KinKal: A Kinematic Kalman Filter Track Fit Package

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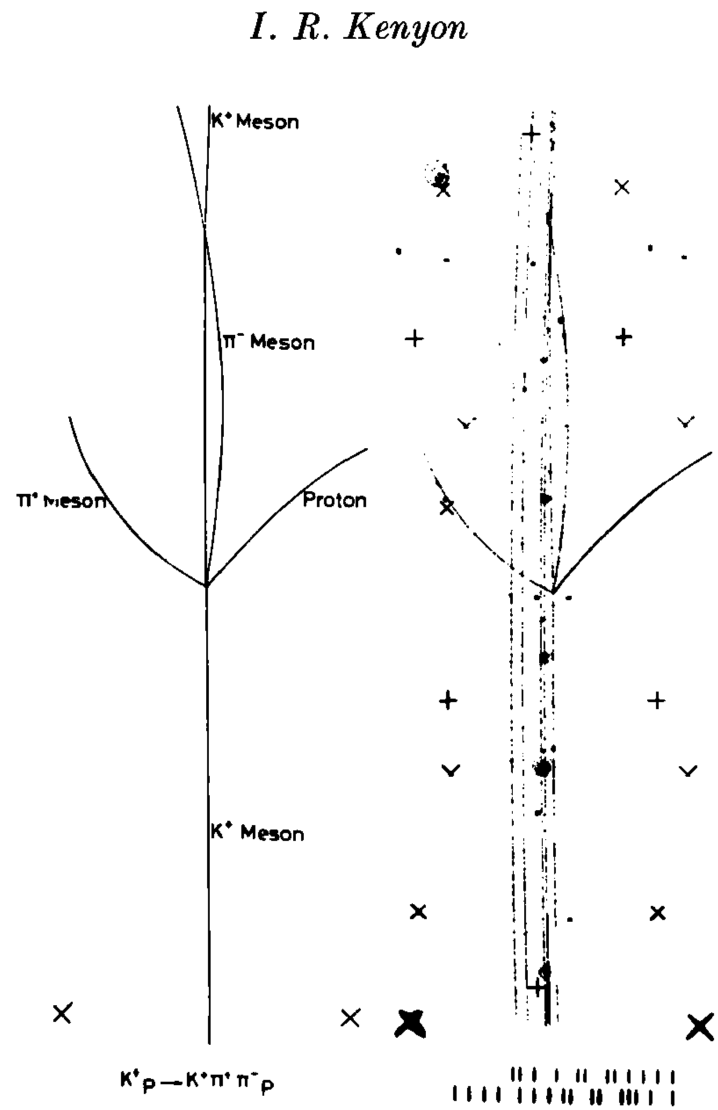
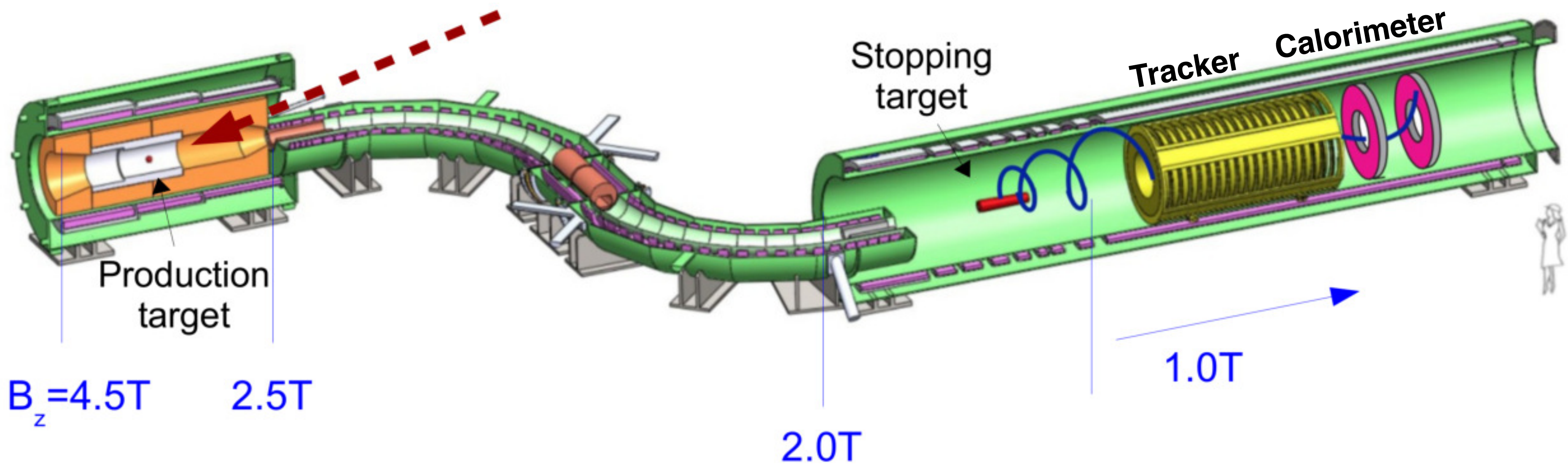
Kinematic Track Fit History

- Invented for bubble chamber analysis
- Allows precision reconstruction of low-momentum particles
- Integrated track fit + particle identification (PID)
- Replaced by geometric track fitting in HEP
  - mass effects are small when $\beta\gamma \gg 1$
  - PID is performed in analysis using non-tracking information



I. R. Kenyon (1972) Bubble Chamber Film Analysis, Contemporary Physics, 13:1, 75-104, DOI: 10.1080/00107517208205669

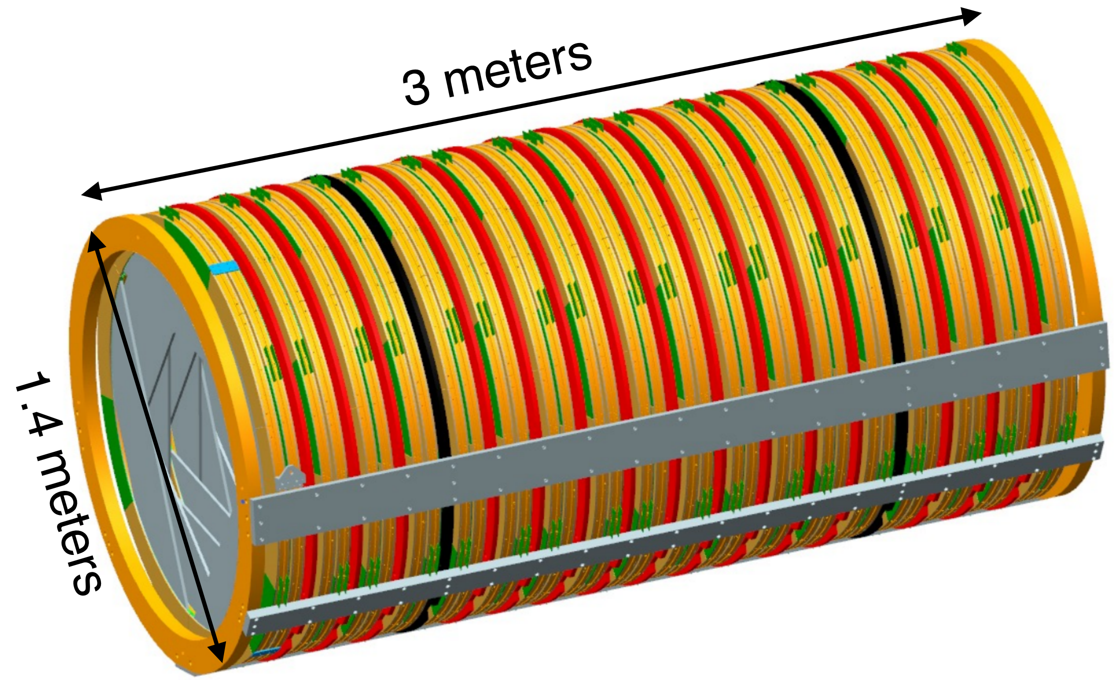
The Mu2e Experiment

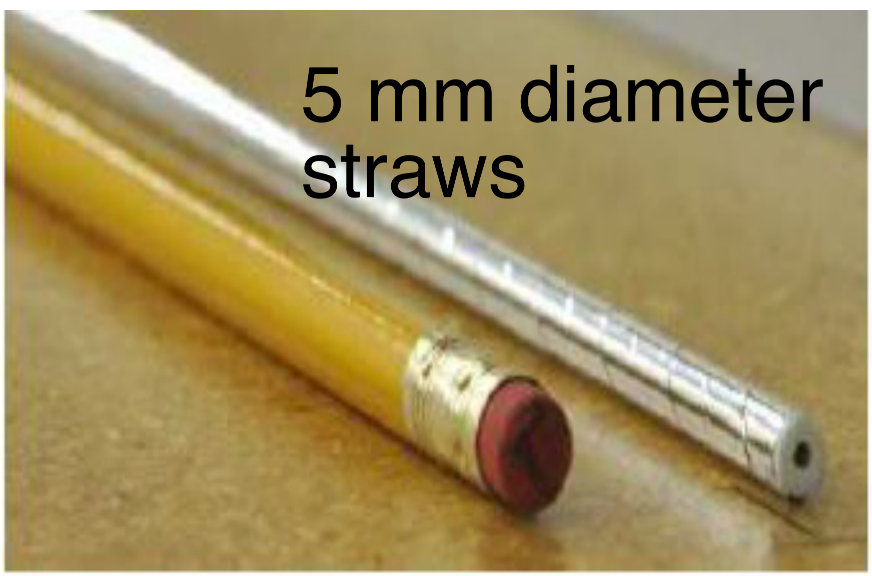


- A search for Charged Lepton Flavor Violation (CLFV)
  - $\mu^- \rightarrow e^\pm$ conversion on (Al) nuclei
- Sensitivity goal: $\Gamma_{\mu\rightarrow e}/\Gamma_{\mu\ capture} \sim 10^{-16}$
  - Requires $\sim 10^{20}$ muons
- Experimental Signature: a single 105 MeV electron
  - ~1‰ momentum resolution needed to suppress beam μ^- backgrounds
  - Particle ID and directionality needed to suppress cosmic $\mu^\pm$ backgrounds

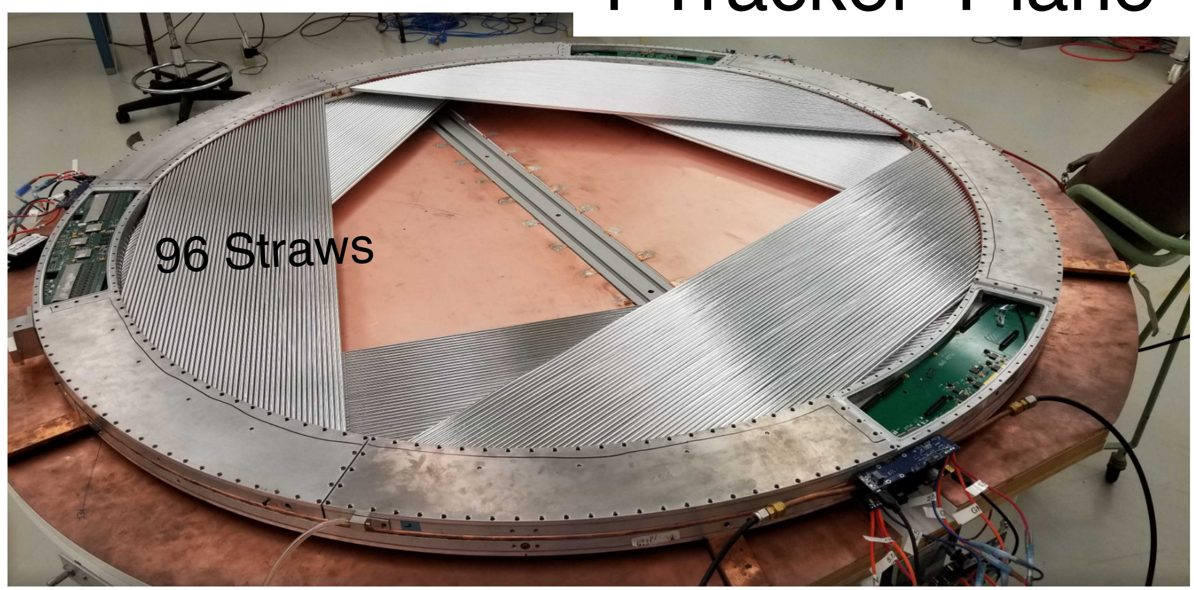
The Mu2e Straw Tracker

- 36 'planes' of 5mm straws in vacuum
  - 15µm mylar walls
  - ~1% X_0 total mass
- Central hole reduces beam background
- < 200µm σ R_{drift} required

