# Investigating mixed-precision for AGATA pulse-shape analysis

Roméo Molina<sup>12</sup> Joint work with David Chamont<sup>1</sup>, Fabienne Jézéquel<sup>2</sup>, Vincent Lafage<sup>1</sup>

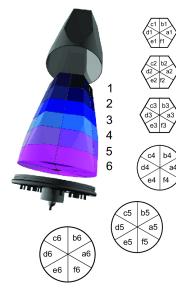
> <sup>1</sup>IJCLab, Paris-Saclay, France <sup>2</sup>LIP6, Sorbonne Université, France

> > CHEP 2023 8 May 2023

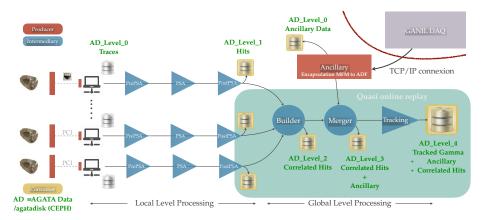


## AGATA Advanced GAmma Tracking Array





# AGATA Data flow<sup>1</sup>



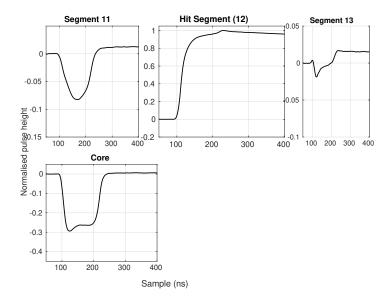
#### <sup>1</sup>O. Stézowski, AGATA Meeting 2022

#### perf allows to count the number of CPU cycles per function

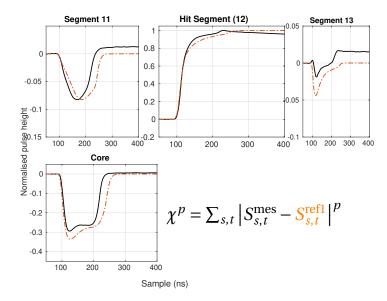
Samples:	58K of ev	ent 'cycles', Event	count (approx.): 68053389580
0verhead	Command	Shared Object	Symbol
68,56%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
5,69%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
4,83%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
2,80%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
1,93%	femul	libPSAFilter.so	[.] pointExp::AddBaseTrace
1,67%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB
1,59%	femul	libPSAFilter.so	<pre>[.] pointPsa::addXT</pre>
1,12%	femul	libPSAFilter.so	[.] SignalBasis::FindNeighbours
1,05%	femul	libPSAFilter.so	[.] SignalBasis::CalcPtPtDistance
0,87%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::FitT0AfterPSA
0,76%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::ShiftMoveTrace
0,73%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,71%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution

 $\Rightarrow$  We shall optimize Chi2InnerLoop!

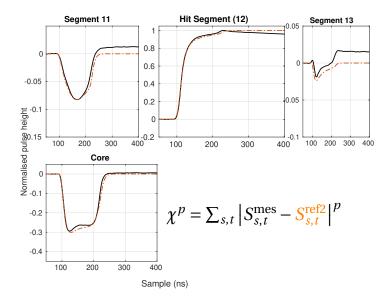
## **PSA Pulse Shape Analysis**



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#### perf allows to analyse the memory usage

Samples:	52K of ev	<pre>vent 'cache-references',</pre>	, Event count (approx.): 742466595
<b>Overhead</b>	Command	Shared Object	Symbol
80,89%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
3,02%	femul	[unknown]	[k] 0xffffffffa005e23e
2,60%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
1,69%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
0,85%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
0,85%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,72%	femul	libPSAFilter.so	<pre>[.] pointPsa::addXT</pre>
0,64%	femul	libPSAFilter.so	<pre>[.] pointExp::AddBaseTrace</pre>
0,62%	femul	libc-2.31.so	<pre>[.]memmove_avx_unaligned_erms</pre>
0,55%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution
0,55%	femul	libc-2.31.so	<pre>[.]memset_avx2_erms</pre>
0,51%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB

 $\Rightarrow$  consistant with cycles analysis

#### Cache-misses

Cache-misses happen when the data is not in cache memory. The application has to attempt to find the data in slower memory that causes massive performance reduction.

