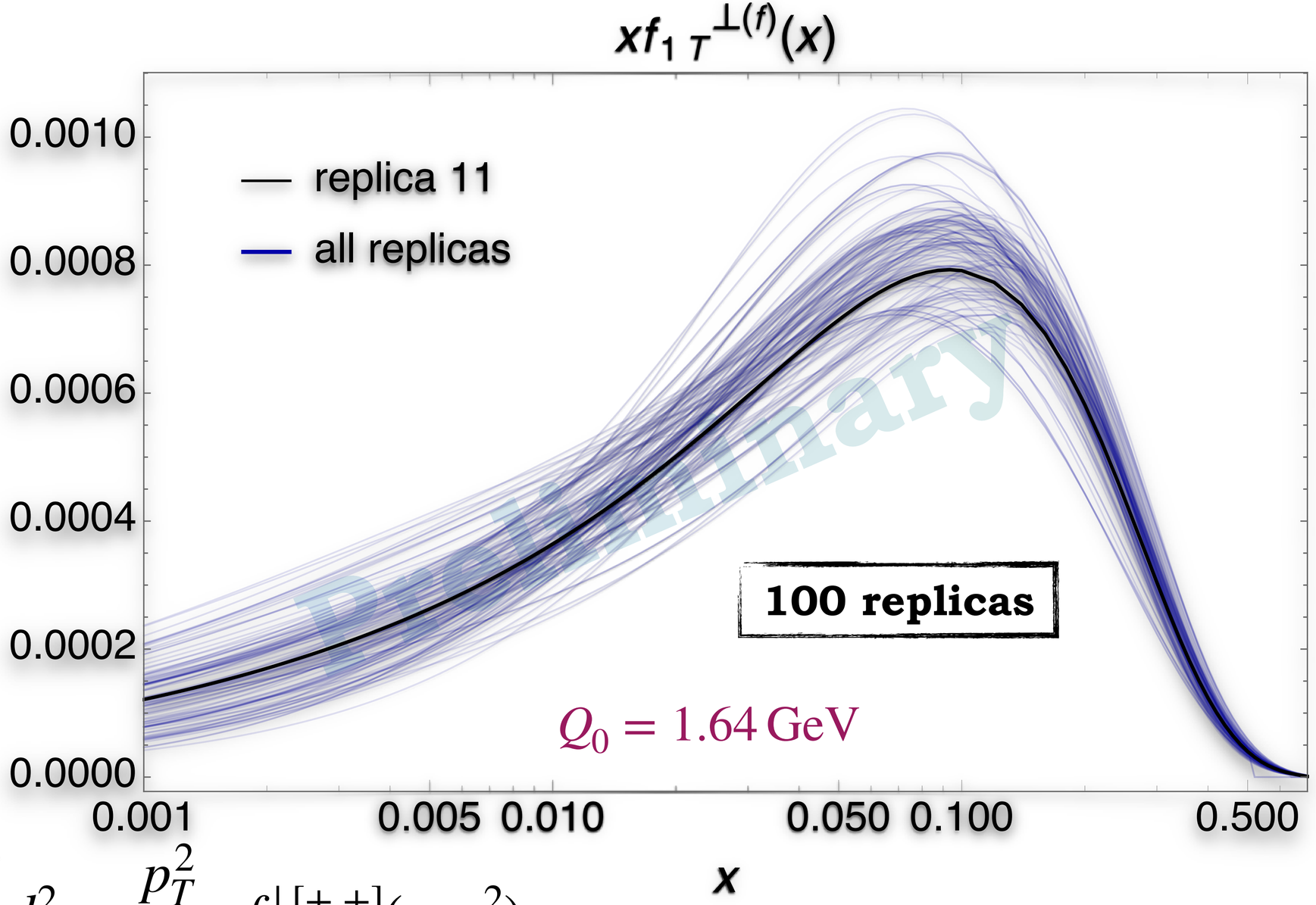
Assumption: $\quad g^d_{1,2}(p^2) = g^f_{1,2}(p^2) \equiv g_{1,2}(p^2) \quad \Leftrightarrow \quad f_{1T}^{\perp[+,-]} = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv -\frac{5}{18} f_{1T}^{\perp[+,+]}$

Backup

# $f$-type Qiu-Sterman twist-3 gluon PDF




$$f_{1T}^{\perp(f)}(x) = \int d^2p_T \frac{p_T^2}{2M^2} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

Backup

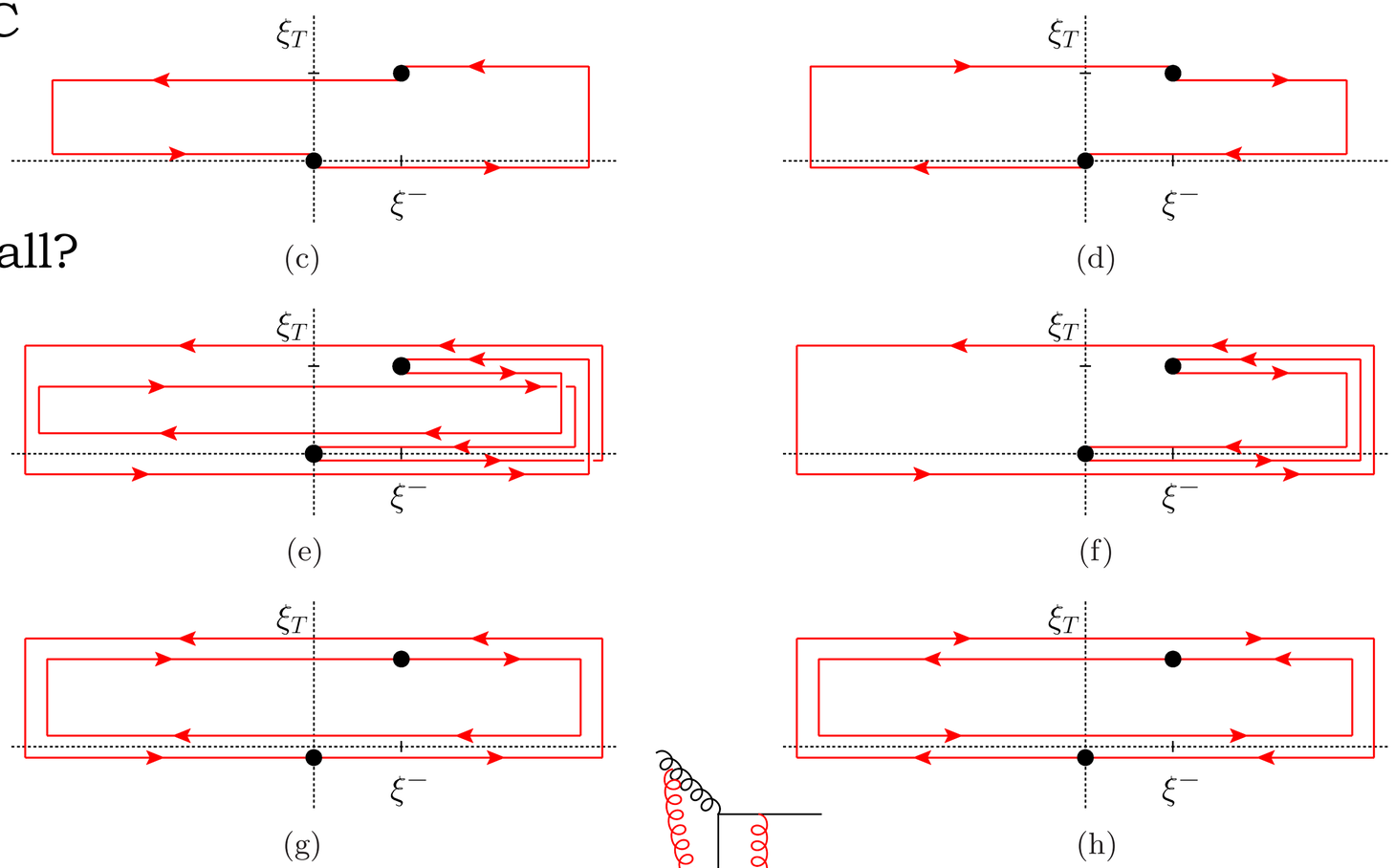
# Dihadron hadroproduction and factorization breaking

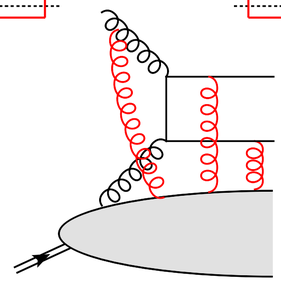
* Proof of factorization violation 🔗 [T. J. Rogers, P. J. Mulders (2010)]
* Assumed factorization in SCET and CGC
* Significance of low-$x$ studies
* Size of factorization-breaking effects small?
* DP TMDs:

  (c) $[+,-]$ and (d) $[-,+]$
* Appearance of new gauge **loop links**:

  (e) $[+\square,+\square]$, (f) $[+,+\square]$,

  (g) $[\square,\square]$, and (h) $[\square,\square]$



Backup

# Assumptions of the model

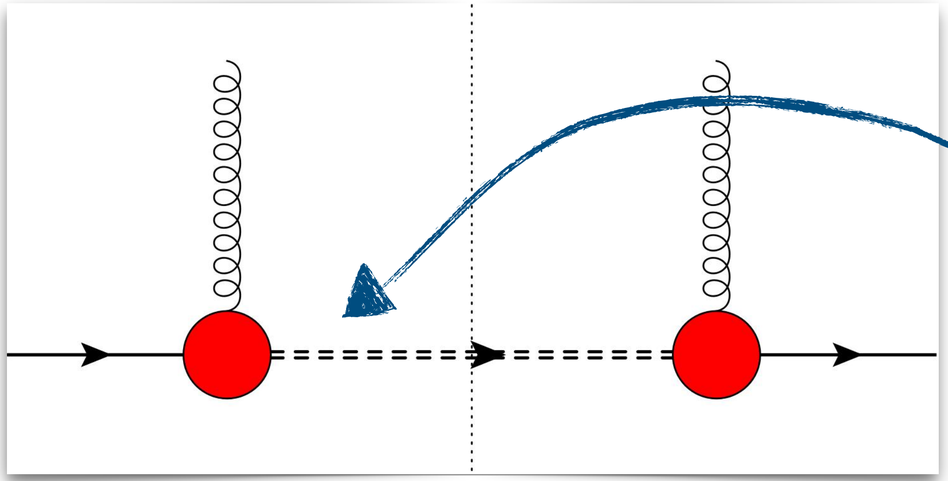
## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



$\mathcal{Y}_g(p^2)$

Instead of a single on-shell spectator, a continuum of spectators

Backup

# Assumptions of the model

## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X\left(M_X; \{X^{(\mathrm{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}\right) = \mu^{2a}\left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

