Measurement of the neutron charge radius through the study of the nucleon excitation

A proposal for Jlab PAC49

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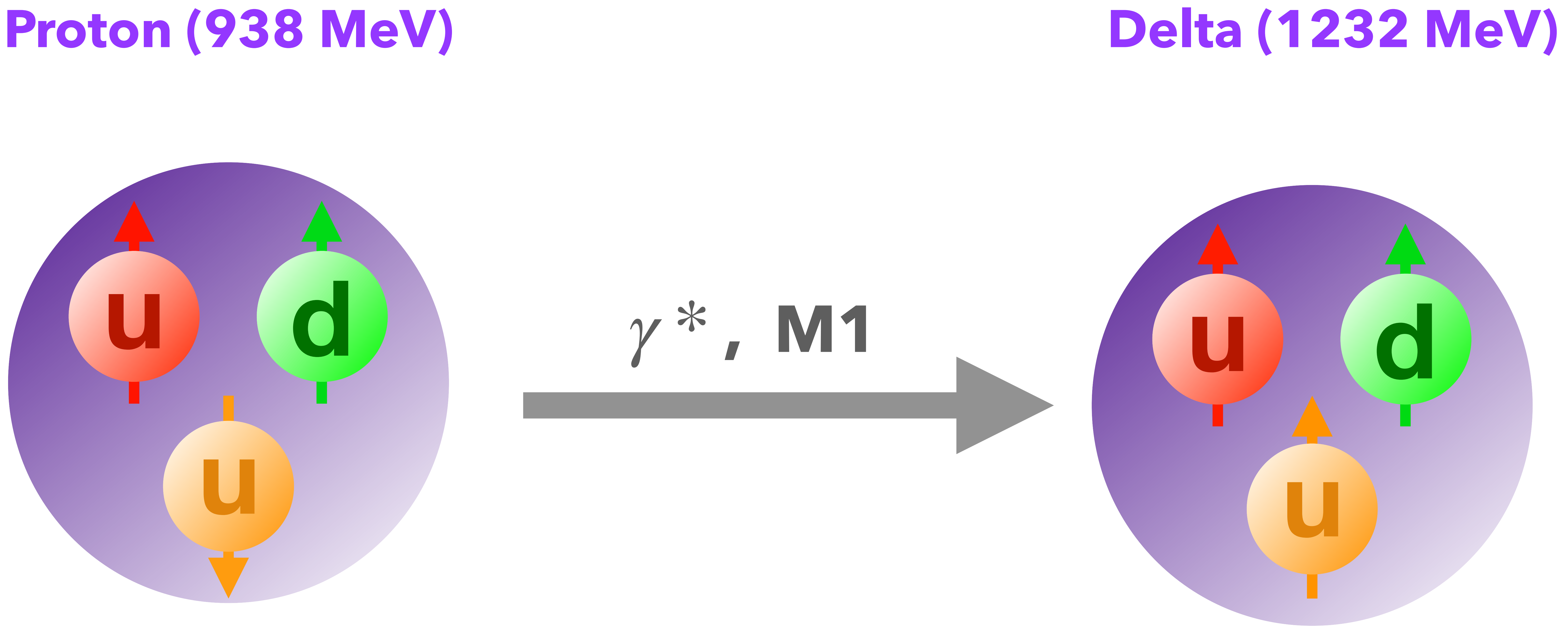
Primary Physics Goals

- **Proton N-Δ Transition Form Factors:**
  - JLab has invested significantly to the physics program of the N-Δ TFFs, with multiple experiments (in Halls A, B, and C).
  - TFFs have been measured up to Q^2=6 GeV^2 Here we aim to push the limits of the low Q^2, where the mesonic cloud dynamics is predicted to be dominant and rapidly changing
  - Test bed for ChEFT and LQCD calculations

