Smart Lookup Tables of Compton Form Factors

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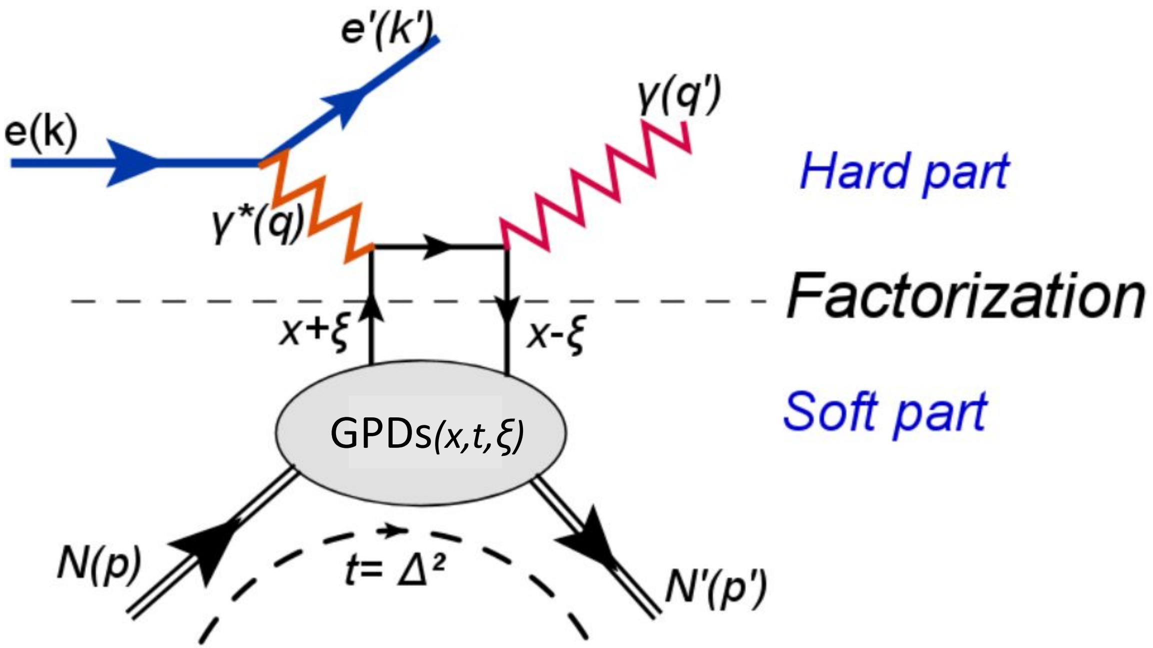
Deep Virtual Exclusive Scattering & Nuclear Femtography

- Deep Virtual Compton Scattering(DVCS) or Deep Exclusive Meson Production

$e + p \rightarrow e' + p' + \gamma$ *or* $e + p \rightarrow e' + p' + m$

- For DVCS $\frac{d\sigma}{dx_{Bj}dQ^2dtd\phi} = |T_{BH}|^2 + |T_{DVCS}|^2 + \mathcal{I}$
- The Bethe-Heitler amplitude is well known and the DVCS amplitude is parameterized by Generalized Parton Distributions (GPDs)
- Process involves a single hadron, providing a clean tool for constraining GPDs
- Fourier transform of GPDs w.r.t $\sqrt{t}$ at fixed-x and $\xi=0$ can give quark-distributions as a function of transverse distance



DVCS Amplitude

Compton Form Factors $\mathcal{F}$

- Convolutions over *x* of calculable coefficient functions and the GPDs (*$\mathcal{F}$=H,E, ..*)
- Integrating reduces the dimensionality from (x,ξ,t) $\longrightarrow$ (ξ,t) $\otimes$ Q^2 evolution
- DVCS amplitude is linear in the CFFs
- CFFs are in-principle directly accessed from experiment
- A more natural choice bridging between experimental data and theory compared to GPDs

$$\mathcal{F}_q(\xi,t) = \lim_{\epsilon \to 0} \int_{-1}^{1} F_q(x,\xi,t)\left[\frac{1}{x-\xi-i\epsilon} \pm \frac{1}{x+\xi-i\epsilon}\right]dx$$

$$\Im[\mathcal{F}_q] = i\pi[F(\xi,\xi,t) \pm F(-\xi,\xi,t)]$$

$$\Re[\mathcal{F}_q] = PV\int_{-1}^{1} F(x,\xi,t)\left[\frac{1}{x-\xi} \pm \frac{1}{x+\xi}\right]dx$$

Monte-Carlo Event Generators

- To predict DVES measurements, plan experiments, and analyze experimental data, extensive simulations require a large number of Monte-Carlo events
- Calculating the DVCS cross-section from GPD models is computationally expensive
- Cross-section codes are not always general for use in collider vs fixed-target collisions, or changing the GPD models

