Investigating mixed-precision for AGATA pulse-shape analysis

Roméo Molina¹² Joint work with David Chamont¹, Fabienne Jézéquel², Vincent Lafage¹

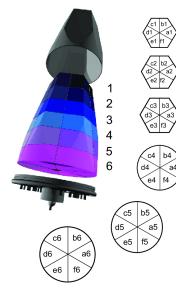
> ¹IJCLab, Paris-Saclay, France ²LIP6, Sorbonne Université, France

> > CHEP 2023 8 May 2023

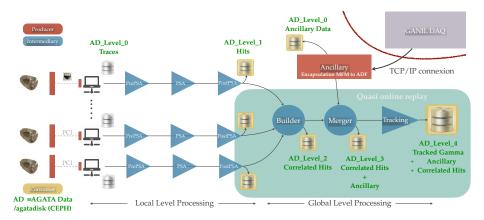


AGATA Advanced GAmma Tracking Array





AGATA Data flow¹



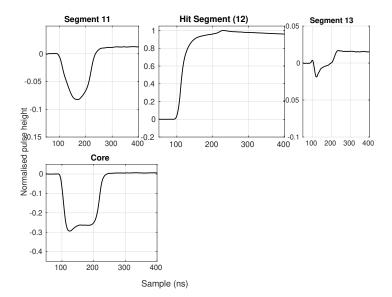
¹O. Stézowski, AGATA Meeting 2022

perf allows to count the number of CPU cycles per function

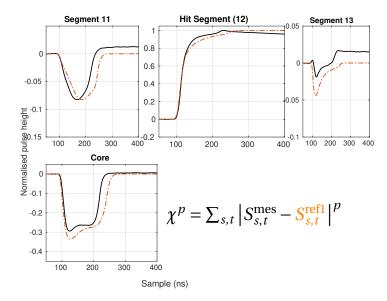
Samples:	58K of ev	ent 'cycles', Event	count (approx.): 68053389580
0verhead	Command	Shared Object	Symbol
68,56%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
5,69%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
4,83%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
2,80%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
1,93%	femul	libPSAFilter.so	[.] pointExp::AddBaseTrace
1,67%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB
1,59%	femul	libPSAFilter.so	<pre>[.] pointPsa::addXT</pre>
1,12%	femul	libPSAFilter.so	[.] SignalBasis::FindNeighbours
1,05%	femul	libPSAFilter.so	[.] SignalBasis::CalcPtPtDistance
0,87%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::FitT0AfterPSA
0,76%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::ShiftMoveTrace
0,73%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,71%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution

 \Rightarrow We shall optimize Chi2InnerLoop!

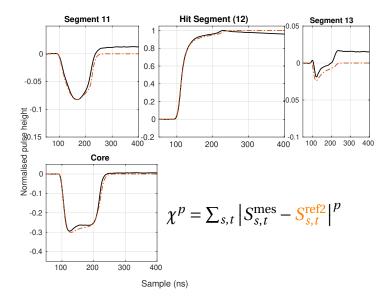
PSA Pulse Shape Analysis



PSA Pulse Shape Analysis



PSA Pulse Shape Analysis



perf allows to analyse the memory usage

Samples:	52K of ev	<pre>vent 'cache-references',</pre>	, Event count (approx.): 742466595
Overhead	Command	Shared Object	Symbol
80,89%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
3,02%	femul	[unknown]	[k] 0xffffffffa005e23e
2,60%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
1,69%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
0,85%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
0,85%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,72%	femul	libPSAFilter.so	<pre>[.] pointPsa::addXT</pre>
0,64%	femul	libPSAFilter.so	<pre>[.] pointExp::AddBaseTrace</pre>
0,62%	femul	libc-2.31.so	<pre>[.]memmove_avx_unaligned_erms</pre>
0,55%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution
0,55%	femul	libc-2.31.so	<pre>[.]memset_avx2_erms</pre>
0,51%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB

 \Rightarrow consistant with cycles analysis

Cache-misses

Cache-misses happen when the data is not in cache memory. The application has to attempt to find the data in slower memory that causes massive performance reduction.