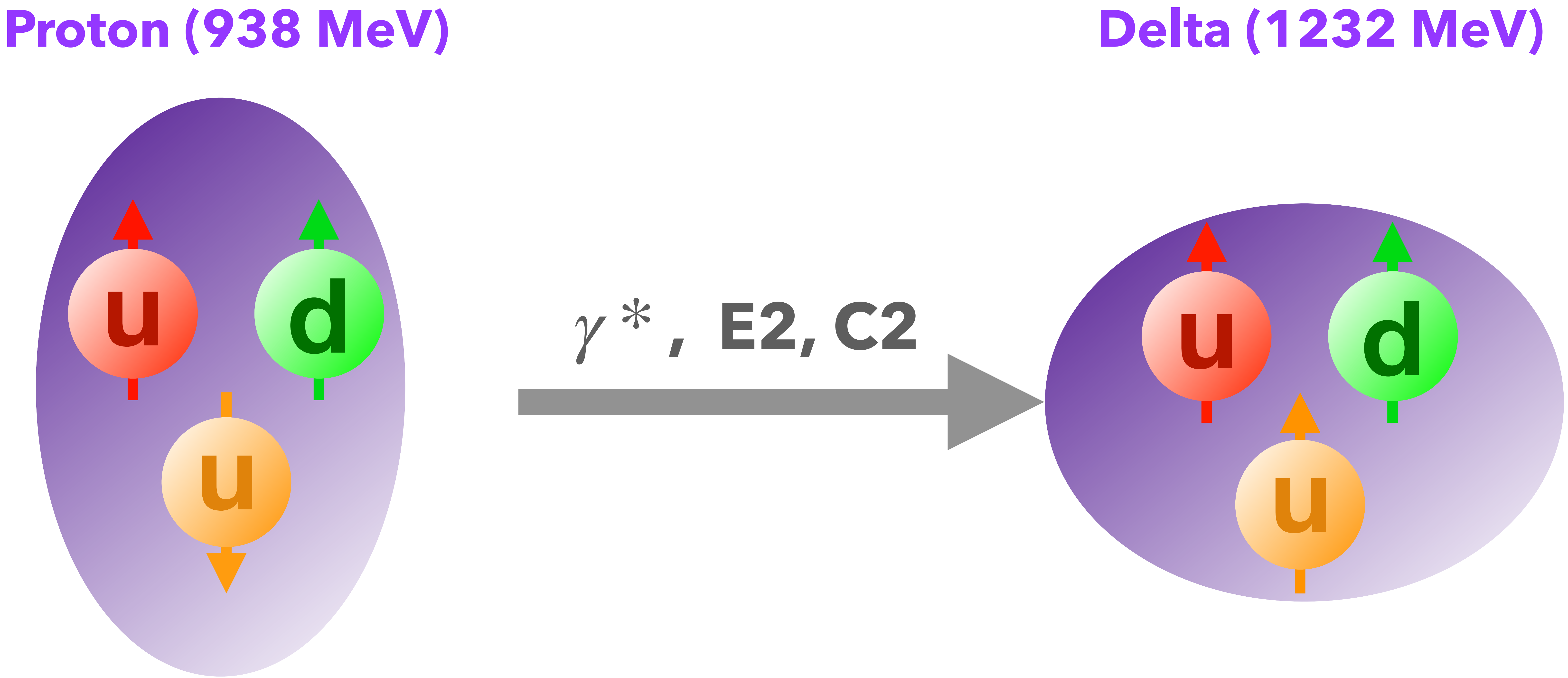
- **Neutron charge radius:**
  - One of the system's most basic properties.
  - Measured with only one (rather indirect) method.
  - World data exhibit tensions. Underestimated systematics.
  - Cross checking with a different method, whenever nature allows a path for it, is a *scientific obligation*.

