

ACCESSING COMPTON AMPLITUDES IN A QUANTUM COMPUTER

Juan Guerrero

EICUG early career

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Based on:

Briceño, JG, Hansen & Sturzu: PRD 103 (2020) 014506

Briceño, Carrillo, JG, Hansen & Sturzu: PoS Lattice 2021, 315

A BIT OF LATTICE QCD

Lattice QCD: first principle non-perturbative approach to QCD

$$S[\phi] = \int dt \int d^3\mathbf{x} \mathcal{L}[\phi]$$

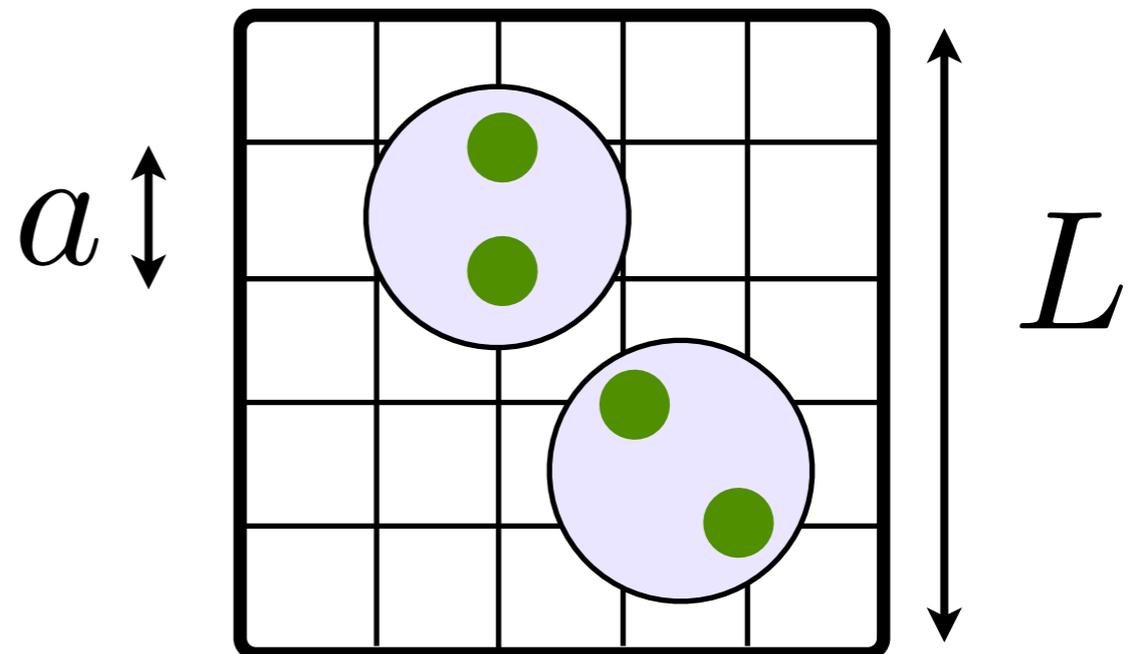
❑ Euclidean space time: $t^M \rightarrow -it^E$

$$Z = \int d\phi e^{iS^M[\phi]} \longrightarrow \int d\phi e^{-S^E[\phi]} \longleftarrow S^E \geq 0 : \text{weight!}$$

☑ Importance sampling

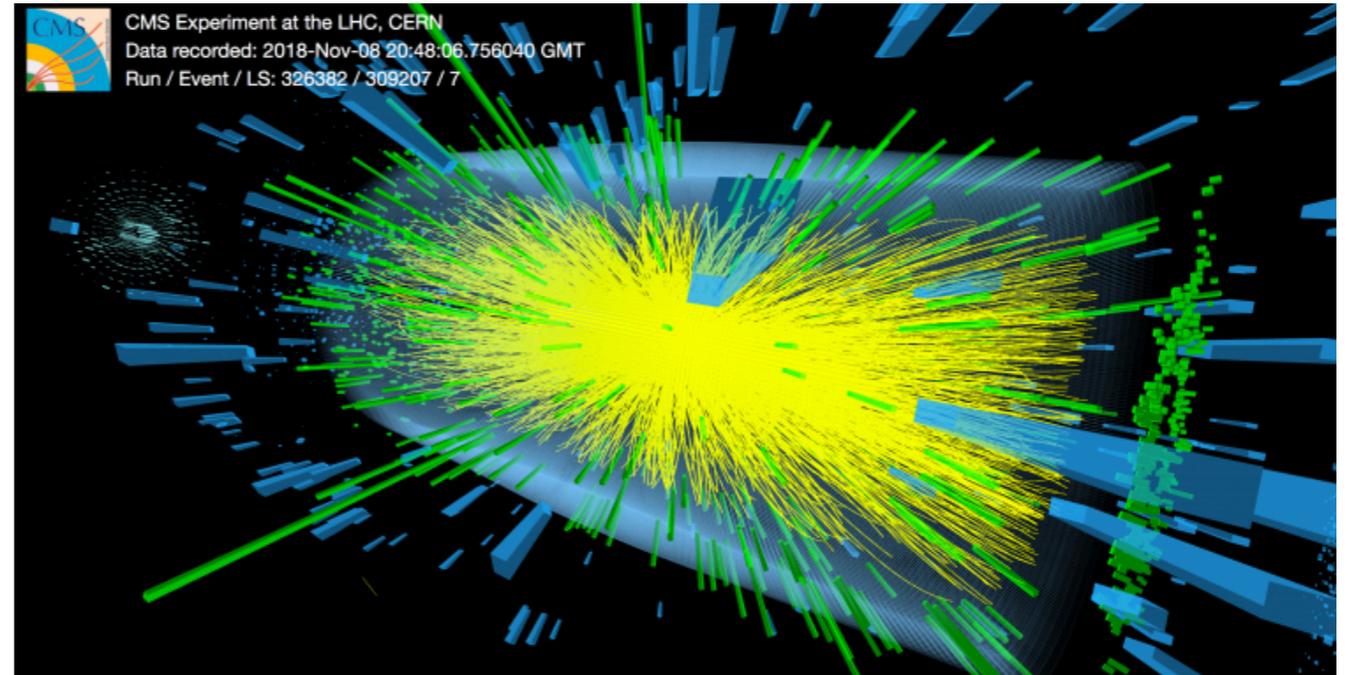
❑ Discrete space a

❑ Finite volume L



REAL-TIME DYNAMICS: SIGN PROBLEM IN LQCD

- Heavy-ion collisions
- Parton showers
- Fragmentation
- Long-range processes in the SM:
Compton scattering...

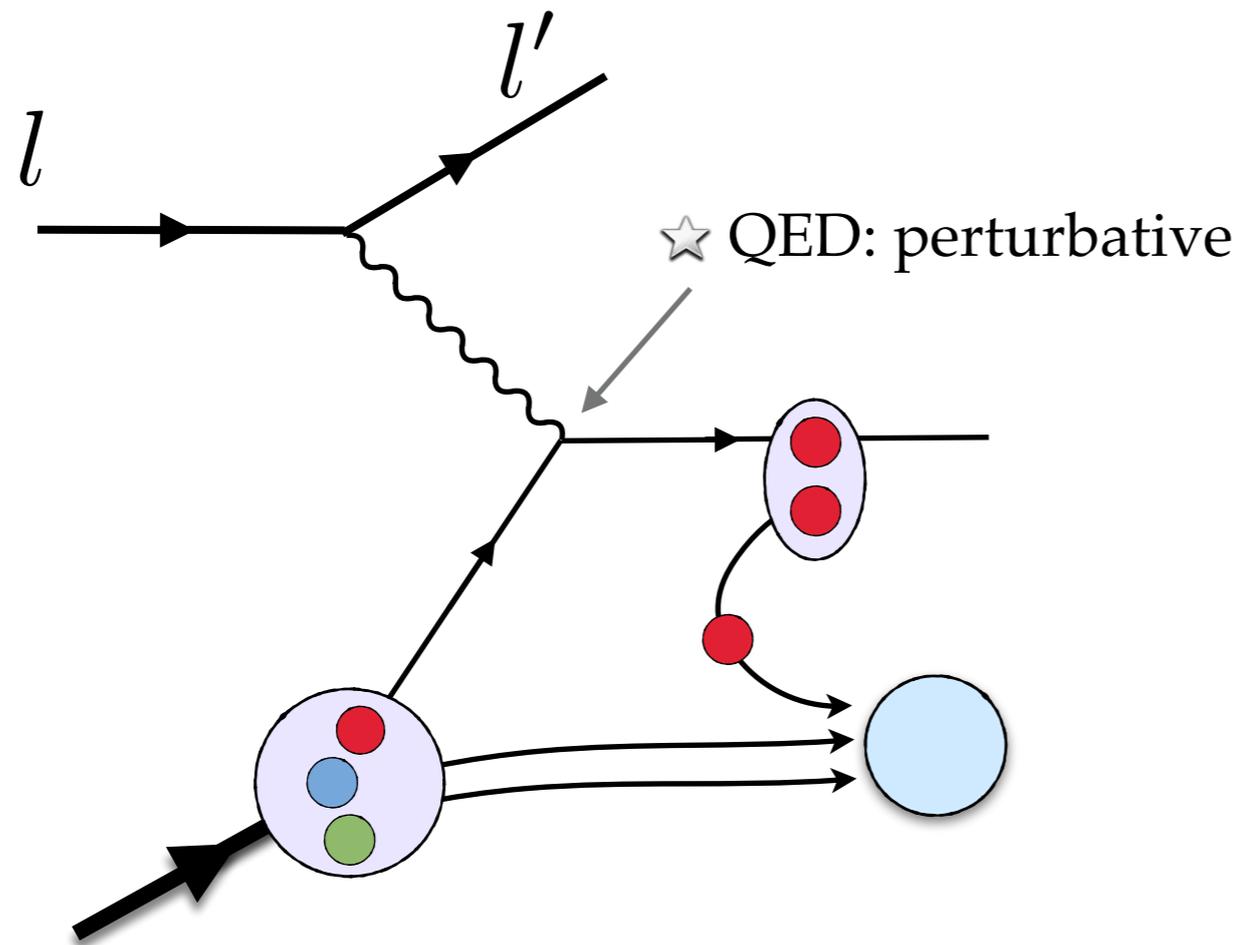


Path integral formulation: $Z = \int d\phi e^{iS^M[\phi]}$ ← **sign problem!**

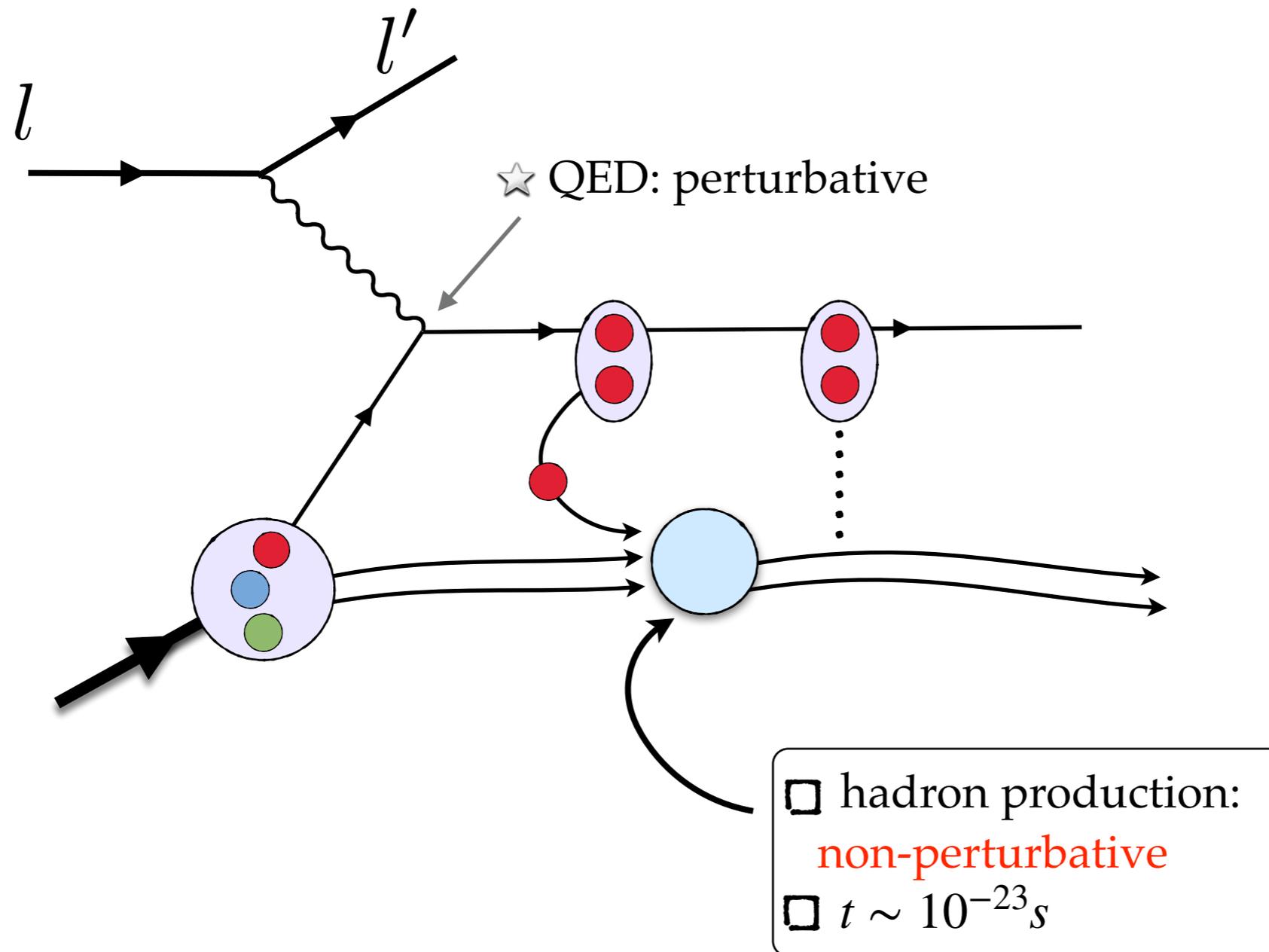
Quantum computing: prospective non-perturbative way to study QCD