Project Goals

- Work with the ODU Center for Real-Time Computing(CRTC) to create lookup tables of adaptive tessellations of the CFFs, mesh of tetrahedra in 3D: (ξ, t, Q^2)
- Develop software tools to extract the CFFs through interpolation
- Develop a *flexible* code to calculate the DVCS cross-section from the CFF lookup tables
- Optimize tessellation to achieve desired lookup table interpolation precision

Data Generation

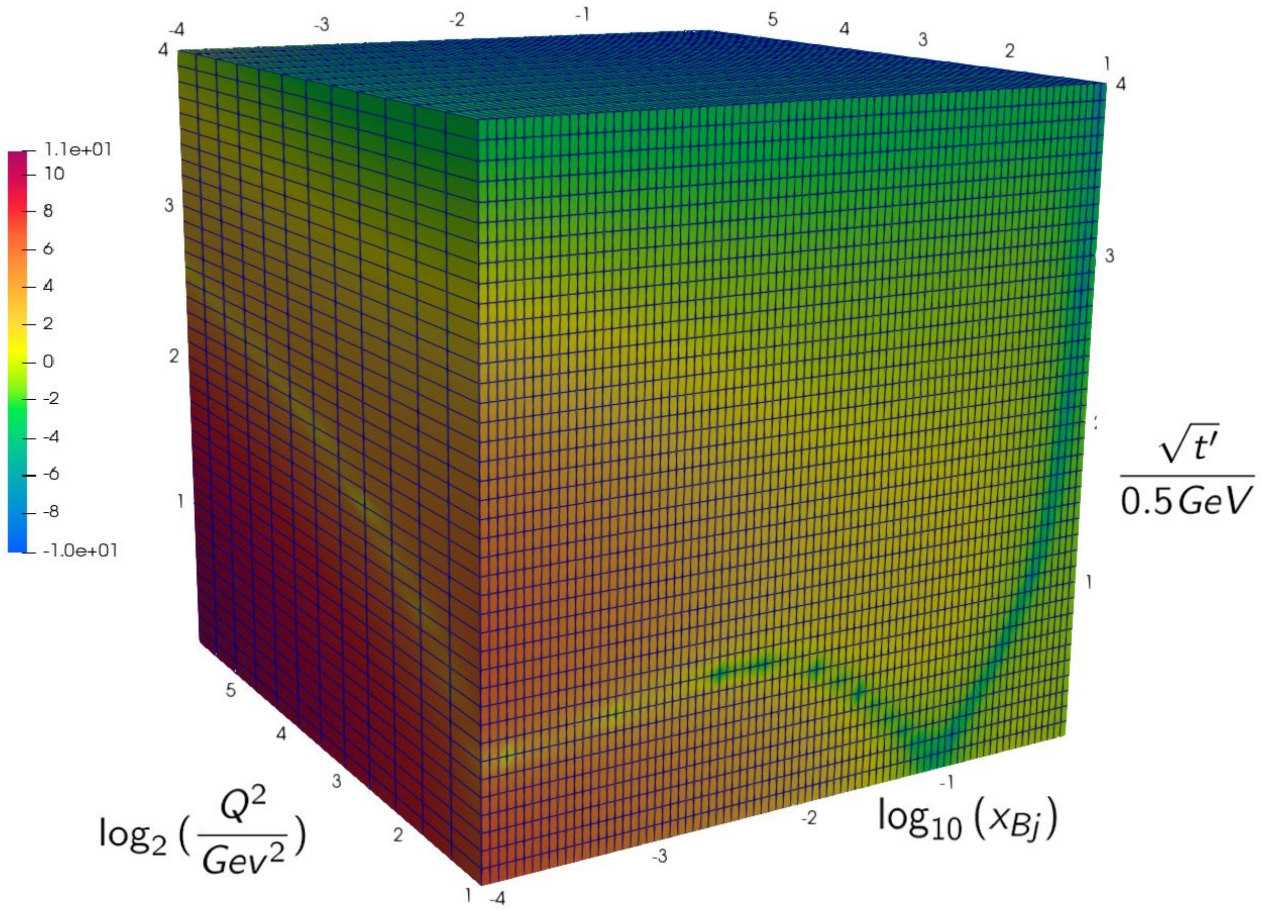
- Define a uniform cartesian grid in the kinematic variables:

$$\log_{10}(x_{Bj}) \qquad \log_2\left(\frac{Q^2}{Gev^2}\right) \qquad \frac{\sqrt{t'}}{0.5\,GeV}$$

- Transform variables into required parameters for the GPD model (PARTONs GPDGK16)

