ACCESSING COMPTON AMPLITUDES IN A QUANTUM COMPUTER

Juan Guerrero

EICUG early career

July, 2022

Based on:
Briceño, JG, Hansen & Sturzu: PRD 103 (2020) 014506
Briceño, Carrillo, JG, Hansen & Sturzu: PoS Lattice 2021, 315

Jefferson Lab
Thomas Jefferson National Accelerator Facility

A BIT OF LATTICE QCD

Lattice QCD: first principle non-perturbative approach to QCD

$$S[\phi] = \int dt \int d^3\mathbf{x}\, \mathscr{L}[\phi]$$

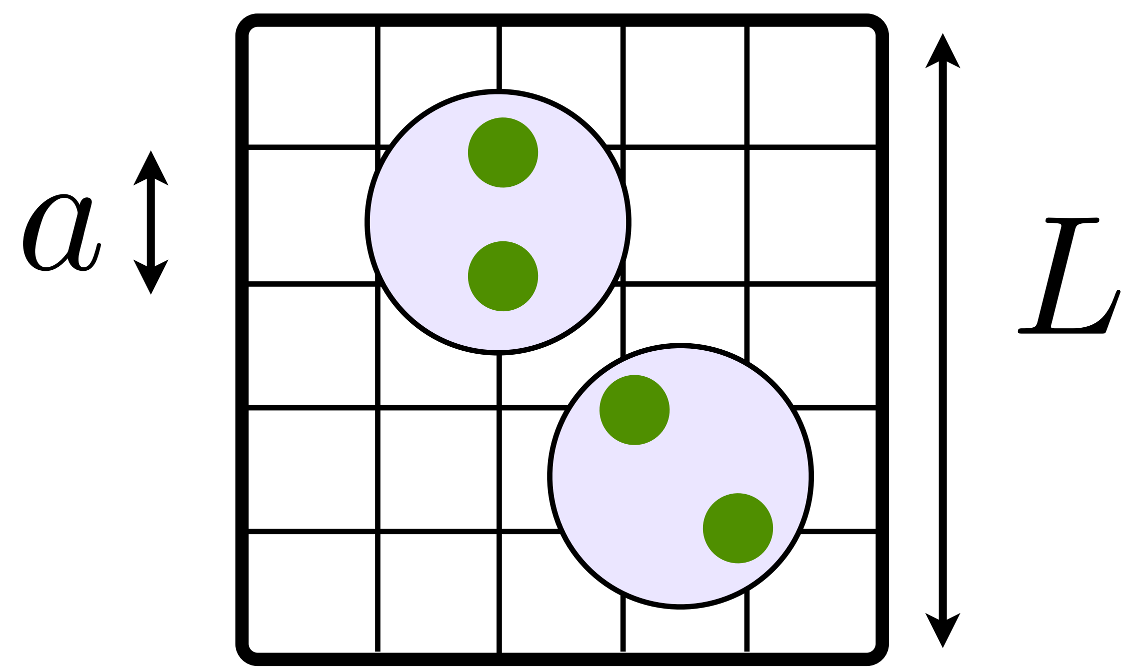
- [ ] Euclidean space time: $t^M \to -it^E$

$$Z = \int d\phi\, e^{iS^M[\phi]} \longrightarrow \int d\phi\, e^{-S^E[\phi]} \longleftarrow S^E \geq 0 : \text{weight!}$$

- [x] Importance sampling

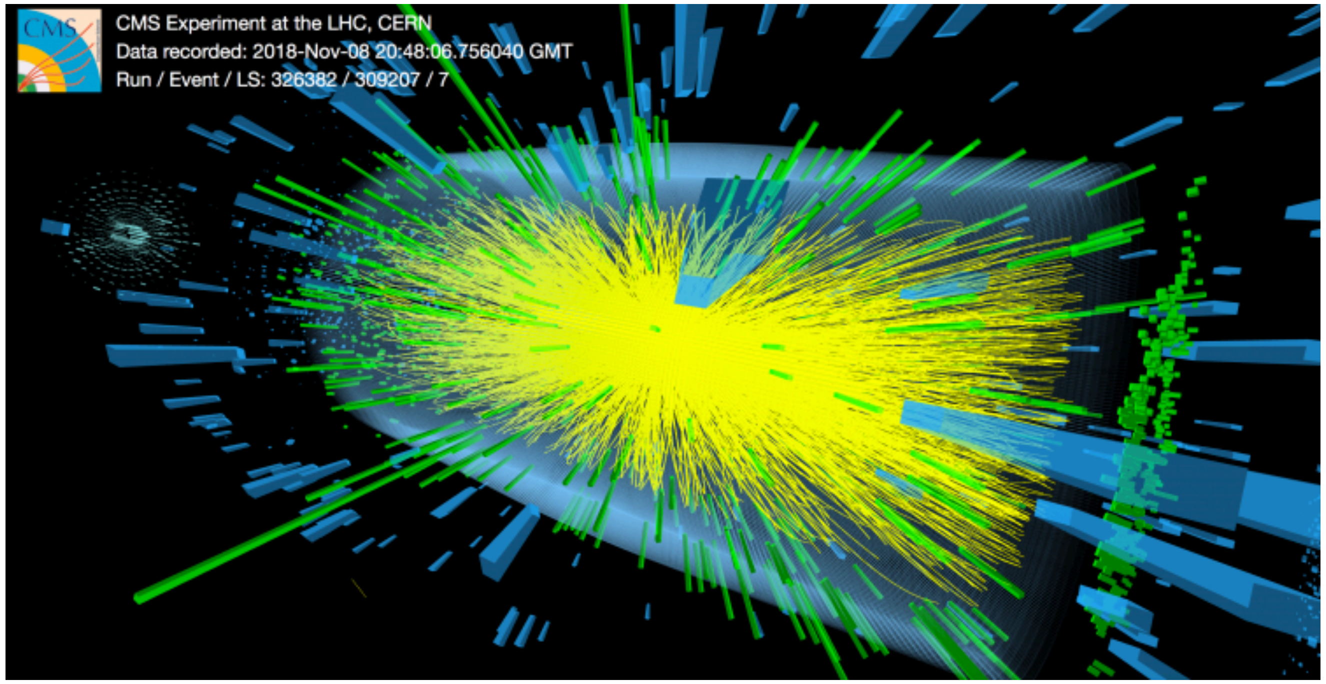
- [ ] Discrete space a

- [ ] Finite volume L



REAL-TIME DYNAMICS: SIGN PROBLEM IN LQCD

- ☐ Heavy-ion collisions
- ☐ Parton showers
- ☐ Fragmentation
- ☑ Long-range processes in the SM: Compton scattering…

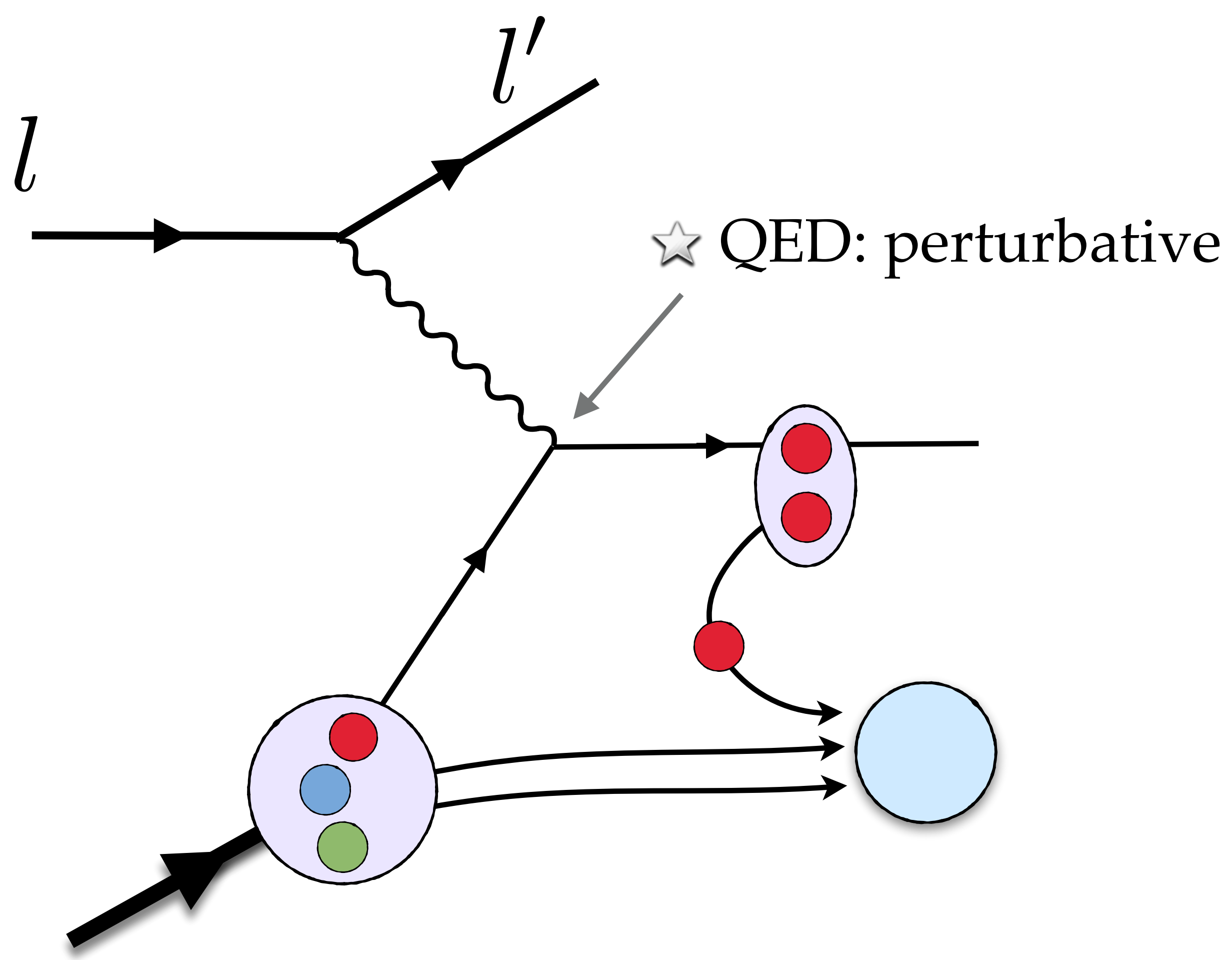


Path integral formulation: $Z = \int d\phi\, e^{iS^M[\phi]} \longleftarrow$ **sign problem!**

Quantum computing: prospective non-perturbative way to study QCD

$$U(t, t_0) = e^{-iH(t-t_0)} \longleftarrow \text{Hamiltonian time evolution}$$

COMPTON SCATTERING



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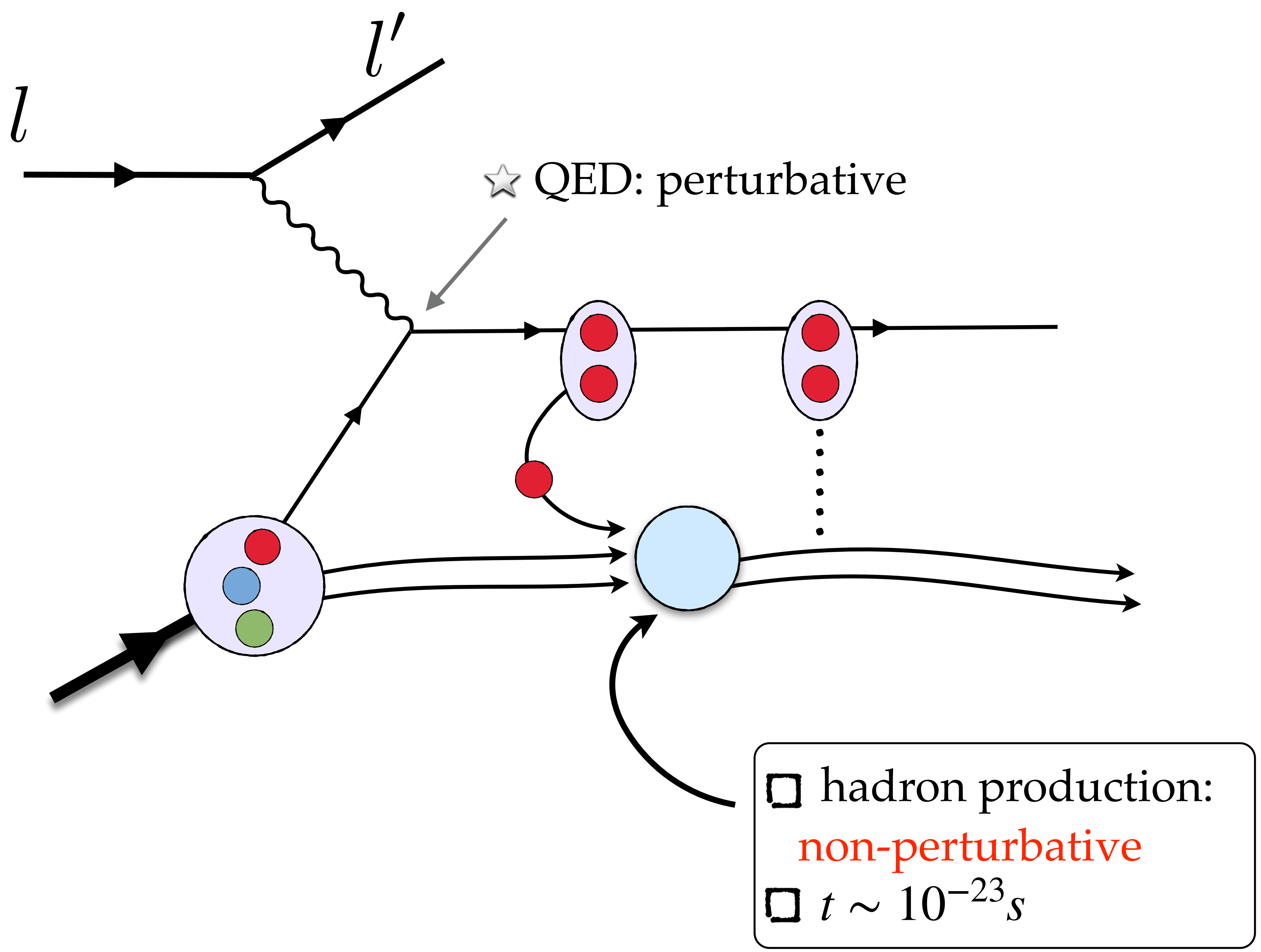

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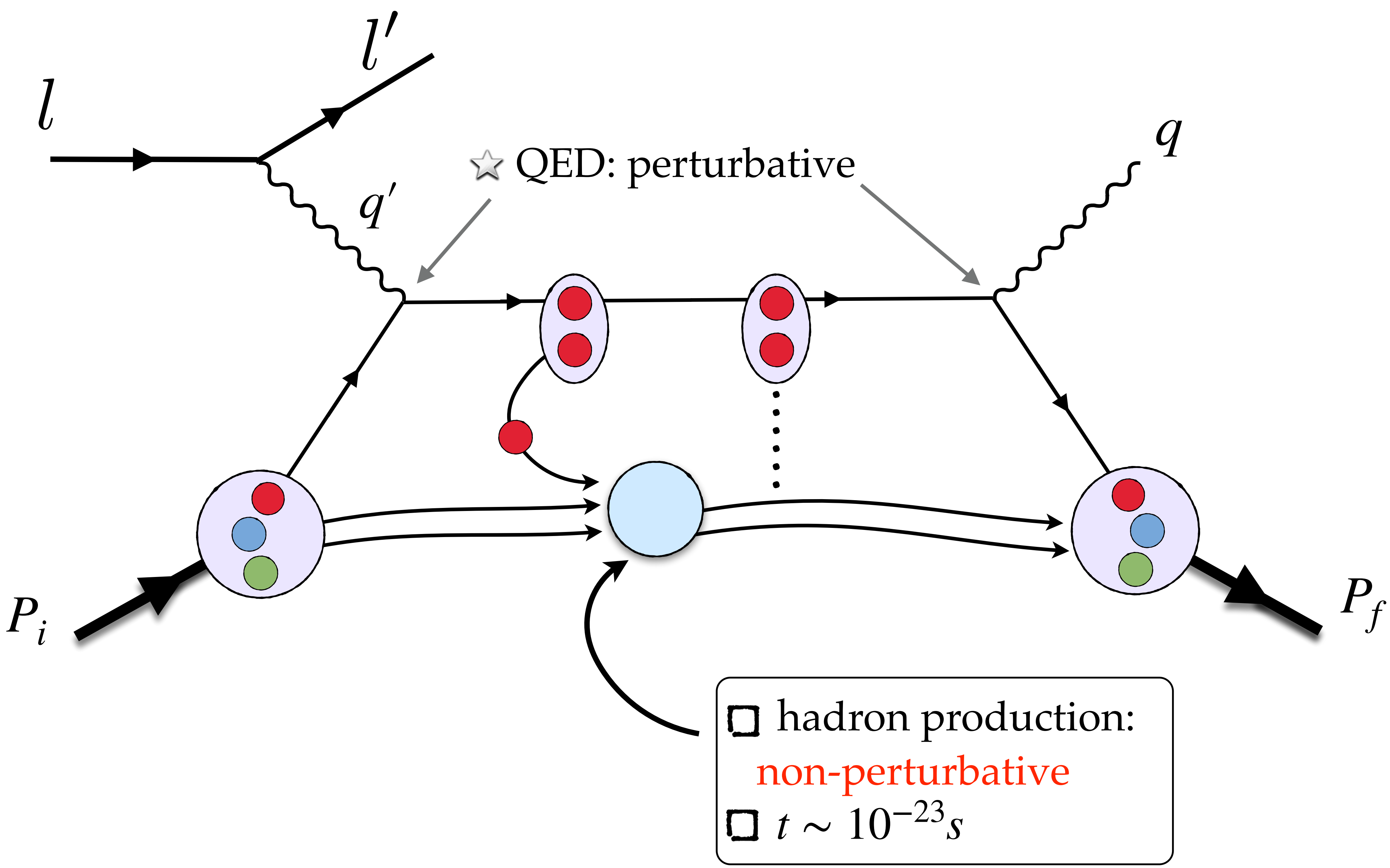