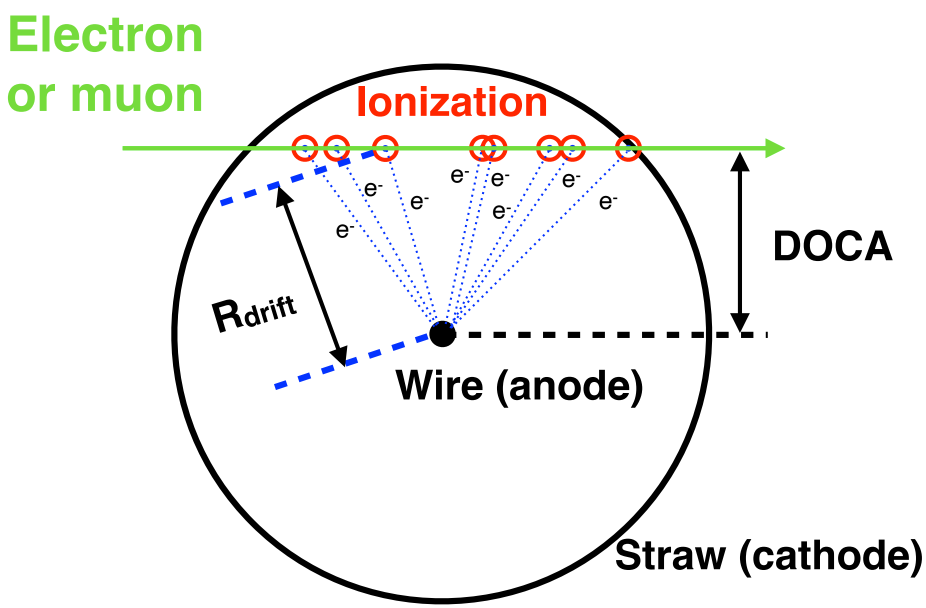


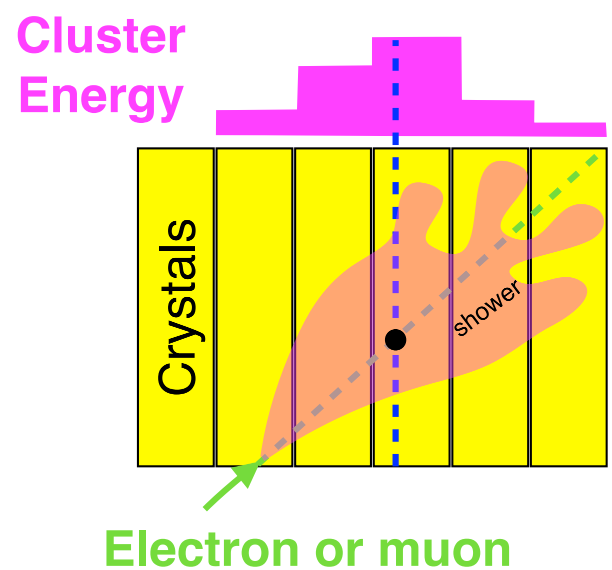


1 Tracker 'Plane'



Tracking Hits in Mu2e



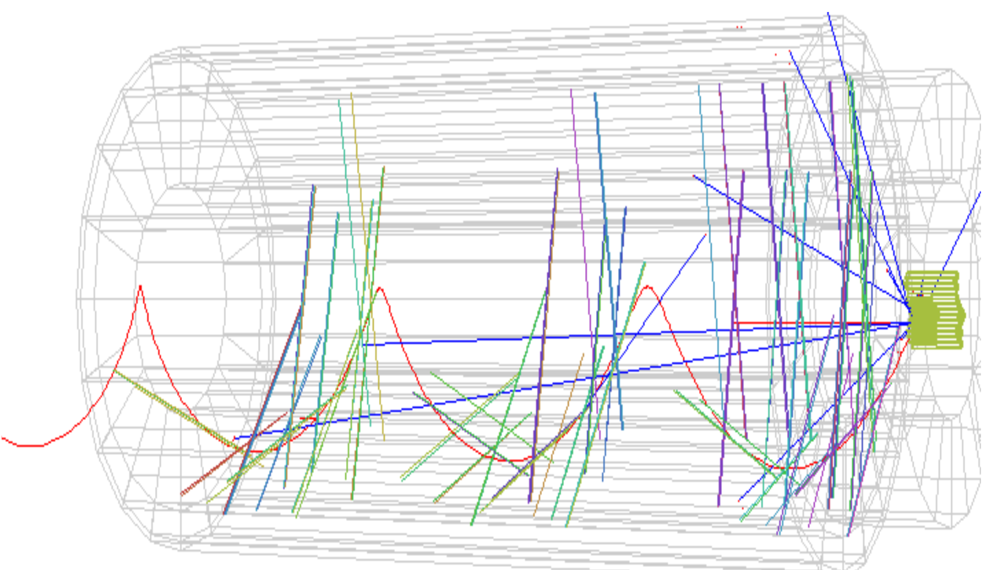


D. Brown, J. Ilic, G. Mohanty, Nucl. Instrum. Meth. A 592, 254 (2008)

- Straw hits constrain the trajectory WRT the wire position
  - Distance to wire (DOCA) approximated by $R_{drift} \approx V_{drift} \times (t_{hit} - t_0)$
    - Strongly couples geometric parameters and t_0
    - Discrete ionization effects introduce biases and cause non-Gaussian errors
- The calorimeter provides a (nearly) direct time constraint
  - Stereo track overlay determines particle and light propagation times
  - Constrains t_0 with a net resolution < 500 ps

The Mu2e Tracking Environment

- No external measurement of t_0
  - No other associated tracks or relevant accelerator timing signal
- Large magnetic field gradients
  - $dB_z/B_z/dZ$ = 1.4%/meter near the tracker, 10%/meter near the target
- Low intrinsic signal/noise
  - >2 GHz of stopped muons, ~100 KHz hits/straw
  - ~40 signal particle hits vs ~2000 pileup hits each 1 μsec readout window



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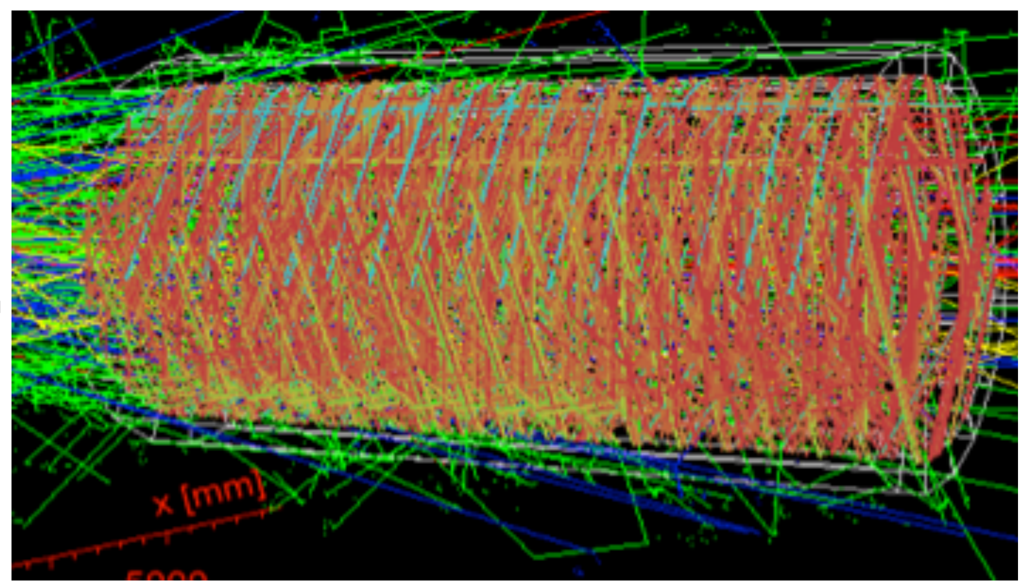
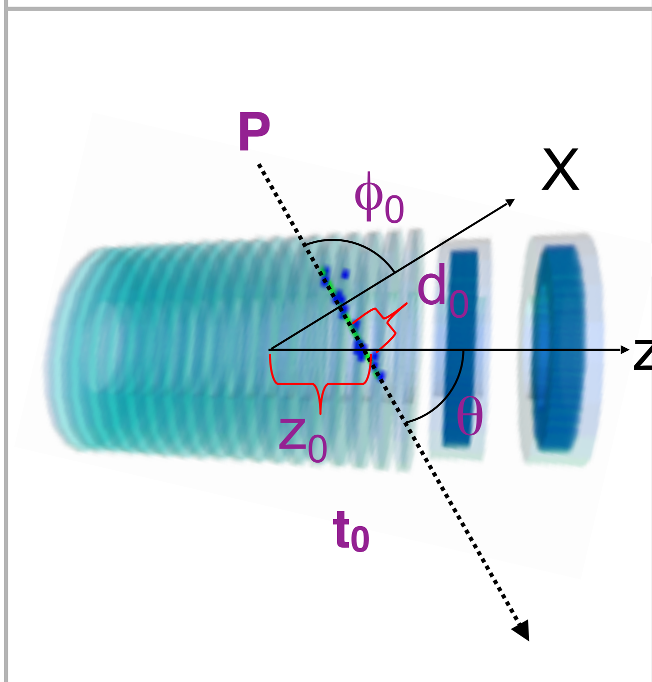
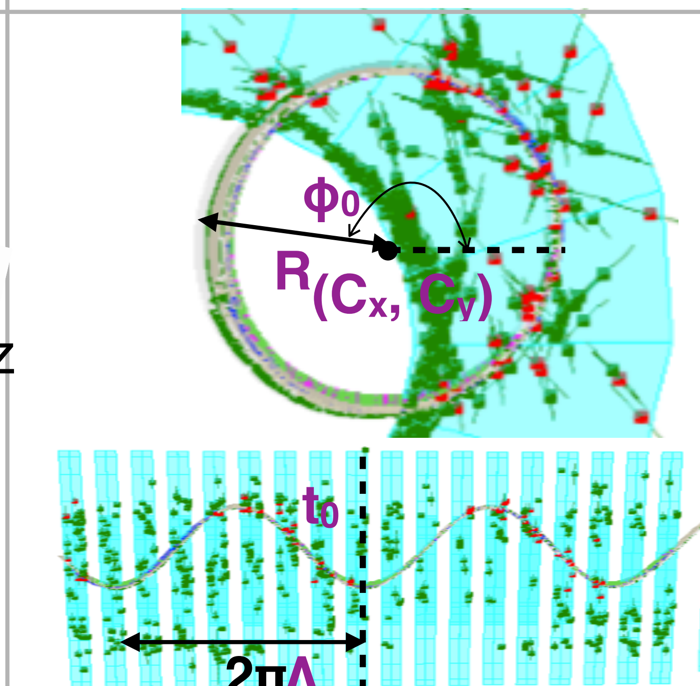
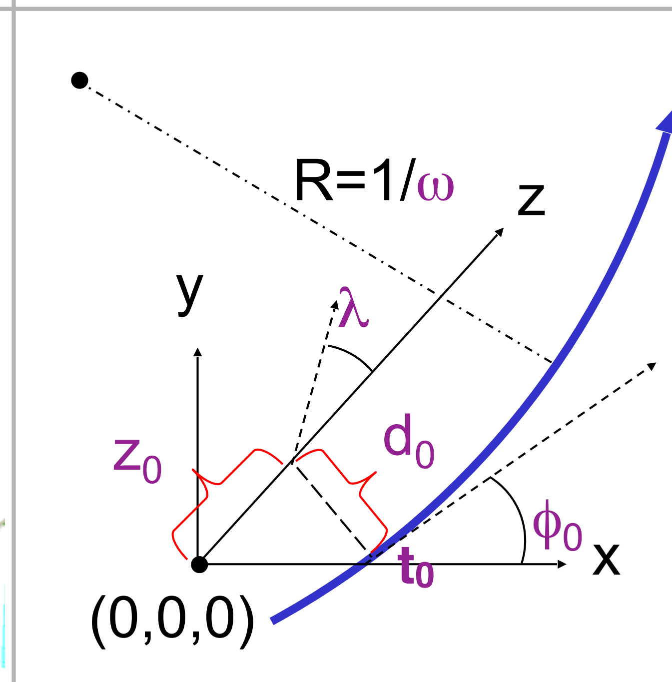


