Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

Meld: Exploring the feasibility of a framework-less framework

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DUNE Offline Computing

Conceptual Design Report

https://doi.org/10.48550/arXiv.2210.15665

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😧 ***That sounds like a framework-less framework...***

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  - Regular discussions with DUNE experts
  - Existing framework capabilities and limitations
  - Functional programming (e.g. Haskell)
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Prerequisites

Support user-provided algorithms written in C++20 or newer

Design for concurrency

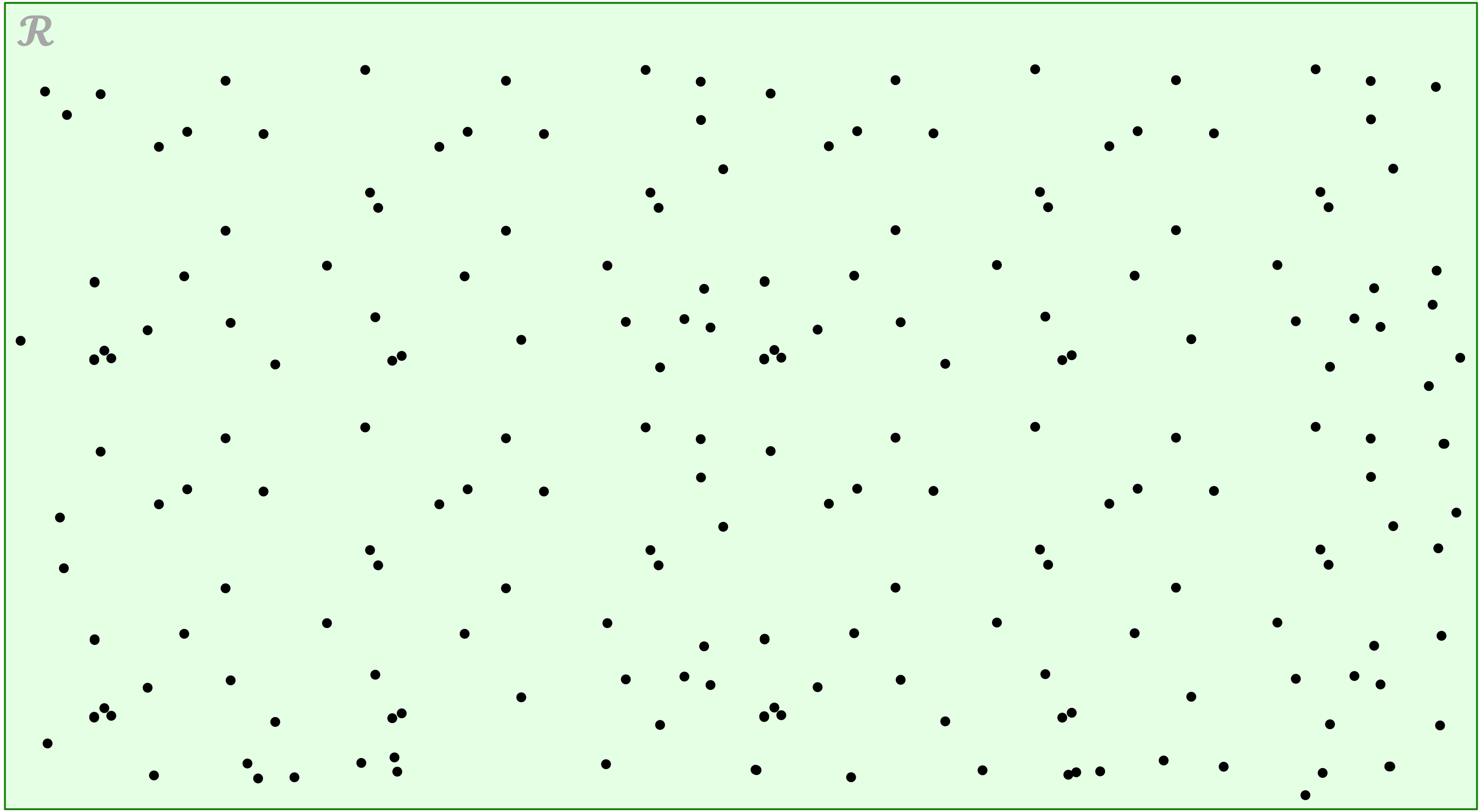
Favor community-provided software

Looking at the data

The following discussion describes a logical organization of data.

It does not imply a specific in-memory representation of data.

Looking at the data (set)



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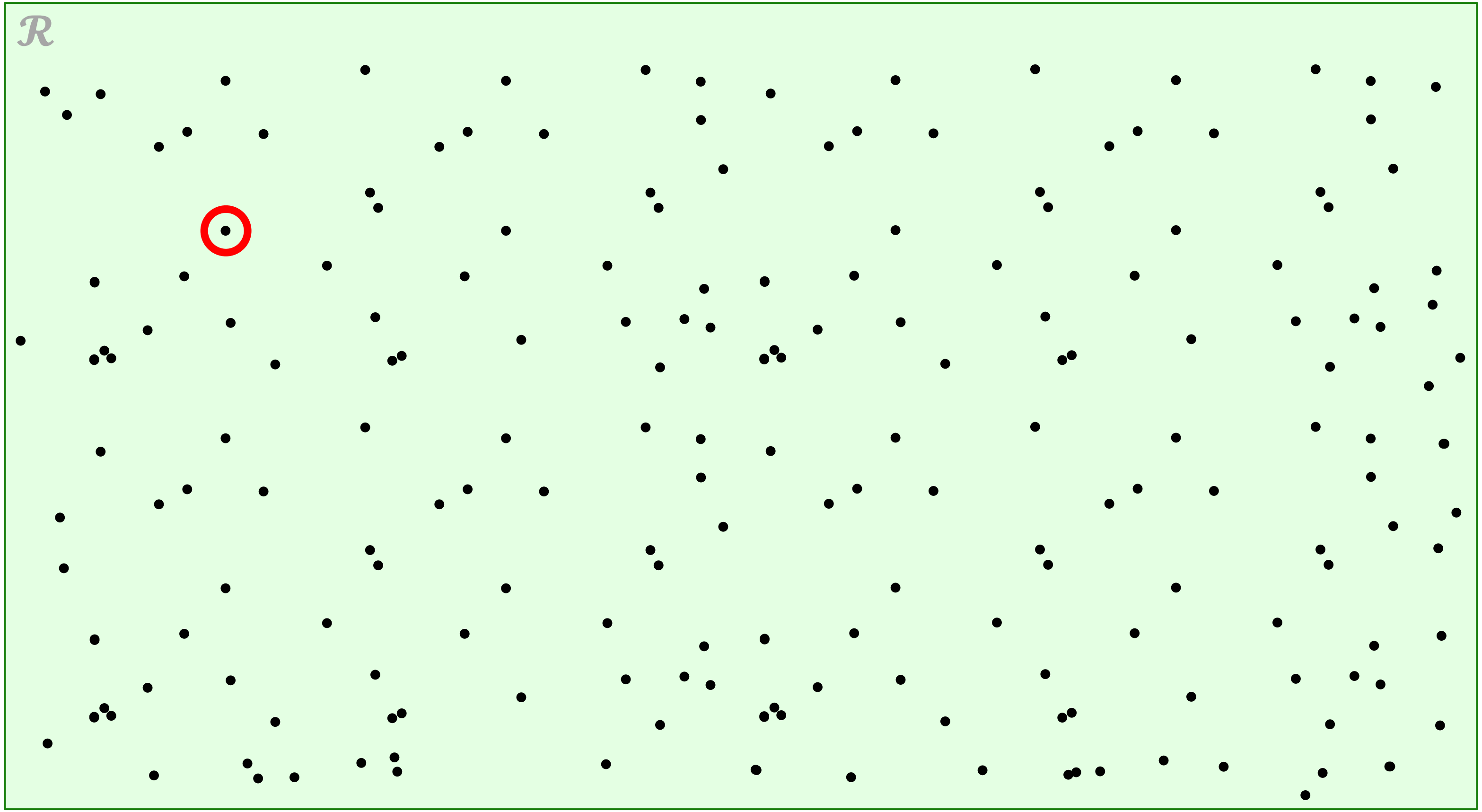

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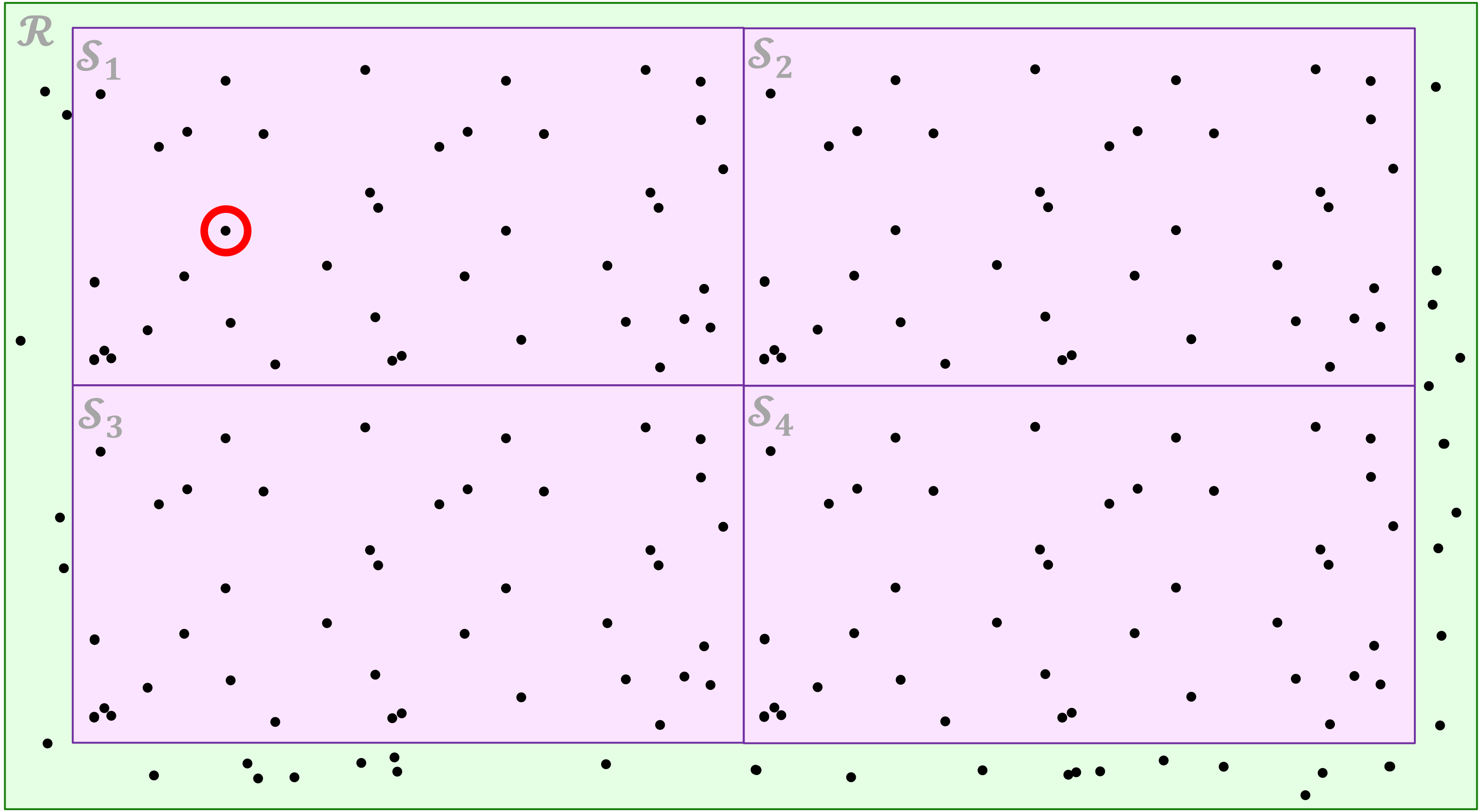