N-Δ transition as a pathway to nucleon structure



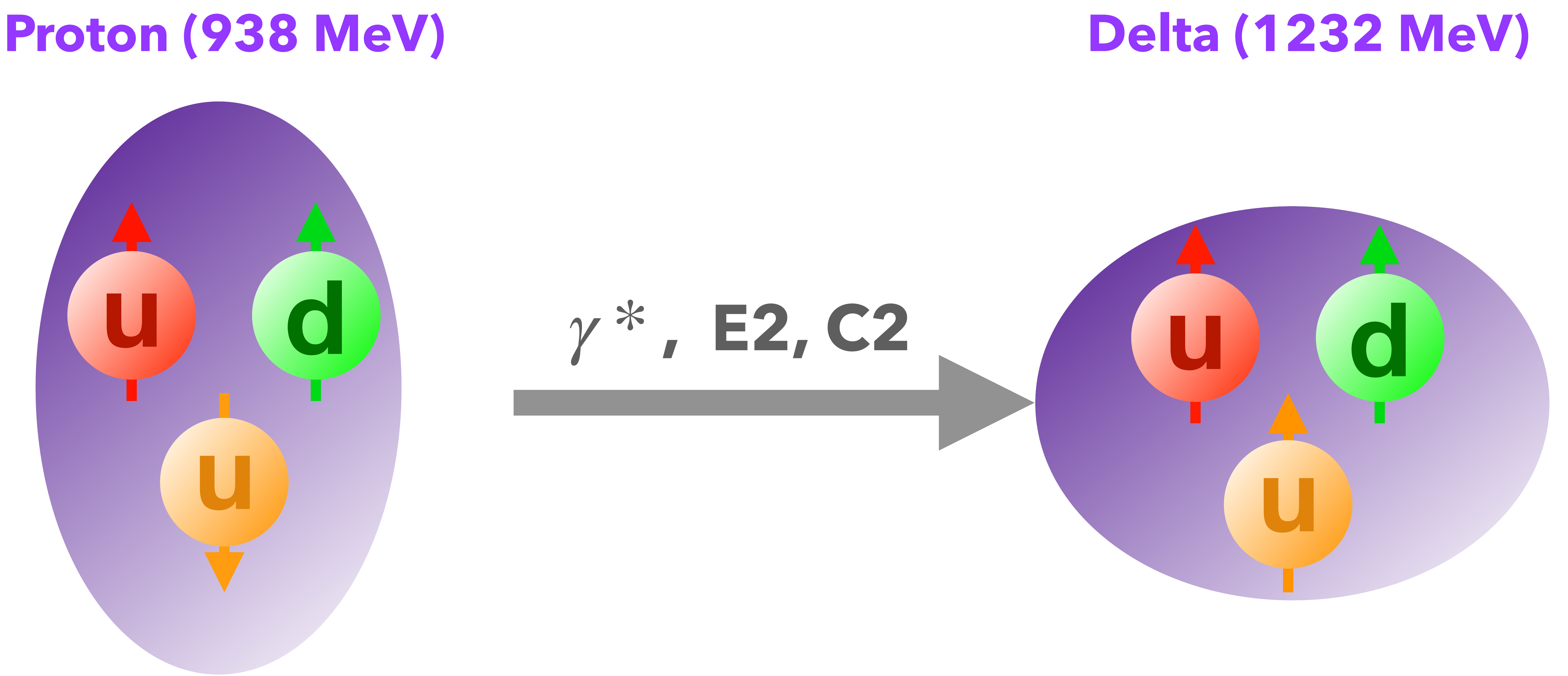
The dominant transition from proton to delta involves a dipole (M1) transition (spherical S-wave proton WF -> spherical S-wave Delta WF)

N-Δ transition as a pathway to nucleon structure



There also exists a quadrupole (E2 or C2) transition from proton to delta.
(non-spherical proton WF -> non-spherical Delta WF)

N-Δ transition as a pathway to nucleon structure



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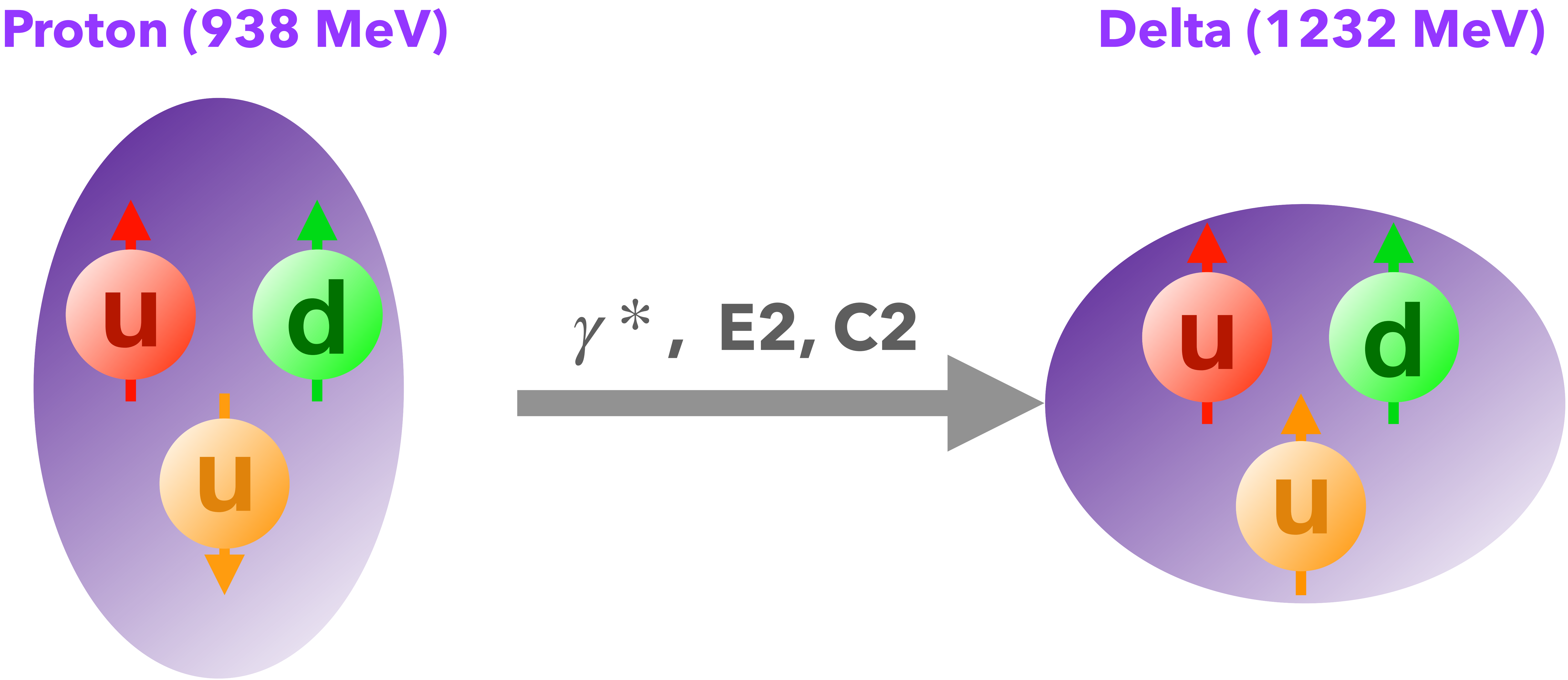
The quadrupole to dipole ratio (E2/M1 or C2/M1) is non-zero… Why?