Kinematic Track Fitting

- Time (t_0) is an explicit fit parameter
  - eg parameters $\P \equiv \{d_0, \phi_0, z_0, \omega, \lambda, t_0\}$
- Time is the trajectory parametric variable
  - ie particle position = $\overrightarrow{X}(\P : t)$
- The time flow is explicitly chosen in the fit
  - Particle direction = hit time order = energy loss direction
  - Time order has a first-order effect on drift measurements
- Time measurements constraints are used in the fit
- Particle mass is a (static) track parameter
  - $m \in \{e, \mu, \pi, K, P, \ldots\}$
  - Kinematics $E(\P, m : t), \vec{v}(\P, m : t)$ are part of the trajectory

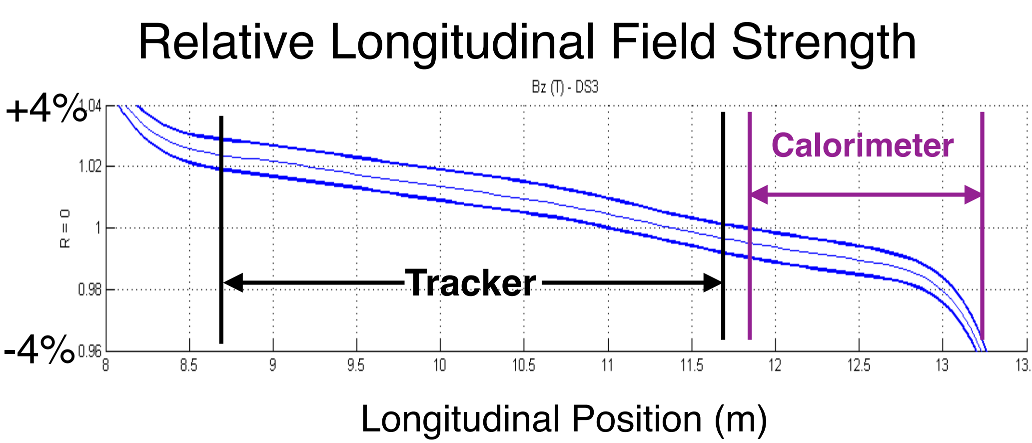
KinKal = Kinematic Kalman Filter Track Fit

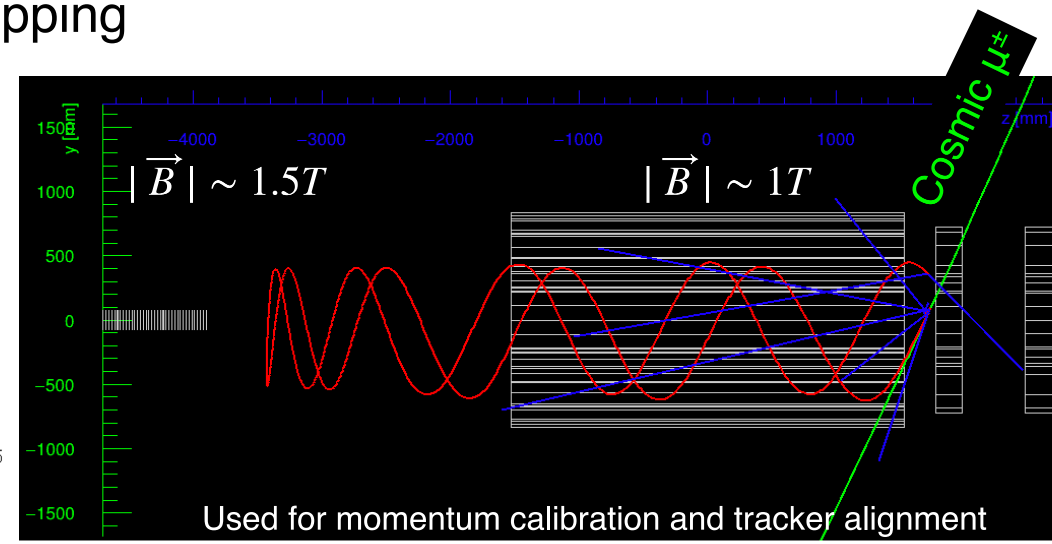
- Kalman Filter implementation adapted from the BaBar track fit
- Templated fitter with 3 representations optimized for different track types
- MVA-based pattern recognition tools
- Transport in high-gradient magnetic fields
- Straw hits and calorimeter cluster hit representations

Field-off cosmic μ±	Conversion e±	Field-on cosmic μ±
		
KinematicLine	**LoopHelix**	**CentralHelix**

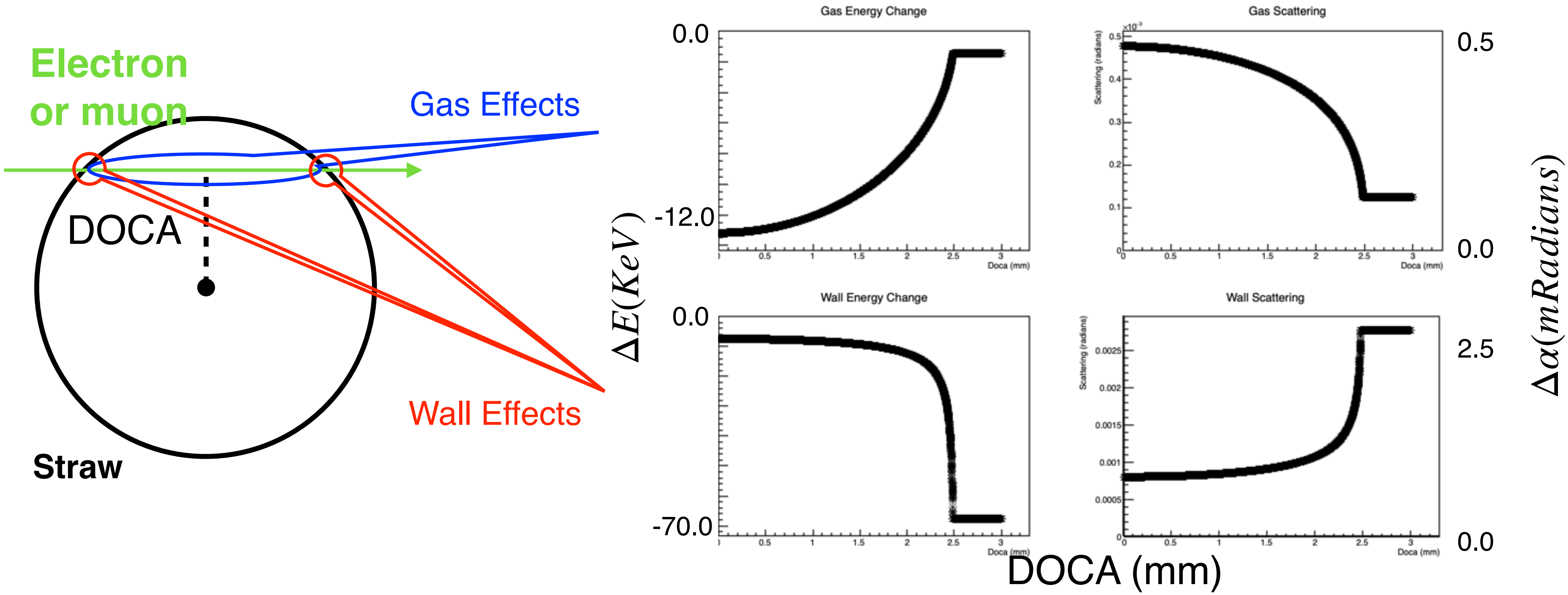
Magnetic Field Transport

- Trajectories are divided into magnetic 'domains' by requiring:
  - $|q\vec{v} \times \overrightarrow{\Delta B}| / |q\vec{v} \times \overrightarrow{B}| < \epsilon$ within each domain
    - ϵ = configurable threshold, set to 10^{-4} for Mu2e
  - Domains provide a dynamically-computed step size
- Each domain's trajectory references its local (midpoint) $\overrightarrow{B}$
- Trajectory parameters are transported across domain boundaries
  - 1st-order correction, assuming continuity of position and momentum
    - $\overrightarrow{X}(\P, \overrightarrow{B} : t_d) = \overrightarrow{X}(\P', \overrightarrow{B}' : t_d)$ and $\overrightarrow{P}(\P, \overrightarrow{B} : t_d) = \overrightarrow{P}(\P', \overrightarrow{B}' : t_d)$
    - Calculated using analytic derivative tensors $d\P/d\overrightarrow{B}$
  - Effectively uses helix basis for RK stepping