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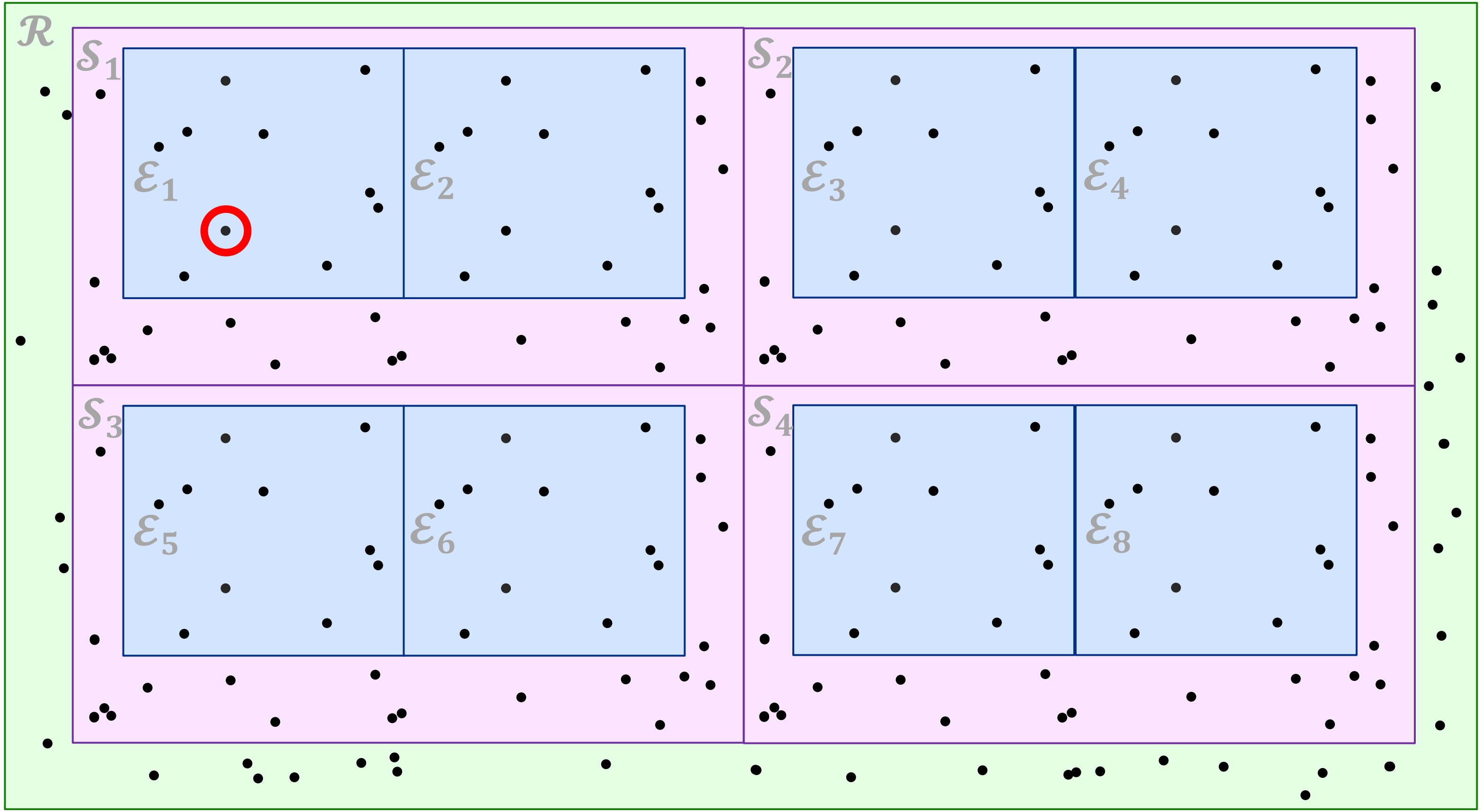

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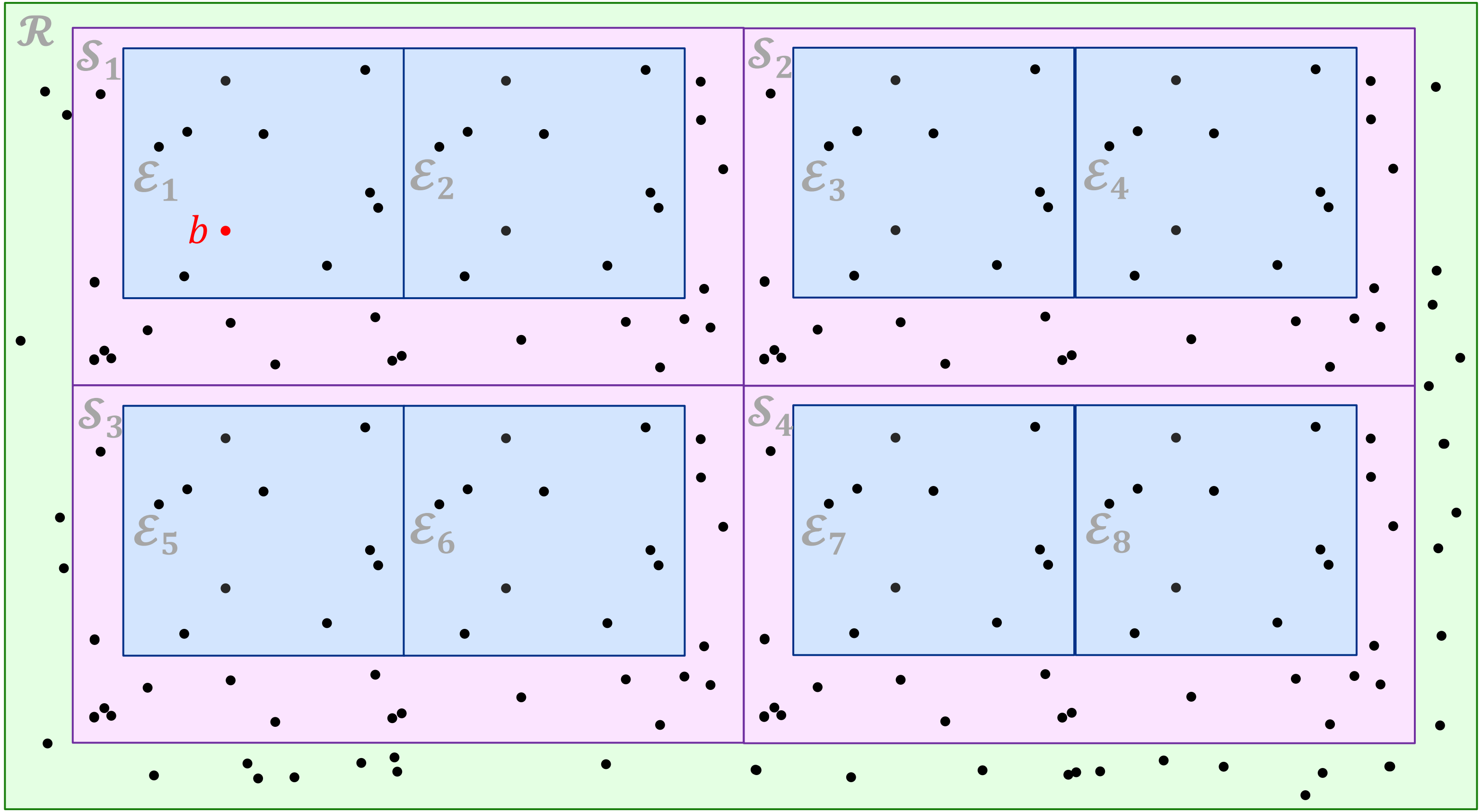

