國立交通大學
National Chiao Tung University

Meson Electroproduction & Hadron Structure

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with
Ayşe Kızılersü, Anthony W. Thomas

arXiv:1811.09356, arXiv:2005.01395

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

THE UNIVERSITY of ADELAIDE

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Outline

- Theoretical Background: Hadron EM Form Factor
- Predictions
- Model dependence on pion form factor. arXiv:1811.09356
- Alternative implementation of gauge invariance. arXiv:2005.01395
- Generalization to kaon electroproduction.
- Conclusions, Further Work

Electromagnetic Form Factors: Theory

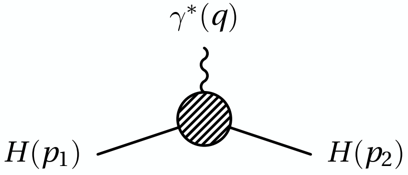
- LSZ: Form factors appear as residues of pole terms:

$$\int d^4x_1 d^4x_2 e^{-ip_1\cdot x_1} e^{ip_2\cdot x_2} \langle\Omega| T\{\mathcal{O}_H(x_2)J^\mu(z)\mathcal{O}_H(x_1)\} |\Omega\rangle$$

$$\sim \frac{1}{[p_2^2 - m_{H_2}^2]} \langle H(p_2)| J^\mu(z) |H(p_1)\rangle \frac{1}{[p_1^2 - m_{H_1}^2]}$$

- Performing Fourier transform:

$$\int d^4z e^{iq\cdot z} \langle H(p_2)| J^\mu(z) |H(p_1)\rangle = (2\pi)^4\delta(p_1 + q - p_2) \langle H(p_2)| J^\mu(0) |H(p_1)\rangle$$



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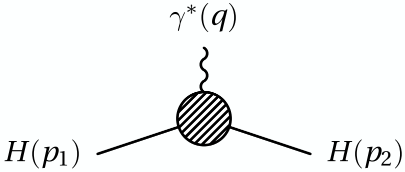


Pion & Kaon Form Factor

- Lorentz decomposition

$$\langle H(p_2)| J^\mu(0) |H(p_1)\rangle = f_1(q^2)(p_1+p_2)^\mu + f_2(q^2)(p_1-p_2)^\mu$$

- Gauge invariance $\implies$ Ward-Takahashi-Green Identity:

$$q_\mu \langle H(p_2)| J^\mu(0) |H(p_1)\rangle = 0 \implies f_2(q^2) = 0$$

$$\langle \pi(p_2)| J^\mu(0) |\pi(p_1)\rangle = F_\pi(q^2)(p_1+p_2)^\mu$$

$$\langle K(p_2)| J^\mu(0) |K(p_1)\rangle = F_K(q^2)(p_1+p_2)^\mu$$

- Rigorously defined QFT matrix elements.
- Initial and final hadronic states *on-shell*.

Asymptotic Prediction

- In asymptotically free theory, quark counting argument predicts

$$F_\pi(Q^2) \sim Q^{-2}, \qquad F_K(Q^2) \sim Q^{-2}$$

DEEP ELASTIC PROCESSES OF COMPOSITE PARTICLES IN FIELD THEORY AND ASYMPTOTIC FREEDOM*

A.V. Radyushkin**

** The investigation has been performed (and completed in June 1977) at the Laboratory of Theoretical Physics, JINR, Dubna, Russian Federation*

English translation and comments: October 2004

*** Present address: Physics Department, Old Dominion University, Norfolk, VA 23529, USA and Theory Group, Jefferson Lab, Newport News, VA 23606, USA*

This is an English translation of my 1977 Russian preprint. It contains the first explicit definition of the pion distribution amplitude (DA), the expression for the pion form factor asymptotics in terms of the pion DA, and formulates the pQCD parton picture for hard exclusive processes.
Abstract of the original paper:
The large Q^2 behavior of the pion electromagnetic form factor is explicitly calculated in the non-Abelian gauge theory to demonstrate a field-theoretical approach to the deep elastic processes of composite particles. The approach is equivalent to a new type of parton model.

HYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

Exclusive processes in perturbative quantum chromodynamics

G. Peter Lepage
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Stanley J. Brodsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 27 May 1980)

We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon "distribution amplitudes" $\phi(x_i,Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

Pion LCDA

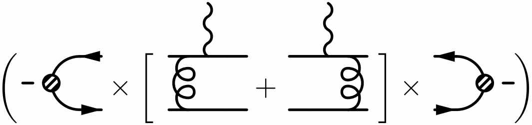
- $\phi_M(x,\mu^2)$: Light Cone Distribution Amplitude (LCDA)

$$\phi_M(x,\mu^2) = 6x(1-x)\sum_{n=0}^{\infty} a_n(\mu^2)C_n^{(3/2)}(2x-1) \underset{\mu^2\to\infty}{\to} 6x(1-x)$$

- Factorization theorem:

$$F_\pi(Q^2) \underset{\text{large } Q^2}{=} \int_0^1 dxdy\phi_{\overline{M}}(y,Q^2)T_H(x,y,Q^2)\phi_M(x,Q^2)$$

$$\underset{\text{large } Q^2}{=} \int_0^1 dxdy$$



$$\underset{\text{large } Q^2}{=} \frac{16\pi\alpha_S(Q^2)}{Q^2}f_\pi^2$$

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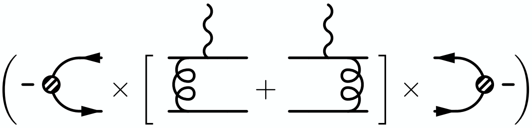

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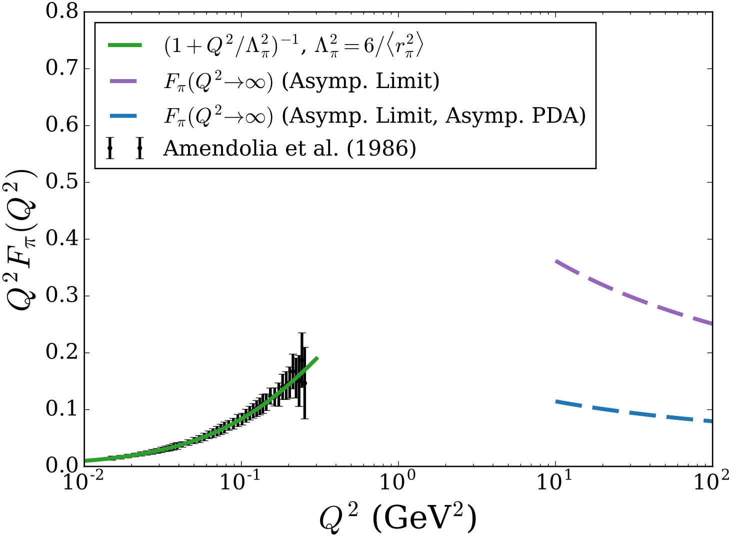
Why is this still interesting?



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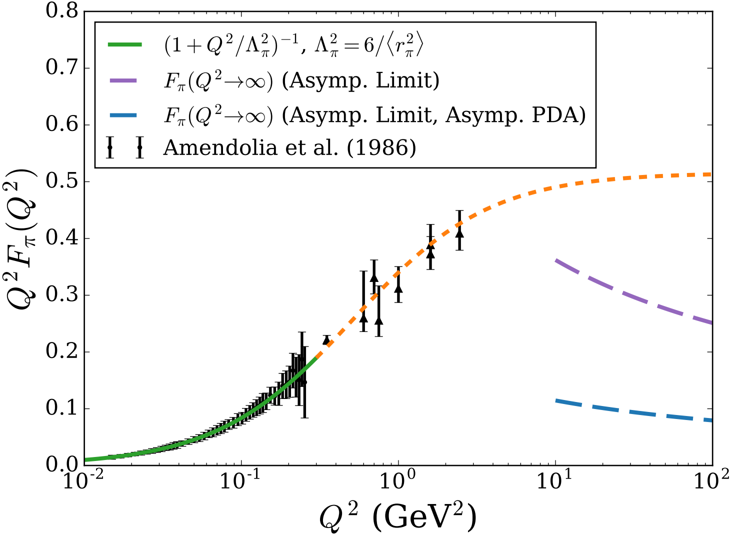
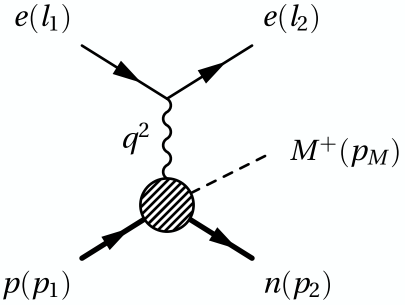


EXTRACTING F_π FROM ELECTROPRODUCTION DATA

Meson Electroproduction



- Kinematic variables:

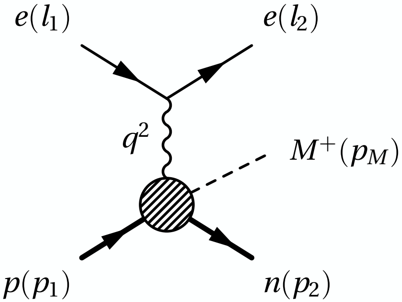
$$s = (p_1 + q)^2 = W^2$$
$$t = (p_1 - p_2)^2$$
$$Q^2 = -q^2$$