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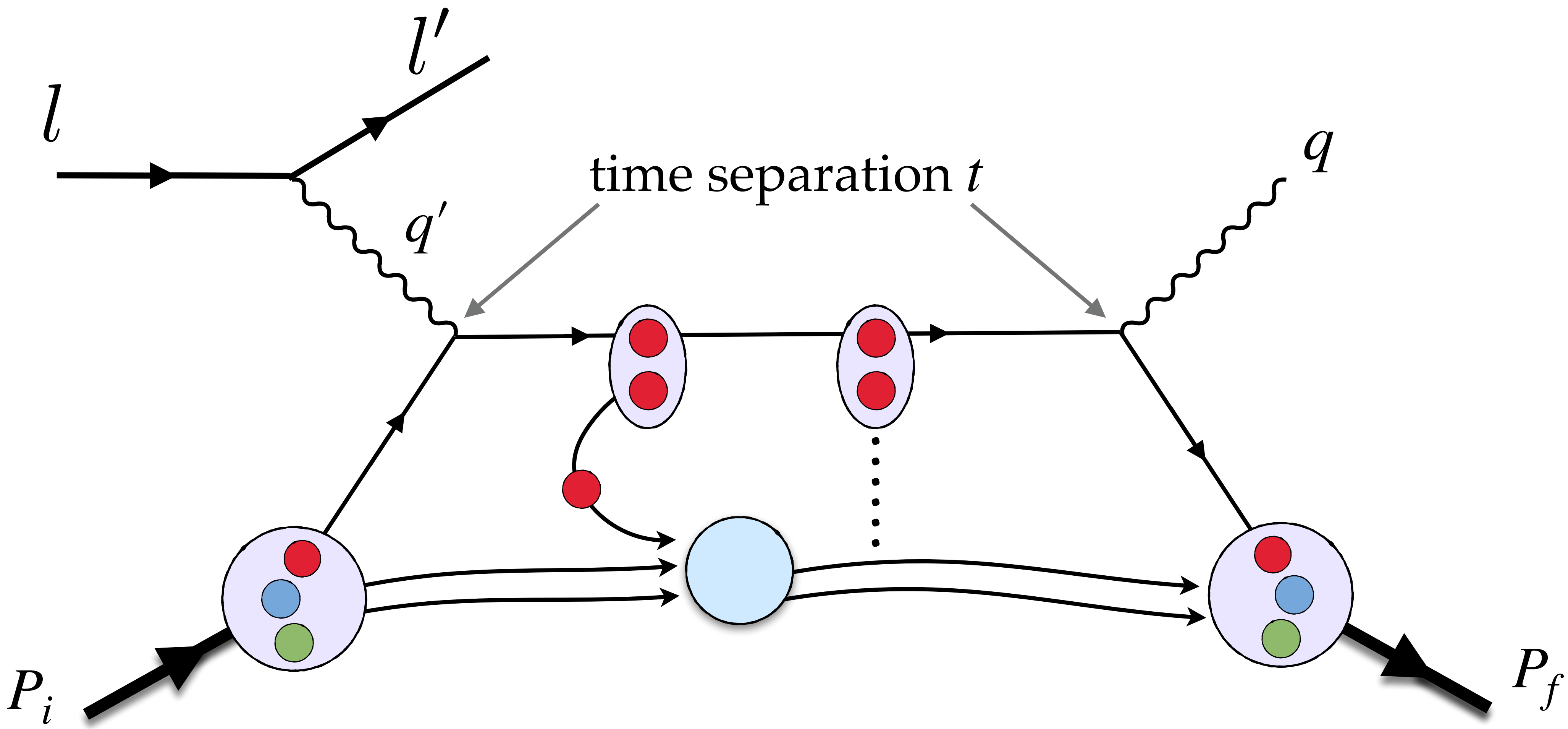


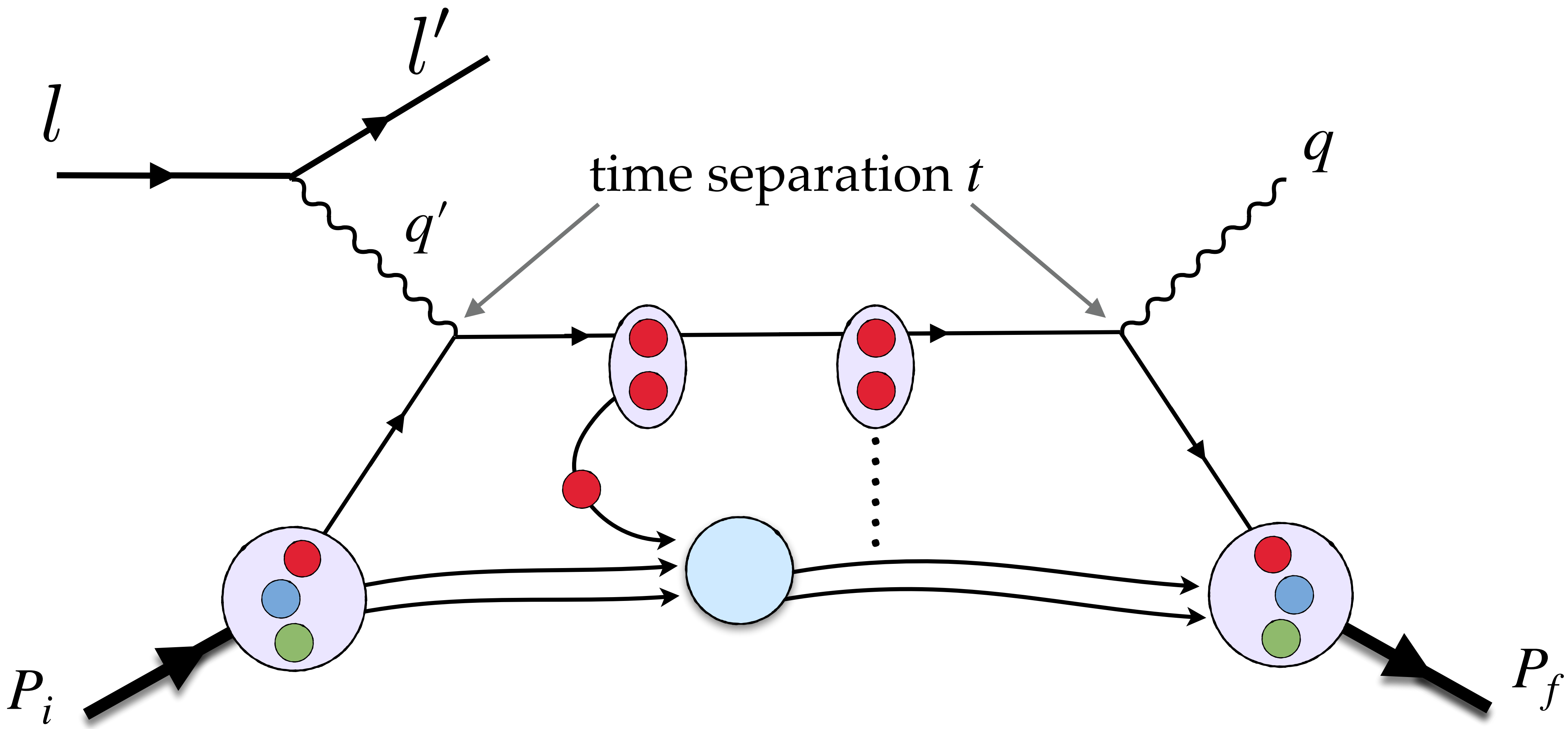
☑ Compton Amplitude: *master* of the inclusive structure amplitudes

$$\mathscr{T} = i\int d^4x\, e^{ix\cdot q}\, \langle N(P_f)\,|\, T\left[\mathscr{J}(t)\,\mathscr{J}'(0)\right]|\,N(P_i)\rangle_\infty$$

☐ Generalized Parton Distributions (GPDs): 3D- nucleon structure

☐ Im($\mathscr{T}$) gives access to Parton Distribution Functions (PDFs)

COMPTON SCATTERING



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☑ Quantum Computer: test of convergence of factorization theorems directly from the Standard Model

QUANTUM COMPUTERS

Quantum computing: prospective non-perturbative way to study QCD

$$U(t, t_0) = e^{-iH(t-t_0)} \longleftarrow \text{Hamiltonian time evolution}$$

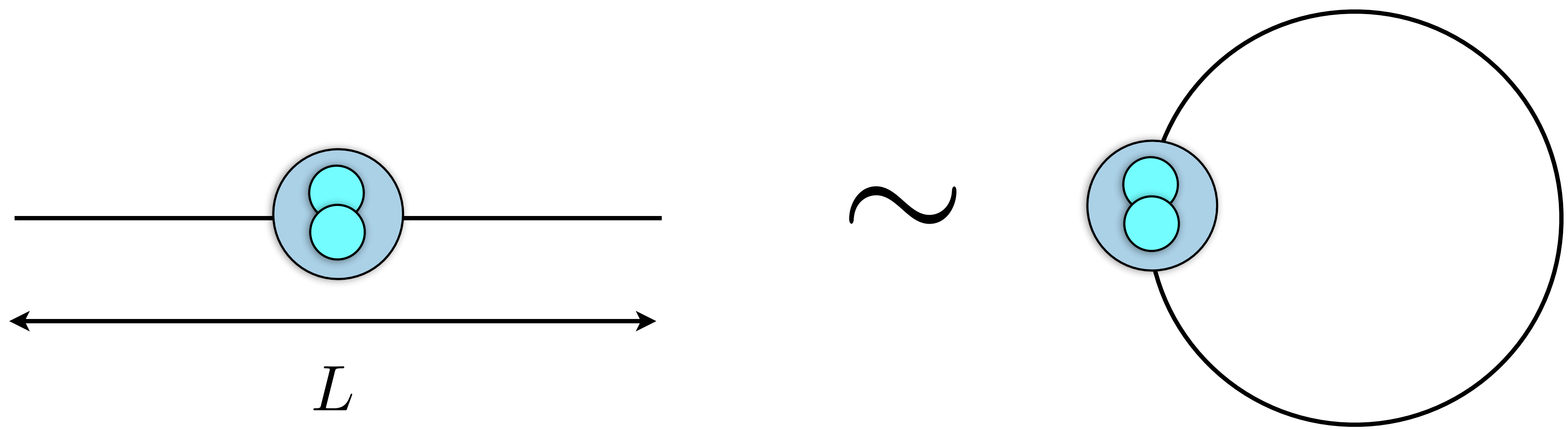
☐ Minkowski space time

☐ Discrete space a

☑ Finite volume L

PHYSICS IN A 1D FINITE BOX

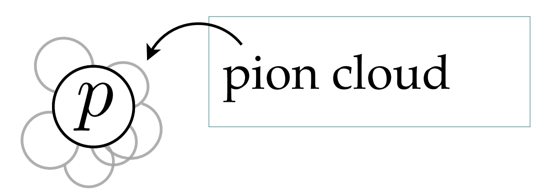
- Free particle wave function: $\varphi_p(x) = e^{ipx}$



$$\varphi_p(L+x) = e^{ip(x+L)} = \varphi_p(x) = e^{ipx}$$

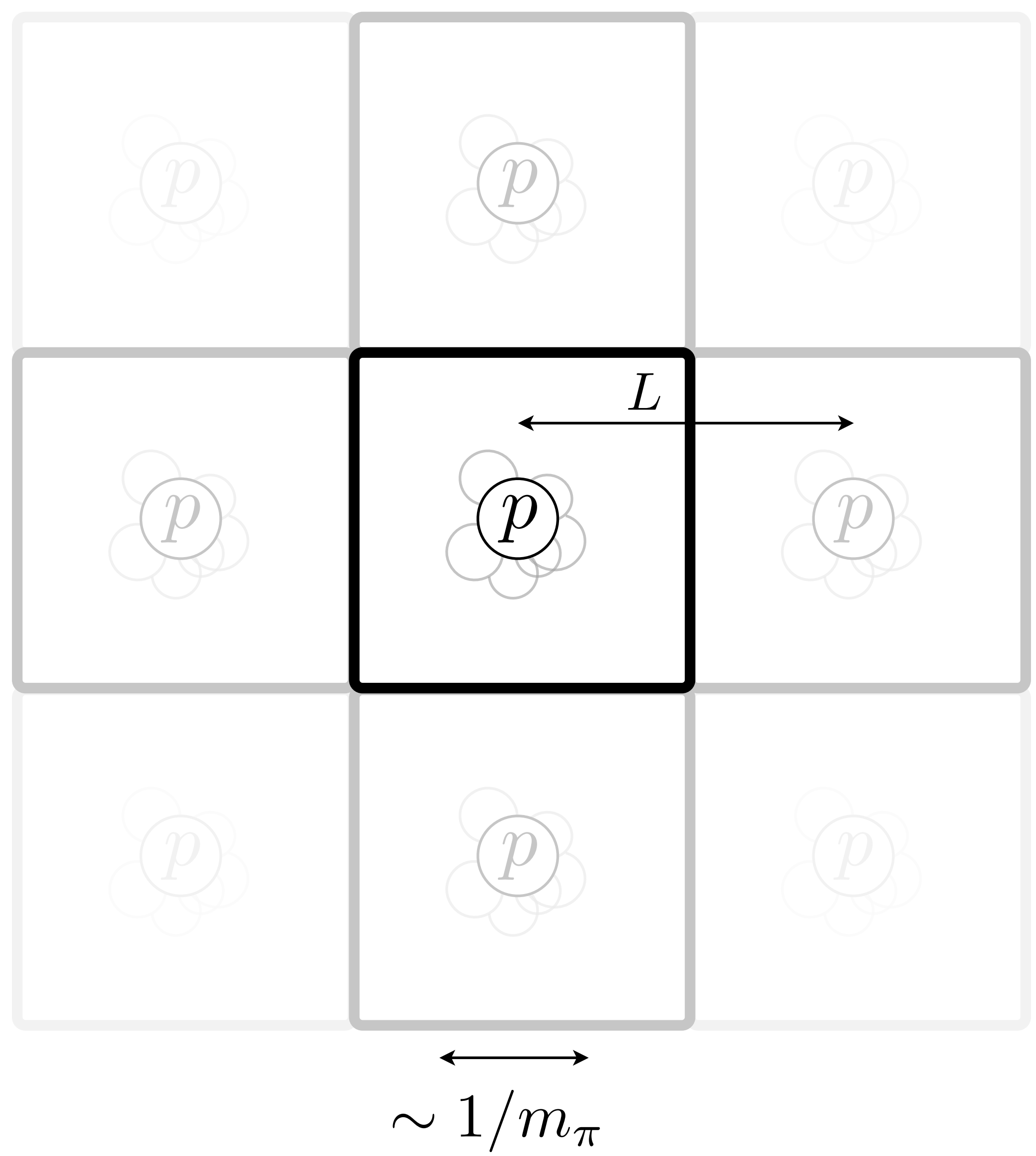
- Discretized momentum and spectrum: $p = \dfrac{2\pi n}{L}$

FINITE VOLUME: INFRARED LIMIT OF THE THEORY



FINITE VOLUME: INFRARED LIMIT OF THE THEORY

- Finite-volume artifacts arise from the interactions with mirror images



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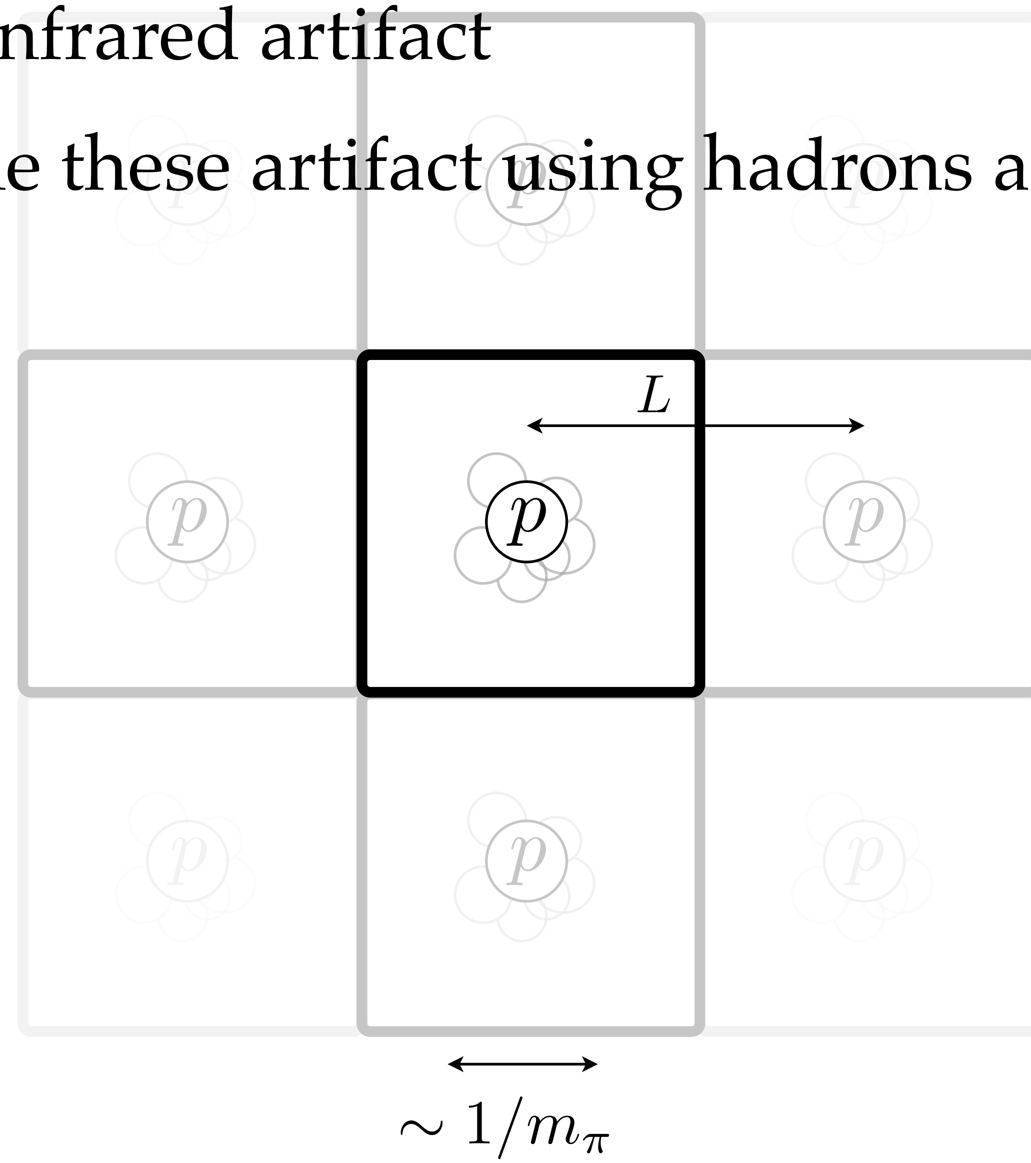
- Finite-volume artifacts arise from the interactions with mirror images
- Assuming $L >> $ size of the hadrons ~ $1/m_\pi$
  - This is a purely infrared artifact
  - We can determine these artifact using hadrons as the degrees of freedom



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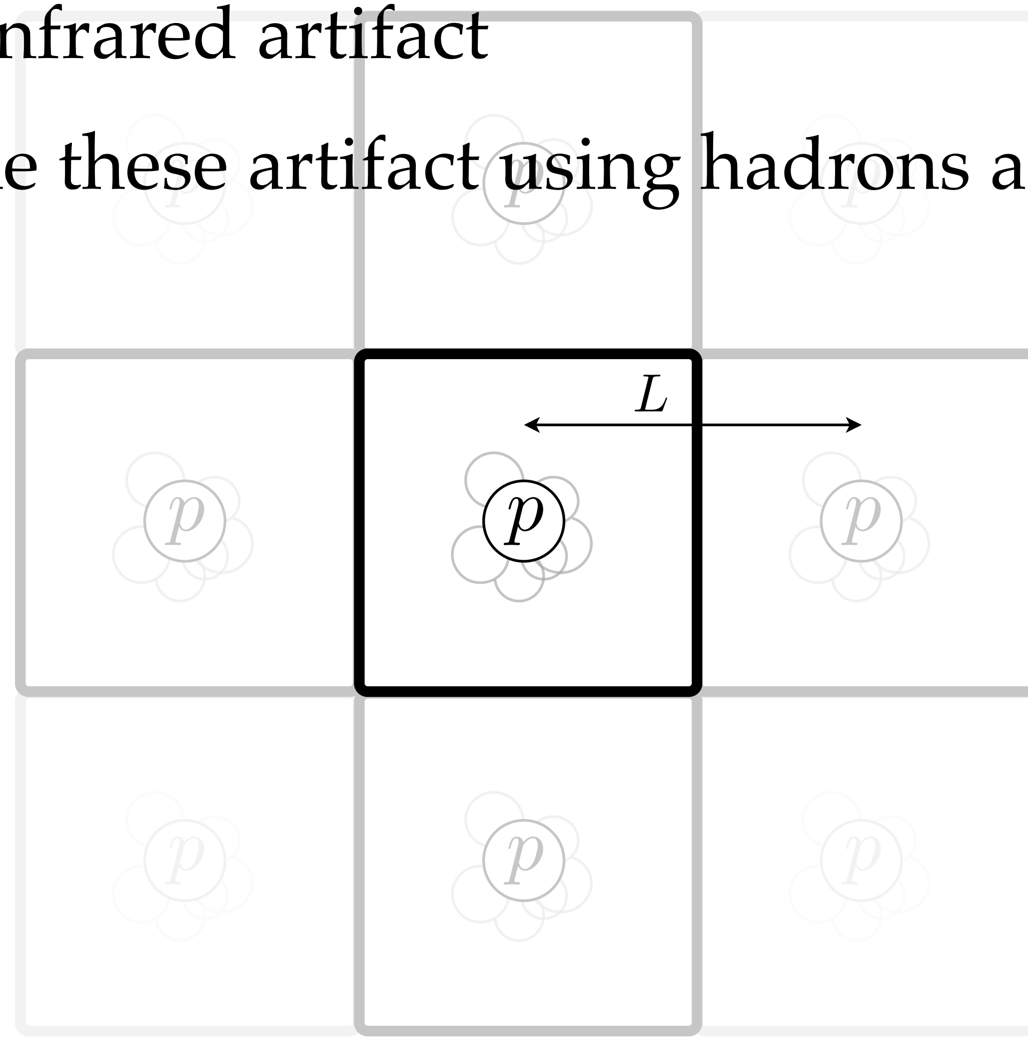