**low-$x$ (high-$\mu^2$) tail** $\propto (a - b)$

$q\bar{q}$ contributions energetically available at large $M_X$

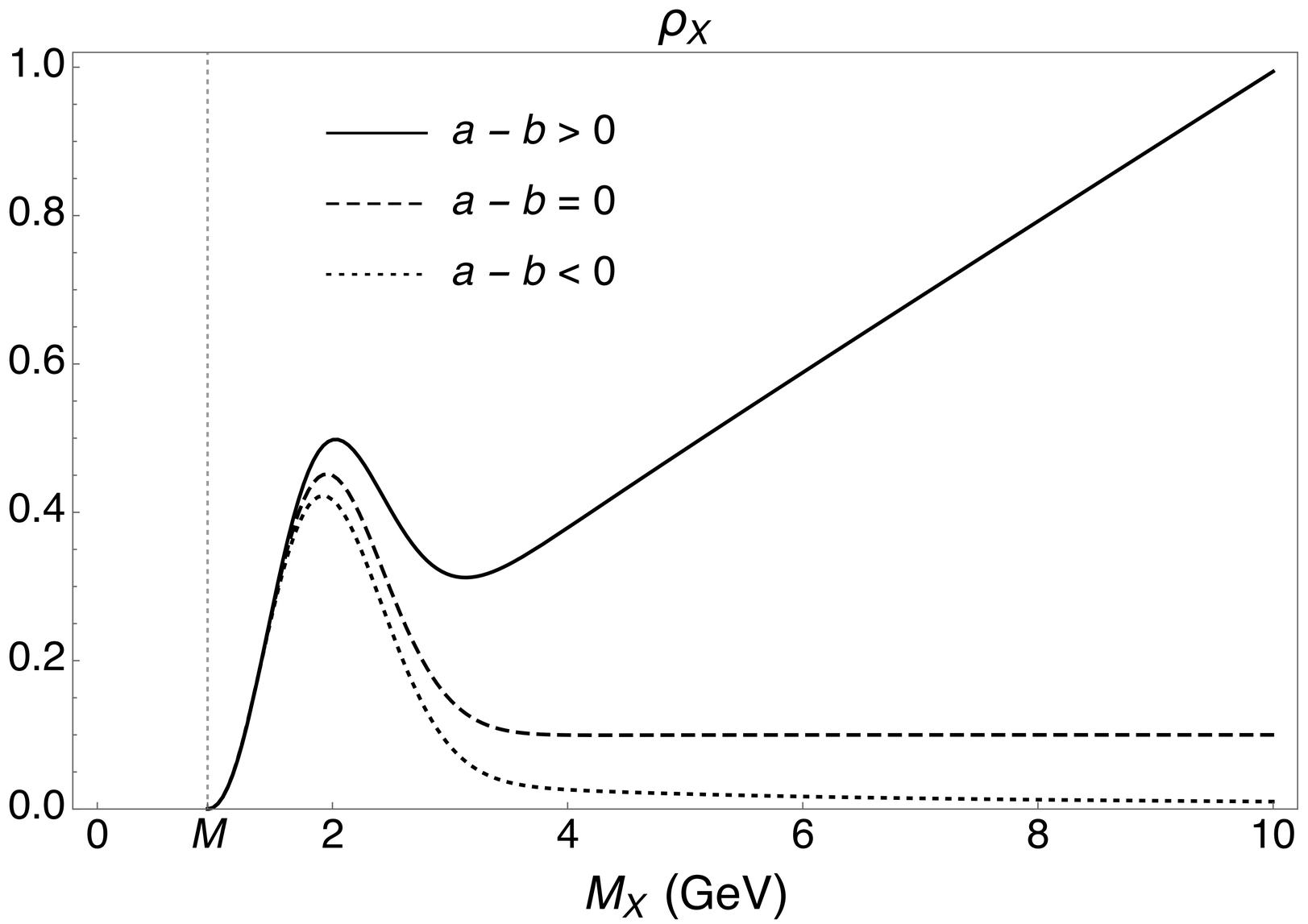
$$\mu^2 = M_X^2 - M^2$$

**moderate-$x$ trend**

pure tri-quark contribution at low $M_X$

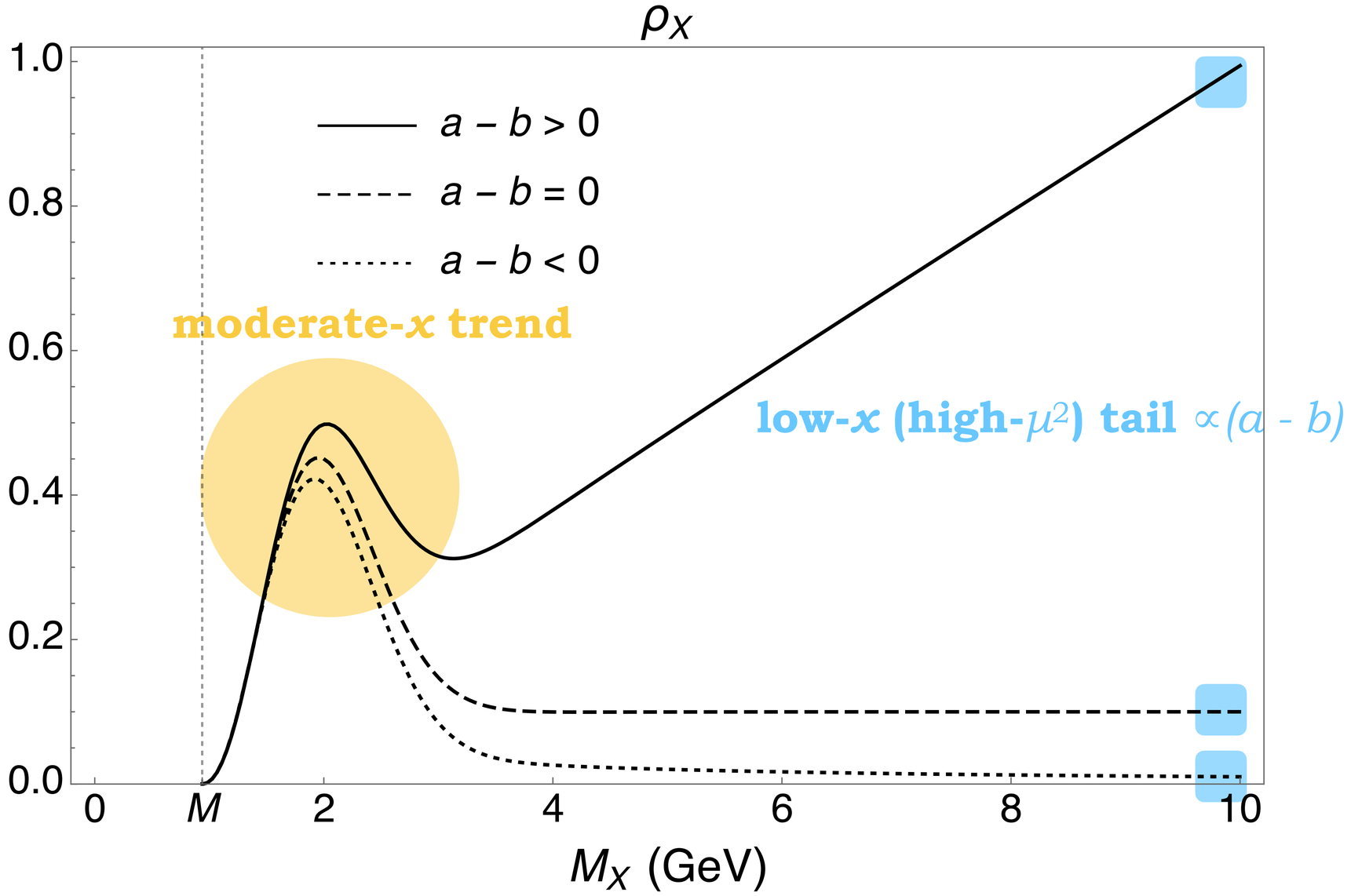
Backup

# Spectral function vs *(a - b)*



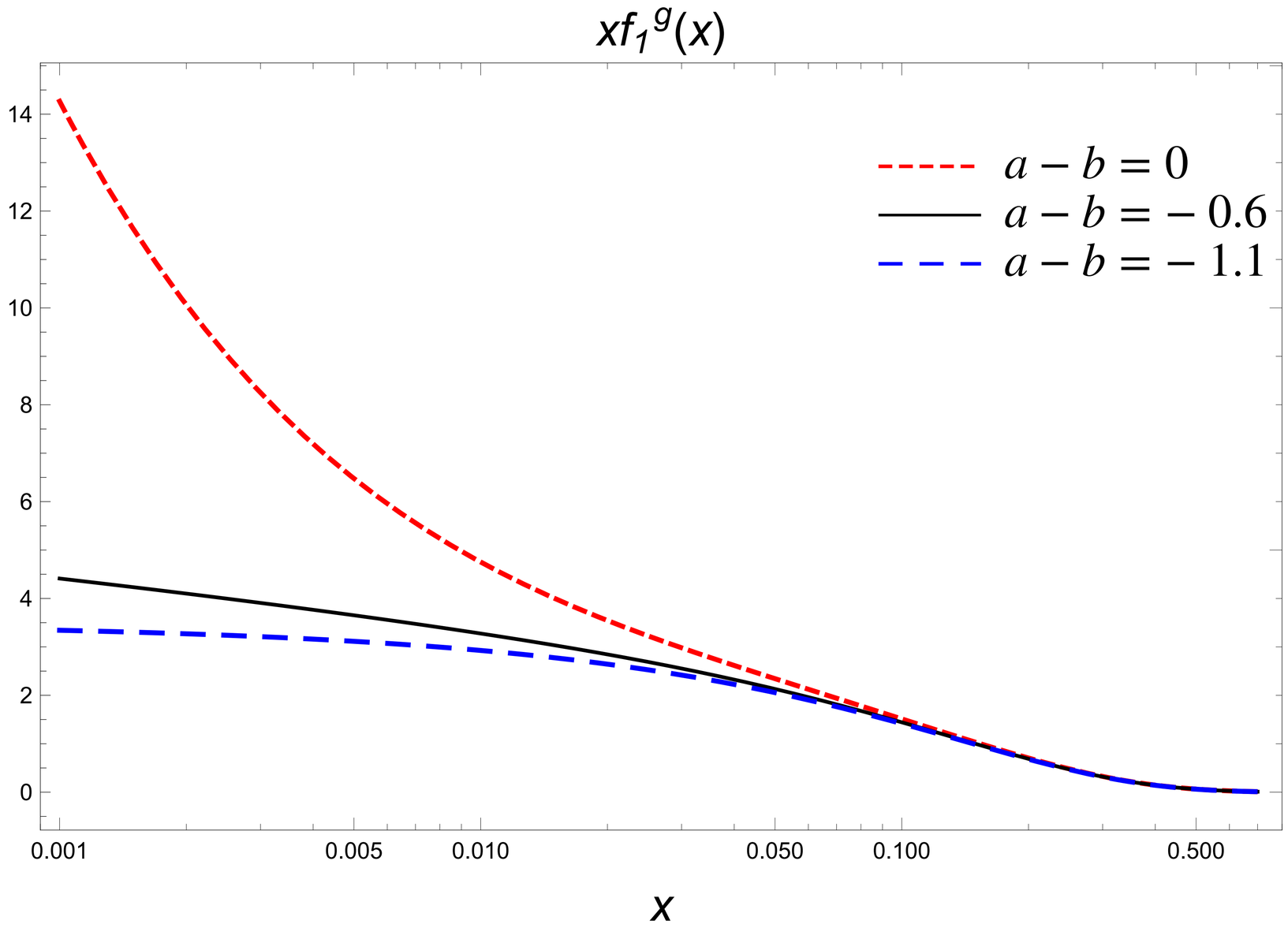

Backup

# Spectral function vs $(a - b)$



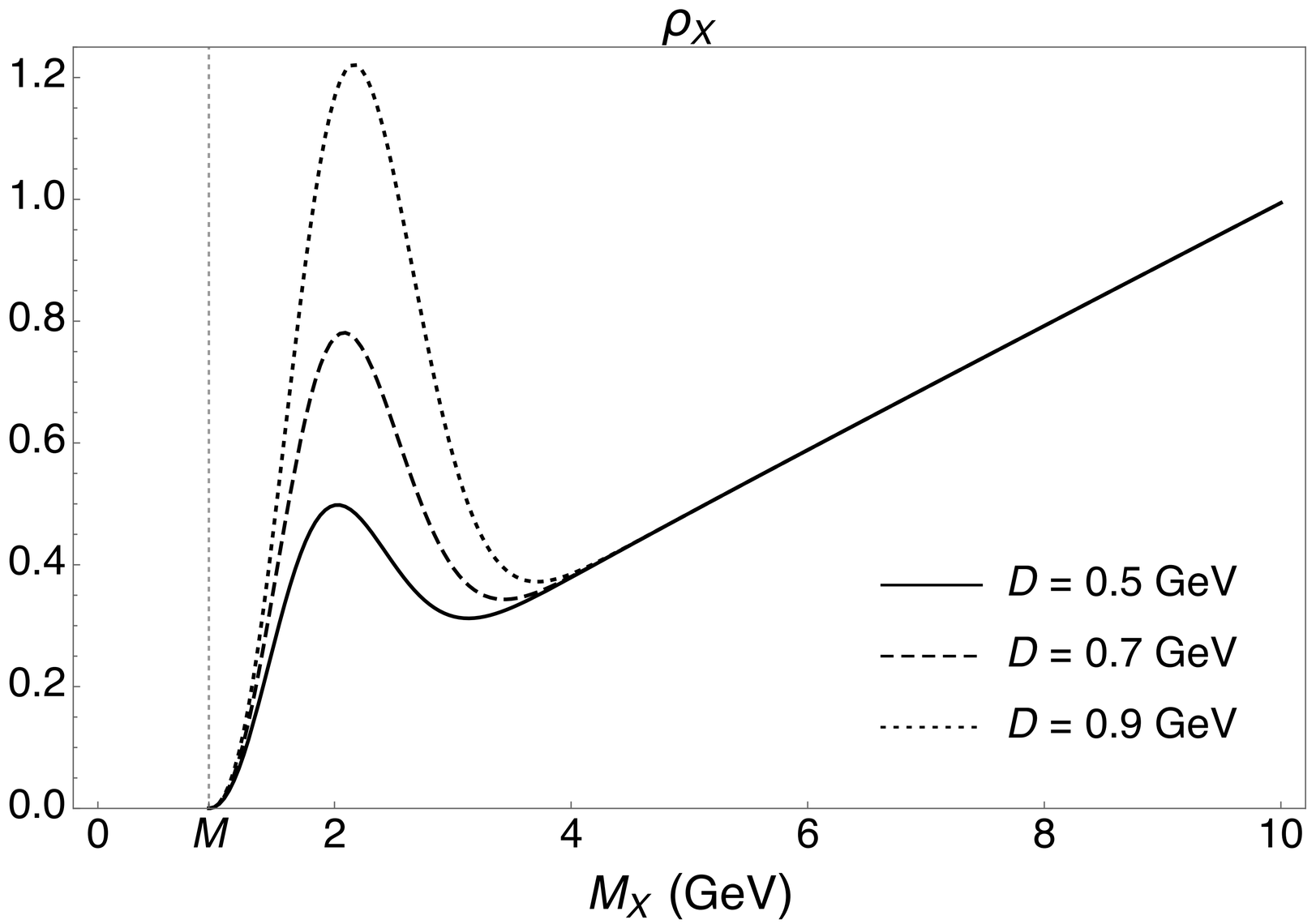

Backup

# $xf_1$ collinear PDF vs $(a - b)$



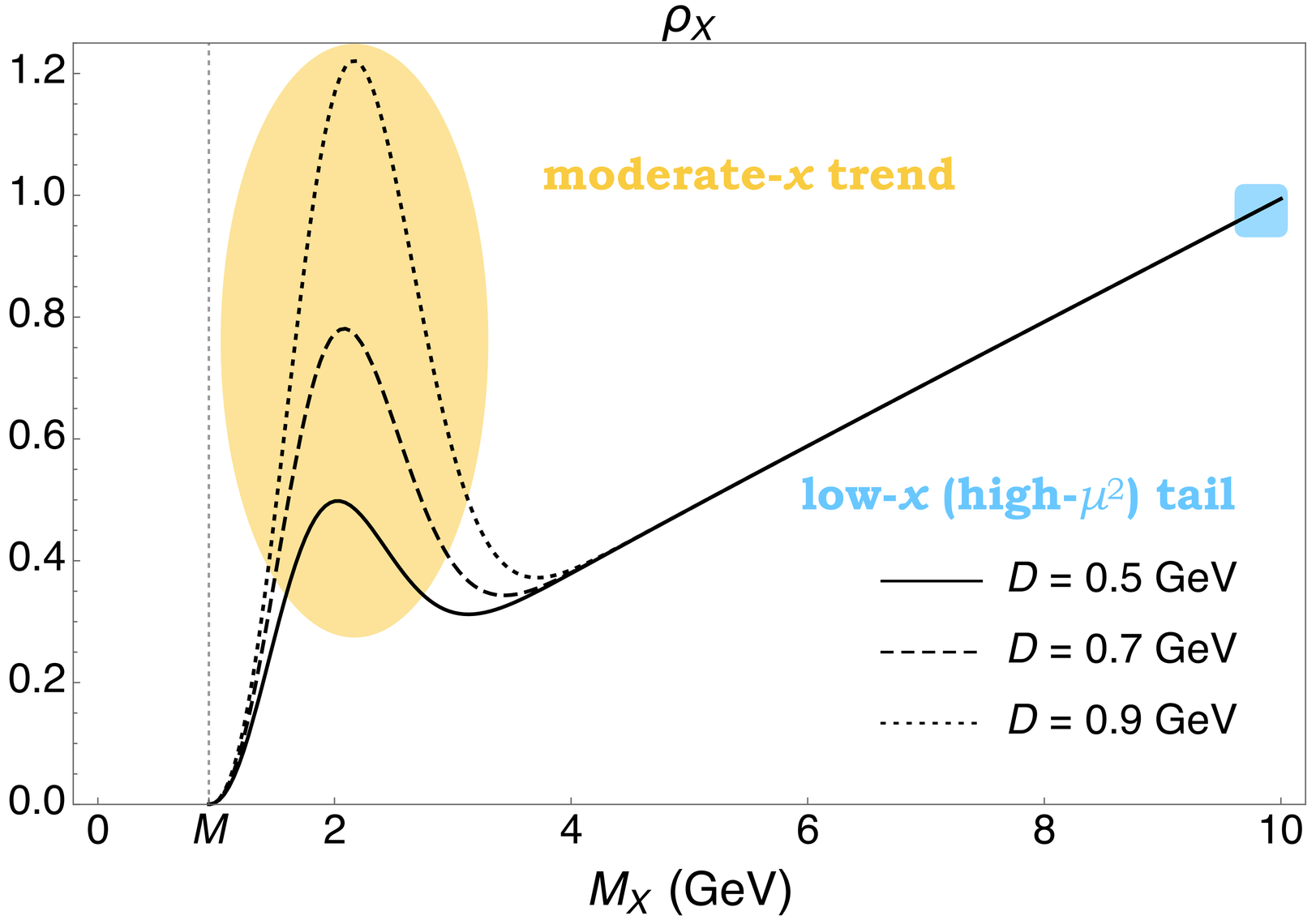

Backup

# Spectral function vs $D$




Backup

# Spectral function vs $D$




Backup

# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for $g_1$

Backup

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$$\langle x \rangle_g = \int_0^1 dx\, x f_1^g(x, Q_0) \qquad S_g = \frac{1}{2}\langle 1 \rangle_{\Delta g} = \int_0^1 dx\, g_1^g(x, Q_0)$$

Backup

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Our model @ $Q_0 = 1.64$ GeV

$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

Lattice @ $Q_0 = 2$ GeV

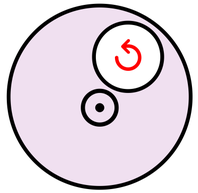
$$\langle x \rangle_g = 0.427(92)$$

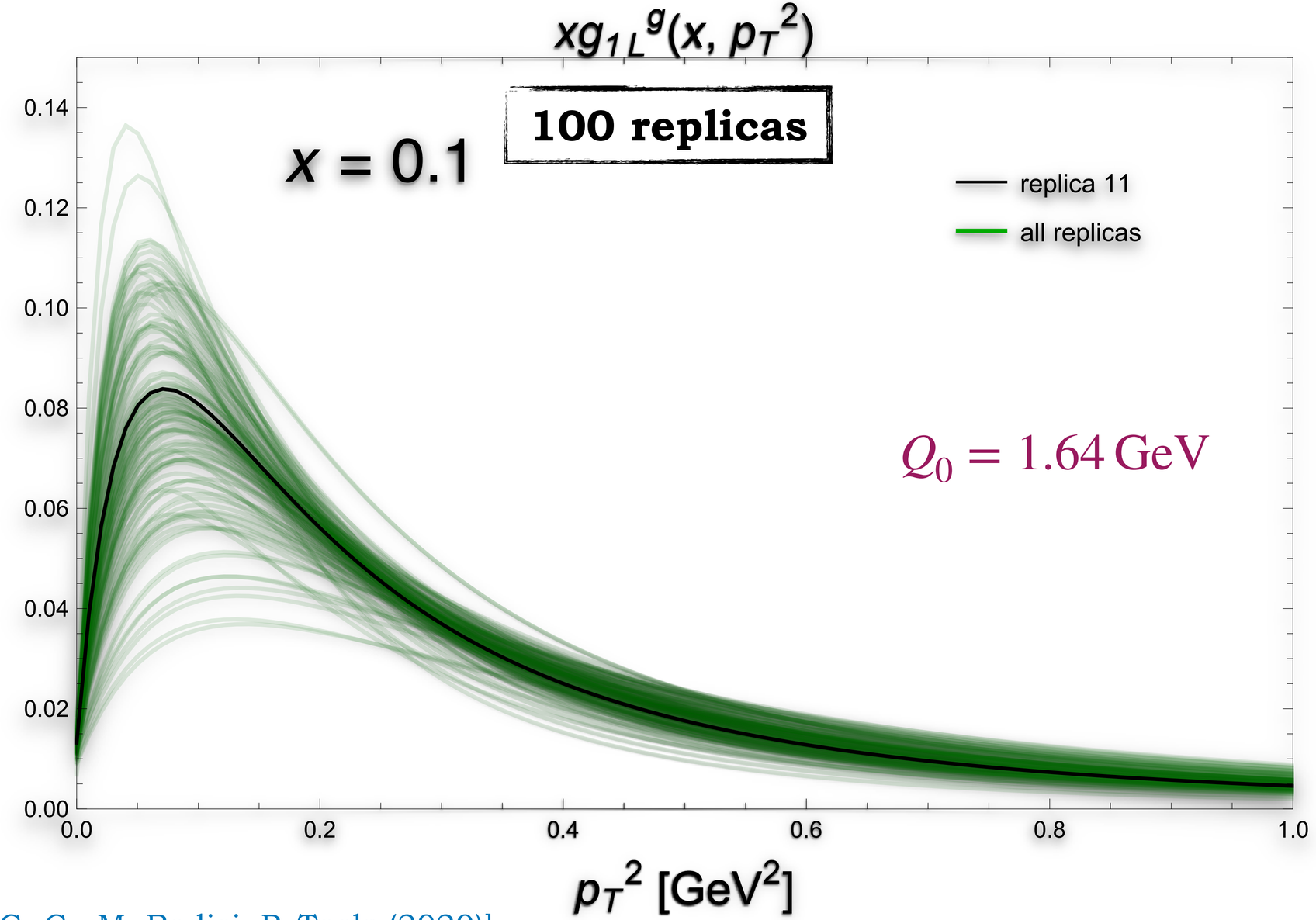
$$\langle J \rangle_g = 0.187(46)$$

[C. Alexandrou *et al.* (2020)]

