KinKal: A Kinematic Kalman Filter Track Fit Package

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Kinematic Track Fit History

- Invented for bubble chamber analysis
- Allows precision reconstruction of lowmomentum particles
- Integrated track fit + particle identification (PID)
- Replaced by geometric track fitting in HEP
 - mass effects are small when $\beta\gamma\gg 1$
 - PID is performed in analysis using non-tracking information





I. R. Kenyon (1972) Bubble Chamber Film Analysis, Contemporary Physics, 13:1, 75-104, DOI: 10.1080/00107517208205669

The Mu2e Experiment



- A search for Charged Lepton Flavor Violation (CLFV)
 - μ ---> e[±] conversion on (AI) nuclei
- Sensitivity goal: $\Gamma_{\mu \to e}/\Gamma_{\mu \text{ capture}} \sim 10^{-16}$
 - Requires ~10²⁰ muons
- Experimental Signature: a single 105 MeV electron
 - ~1‰ momentum resolution needed to suppress beam $\mu^{\scriptscriptstyle 2}$ backgrounds
 - Particle ID and directionality needed to suppress cosmic μ^{\pm} backgrounds

The Mu2e Straw Tracker

- 36 'planes' of 5mm straws in vacuum
 - 15µm mylar walls
 - ~1% X₀ total mass
- Central hole reduces beam background
- < 200 μ m σ R_{drift} required



1 Tracker 'Plane'





Tracking Hits in Mu2e



- Straw hits constrain the trajectory WRT the wire position
 - Distance to wire (DOCA) approximated by $R_{drift} \approx V_{drift} \times (t_{hit} t_0)$
 - Strongly couples geometric parameters and t₀
 - Discrete ionization effects introduce biases and cause non-Gaussian errors
- The calorimeter provides a (nearly) direct time constraint
 - Stereo track overlay determines particle and light propagation times
 - Constrains t_0 with a net resolution < 500 ps

The Mu2e Tracking Environment

- No external measurement of t₀
 - No other associated tracks or relevant accelerator timing signal
- Large magnetic field gradients
 - $dB_z/B_z/dZ = 1.4\%$ /meter near the tracker, 10%/meter near the target
- Low intrinsic signal/noise
 - >2 GHz of stopped muons, ~100 KHz hits/straw
 - ~40 signal particle hits vs ~2000 pileup hits each 1 μsec readout window





Kinematic Track Fitting

- Time (t₀) is an explicit fit parameter
 - eg parameters $\mathscr{J} \equiv \{d_0, \phi_0, z_0, \omega, \lambda, t_0\}$
- Time is the trajectory parametric variable • ie particle position = $\overrightarrow{X}(\mathscr{T}: t)$
- The time flow is explicitly chosen in the fit
 - Particle direction = hit time order = energy loss direction
 - Time order has a first-order effect on drift measurements
- Time measurements constraints are used in the fit
- Particle mass is a (static) track parameter
 - $m \in \{e, \mu, \pi, K, P, \dots\}$
 - Kinematics $E(\P, m : t), \vec{v}(\P, m : t)$ are part of the trajectory

KinKal = Kinematic Kalman Filter Track Fit

- Kalman Filter implementation adapted from the BaBar track fit
- Templated fitter with 3 representations optimized for different track types
- MVA-based pattern recognition tools
- Transport in high-gradient magnetic fields
- Straw hits and calorimeter cluster hit representations



Magnetic Field Transport

- Trajectories are divided into magnetic 'domains' by requiring:
 - $|q\vec{v} \times \overrightarrow{\Delta B}| / |q\vec{v} \times \overrightarrow{B}| < \epsilon$ within each domain
 - ϵ = configurable threshold, set to 10⁻⁴ for Mu2e
 - Domains provide a dynamically-computed step size
- Each domain's trajectory references its local (midpoint) \overrightarrow{B}
- Trajectory parameters are transported across domain boundaries
 - 1st-order correction, assuming continuity of position and momentum
 - $\overrightarrow{X}(\mathcal{T}, \overrightarrow{B}: t_d) = \overrightarrow{X}(\mathcal{T}', \overrightarrow{B'}: t_d)_e and \overrightarrow{P}(\mathcal{T}, \overrightarrow{B}: t_d)_e and \overrightarrow{P}(\mathcal{T}, \overrightarrow{B}: t_d)_e and \overrightarrow{P}(\mathcal{T}, \overrightarrow{B}: t_d)$
 - Calculated using analytic derival Must E Calculated using analytic derival \overline{B}
 - Effectively uses helix basis for RK stepping 15 of 116