- ☑ interactions with mirror images: Yukawa

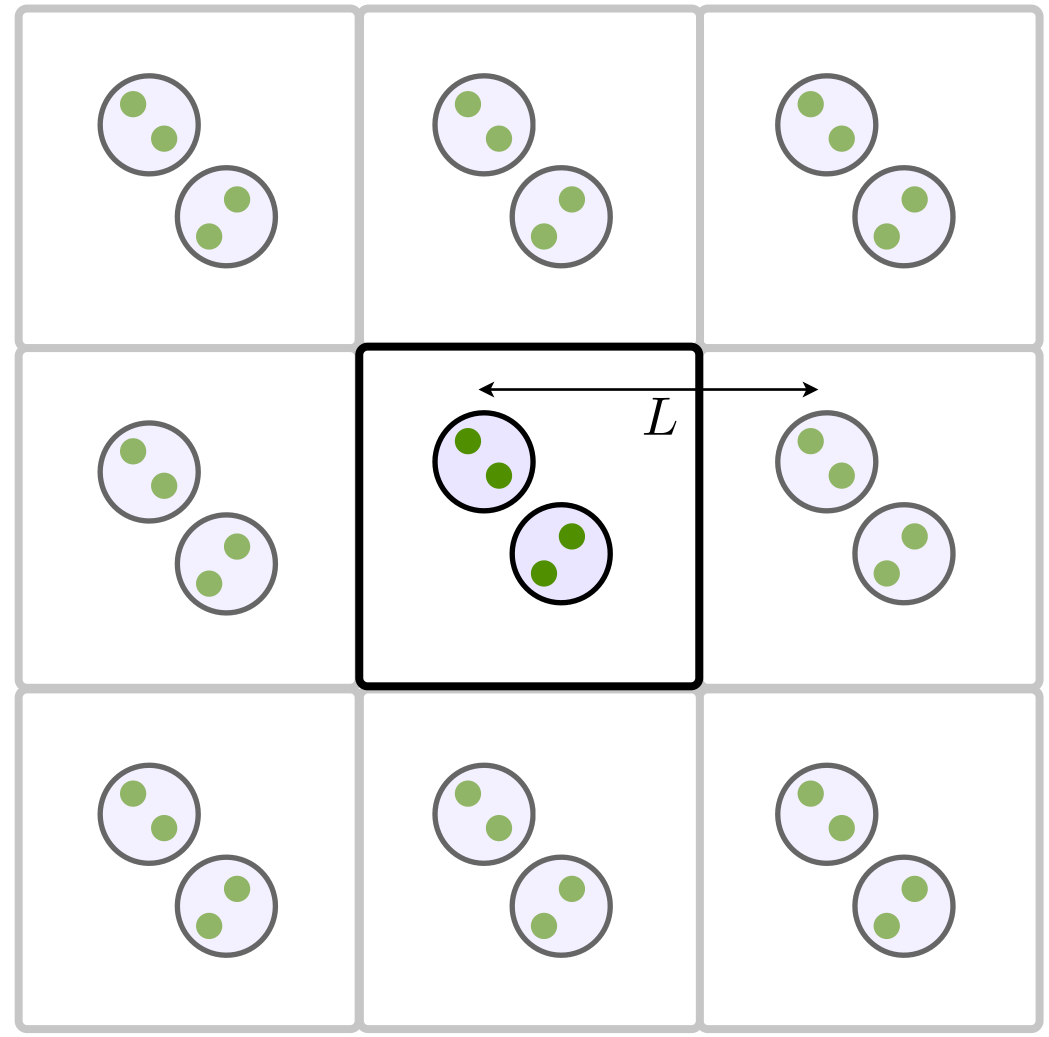
$$m_N(L) - m_N(\infty) \sim \langle N | \hat{V} | N \rangle_L \sim e^{-m_\pi L}$$

Lüscher (1985)

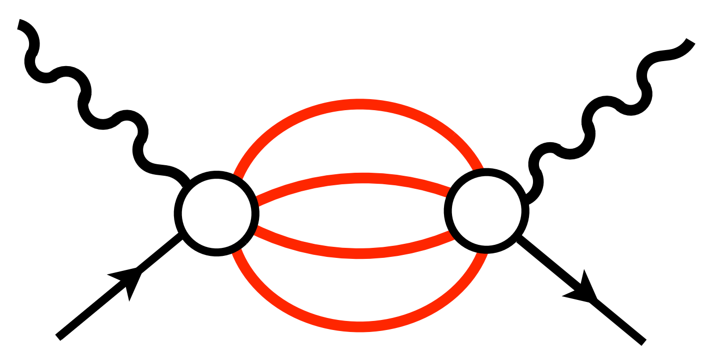
REAL TIME CALCULATIONS: FINITE VOLUME SYSTEMATICS

- **Finite volume L:**
  - Particles are never free
  - No asymptotic states
  - No scattering
  - Multi-particle states:

$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^{\#}}$$



Goal: study inclusive processes at arbitrary kinematics



- Involve multi-particle states
- Finite volume: **expected to be dominant**

COMPTON SCATTERING IN A FINITE BOX

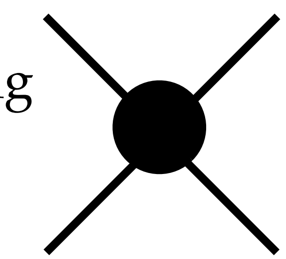
- [x] Use known formalism as a diagnostic tool
- [x] Finite-volume long range matrix elements:

$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\,\frac{1}{F^{-1}(P, L) + \mathcal{M}(s)}\,\mathcal{H}'(s, Q^2_{if})}$$

Briceño et al,
PRD 101 (2020) 014509

geometric function
encoding FV effects

2-body scattering
amplitude:



- [ ] $\mathscr{T}_L$ is purely real:
    - [ ] Note: all terms in the right hand side are complex

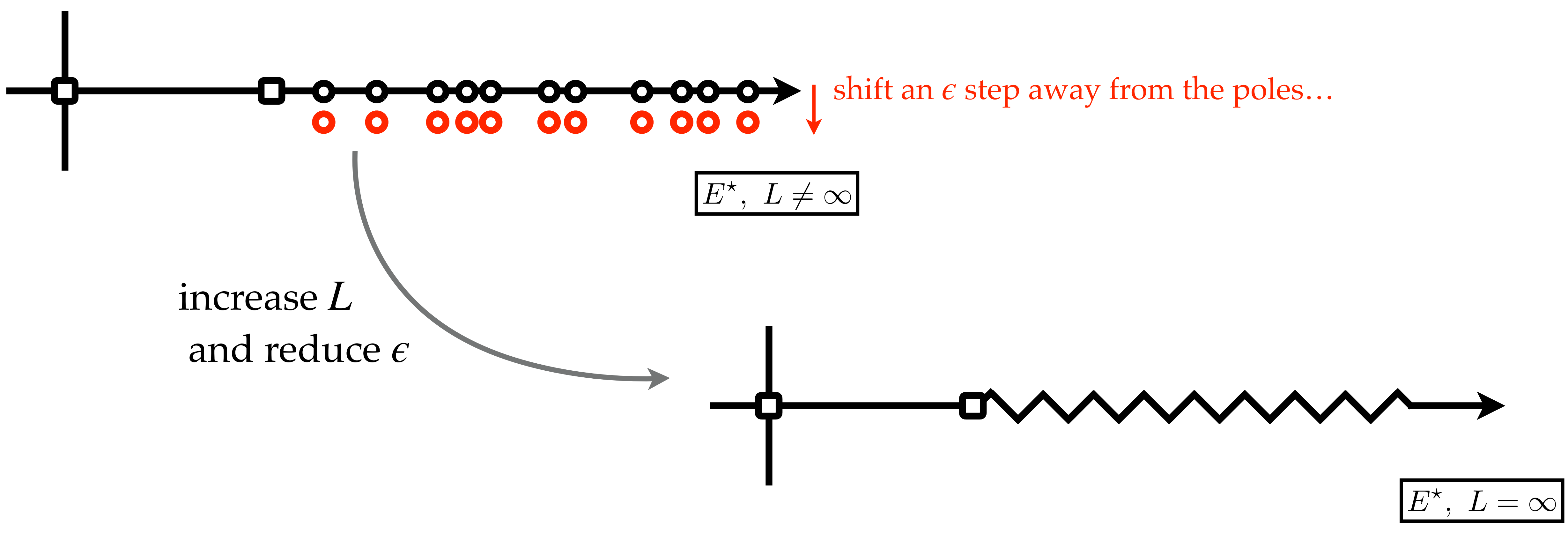
RECOVERING THE INFINITE VOLUME AMPLITUDE

$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s,Q^2)\,\frac{1}{F^{-1}(P,L)+\mathcal{M}(s)}\,\mathcal{H}'(s,Q^2_{if})}$$

- [ ] Introduce an $i\epsilon$ by hand ($i\epsilon$ prescription)

accesible in a Quantum computer

$$\mathscr{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} dt\, e^{iq_0 t-\epsilon|t|}\,\langle n_f|\,T[\mathscr{J}_2(t)\,\mathscr{J}_1(0)]\,|n_i\rangle_L$$



RECOVERING THE INFINITE VOLUME AMPLITUDE

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- [ ] Introduce an $i\epsilon$ by hand ($i\epsilon$ prescription)

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} dt\, e^{iq_0 t - \epsilon|t|}\, \langle n_f |\, T[\mathcal{J}_2(t)\,\mathcal{J}_1(0)]\, | n_i \rangle_L$$

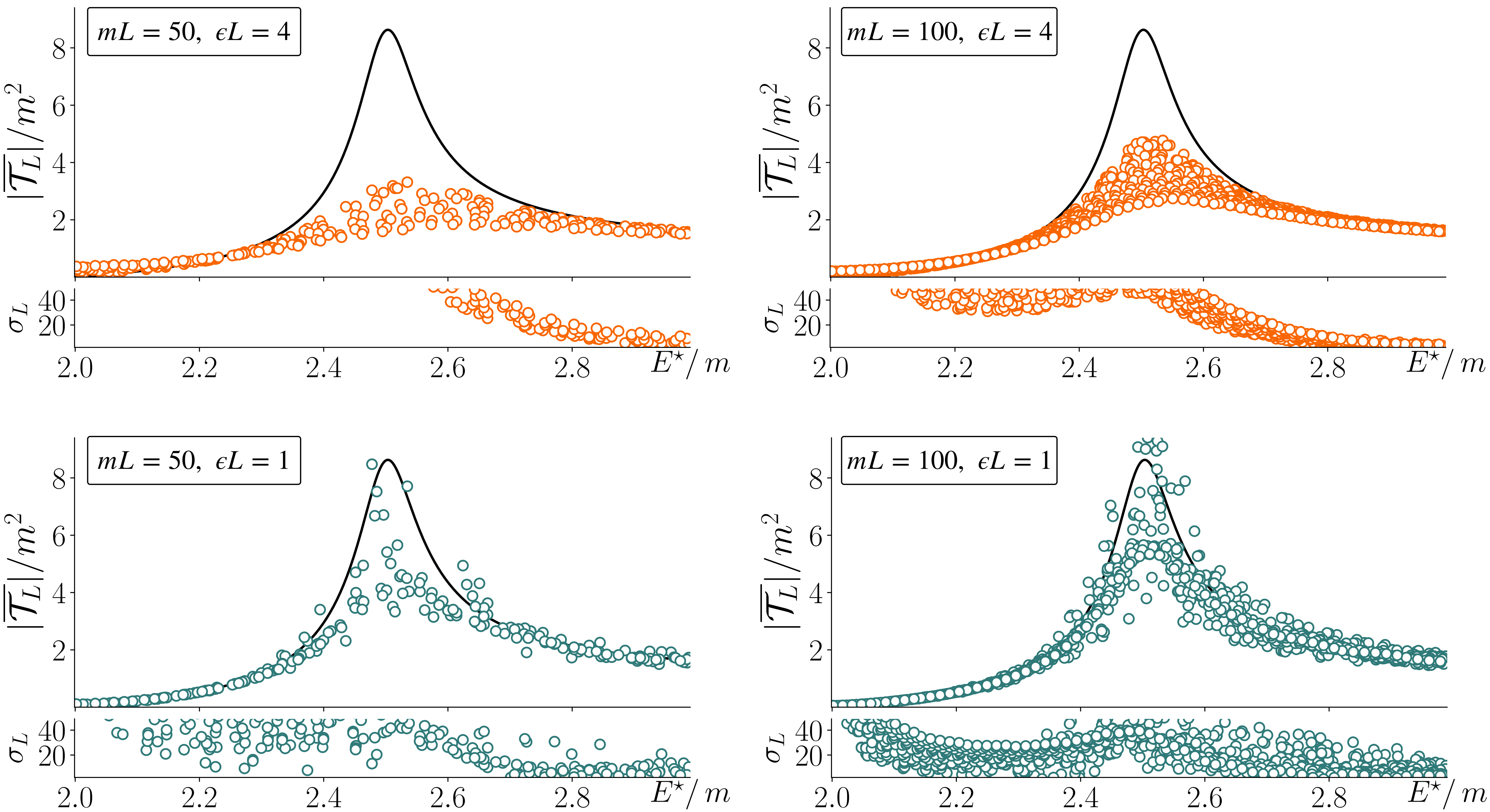
- [ ] Then consider the double limit:

$$\mathcal{T} = \lim_{\epsilon\to 0}\lim_{L\to\infty} \mathcal{T}_L(\epsilon)$$

- [x] Test this idea using a toy model
    - [ ] What volume sizes do we need?
    - [ ] How quickly do we recover the asymptotic behavior?
    - [ ] Does this depend on the dynamics of the system?

COMPTON-LIKE AMPLITUDES

$$\mathcal{T}_L(p_f, q, p_i) = \mathcal{T}(E^\star, Q^2, Q^2_{if}) - \mathcal{H}(E^\star, Q^2)\,\frac{1}{F^{-1}(E^\star, \boldsymbol{P}, L) + \mathcal{M}(E^\star)}\,\mathcal{H}'(E^\star, Q^2_{if})$$



EXPLOITING SYMMETRY: BOOST AVERAGING

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}\,\frac{F(P,L)}{1+\mathcal{M}F(P,L)}\,\mathcal{H}'$$

not Lorentz scalar   Lorentz scalar

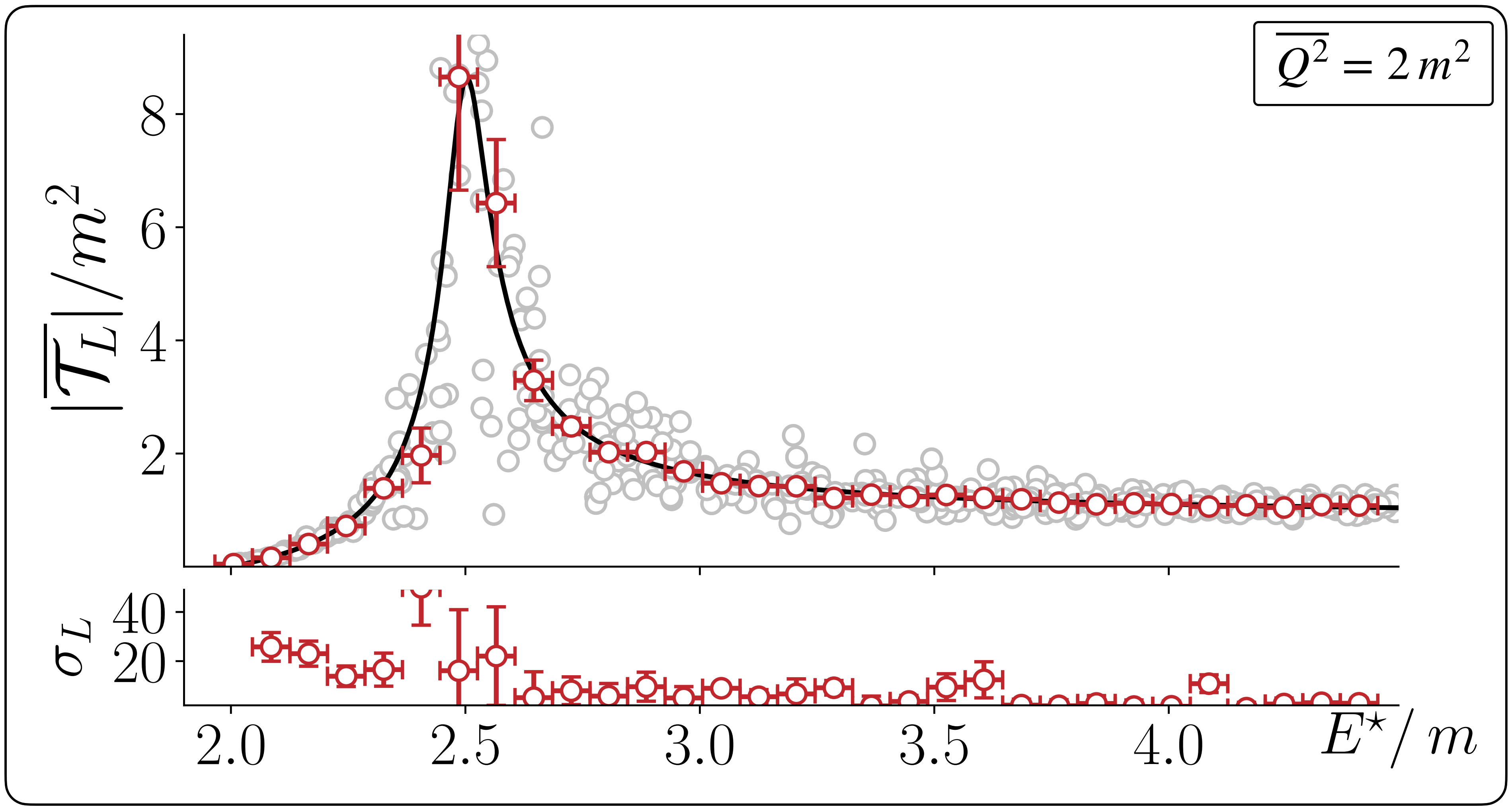
- Take advantage and exploit the symmetry:
  - The physical amplitudes only depend on Lorentz scalars.
  - Boost average