- Four structure functions:

$$(2\pi)\frac{d^2\sigma}{dt d\phi} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi.$$

- ϵ is a measure of the virtual photon polarization

Meson Electroproduction



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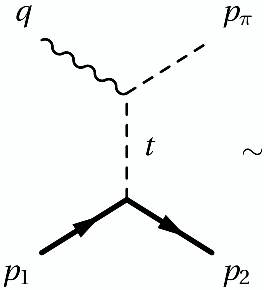
- ϵ is a measure of the virtual photon polarization

$\gamma^* p \to \pi^+ n$

- One-photon-exchange approximation: Hadronic and leptonic vertices factorize.
- Consider $t/s \to 0$, ie peripheral scattering: pion cloud.
- Natural to consider pion exchange process

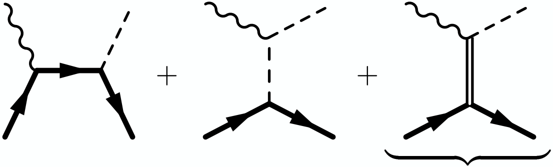


$$\sim \frac{1}{t - m_\pi^2} \sim$$

- Initial pion off-shell.
- Amplitude not gauge invariant,
- Cross section vanishes in forward limit.

Recap of VGL Model

- Construct Gauge invariant amp from Effective Lagrangian

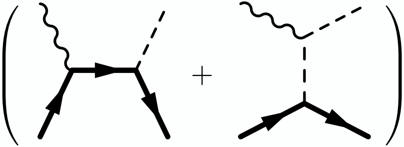
$\mathcal{M}_{\text{BTM}} =$ 

ρ exchange

- $t/s \to 0$: Regge theory:

$$\mathcal{R}_{\pi}(s,t) = \frac{\pi \alpha'_{\pi} \phi(t)}{\sin(\pi \alpha_{\pi}(t) \Gamma(1+\alpha_{\pi}(t)))} \left(\frac{s}{s_0}\right)^{\alpha_{\pi}(t)}$$

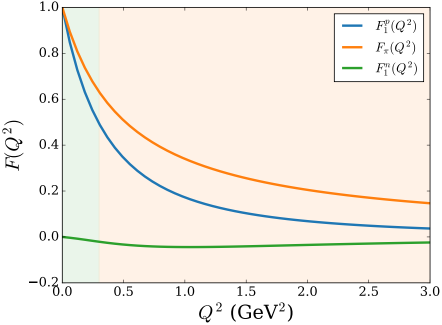
- Incorporate structure

$\mathcal{M}_{\text{VGL}} = F_{\pi}(Q^2) \times (t - m_{\pi}^2) \mathcal{R}_{\pi}(s,t) \times$ 

Possible Improvements to the Model

$$\left.\frac{d\sigma_L}{dt}\right|_{\mathrm{VGL}} \propto |F_\pi(Q^2)|^2$$

- Background should not be sensitive to $F_\pi(Q^2)$: Model dependence?
- How does this effect extraction?



Examining the Model Dependence of F_π in a Simple Model

A Bosonic Model of Pion Electro-production

- Inspired by a simple model due to Miller.

PHYSICAL REVIEW C **80**, 045210 (2009)

Electromagnetic form factors and charge densities from hadrons to nuclei

Gerald A. Miller
Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA
(Received 18 August 2009; published 22 October 2009)

A simple exact covariant model in which a scalar particle Ψ is modeled as a bound state of two different particles is used to elucidate relativistic aspects of electromagnetic form factors $F(Q^2)$. The model form factor is computed using an exact covariant calculation of the lowest order triangle diagram. The light-front

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_N)^2 - \frac{1}{2} m_N^2 \Psi_N^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 - g_{\pi N} \Psi_N^\dagger \boldsymbol{\tau} \cdot \boldsymbol{\pi} \Psi_N$$

- Include electromagnetic interactions via $\partial_\mu \to \partial_\mu + ieA_\mu$

Proposal

	Current Extraction	This Analysis
Model	$i\mathcal{M}^{\mu}_{\text{VGL}} = F_\pi(Q^2) D_F^{\pi -1} D_\pi^{\text{R}}(t)[i\mathcal{M}^{\mu}_{\text{BTM}}]$	$i\mathcal{M}^{\mu} = F_\pi(Q^2)[i\mathcal{M}^{\mu}_{\text{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^1\text{H}(e, e'\pi^+)n$	$i\mathcal{M}^{\mu}_{\text{1-Loop}}$

