Meld: Exploring the feasibility of a framework-less framework

Kyle J. Knoepfel
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What type of framework does DUNE need?
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• What DUNE has:
  
  DUNE’s current framework (art) originates from a collider-physics experiment, steeped in event-based concepts.

• But:
  
  The “event” is not always a helpful concept for neutrino experiments.

• What DUNE needs…
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  DUNE’s current framework (art) originates from a collider-physics experiment, steeped in event-based concepts.

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• What DUNE needs…
Some DUNE framework requirements (paraphrased)
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The framework should make minimal assumptions about the data model.

That sounds like a framework-less framework...
But is it so crazy?

- How many of art’s assumptions can be relaxed/removed to meet DUNE’s needs?
But is it so crazy?

• How many of art’s assumptions can be relaxed/removed to meet DUNE’s needs?
• Asking this question has resulted in a 2-year project called Meld, a laboratory-directed R&D project based at Fermilab.
• The goal is to explore options, not necessarily to provide software.
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- The goal is to explore options, not necessarily to provide software.

- Meld has been heavily influenced by:
  - Regular discussions with DUNE experts
  - Existing framework capabilities and limitations
  - Functional programming (e.g. Haskell)
  - Mathematics (set, graph, and category theory)
But is it so crazy?

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Prerequisites
- Support user-provided algorithms written in C++20 or newer
- Design for concurrency
- Favor community-provided software
Looking at the data

The following discussion describes a logical organization of data.  

*It does not imply a specific in-memory representation of data.*
Looking at the data (set)
Looking at the data (products)
Looking at the data (products)
Looking at the data (products)
Looking at the data (products)
Looking at the data (products)
Looking at the data (products)
Looking at the data (products)

\[ \mathcal{R} \]

\[ S_1 \]

\[ E_1 \]

\[ a \cdot b \cdot \cdot J \]

\[ E_2 \]

\[ a \cdot b \cdot \cdot K \]

\[ S_2 \]

\[ E_3 \]

\[ a \cdot b \cdot \cdot J \]

\[ E_4 \]

\[ a \cdot b \cdot \cdot K \]

\[ S_3 \]

\[ E_5 \]

\[ a \cdot b \cdot \cdot J \]

\[ E_6 \]

\[ a \cdot b \cdot \cdot K \]

\[ S_4 \]

\[ E_7 \]

\[ a \cdot b \cdot \cdot J \]

\[ E_8 \]

\[ a \cdot b \cdot \cdot K \]
Looking at the data (product mappings)
Looking at the data (product sequences)
Looking at the data (product sequences)
Looking at the data (product sequences)
Looking at the data (product sequences)

We can make the following replacement (e.g.):

\[ C = (C)_8 \]

depicting the data products labeled \( c \) from 8 events as a sequence.
Looking at the data (product sequences)
What type of things are we dealing with?

- An operation that converts a sequence of elements \((a)_8\) to a sequence of elements \((b)_8\) of the same length using a function \(f\):
What type of things are we dealing with?

• An operation that converts a sequence of elements $(a)_8$ to a sequence of elements $(b)_8$ of the same length using a function $f$:

This is a map or transform.
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  This is a fold or reduction.
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- An operation that pairs element of two sequences \((J)_4\) and \((K)_4\) into one sequence \((J,K)_4\):

\[
\begin{align*}
(a)_8 & \xrightarrow{f} (b)_8 \\
(c)_8 & \xrightarrow{g_0} (J)_4, (K)_4 \\
(J)_4 & \xrightarrow{h_0} (W)_1
\end{align*}
\]
What type of things are we dealing with?

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- An operation that pairs element of two sequences \((J)_4\) and \((K)_4\) into one sequence \((J,K)_4\):
  
  This is a zip.
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- An operation that converts a sequence of elements \((a)_8\) to a sequence of elements \((b)_8\) of the same length using a function \(f\):
  
  ![Diagram showing the map or transform process](image)
  
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- An operation that converts a sequence of elements \((c)_8\) to a shorter sequence of elements \(K_0\) at a higher level of nesting, using a function \(g_0\):

  ![Diagram showing the fold or reduction process](image)

  This is a fold or reduction.

- An operation that pairs elements of two sequences \((J)_4\) and \((K)_4\) into one sequence \((J,K)_4\):

  ![Diagram showing the zip process](image)

  This is a zip.