BOOST AVERAGING: NUMERICAL RESULTS

- Boost averaging: $mL = [20,25,30]$ with $d \leq mL$
- Binning in virtualities and energies:

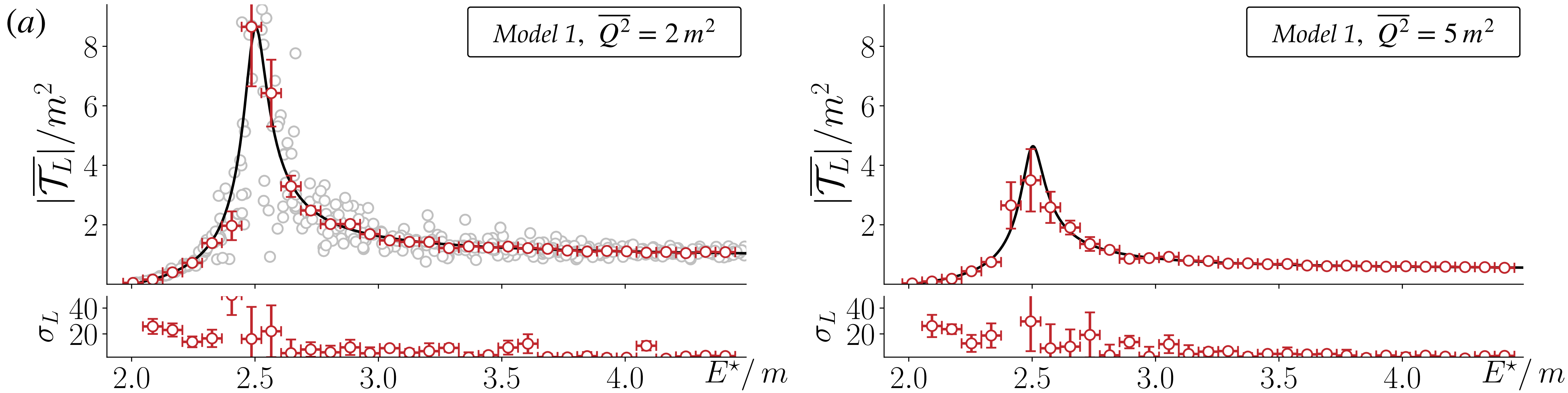
$$\left|\overline{Q^2} - Q^2\right| < \Delta_{Q^2}, \left|Q^2_{if} - Q^2\right| < \Delta_{Q^2} \text{ and } \left|\overline{E^\star} - E^\star\right| \leq \Delta_{E^\star}$$

$$\boxed{\overline{\mathcal{T}_L}\left(\overline{E^\star}, \overline{Q^2}\right) = \frac{1}{\mathcal{N}} \sum_{L,\epsilon} \sum_{\{\boldsymbol{q},\boldsymbol{p}_f,\boldsymbol{p}_i,\omega\}\in\Omega} \delta\left(\boldsymbol{q},\boldsymbol{p}_f,\boldsymbol{p}_i,\omega \middle| \overline{E^\star}, \overline{Q^2}\right) \mathcal{T}_L(p_f, q, p_i)}$$

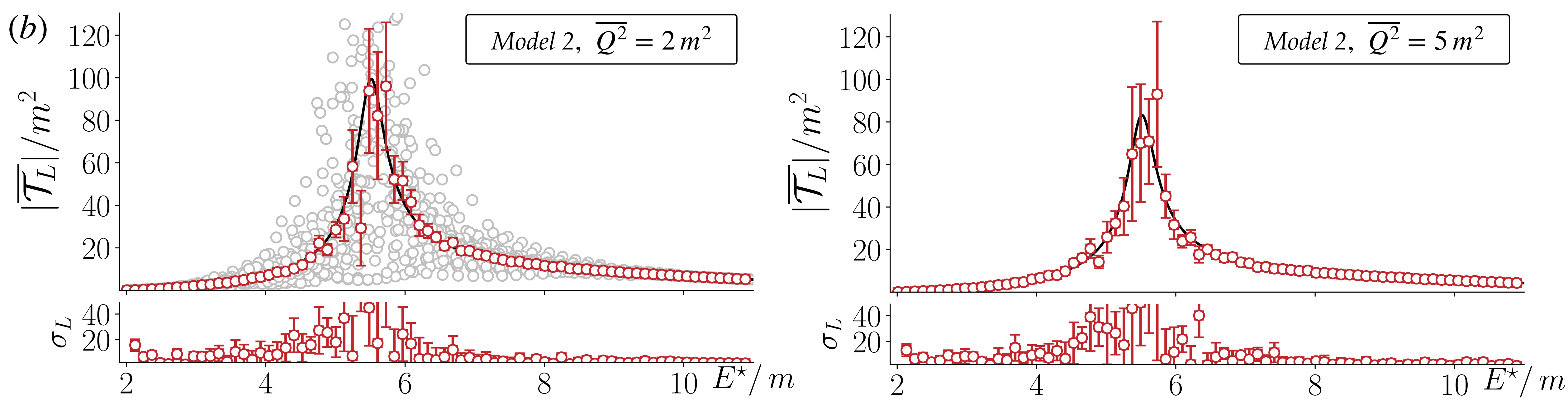


BOOST AVERAGING: NUMERICAL RESULTS

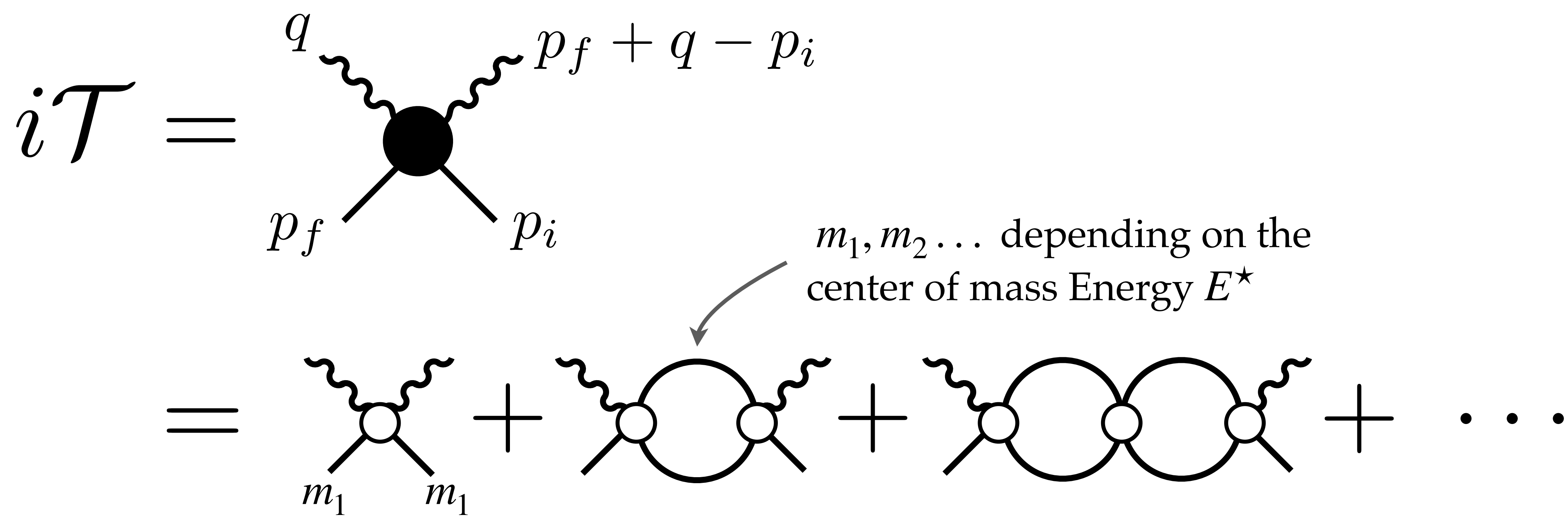
- [x] Average over $mL = [20,25,30]$, with $d \le mL$
- [ ] Model 1



- [ ] Model 2



TOY MODEL: COUPLED CHANNELS

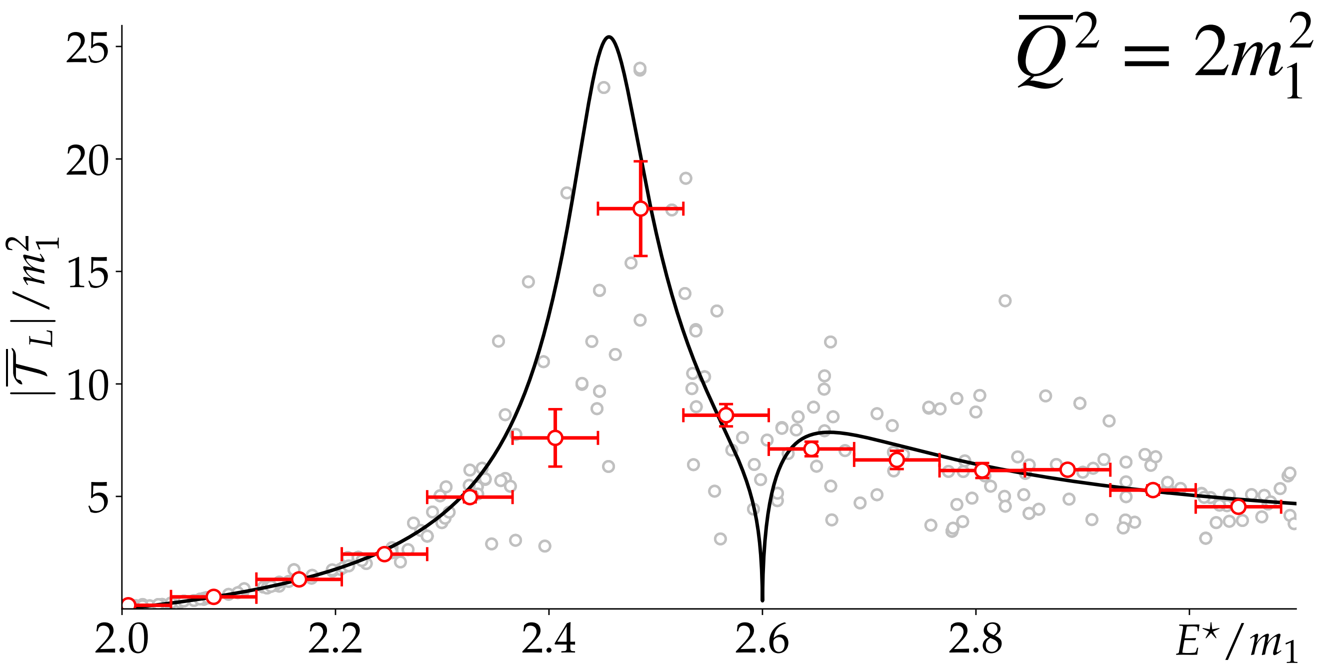


Consider toy theory for energies: $2m_1 < E^\star < 3m_1$

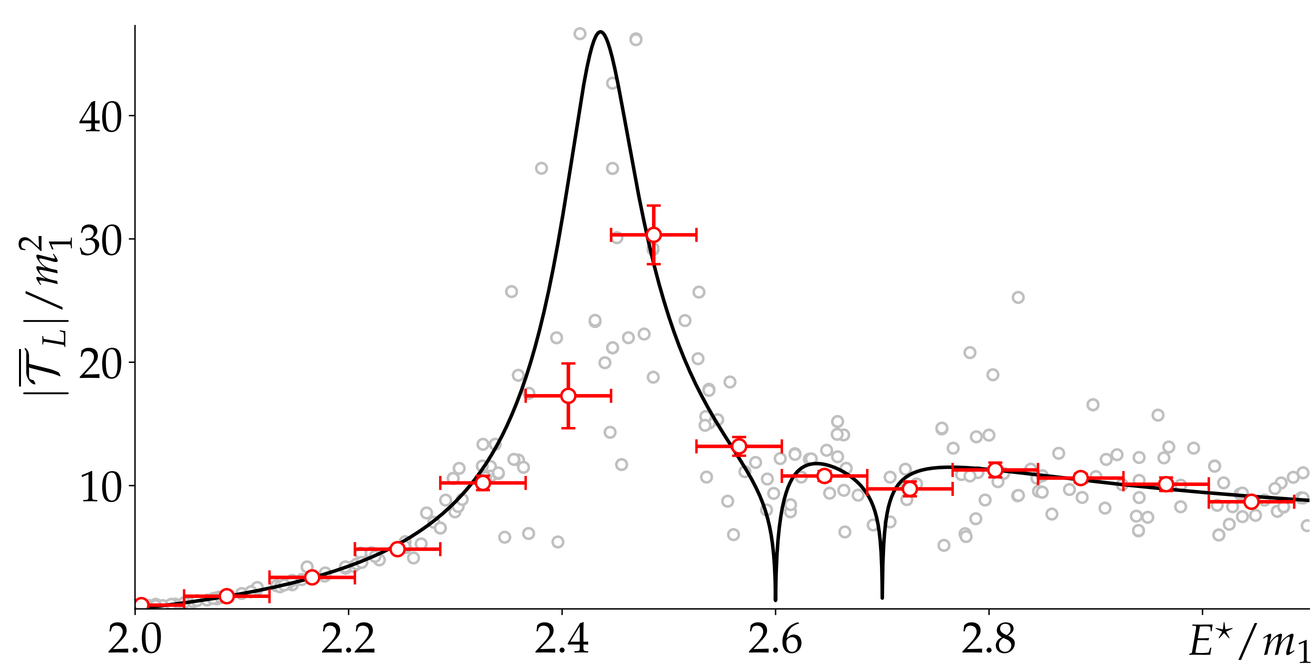
- [x] Previously: only 1 channel of mass m_1 can go on-shell in the loop
- [ ] Lift this assumption:
    - [ ] 2 coupled channels can go on-shell: $2m_2 < E^\star < 3m_1$
    - [ ] 3 or more coupled channels can go on-shell

MULTIPLE COUPLED CHANNELS

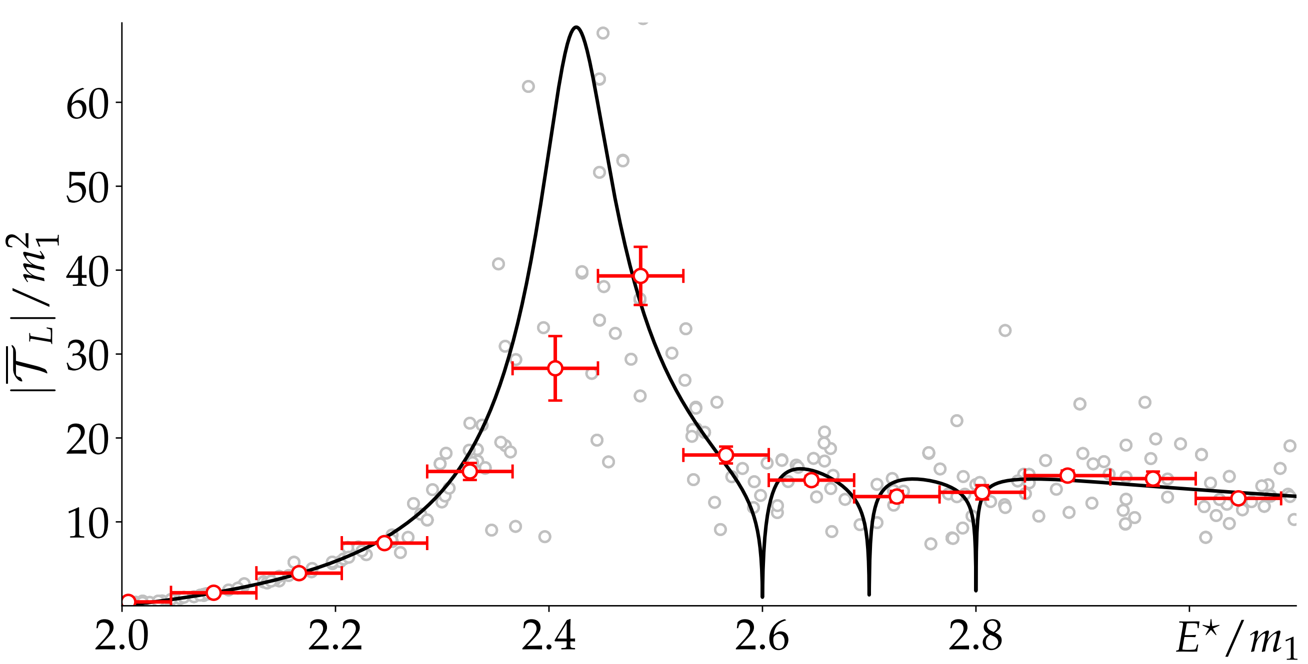
2 coupled channels



3 coupled channels



4 coupled channels



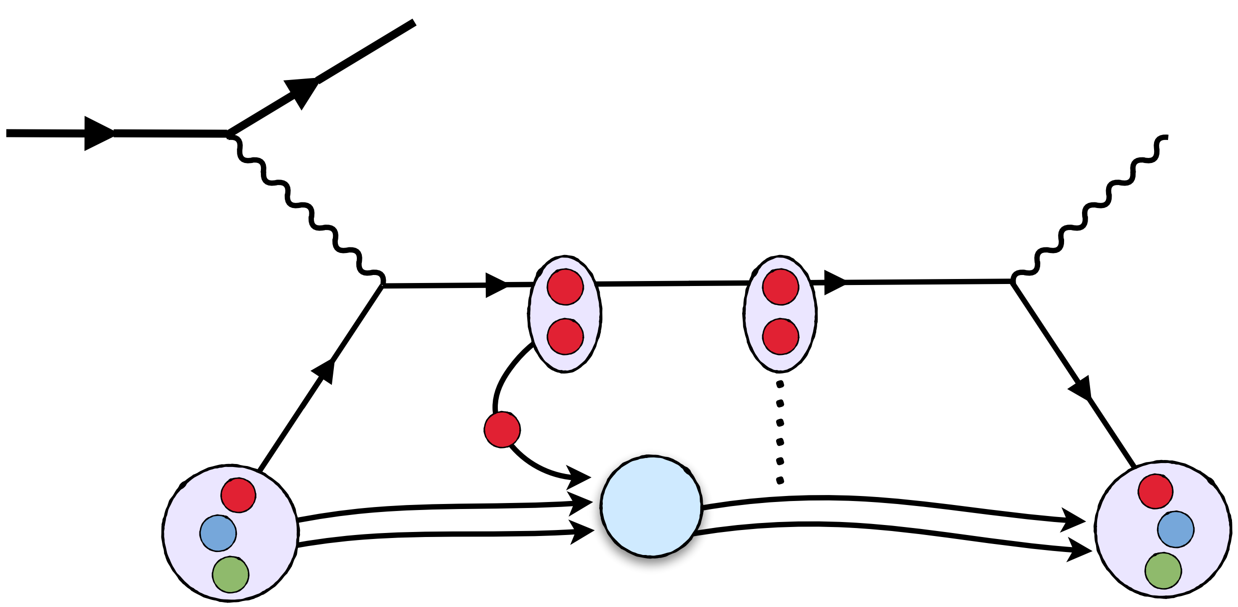
- ☑ Binning + averaging: still mostly recovers multi-channel case
- ☑ Optimal Binning becomes necessary