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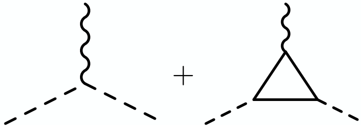
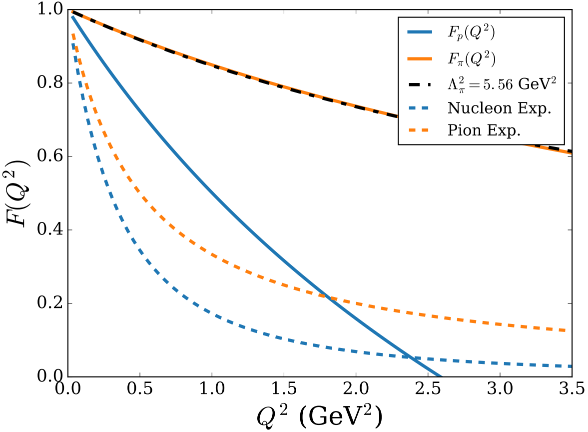
Data	$^1\mathsf{H}(e, e'\pi^+)n$	$i\mathcal{M}^{\mu}_{\text{1-Loop}}$

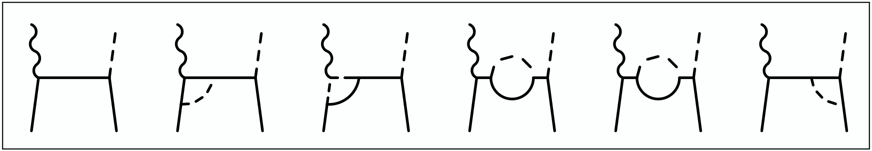
Form Factors in Simple Model

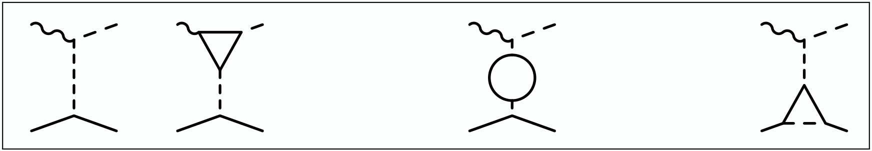
$$\langle p_2 | J^\mu(0) | p_1 \rangle =$$