Samples:	49K of ev	ent 'cache-misses',	Event count (approx.): 311766482
0verhead	Command	Shared Object	Symbol
73,26%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
6,44%	femul	[unknown]	[k] 0xfffffffa005e23e
4,49%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
1,16%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
1,07%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB
1,05%	femul	libc-2.31.so	<pre>[.]memmove_avx_unaligned_erms</pre>
0,98%	femul	libc-2.31.so	<pre>[.]memset_avx2_erms</pre>
0,67%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,57%	femul	[unknown]	[k] 0xfffffffa005e240
0,56%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
0,51%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution

⇒ Memory bound algorithm

\Rightarrow reduce the amount of data to make it fit in the cache

 \Rightarrow use smaller formats while maintaining the same accuracy \Rightarrow what gains for what risks?

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Using low precisions is promising

	Number of bits													
		Signif. (t)	Exp.	Range	$u = 2^{-t}$									
fp128	quadruple	113	15	$10^{\pm 4932}$	1×10^{-34}									
fp64	double	53	11	$10^{\pm 308}$	1×10^{-16}									
fp32	single	24	8	$10^{\pm 38}$	6×10^{-8}									
fp16	half	11	5	$10^{\pm 5}$	5×10^{-4}									
bfloat16	half	8	8	$10^{\pm 38}$	4×10^{-3}									
fp8 (e4m3)	quartar	4	4	$10^{\pm 2}$	6×10^{-2}									
fp8 (e5m2)	quarter	3	5	$10^{\pm 5}$	1×10^{-1}									

Low precision increasingly supported by hardware

Great benefits:

- Reduced storage, data movement, and communications
- Reduced energy consumption (5× with fp16, 9× with bfloat16)
- Increased **speed** (16× on A100 from fp32 to fp16/bfloat16)

Floating-point computation \neq mathematical evaluation

- rounding $a \oplus b \neq a + b$
- no more associativity $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$

Consequences:

- invalid results
- non reproducibility
- performance issue (useless iterations)

Some limitations to the low precisions:

- Low accuracy (large *u*)
- Narrow range

Assess the accuracy

cadna.lip6.fr



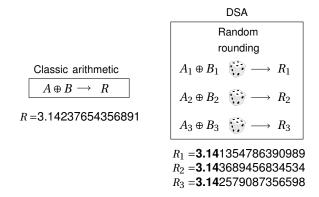
- implements stochastic arithmetic for C/C++ or Fortran codes
- all operators and mathematical functions overloaded ⇒ little code rewriting
- support for MPI, OpenMP, GPU, vectorised codes
- supports emulated ou native half precision
- in one CADNA execution: accuracy of any result, complete list of numerical instabilities

CADNA cost

- memory: 4
- In time ≈ 10

[Chesneaux'90], [Jézéquel & al'08], [Lamotte & al'10], [Eberhart & al'18],...

Discrete Stochastic Arithmetic (DSA) [Vignes'04]



- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%
- operations executed synchronously
 - \Rightarrow detection of numerical instabilities (ex: if (A>B) with A-B numerical noise)
 - ⇒ optimization of stopping criteria to avoid useless iterations

- PSA performed natively in fp32
- minimum search in a 504-dimensional space
- risk to accumulate catastrophic cancellations
- requires instrumentation to assess the accuracy results
- \Rightarrow code sensitive to perturbations?
 - 0.02% among points matched differently between fp64 version and original version
 - 0.02% between CADNA version and original version
- \Rightarrow Satisfactory original fullgrid PSA results!