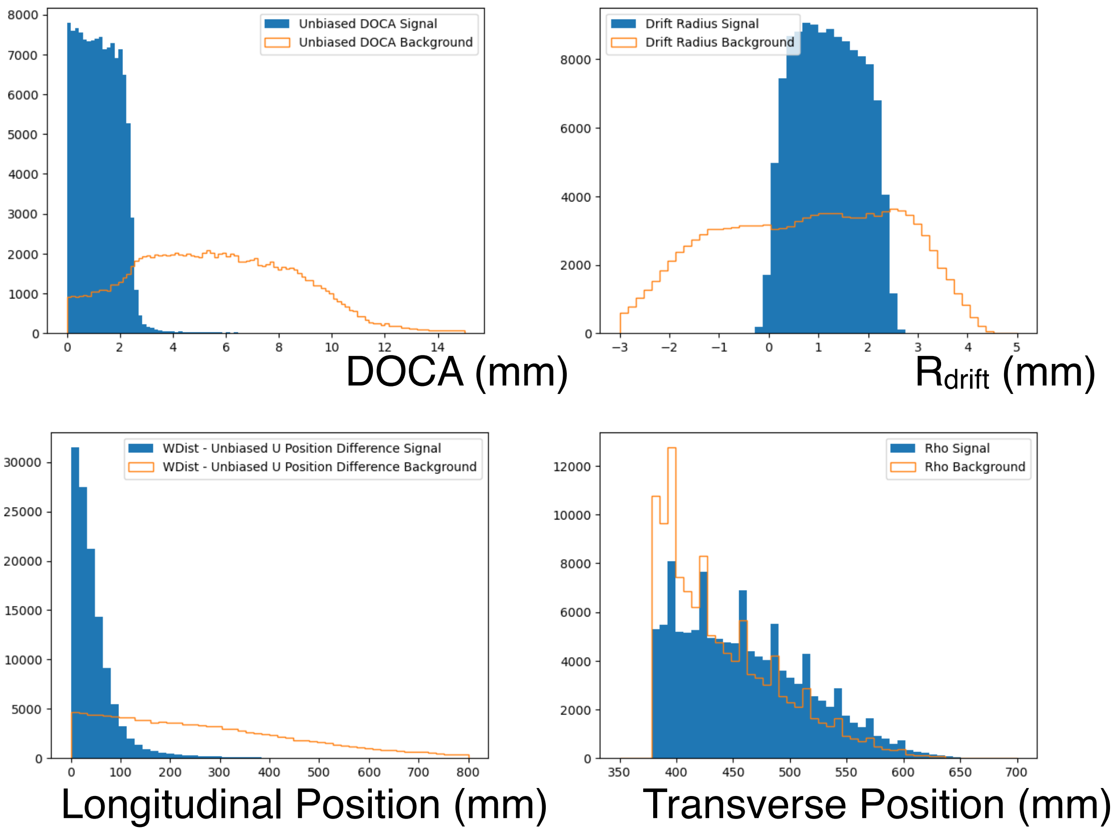


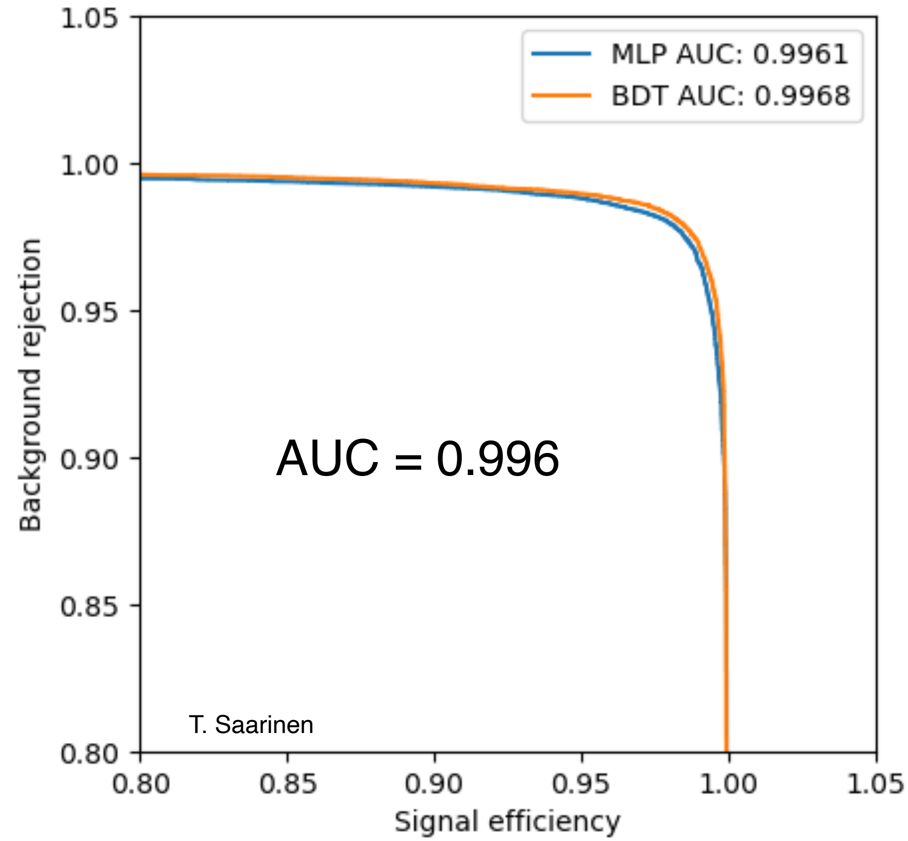
Straw Material Modeling



- Straw material effects are estimated using DOCA to the wire
  - Moyal approximation to mean ionization energy loss
  - Lynch-Dahl approximation to Moliere scattering
  - 1st order parameter transport using closed-form derivatives
- Transport updated as part of the annealing schedule
  - Using a dedicated 'straw material' updater

Background Hit Filtering

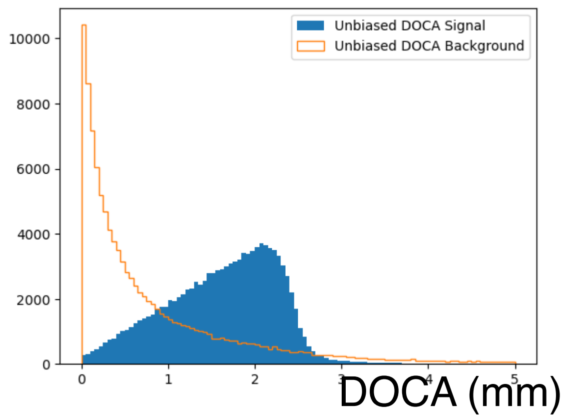


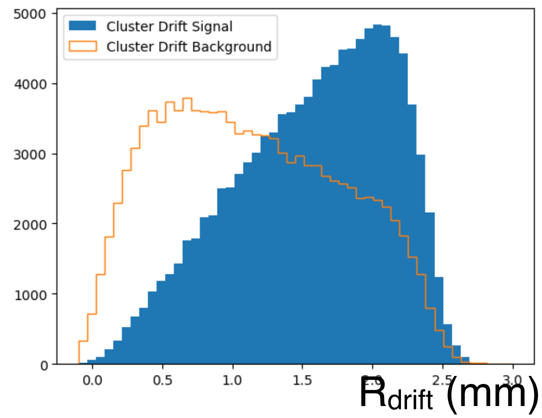


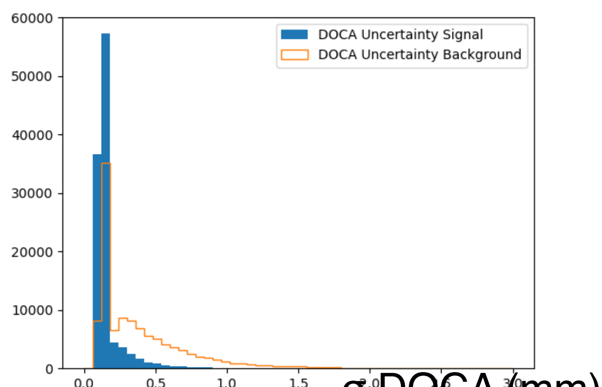
- KERAS dense layer ANN used to filter background hits
  - Variables sensitive to consistency with straw physics models
- Filtering applied iteratively as part of the annealing schedule
  - Inference function generated by ROOT::SOFIE
- Implemented as an instance of hit updating
  - Generic 'updaters' can be applied selectively to any fit object

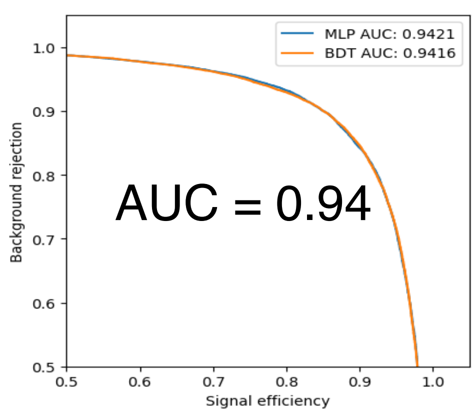
R_{drift} Constraint

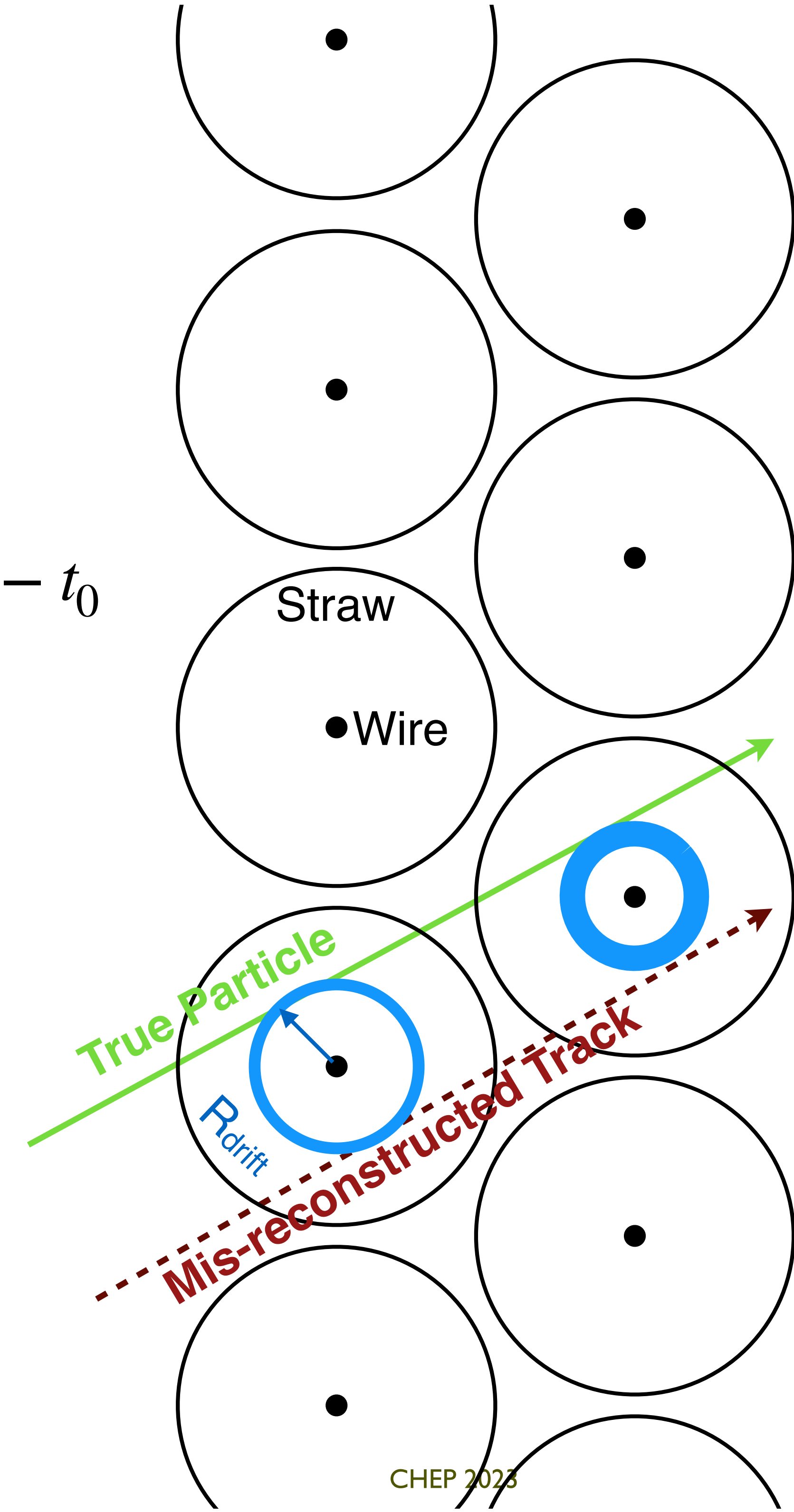
- R_{drift} has a discrete ambiguity
  - The dominant cause of momentum resolution tails
- A dense-layer ANN selects resolvable cases
  - Using consistency with physical drift model
  - Unresolved cases constrain to the wire
- A subsequent ANN infers DOCA from $\Delta t = t_{hit} - t_0$
  - Solves non-linear Time to Distance relation