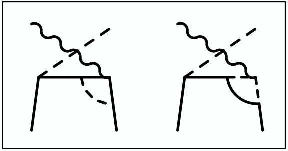


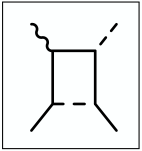


Diagrams

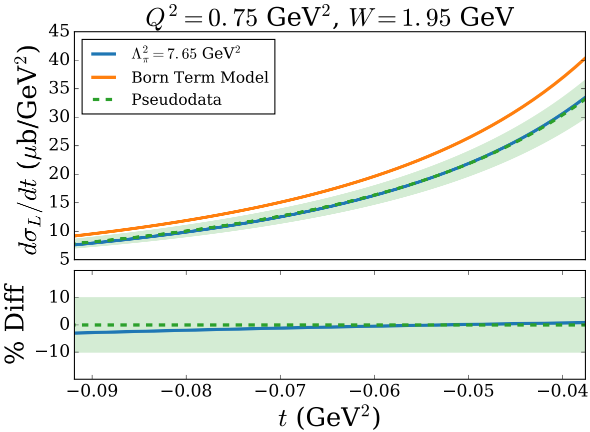






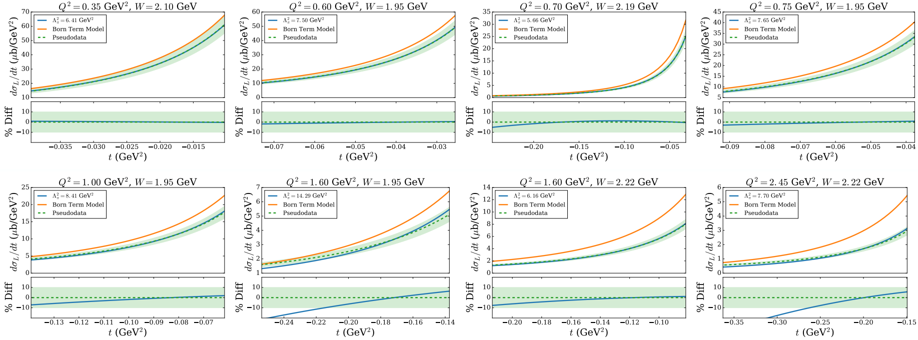


A Specific Example

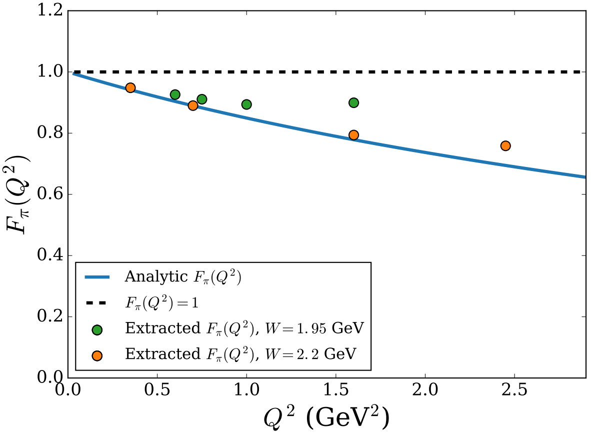


- t range chosen to be same as experiment.

Cross Section

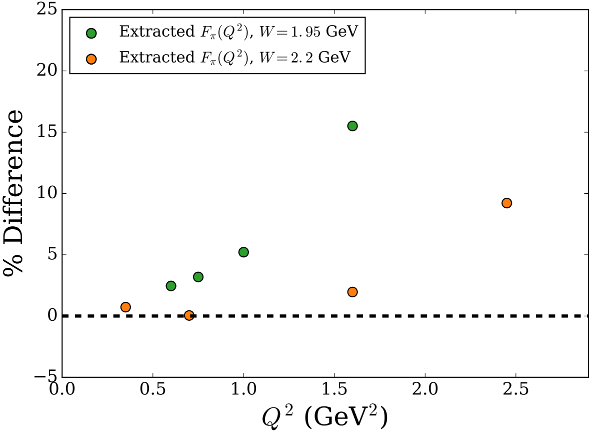


RESULTS



- Apart from possibly point at $(Q^2, W) = (1.6, 1.95)$, results look ok.

Systematic Overestimate?



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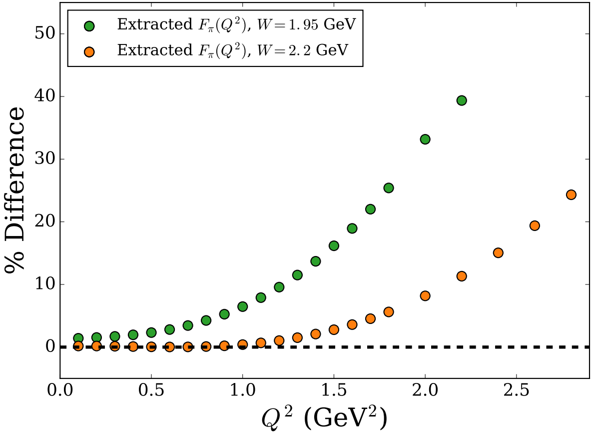
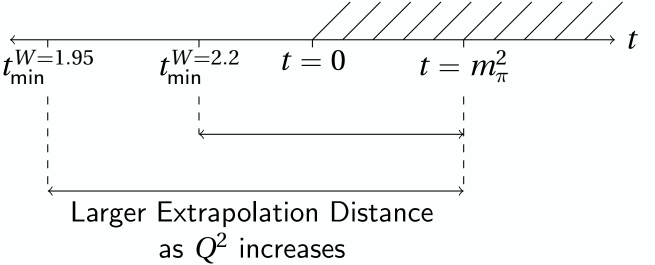


- Fit first five percent of allowed t.

How Do We Understand the W Dependence?

- A well known fact: Try to extract pion form factor close to pion pole!



- Specific details of model more important for larger $|t_{\text{min}}|$.

Isolating the Pion Exchange Contribution

$F_i^p \neq F_\pi$

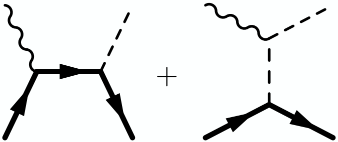
- Note that

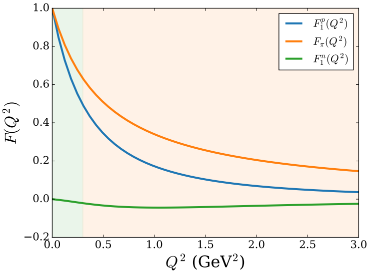
$$\left.\frac{d\sigma_L}{dt}\right|_{\mathsf{VGL}} \propto |F_\pi(Q^2)|^2$$

- LSZ: residue of pion-pion matrix element:

$$\langle \pi(p_2)|\, J^\mu(0)\, |\pi(p_1)\rangle = F_\pi(Q^2)(p_1 + p_2)^\mu$$

- Should not be sensitive to nucleon term.