Backup

# Helicity gluon TMD

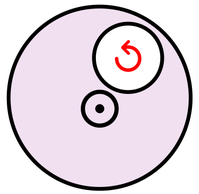


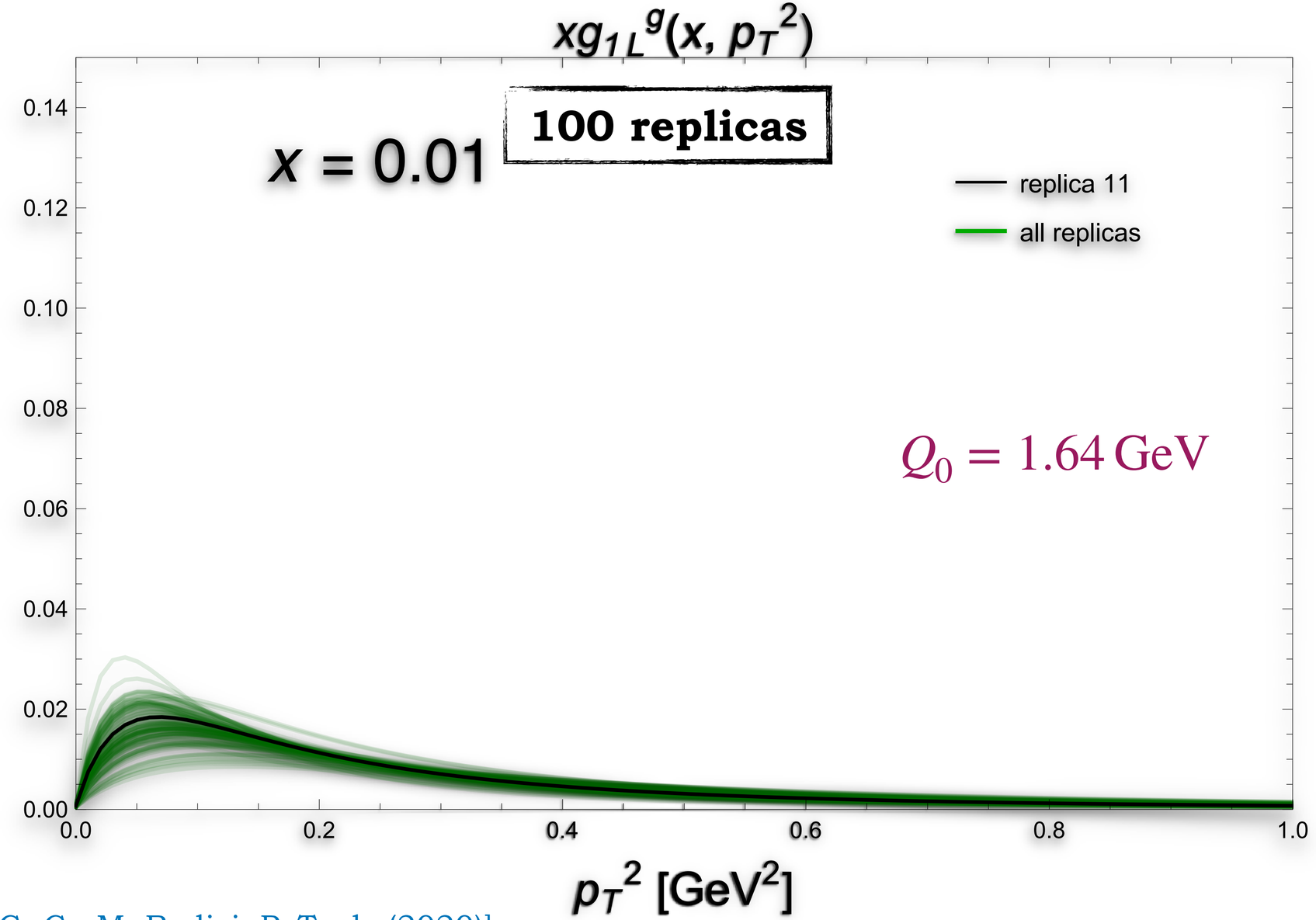



[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Backup

# Helicity gluon TMD

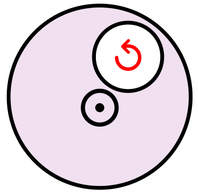


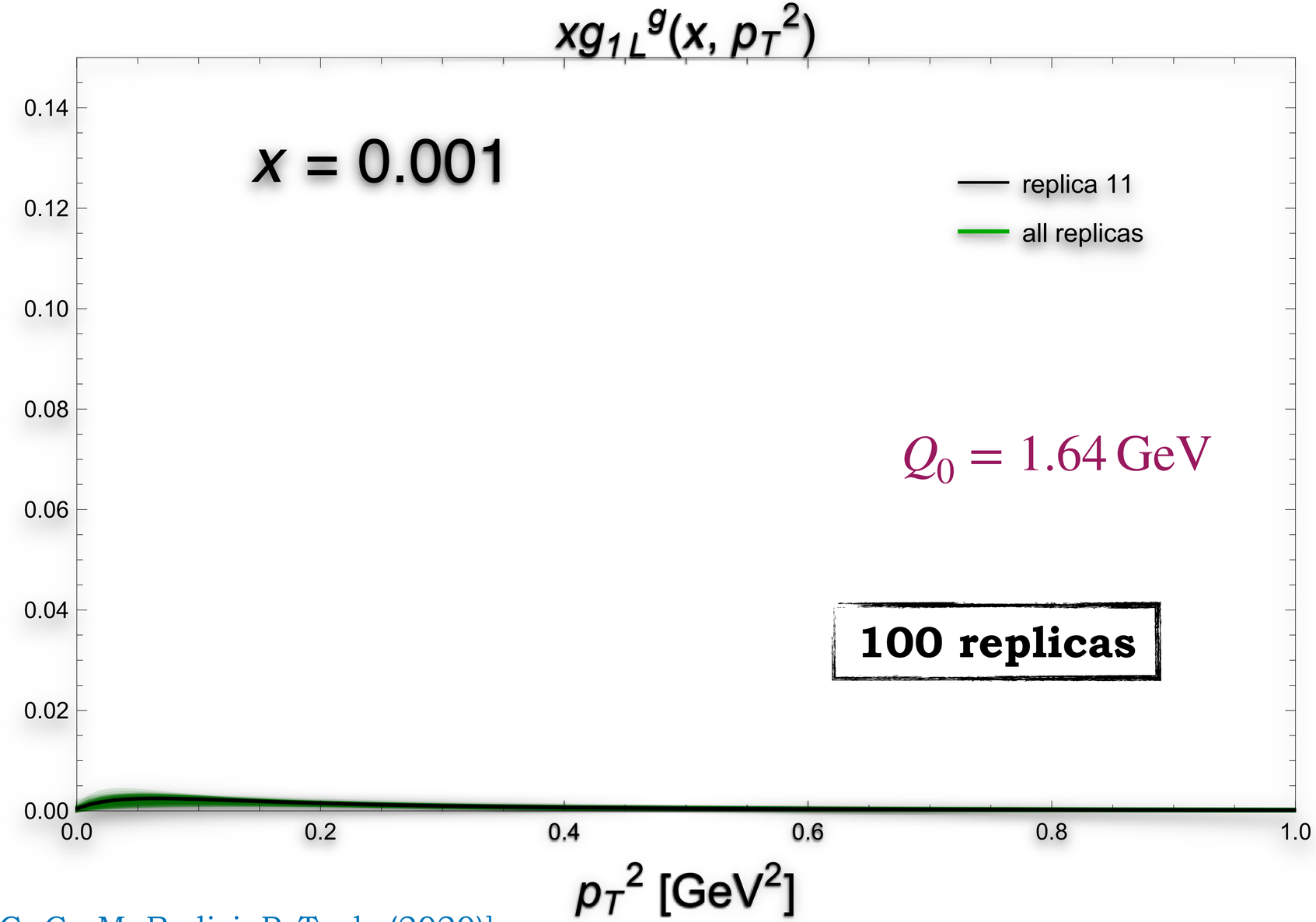



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Backup

# Helicity gluon TMD

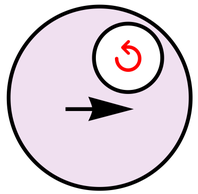