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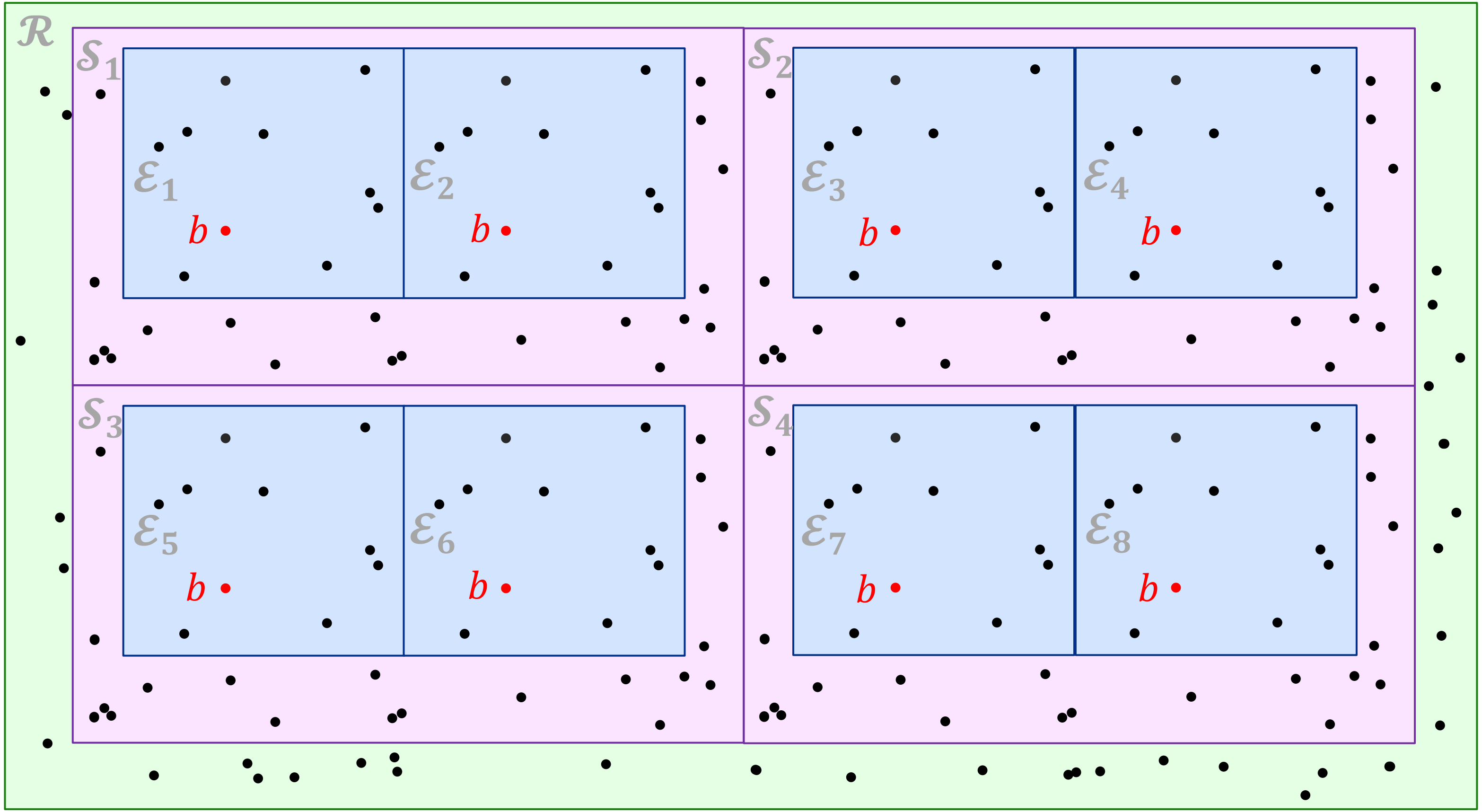

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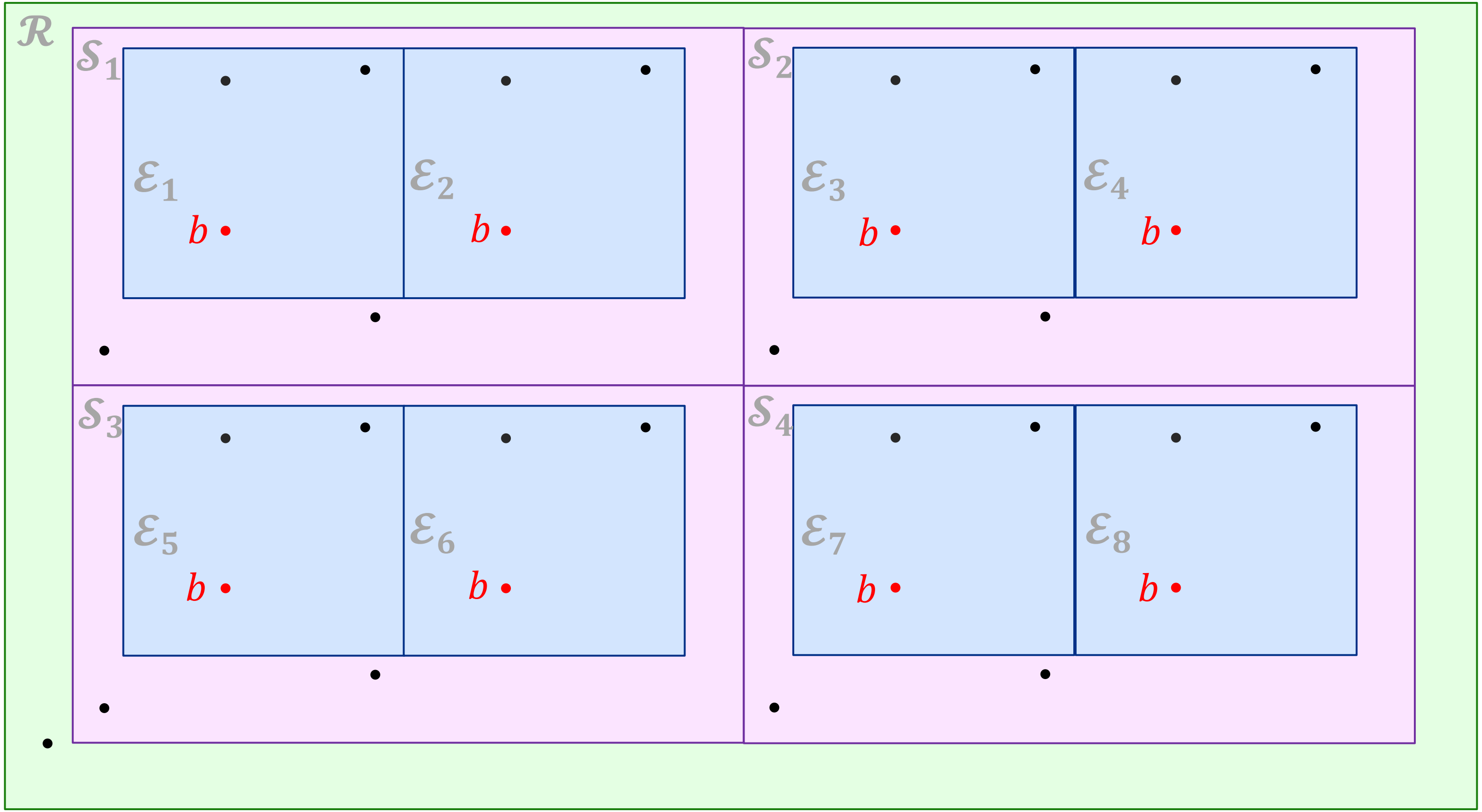

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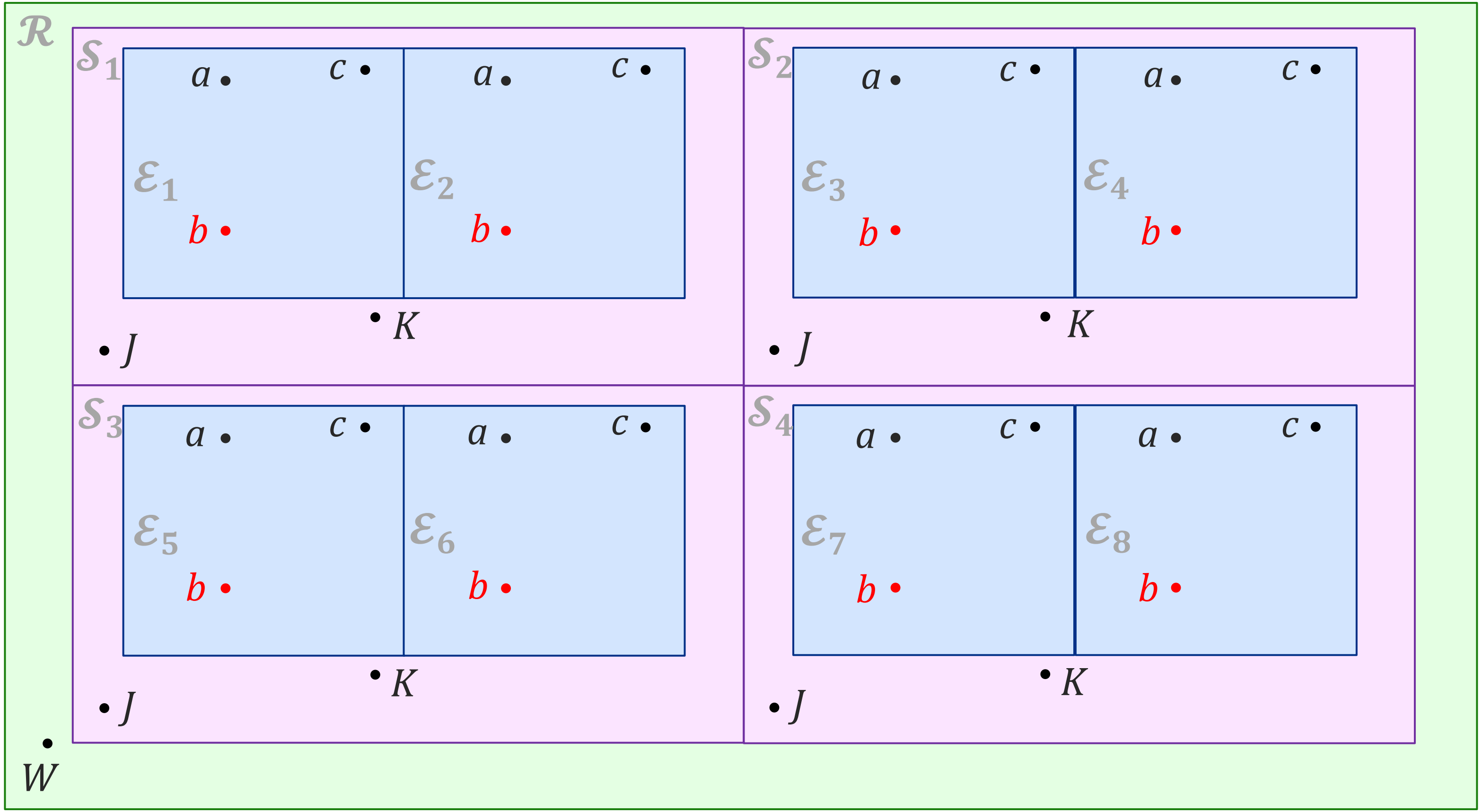