SUMMARY

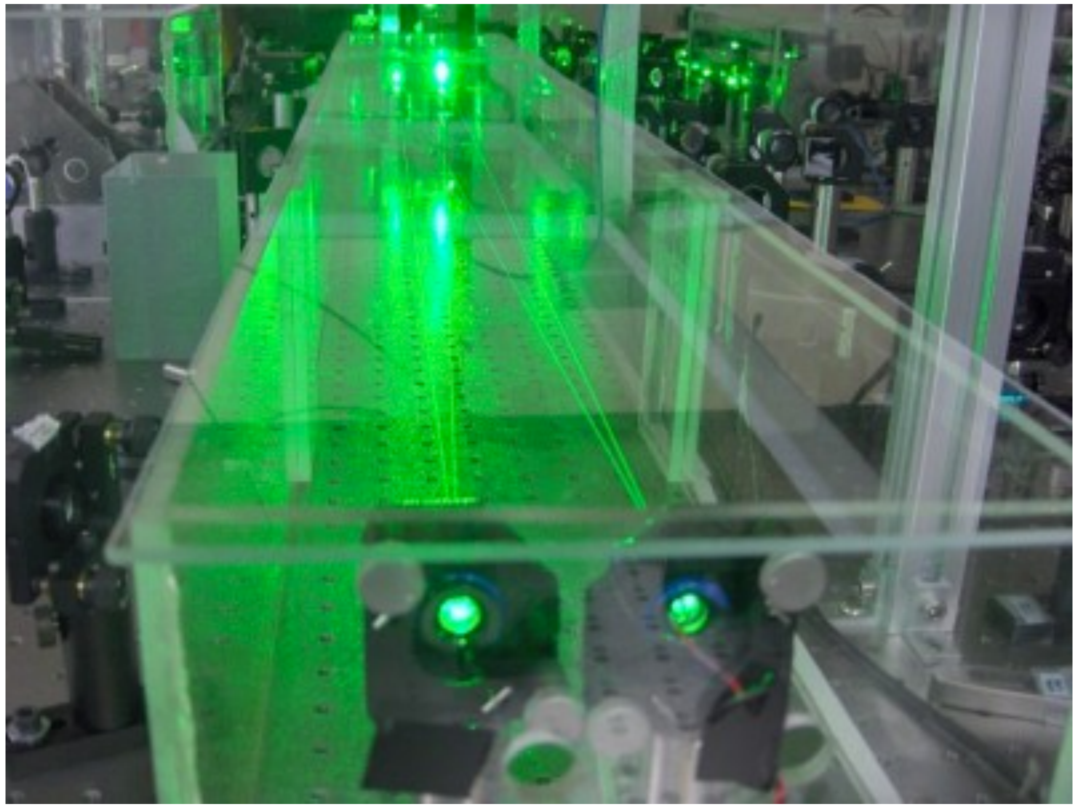
- ☑ Prospects for extracting scattering amplitudes from a Quantum Computer
- ☐ Finite volume effects: large as expected
  - ☐ Naïve analysis: $mL \sim 100$
- ☑ Solution: Exploit symmetry and binning (wave packets)
  - ☑ $mL \sim 20 - 30$
- ☑ Inclusive scattering amplitudes: in principle accesible
  - ☑ Simple tools needed
  - ☑ Tested this idea using a toy model

OUTLOOK

- ☐ Optimal choices of ϵ/boosting/binning?
- ☐ Inelastic processes?
- ☐ Test on toy theory [quantum simulation vs. lattice]
- ☐ 3+1D
- ☐ ….
- ☑ LDRD proposal: The EIC on a top table



Optics based QC system at UVA



THANK YOU!

TOY MODEL: COMPTON-LIKE AMPLITUDES

Scattering amplitude parametrization

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}, \qquad \mathcal{K}(E^\star) = m^2 q^{\star\,2} \frac{g^2}{m_R^2 - E^{\star 2}}$$

All-order perturbation theory implies that the Compton-like amplitudes satisfy

$$\mathcal{T}(s, Q^2, Q_{if}^2) = w_2(s, Q^2, Q_{if}^2) + \mathcal{A}_{12}(s, Q^2)\mathcal{M}(s)\mathcal{A}'_{21}(s, Q_{if}^2)$$

Our parametrization:

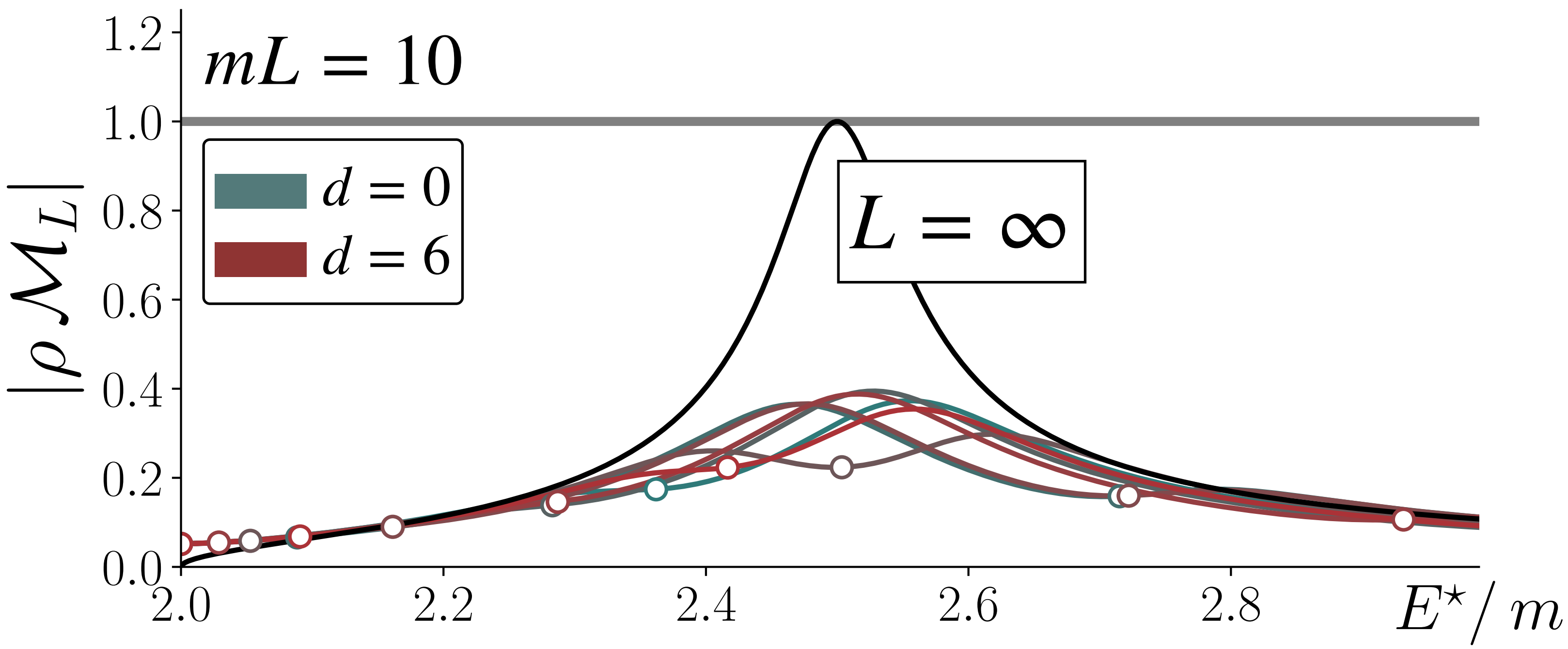
$$w_2(s, Q^2, Q_{if}^2) = 0, \qquad \mathcal{A}_{12}(s, Q^2) = \mathcal{A}'_{21}(s, Q^2) = \frac{1}{1 + Q^2/M^2}$$

FINITE VOLUME TWO BODY AMPLITUDE

$$\mathcal{M}_L = \frac{1}{\mathcal{M}^{-1} + F(P, L)}$$

Finite-volume amplitude depends on the total momentum $P = 2\pi d/L$

Consider $\epsilon L = 1$:



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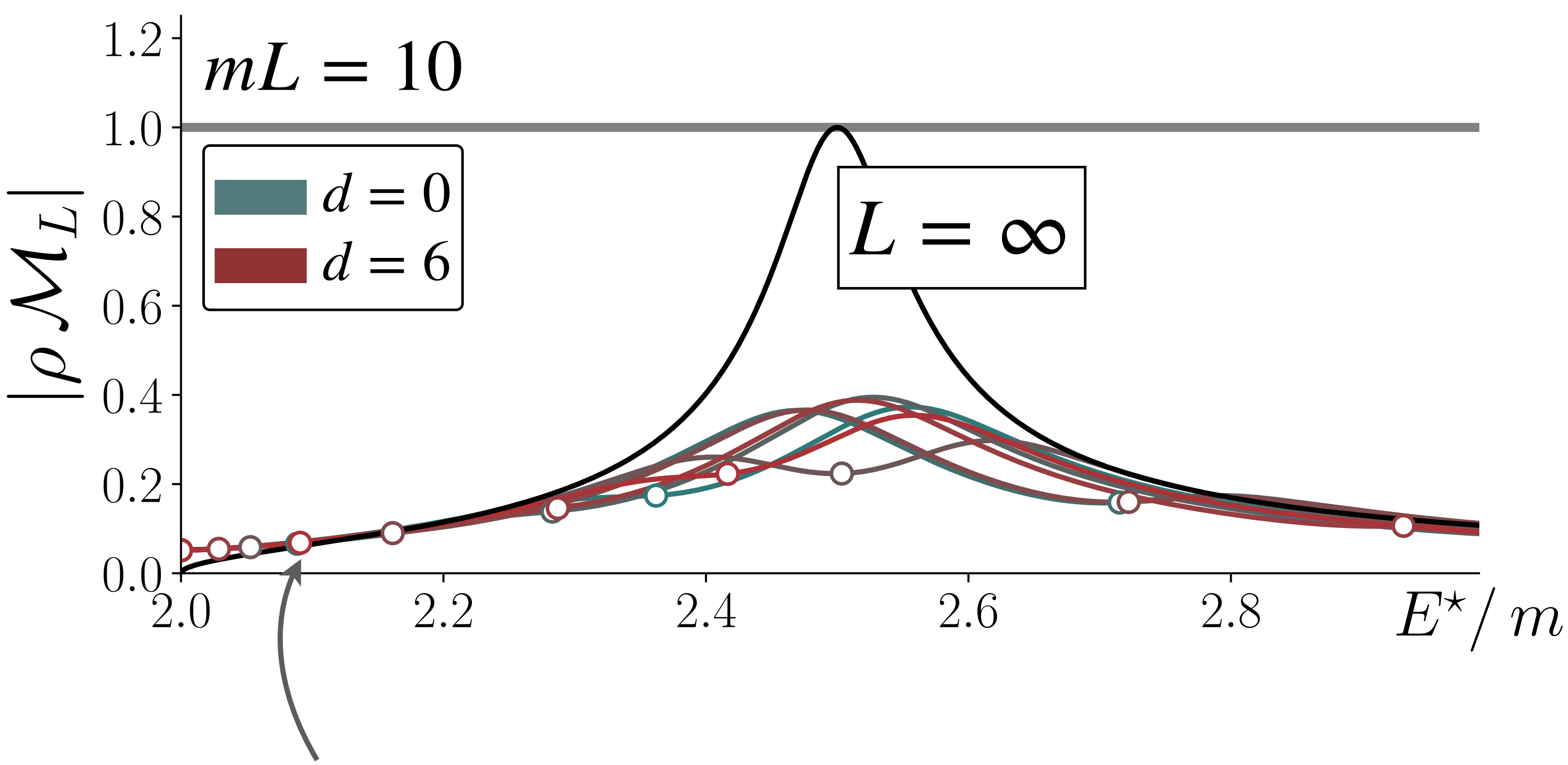

free points:

$$E_n = \sqrt{(2\pi n/L)^2 + m^2} + \sqrt{\left(2\pi(n-d)/L\right)^2 + m^2}$$

To get this $\mathscr{M}_L$ from a simulation:

- ☐ 4pt Corr :$\langle p_f | \mathcal{O}(t)\mathcal{O}^\dagger(0) | p_i \rangle$
- ☐ LSZ reduction:
  - ☐ Amputate external legs
  - ☐ external legs on-shell
- ☐ Free points: only points accesible in a simulation

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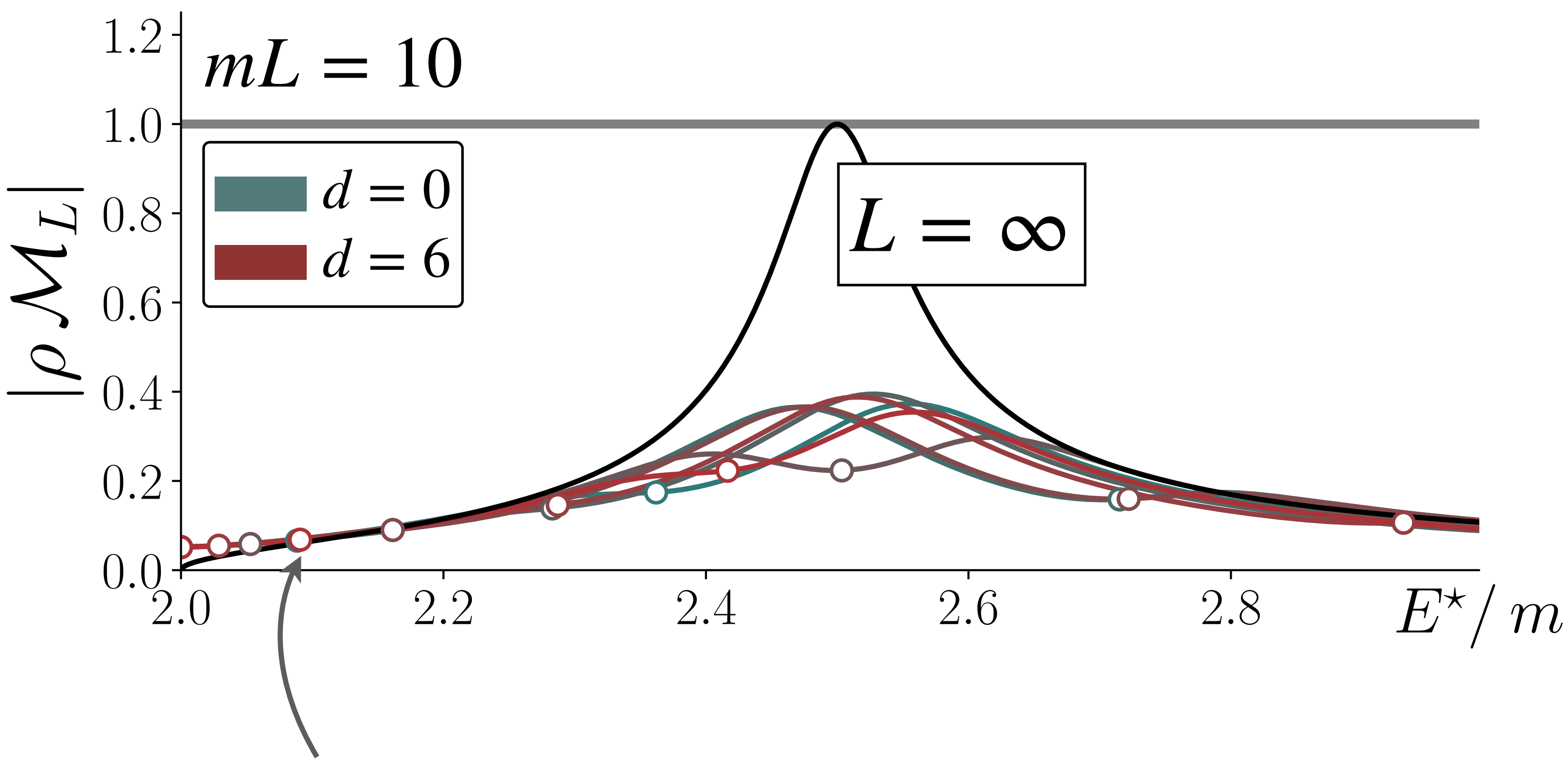

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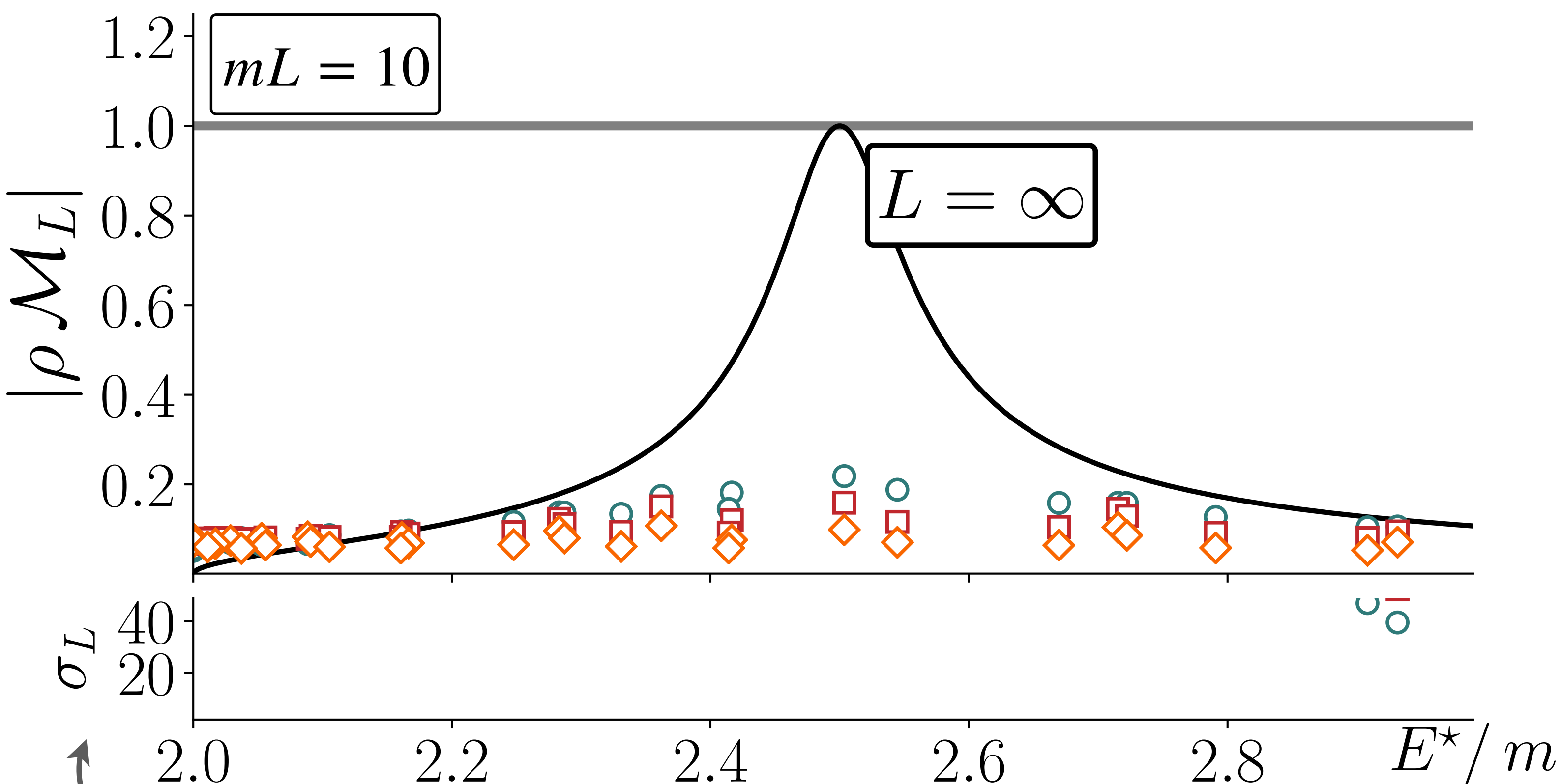
ϵ and volume dependence: $\epsilon L = 1$ $\epsilon L = 2$ $\epsilon L = 4$



$$\sigma_L = 100 \times \left| \frac{\mathcal{M}_L - \mathcal{M}(E^\star)}{\mathcal{M}(E^\star)} \right|$$