$U(t, t_0) = e^{-iH(t-t_0)}$ ← Hamiltonian time evolution

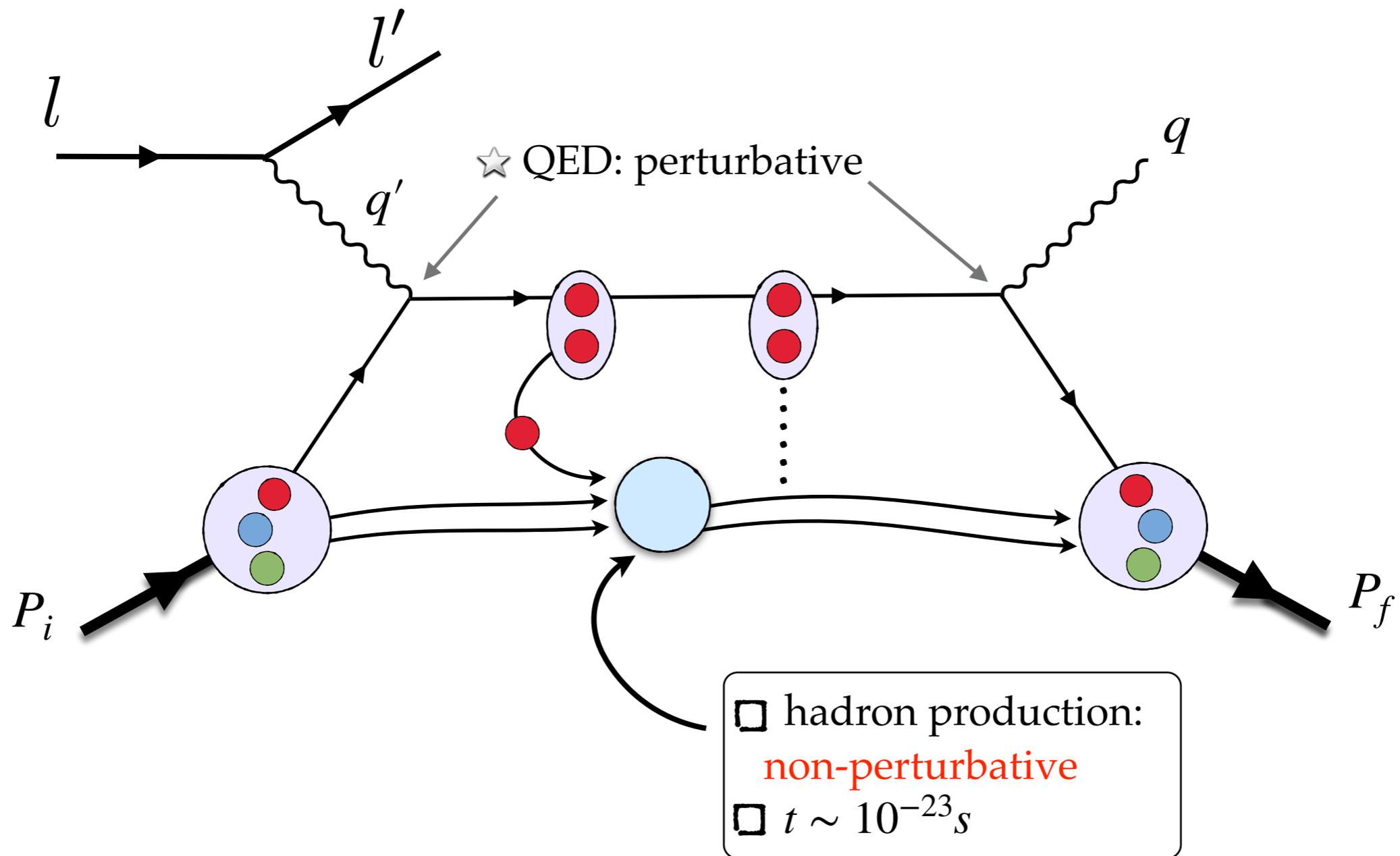
COMPTON SCATTERING



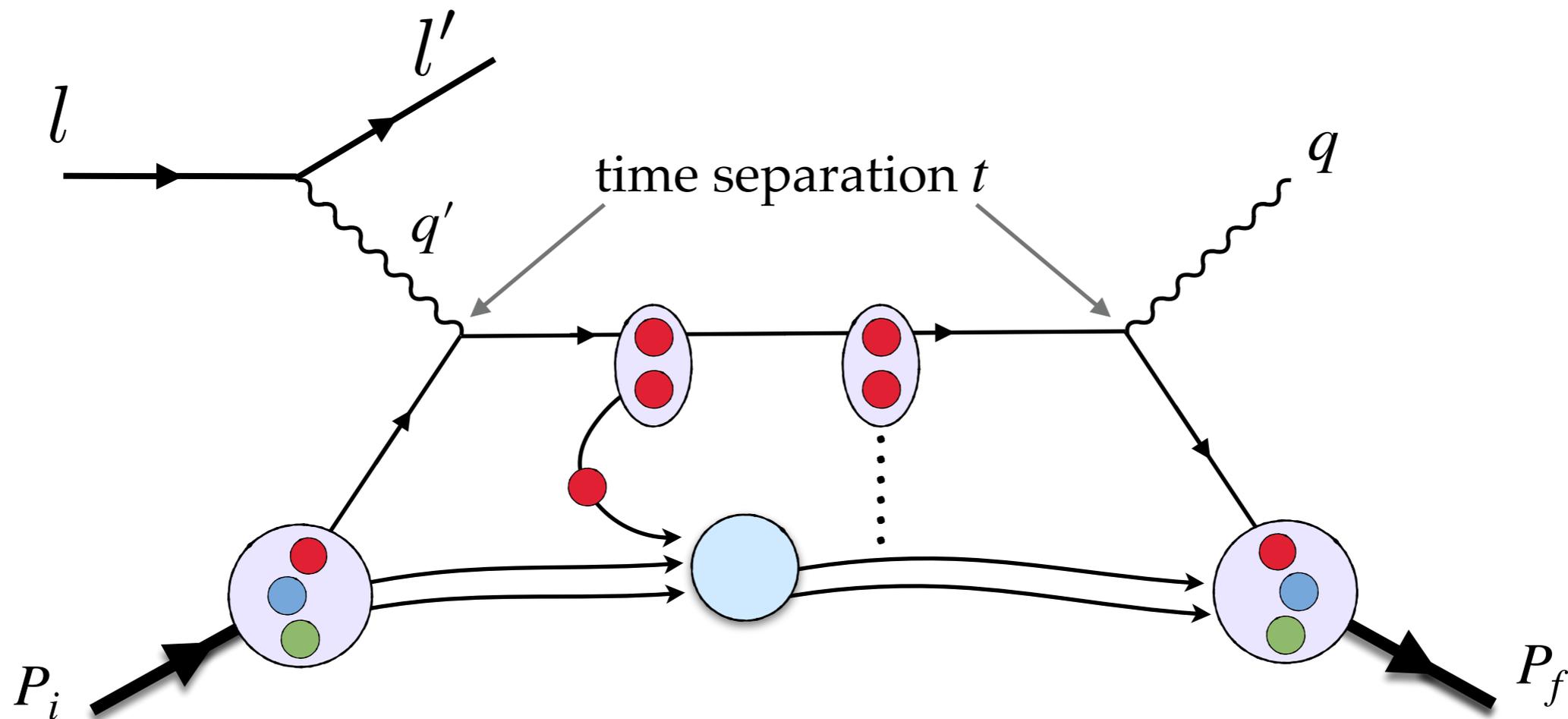
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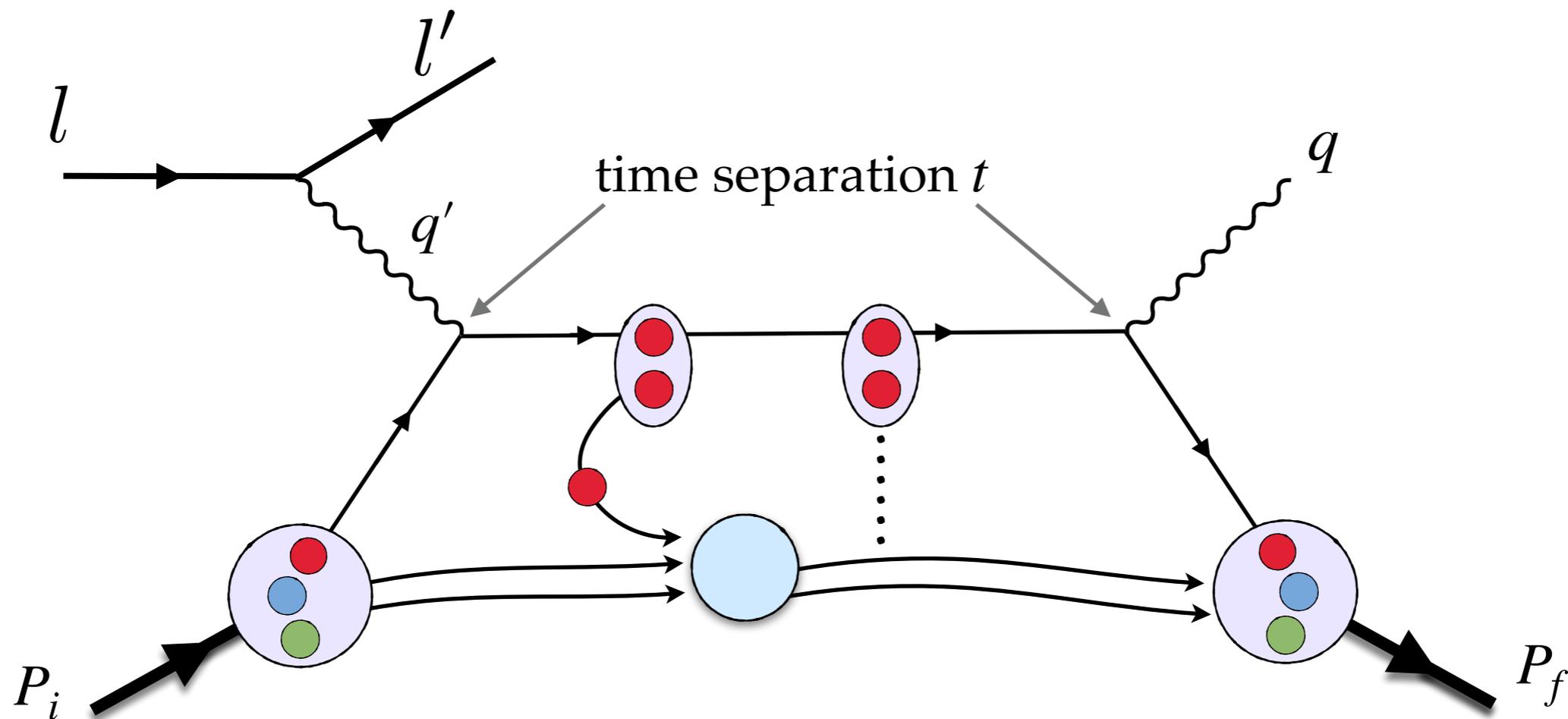


- Compton Amplitude: *master* of the inclusive structure amplitudes

$$\mathcal{T} = i \int d^4x e^{ix \cdot q} \langle N(P_f) | T [\mathcal{J}(t) \mathcal{J}'(0)] | N(P_i) \rangle_\infty$$

- Generalized Parton Distributions (GPDs): 3D- nucleon structure
- $\text{Im}(\mathcal{T})$ gives access to Parton Distribution Functions (PDFs)

COMPTON SCATTERING



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- ☑ Quantum Computer: test of convergence of factorization theorems directly from the Standard Model

QUANTUM COMPUTERS

Quantum computing: prospective non-perturbative way to study QCD

$$U(t, t_0) = e^{-iH(t-t_0)} \longleftarrow \text{Hamiltonian time evolution}$$

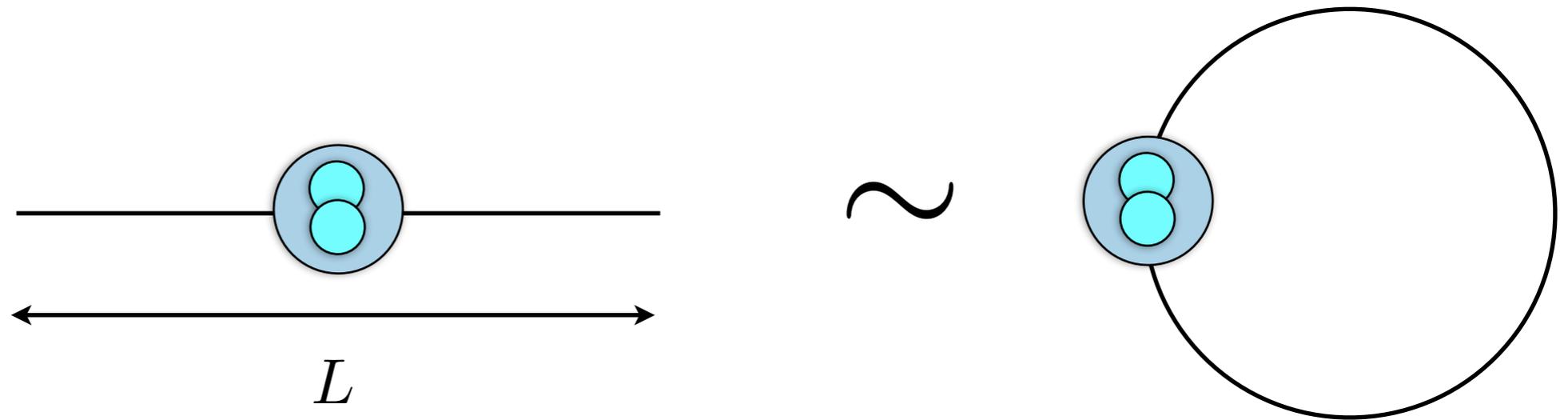
Minkowski space time

Discrete space a

Finite volume L

PHYSICS IN A 1D FINITE BOX

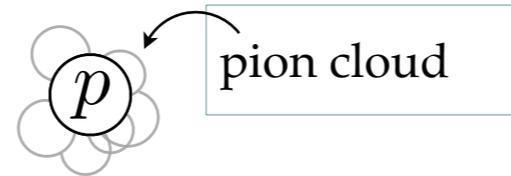
- Free particle wave function: $\varphi_p(x) = e^{ipx}$



$$\varphi_p(L + x) = e^{ip(x+L)} = \varphi_p(x) = e^{ipx}$$

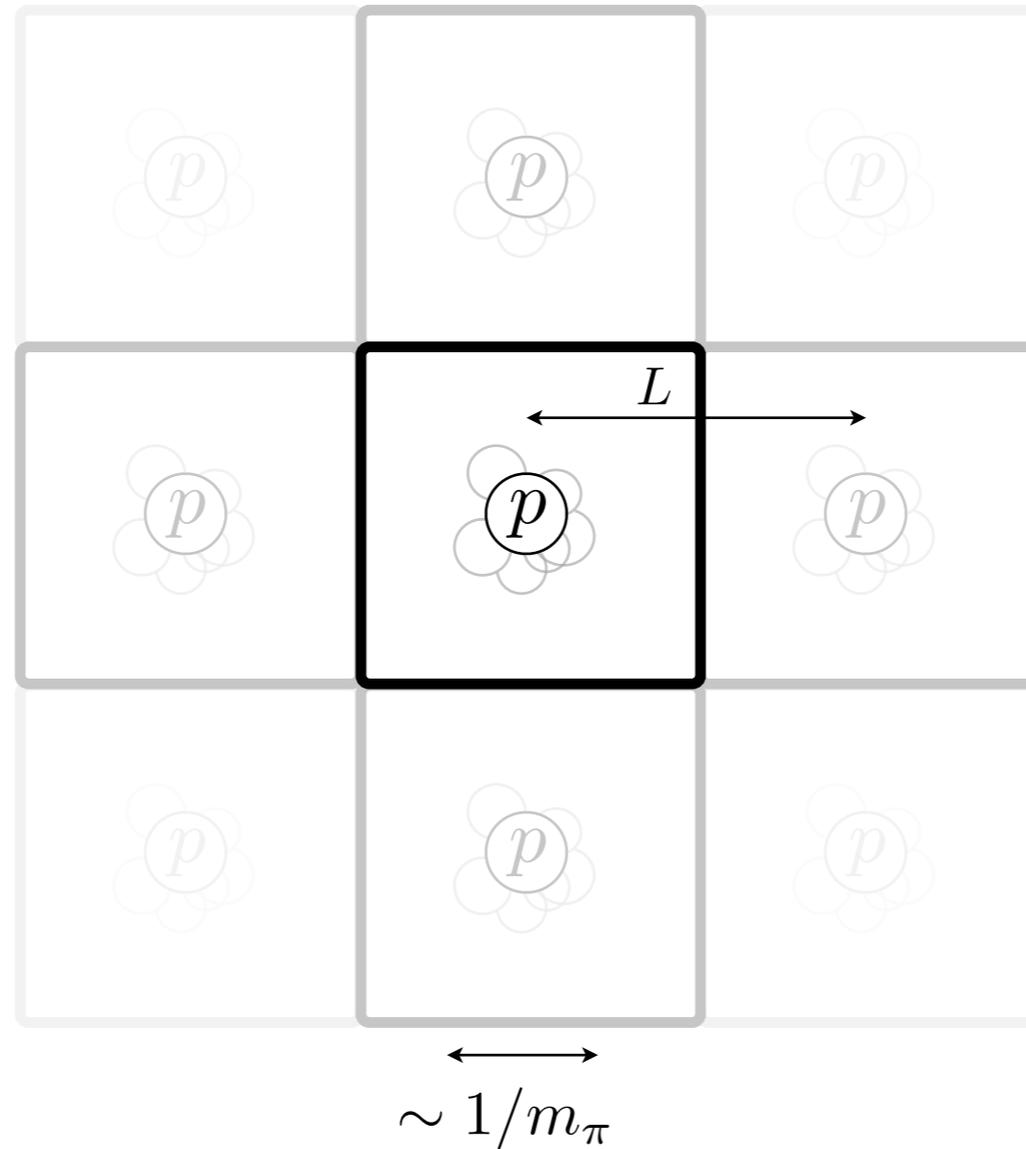
- Discretized momentum and spectrum: $p = \frac{2\pi n}{L}$

FINITE VOLUME: INFRARED LIMIT OF THE THEORY



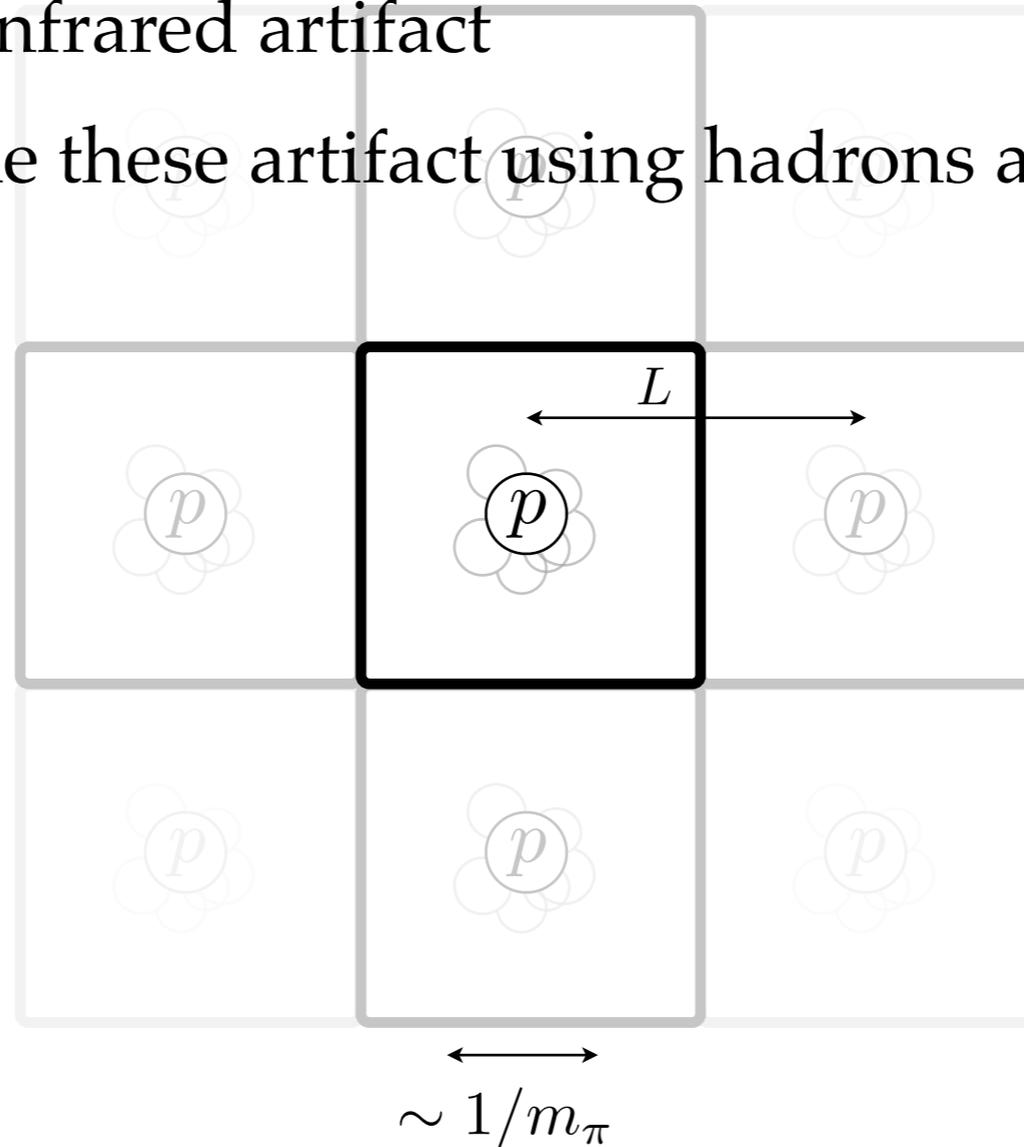
FINITE VOLUME: INFRARED LIMIT OF THE THEORY

- Finite-volume artifacts arise from the interactions with mirror images



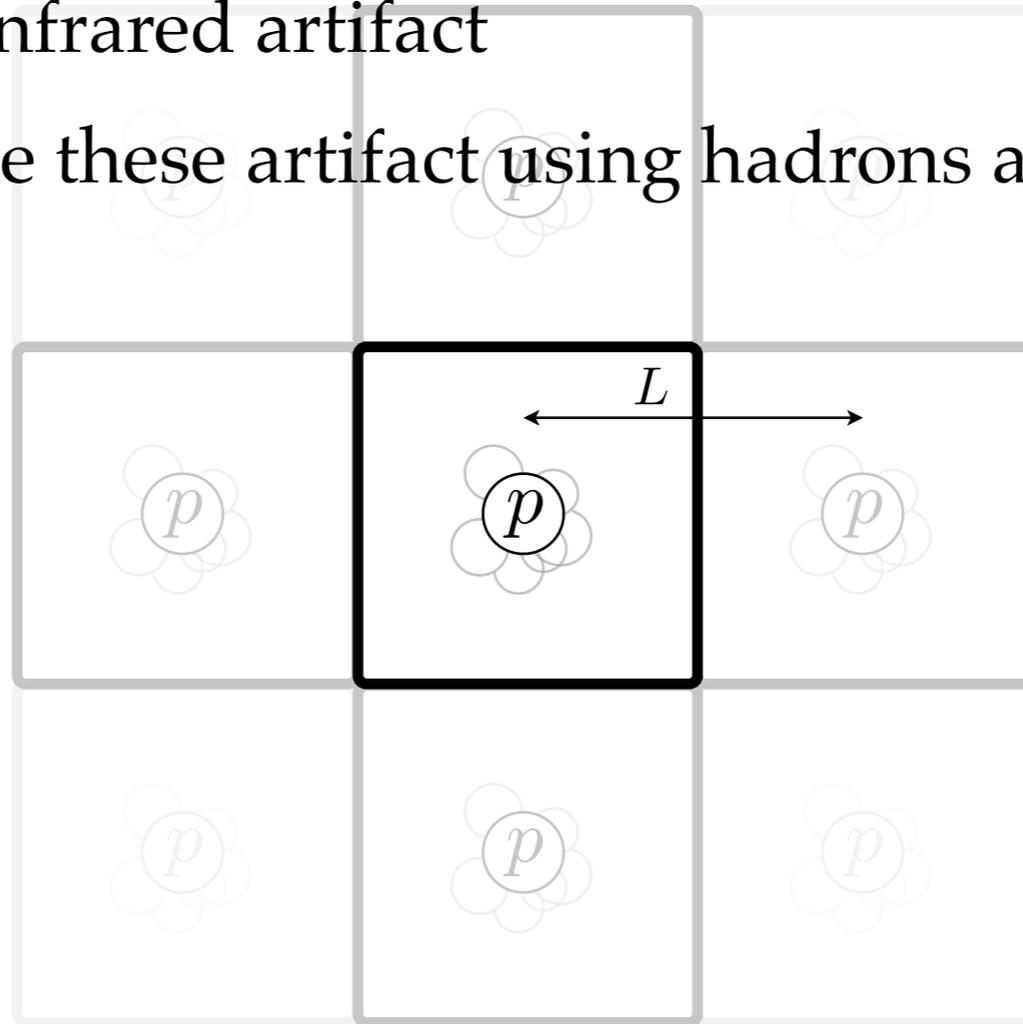
FINITE VOLUME: INFRARED LIMIT OF THE THEORY

- ❑ Finite-volume artifacts arise from the interactions with mirror images
- ❑ Assuming $L \gg$ size of the hadrons $\sim 1/m_\pi$
- ❑ This is a purely infrared artifact
- ❑ We can determine these artifact using hadrons as the degrees of freedom