Alternative Prescription for Gauge Invariance

- Return to the Born Term Model:

$$i\mathcal{M}^{\mu}_{\mathsf{BTM}} \propto \overline{u}_N(p_2)\gamma_5\left[\frac{(\not{p}_1 + \not{q} + m_N)}{s - m_N^2}\gamma^{\mu} + \frac{(2p_{\pi} - q)^{\mu}}{t - m_{\pi}^2}\right]u_N(p_1)$$

- Take inspiration from Ward-Green-Takahashi Identity:

$$iq_{\mu}\Gamma^{\mu}(p_1, p_2; q) = D_F^{-1}(p_2) - D_F^{-1}(p_1)$$

- General form of scalar propagator

$$D_F(p) = \frac{i}{p^2 - m^2 - \Sigma(p^2)}$$

A Simple Modification

- General decomposition for Γ^μ

$$\Gamma^\mu_\pi(p_\pi, q) = f_1(t, p_\pi^2; q^2)(2p_\pi - q)^\mu - f_2(t, p_\pi^2; q^2)q^\mu$$

where $t = (p_\pi - q)^2$

- Relate the two form factors as

$$f_2(t^2, m_\pi^2; q^2) = \frac{[t^2 - m_\pi^2 - \Sigma_\pi(t^2)] - (t^2 - m_\pi^2)f_1(t, m_\pi^2; q^2)}{q^2}.$$

- In this way, we may have an arbitrary pion form factor

A Simple Modification

- Pion-exchange diagram becomes

$$\overline{u}(p_2)\gamma_5 \frac{(2p_\pi - q)^\mu}{t - m_\pi^2} u(p_2)$$
$$\to \overline{u}(p_2)\gamma_5 \left[\frac{f_1(t, m_\pi^2, q^2)}{t - m_\pi^2}(2p_\pi - q)^\mu - \frac{f_2(t, m_\pi^2, q^2)}{t - m_\pi^2} q^\mu \right] u(p_1)$$

- where

$$f_2(t^2, m_\pi^2; q^2) = \frac{[t^2 - m_\pi^2 - \Sigma_\pi(t^2)] - (t^2 - m_\pi^2) f_1(t, m_\pi^2; q^2)}{q^2}.$$

Taylor Series

- Assume deviations from on-shell limit analytic:

$$f_1(t, p_\pi^2; q^2) = f_1(m_\pi^2, m_\pi^2; q^2) + (t - m_\pi^2)\frac{d}{dt} f_1(t, p_\pi^2; q^2)\bigg|_{t=m_\pi^2} + \ldots$$

- Define

$$g_1(q^2) = \frac{d}{dt} f_1(t, p_\pi^2; q^2)\bigg|_{t=m_\pi^2}$$

- Treat as fitting parameter: absorbs non-pion pole contributions.

Taylor Series

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- Treat as fitting parameter: absorbs non-pion pole contributions.

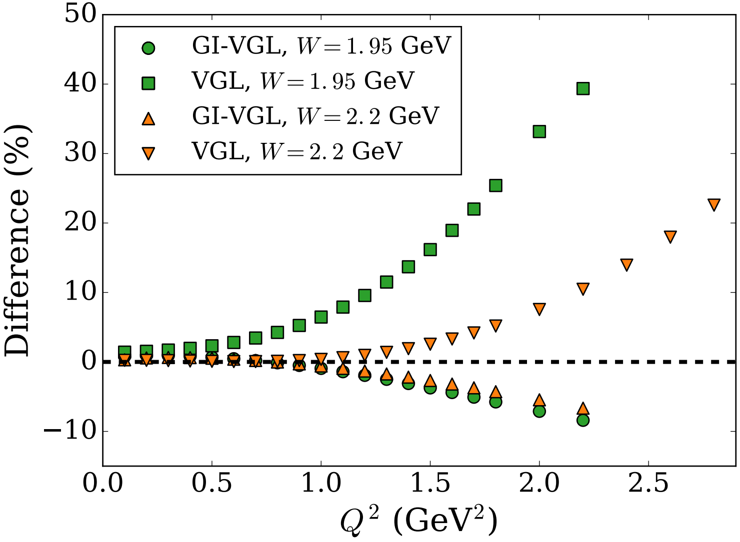
Statement of model

- Model is

$$i\mathcal{M}^{\mu}_{\mathsf{GIVGL}} \propto \overline{u}_N(p_2)\gamma_5 F_1^p(Q^2)\left[\frac{(\not{p}_1+\not{q}+m_N)}{s-m_N^2}\gamma^\mu + \frac{f_1(q^2)}{t-m_\pi^2}(2p_\pi - q)^\mu + g_1(q^2)(2p_\pi - q)^\mu - \frac{f_2(t,m_\pi^2,q^2)}{t-m_\pi^2}q^\mu\right]u_N(p_1)$$

- Multiply by $F_1^p(Q^2)$. Identify $F_\pi(Q^2) = F_1^p(Q^2) f_1(q^2)$
- Other options are possible: motivated by simplicity.