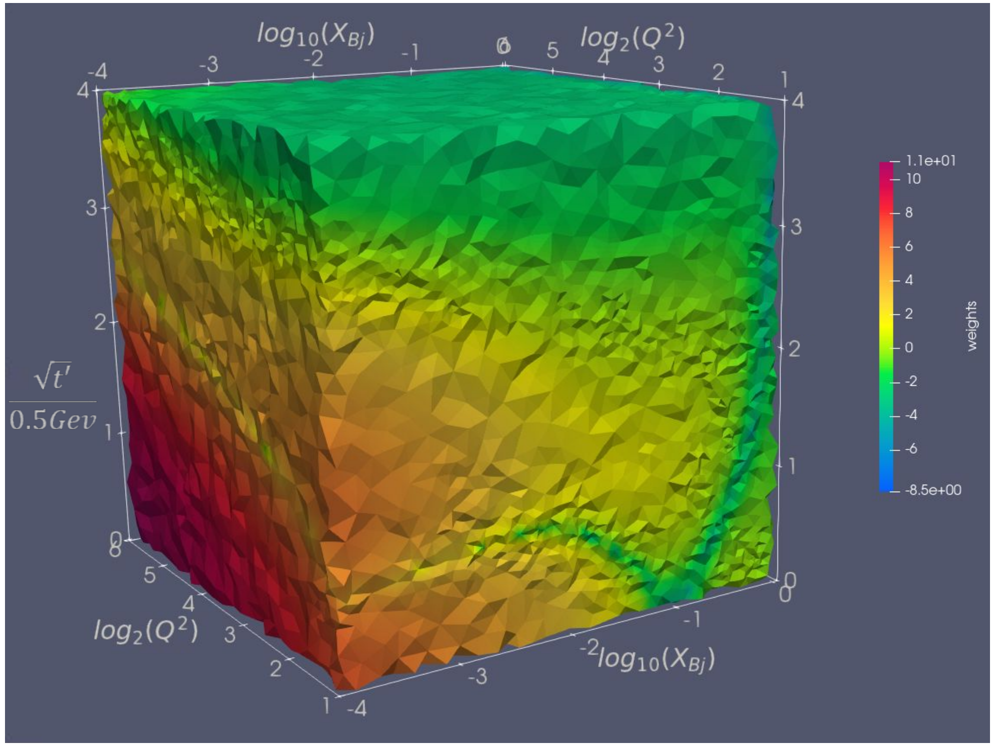
$$\xi \approx \frac{x_{Bj}}{2 - x_{Bj}} \qquad t = t_{min} - t' \qquad Q^2$$

Input image of $\ln|\Re(H_u)|$



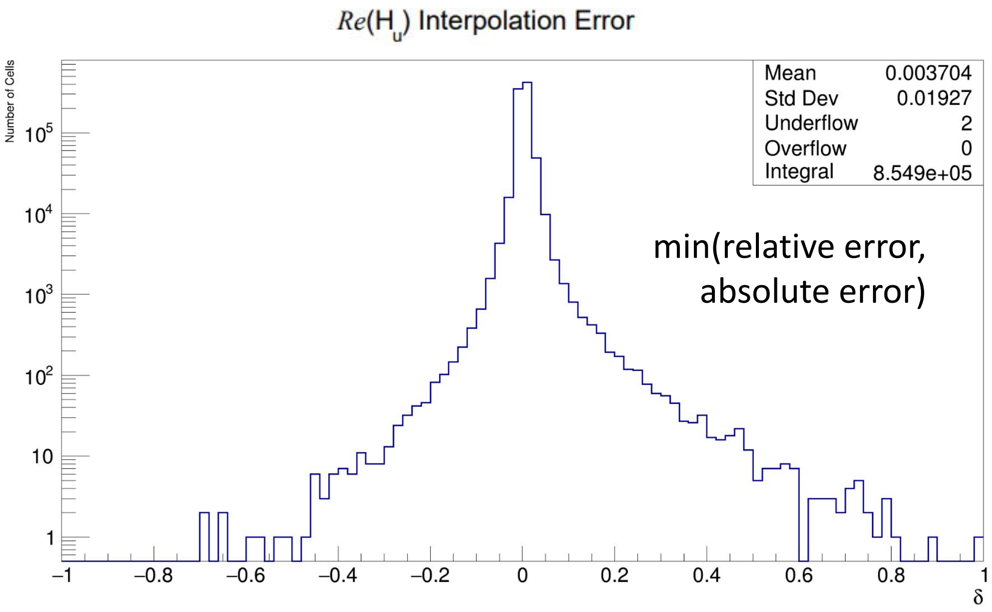
Initial Tessellations

- The cartesian grid is passed to PODM (grid to mesh software) to generate an adaptive tessellation
- Original adaptivity metric: the relative range of the function within a simplex