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- ☑ interactions with mirror images: Yukawa

$$m_N(L) - m_N(\infty) \sim \langle N | \hat{V} | N \rangle_L \sim e^{-m_\pi L}$$

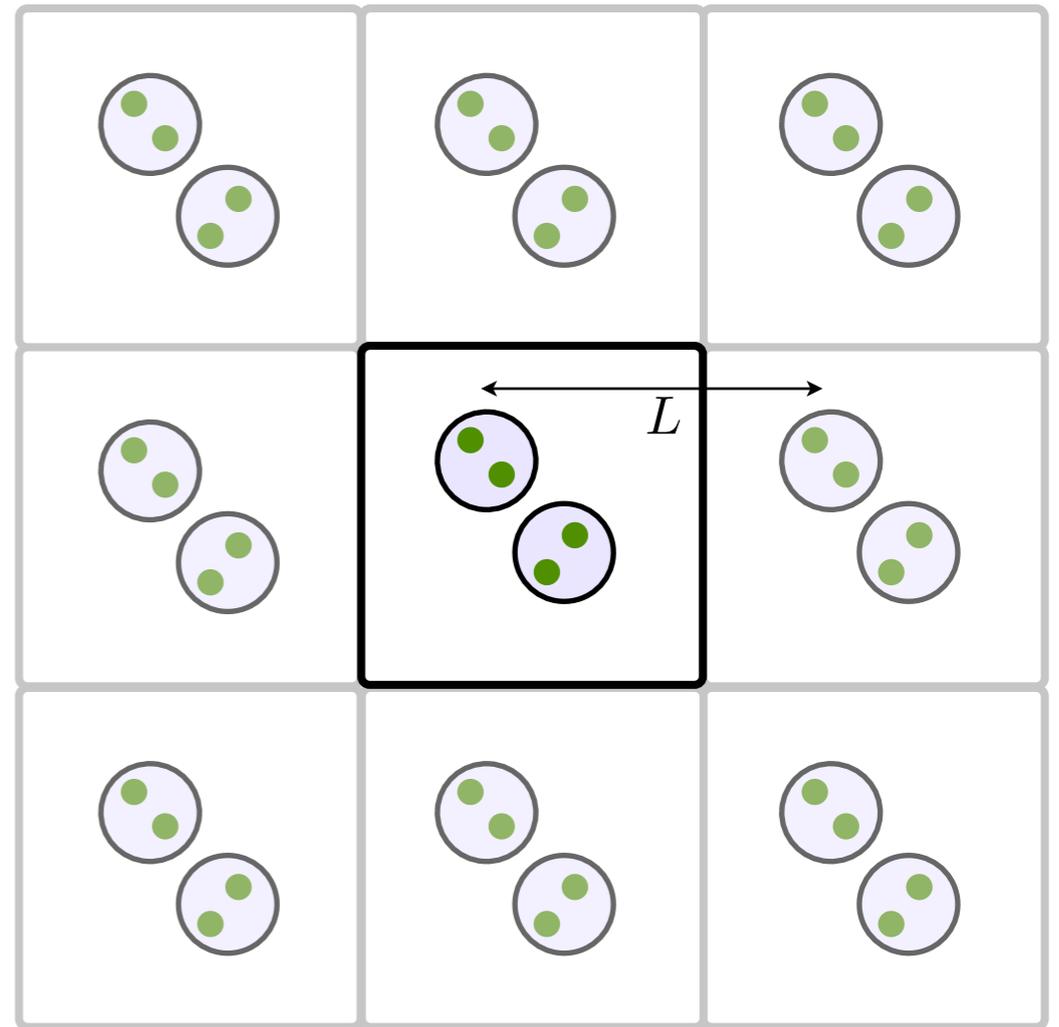
Lüscher (1985)

REAL TIME CALCULATIONS: FINITE VOLUME SYSTEMATICS

□ Finite volume L :

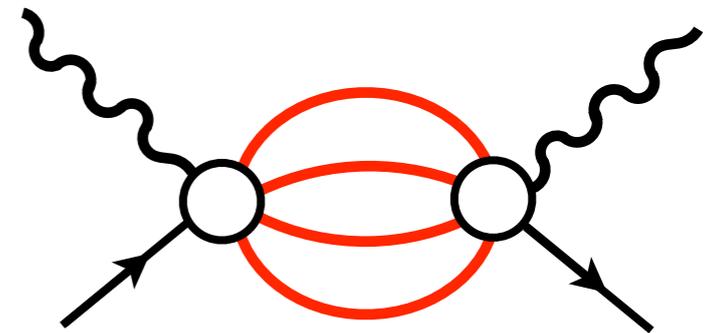
- Particles are never free
- No asymptotic states
- No scattering
- Multi-particle states:

$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^\#}$$



Goal: study inclusive processes at arbitrary kinematics

- Involve **multi-particle states**
- Finite volume: **expected to be dominant**



COMPTON SCATTERING IN A FINITE BOX

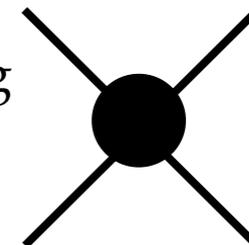
- ☑ Use known formalism as a diagnostic tool
- ☑ Finite-volume long range matrix elements:

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2) \frac{1}{F^{-1}(P, L) + \mathcal{M}(s)} \mathcal{H}'(s, Q_{if}^2)$$

Briceño et al,
PRD 101 (2020) 014509

geometric function
encoding FV effects

2-body scattering
amplitude:



☐ \mathcal{T}_L is purely real:

☐ Note: all terms in the right hand side are complex

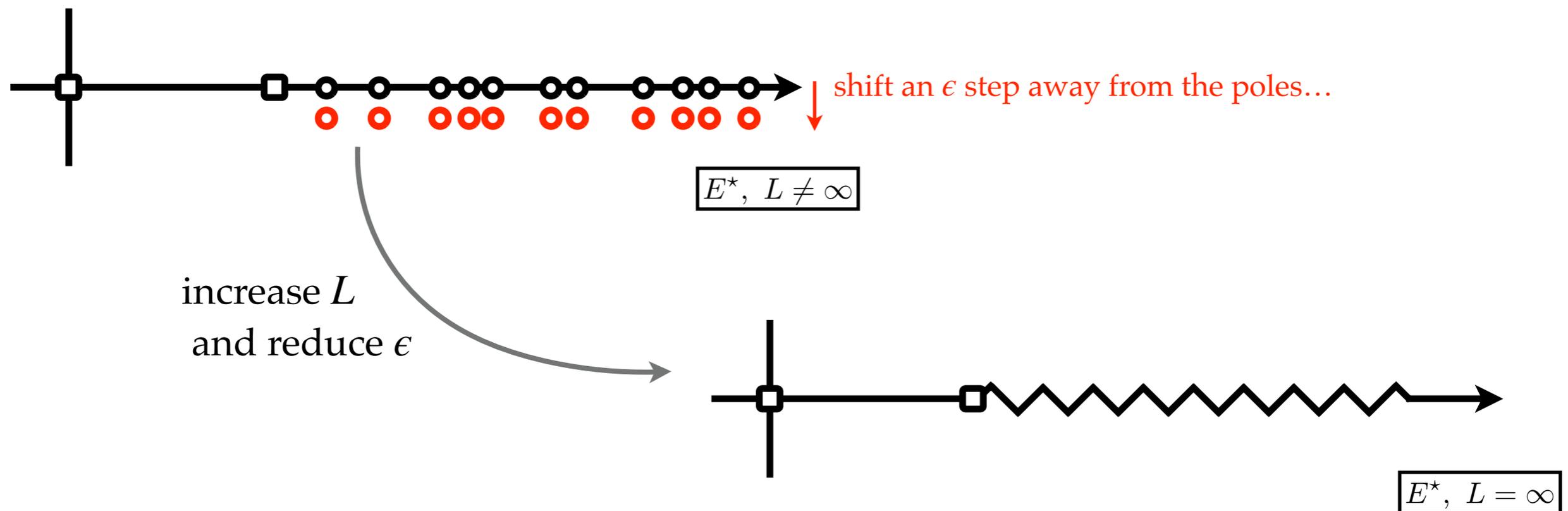
RECOVERING THE INFINITE VOLUME AMPLITUDE

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2) \frac{1}{F^{-1}(P, L) + \mathcal{M}(s)} \mathcal{H}'(s, Q_{if}^2)$$

□ Introduce an $i\epsilon$ by hand ($i\epsilon$ prescription)

accessible in a Quantum computer

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} dt e^{iq_0 t - \epsilon|t|} \langle n_f | T[\mathcal{J}_2(t) \mathcal{J}_1(0)] | n_i \rangle_L$$



RECOVERING THE INFINITE VOLUME AMPLITUDE

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- Then consider the double limit:

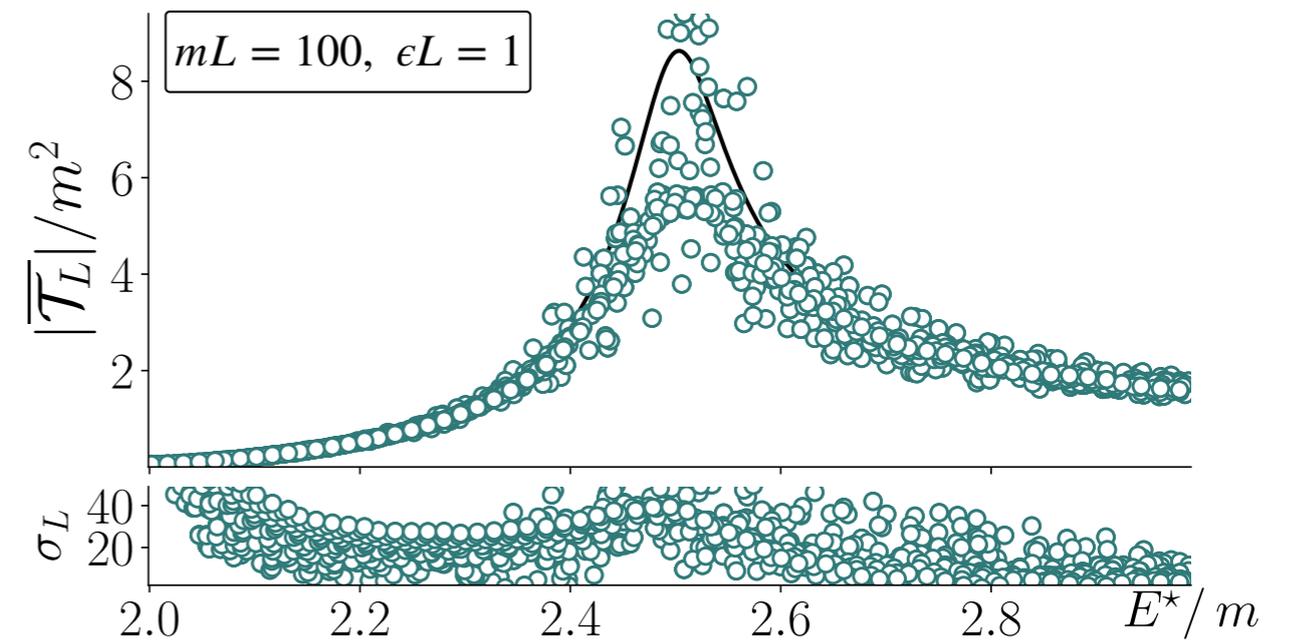
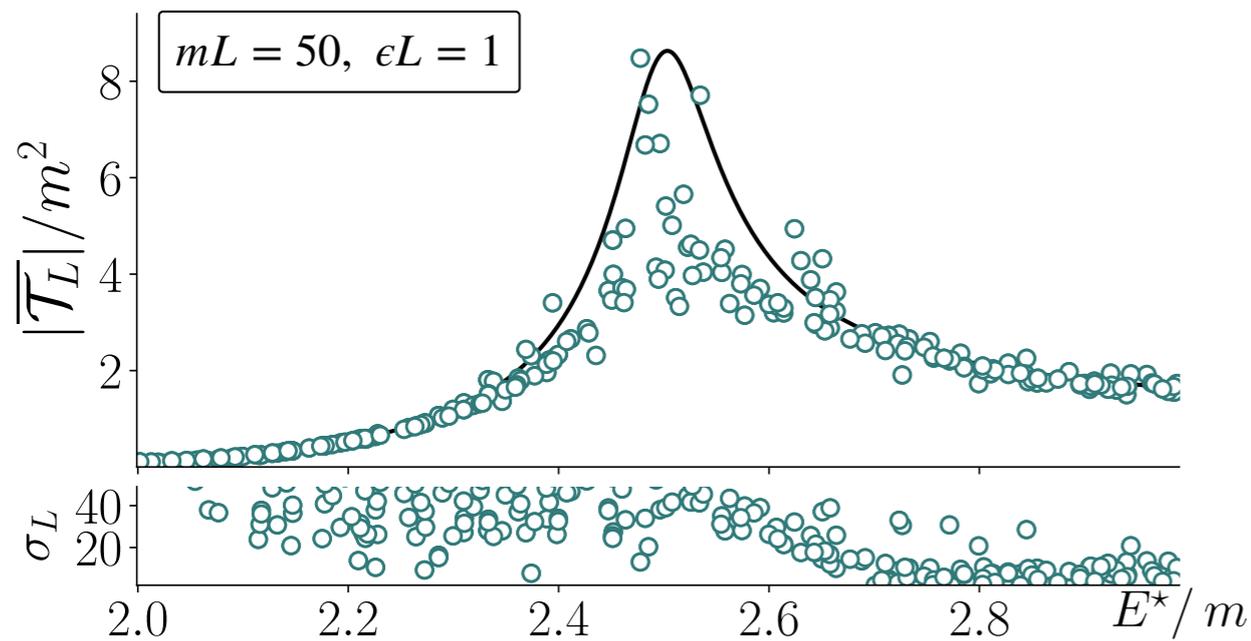
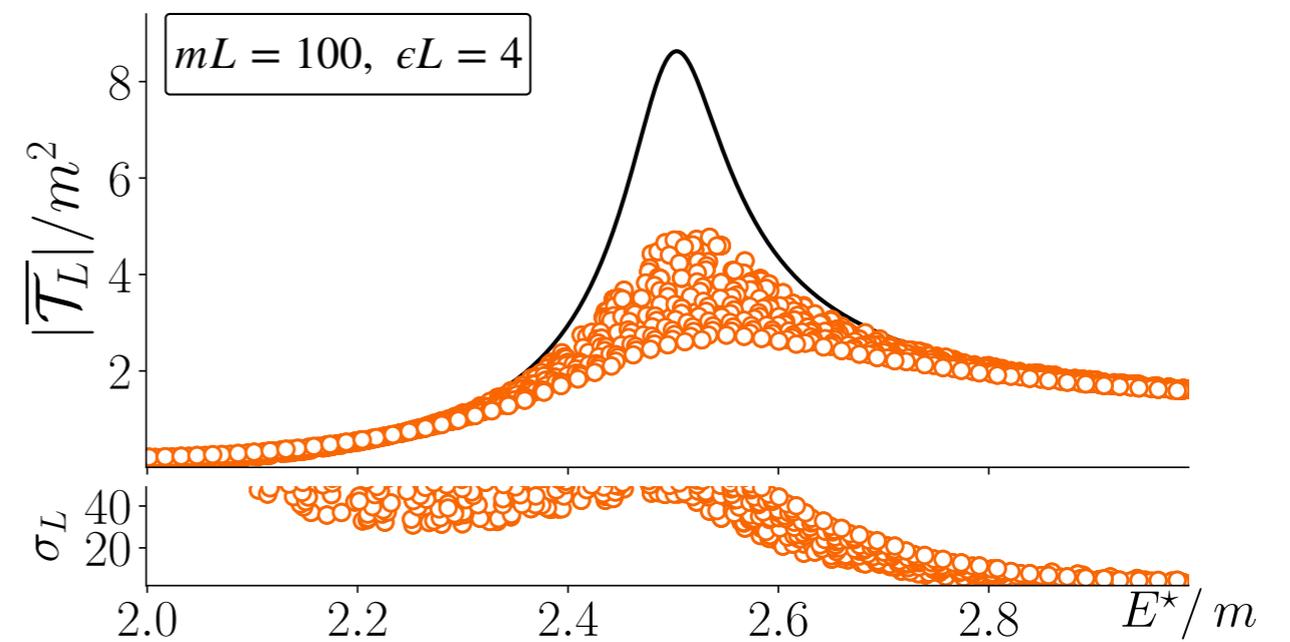
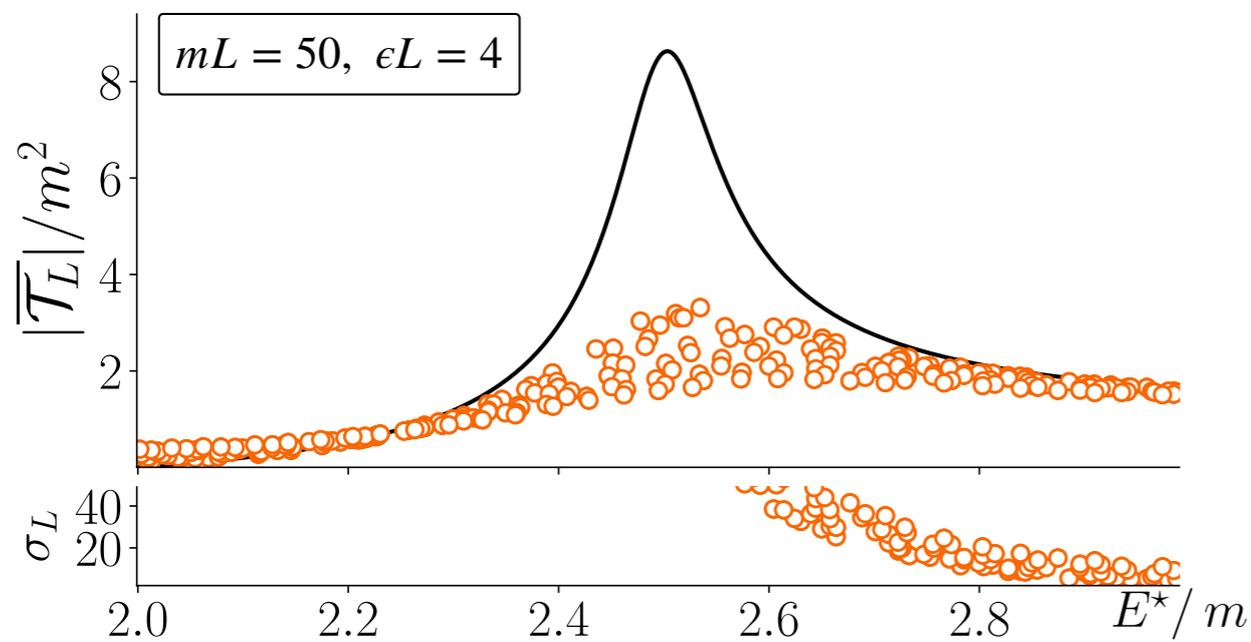
$$\mathcal{T} = \lim_{\epsilon \rightarrow 0} \lim_{L \rightarrow \infty} \mathcal{T}_L(\epsilon)$$