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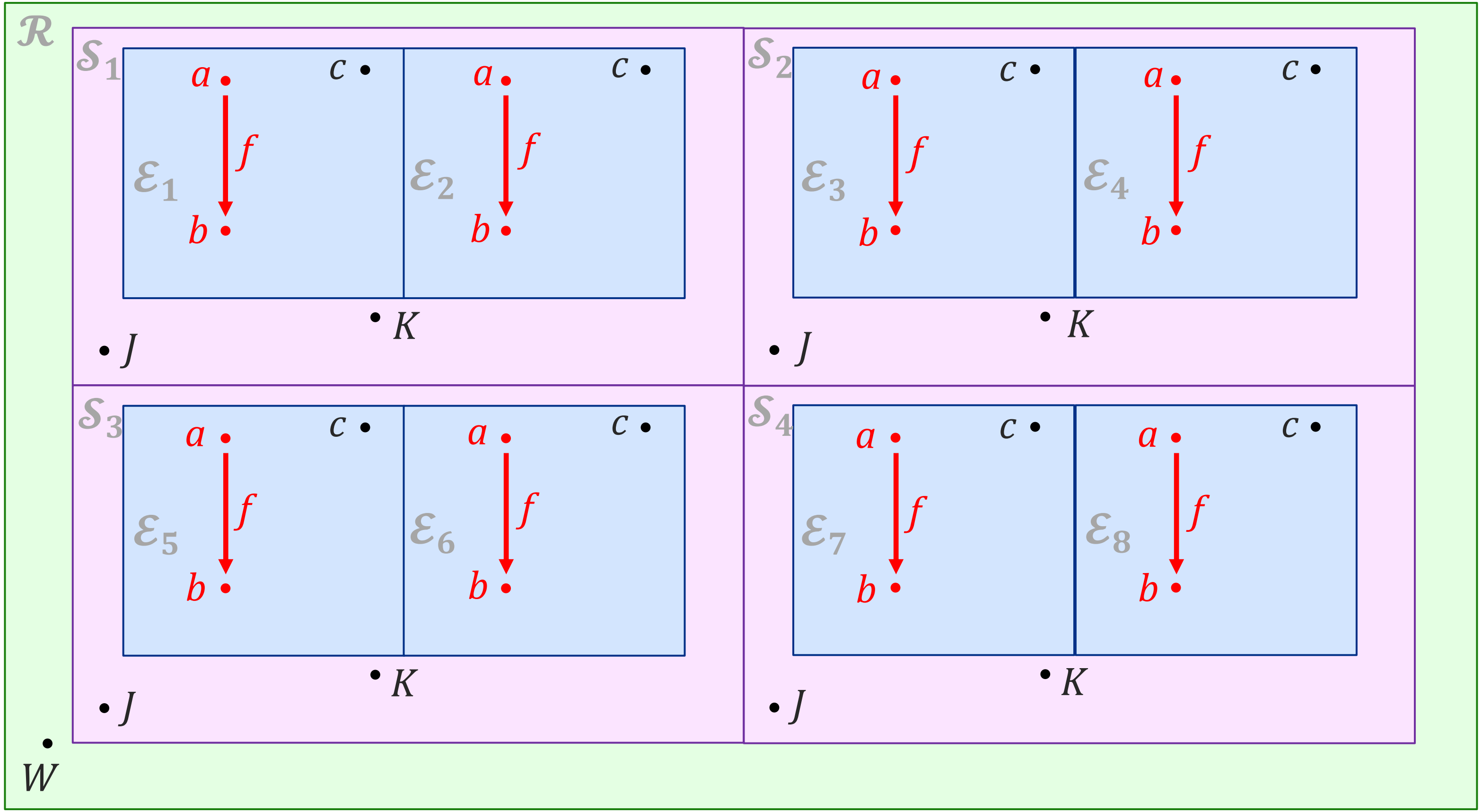

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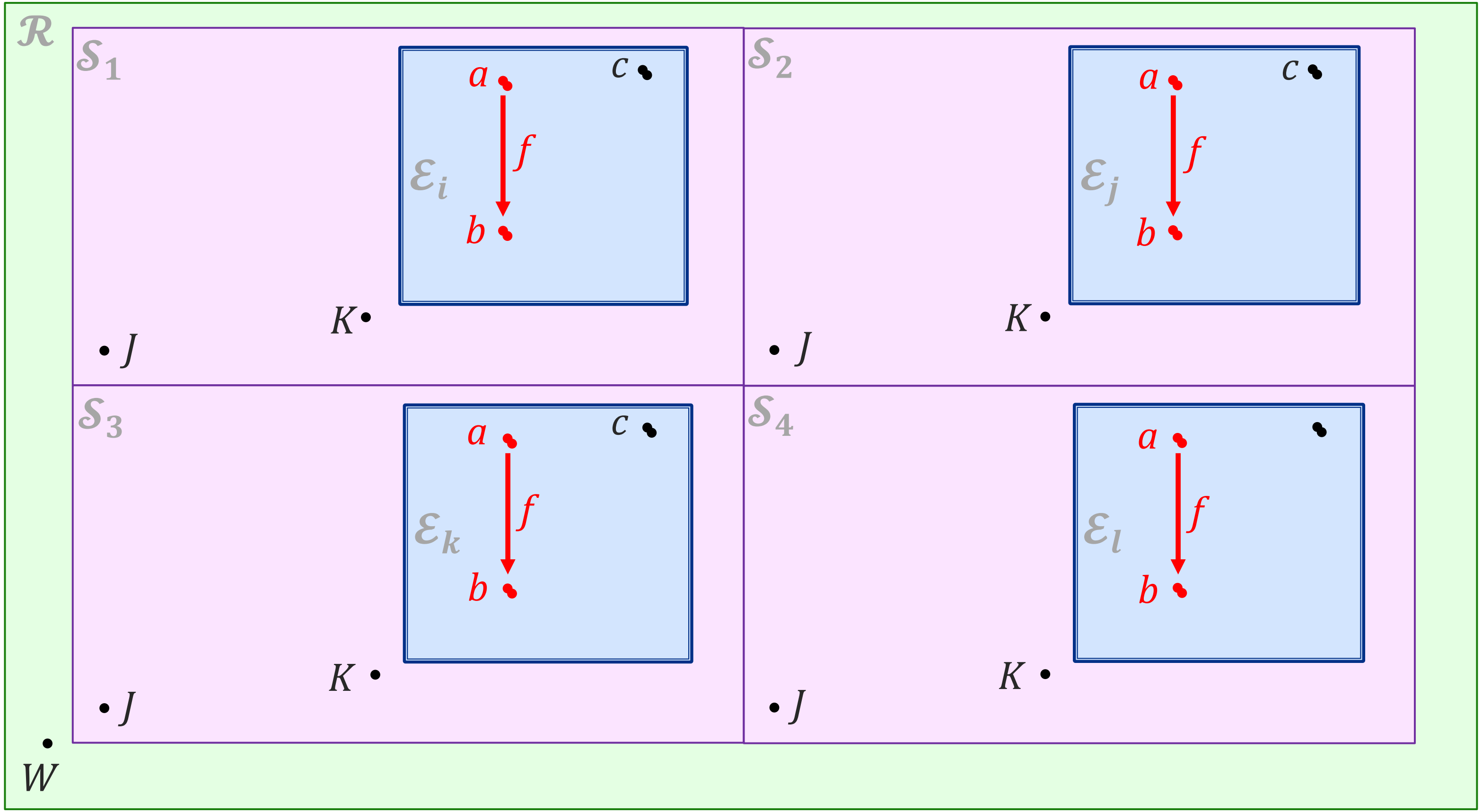

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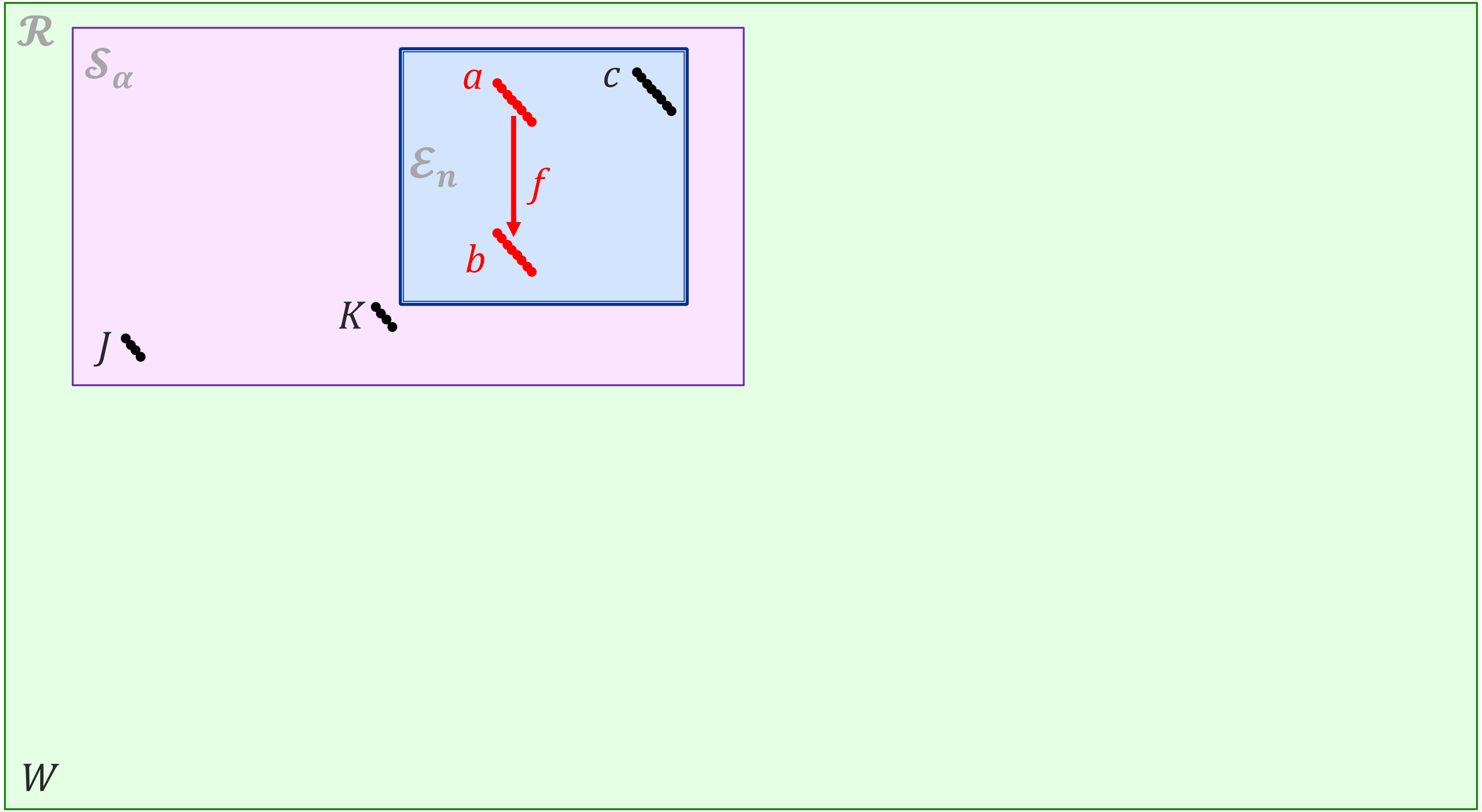