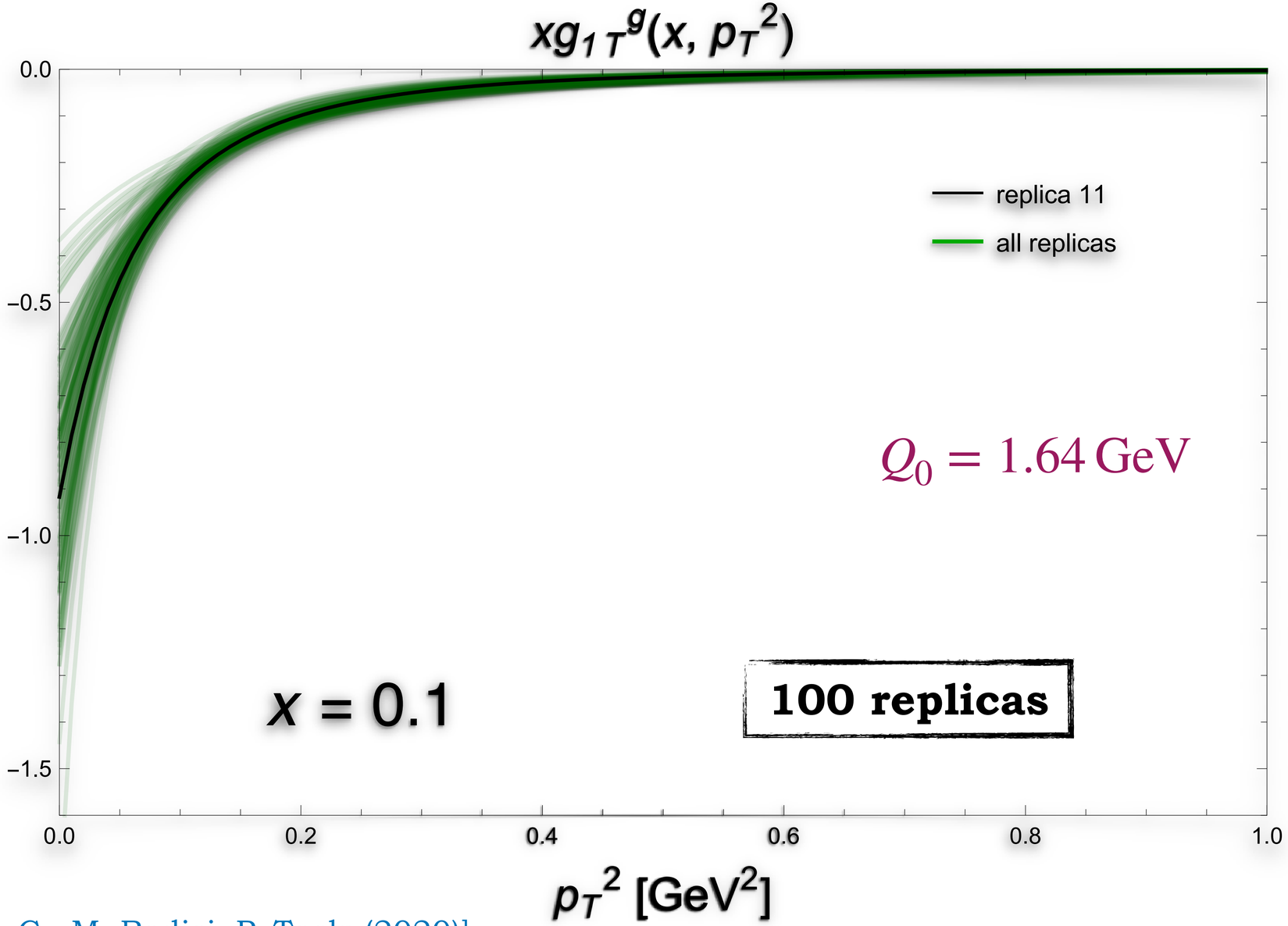



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Backup

# Worm-gear gluon TMD

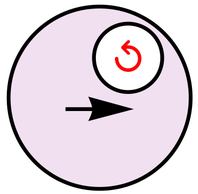


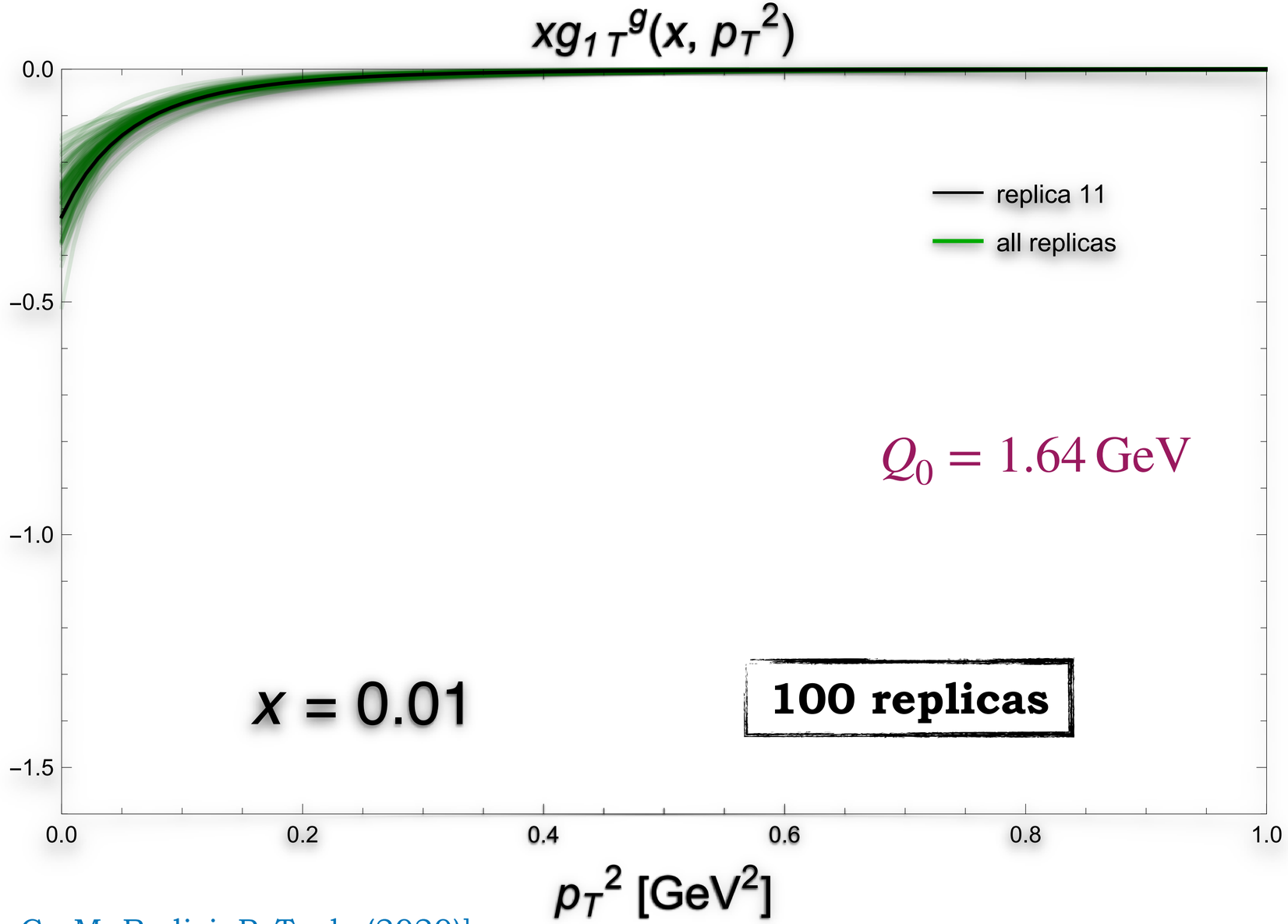



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Backup

# Worm-gear gluon TMD

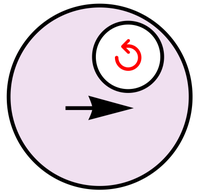