- Test this idea using a toy model

- What volume sizes do we need?
- How quickly do we recover the asymptotic behavior?
- Does this depend on the dynamics of the system?

COMPTON-LIKE AMPLITUDES

$$\mathcal{T}_L(p_f, q, p_i) = \mathcal{T}(E^*, Q^2, Q_{if}^2) - \mathcal{H}(E^*, Q^2) \frac{1}{F^{-1}(E^*, \mathbf{P}, L) + \mathcal{M}(E^*)} \mathcal{H}'(E^*, Q_{if}^2)$$



EXPLOITING SYMMETRY: BOOST AVERAGING

$$\mathcal{T}_L = \mathcal{T} - \mathcal{H} \frac{F(P, L)}{1 + \mathcal{M}F(P, L)} \mathcal{H}'$$

not Lorentz scalar Lorentz scalar

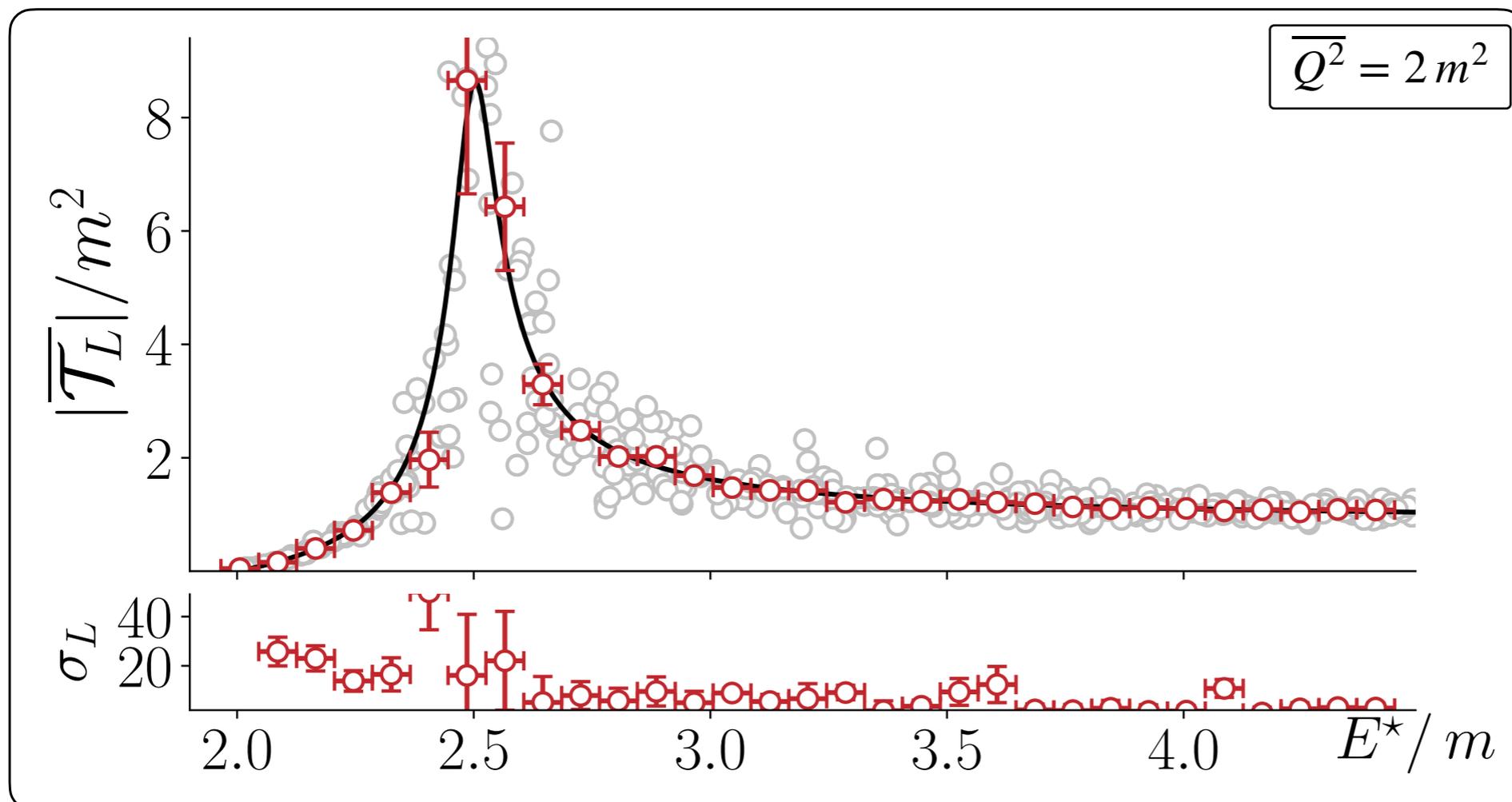
- ❑ Take advantage and exploit the symmetry:
 - ❑ The physical amplitudes only depend on Lorentz scalars.
 - ❑ Boost average

BOOST AVERAGING: NUMERICAL RESULTS

- ✓ Boost averaging: $mL = [20,25,30]$ with $d \leq mL$
- ✓ Binning in virtualities and energies:

$$\left| \overline{Q^2} - Q^2 \right| < \Delta_{Q^2}, \quad \left| Q_{if}^2 - Q^2 \right| < \Delta_{Q^2} \text{ and } \left| \overline{E^*} - E^* \right| \leq \Delta_{E^*}$$

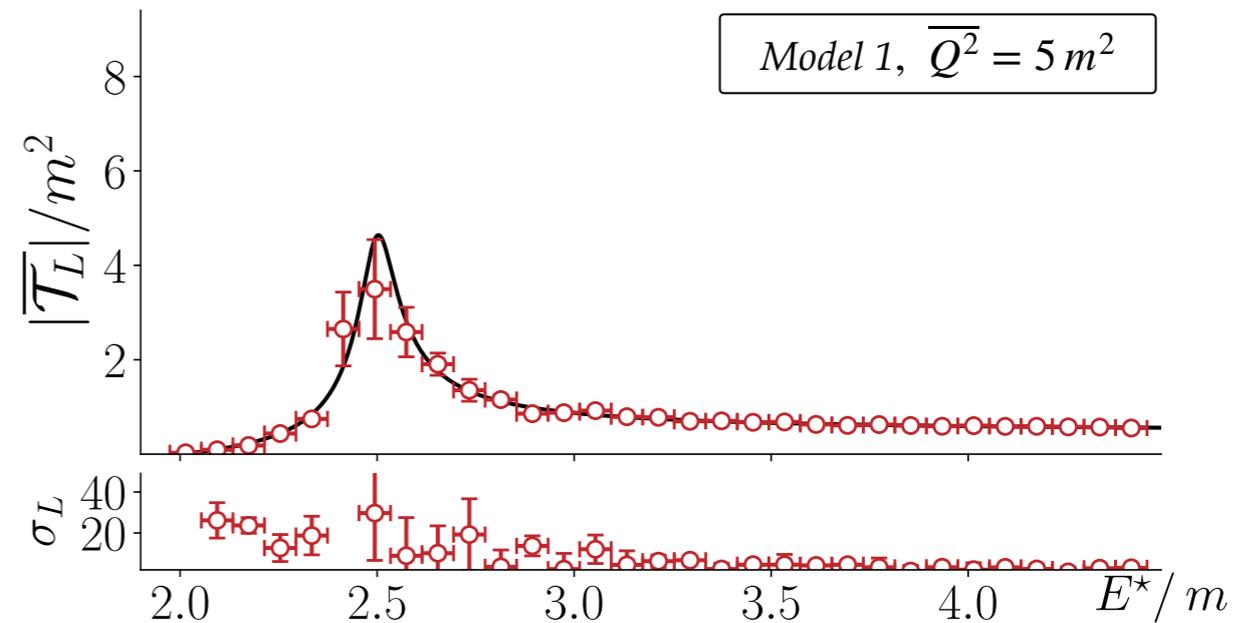
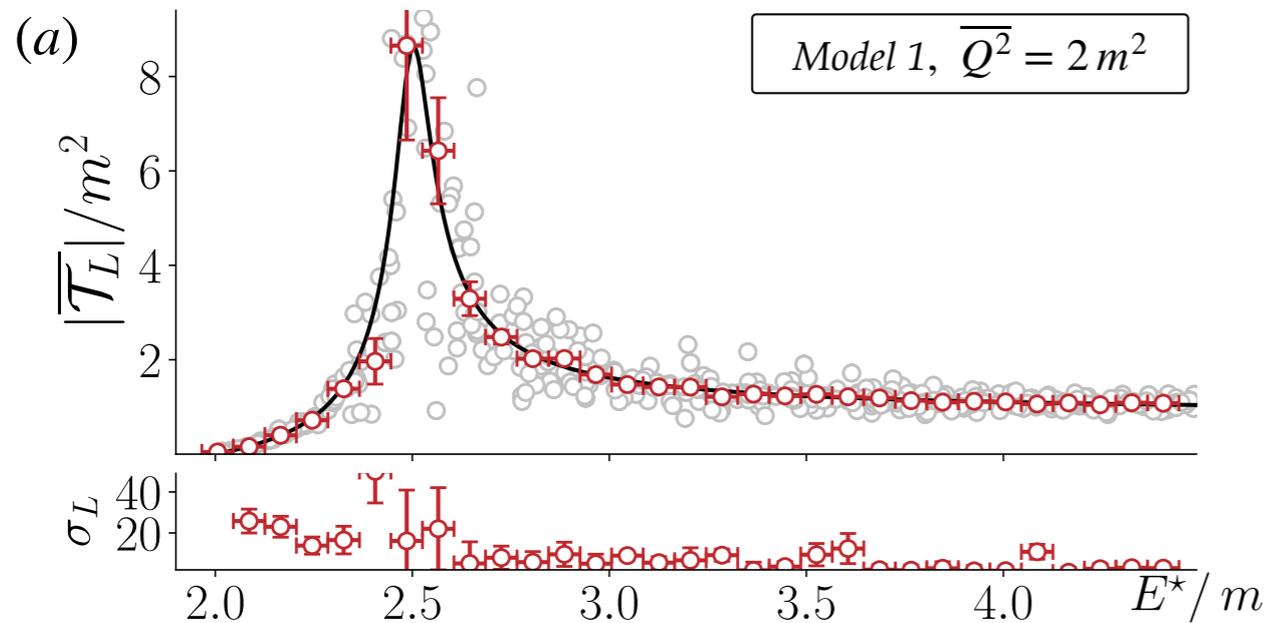
$$\overline{\mathcal{T}}_L(\overline{E^*}, \overline{Q^2}) = \frac{1}{\mathcal{N}} \sum_{L, \epsilon} \sum_{\{\mathbf{q}, \mathbf{p}_f, \mathbf{p}_i, \omega\} \in \Omega} \delta(\mathbf{q}, \mathbf{p}_f, \mathbf{p}_i, \omega | \overline{E^*}, \overline{Q^2}) \mathcal{T}_L(\mathbf{p}_f, \mathbf{q}, \mathbf{p}_i)$$



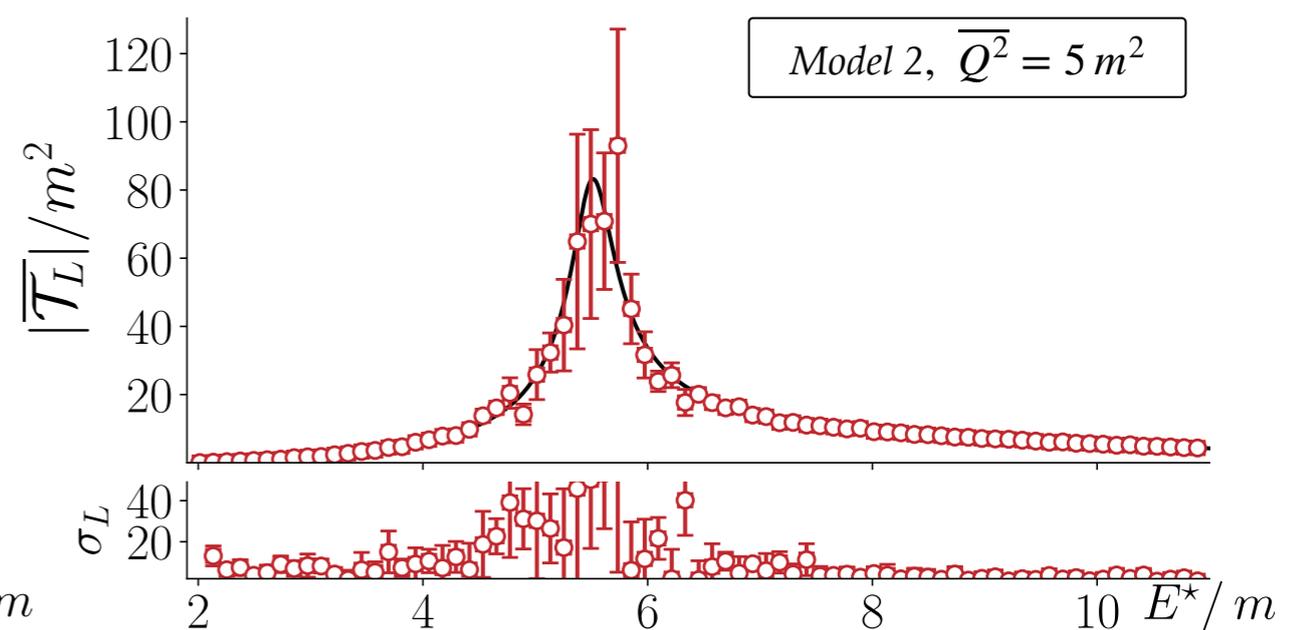
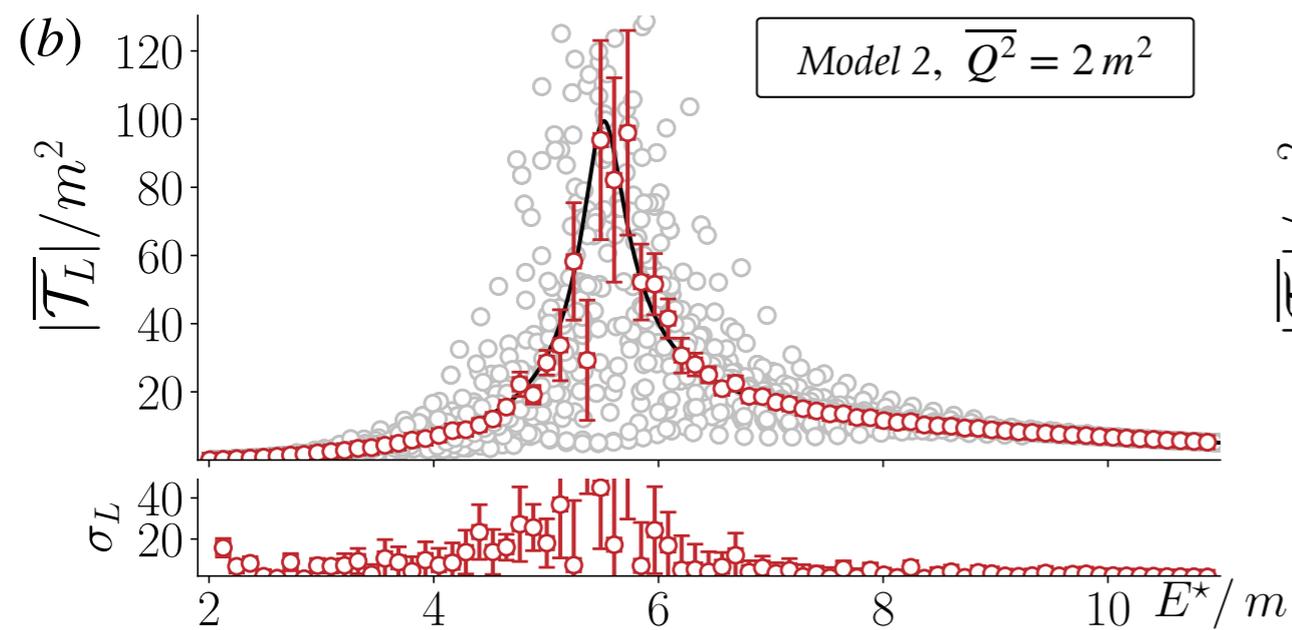
BOOST AVERAGING: NUMERICAL RESULTS