These have to do with higher-order functions.
Graph of data-product sequences

View | Nodes | Edges
--- | --- | ---
Data-centric | Data products | Mappings

This work
Graph of data-product sequences

<table>
<thead>
<tr>
<th>View</th>
<th>Nodes</th>
<th>Edges</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data-centric</td>
<td>Data products</td>
<td>Mappings</td>
<td>This work</td>
</tr>
<tr>
<td>Map-centric</td>
<td>Mappings</td>
<td>Data products</td>
<td>More common</td>
</tr>
</tbody>
</table>
The user specifications are the same with either view:

- Which data products to process
- The data set(s) that contain those products (event, etc.)
- Which higher-order function to use (transform, etc.)
- Which user-defined function to serve as the operation to the higher-order function.
- Allowed concurrency of each function.

The focus is just different.
How are data products and their mappings supported now?

With art, users do not transparently interact with data products. They instead:

- Implement functions based on datasets (e.g. event)
- “Open” the dataset to retrieve and insert products
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- Implement functions based on datasets (e.g. event)
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Some of this is historical and due to:

- The object-oriented nature of the framework.
- Technical limitations of C++ whenever the framework was designed.
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- Implement functions based on datasets (e.g. event)
- "Open" the dataset to retrieve and insert products

Some of this is historical and due to:

- The object-oriented nature of the framework.
- Technical limitations of C++ whenever the framework was designed.

Results in a lot of software mechanics...
Example

- Create tracks from hits for each event.
Example

- Create tracks from hits for each event.

\[
\begin{array}{c}
\mathcal{E}_n \\
\xrightarrow{f} \\
(b)_8 \\
\end{array}
\]

\[
\begin{array}{c}
\mathcal{E}_n \\
\xrightarrow{\text{make\_tracks}} \\
(GoodTracks)_8 \\
\end{array}
\]

\[
\text{Tracks \, make\_tracks(Hits \, const& \, hits) \, \{ \, ... \, \}}
\]
Create tracks from hits for each event.

\[
\begin{align*}
\mathcal{E}_n & \xrightarrow{(a)_8} f \\
\mathcal{E}_n & \xrightarrow{(b)_8}
\end{align*}
\]

Example

```cpp
#include "art/Framework/Core/SharedProducer.h"
#include "art/Framework/Principal/Event.h"

namespace {
    Tracks make_tracks(Hits const& hits) { ... }
}

namespace expt {
    class TrackMaker : public art::SharedProducer {
        public:
            TrackMaker(fhicl::ParameterSet const&)
            {
                consumes<Hits, art::InEvent>("GoodHits");
                produces<Tracks, art::InEvent>("GoodTracks");
                async<art::InEvent>();
            }
            void produce(art::Event& e, art::ProcessingFrame const&) override
            {
                auto const& hits = e.getProduct<Hits>("GoodHits");
                auto tracks = make_tracks(hits);
                e.put(std::make_unique<Tracks>(std::move(tracks)), "GoodTracks");
            }
    }
}

DEFINE_ART_MODULE(expt::TrackMaker)
```
• Create tracks from hits for each event.

Example

This is just a transform? 😳

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#include "art/Framework/Core/SharedProducer.h"
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        }
    }
}

DEFINE_ART_MODULE(expt::TrackMaker)
```
• Create tracks from hits for each event.

\[ \mathcal{E}_n \xrightarrow{f} (a)_8 \]

\[ (a)_8 \xrightarrow{f} \mathcal{E}_n \xrightarrow{\text{make_tracks}} (b)_8 \]

Example: Create tracks from hits for each event.

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        void produce(art::Event& e, art::ProcessingFrame const&) override {
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        }
    }
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This is just a transform? 😬
Create tracks from hits for each event.

Example

\[ \mathcal{E}_n \xrightarrow{f} \mathcal{E}_n \]

(a) \[ \mathcal{E}_n \xrightarrow{f} \mathcal{E}_n \]

(GoodHits) \[ \mathcal{E}_n \xrightarrow{f} \mathcal{E}_n \]

(GoodTracks)

This is just a transform? 😞 Nobody wants this.

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Example

- Create tracks from hits for each event.

\[
\begin{align*}
&\mathcal{E}_n \\
\xrightarrow{f} & (a)_8 \\
\xrightarrow{(b)_8} & (b)_8
\end{align*}
\]

\[
\begin{align*}
&\mathcal{E}_n \\
\xrightarrow{\text{make\_tracks}} & (\text{GoodHits})_8 \\
\xrightarrow{} & \text{(GoodTracks)}_8
\end{align*}
\]

A better way...