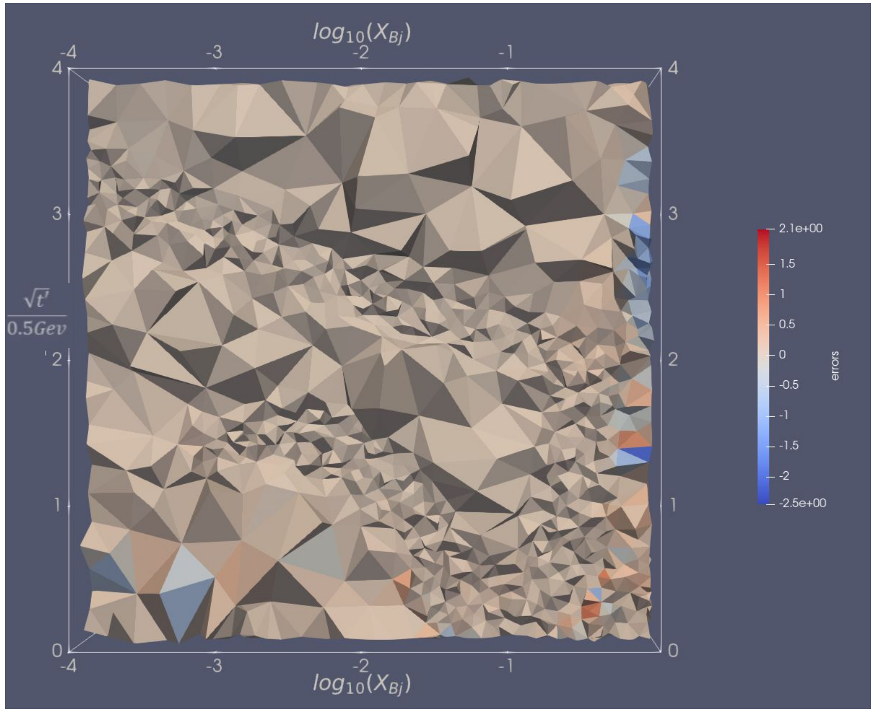
Adaptive Tessellation of $ln|\Re(H_u)|$

- Points are interpolated at the barycenter of each simplex and calculated by PARTONS to evaluate interpolation error of every simplex