Straw Material Modeling



- Straw material effects are estimated using DOCA to the wire
 - <u>Moyal</u> approximation to mean ionization energy loss
 - Lynch-Dahl approximation to Moliere scattering
 - 1st order parameter transport using closed-form derivatives
- Transport updated as part of the annealing schedule
 - Using a dedicated 'straw material' updater

Background Hit Filtering



- KERAS dense layer ANN used to filter background hits
 - Variables sensitive to consistency with straw physics models
- Filtering applied iteratively as part of the annealing schedule
 - Inference function generated by ROOT::SOFIE
- Implemented as an instance of hit updating
 - Generic 'updaters' can be applied selectively to any fit object

Rdrift Constraint

- Rdrift has a discrete ambiguity
 - The dominant cause of momentum resolution tails
- A dense-layer ANN selects resolvable cases
 - Using consistency with physical drift model
 - Unresolved cases constrain to the wire
- A subsequent ANN infers DOCA from $\Delta t = t_{hit} t_0$







Particle and Direction Identification



- Fit χ^2 /NDOF provides e, μ separation power
- Fit success + (N active hits) separates $\rightarrow e^{-}$ from $\leftarrow e^{+}$
 - Same helicity (geometric parameters), wrong time ordering

Conclusions

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Offline software for the Mu2e

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Suggested Workflows

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mplements a kinematic Kalman filter track fit (future ref to CTD/pub). The primary class of KinKal is Track, hares the state describing the fit inputs (hits, material interactions, BField corrections, etc), and owns the f the fit, and the methods for computing it. The fit result is expressed as a piccewise kinematic covariant						CaloMC	Faster CaloDigiMaker for large empty wa	
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trajectory, providing 4-vector position and momentum information about the particle with covariance as a			iance as a			CalorimeterGeom	global whitespace cleanup	
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ck is templated on a simple kinematic trajectory class representing the 1-dimensional path and momentum ticle traveling through empty space in a constant magnetic field as a function of physical time. The							Expand compressRecoMCs to be able to	take multiple CryCoincid

• KinKal is a toolkit for precision low-momentum track fitting

• Mu2e will use it for triggering and analysis

piecewise kinematic trajectory fit result is expressed as a time sequence of these simple trajectory objects.

Material effects and spatial variations of magnetic fields are modeled through changes between adjacent simple trajectories. The simple kinematic trajectory class must satisfy the interface defined in the Track header, including ConditionsBase/inc

ConditionsService

Backup

Track Quality Selection



- Momentum resolution tails persist even after hit filtering
 < 10⁻⁵ suppression factor needed to achieve Mu2e science goals
- ANN selects tracks with high-quality momentum
 - Using fit global consistency, hit pattern vs expected, ...
- 90% signal efficiency, tail reduced by a factor $<10^{-3}$

LHelix Parameterization (CTD 2015)

$$\mathbf{P} \equiv \{R, \Lambda, C_x, C_y, \phi_0, t_0\}$$

$$\Delta t = t - t_0 \qquad Q = -qc |\vec{B}|$$

$$\bar{m} = \frac{m}{Q} \qquad \Omega = \frac{qc}{\sqrt{R^2 + \Lambda^2 + \bar{m}^2}}$$

$$x(t) = C_x + R \cdot sin(\Omega \Delta t + \phi_0)$$

$$y(t) = C_y - R \cdot cos(\Omega \Delta t + \phi_0)$$
$$z(t) = \Lambda \Omega \Delta t$$

$$P_{x}(t) = Q \cdot R \cdot cos(\Omega \Delta t + \phi_{0})$$
$$P_{y}(t) = Q \cdot R \cdot sin(\Omega \Delta t + \phi_{0})$$
$$P_{z}(t) = Q \cdot \Lambda$$