```cpp
#include "meld/module.hpp"

namespace {
    Tracks make_tracks(Hits const & hits) { ... }
}

DEFINE_MODULE(m, config) {
    m.with(make_tracks)
        .transform("GoodHits").in_each("Event")
        .to("GoodTracks")
        .using_concurrency(unlimited);
}
```
Example

- Create tracks from hits for each event.

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A better way...

- Minimal boilerplate.
Example

- Create tracks from hits for each event.

\[ \mathcal{E}_n \xrightarrow{f} (a)_b \]

\[ (Good\text{-}Hits)_{\mathcal{E}_n} \xrightarrow{make\text{-}tracks} (Good\text{-}Tracks)_{(a)_b} \]

### A better way…

- Minimal boilerplate.
- Event is now a label.

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Meld
Example

• Create tracks from hits for each event.

\[
\begin{align*}
\mathcal{E}_n & \xrightarrow{f} (b)_8 \\
\mathcal{E}_n & \xrightarrow{\text{make_tracks}} \text{(GoodTracks)}_8
\end{align*}
\]

A better way…

• Minimal boilerplate.
• Event is now a label.
• Higher-order function is now explicit.

```cpp
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```
Meld implementation

- [https://github.com/knoepfel/meld](https://github.com/knoepfel/meld) (not even alpha release)
- Implemented using oneTBB’s flow graph

<table>
<thead>
<tr>
<th>Supported construct</th>
<th>User function</th>
<th>Notes</th>
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<tr>
<td><strong>Transform</strong> (Map)</td>
<td>$f(a) \rightarrow b$</td>
<td><strong>Standard data-processing idioms</strong></td>
</tr>
<tr>
<td>Filter</td>
<td>$f(a) \rightarrow \text{Boolean}$</td>
<td></td>
</tr>
<tr>
<td>Monitor</td>
<td>$f(a) \rightarrow \text{Void}$</td>
<td></td>
</tr>
<tr>
<td><strong>Reduction</strong> (Fold)</td>
<td>$f_c(a) \rightarrow c$</td>
<td><strong>For splitting and then combining events</strong></td>
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<tr>
<td>Splitter (Unfold)</td>
<td>$f_n(a) \rightarrow (d)_n$</td>
<td></td>
</tr>
<tr>
<td>Zip</td>
<td>—</td>
<td><strong>For combining arguments to user functions</strong></td>
</tr>
<tr>
<td>Sliding window</td>
<td>—</td>
<td><strong>To do: For sliding over adjacent events</strong></td>
</tr>
</tbody>
</table>
Sample hierarchies tested by Meld

Performance numbers are preliminary

art-based hierarchy
Sample hierarchies tested by Meld

Performance numbers are preliminary

Non-trivial hierarchy
Sample hierarchies tested by Meld

- Art-based hierarchy
- Non-trivial hierarchy
- Flat hierarchy

Performance numbers are preliminary
Summary

“Ways change, Stil.” —Paul from *Dune* by Frank Herbert

- Supporting DUNE’s framework needs suggests rethinking framework concepts.
Summary

“Ways change, Stil.” —Paul from Dune by Frank Herbert

• Supporting DUNE’s framework needs suggests rethinking framework concepts.
• Meld seeks to address these needs by considering a framework job as a

(1) graph of data products connected by
(2) user-provided operations of
(3) higher-order functions.

• It is not a framework-less framework, but it is less framework coupling.
• Preliminary work indicates this is a productive avenue to pursue.
Summary

“Ways change, Stil.” —Paul from Dune by Frank Herbert

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• It is not a framework-less framework, but it is less framework coupling.
• Preliminary work indicates this is a productive avenue to pursue.