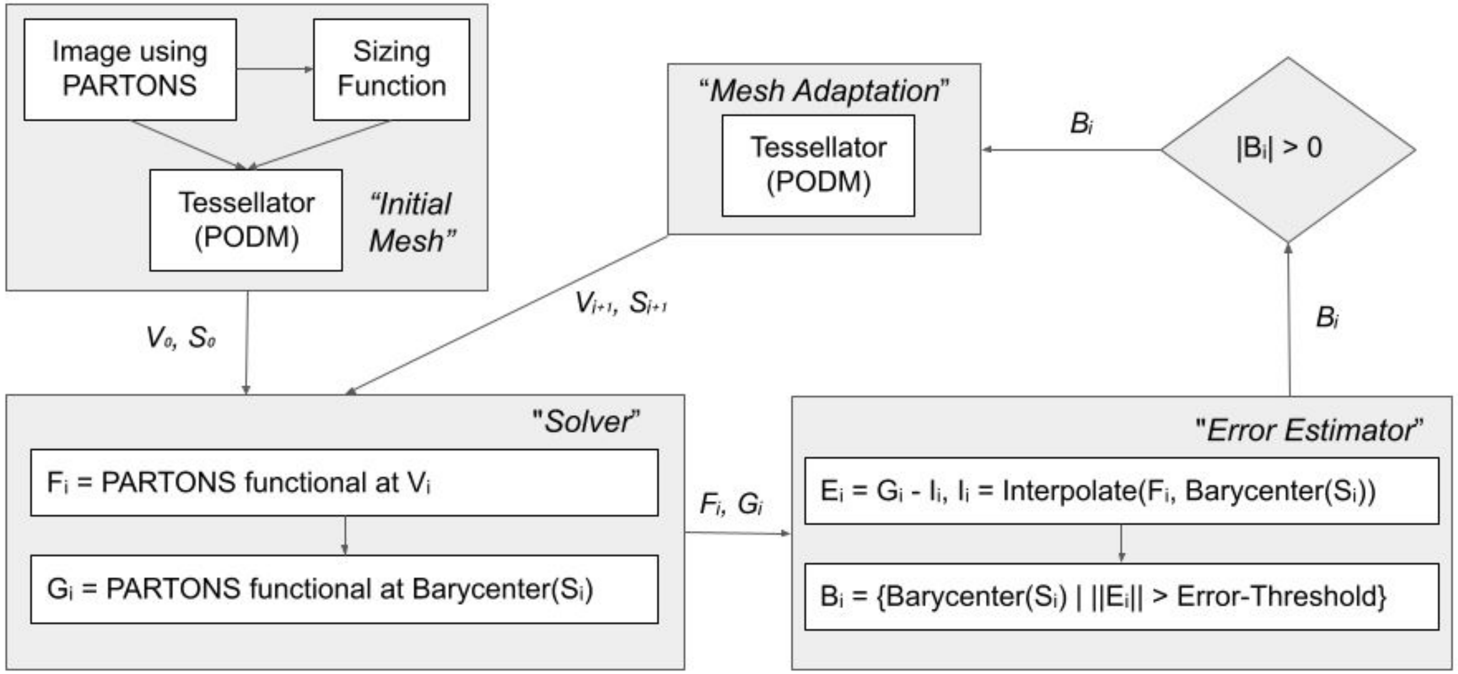


Tessellation Refinement: "Close-the-Loop"

Use interpolation error as a metric to guide the adaptivity



Slice at Q^2= $4GeV^2$ of $ln|\Re(H_u)|$ with the simplex errors as color weight

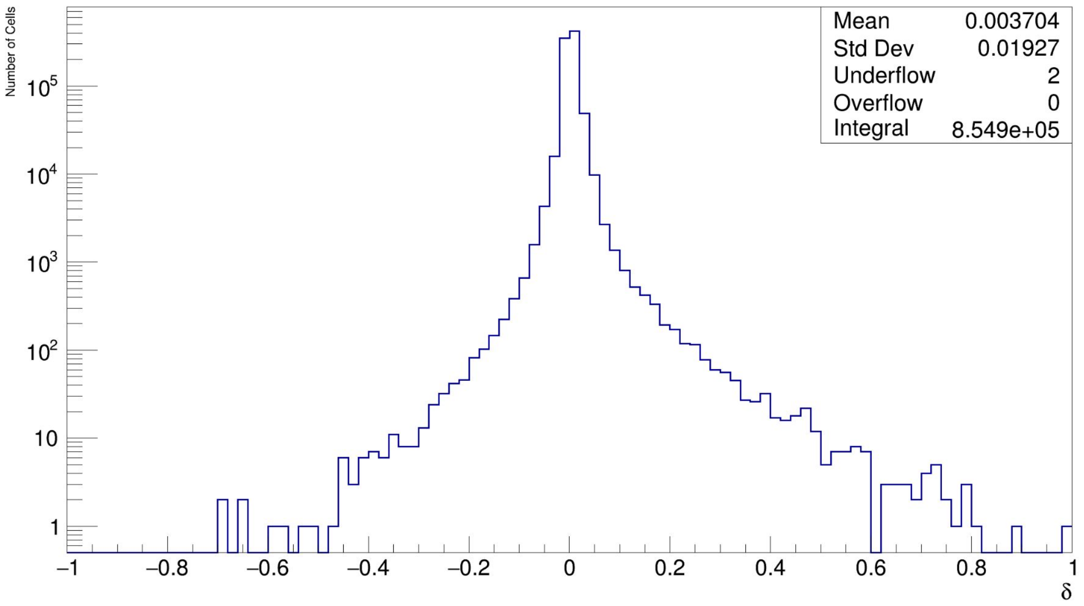


Iterative Refinement

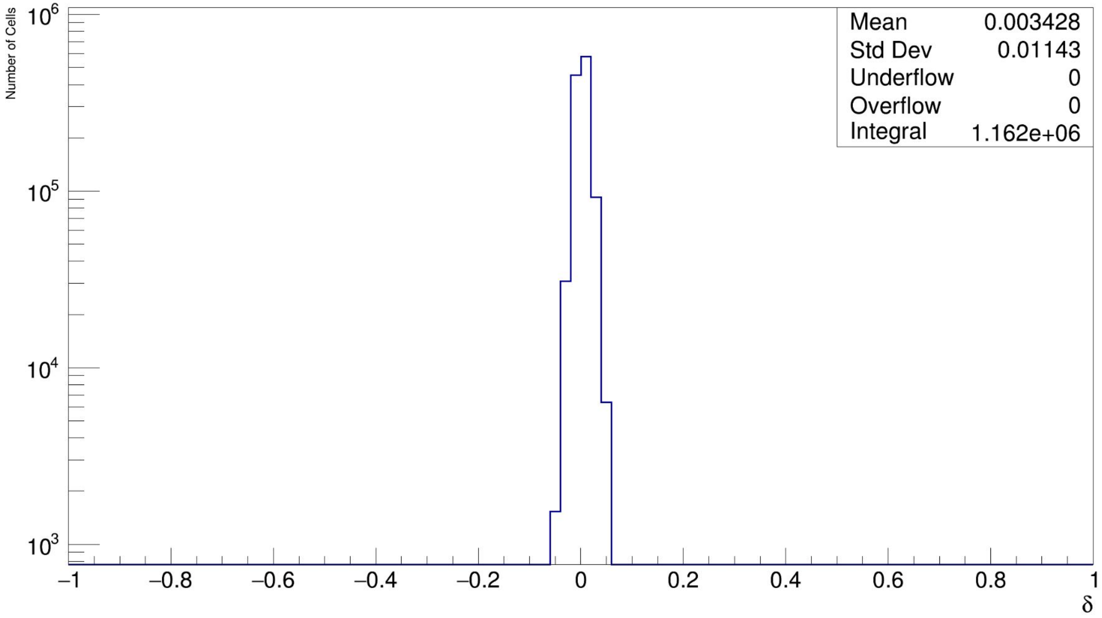
- Tessellations refined by selecting simplices with large interpolation errors and adding new point(s) within it