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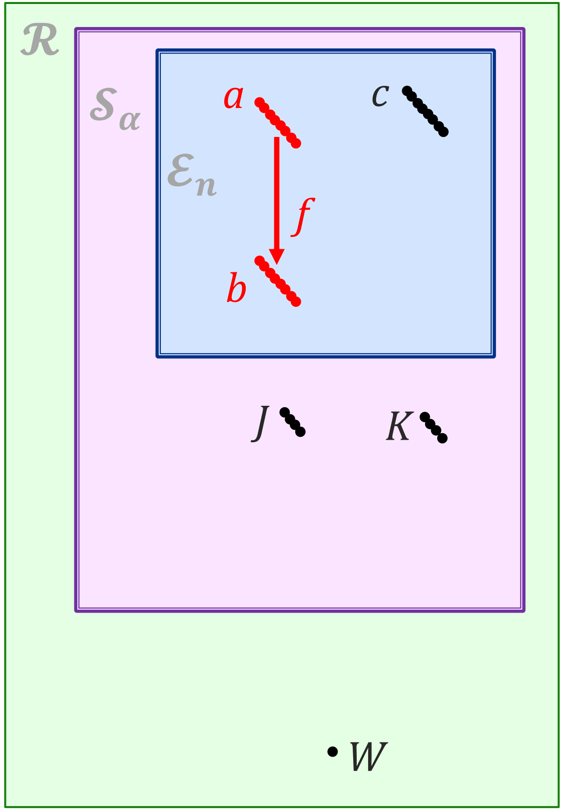

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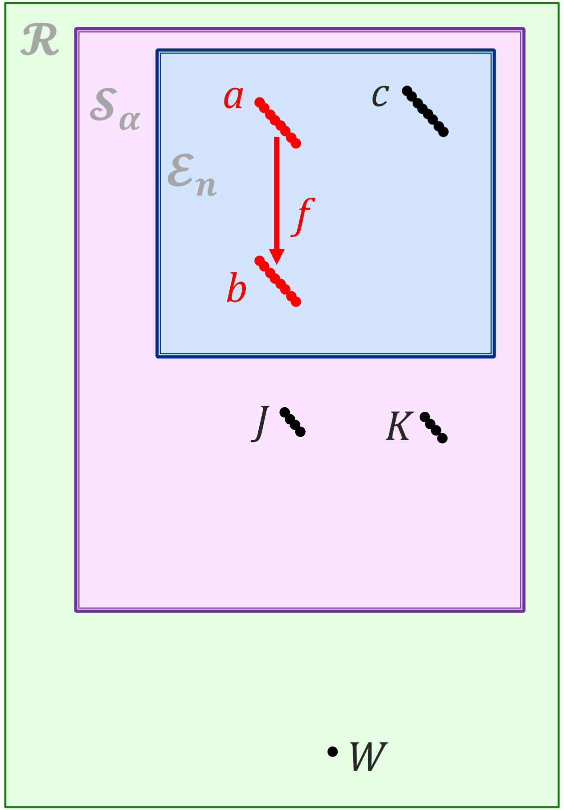

We can make the following replacement (e.g.):

$$= (c)_8$$

depicting the data products labeled c from 8 events as a sequence.

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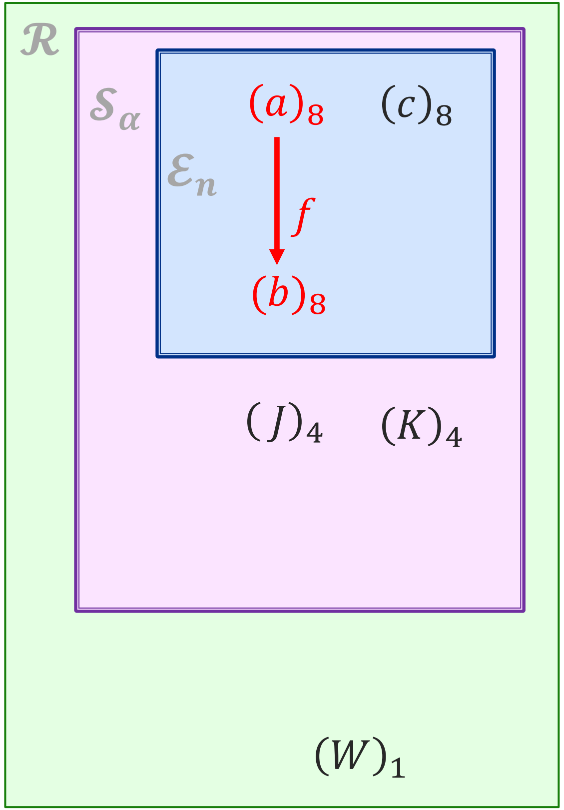

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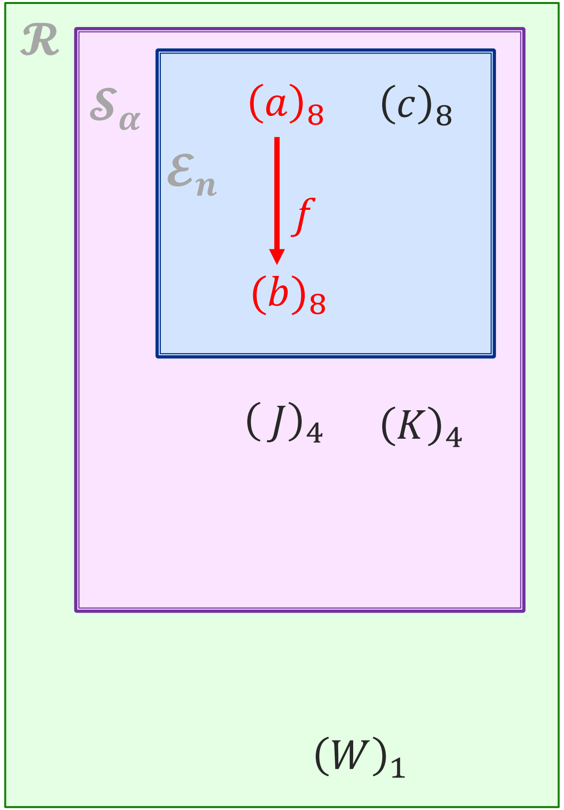

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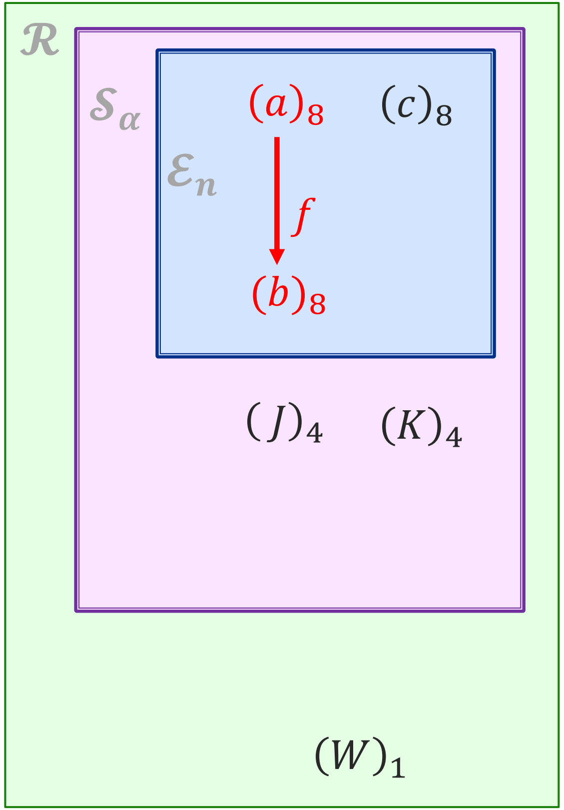

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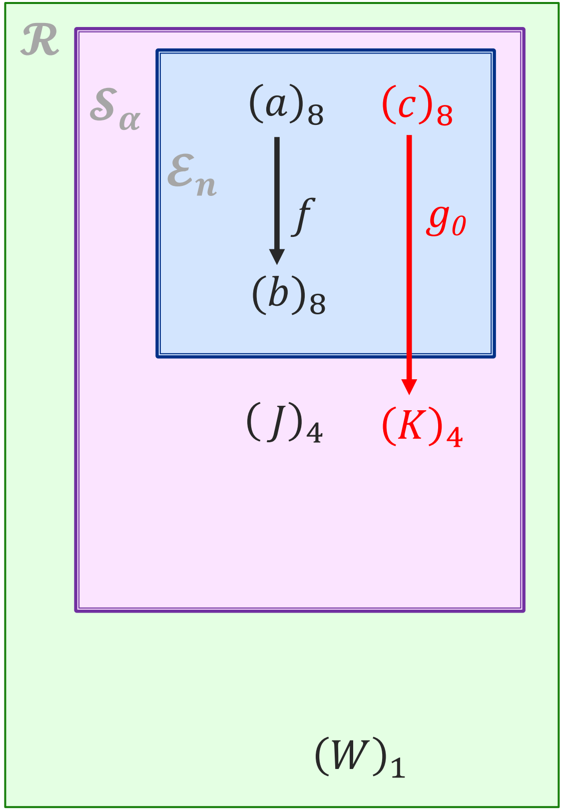

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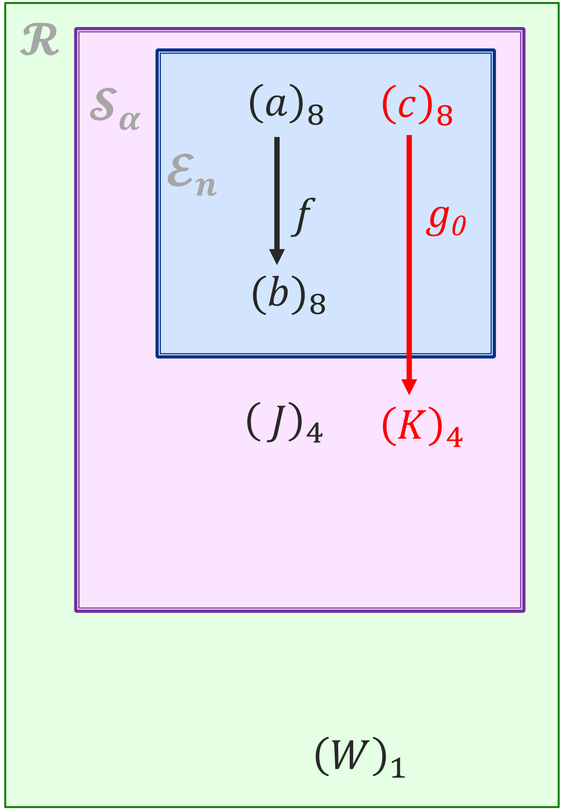