☑ Average over $mL = [20,25,30]$, with $d \leq mL$

☐ Model 1



☐ Model 2



TOY MODEL: COUPLED CHANNELS

$$\begin{aligned}
 i\mathcal{T} &= \text{Diagram with a black vertex and external lines } q, p_f, p_i, p_f + q - p_i \\
 &= \text{Diagram with a white vertex and mass } m_1 + \text{Diagram with a loop and mass } m_1 + \text{Diagram with two loops and mass } m_1 + \dots
 \end{aligned}$$

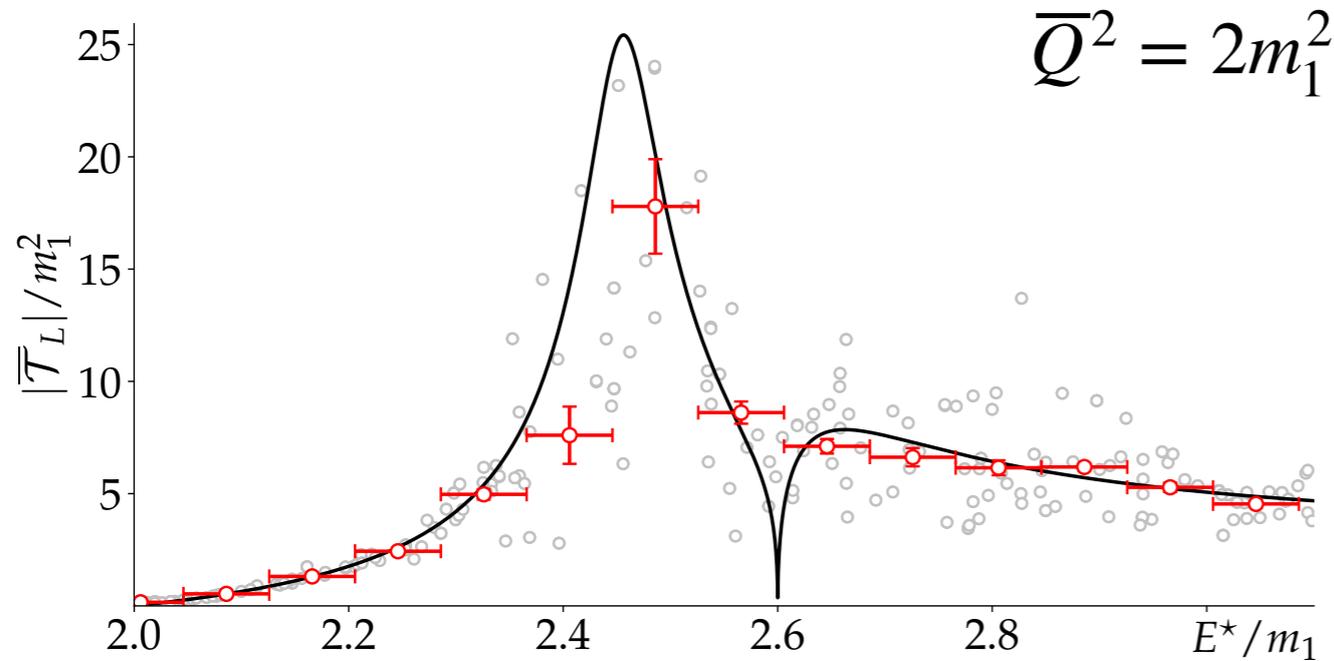
$m_1, m_2 \dots$ depending on the center of mass Energy E^*

Consider toy theory for energies: $2m_1 < E^* < 3m_1$

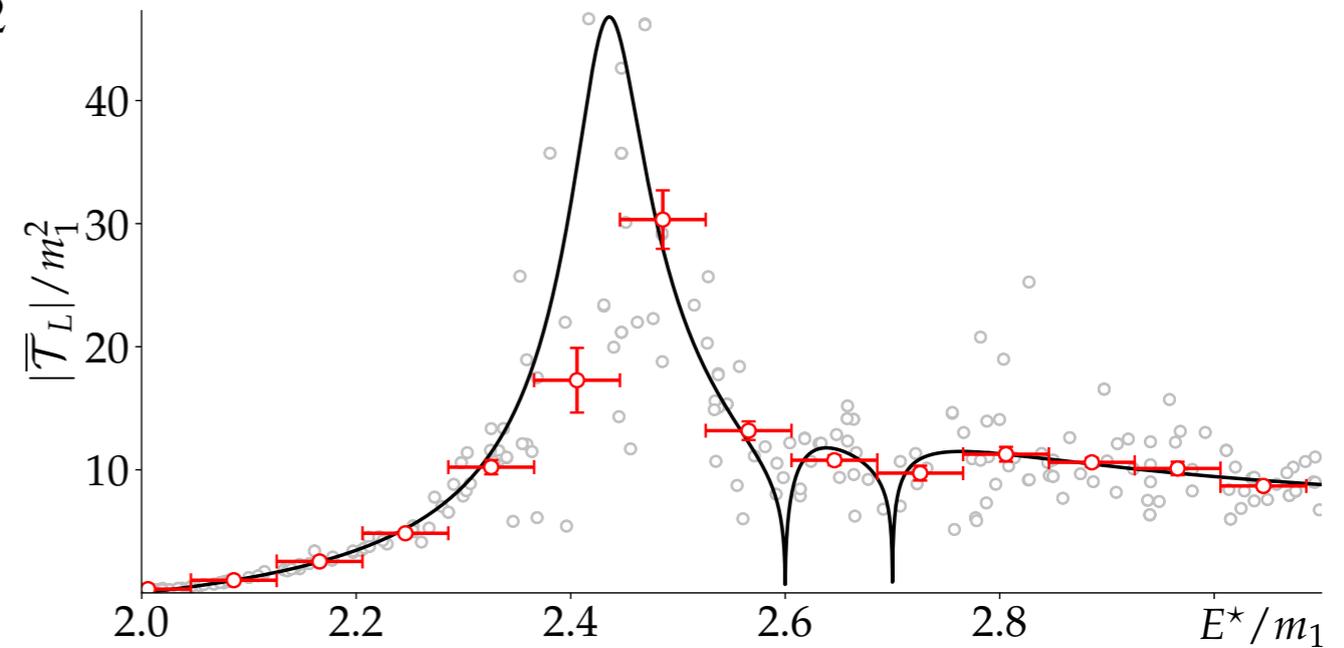
- Previously: only 1 channel of mass m_1 can go on-shell in the loop
- Lift this assumption:
 - 2 coupled channels can go on-shell: $2m_2 < E^* < 3m_1$
 - 3 or more coupled channels can go on-shell

MULTIPLE COUPLED CHANNELS

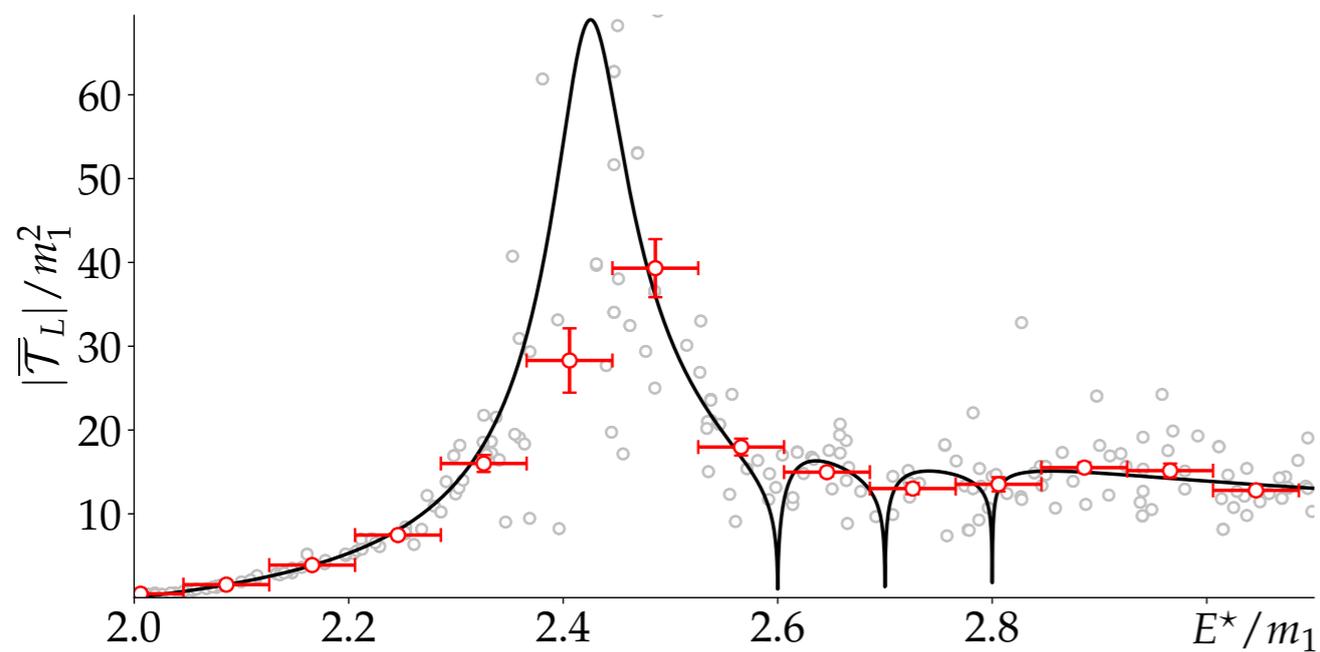
2 coupled channels



3 coupled channels



4 coupled channels



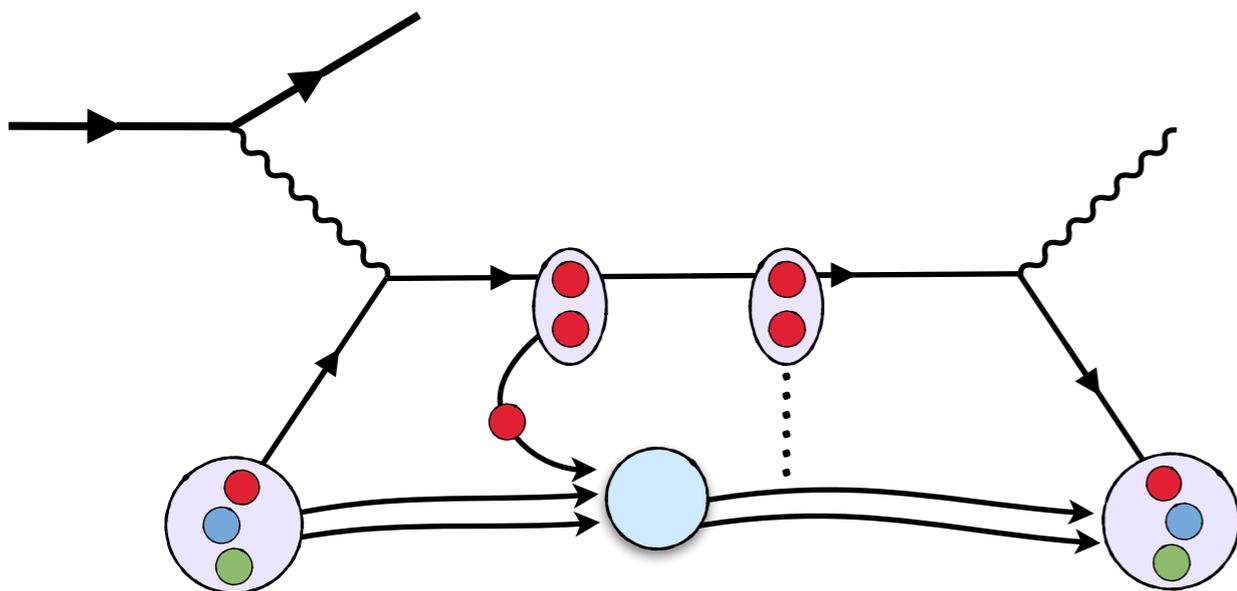
- Binning + averaging: still mostly recovers multi-channel case
- Optimal Binning becomes necessary

SUMMARY

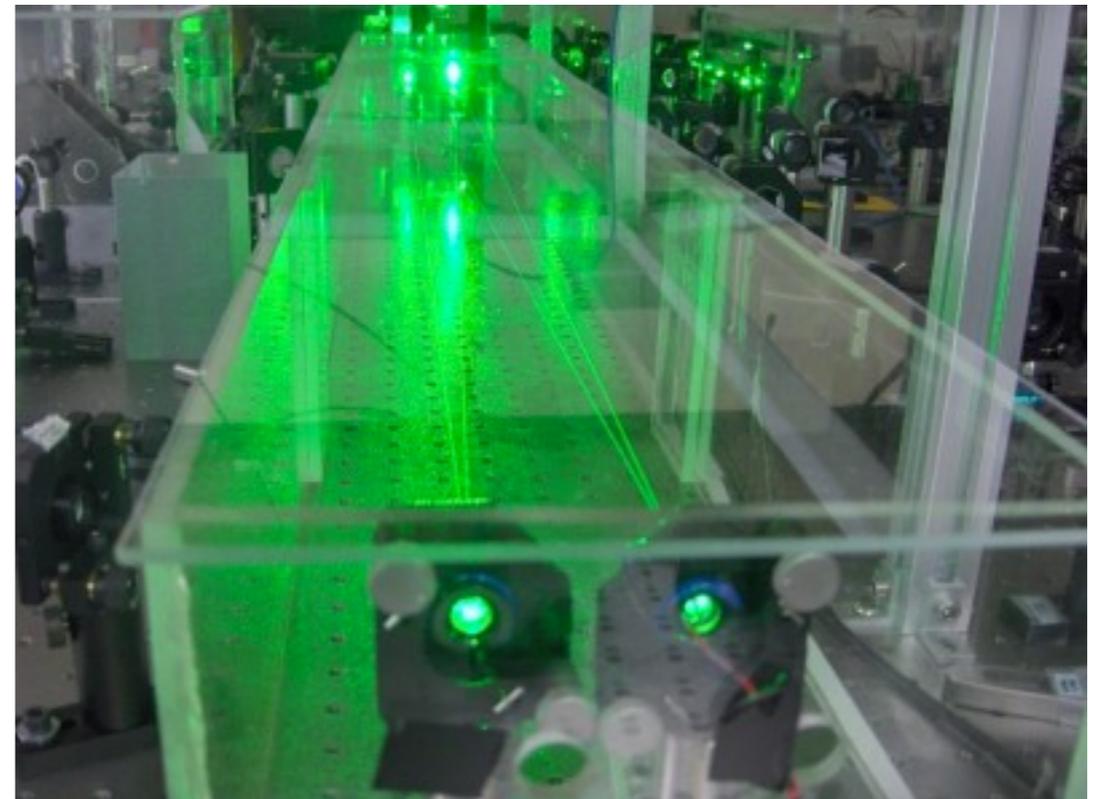
- Prospects for extracting scattering amplitudes from a Quantum Computer
 - Finite volume effects: large as expected
 - Naïve analysis: $mL \sim 100$
 - Solution: Exploit symmetry and binning (wave packets)
 - $mL \sim 20 - 30$
- Inclusive scattering amplitudes: in principle accessible
 - Simple tools needed
 - Tested this idea using a toy model

OUTLOOK

- Optimal choices of ϵ / boosting / binning?
- Inelastic processes?
- Test on toy theory [quantum simulation vs. lattice]
- 3+1D
-
- LDRD proposal: The EIC on a top table



Optics based QC system at UVA



THANK YOU!

