Generalized GDH Sum Rules for Neutron and $^3$He at Low $Q^2$

Chao Peng (Argonne National Laboratory)
For E97-110 and Hall A Collaborations
HALL A/C COLLABORATION MEETING, JUNE 08, 2021
Outline

Introduction

Experiment E97-110

E97-110 Results
Generalized GDH Sum Rules

Virtual Compton amplitudes are related to moments of spin dependent structure functions

- Connect moments of spin-dependent structure functions with the Compton amplitudes

\[
I_{TT}(Q^2) = \frac{M^2}{4\pi^2\alpha} \int_{v_{th}}^{\infty} \frac{K\sigma_{TT}(v, Q^2)}{v^2} dv
\]

\[
= \frac{2M^2}{Q^2} \int_{0}^{x_{th}} \left[ g_1(x, Q^2) - \frac{4M^2x^2}{Q^2} g_2(x, Q^2) \right] dx
\]

\( g_1 \) and \( g_2 \) are experimentally accessible, \( I_{TT}(Q^2) \) predictions are given by theories

- Chiral Effective Field Theory (ChEFT)
- Lattice QCD (not available yet)
Generalized Spin Polarizabilities

Longitudinal-Transverse (LT) interference polarizability

\[ \delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_{th}} [g_1(x, Q^2) + g_2(x, Q^2)] x^2 dx \]

- Quantifies the spin precession from LT interference (analogous in classical view)
- Arises because of virtual photon \((Q^2 \neq 0)\) can be longitudinally polarized
- “Gold-plated” observable for ChEFT because of suppression in \(\Delta(1232)\) contributions

Forward spin polarizability

\[ \gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_{th}} \left[ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right] x^2 dx \]
Outline

Introduction

Experiment E97-110

E97-110 Results
E97-110 at Jefferson Lab

Inclusive measurement, $^3\text{He}(e,e')X$
- Scattering angles: $6^\circ$ and $9^\circ$
- Polarized electron beam, $P_{\text{beam}} = 75\%$
- Polarized $^3\text{He}$ target, $P_{\text{target}} = 40\%$

Measured the differences of polarized cross sections
- Parallel (anti-parallel)
- Perpendicular

Spokespersons: J.-P. Chen, A. Deur, F. Garibaldi
Graduate students: J. Singh, V. Sulkosky, J. Yuan, C. Peng, N. Ton
E97-110 at Jefferson Lab

<table>
<thead>
<tr>
<th>Target Cell</th>
<th>Angle</th>
<th>Beam Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penelope</td>
<td>6.10°</td>
<td>2134.2</td>
</tr>
<tr>
<td>Priapus</td>
<td>6.10°</td>
<td>2134.9</td>
</tr>
<tr>
<td>Priapus</td>
<td>6.10°</td>
<td>2844.8</td>
</tr>
<tr>
<td>Priapus</td>
<td>6.10°</td>
<td>4208.8</td>
</tr>
<tr>
<td>Priapus</td>
<td>9.03°</td>
<td>1147.3</td>
</tr>
<tr>
<td>Priapus</td>
<td>9.03°</td>
<td>2233.9</td>
</tr>
<tr>
<td>Priapus</td>
<td>9.03°</td>
<td>3318.8</td>
</tr>
<tr>
<td>Priapus</td>
<td>9.03°</td>
<td>3775.4</td>
</tr>
<tr>
<td>Priapus</td>
<td>9.03°</td>
<td>4404.2</td>
</tr>
</tbody>
</table>
Radiative Correction

Iterative correction
- Build pseudo-model with experimental data
- Interpolation and extrapolation (or filled by other models) for unmeasured points
- Calculate radiative effects with this pseudo-model
- Unfold Born cross sections, and then update the pseudo-model
- Repeat until results are converged
Radiative Correction

Peter-bosted model for unmeasured extrapolation

Fit for interpolation

\[ \sigma \left( \text{nb} \cdot \text{MeV}^{-1} \cdot \text{sr}^{-1} \right) \]

\[ E - E' \ (\text{MeV}) \]

\[ \chi^2/n.d.f = 1.364091 \]
Radiative Correction

$\Delta \sigma = (\sigma - \sigma_{\text{no radiation}})$ (nb \cdot MeV$^{-1} \cdot$ sr$^{-1}$)

$E - E'$ (MeV)

$E = 2135$ MeV @ 6°
Radiative Correction

Systematic uncertainties

- Internal effects by comparing different approaches < 3%
- Extrapolation or model dependency for the unmeasured region
  - Cross-check with each other < 3%
- Free parameter $\Delta$ for singular integral of $I(E, E', l)$
  - $\Delta = 1 \pm 0.5$ MeV tested, negligible
- Material thickness uncertainty
- Particle trajectory uncertainty
  - Varied the central angle by $\pm 0.1^\circ$

![Graph showing energy differences](image)
Outline

Introduction

Experiment E97-110

E97-110 Results
$^3\text{He}$
Spin-dependent Structure functions
$^3$He
Spin-dependent
Structure functions
$^3$He
Spin-dependent
Structure
functions
Interpolation to constant $Q^2$

$Q^2 = 0.032 \sim 0.23 \text{ GeV}^2$

Blue: 9 degree
Red: 6 degree
Black points: interpolated data points

$Q^2 = 0.032 \sim 0.23 \text{ GeV}^2$
$^{3}\text{He Results}$

$$I_{GDH}(Q^2) = \frac{8\pi e^2}{M^2} I_{TT}(Q^2)$$

$$4I_{LT} = \frac{8M^2}{Q^2} \int_0^{x_{thres}} (g_1(x, Q^2) + g_2(x, Q^2)) \, dx$$

![Graphs showing $I_{GDH}(Q^2)$ and $I_{LT}(Q^2)$ with data points and error bars for different categories, including E97 data, E97 syst., E94 data, E94 syst., and Real Photon Value.]
$^3$He
Spin-dependent
Structure functions
QE subtracted
Neutron Results

Nuclear corrections follow the recipe from C. Ciofi degli Atti and S. Scopetta (1997)
Neutron Results
Neutron Spin Polarizabilities

V. Sulkosky et al., Nature Physics volume 17, p687–692 (2021)
Summary

Generalized GDH integrals are extracted at low $Q^2$
- Neutron GDH shows reasonable agreement with ChEFT calculations
- $^3$He GDH integral exhibits a turning point to recover real photon point

Spin polarizabilities for neutron
- Surprising disagreement with ChEFT calculations at lowest $Q^2$
- Motivates lattice QCD calculations

This work is supported in part by the U.S. Department of Energy, under Contract No. DE-AC02-06CH11357, and DE-FG02-03ER41231