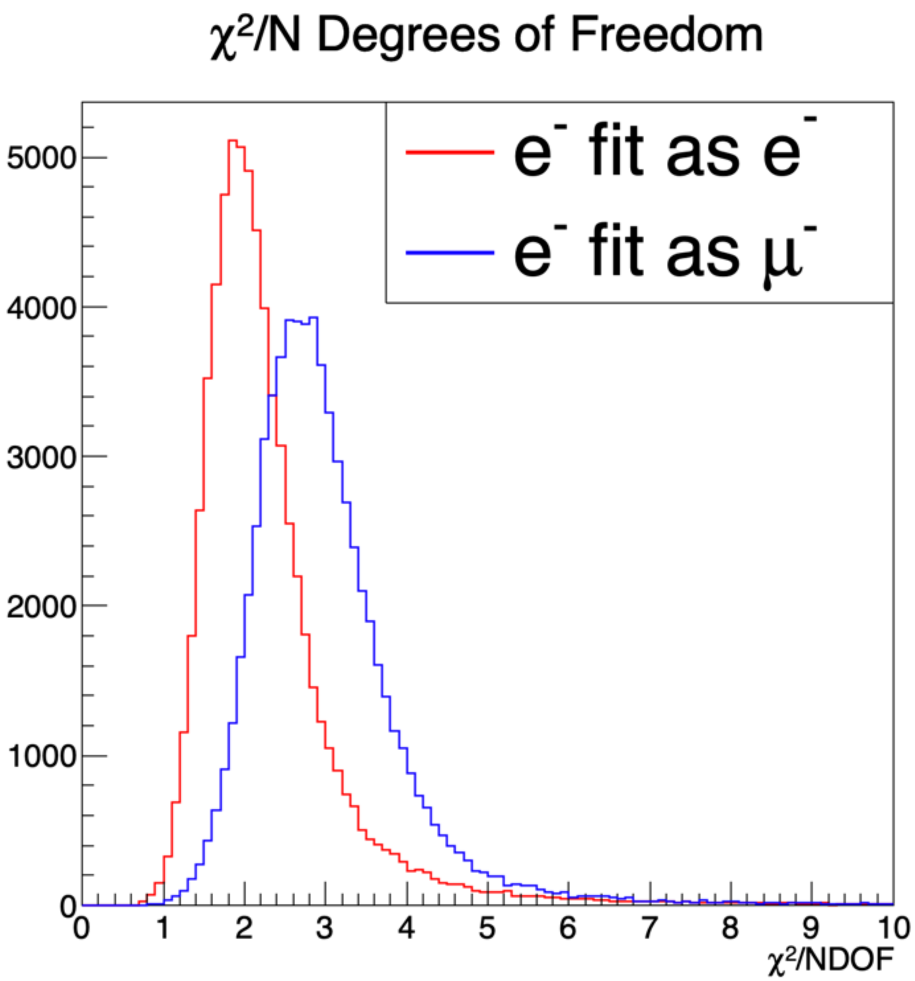


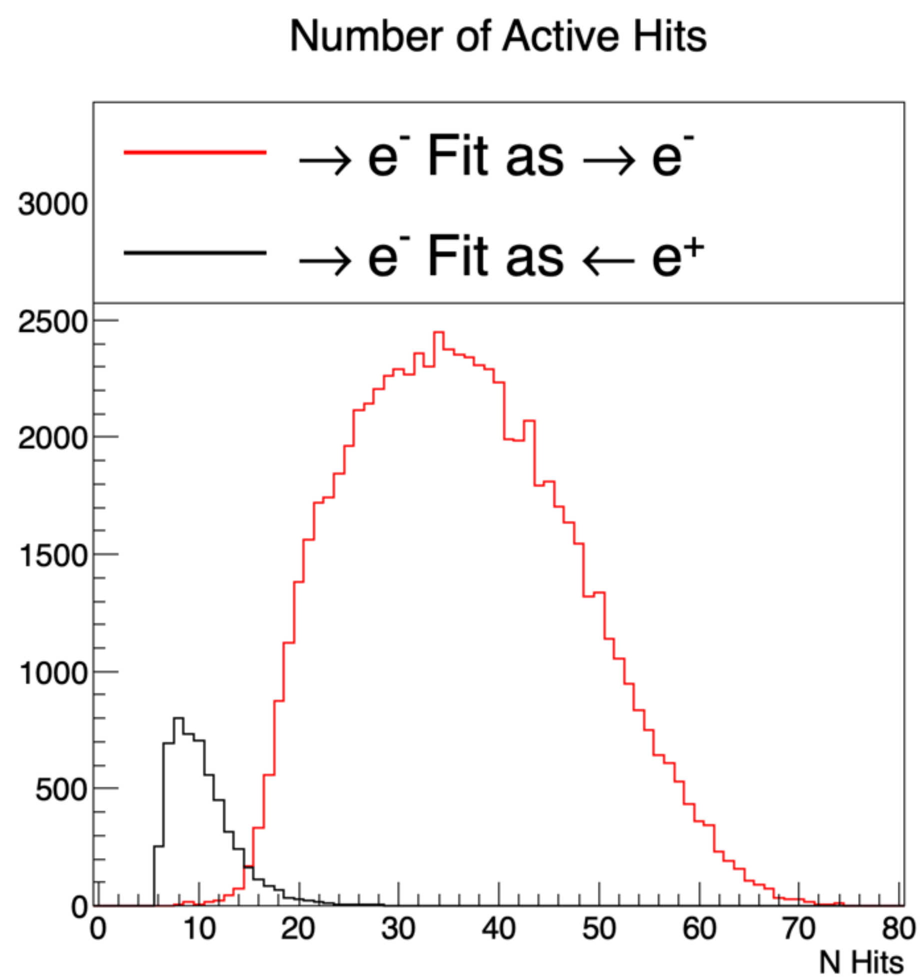






Particle and Direction Identification

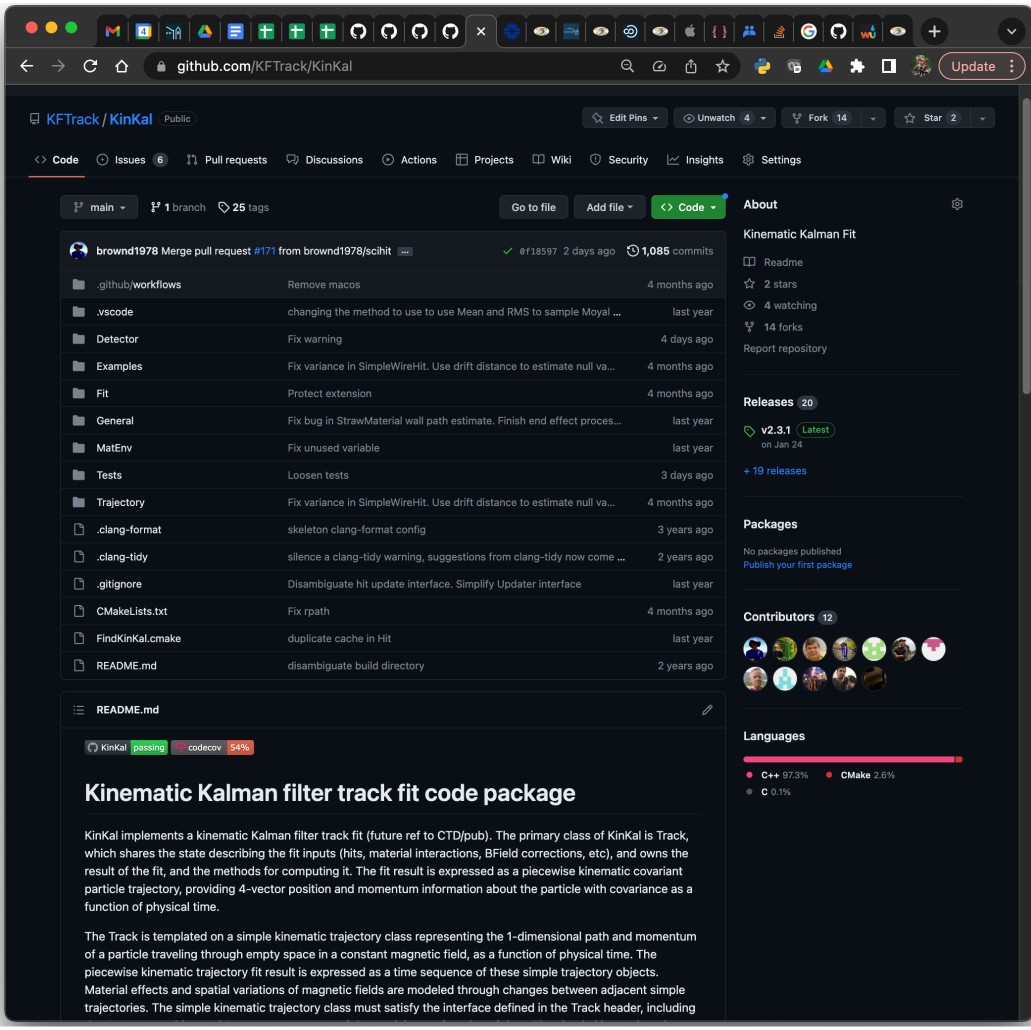




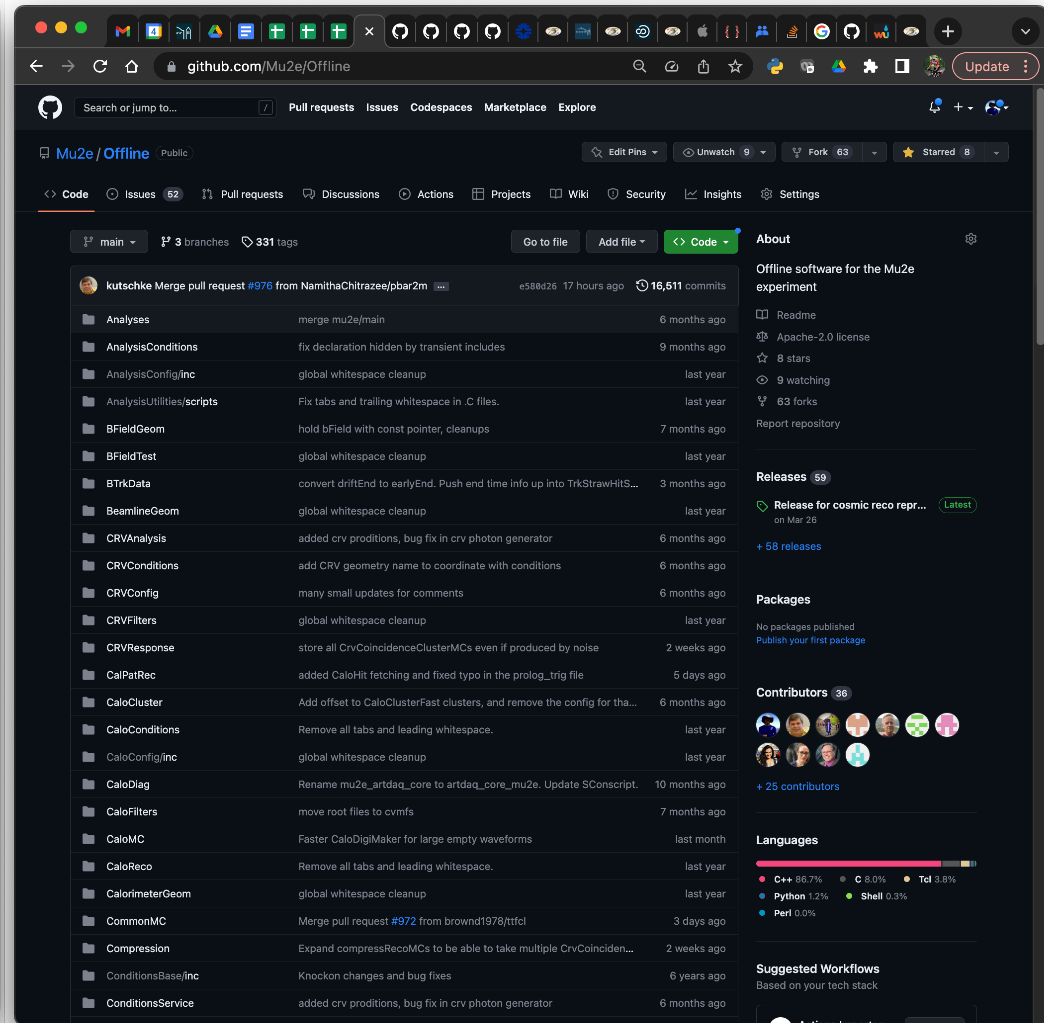
- Fit χ^2/NDOF provides e, μ separation power
- Fit success + (N active hits) separates →e^- from ←e^+
  - Same helicity (geometric parameters), wrong time ordering