TOY MODEL: COMPTON-LIKE AMPLITUDES

Scattering amplitude parametrization

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}, \quad \mathcal{K}(E^*) = m^2 q^{*2} \frac{g^2}{m_R^2 - E^{*2}}$$

All-order perturbation theory implies that the Compton-like amplitudes satisfy

$$\mathcal{T}(s, Q^2, Q_{if}^2) = w_2(s, Q^2, Q_{if}^2) + \mathcal{A}_{12}(s, Q^2) \mathcal{M}(s) \mathcal{A}'_{21}(s, Q_{if}^2)$$

Our parametrization:

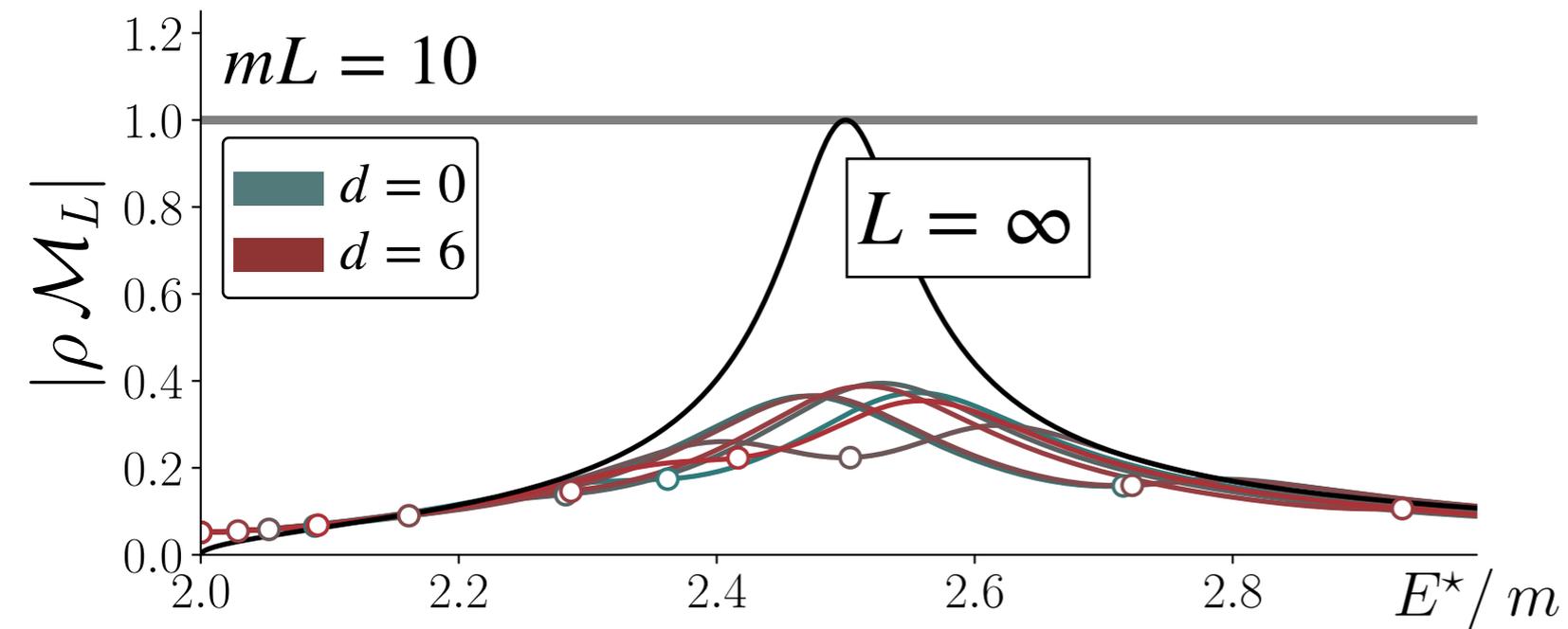
$$w_2(s, Q^2, Q_{if}^2) = 0, \quad \mathcal{A}_{12}(s, Q^2) = \mathcal{A}'_{21}(s, Q^2) = \frac{1}{1 + Q^2/M^2}$$

FINITE VOLUME TWO BODY AMPLITUDE

$$\mathcal{M}_L = \frac{1}{\mathcal{M}^{-1} + F(P, L)}$$

Finite-volume amplitude depends on the total momentum $P = 2\pi d/L$

Consider $\epsilon L = 1$:

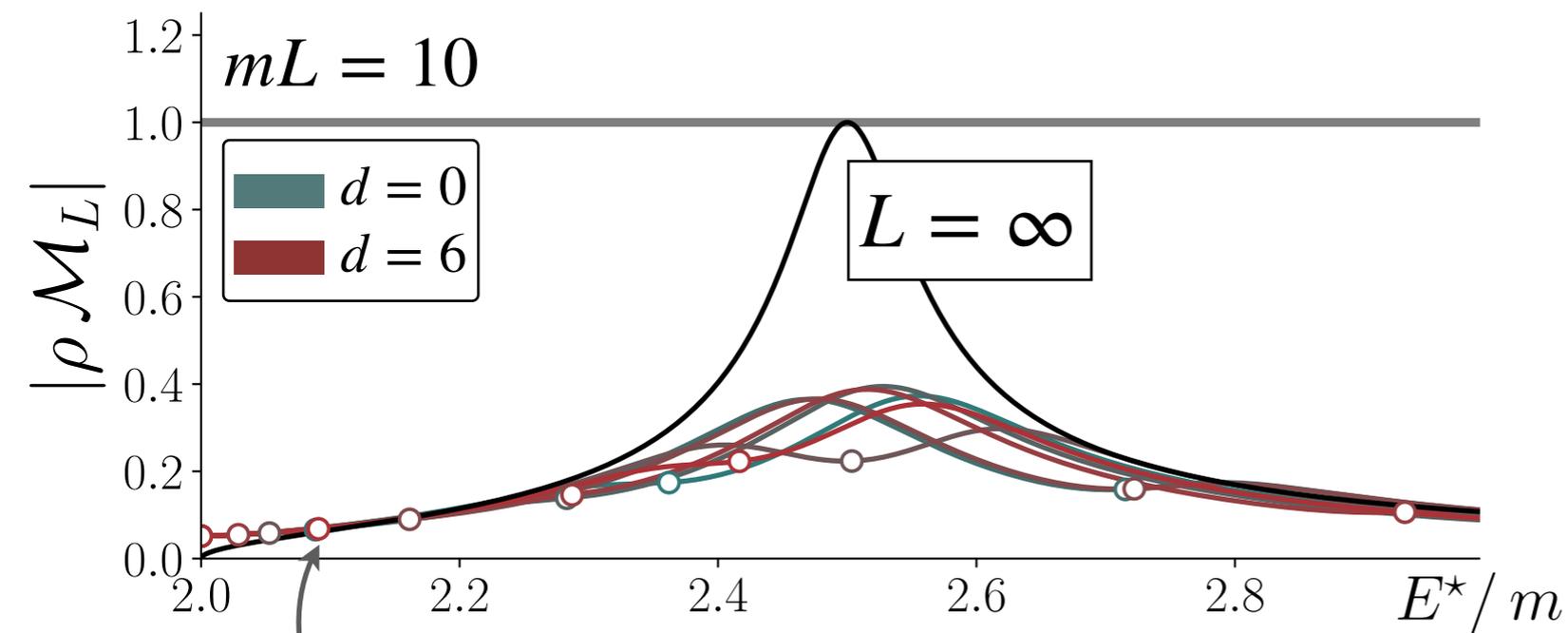


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free points:

$$E_n = \sqrt{(2\pi n/L)^2 + m^2} + \sqrt{(2\pi(n-d)/L)^2 + m^2}$$

To get this \mathcal{M}_L from a simulation:

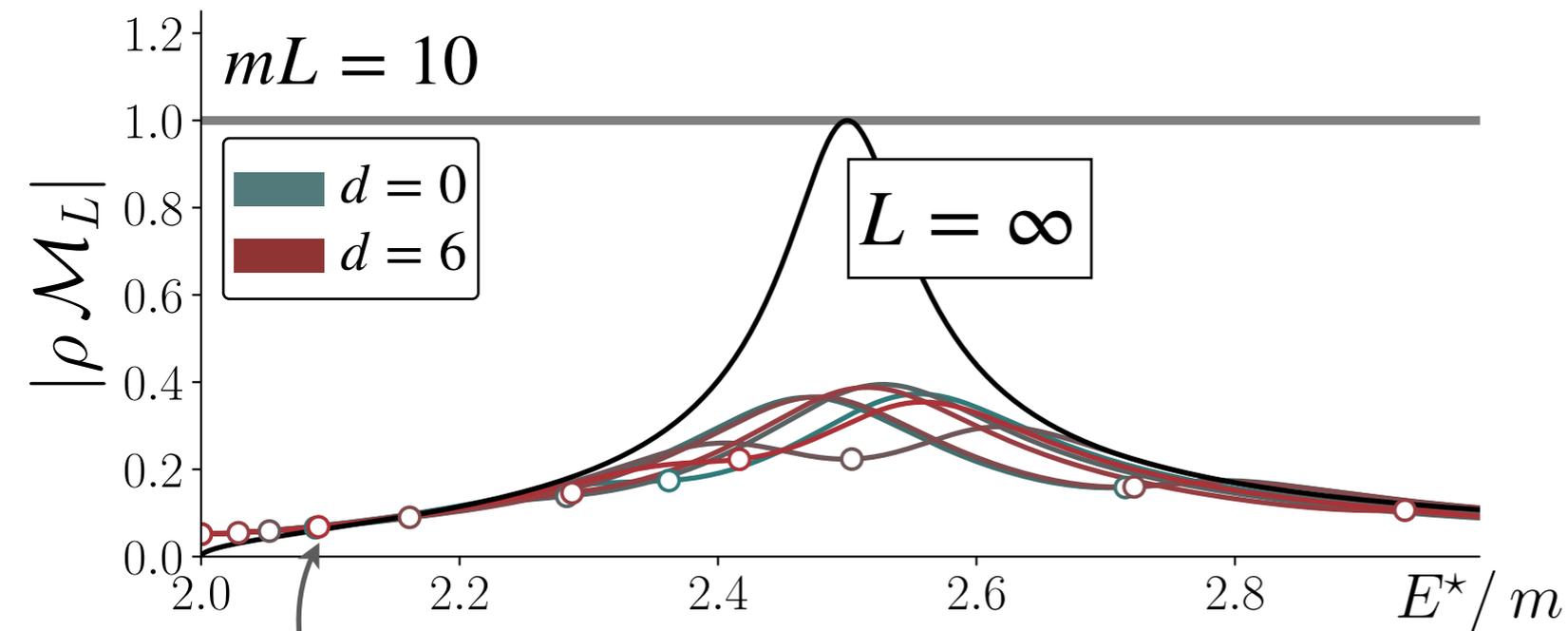
- 4pt Corr : $\langle p_f | \mathcal{O}(t) \mathcal{O}^\dagger(0) | p_i \rangle$
- LSZ reduction:
 - Amputate external legs
 - external legs on-shell
- Free points: only points accessible in a simulation

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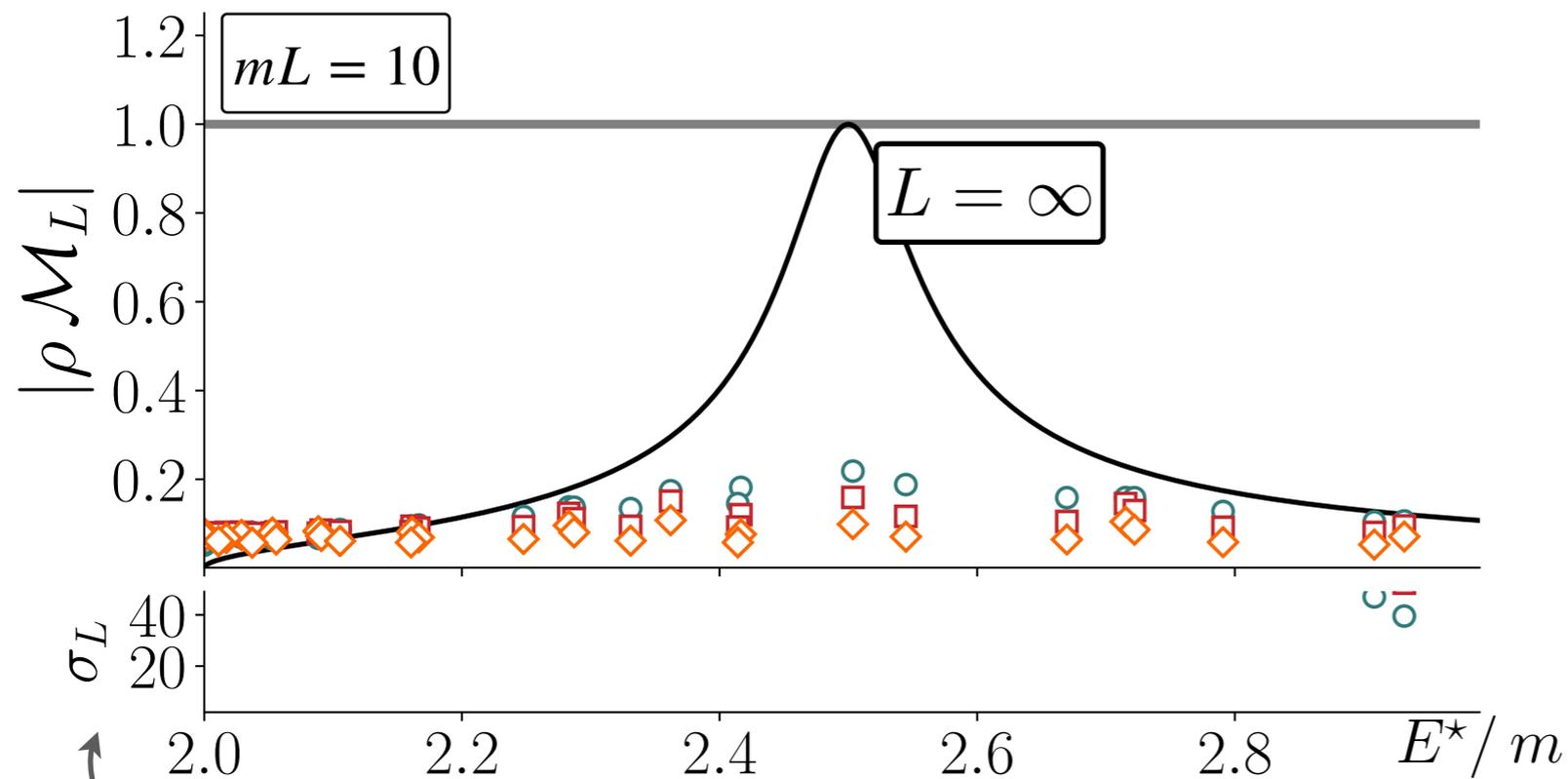
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TWO BODY SCATTERING AMPLITUDE: NUMERICAL RESULTS

$$\mathcal{M}_L = \frac{1}{\mathcal{M}^{-1} + F(P, L)}$$

Finite-volume amplitude depends on the total momentum $P = 2\pi d/L$

ϵ and volume dependence: $\epsilon L = 1$ $\epsilon L = 2$ $\epsilon L = 4$



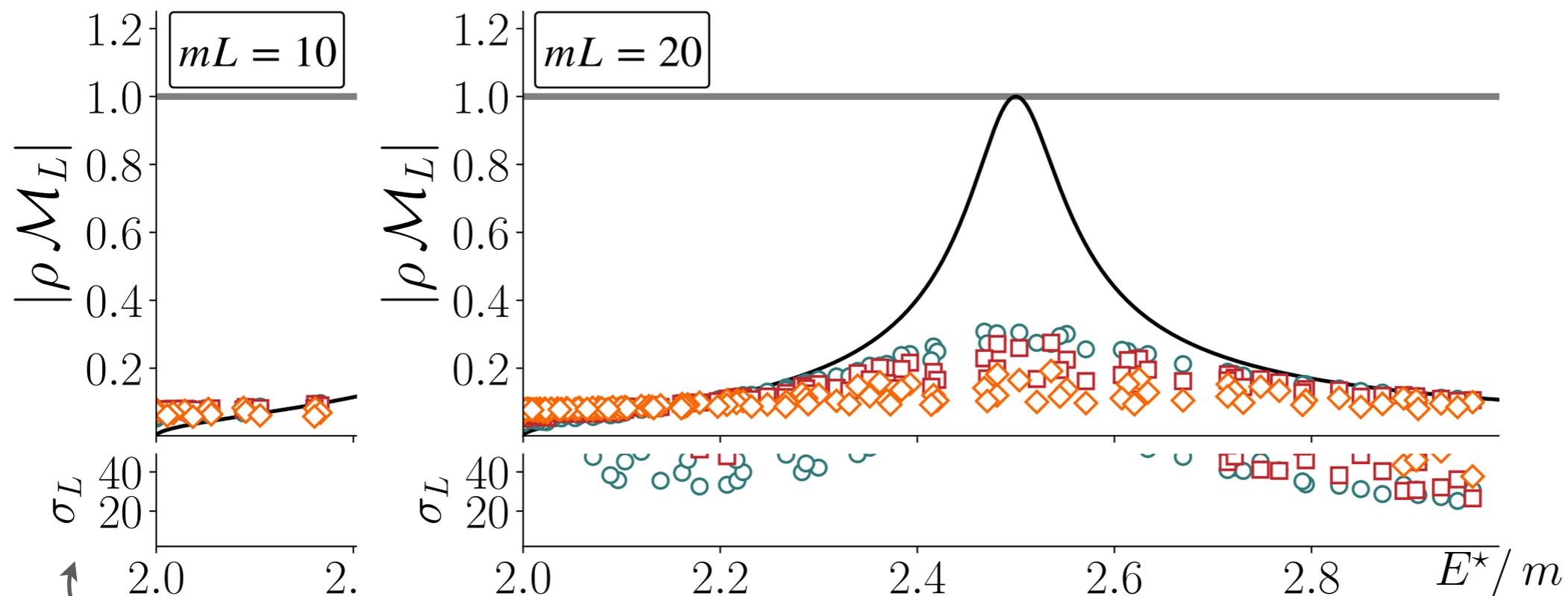
$$\sigma_L = 100 \times \left| \frac{\mathcal{M}_L - \mathcal{M}(E^*)}{\mathcal{M}(E^*)} \right|$$

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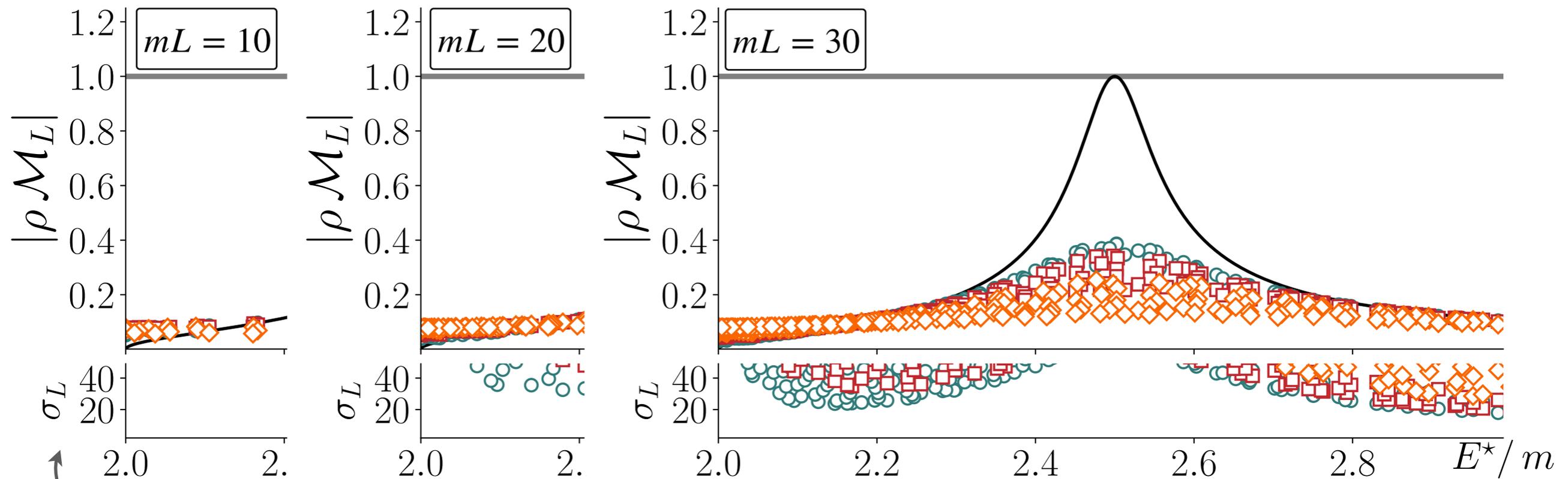
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EXPLOITING SYMMETRY: BOOST AVERAGING