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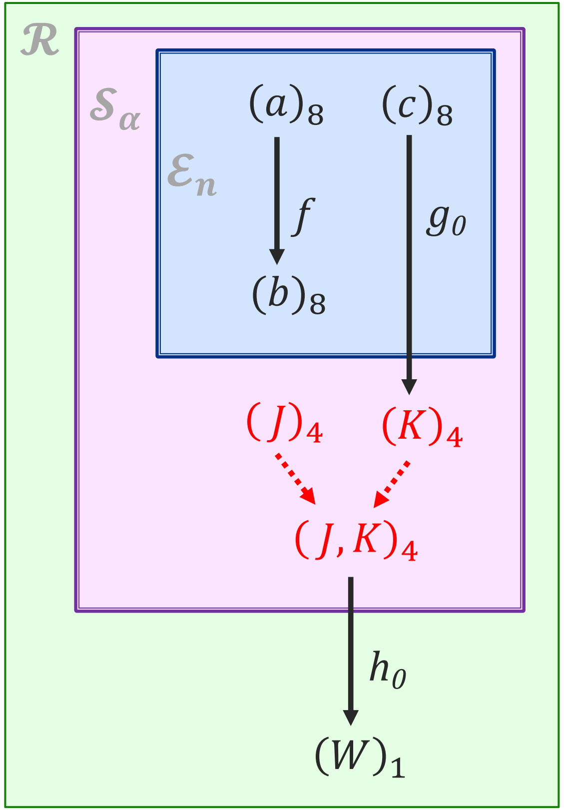

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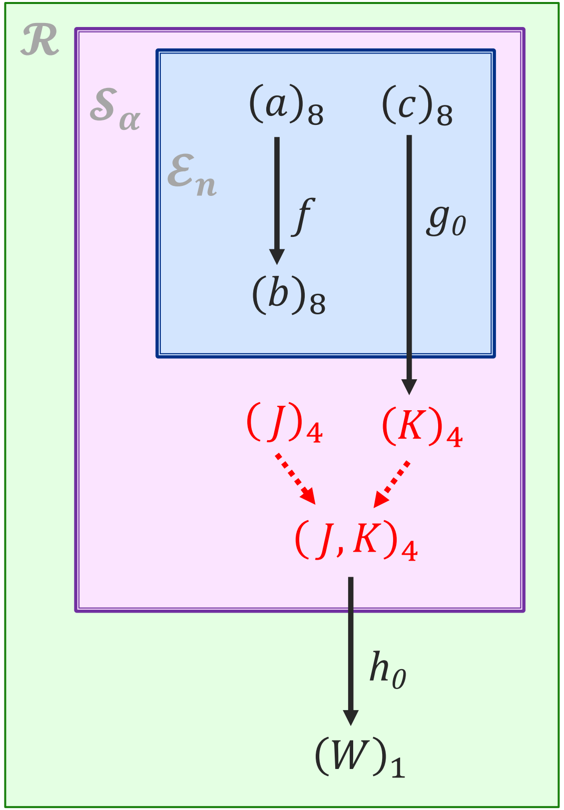

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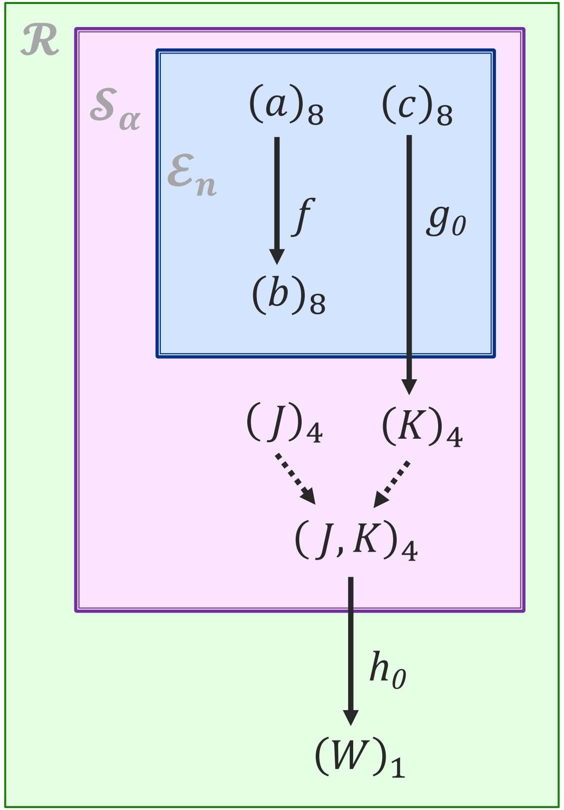

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These have to do with higher-order functions.

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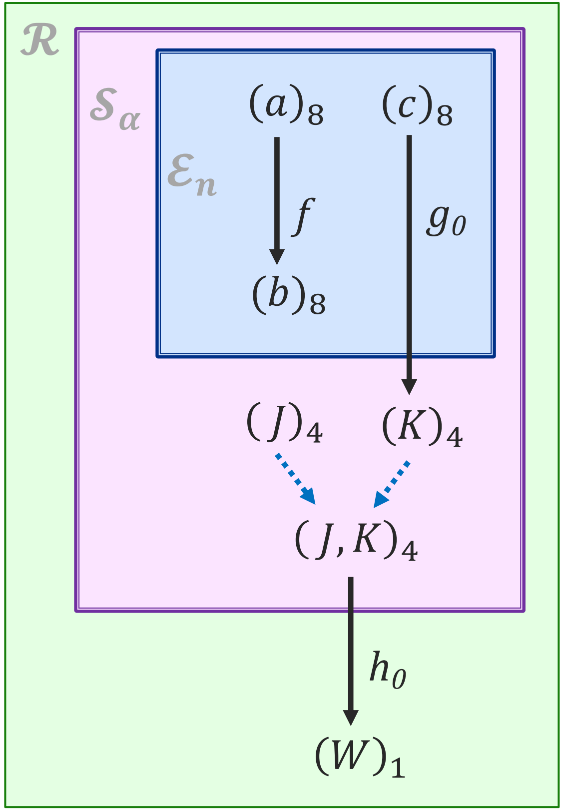
Graph of data-product sequences



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Data-centric	Data products	Mappings	*This work*

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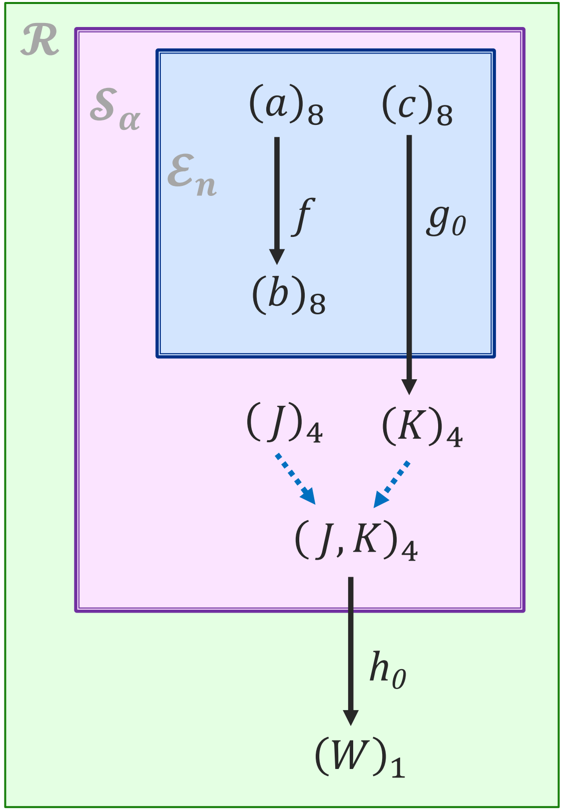

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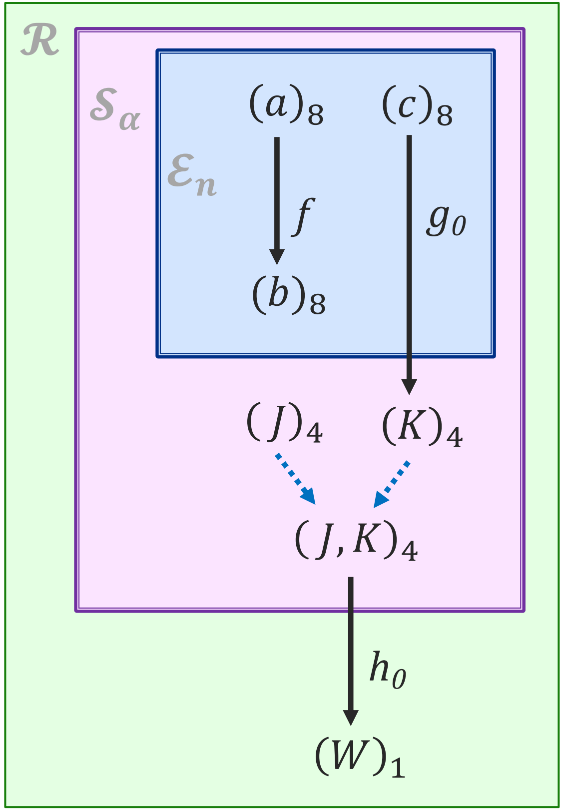

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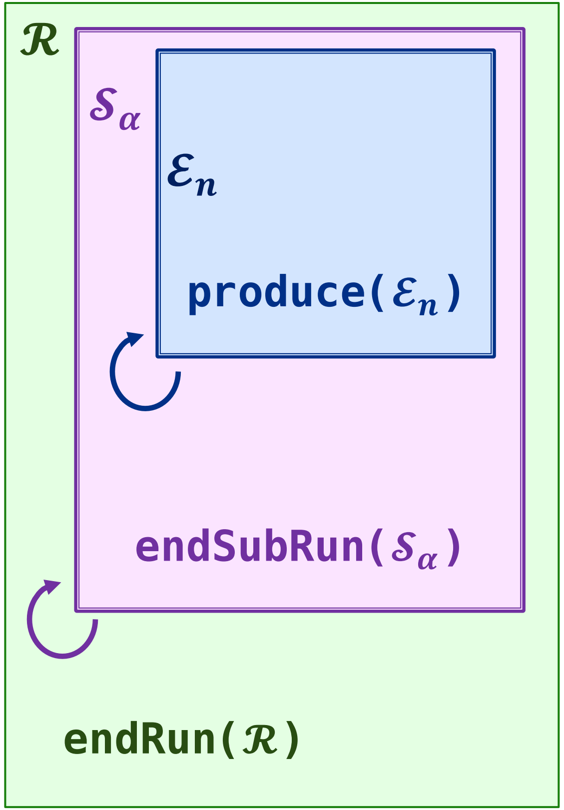
The user specifications are the same with either view:

- Which data products to process
- The data set(s) that contain those products (event, etc.)
- Which higher-order function to use (transform, etc.)
- Which user-defined function to serve as the operation to the higher-order function.
- Allowed concurrency of each function.