Testing the New Approach on our Toy Model



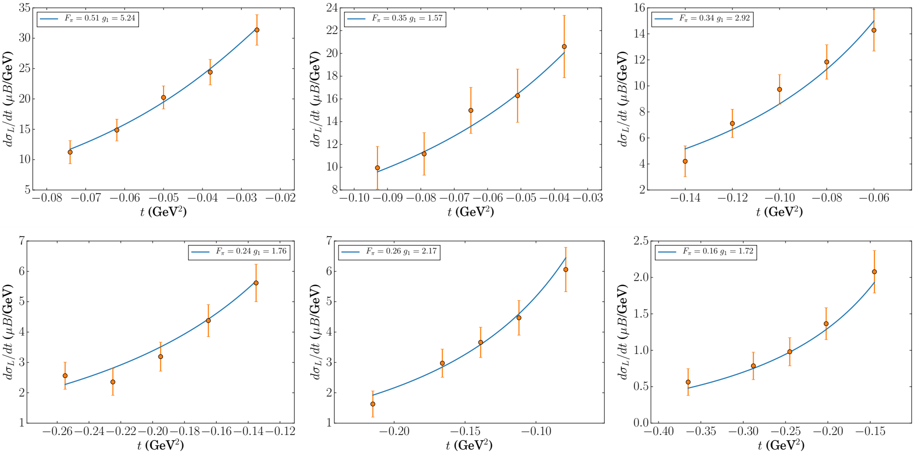
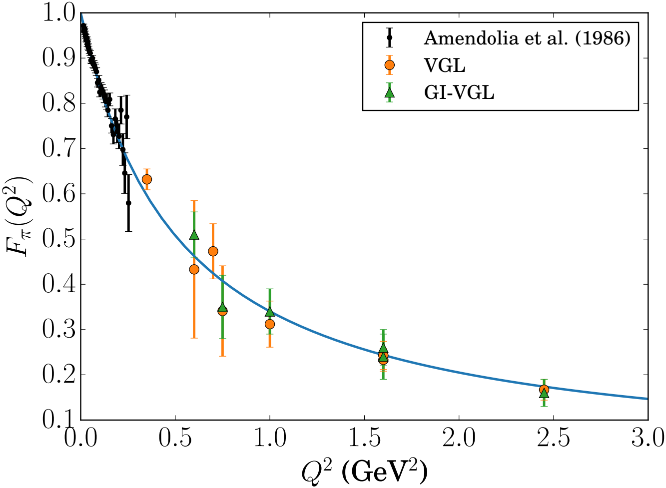


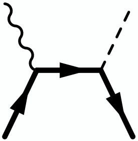
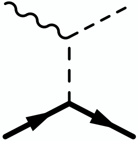
Figure 2: Fits to the experimental cross section data using the modified VGL model.

Extracting the Pion Form Factor From Electroproduction Data



Application to Kaon Electroproduction

- $H(e, e'K^+)\Lambda$
- Construct Gauge invariant amp from Effective Lagrangian

$\mathcal{M}_{\mathsf{BTM}} =$  $+$ 

- Exchange coupling $g_{\pi NN} \to g_{KN\Lambda}$
- Gauge restoration prescription still valid.

Reggeizing Model

- Also possible to retain Reggeization of VGL Model.
- Take 'inspiration' from WTI; use Regge propagator in identity.

$$i\mathcal{M}^{\mu}_{\text{Regge}} \propto \overline{u}_N(p_2,\lambda_2)\gamma_5\Bigg[\frac{(\not{p}_1+\not{q}+m_N)}{s-m_N^2}\gamma^{\mu} + [f_1(t,p_\pi^2;q^2)(2p_\pi-q)^{\mu} - f_2(t,p_\pi^2;q^2)q^{\mu}]\mathcal{R}_\pi(s,t)\Bigg]u_N(p_1,\lambda_1).$$

where

$$f_2(t^2,m_\pi^2;q^2) = \frac{\mathcal{R}_\pi^{-1}(s,t)-(t^2-m_\pi^2)f_1(t,m_\pi^2;q^2)}{q^2}.$$

Conclusions

- Investigated model dependence of F_π in simple model: possible model dependence at large Q^2
  - Motivates extracting F_π with multiple models to study systematics
- Proposed an alternative implementation of gauge invariance
- Led to less model dependence in toy F_π extraction
- Physical F_π agree within errors: extraction is reliable.