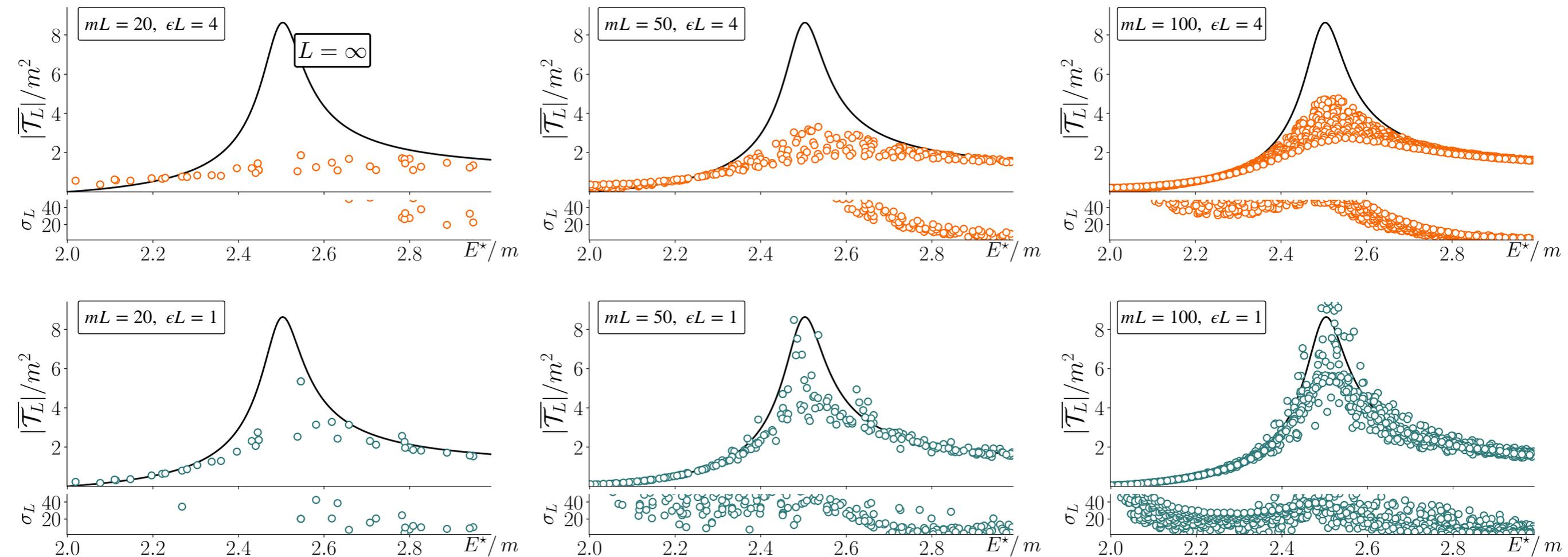
$$\mathcal{T}_L = \mathcal{T} - \mathcal{H} \frac{F(P, L)}{1 + \mathcal{M}F(P, L)} \mathcal{H}'$$

not Lorentz scalar Lorentz scalar

- ❑ Take advantage and exploit the symmetry:
 - ❑ The physical amplitudes only depend on Lorentz scalars.
 - ❑ Boost average

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$$\mathcal{T}_L(p_f, q, p_i) = \mathcal{T}(E^*, Q^2, Q_{if}^2) - \mathcal{H}(E^*, Q^2) \frac{1}{F^{-1}(E^*, \mathbf{P}, L) + \mathcal{M}(E^*)} \mathcal{H}'(E^*, Q_{if}^2)$$

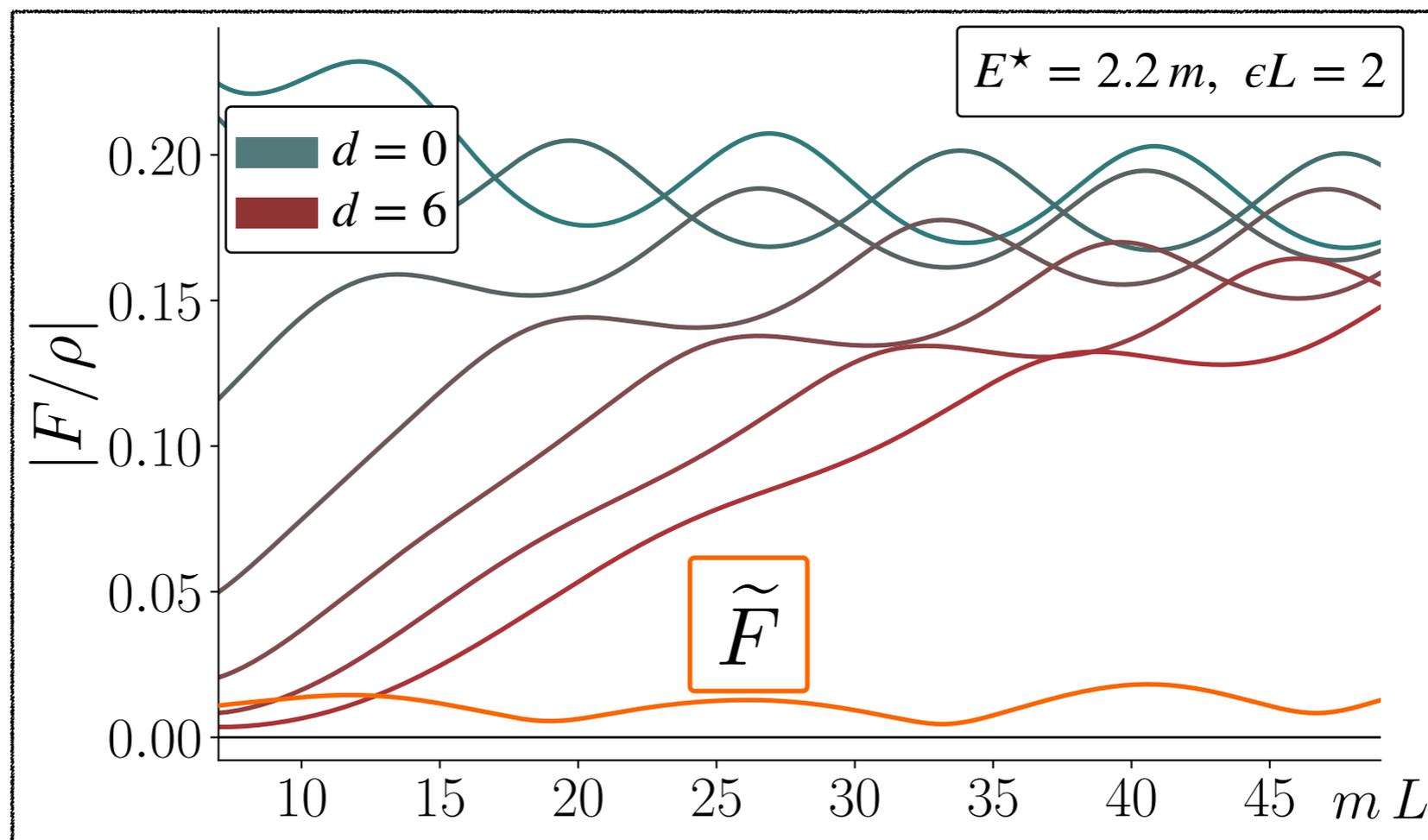


- $m_R = 2.5m$, $g = 2.5$, and $h(E^{*2}) = 0$
- $\bar{Q}^2 = 2m^2$, $\Delta_{Q^2} = 0.01m^2$
- Target virtualities: $|\bar{Q}^2 - Q^2| < \Delta_{Q^2}$ and $|Q_{if}^2 - Q^2| < \Delta_{Q^2}$

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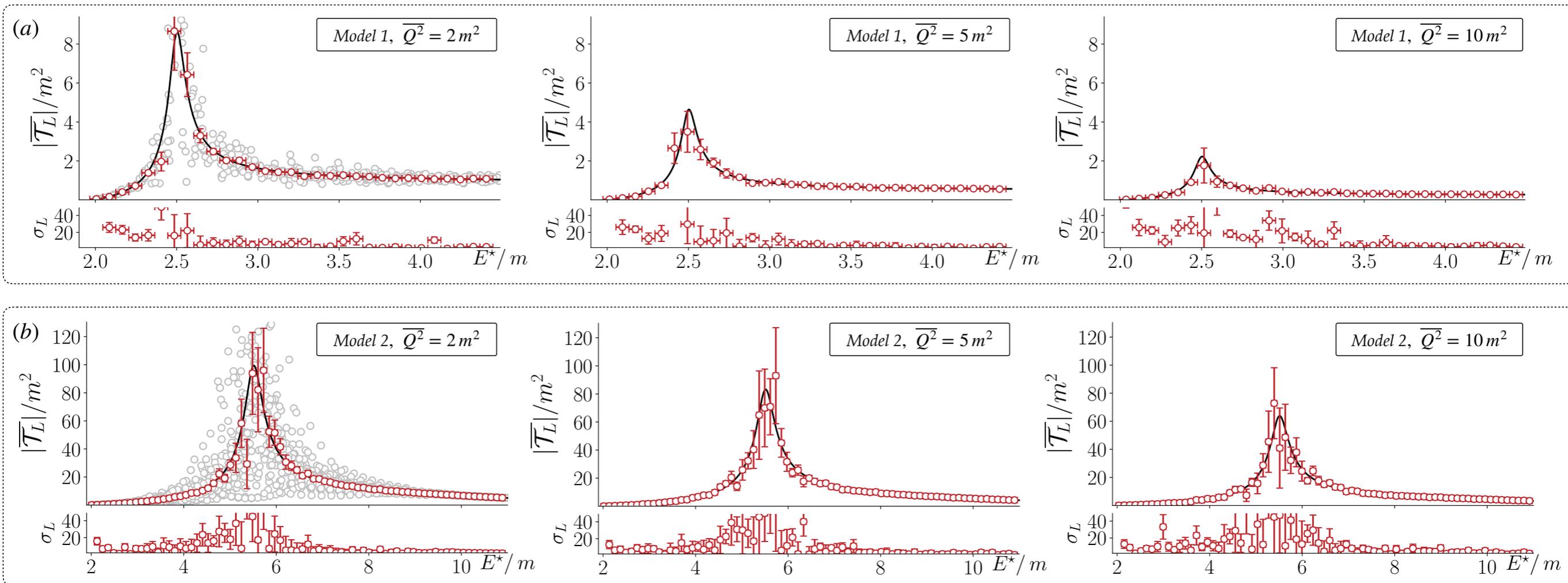
- $F(P, L)$ encodes FV effects: not a Lorentz scalar.
- Asymptotic behavior : $F \sim e^{-L\epsilon\alpha_0} (-1)^d \leftarrow$ alternating sign!
- Boost averaging: should reduce finite volume effects



BOOST AVERAGING: NUMERICAL RESULTS

☑ Average over $mL = [20,25,30]$, with $d \leq mL$

☑ $\Delta_{Q^2} = 0.05m^2$, $\Delta_{E^*} = 0.08m$, $\epsilon = \frac{1}{L(mL)^{1/2}}$



☐ Model 1: $m_R = 2.5m$, $g = 2.5$, $h(E^{*2}) = 0$

☐ Model 2: $m_R = 5.5m$, $g = 6$, $h(E^{*2}) = 0.2/m^2$

INFINITE/FINITE VOLUME ANALYTIC STRUCTURE

Infinite volume:

- Asymptotic states satisfy:

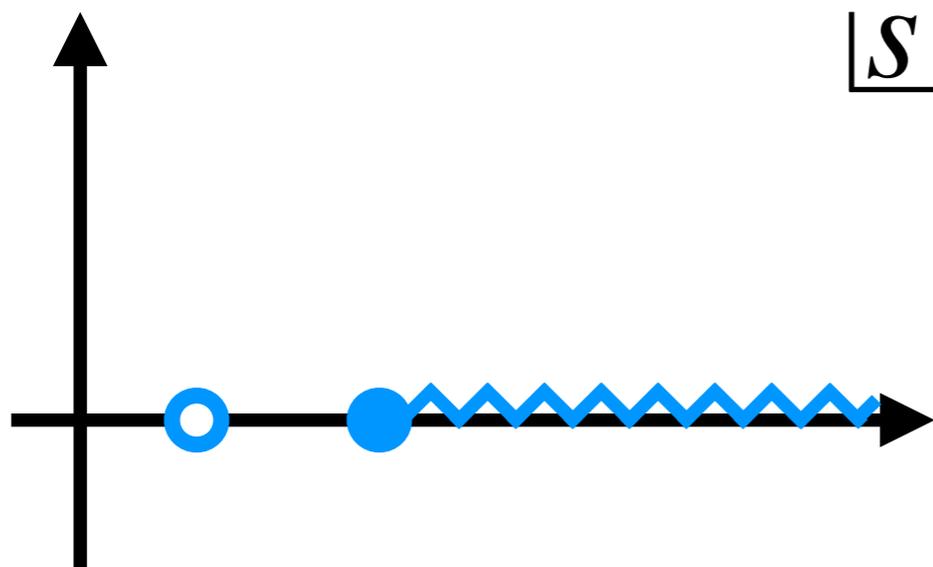
$$\hat{H}_{\infty,0} |p_1, p_2, \dots, p_n\rangle_0 = E(p_1, p_2, \dots, p_n) |p_1, p_2, \dots, p_n\rangle_0$$

- Continue spectrum

$$E(p_1, p_2, \dots, p_n) = \sum_{i=0}^n \sqrt{p_i^2 + m^2}$$

- Branch cut

Infinite-volume analytic structure



Finite volume:

- No asymptotic states:

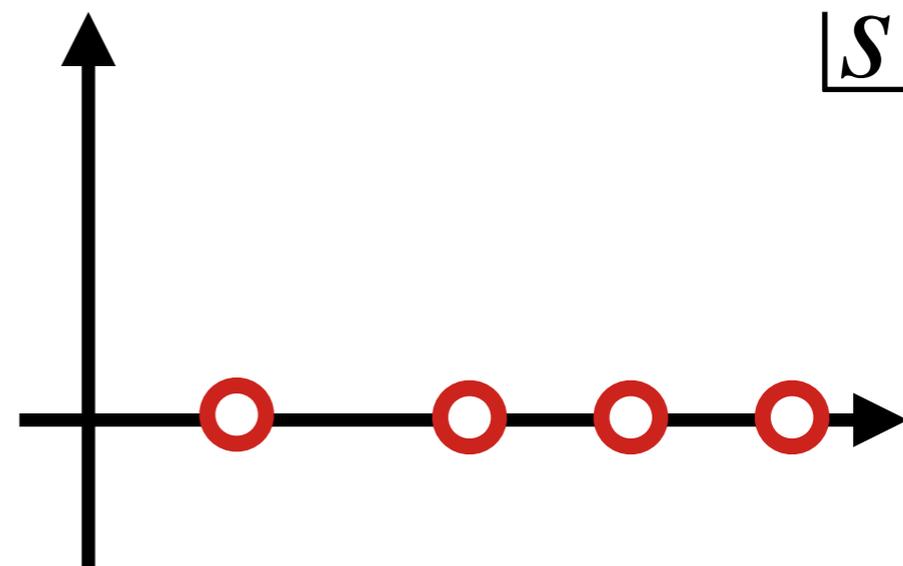
$$\hat{H}_L |n\rangle_L = E_n(L) |n\rangle_L$$