Conclusions

KinKal



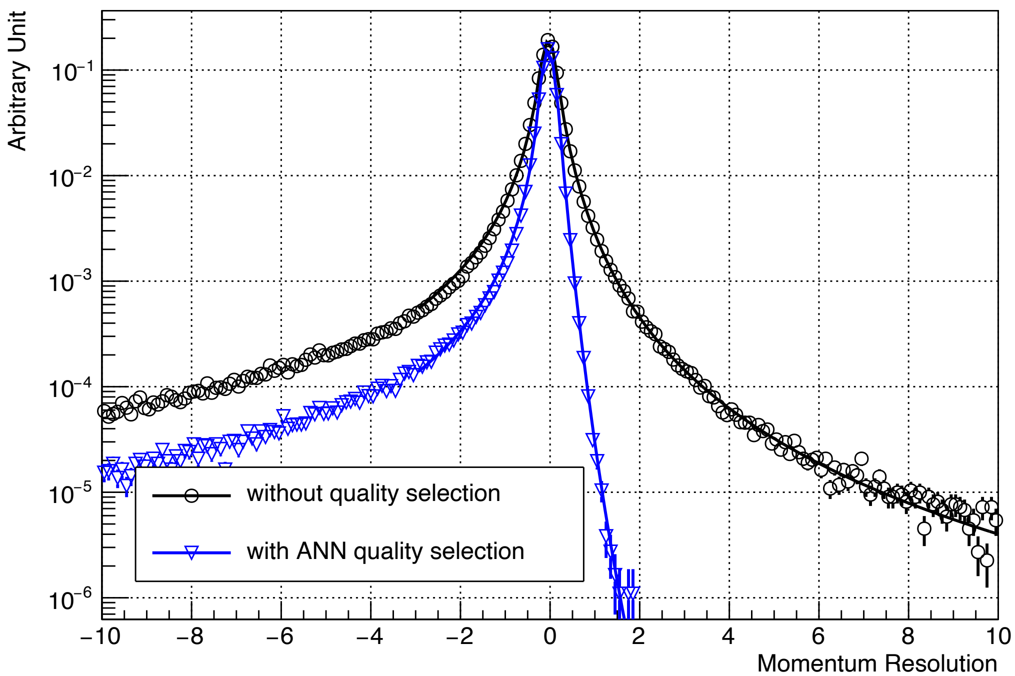
Mu2eKinKal

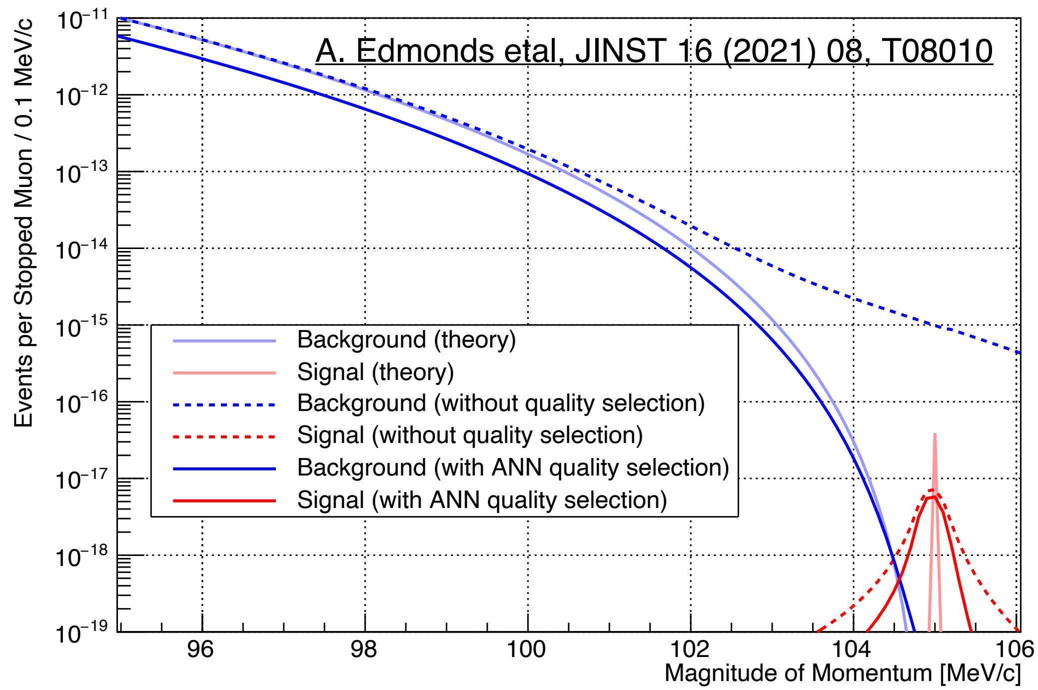


- KinKal is a toolkit for precision low-momentum track fitting
- Mu2e will use it for triggering and analysis

Backup

Track Quality Selection





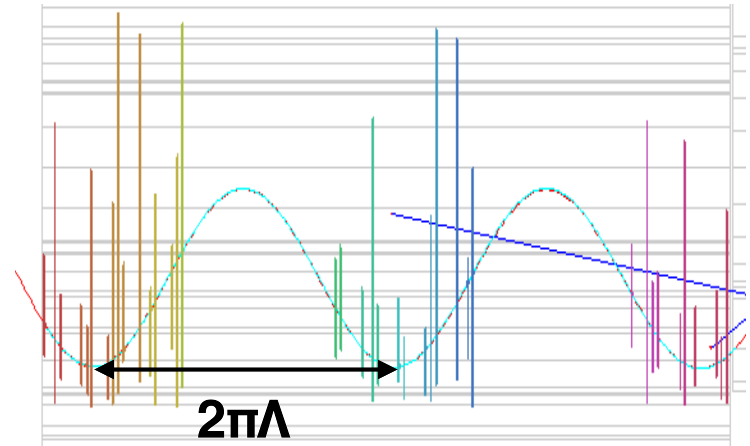
- Momentum resolution tails persist even after hit filtering
  - < 10^{-5} suppression factor needed to achieve Mu2e science goals
- ANN selects tracks with high-quality momentum
  - Using fit global consistency, hit pattern vs expected, …
- 90% signal efficiency, tail reduced by a factor <10^{-3}

LHelix Parameterization (CTD 2015)

$$\mathbf{P} \equiv \{R, \Lambda, C_x, C_y, \phi_0, t_0\}$$

$$\Delta t = t - t_0 \qquad Q = -qc\,|\vec{B}\,|$$

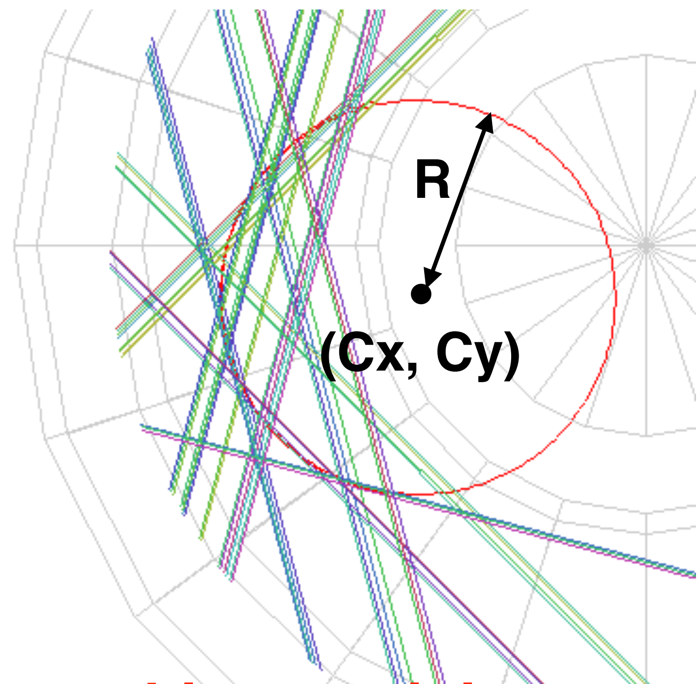
$$\bar{m} = \frac{m}{Q} \qquad \Omega = \frac{qc}{\sqrt{R^2 + \Lambda^2 + \bar{m}^2}}$$



$$x(t) = C_x + R \cdot sin(\Omega\Delta t + \phi_0)$$

$$y(t) = C_y - R \cdot cos(\Omega\Delta t + \phi_0)$$

$$z(t) = \Lambda\Omega\Delta t$$



$$P_x(t) = Q \cdot R \cdot cos(\Omega\Delta t + \phi_0)$$

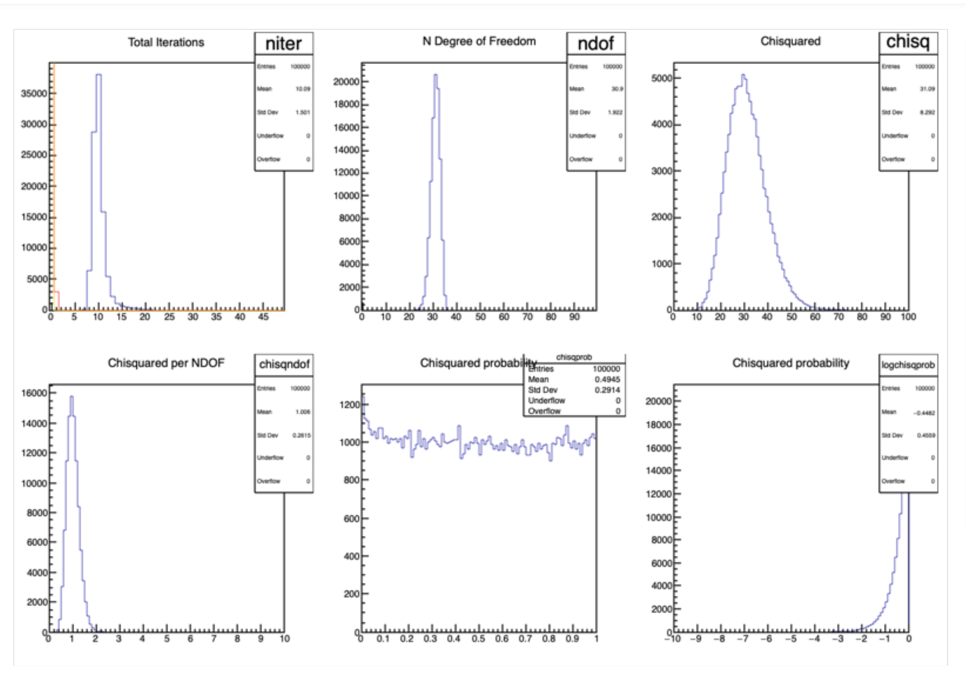
$$P_y(t) = Q \cdot R \cdot sin(\Omega\Delta t + \phi_0)$$

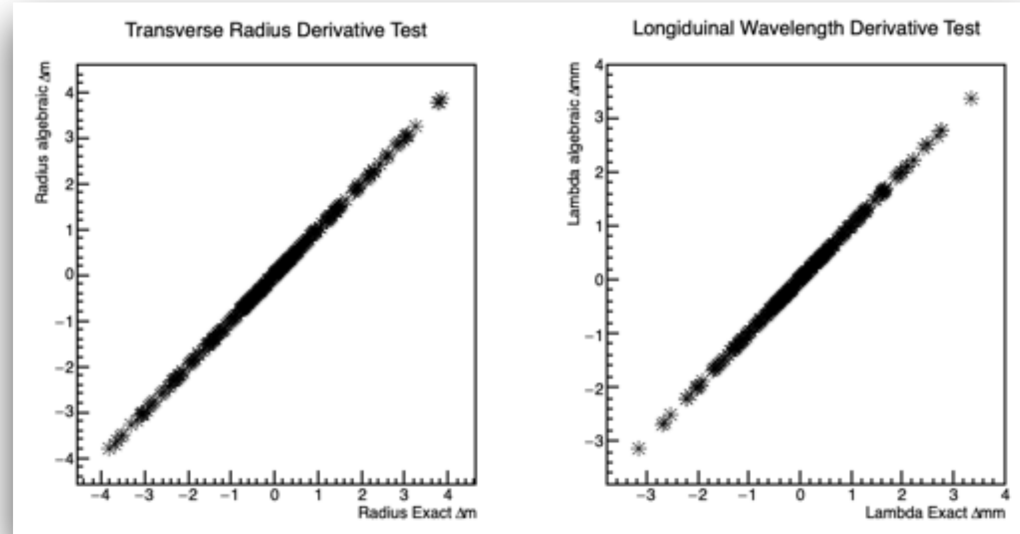
$$P_z(t) = Q \cdot \Lambda$$

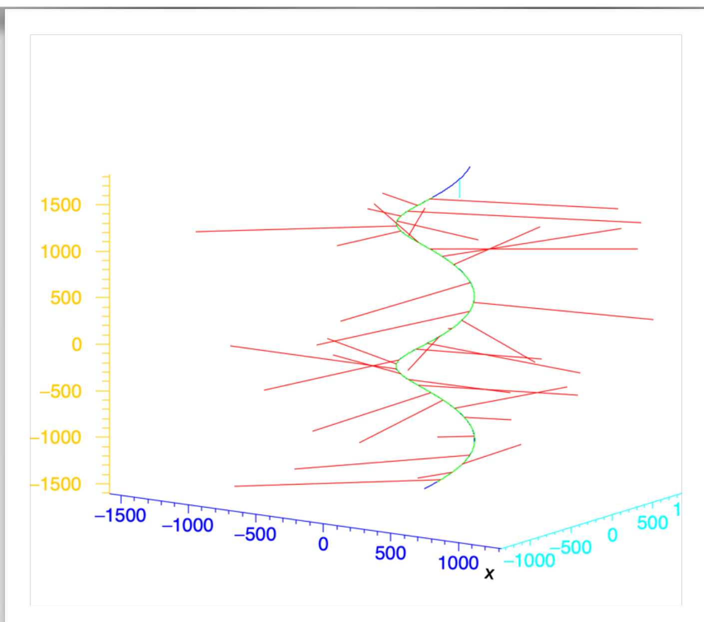
$$P_t(t) = |Q| \cdot \sqrt{R^2 + \Lambda^2 + \bar{m}^2}$$