The focus is just different.

How are data products and their mappings supported now?



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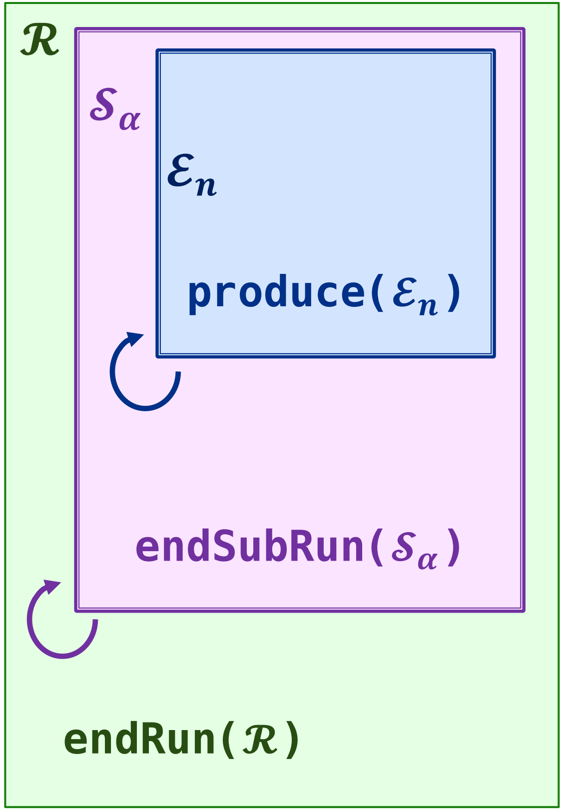

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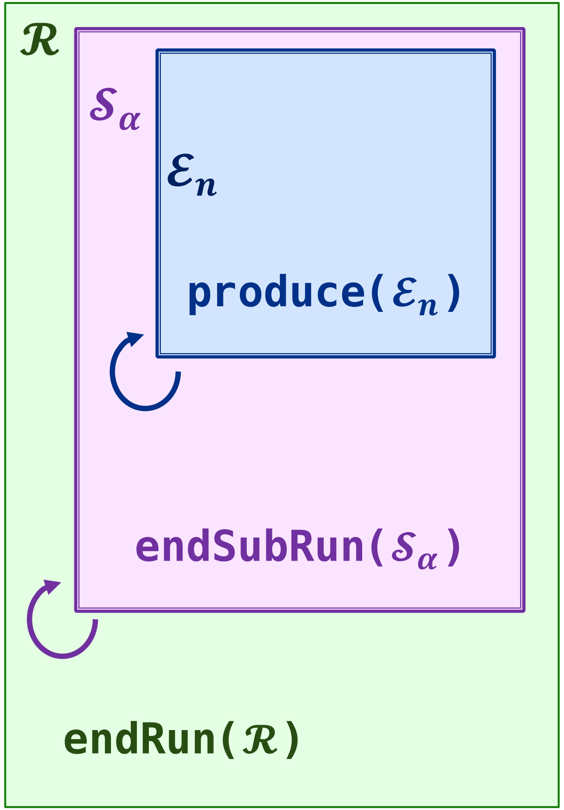

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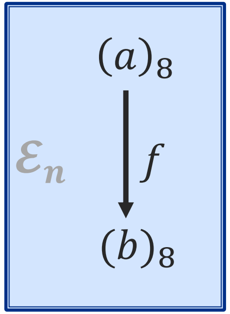
Results in a lot of software mechanics...

Example

- Create tracks from hits for each event.



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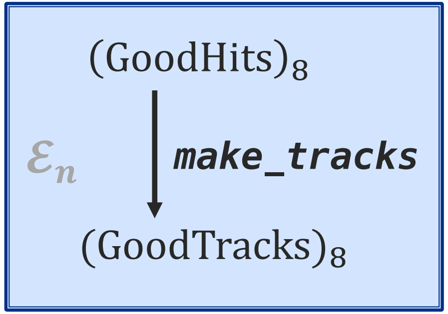


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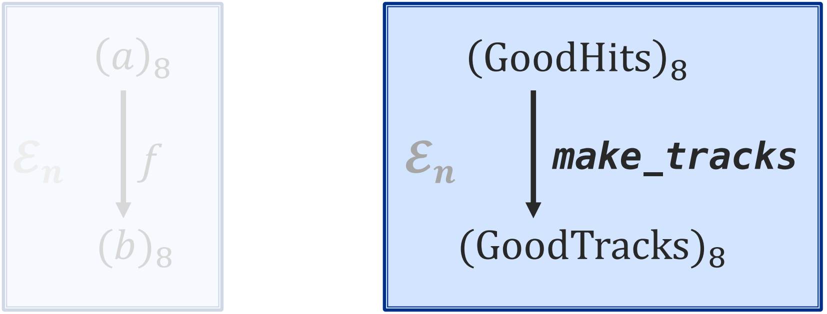


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namespace expt {
  class TrackMaker : public art::SharedProducer {
  public:
    TrackMaker(fhicl::ParameterSet const&) :
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      consumes<Hits, art::InEvent>("GoodHits");
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      async<art::InEvent>();
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    void produce(art::Event& e,
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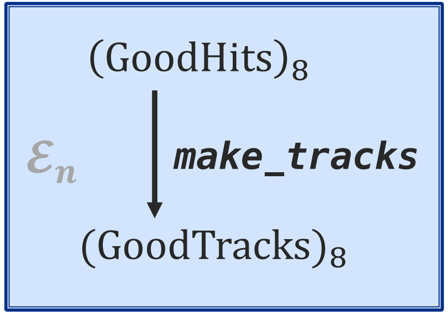


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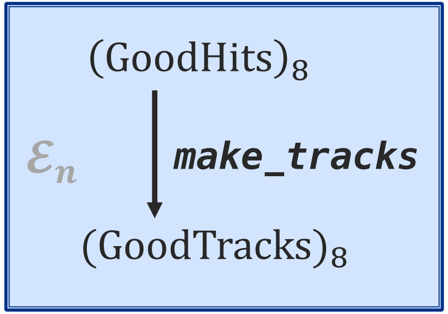


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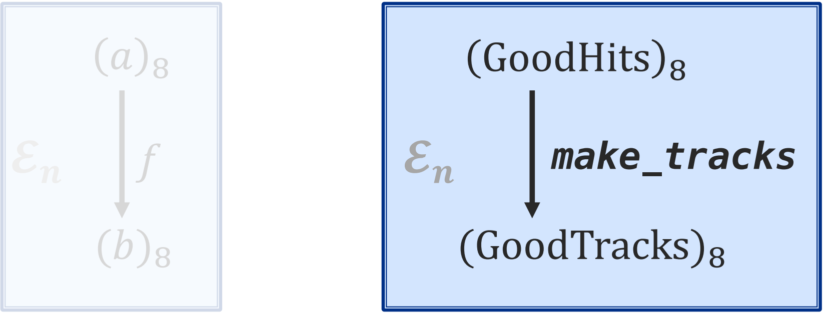


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Nobody wants this.

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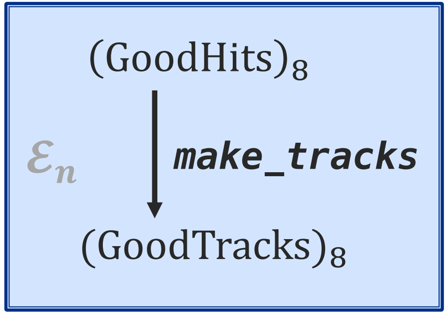
Meld

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```
#include "meld/module.hpp"

namespace {
  Tracks make_tracks(Hits const& hits) { ... }
}

DEFINE_MODULE(m, config) {
  m.with(make_tracks)
   .transform("GoodHits").in_each("Event")
   .to("GoodTracks")
   .using_concurrency(unlimited);
}
```

A better way…

Example

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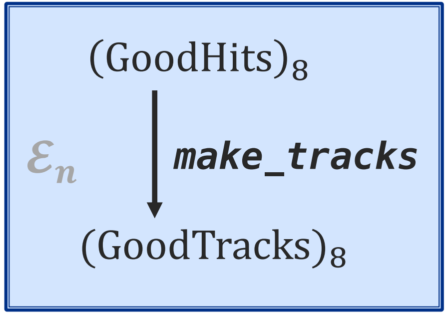
Meld

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```

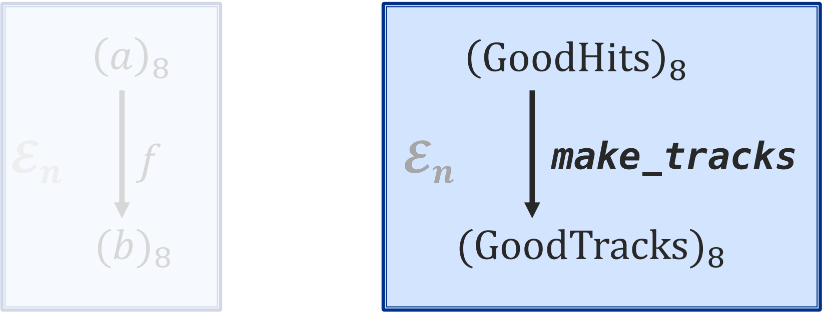
A better way…

- Minimal boilerplate.

Example

- Create tracks from hits for each event.

Meld

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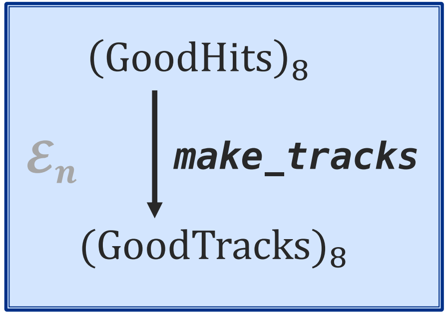


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A better way…

- Minimal boilerplate.
- Event is now a label.
- Higher-order function is now explicit.

Meld implementation

- https://github.com/knoepfel/meld (not even alpha release)
- Implemented using oneTBB's flow graph

Supported construct	User function	
Transform (Map)	$f(\boldsymbol{a}) \rightarrow \boldsymbol{b}$	*Standard data-processing idioms*
Filter	$f(\boldsymbol{a}) \rightarrow \text{Boolean}$	
Monitor	$f(\boldsymbol{a}) \rightarrow \text{Void}$	
Reduction (Fold)	$f_c(\boldsymbol{a}) \rightarrow \boldsymbol{c}$	***For splitting and then combining events***
Splitter (Unfold)	$f_n(\boldsymbol{a}) \rightarrow (\boldsymbol{d})_n$	
Zip	—	***For combining arguments to user functions***
Sliding window	—	*To do:* ***For sliding over adjacent events***

Sample hierarchies tested by Meld