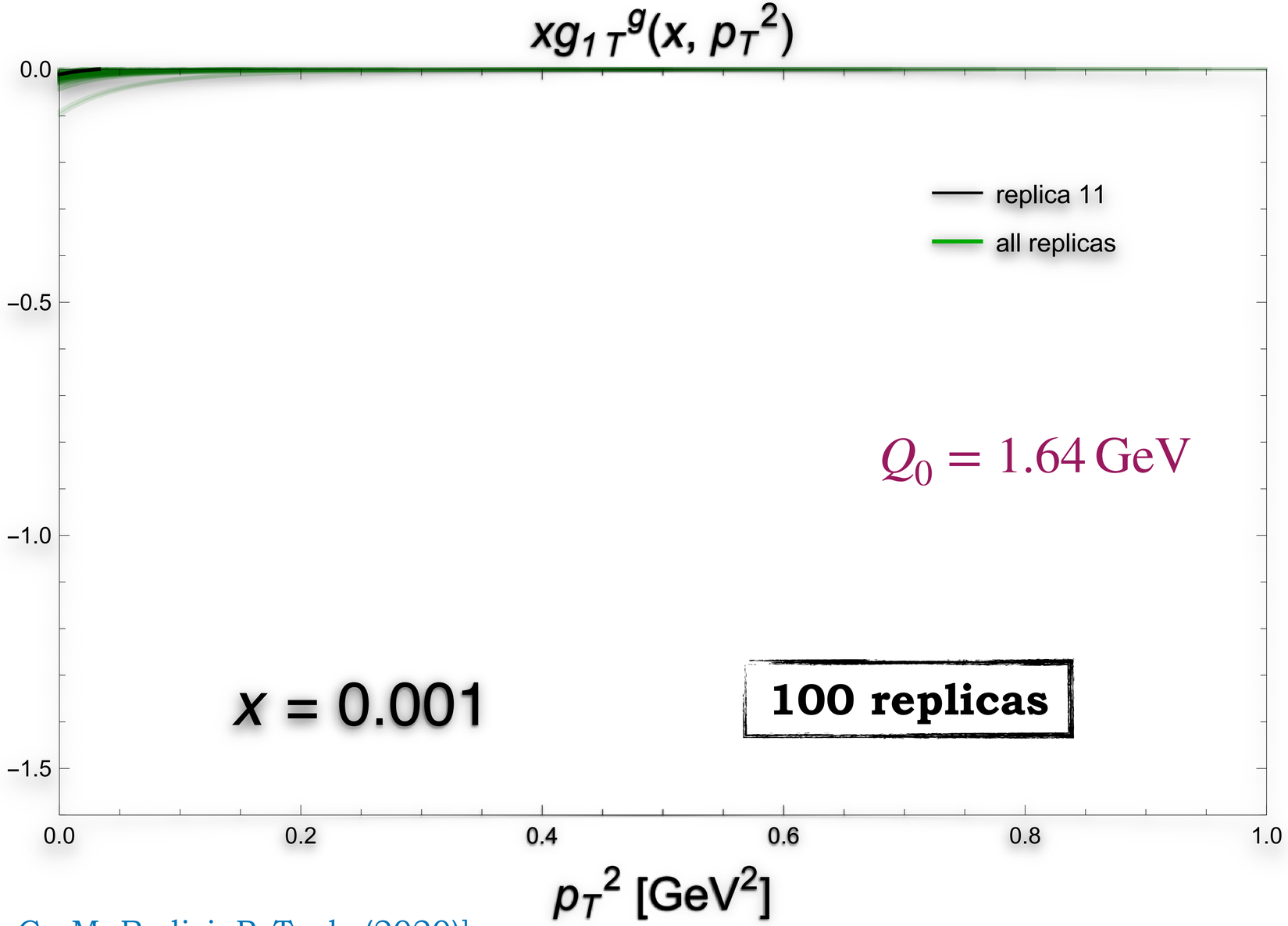



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

Backup

# Worm-gear gluon TMD



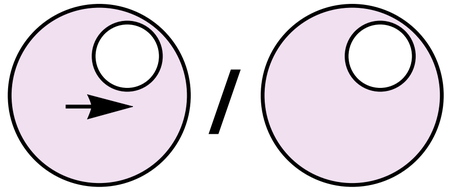



🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

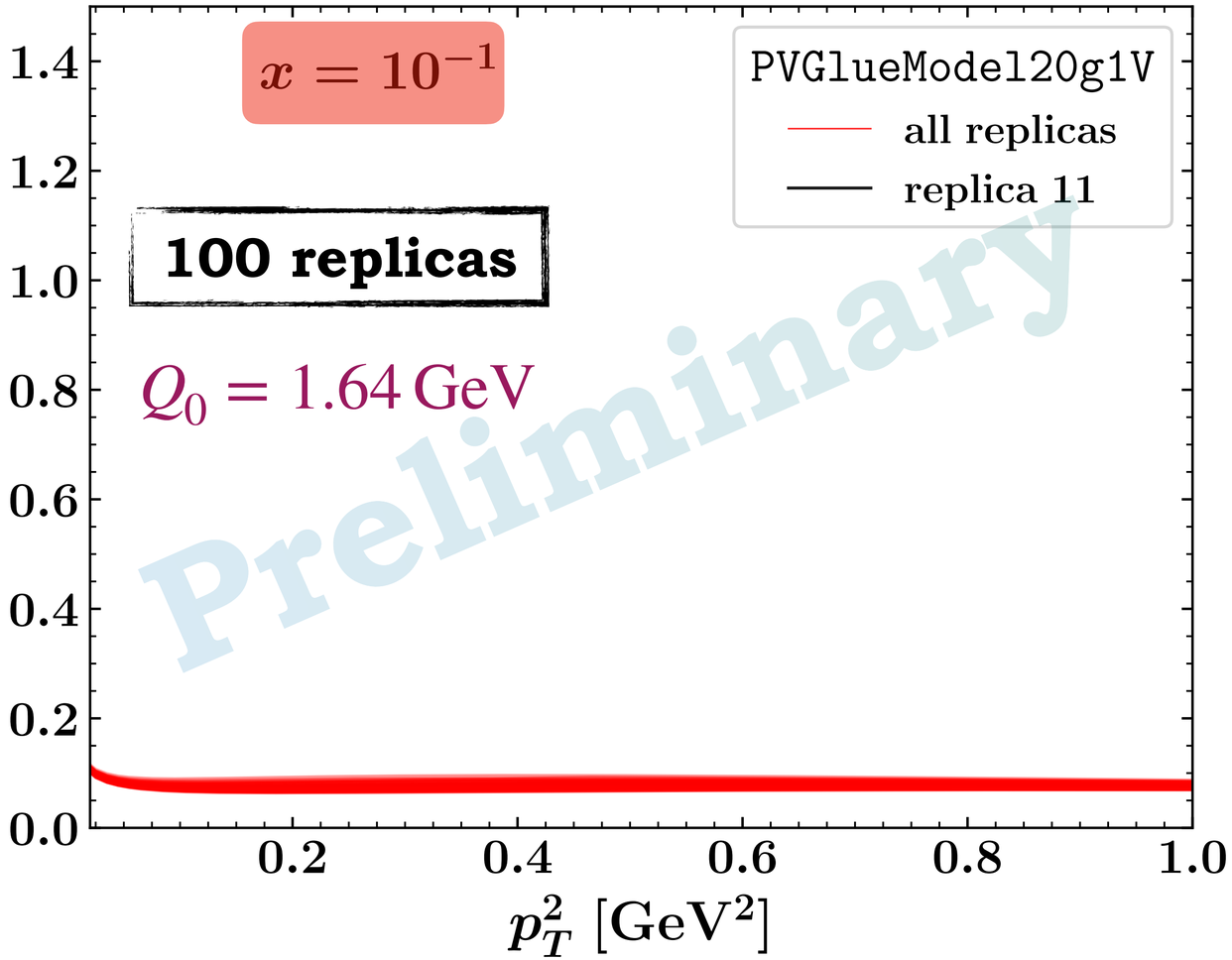
Backup

# $f$-type Sivers/unpol.

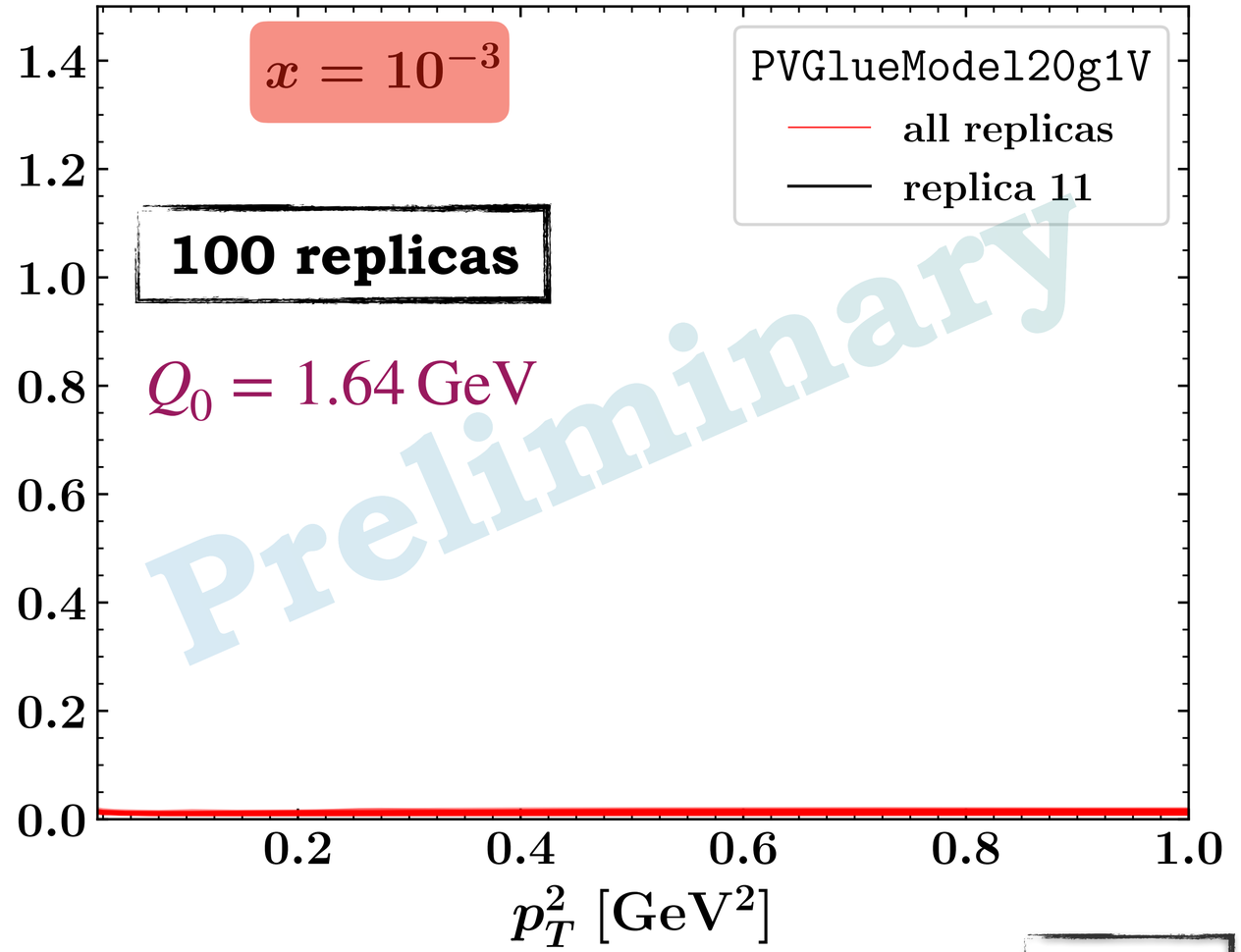
[A. Bacchetta, F.G. C., M. Radici (in preparation)]