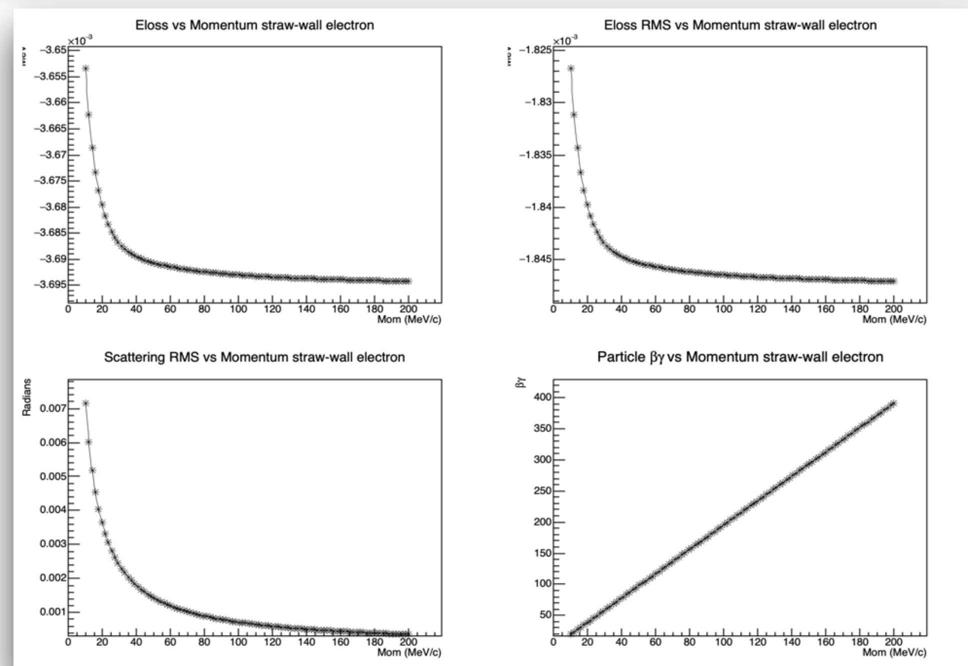
Natural description of a looping track from an arbitrary origin

KinKal Package Unit Tests





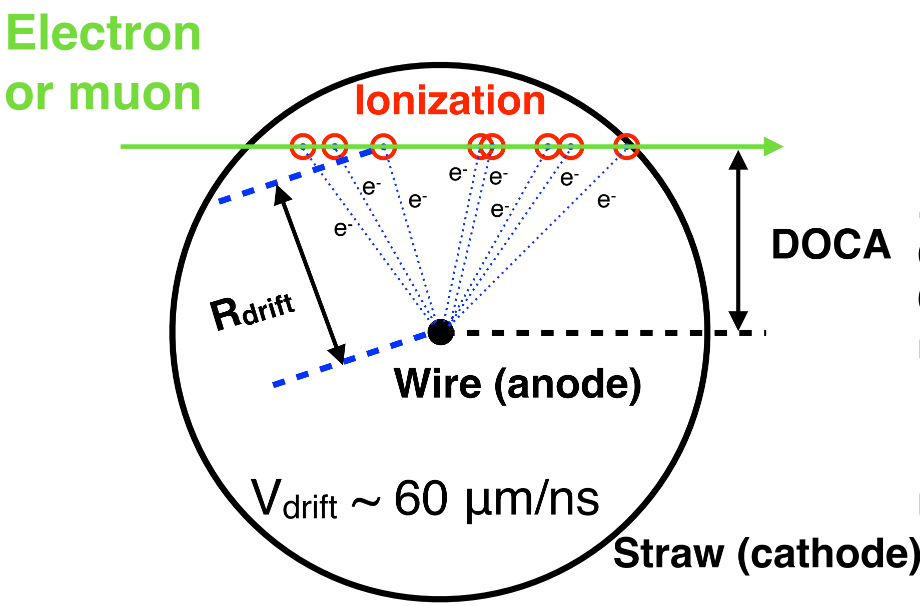


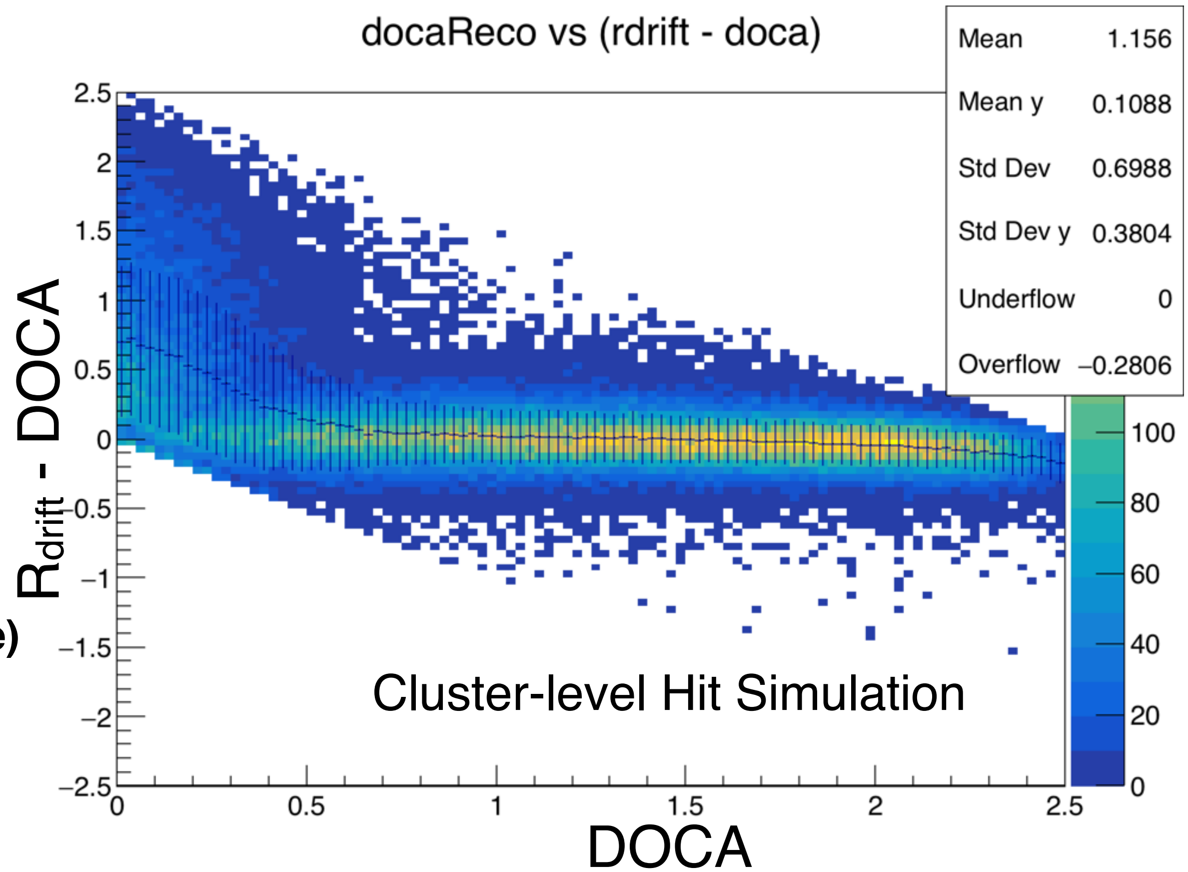


Comparison with BTrk

	BTrk	KinKal
Code	1990s C++ (pre STL)	C++17
Kinematics: mass.Momentum	External interpretation	Internal
parameteric variable	3D flightlength	Physical time
BField Correction	Relative to B_0 ($\Delta B/B < 5\%$)	B →B’ (all $\Delta B/B$)
t_0	external meta-fit parameter	internal parameter
Annealing	External	Internal
Updating Model	Fixed algebraic	ANN-based
Trajectory Models	Central Helix	KLine, CHelix, LHelix
Matrix Algebra	CLHEP::Matrix	Root::Math::SMatrix
Memory	vector<X*>	vector<unique_ptr> …
Linear Algebra	CLHEP, difAlgebra	ROOT::Math::SMatrix, OpenBLAS
SpaceTime	Hep3Vector, BbrLorentzVector	ROOT::Math::GenVector
Execution time	10 ms/track	500 ms/track

Drift Resolution



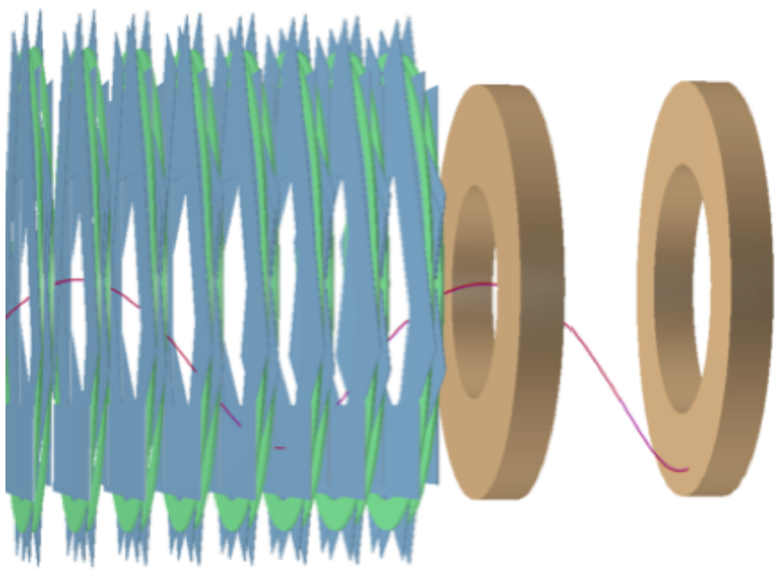


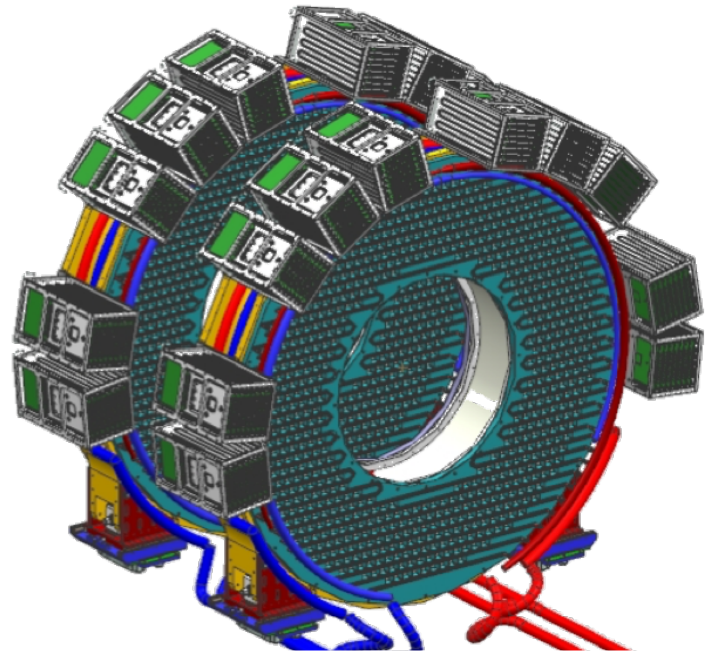
- Far from wire ⇒ hit time directly related to impact parameter
  - Residual = t_{hit} - t_0
- Near to wire ⇒ hit time poor approximation to impact parameter
  - Residual = wire DOCA (no drift information)