```
[info] Number of worker threads: 12
[info] Processed levels:

  job
   │
   └ run: 1
        │
        └ subrun: 2
              │
              └ event: 10

[info] CPU efficiency: 259.55%
[info] Max. RSS: 6.205 MB
```

art-based hierarchy

Performance numbers are preliminary

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Sample hierarchies tested by Meld

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art-based hierarc

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```

Non-trivial hierarchy

Sample hierarchies tested by Meld

Performance numbers are preliminary

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art-based hierarc

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   ├ trigger primitive: 10
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       │
       └ event: 10

[info] CPU efficiency: 230.81%
[info] Max. RSS: 6.136 MB
```

Non-trivial hierarchy

```
[info] Number of worker threads: 12
[info] Processed levels:

  job
   │
   └ event: 100000

[info] CPU efficiency: 882.50%
[info] Max. RSS: 16.527 MB
```

Flat hierarchy

Summary

> ***"Ways change, Stil."*** —Paul from *Dune* by Frank Herbert

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- Supporting DUNE's framework needs suggests rethinking framework concepts.
- Meld seeks to address these needs by considering a framework job as a

    (1) **graph of data products** *connected by*
    (2) **user-provided operations** *of*
    (3) **higher-order functions**.

- It is not a framework-less framework, but it *is* less framework coupling.
- Preliminary work indicates this is a productive avenue to pursue.

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- It is not a framework-less framework, but it *is* less framework coupling.
- Preliminary work indicates this is a productive avenue to pursue.

Thank you for your time and attention.

Backup slides

Accessing provenance information

```
#include "meld/module.hpp"

namespace {
-  Tracks make_tracks(Hits const& hits) { ... }
+  Tracks make_tracks(meld::handle<Hits> hits) { ... }
}

DEFINE_MODULE(m, config) {
  m.with(make_tracks)
   .transform("GoodHits").in_each("Event")
   .to("GoodTracks")
   .using_concurrency(unlimited);
}
```


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Class example using lambda expression

```
#include "meld/module.hpp"

DEFINE_MODULE(m, config)
{
  auto threshold = config.get<unsigned int>("threshold");
  m.with([threshold](Hits const& hits) { return hits.size() > threshold; })
   .filter("GoodHits").in_each("Event")
   .using_concurrency(unlimited);
}
```

Class example registering two member functions

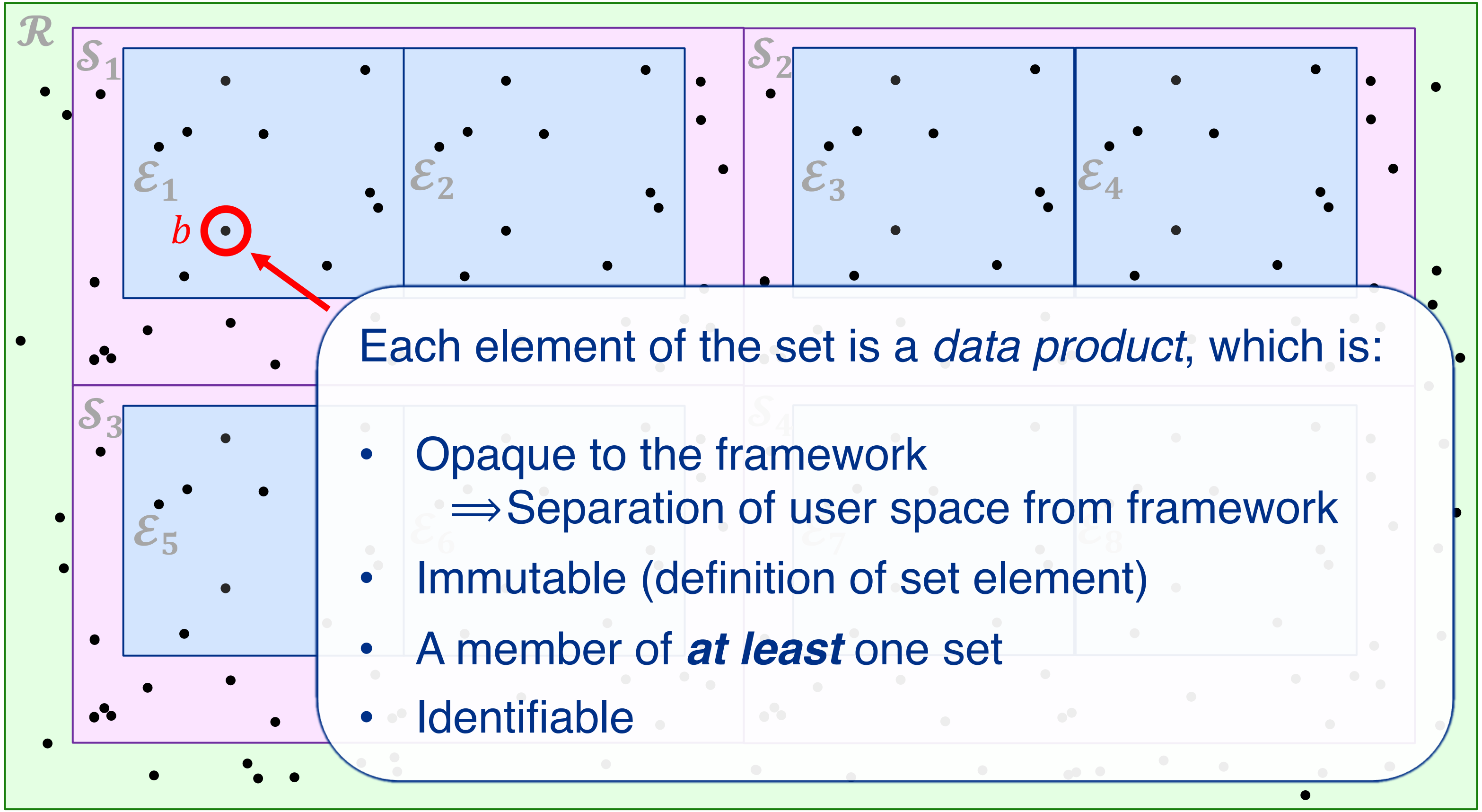
```cpp
#include "meld/module.hpp"

class Selector {
public:
  Selector(unsigned int n) : threshold{n} {}
  bool gt(Hits const& hits) const { return hits.size() > threshold; }
  bool le(Hits const& hits) const { return !gt(hits); }

private:
  unsigned int threshold;
};

DEFINE_MODULE(m, config)
{
  auto threshold = config.get<unsigned int>("threshold");
  auto bound_m = m.make<Selector>(threshold);
  bound_m.with(&Selector::gt).filter("GoodHits").in_each("Event");
  bound_m.with(&Selector::le).filter("GoodHits").in_each("Event");
}
```

Reduction example

```
class MyAccumulator : public art::EDProducer {
public:
  MyAccumulator(ParameterSet const&)
  {
    produces<int, art::InSubRun>("sum");
  }

  void produce(art::Event&) override
  {
    ++counter_;
  }

  void endSubRun(art::SubRun& sr) override
  {
    sr.put(std::make_unique<int>(counter_), "sum");
    counter_ = 0;
  }

private:
  int counter_ = 0;
};

DEFINE_ART_MODULE(MyAccumulator)
```

```
void accumulate(int& counter,
                meld::level_id const&)
{
  ++counter;
}


DEFINE_MODULE(m) {
  m.with(accumulate, 0).for_each("SubRun")
   .reduce("id").in_each("Event")
   .to("sum");
}
```

Looking at the data (products)



Each element of the set is a *data product*, which is:

- Opaque to the framework
  ⟹Separation of user space from framework
- Immutable (definition of set element)
- A member of ***at least*** one set
- Identifiable

Higher-order functions

- We are interested in the mappings of the form:

$$\left\{(\boldsymbol{a})_n \xrightarrow{f} (\boldsymbol{b})_m\right\} \in \mathcal{D}$$

- Each object $\boldsymbol{a}$ corresponds to a tuple of arguments passed to f.
- The signature of f and the value $f(\boldsymbol{a})$, depends on the higher-order function.
- The above mapping happens within a domain $\mathcal{D}$ (e.g. job, run, event).
- Each object $\boldsymbol{a}$ is an element of a subset of the domain $\mathcal{D}$.

Supported higher-order functions

Meld term	CS term	Mathematical description		Domain
Transform	Map	$(\boldsymbol{a})_n \xrightarrow{f} (\boldsymbol{b})_n$	where $f(\boldsymbol{a}) \to \boldsymbol{b}$	Same as $(\boldsymbol{a})_n$
Filter	Filter	$(\boldsymbol{a})_n \xrightarrow{f} (\boldsymbol{a})_m$ where $m \leq n$	where $f(\boldsymbol{a}) \to \text{Boolean}$	Same as $(\boldsymbol{a})_n$
Monitor	—	$(\boldsymbol{a})_n \xrightarrow{f} (\ )_0$	where $f(\boldsymbol{a}) \to \text{Void}$	Same as $(\boldsymbol{a})_n$
Reduction	Fold	$(\boldsymbol{a})_n \xrightarrow{f_c} (\boldsymbol{c})_1$	where $f_c(\boldsymbol{a}) \to \boldsymbol{c}$	**Above** $(\boldsymbol{a})_n$
Splitter	Unfold	$(\boldsymbol{a})_1 \xrightarrow{f_n} (\boldsymbol{d})_m$	where $f_n(\boldsymbol{a}) \to (\boldsymbol{d})_n$	**Below** $(\boldsymbol{a})_n$
Zip	Zip	$\big((a)_n\, , (b)_n\big) \to (a, b)_n$		More nested domain

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