Samples:	49K of ev	ent 'cache-misses',	Event count (approx.): 311766482
0verhead	Command	Shared Object	Symbol
73,26%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::Chi2InnerLoop
6,44%	femul	[unknown]	[k] 0xfffffffa005e23e
4,49%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::SearchAdaptive1
1,16%	femul	libPSAFilter.so	<pre>[.] pointPsa::add</pre>
1,07%	femul	libPSAFilter.so	[.] SignalBasis::ReadBasisFormatBartB
1,05%	femul	libc-2.31.so	<pre>[.]memmove_avx_unaligned_erms</pre>
0,98%	femul	libc-2.31.so	<pre>[.]memset_avx2_erms</pre>
0,67%	femul	libPSAFilter.so	<pre>[.] pointPsa::sumOfSignals</pre>
0,57%	femul	[unknown]	[k] 0xfffffffa005e240
0,56%	femul	libPSAFilter.so	<pre>[.] pointPsa::convDeltaToExp</pre>
0,51%	femul	libPSAFilter.so	[.] PSAFilterGridSearch::AddToSolution

⇒ Memory bound algorithm

#### $\Rightarrow$ reduce the amount of data to make it fit in the cache

 $\Rightarrow$  use smaller formats while maintaining the same accuracy  $\Rightarrow$  what gains for what risks?

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# Using low precisions is promising

	Number of bits													
		Signif. (t)	Exp.	Range	$u = 2^{-t}$									
fp128	quadruple	113	15	$10^{\pm 4932}$	$1 \times 10^{-34}$									
fp64	double	53	11	$10^{\pm 308}$	$1 \times 10^{-16}$									
fp32	single	24	8	$10^{\pm 38}$	$6 \times 10^{-8}$									
fp16	half	11	5	$10^{\pm 5}$	$5 \times 10^{-4}$									
bfloat16	half	8	8	$10^{\pm 38}$	$4 \times 10^{-3}$									
fp8 (e4m3)	quartar	4	4	$10^{\pm 2}$	$6 \times 10^{-2}$									
fp8 (e5m2)	quarter	3	5	$10^{\pm 5}$	$1 \times 10^{-1}$									

Low precision increasingly supported by hardware

#### Great benefits:

- Reduced storage, data movement, and communications
- Reduced energy consumption (5× with fp16, 9× with bfloat16)
- Increased **speed** (16× on A100 from fp32 to fp16/bfloat16)

Floating-point computation  $\neq$  mathematical evaluation

- rounding  $a \oplus b \neq a + b$
- no more associativity  $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$

Consequences:

- invalid results
- non reproducibility
- performance issue (useless iterations)

#### Some limitations to the low precisions:

- Low accuracy (large *u*)
- Narrow range

## Assess the accuracy

cadna.lip6.fr



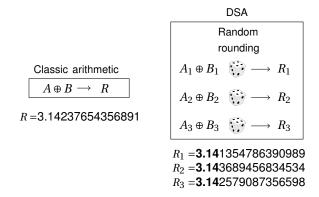
- implements stochastic arithmetic for C/C++ or Fortran codes
- all operators and mathematical functions overloaded ⇒ little code rewriting
- support for MPI, OpenMP, GPU, vectorised codes
- supports emulated ou native half precision
- in one CADNA execution: accuracy of any result, complete list of numerical instabilities

#### CADNA cost

- memory: 4
- In time ≈ 10

[Chesneaux'90], [Jézéquel & al'08], [Lamotte & al'10], [Eberhart & al'18],...

## Discrete Stochastic Arithmetic (DSA) [Vignes'04]



- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%
- operations executed synchronously
  - $\Rightarrow$  detection of numerical instabilities (ex: if (A>B) with A-B numerical noise)
  - ⇒ optimization of stopping criteria to avoid useless iterations

- PSA performed natively in fp32
- minimum search in a 504-dimensional space
- risk to accumulate catastrophic cancellations
- requires instrumentation to assess the accuracy results
- $\Rightarrow$  code sensitive to perturbations?
  - 0.02% among points matched differently between fp64 version and original version
  - 0.02% between CADNA version and original version
- $\Rightarrow$  Satisfactory original fullgrid PSA results!