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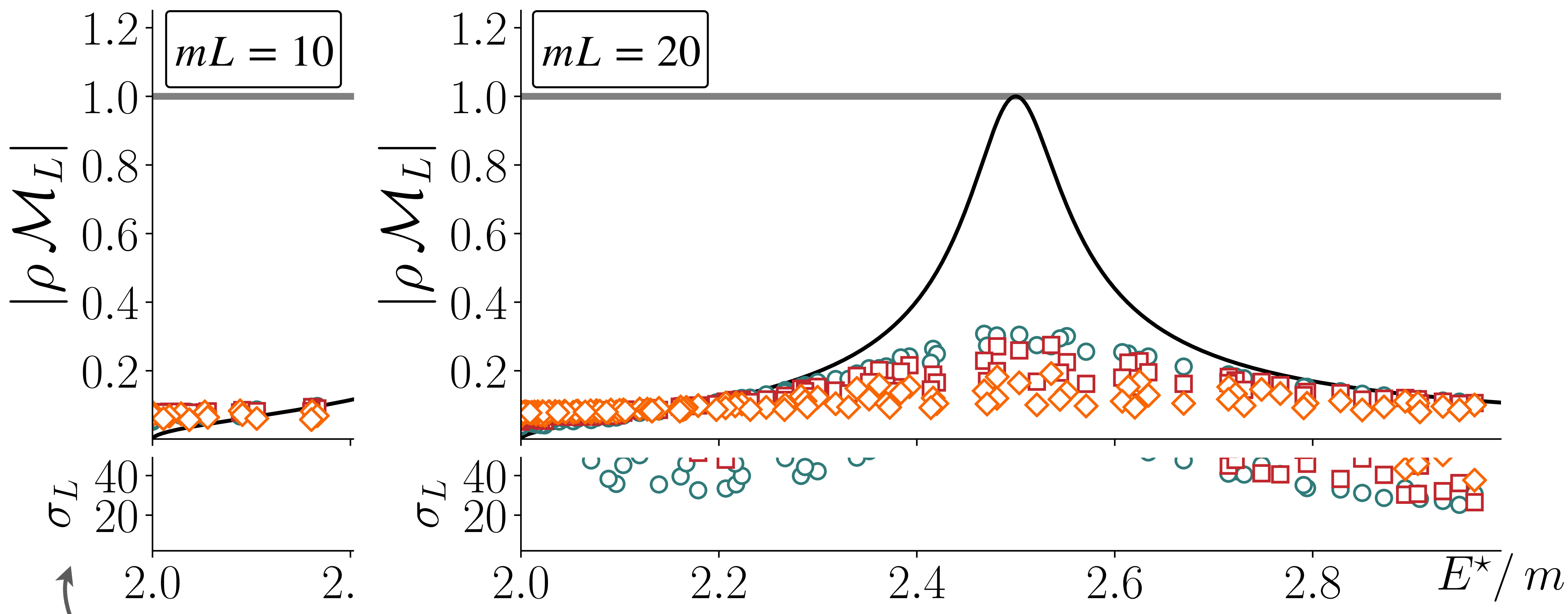

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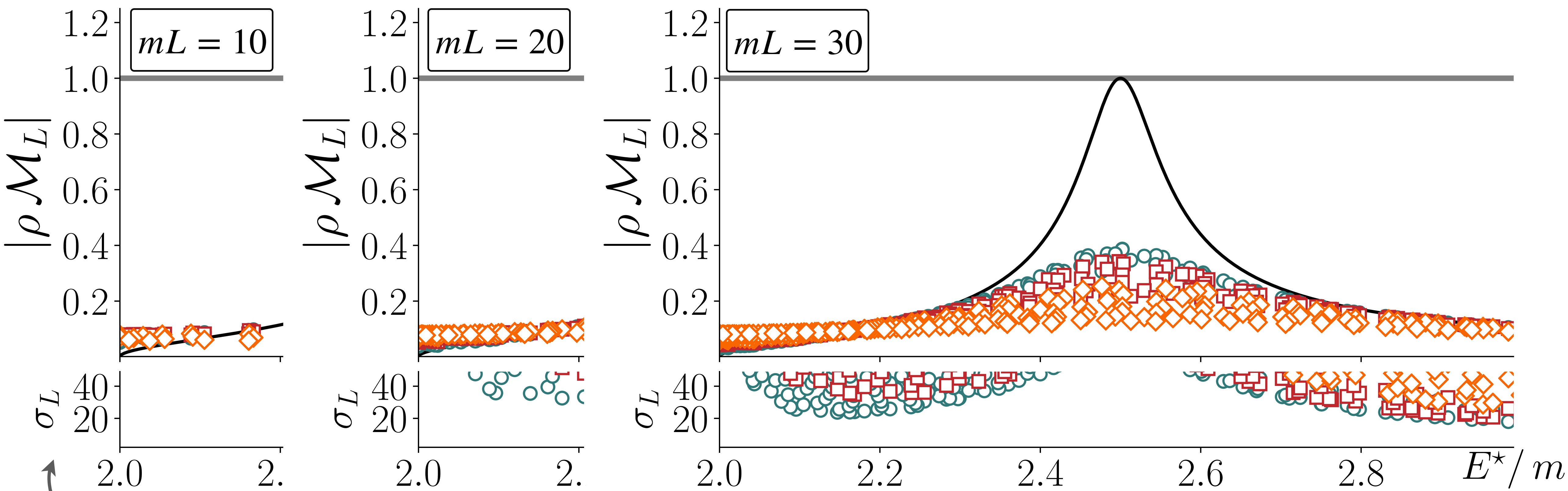

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EXPLOITING SYMMETRY: BOOST AVERAGING

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}\,\frac{F(P,L)}{1+\mathcal{M}F(P,L)}\,\mathcal{H}'$$

not Lorentz scalar   Lorentz scalar

- Take advantage and exploit the symmetry:
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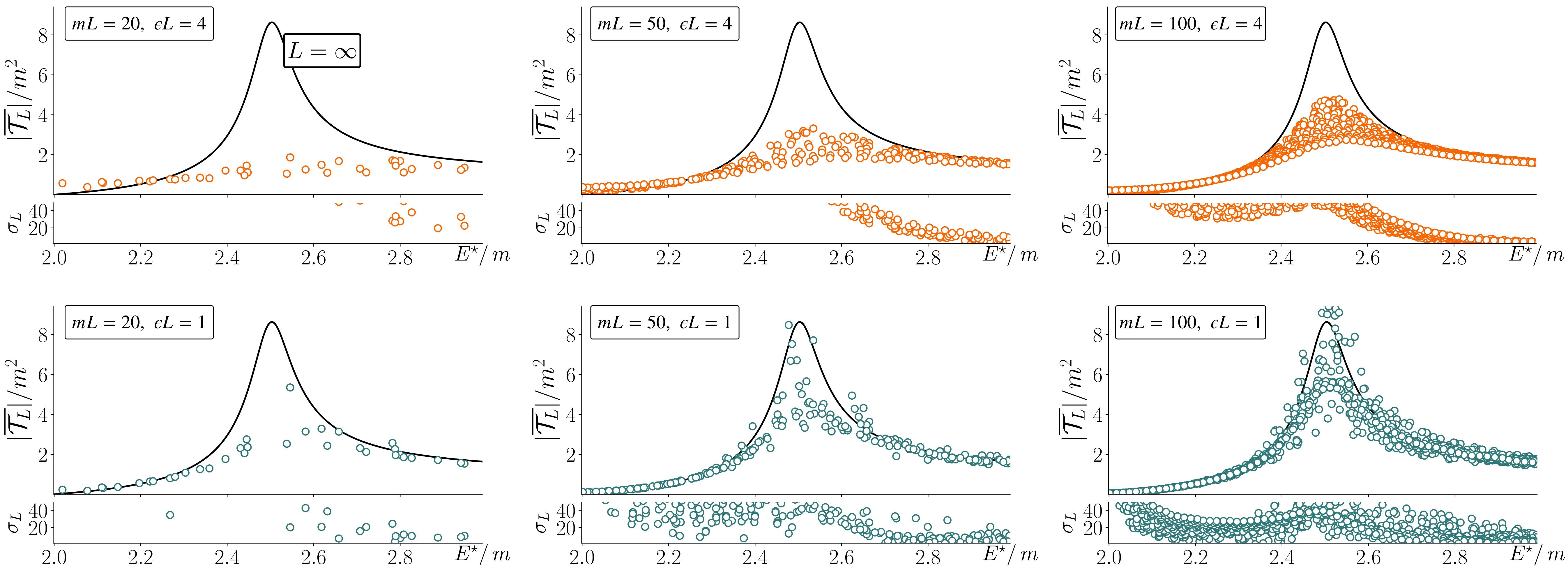
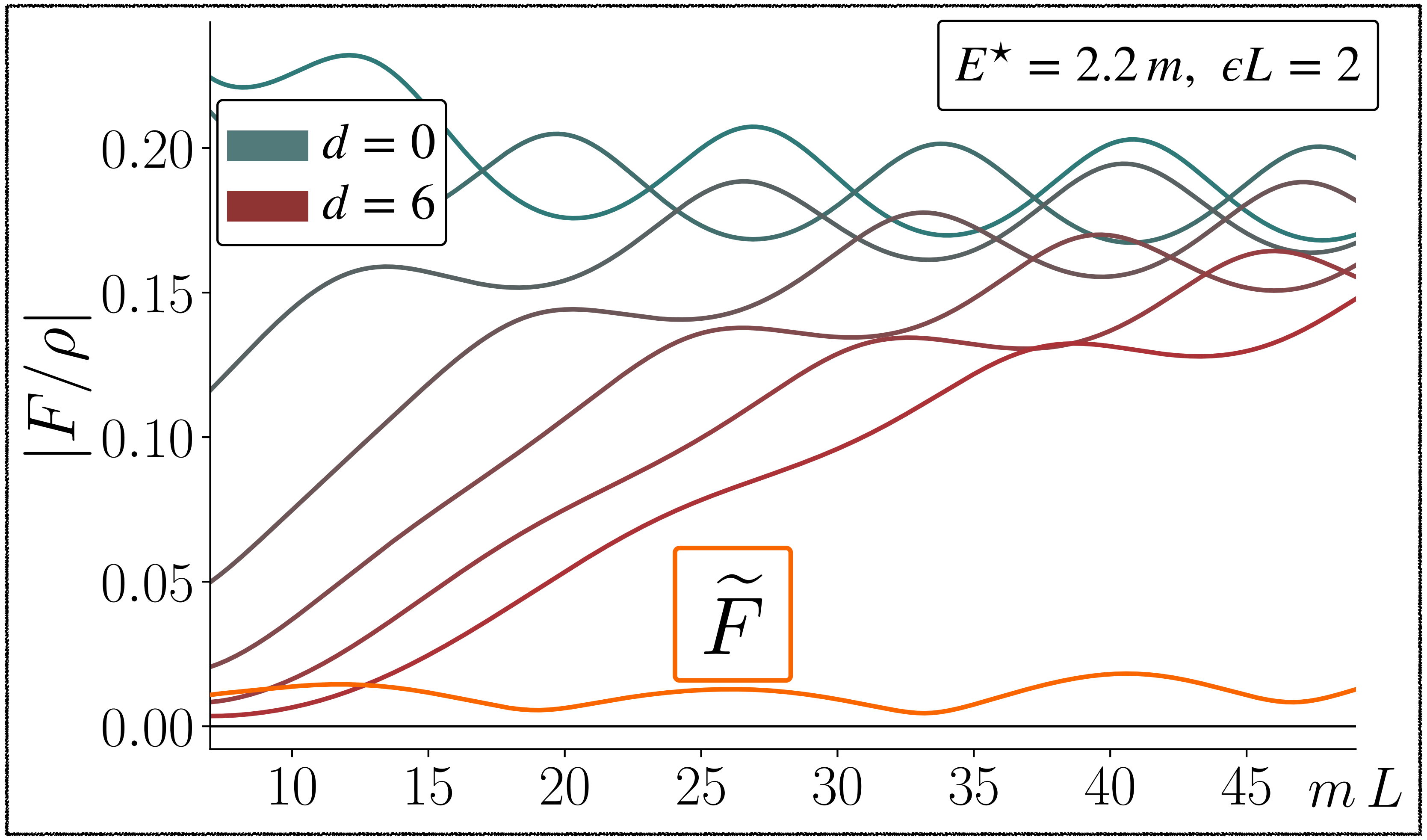


- $m_R = 2.5m$, $g = 2.5$, and $h(E^{*2}) = 0$
- $\bar{Q}^2 == 2m^2$, $\Delta_{Q^2} = 0.01m^2$
- Target virtualities: $\left|\overline{Q^2} - Q^2\right| < \Delta_{Q^2}$ and $\left|Q^2_{if} - Q^2\right| < \Delta_{Q^2}$

EXPLOITING SYMMETRY: BOOST AVERAGING

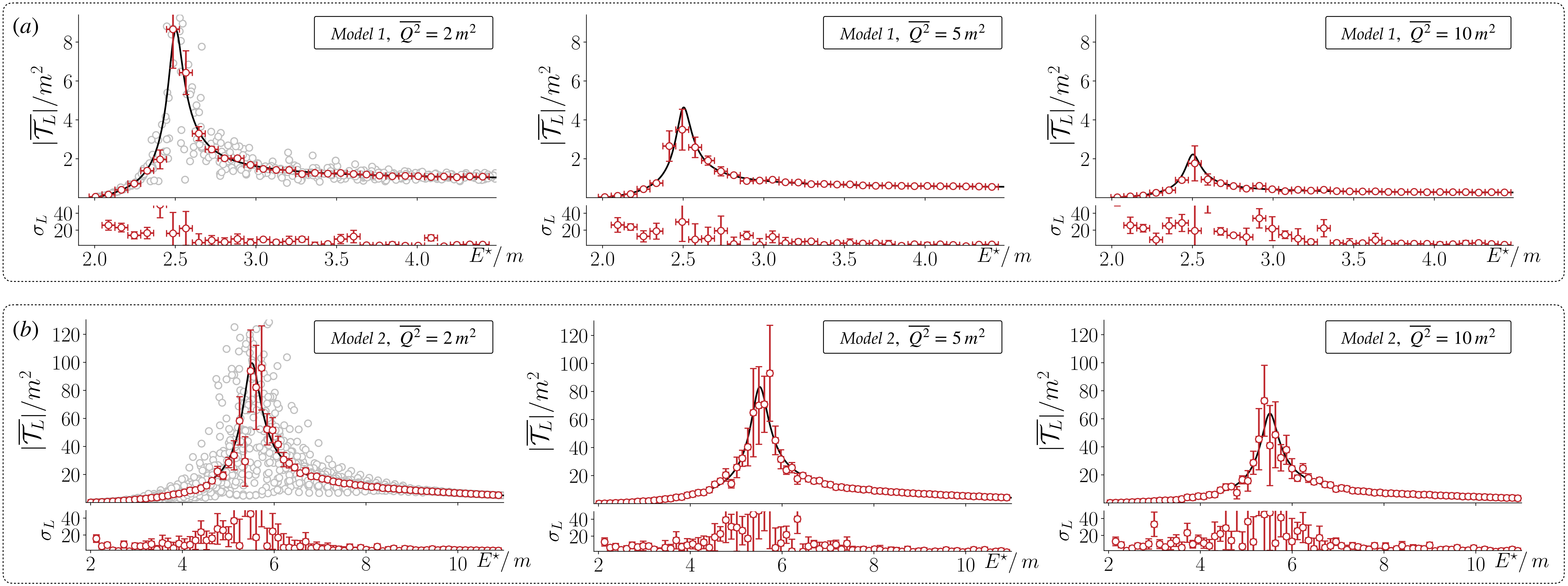
$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}\,\frac{F(P,L)}{1+\mathcal{M}F(P,L)}\,\mathcal{H}'$$

- $F(P,L)$ encodes FV effects: not a Lorentz scalar.
- Asymptotic behavior : $F \sim e^{-L\epsilon\alpha_0}(-1)^{d}$ $\longleftarrow$ alternating sign!
- Boost averaging: should reduce finite volume effects



BOOST AVERAGING: NUMERICAL RESULTS

- [x] Average over $mL = [20,25,30]$, with $d \leq mL$
- [x] $\Delta_{Q^2} = 0.05m^2$, $\Delta_{E^*} = 0.08m$, $\epsilon = \dfrac{1}{L(mL)^{1/2}}$



- [ ] Model 1: $m_R = 2.5m, g = 2.5, h(E^{*2}) = 0$
- [ ] Model 2: $m_R = 5.5m, g = 6, h(E^{*2}) = 0.2/m^2$

INFINITE/FINITE VOLUME ANALYTIC STRUCTURE

Infinite volume:

- Asymptotic states satisfy:

$$\hat{H}_{\infty,0}\,|p_1, p_2, \cdots p_n\rangle_0 = E(p_1, p_2, \cdots p_n)\,|p_1, p_2, \cdots p_n\rangle_0$$

- Continue spectrum

$$E(p_1, p_2, \cdots p_n) = \sum_{i=0}^{n} \sqrt{p_i^2 + m^2}$$

- Branch cut

Finite volume:

- No asymptotic states:

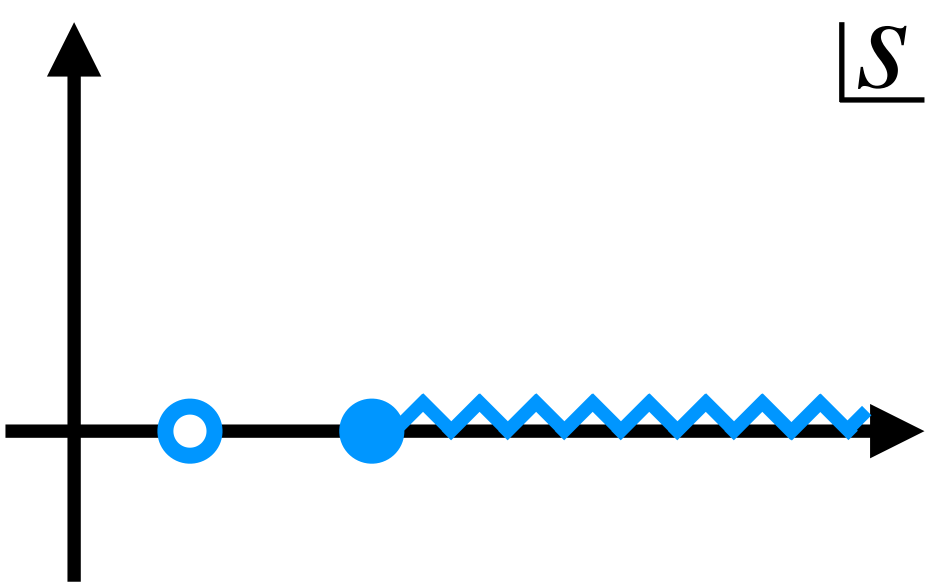
$$\hat{H}_L\,|n\rangle_L = E_n(L)\,|n\rangle_L$$