Thank you for your time and attention.
Backup slides
Accessing provenance information

```cpp
#include "meld/module.hpp"

namespace {
  Tracks make_tracks(Hits const& hits) { ... }
  Tracks make_tracks(meld::handle<Hits> hits) { ... }
}

DEFINE_MODULE(m, config) {
  m.with(make_tracks)
    .transform("GoodHits").in_each("Event")
    .to("GoodTracks")
    .using_concurrency(unlimited);
}
```
#include "meld/module.hpp"

DEFINE_MODULE(m, config)
{
    auto threshold = config.get<unsigned int>("threshold");
    m.with([threshold](Hits const& hits) { return hits.size() > threshold; })
        .filter("GoodHits").in_each("Event")
        .using_concurrency(unlimited);
}
#include "meld/module.hpp"

class Selector {
public:
    Selector(unsigned int n) : threshold{n} {}
    bool gt(Hits const& hits) const { return hits.size() > threshold; }
    bool le(Hits const& hits) const { return !gt(hits); }

private:
    unsigned int threshold;
};

DEFINE_MODULE(m, config)
{
    auto threshold = config.get<unsigned int>("threshold");
    auto bound_m = m.make<Selector>(threshold);
    bound_m.with(&Selector::gt).filter("GoodHits").in_each("Event");
    bound_m.with(&Selector::le).filter("GoodHits").in_each("Event");
}
Reduction example

class MyAccumulator : public art::EDProducer {
public:
    MyAccumulator(ParameterSet const&)
    {
        produces<int, art::InSubRun>("sum");
    }

    void produce(art::Event&) override
    {
        ++counter_;
    }

    void endSubRun(art::SubRun& sr) override
    {
        sr.put(std::make_unique<int>(counter_), "sum");
        counter_ = 0;
    }

private:
    int counter_ = 0;
};

DEFINE_ART_MODULE(MyAccumulator)

void accumulate(int& counter, meld::level_id const&)
{
    ++counter;
}

DEFINE_MODULE(m) {
    m.with(accumulate, 0).for_each("SubRun")
        .reduce("id").in_each("Event")
        .to("sum");
}
Each element of the set is a *data product*, which is:

- Opaque to the framework
  \[\Rightarrow\text{Separation of user space from framework}\]
- Immutable (definition of set element)
- A member of *at least* one set
- Identifiable
Higher-order functions

• We are interested in the mappings of the form:

\[ \left\{ (a)_n \xrightarrow{f} (b)_m \right\} \in \mathcal{D} \]

• Each object \( a \) corresponds to a tuple of arguments passed to \( f \).
• The signature of \( f \) and the value \( f(a) \), depends on the higher-order function.
• The above mapping happens within a domain \( \mathcal{D} \) (e.g. job, run, event).
• Each object \( a \) is an element of a subset of the domain \( \mathcal{D} \).
## Supported higher-order functions

<table>
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<tr>
<th>Meld term</th>
<th>CS term</th>
<th>Mathematical description</th>
<th>Domain</th>
</tr>
</thead>
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<tr>
<td><strong>Transform</strong></td>
<td>Map</td>
<td>( (a)_n \xrightarrow{f} (b)_n ) where ( f(a) \to b )</td>
<td>Same as ( (a)_n )</td>
</tr>
<tr>
<td><strong>Filter</strong></td>
<td>Filter</td>
<td>( (a)_n \xrightarrow{f} (a)_m ) where ( m \leq n ) where ( f(a) \to \text{Boolean} )</td>
<td>Same as ( (a)_n )</td>
</tr>
<tr>
<td><strong>Monitor</strong></td>
<td>—</td>
<td>( (a)_n \xrightarrow{f} ( )_0 ) where ( f(a) \to \text{Void} )</td>
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</tr>
<tr>
<td><strong>Reduction</strong></td>
<td>Fold</td>
<td>( (a)_n \xrightarrow{f_c} (c)_1 ) where ( f_c(a) \to c )</td>
<td>Above ( (a)_n )</td>
</tr>
<tr>
<td><strong>Splitter</strong></td>
<td>Unfold</td>
<td>( (a)_1 \xrightarrow{f_n} (d)_m ) where ( f_n(a) \to (d)_n )</td>
<td>Below ( (a)_n )</td>
</tr>
<tr>
<td><strong>Zip</strong></td>
<td>Zip</td>
<td>( ((a)_n, (b)_n) \to (a, b)_n )</td>
<td>More nested domain</td>
</tr>
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