- Discretizes the spectrum

$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^\#}$$

- Eliminates the branch cut

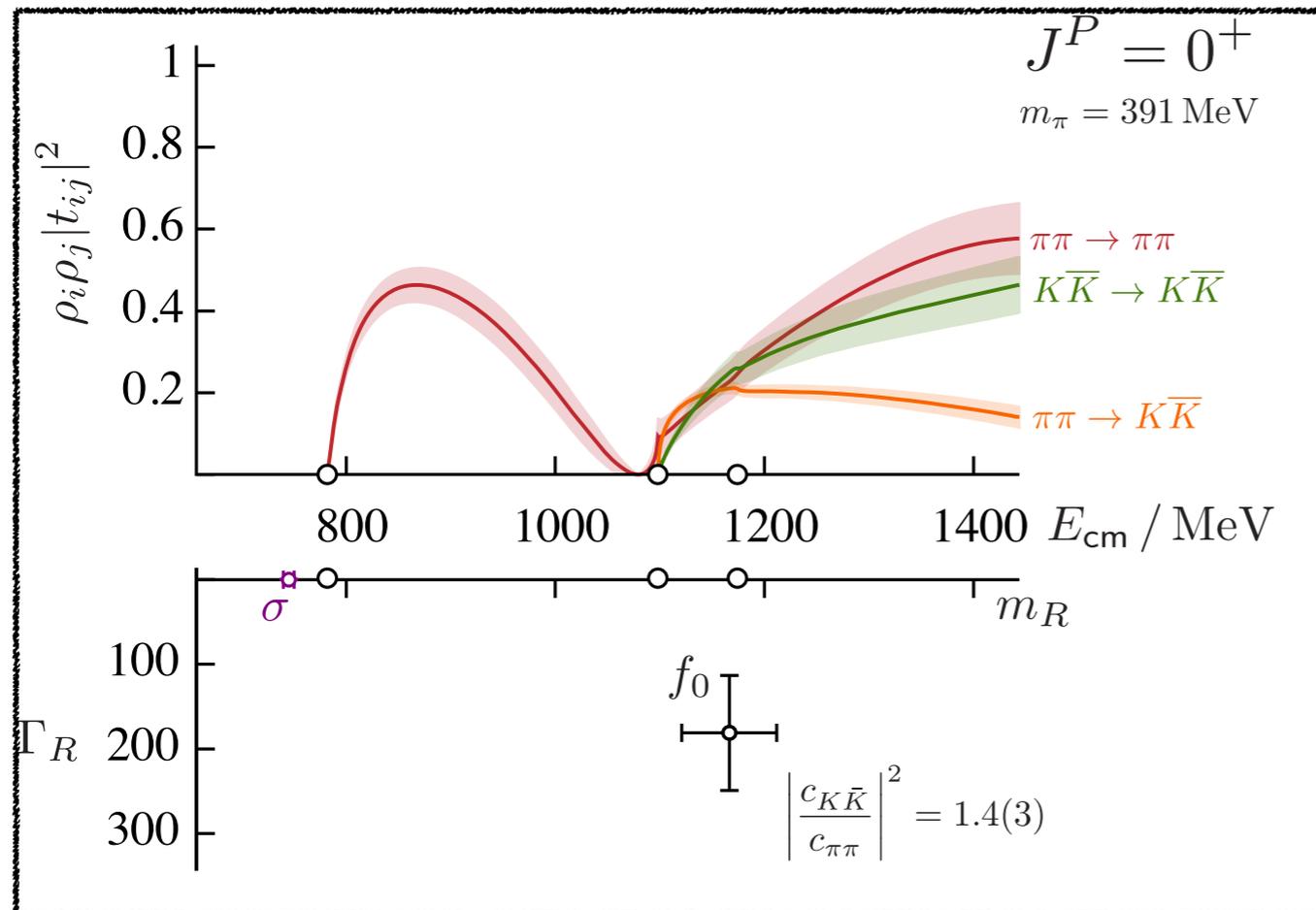
Finite-volume analytic structure



“SCATTERING” IN A FINITE VOLUME

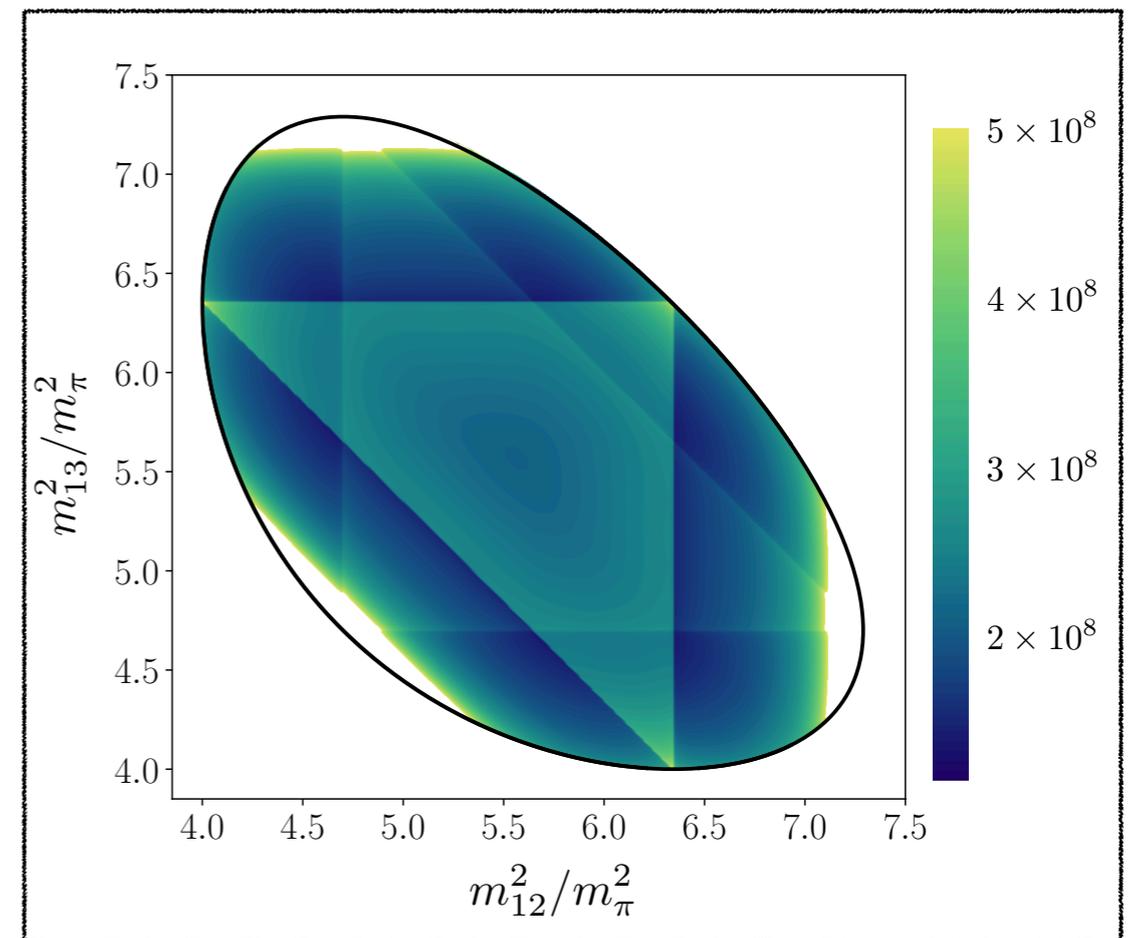
Finite volume: scattering is **not well** defined!

- Use finite-volume as a tool in LQCD (Lüscher-like techniques)
 - Relate the spectrum $E_n(L)$ to physical amplitudes
 - Exclusive processes



2 → 2 amplitude

HadSpec collaboration
PRD 97 (2019) 054513



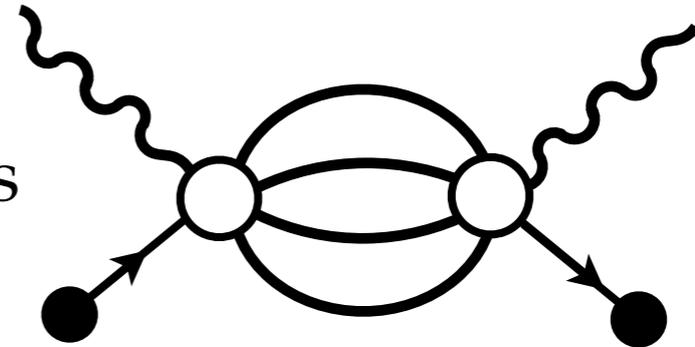
3 → 3 amplitude

HadSpec collaboration
PRL 126 (2021) 012001

“SCATTERING” IN A FINITE VOLUME

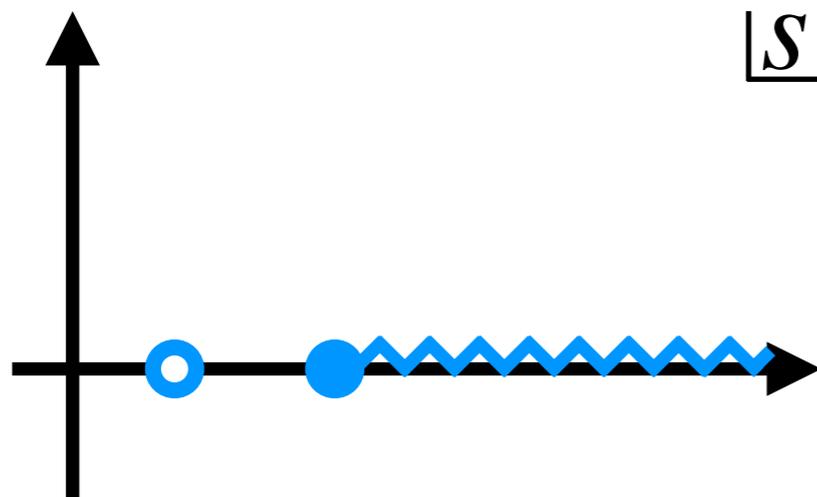
Finite volume: scattering is **not well** defined!

- Use finite-volume as a tool in LQCD (Luscher-like techniques)
 - Relate the spectrum $E_n(L)$ to physical amplitudes
 - Exclusive processes
 - Challenging for inclusive reactions



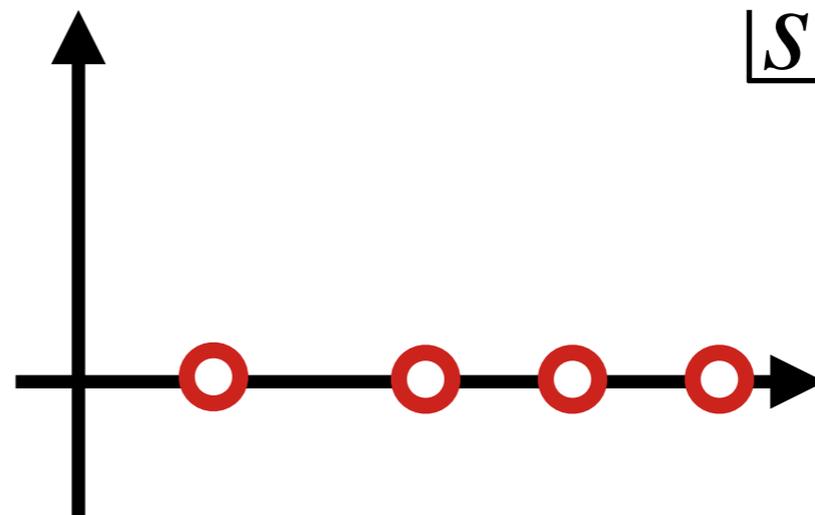
Infinite-volume amplitudes

- complex functions
- kinematic singularities



Finite-volume “amplitudes”

- real functions
- power-law finite-volume errors



“SCATTERING” IN A FINITE VOLUME

Finite volume: scattering is **not well** defined!

QC may be an useful tool to approach inclusive reactions

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} dt e^{iq_0 t - \epsilon|t|} \langle n_f | T[\mathcal{J}_2(t) \mathcal{J}_1(0)] | n_i \rangle_L$$

Direct extraction of the amplitude $\mathcal{T}_L(\epsilon)$

Extract $\langle n_f | \mathcal{J}_{2,M}(t) \mathcal{J}_1(0) | n_i \rangle_L \leftarrow$ accessible in a QC

Construct $|n_i\rangle$ and $|n_f\rangle$

Evaluate $\mathcal{J}(t)$ for different times: $e^{iHt} \mathcal{J}(0) e^{-iHt}$

$i\epsilon$ prescription: consider several ϵ values

consider different L

Finally consider the double limit:

$$\mathcal{T} = \lim_{\epsilon \rightarrow 0} \lim_{L \rightarrow \infty} \mathcal{T}_L(\epsilon)$$

TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:

$$\begin{aligned} i\mathcal{M} &= \text{[Contact vertex]} \\ &= \text{[Tree]} + \text{[Loop]} + \text{[2-loop]} + \dots \\ &= \text{[Tree]} + \text{[Loop]} + \text{[2-loop]} + \dots \end{aligned}$$

$i\mathcal{K}$ encodes dynamics

$i\mathcal{B}$

ρ

The diagram illustrates the expansion of the two-body scattering amplitude $i\mathcal{M}$ at all orders. It is shown as a sum of Feynman diagrams. The first row shows the contact vertex, which is a black circle with four external lines. The second row shows the tree-level diagram (a white circle with four external lines) and the first two loop diagrams (one-loop and two-loop). The third row shows the tree-level diagram and the first two loop diagrams with a dashed line representing a propagator. Labels include $i\mathcal{K}$, $i\mathcal{B}$, and ρ .

TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:

$$\begin{aligned}
 i\mathcal{M} &= \text{[Contact vertex]} \\
 &= \text{[Tree]} + \text{[Loop]} + \text{[2-loop]} + \dots \\
 &= \text{[Tree]} + \text{[Loop with } \rho \text{]} + \text{[2-loop with } \rho \text{]} + \dots
 \end{aligned}$$

E^* : center of mass energy

q^* : single particle momentum (c.m.)

$$q^* = \sqrt{\frac{E^{*2}}{4} - m^2}$$

$$\rho = \frac{1}{8E^*q^*} \sim \frac{1}{\sqrt{s - s_{\text{th}}}}$$

kinematic singularity

1+1D

TWO-BODY SCATTERING AMPLITUDE

Two-body scattering amplitude at all orders:

$$\begin{aligned}
 i\mathcal{M} &= \text{[Contact vertex]} \\
 &= \text{[Tree]} + \text{[Loop]} + \text{[2-loop]} + \dots \\
 &= \text{[Square]} + \text{[Loop with } \rho \text{]} + \text{[2-loop with } \rho \text{]} + \dots \\
 &= \frac{i}{\mathcal{K}^{-1} - i\rho}
 \end{aligned}$$

The diagrammatic expansion shows the scattering amplitude $i\mathcal{M}$ as a sum of terms. The first row shows the contact term (a black dot) and the first two rows show the expansion in terms of loop diagrams. The first row of diagrams includes a tree-level vertex (a white circle) and a one-loop diagram (a circle with an infinity symbol). The second row of diagrams includes a tree-level vertex (a white square) and a one-loop diagram (a circle with a vertical dashed line and a ρ label). The third row of diagrams includes a two-loop diagram (two circles with vertical dashed lines and ρ labels). The labels $i\mathcal{K}$ and $i\mathcal{B}$ are shown in boxes with arrows pointing to the tree-level vertices in the first and second rows, respectively.

TWO-BODY SCATTERING AMPLITUDE IN A BOX

Finite volume: integral in loops become sum over discrete momenta.

Two-body scattering amplitude in a finite volume:

$$\begin{aligned}
 i\mathcal{M}_L &= \text{Diagram with a central black dot and four external lines} \\
 &= \text{Diagram with a central white dot and four external lines} + \text{Diagram with a central white dot, two external lines, and a loop labeled } v \\
 &\quad + \text{Diagram with a central white dot, two external lines, and two loops labeled } v \\
 &\quad + \dots \\
 &= \text{Diagram with a central black dot and four external lines} + \text{Diagram with a central black dot, two external lines, and a loop with a vertical dashed line labeled } iF \\
 &\quad + \text{Diagram with a central black dot, two external lines, and two loops with vertical dashed lines labeled } iF \\
 &\quad + \dots \\
 &= \frac{i}{\mathcal{M}^{-1} + F}
 \end{aligned}$$

$i\mathcal{M}$ (points to the first diagram in the second row)
 $i\mathcal{B}$ (points to the top loop in the third diagram of the second row)
 iF (points to the vertical dashed lines in the diagrams of the third row)
 geometric function encoding FV effects (points to the iF labels)

TWO-BODY SCATTERING AMPLITUDE IN A BOX

Two body scattering amplitude analog:

$$\mathcal{M}_L = \frac{1}{\mathcal{M}^{-1} + F}$$

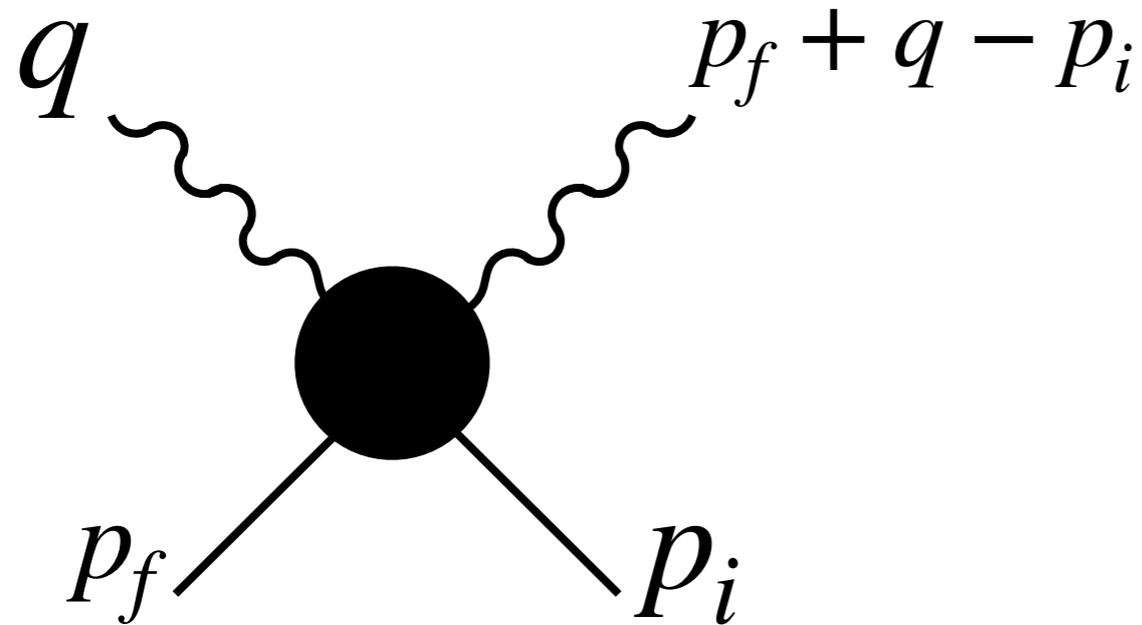
where the geometric function F is defined as:

$$F(E, \mathbf{P}, L) = \lim_{\epsilon \rightarrow 0^+} \frac{1}{2} \left[\frac{1}{L} \sum_{\mathbf{k}} - \int \frac{d\mathbf{k}}{2\pi} \right] \frac{1}{2\omega_{\mathbf{k}}} \frac{1}{(P - k)^2 - m^2 + i\epsilon}$$

the total momentum $\mathbf{P} = \frac{2\pi d}{L}$

- ❑ $F(P, L)$ encodes the finite volume effects!
- ❑ $F(P, L)$: not a Lorentz scalar.

COMPTON SCATTERING: KINEMATICS



Total momentum of the system:

$$P^\mu \equiv (q + p_f) = (E, \mathbf{P})$$

Invariants:

$$p_i^2 = p_f^2 = m^2$$

$$Q^2 = -q^2$$

$$Q_{if}^2 = -(p_f + q - p_i)^2$$

Center of mass frame:

Energy:

$$E^{*2} = P_\mu P^\mu = E^2 - \mathbf{P}^2 = s$$

Single particle momentum:

$$q^* = \sqrt{\frac{E^{*2}}{4} - m^2}$$

COMPTON-LIKE AMPLITUDES

Compton-like amplitudes [similar analytic structure compared to two body case]:

$$\begin{aligned}
 i\mathcal{T} &= \text{Diagram: A black vertex with four external lines. Top-left: wavy line labeled } q. \text{ Top-right: wavy line labeled } p_f + q - p_i. \text{ Bottom-left: straight line labeled } p_f. \text{ Bottom-right: straight line labeled } p_i. \\
 &= \text{Diagram: A white vertex with four external lines. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \text{Diagram: A white vertex with a loop. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \text{Diagram: A white vertex with two loops. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \dots \\
 &= \text{Diagram: A white square vertex with four external lines. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \text{Diagram: A white square vertex with a loop. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \text{Diagram: A white square vertex with two loops. Top-left: wavy line. Top-right: wavy line. Bottom-left: straight line. Bottom-right: straight line.} + \dots
 \end{aligned}$$

$$\mathcal{T}(s, Q^2, Q_{if}^2) = w_2(s, Q^2, Q_{if}^2) + \mathcal{A}_{12}(s, Q^2) \mathcal{M}(s) \mathcal{A}'_{21}(s, Q_{if}^2)$$

real and "smooth" for
 $Q^2, Q_{if}^2 > 0$

COMPTON-LIKE AMPLITUDES IN A BOX

Finite volume: integral in loops become sum over discrete momenta.

Compton-like amplitudes in a finite volume:

$$i\mathcal{T}_L = \text{tree} + \text{loop}(v) + \text{loop}(v,v) + \dots$$

$$= \text{tree}(i\mathcal{T}) + \text{loop}(i\mathcal{H}) + \text{loop}(iF, iF) + \dots$$

geometric function
encoding FV effects

$$= i\mathcal{T} - i\mathcal{H} \frac{1}{F^{-1} + \mathcal{M}} \mathcal{H}'$$