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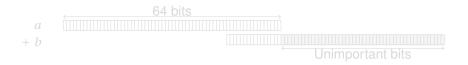
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- 7.76% differences between original and fp16 version
- too much?
- need to find another way to exploit low precision

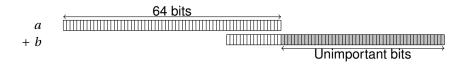
Mix several precisions in the same code with the goal of

- Getting the performance benefits of low precisions
- While preserving the accuracy and stability of high precision
- ⇒ Why does it make sense to make the precision vary?
- Because not all computations are equally "important"! Example:

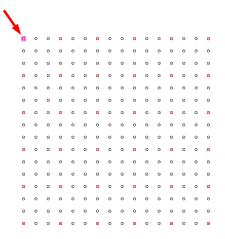


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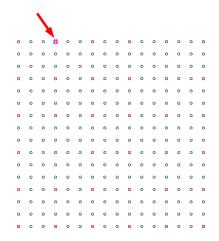
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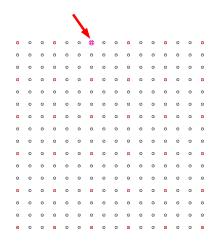
- the algorithm used in practice is smarter than previously presented
- coarse-fine algorithm



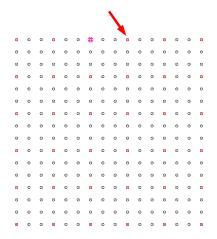
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0	Э	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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4	Э	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	٥	0	٥	0	0	٥	0	0	0	0	0
٥	0	٥	0	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	٥	0	٥	0	0	٥	0	0	0	0	0
٥	0	٥	0	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	٥	0	0	0	0	0	0	0	$^{\odot}$	0	0
0	0	0	0	0	٥	0	0	0	0	٥	0	0	0	0	0
٥	o	0	٥	0	⊙	٥	o	0	٥	o	0	٥	o	0	٥

- the algorithm used in practice is smarter than previously presented
- coarse-fine algorithm

٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	⊙	0	O
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	٥	0	0	0	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	0	0	0
٥	0	0	0	0	0	0	٥	0	0	0	0	0	٥	0	0
٥	0	0	٥	ø	0	٥	0	0	٥	0	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	0	0	o	0	O	o	0	0	⊙	0	O
٥	0	$^{\odot}$	0	0	0	0	٨	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	۲	٥	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	٥	0	٥	0	0	0	٥	0	٥	٥	0	٥	٥	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	0	0	0
٥	0	$^{\odot}$	0	0	\odot	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	0	0	0	0	0	0
٥	0	$^{\odot}$	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	ø	0	0	ø	0	0	ø	0	٥	0	0	٥	0	0	٥
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	0	0	٥	0	0	0	0	0	0	0	٥	٥	0	0
٥	o	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	⊙	0	o	0	0	o	0	0	⊙	0	O
٥	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	٥	٥	0	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	٥	0	0	0	0	0	0	0	٥	0	0
٥	٥	٥	٥	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	$^{\odot}$	0	0	٥	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	٥	0	٥	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	0	0	0	0	0	0
٥	0	$^{\odot}$	0	0	٥	0	0	0	0	0	0	0	٥	0	0
٥	ø	0	0	ø	0	0	0	0	٥	0	0	٥	0	0	٥
0	0	\odot	0	0	0	0	0	0	0	$^{\odot}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
٥	o	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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8.22% differences with fullgrid fp32 version, validated by the physicists
⇒ provides an opportunity for mixed precision

- first step in half
- second step in float
- 8.55% differences with fullgrid fp32 version
- under the same conditions, half-half produces 14.04% differences!

- low precision is beneficial (speed, energy, storage) but you should be careful
- accuracy control is mandatory
- CADNA is well designed to do so
- mixed-precision is a way to benefit from low precision in fields that require high accuracy

- varying the coarse/fine gridsize allocation
- introducing a hierarchy of intermediate grids
- implement it on GPUs to improve performance

Thank you for your attention!