$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$




$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$




Backup

# Gluon TMD correlator and T-odd gluon densities

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij} f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp}(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^{\perp}(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{k}^2)\right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\right]$$

Backup

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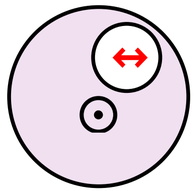
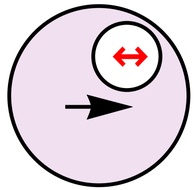
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\right]$$

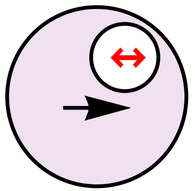


pseudo worm-gear



linearity TMD

pretzelosity



Backup

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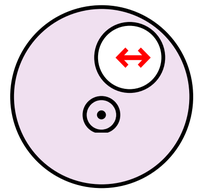
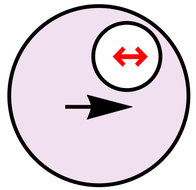
$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^{\perp}(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{k}^2)\right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\right]$$

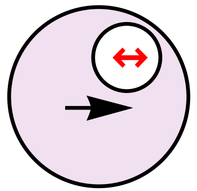
pseudo worm-gear



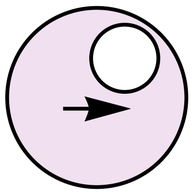
linearity TMD



pretzelosity



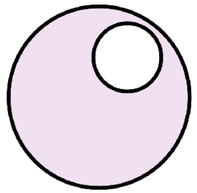
Sivers



$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) = \frac{1}{2}\,\delta_{Tij}\,\Gamma_T^{ij}(x,\boldsymbol{k})$$

Backup

# $\rho$-densities

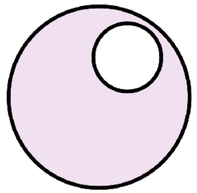


**Unpolarized [u/u]**

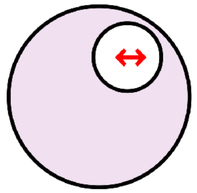
$$f_1(x, p_x, p_y)$$

Backup

# $\rho$-densities



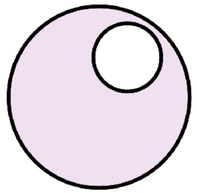
**Unpolarized [u/u]** $f_1(x, p_x, p_y)$




**Boer-Mulders [↔/u]** $f_1(x, p_x, p_y) + \dfrac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x, p_x, p_y)$

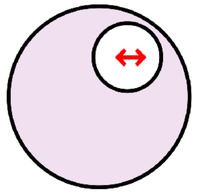
Backup

# $\rho$-densities



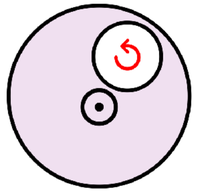
**Unpolarized [u/u]**

$$f_1(x, p_x, p_y)$$




**Boer-Mulders [↔/u]**

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x, p_x, p_y)$$

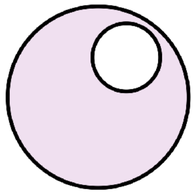



**Helicity [↻/+]**

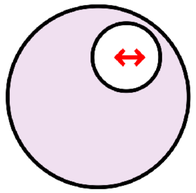
$$\frac{1}{2}\left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y)\right]$$

Backup

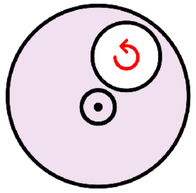
# $\rho$-densities



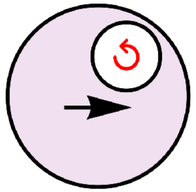
**Unpolarized [u/u]** $f_1(x, p_x, p_y)$




**Boer-Mulders [↔/u]** $f_1(x, p_x, p_y) + \dfrac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x, p_x, p_y)$




**Helicity [↺/+]** $\dfrac{1}{2}\left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y)\right]$




**Worm-gear [↺/→]** $f_1(x, p_x, p_y) - \dfrac{p_x}{M} g_{1T}(x, p_x, p_y)$

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# $\eta_{b,c}$ production in unpolarized $pp$ collisions

**TMD phenomenology:**

**from JLab to the LHC**

Andrea Signori

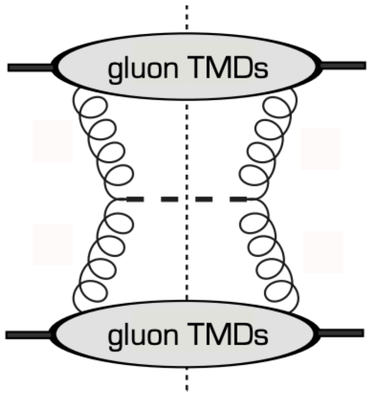
Spatial and momentum tomography of hadrons and nuclei

INT 17-3
Sept 25 2017

**NRQCD**

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$

## gluon TMD PDFs




pseudoscalar quarkonium production:

$p\ p \to \eta_b\ X$ M = 9.39 GeV

$p\ p \to \eta_c\ X$ M = 2.98 GeV

(see also talk by C. Pisano week 4)

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U\ \Phi_B^U\ |\mathcal{M}|^2$$

unpolarized cross section at low transverse momentum for (pseudo)scalar state

$$\sim \mathcal{C}[\ f_1^{g/A}\ f_1^{g/B}\ ] \pm \mathcal{C}[\ h_1^{\perp g/A}\ h_1^{\perp g/B}\ ]$$

**unpolarized** gluons — **lin. polarized** gluons

$C_{ff}$ $C_{hh}$



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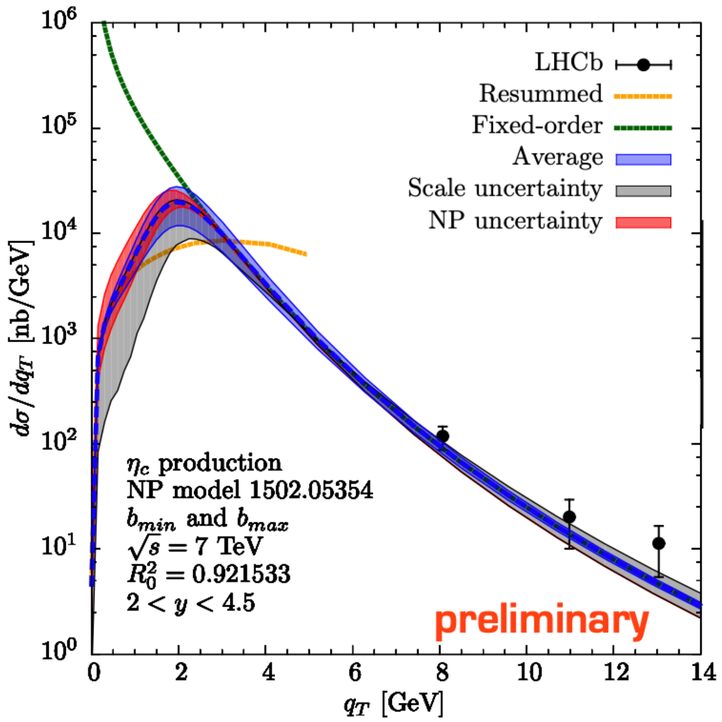
# $\eta_c$ production @ 7TeV LHC

## $\eta_c$ production at LHC

full transverse momentum spectrum:
low qT matched with high qT region




**blue band**: uncertainty from matching

**grey band**: scale uncertainty

**red band**: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = -\left[\frac{a_1}{2} + \frac{a_2}{2}\ \ln Q^2\right]\ \bar{b}_T^2$$

$a_i$ = 0.5 GeV$^2$, var. 50%, envelope

both for unpolarized and
linearly polarized distributions

**the formalism is in good shape!**
we need the data at low qT

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25

slide by **Andrea Signori** (INT 2017)

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