- Start with a coarse mesh and iterate this refinement process until maximum error of all simplices is below the user-specified value

Error Threshold of 5%



Interpolation Errors for first iteration



Interpolation Errors after 10 iterations

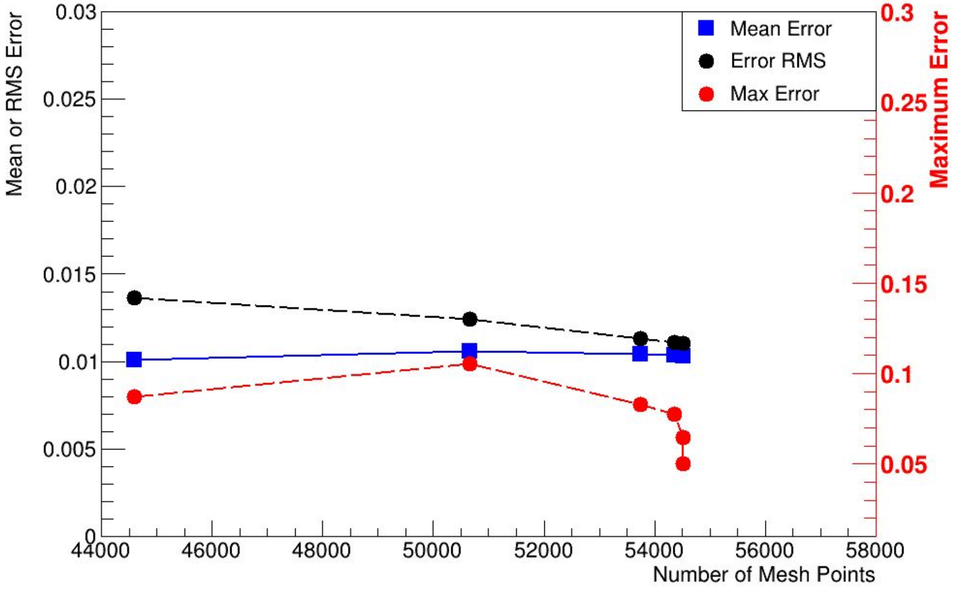
Iterative Refinement

- With a maximum interpolation error of 5% , the tessellation adaptivity for *Im(H)* converged after 5 iterations going from 44,000 vertices to 55,000

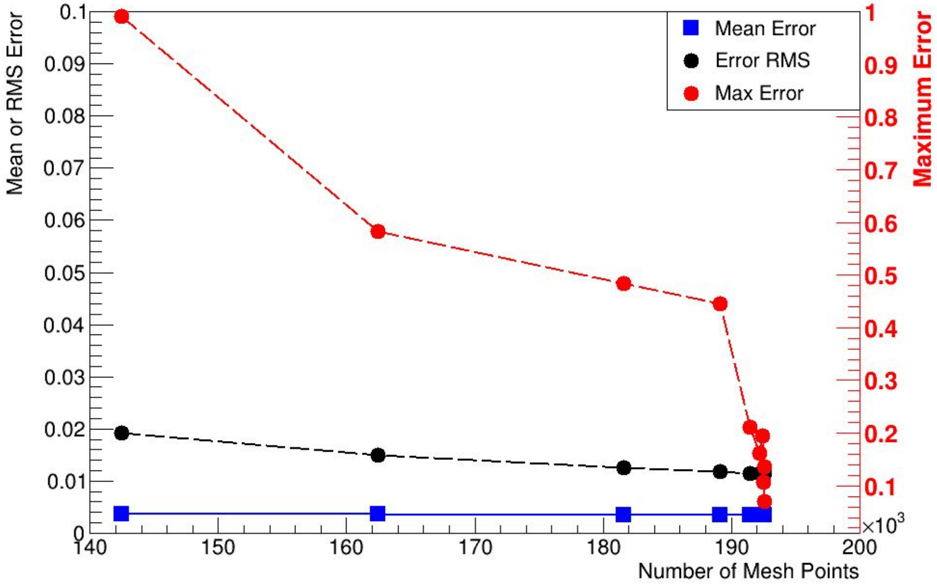
- The *Re(H)* tessellation converged after 10 iterations, going from 140,000 to 191,000 vertices