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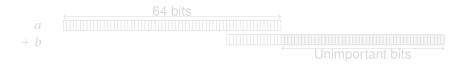
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- 7.76% differences between original and fp16 version
- too much?
- need to find another way to exploit low precision

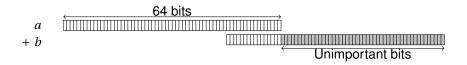
Mix several precisions in the same code with the goal of

- Getting the performance benefits of low precisions
- While preserving the accuracy and stability of high precision
- ⇒ Why does it make sense to make the precision vary?
- Because not all computations are equally "important"! Example:

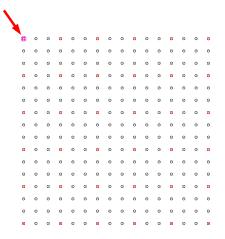


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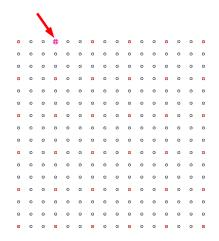
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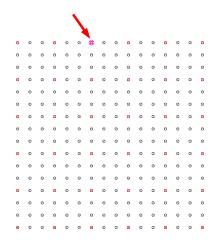
- the algorithm used in practice is smarter than previously presented
- coarse-fine algorithm



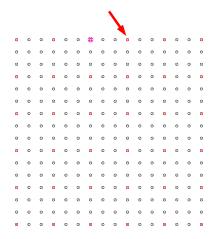
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	٥	0	0	0	0	٥	0	0	0	0	0
٥	0	0	٥	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	٥	0	٥	0	0	٥	0	0	0	0	0
٥	0	٥	0	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	٥	0	٥	0	0	٥	0	0	0	0	0
٥	0	٥	0	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	٥	0	0	0	0	0	0	0	$^{\odot}$	0	0
0	0	0	0	0	٥	0	0	0	0	٥	0	0	0	0	0
٥	o	0	٥	0	⊙	٥	o	0	٥	o	0	٥	o	0	٥

- the algorithm used in practice is smarter than previously presented
- coarse-fine algorithm

٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	⊙	0	O
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	٥	0	0	0	0	0	٥	0	0
٥	0	0	٥	0	0	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	0	0	0
٥	0	0	0	0	0	0	٥	0	0	0	0	0	٥	0	0
٥	0	0	٥	ø	0	٥	0	0	٥	0	0	٥	٥	0	٥
0	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	0	0	o	0	O	o	0	0	⊙	0	O
٥	0	$^{\odot}$	0	0	0	0	٨	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	۲	٥	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	0	0	0
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	٥	0	٥	0	0	0	٥	0	٥	٥	0	٥	٥	0	٥
٥	0	0	0	o	0	0	o	0	0	o	0	0	0	0	0
٥	0	$^{\odot}$	0	0	$\odot$	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	0	٥	0	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	0	0	0	0	0	0
٥	0	$^{\odot}$	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	ø	0	0	ø	0	0	ø	0	٥	0	0	٥	0	0	٥
٥	0	0	0	0	0	0	0	0	0	0	0	0	٥	0	0
٥	0	0	0	٥	0	0	0	0	0	0	0	٥	٥	0	0
٥	o	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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٥	0	0	٥	0	0	0	0	0	٥	0	0	٥	0	0	٥
٥	0	0	0	o	⊙	0	o	0	0	o	0	0	⊙	0	O
٥	0	$^{\odot}$	0	0	0	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	٥	٥	0	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	0	0	0	٥	0	0	0	0	0	0	0	٥	0	0
٥	٥	٥	٥	0	٥	0	0	0	٥	٥	0	٥	٥	0	٥
0	0	0	0	o	0	0	0	0	0	o	0	0	0	0	0
٥	0	$^{\odot}$	0	0	٥	0	0	0	0	$^{\odot}$	0	0	٥	0	0
٥	0	0	٥	0	٥	0	٥	0	٥	0	0	٥	٥	0	٥
٥	0	0	0	o	0	0	0	0	0	0	0	0	0	0	0
٥	0	$^{\odot}$	0	0	٥	0	0	0	0	0	0	0	٥	0	0
٥	ø	0	0	ø	0	0	0	0	٥	0	0	٥	0	0	٥
0	0	$\odot$	0	0	0	0	0	0	0	$^{\odot}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
٥	o	o	٥	o	0	٥	o	0	٥	o	0	٥	o	0	٥

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8.22% differences with fullgrid fp32 version, validated by the physicists
⇒ provides an opportunity for mixed precision

- first step in half
- second step in float
- 8.55% differences with fullgrid fp32 version
- under the same conditions, half-half produces 14.04% differences!

- low precision is beneficial (speed, energy, storage) but you should be careful
- accuracy control is mandatory
- CADNA is well designed to do so
- mixed-precision is a way to benefit from low precision in fields that require high accuracy

- varying the coarse/fine gridsize allocation
- introducing a hierarchy of intermediate grids
- implement it on GPUs to improve performance

Thank you for your attention!