$$P_t(t) = |Q| \cdot \sqrt{R^2 + \Lambda^2 + \bar{m}^2}$$

Natural description of a looping track from an arbitrary origin





KinKal Package Unit Tests



Comparison with BTrk

	BTrk	KinKal	
Code	1990s C++ (pre STL)	C++17	
Kinematics: mass.Momentum	External interpretation	Internal	
parameteric variable	3D flightlength	Physical time	
BField Correction	Relative to $B_0 (\Delta B/B < 5\%)$	B →B' (all ΔB/B)	
to	external meta-fit parameter	internal parameter	
Annealing	External	Internal	
Updating Model	Fixed algebraic	ANN-based	
Trajectory Models	Central Helix	KLine, CHelix, LHelix	
Matrix Algebra	CLHEP::Matrix	Root::Math::SMatrix	
Memory	vector <x*></x*>	vector <unique_ptr></unique_ptr>	
Linear Algebra	CLHEP, difAlgebra	ROOT::Math::SMatrix, OpenBLAS	
SpaceTime	Hep3Vector, BbrLorentzVector	ROOT::Math::GenVector	
Execution time	10 ms/track	500 ms/track	

Drift Resolution



- Far from wire \Rightarrow hit time directly related to impact parameter
 - Residual = $t_{hit} t_0$
- Near to wire \Rightarrow hit time poor approximation to impact parameter
 - Residual = wire DOCA (no drift information)

LHelix DOCA Derivatives

- Consider wire \perp to z at W, azimuth = η
- $\bar{\Phi} \equiv \Phi_0 + W_z / \Lambda \eta$
- $\Delta = -\sin(\eta)(C_x W_x) + \cos(\eta)(C_y W_y)$
- $F = \Lambda/sqrt(\Lambda^2 + R^2sin^2(\bar{\Phi}))$
- DOCA = -F(Rcos($\bar{\phi}$) Δ) + O(r²_{straw}/(R, Λ))
- $\delta DOCA/\delta C_x = -Fsin(\eta)$
- $\delta DOCA/\delta C_y = Fcos(\eta)$
- $\delta DOCA/\delta \phi_0 = FRsin(\bar{\phi})$
- $\delta DOCA/\delta R = -F(cos(\bar{\phi})\Lambda^2 + sin^2(\bar{\phi})R\Delta)/(\Lambda^2 + R^2sin^2(\bar{\phi}))$
- $\delta DOCA/\delta \Lambda = -FRsin(\bar{\phi})W_z/\Lambda^2$
- $\delta DOCA/\delta t_0 = 0$



- ~2000 rectangular CsI crystals arranged in 2 disks
 - 5% energy measurement, ~0.5 ns timing measurement
- Primary function: muon-electron separation
 - electron E/p ~ 0.95 (with long tails), muon E/p ~0.4
 - Muon rest mass energy is released in decay, but delayed
 - Dominant discriminant comes from relative timing
 - 105 MeV/c $\mu^{\mbox{-}}$ has $\beta\mbox{=}0.7,$ arrives \sim 3 ns later at calorimeter WRT $e^{\mbox{-}}$







- Goal: < 1 background event over the experiment lifetime
- Suppressing the radiative tail of Michel decay electrons
 - Requires excellent momentum resolution (σ < 500 KeV/c)
- Suppressing cosmic muons
 - Requires an external veto with 99.99% efficiency
 - Requires e/µ separation
 - Requires separating upstream from downstream tracks

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Conversion e- Trigger Fit

Momentum at tracker middle



- 3 msec/track execution time
- 600 KeV/c Momentum resolution
 - Pileup event rejection power of ~10⁻⁴
- 2 redundant track fit seed algorithms

Lynch-Dahl Scattering Model

- \bullet Used to compute σ used in Kalman filter fit
- Integral screened Rutherford scattering model, parameterized by 'tail fraction'
 - larger fraction→larger sigma
- Can be tuned to give 'flat' probability distribution
 - larger → narrower core
 - smaller→smaller tails
 - Hit Straws: tail<0.9999
 - Add Straws: tail<0.995