LHelix DOCA Derivatives

- Consider wire ⊥ to z at W, azimuth = η
- $\bar{\phi} \equiv \phi_0 + W_z/\Lambda - \eta$
- $\Delta \equiv -\sin(\eta)(C_x - W_x) + \cos(\eta)(C_y - W_y)$
- $F \equiv \Lambda/\mathrm{sqrt}(\Lambda^2 + R^2\sin^2(\bar{\phi}))$
- $DOCA = -F(R\cos(\bar{\phi}) - \Delta) + O(r^2_{straw}/(R,\Lambda))$
- $\delta DOCA/\delta C_x = -F\sin(\eta)$
- $\delta DOCA/\delta C_y = F\cos(\eta)$
- $\delta DOCA/\delta \phi_0 = FR\sin(\bar{\phi})$
- $\delta DOCA/\delta R = -F(\cos(\bar{\phi})\Lambda^2 + \sin^2(\bar{\phi})R\Delta)/(\Lambda^2 + R^2\sin^2(\bar{\phi}))$
- $\delta DOCA/\delta \Lambda = -FR\sin(\bar{\phi})W_z/\Lambda^2$
- $\delta DOCA/\delta t_0 = 0$

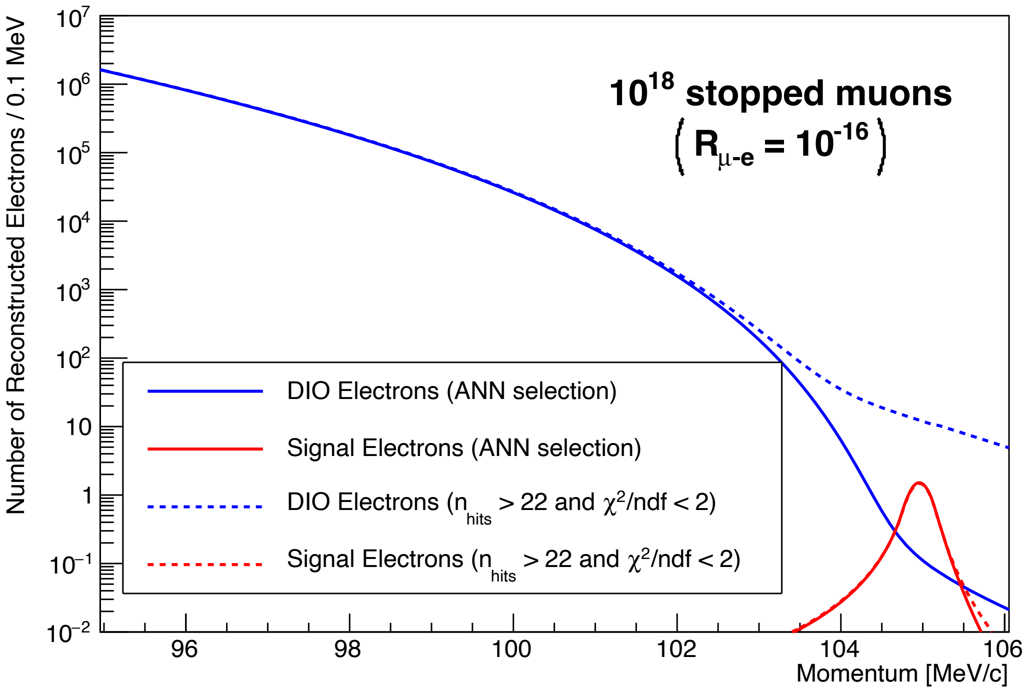
The Mu2e Calorimeter

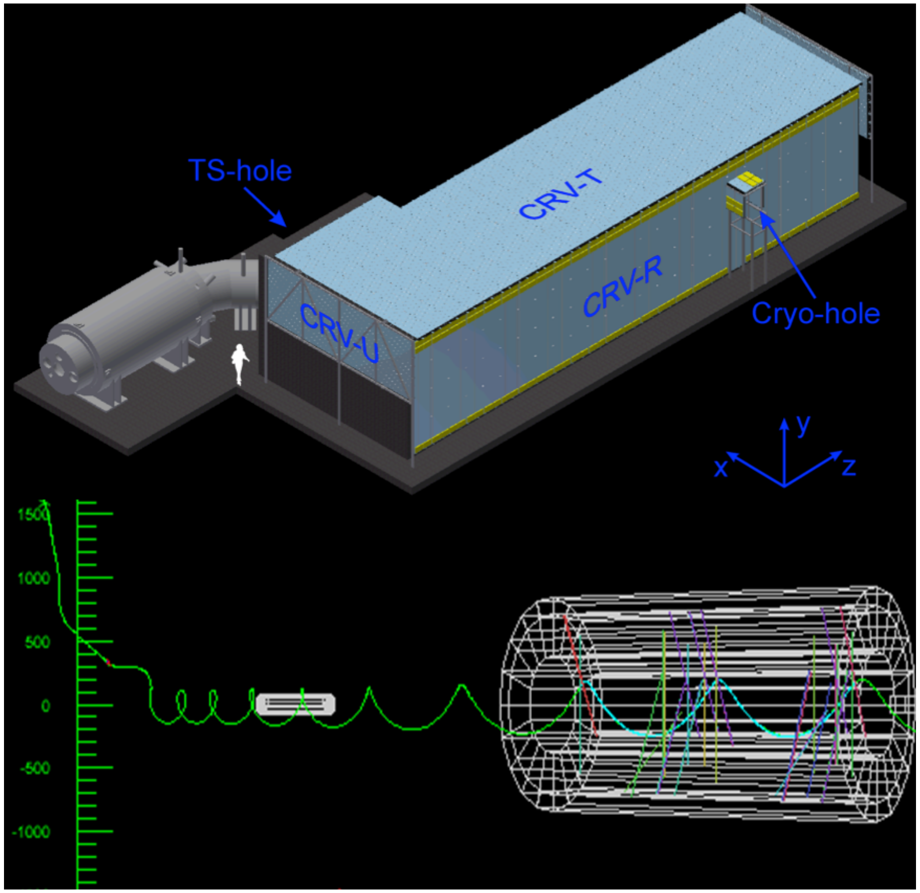




- ~2000 rectangular CsI crystals arranged in 2 disks
  - 5% energy measurement, ~0.5 ns timing measurement
- Primary function: muon-electron separation
  - electron E/p ~ 0.95 (with long tails), muon E/p ~0.4
    - Muon rest mass energy is released in decay, but delayed
  - Dominant discriminant comes from relative timing
    - 105 MeV/c μ^- has β=0.7, arrives ~ 3 ns later at calorimeter WRT e^-

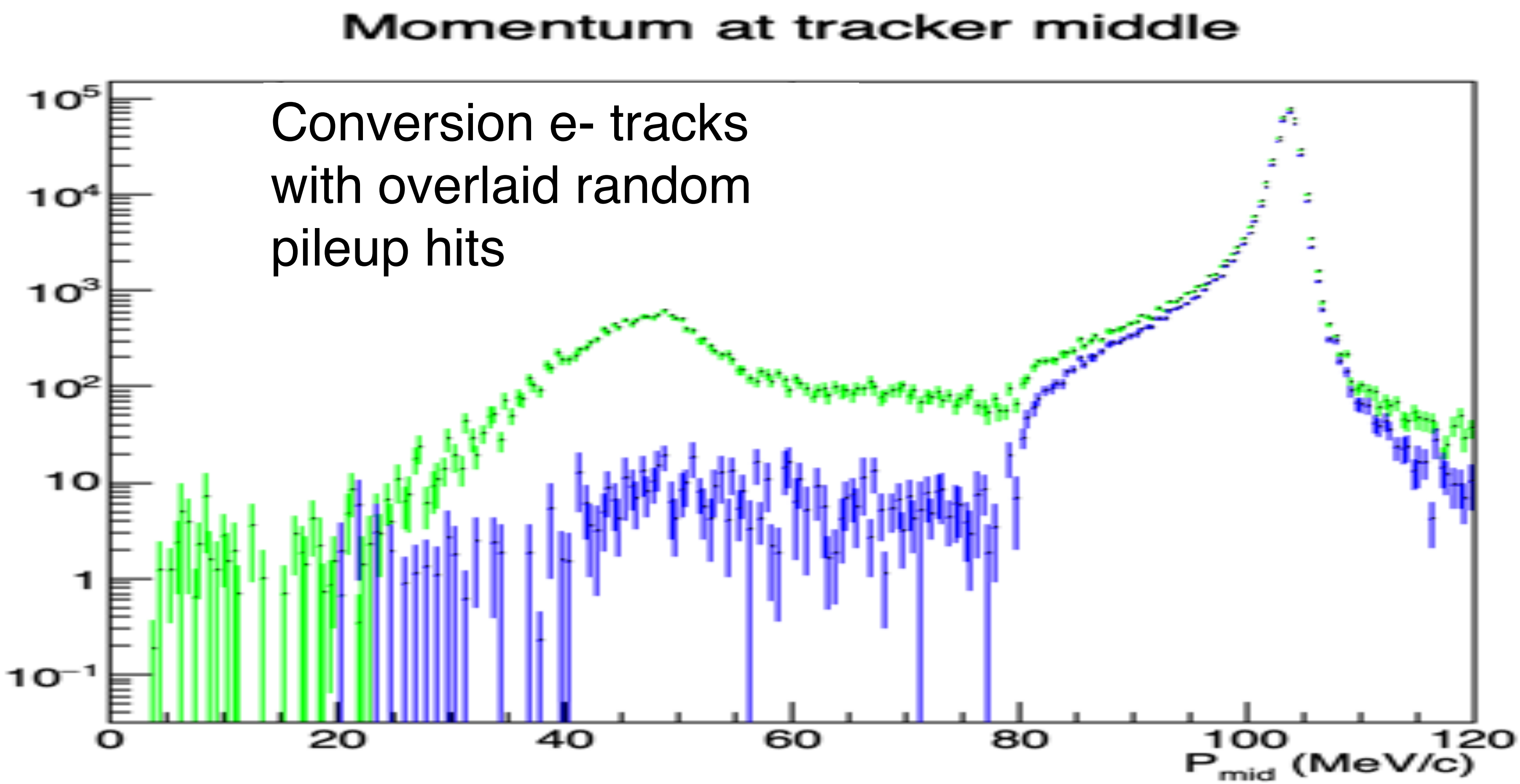
Mu2e Backgrounds





- Goal: < 1 background event over the experiment lifetime
- Suppressing the radiative tail of Michel decay electrons
  - Requires excellent momentum resolution (σ< 500 KeV/c)
- Suppressing cosmic muons
  - Requires an external veto with 99.99% efficiency
  - Requires e/μ separation
  - Requires separating upstream from downstream tracks

Conversion e^- Trigger Fit



- 3 msec/track execution time
- 600 KeV/c Momentum resolution
  - Pileup event rejection power of ~10^{-4}
- 2 redundant track fit seed algorithms

Lynch-Dahl Scattering Model

- Used to compute σ used in Kalman filter fit
- Integral screened Rutherford scattering model, parameterized by ‘tail fraction’
    - larger fraction→larger sigma
- Can be tuned to give ‘flat’ probability distribution
    - larger→ narrower core
    - smaller→smaller tails
    - Hit Straws: tail<0.9999
    - Add Straws: tail<0.995