- Both tessellations achieve a mean error <1%

Im(H)



Re(H)



Computational Speedup

- The mesher creates/shifts many points in the mesh, needing many more model calls

- Reduce time and unnecessary calculations by storing known points from previous iterations!

- Mesh generation time depends only on the desired level of interpolation precision and is independent of number of events in the MC
    - Computation speedup potential increases with the number of events

Computation Time of Adaptive Pipeline

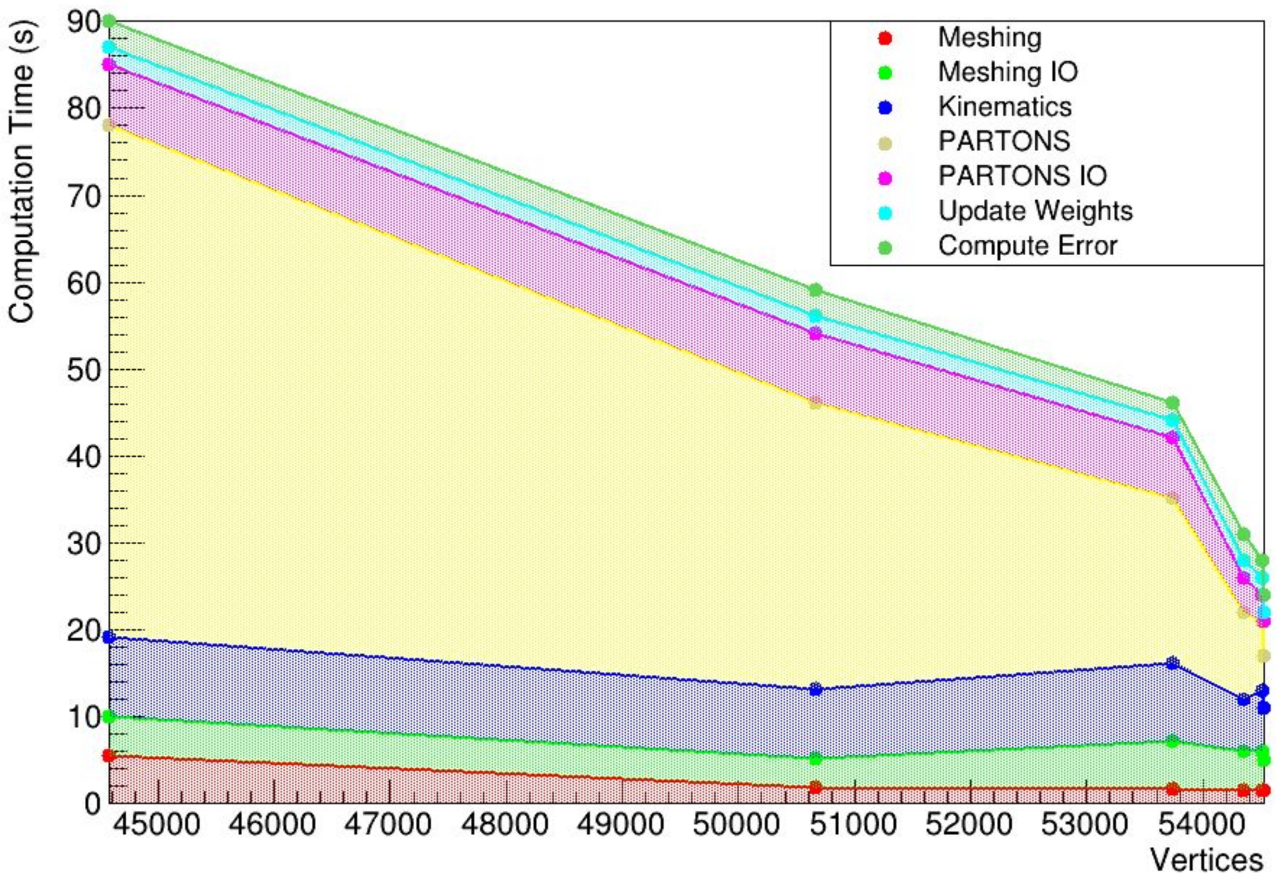
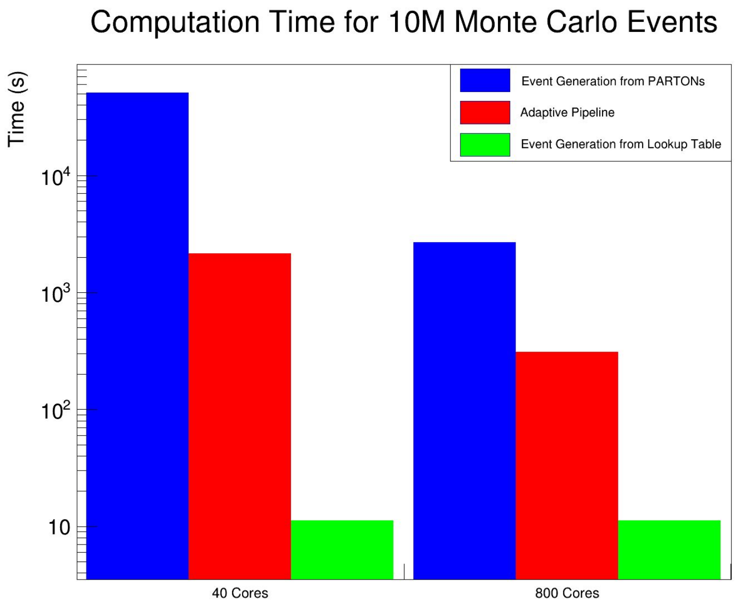
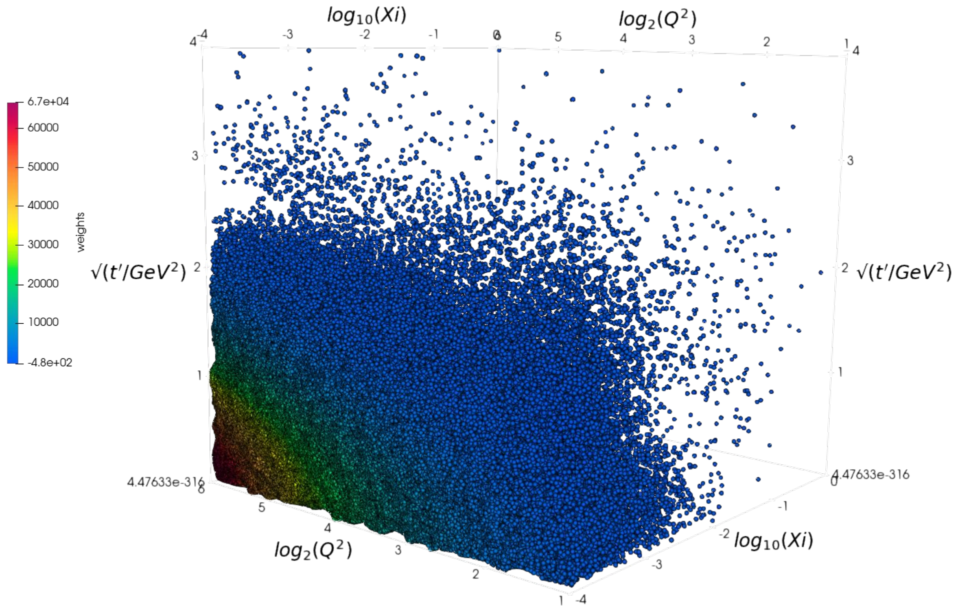


Illustration of MC Generator from Extracting CFFs

Smart Lookup Table Utilities provides an efficient method of extraction of the CFFs for use in cross section calculations and event generators.





MC event generation according to distribution of CFF using importance sampling

- Time to compute 10,000,000 events using PARTONS =14.1 hours on 40 cores
- Time to generate 10,000,000 MC events using a tessellation as a Smart Lookup Table = 35.8 minutes on 40 cores
  - 35.7 minutes to generate initial tessellation and refine
  - 9.2s for generation of 10^7 MC events (Scales with number of events).
  - Any subsequent event generations will not need to re-generate a new tessellation!

Conclusion

- Adaptive tessellations as a smart lookup table can decrease computational time in event generation by up to several orders of magnitude while still maintaining an maximum interpolation error of 5% and an average error of <1%

- Adaptive tessellation generation software and smart lookup table utilities will be made available as a Docker image

Future Work

- Generate the entire table of all CFFs

- Add DVCS cross-section calculation into the event generator using the CFF tessellations and the formulation described in B. Kriesten and S. Liuti in https://arxiv.org/2004.08890

Thank you!

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