Thanks

References

Spare Slides

Kinematics and Conventions

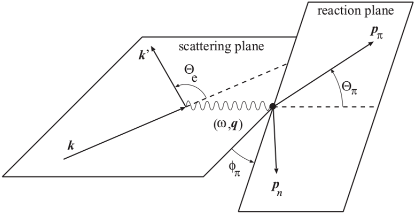


Figure 3: Blok et al., 2008

Mandelstam Variables:

$$s = p_s^2 = (p+q)^2 = (p' + p_\pi)^2 \equiv W^2$$
$$t = p_t^2 = (p_\pi - q)^2 = (p - p')^2 < 0$$
$$u = p_u^2 = (p - p_\pi)^2 = (p' - q)^2$$

Experimentally, use Q^2, W and t.

Evidence for Enhanced Form Factor

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Off-shell persistence of composite pions and kaons

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In order for a Sullivan-like process to provide reliable access to a meson target as t becomes spacelike, the pole associated with that meson should remain the dominant feature of the quark-antiquark scattering matrix and the wave function describing the related correlation must evolve slowly and smoothly. Using continuum methods for the strong-interaction bound-state problem, we explore and delineate the circumstances under which these conditions are satisfied: for the pion, this requires $-t \lesssim 0.6\,\text{GeV}^2$, whereas $-t \lesssim 0.9\,\text{GeV}^2$ will suffice for the kaon. These results should prove useful in planning and evaluating the potential of numerous experiments at existing and proposed facilities.

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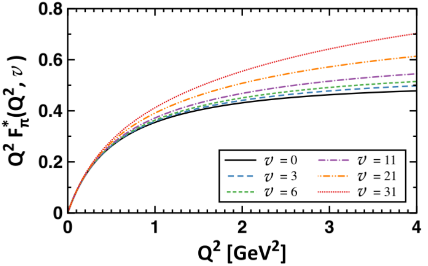
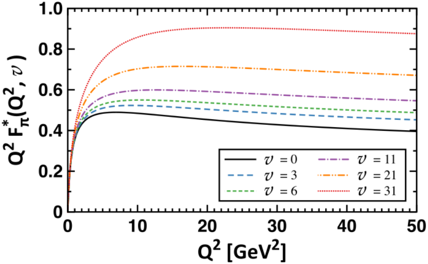
Evidence for Enhanced Form Factor

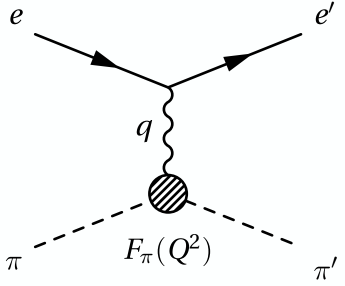
- How does pion form factor vary off-shell?
- Although off-shell pion is not well defined, can attempt to address question using BSE.
- $\nu \geq 0$ parameterizes "off-shellness" in units of m_π^2
- $t = 0.015 \approx m_\pi^2$ GeV^2, $\nu = 1$
- $t = 0.35 \approx 18 m_\pi^2$ GeV^2, $\nu = 18$





Experimental Measurements

- At low energy, scatter pion beam from electrons in liquid hydrogen target.
- Measure recoiling pion and electron.



- Differential cross section is

$$\frac{d\sigma}{dq^2} \propto |F_\pi|^2 \frac{1}{q^4}\left(1 - \frac{q^2}{q^2_{\mathsf{max}}}\right)$$

Kinematic Limitations

- Direct measurement has kinematic limitation.

$$\frac{d\sigma}{dq^2} \propto |F_\pi|^2 \frac{1}{q^4}\left(1 - \frac{q^2}{q^2_{\text{max}}}\right)$$

- Where q^2_{max} corresponds to backward scattering in CM frame.
  - roughly proportional to pion beam momentum
- For 300 GeV pion beam, $q^2_{\text{max}} = 0.288$ GeV^2.
- Close to this momentum, the cross section is suppressed, and an extraction becomes difficult.
- Thus could only measure pion form factor up to about 0.3 GeV.

Extracting Pion Form Factor from Data

- Measure cross section at a range of t values for fixed Q^2 and W.
- Fit model to cross section.
- If required...
  - Fit each data point.
  - Extrapolate these points to $t = t_{\text{min}}$, where there is least contamination from interfering backgrounds not included in the VGL model.
- More on this later...