- Discretizes the spectrum

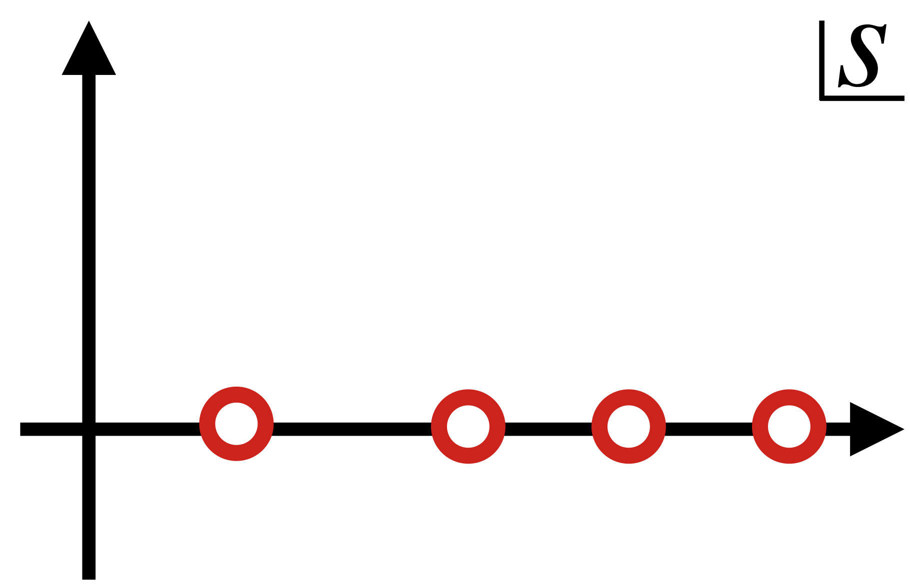
$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^\#}$$

- Eliminates the branch cut

Infinite-volume analytic structure



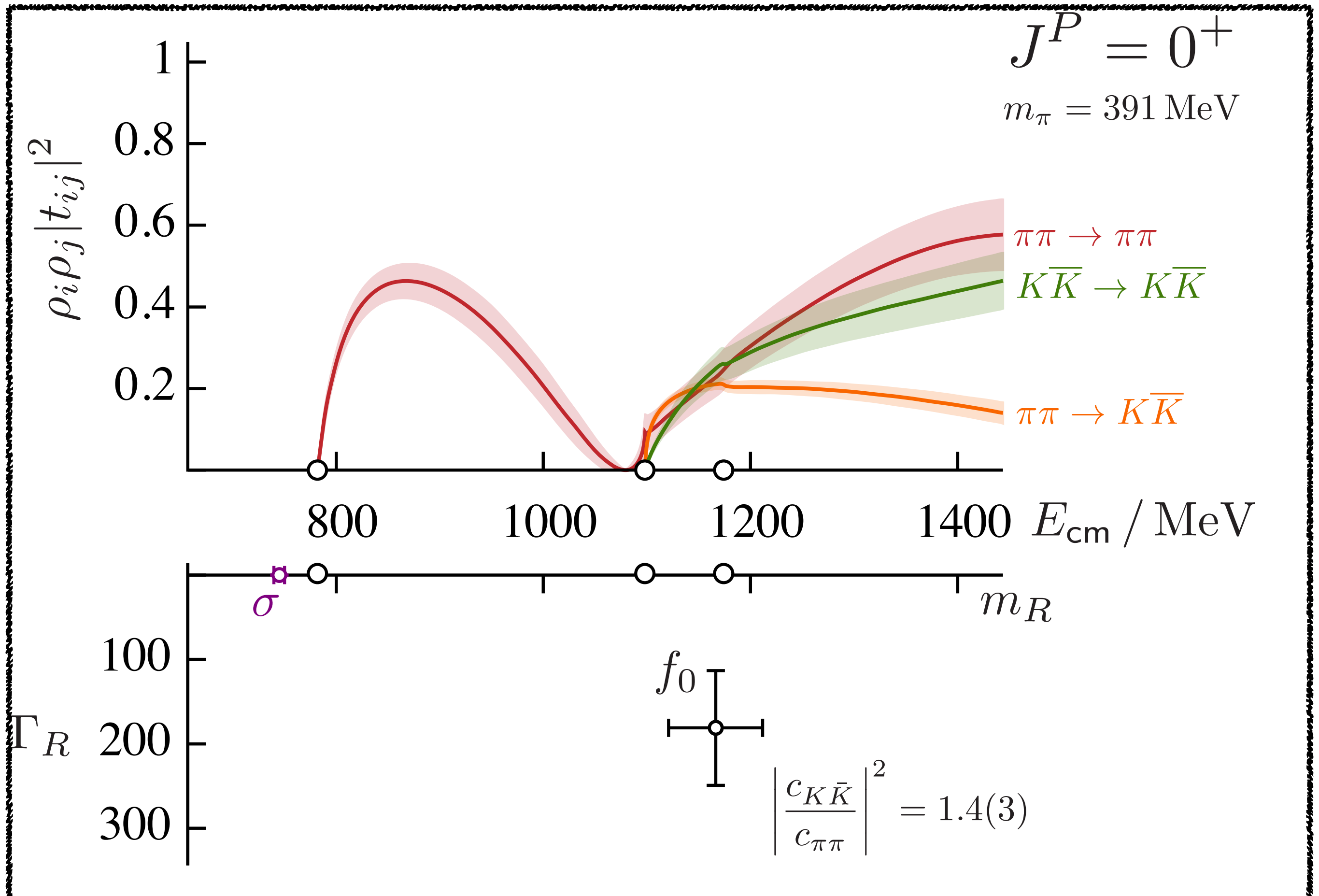
Finite-volume analytic structure



"SCATTERING" IN A FINITE VOLUME

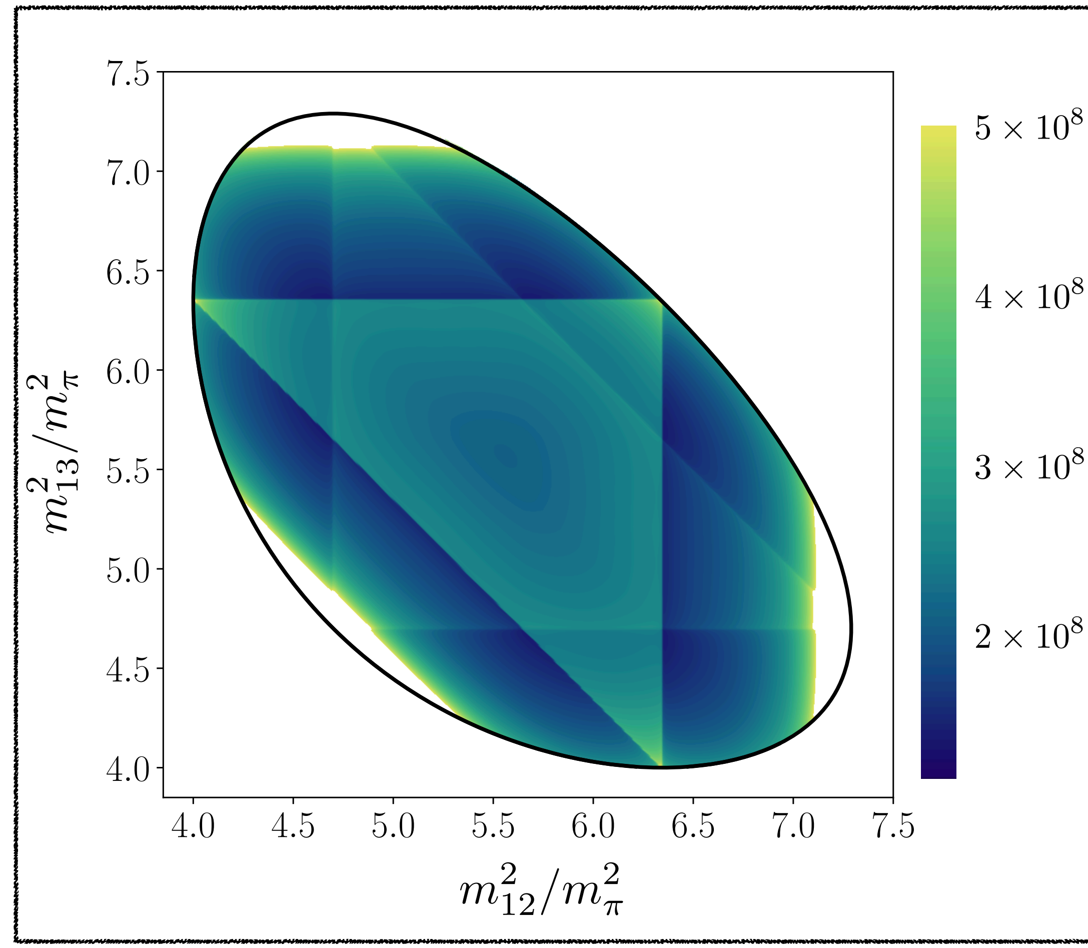
Finite volume: scattering is **not well** defined!

- ☐ Use finite-volume as a tool in LQCD (Luscher-like techniques)
  - ☐ Relate the spectrum $E_n(L)$ to physical amplitudes
  - ☑ Exclusive processes



2 → 2 amplitude

HadSpec collaboration
PRD 97 (2019) 054513



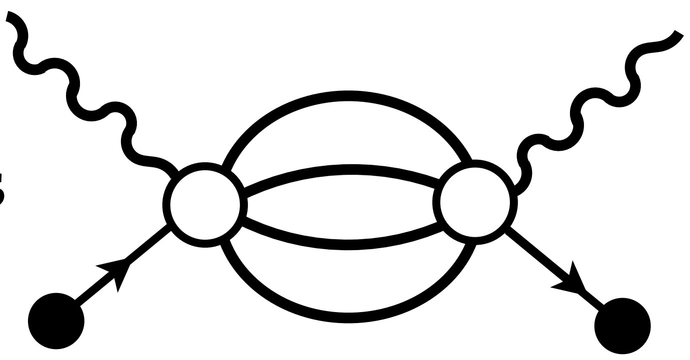
3 → 3 amplitude

HadSpec collaboration
PRL 126 (2021) 012001

"SCATTERING" IN A FINITE VOLUME

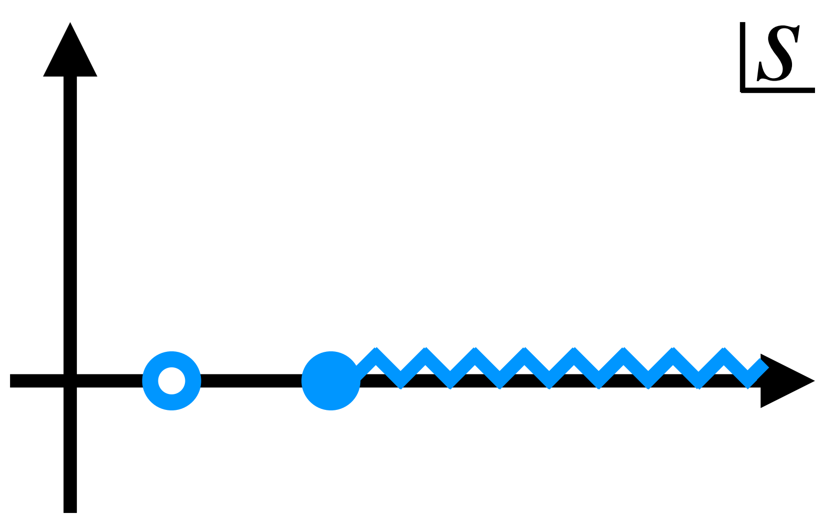
Finite volume: scattering is **not well** defined!

- ☐ Use finite-volume as a tool in LQCD (Luscher-like techniques)
  - ☐ Relate the spectrum $E_n(L)$ to physical amplitudes
  - ☑ Exclusive processes
  - ☐ Challenging for inclusive reactions



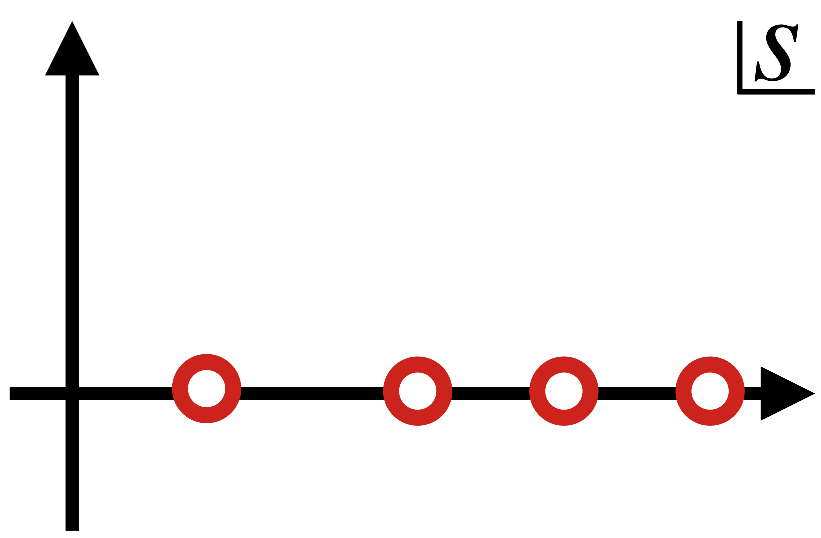
Infinite-volume amplitudes

- ☐ complex functions
- ☐ kinematic singularities



Finite-volume "amplitudes"

- ☐ real functions
- ☐ power-law finite-volume errors



"SCATTERING" IN A FINITE VOLUME

Finite volume: scattering is **not well** defined!

- [x] QC may be an useful tool to approach inclusive reactions

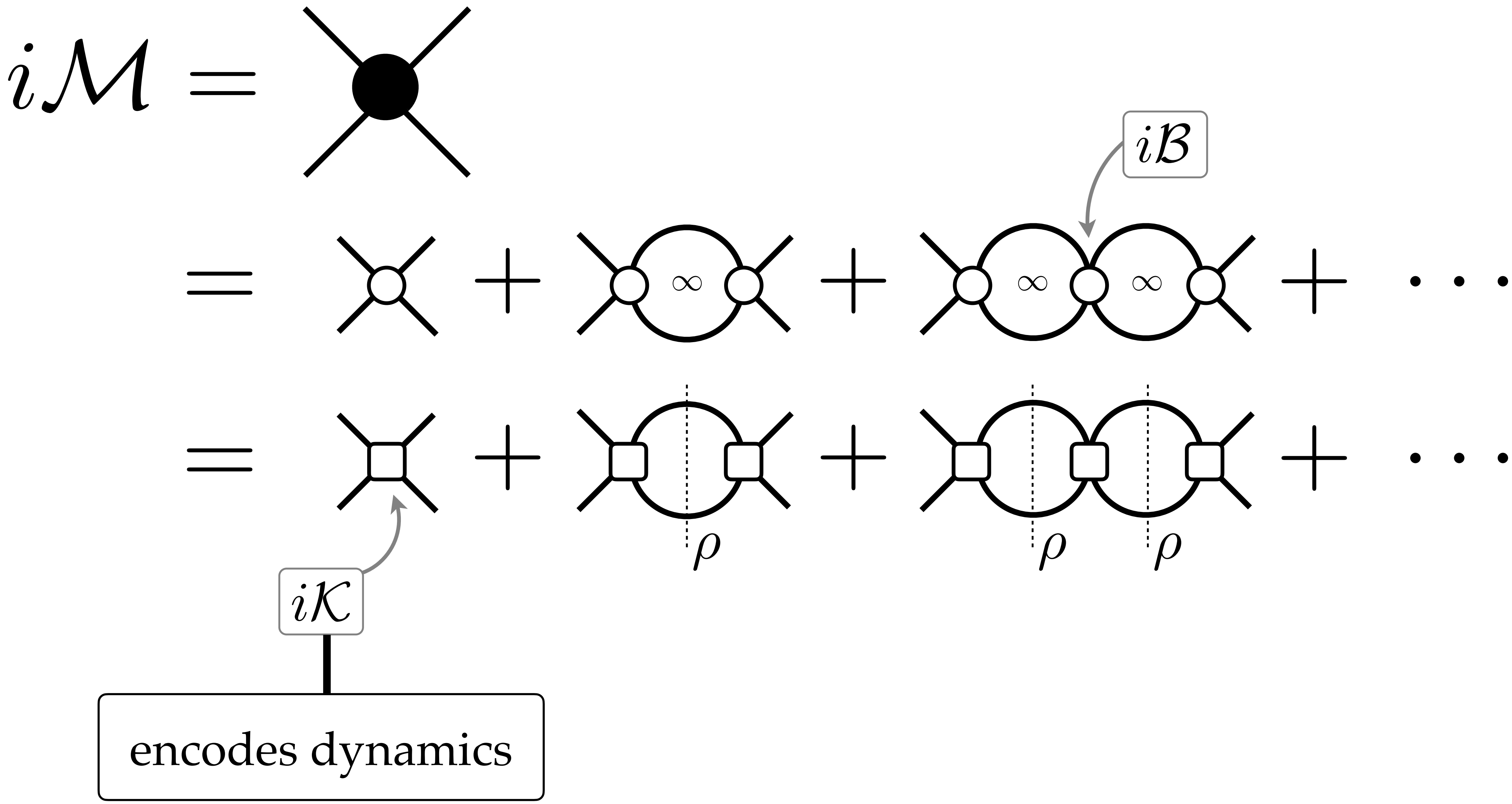
$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} dt\, e^{iq_0 t - \epsilon|t|} \langle n_f | \, T[\mathcal{J}_2(t)\, \mathcal{J}_1(0)] \, | n_i \rangle_L$$

- [x] Direct extraction of the amplitude $\mathcal{T}_L(\epsilon)$
    - [ ] Extract $\langle n_f | \, \mathcal{J}_{2,M}(t)\, \mathcal{J}_1(0) \, | n_i \rangle_L \longleftarrow$ accesible in a QC
        - [ ] Construct $|n_i\rangle$ and $|n_f\rangle$
        - [ ] Evaluate $\mathcal{J}(t)$ for different times: $e^{iHt}\mathcal{J}(0)e^{-iHt}$
    - [ ] $i\epsilon$ prescription: consider several ϵ values
    - [ ] consider different L
    - [ ] Finally consider the double limit:

$$\mathcal{T} = \lim_{\epsilon \to 0} \lim_{L \to \infty} \mathcal{T}_L(\epsilon)$$

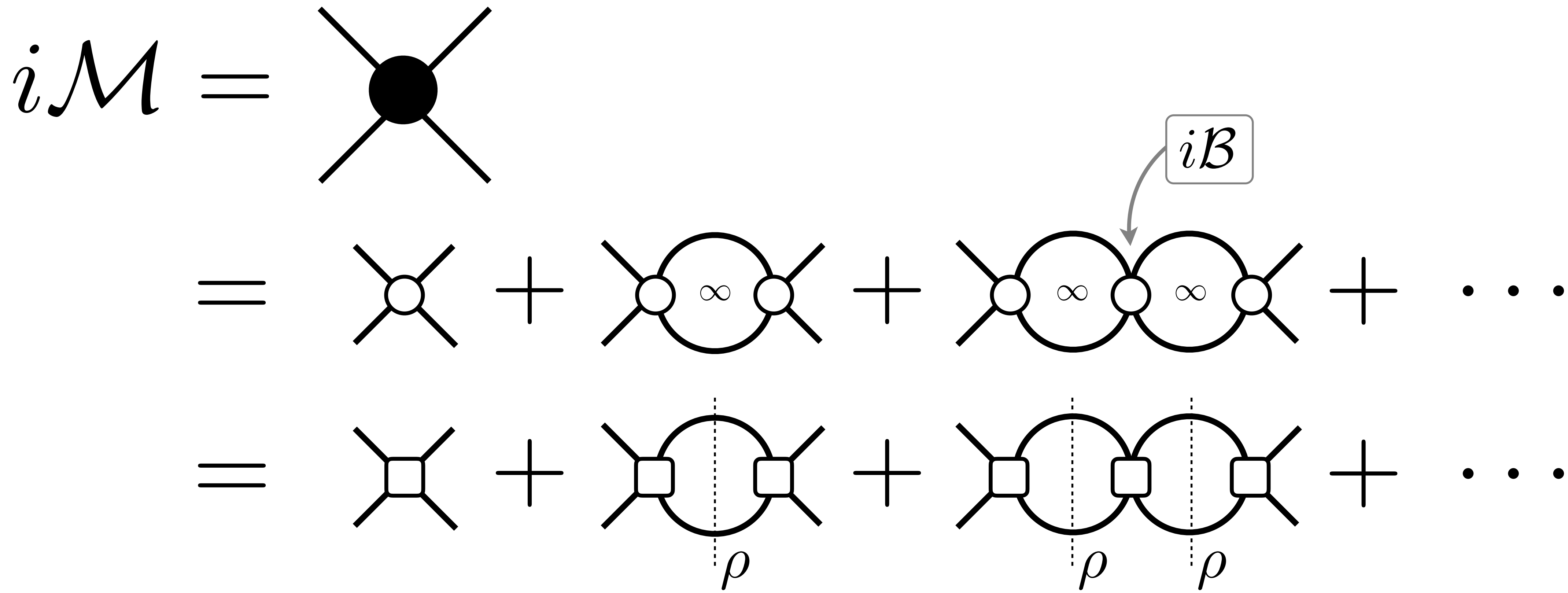
TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:



TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:



$E^\star$: center of mass energy

$q^\star$: single particle momentum (c.m.)

$$q^\star = \sqrt{\frac{E^{\star 2}}{4} - m^2}$$

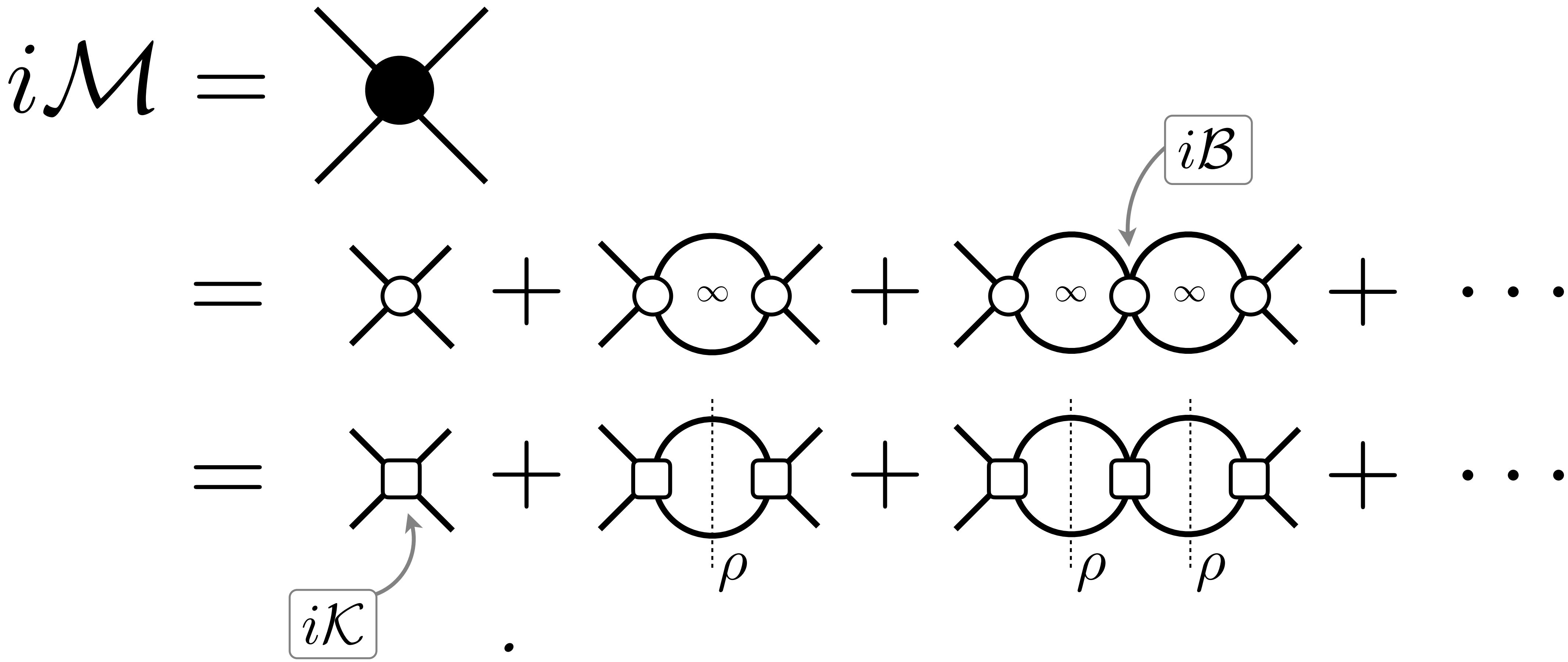
$$\boxed{\rho = \frac{1}{8E^\star q^\star} \sim \frac{1}{\sqrt{s - s_{\mathrm{th}}}}}$$

kinematic singularity

1+1D

TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:



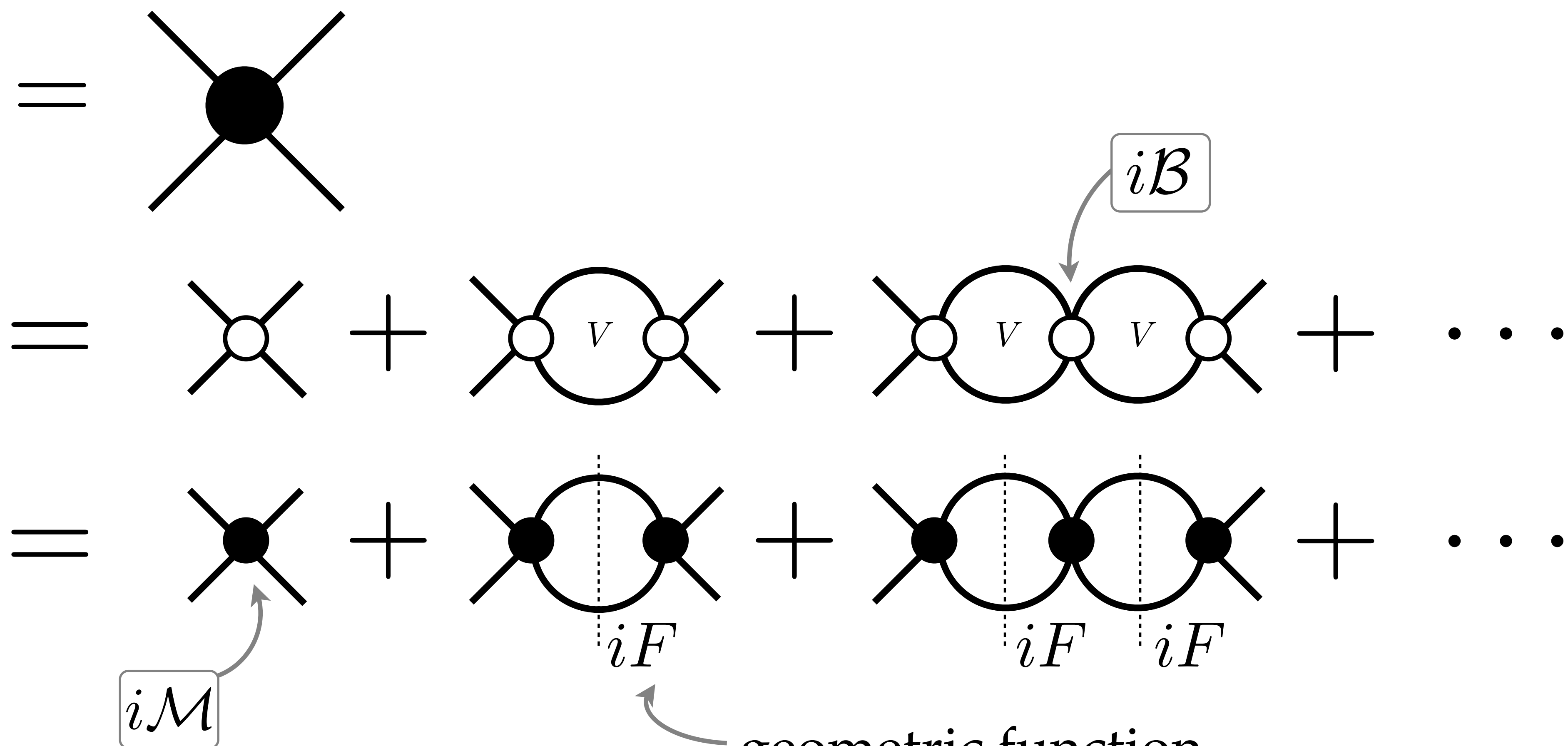
$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$

TWO-BODY SCATTERING AMPLITUDE IN A BOX

Finite volume: integral in loops become sum over discrete momenta.

Two-body scattering amplitude in a finite volume:

$i\mathcal{M}_L =$



geometric function encoding FV effects

$$= \frac{i}{\mathcal{M}^{-1} + F}$$

TWO-BODY SCATTERING AMPLITUDE IN A BOX

Two body scattering amplitude analog:

$$\mathcal{M}_L = \frac{1}{\mathcal{M}^{-1} + F}$$

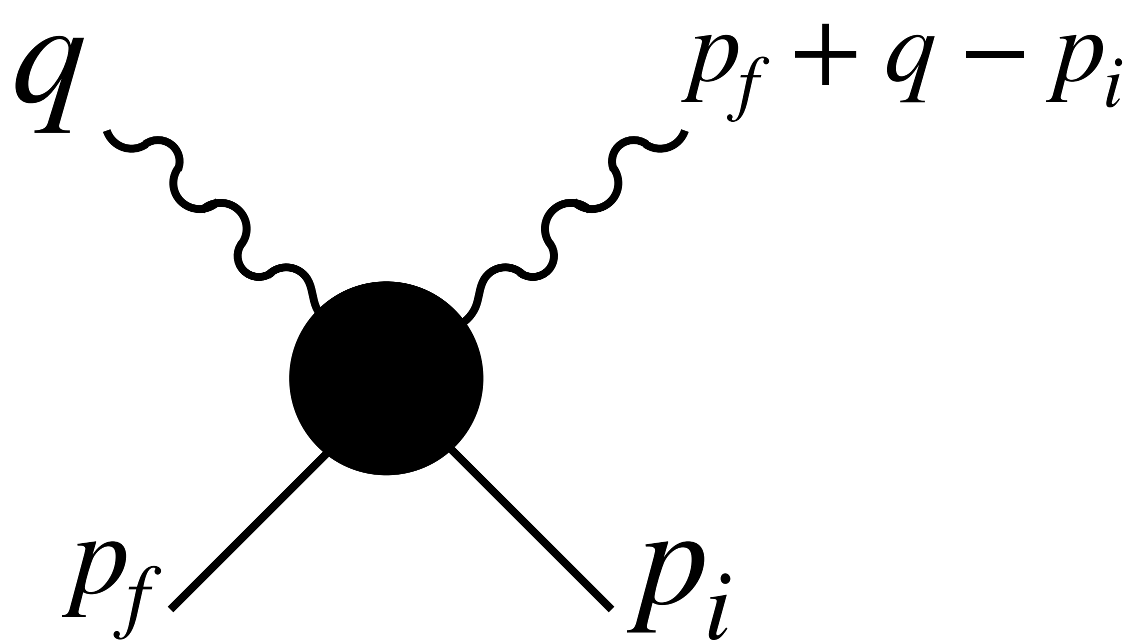
where the geometric function F is defined as:

$$F(E, \boldsymbol{P}, L) = \lim_{\epsilon \to 0^+} \frac{1}{2} \left[\frac{1}{L} \sum_{\boldsymbol{k}} - \int \frac{d\boldsymbol{k}}{2\pi} \right] \frac{1}{2\omega_k} \frac{1}{(P-k)^2 - m^2 + i\epsilon}$$

the total momentum $\mathbf{P} = \dfrac{2\pi d}{L}$

- $F(P, L)$ encodes the finite volume effects!
- $F(P, L)$: not a Lorentz scalar.

COMPTON SCATTERING: KINEMATICS



Total momentum of the system:

$$P^\mu \equiv (q + p_f) = (E, \mathbf{P})$$

<u>Invariants:</u>

$$p_i^2 = p_f^2 = m^2$$

$$Q^2 = -q^2$$

$$Q_{if}^2 = -(p_f + q - p_i)^2$$

<u>Center of mass frame:</u>

Energy:

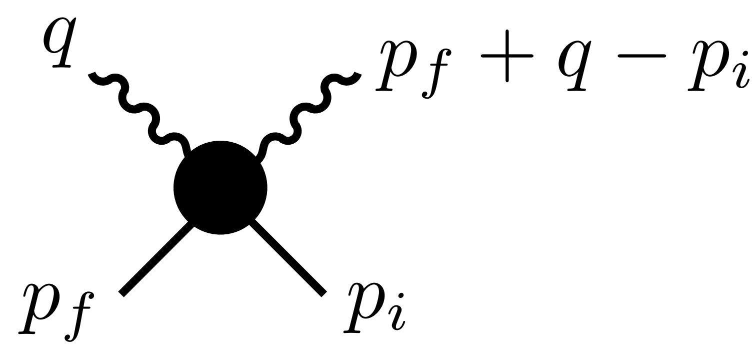
$$E^{\star 2} = P_\mu P^\mu = E^2 - \mathbf{P}^2 = s$$

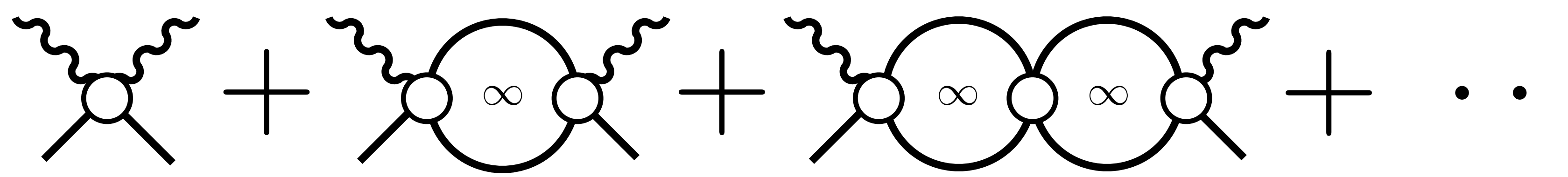
Single particle momentum:

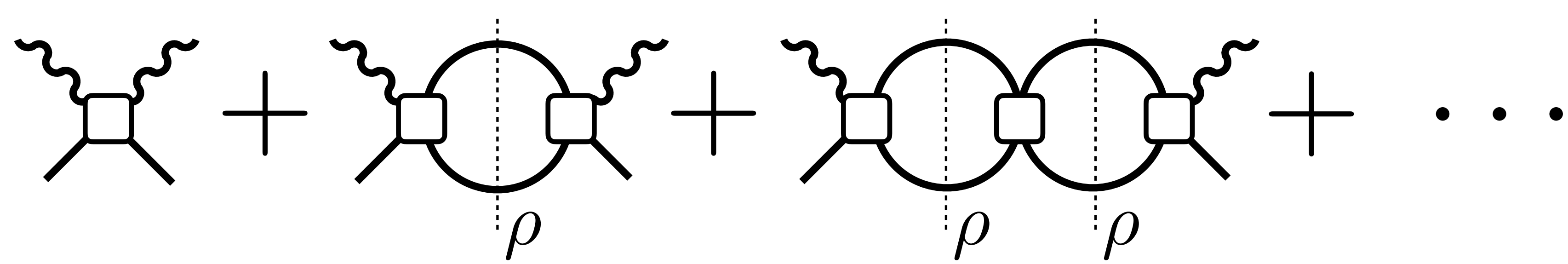
$$q^\star = \sqrt{\frac{E^{\star 2}}{4} - m^2}$$

COMPTON-LIKE AMPLITUDES

Compton-like amplitudes [similar analytic structure compared to two body case]:

$i\mathcal{T} =$ 

$=$ 

$=$ 

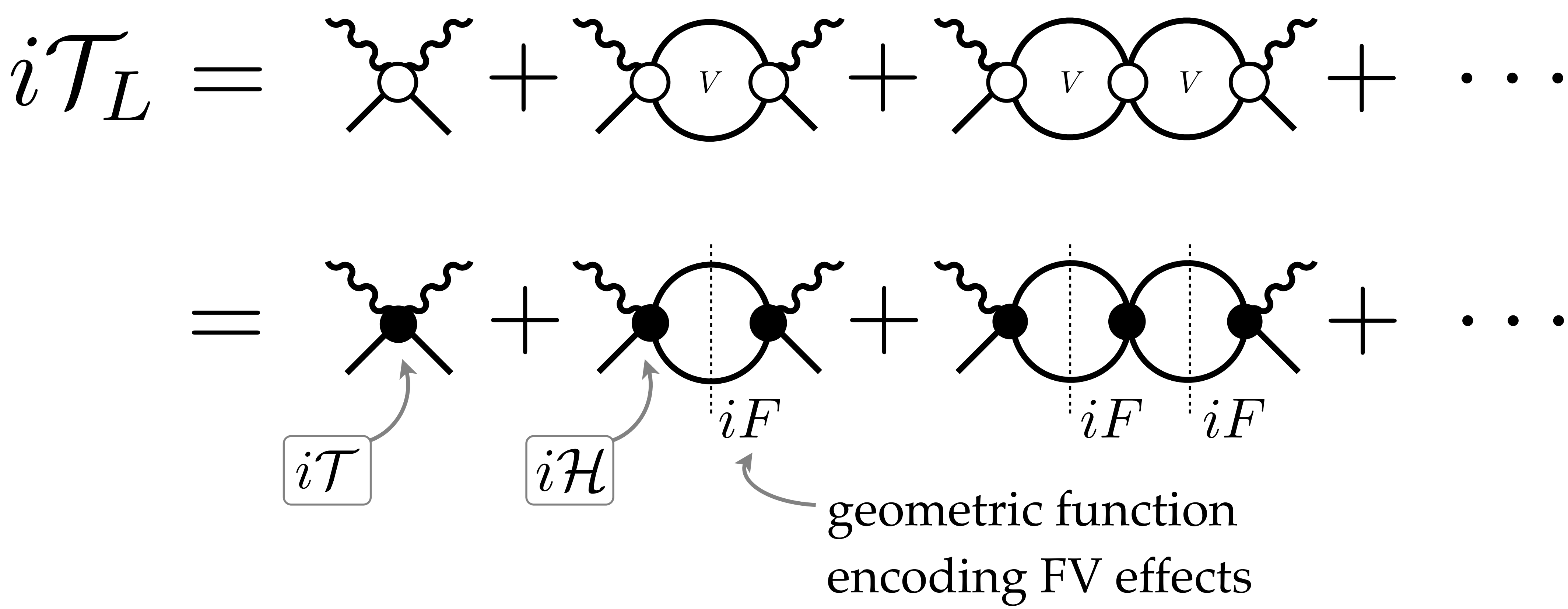
$$\mathcal{T}(s, Q^2, Q^2_{if}) = w_2(s, Q^2, Q^2_{if}) + \mathcal{A}_{12}(s, Q^2)\mathcal{M}(s)\mathcal{A}'_{21}(s, Q^2_{if})$$

real and "smooth" for $Q^2, Q^2_{if} > 0$

COMPTON-LIKE AMPLITUDES IN A BOX

Finite volume: integral in loops become sum over discrete momenta.

Compton-like amplitudes in a finite volume:



$$= i\mathcal{T} - i\mathcal{H}\,\frac{1}{F^{-1} + \mathcal{M}}\,\mathcal{H}'$$