EMR CMR

N-Δ transition as a pathway to nucleon structure



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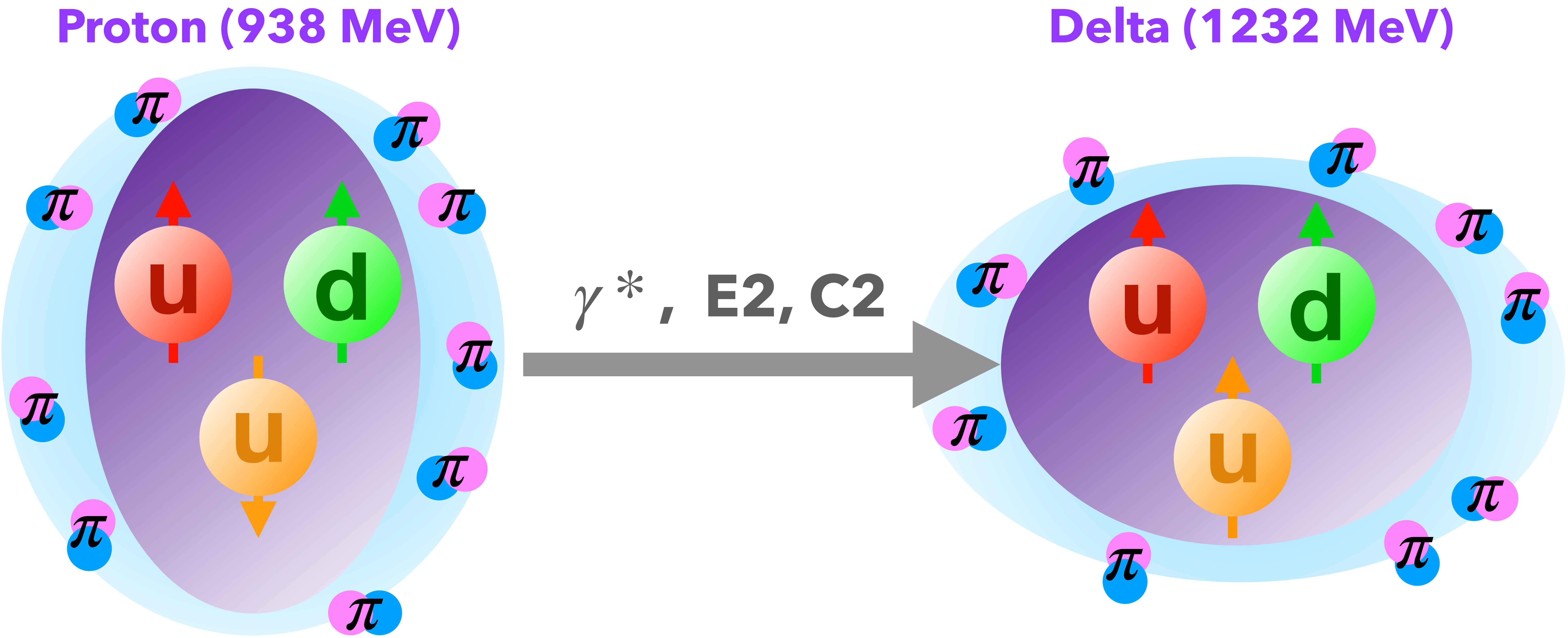
The quadrupole to dipole ratio (E2/M1 or C2/M1) is non-zero... Why?

Non-central (tensor) interactions between quarks can account for some of the spherical deviation, but not all...

N-Δ transition as a pathway to nucleon structure



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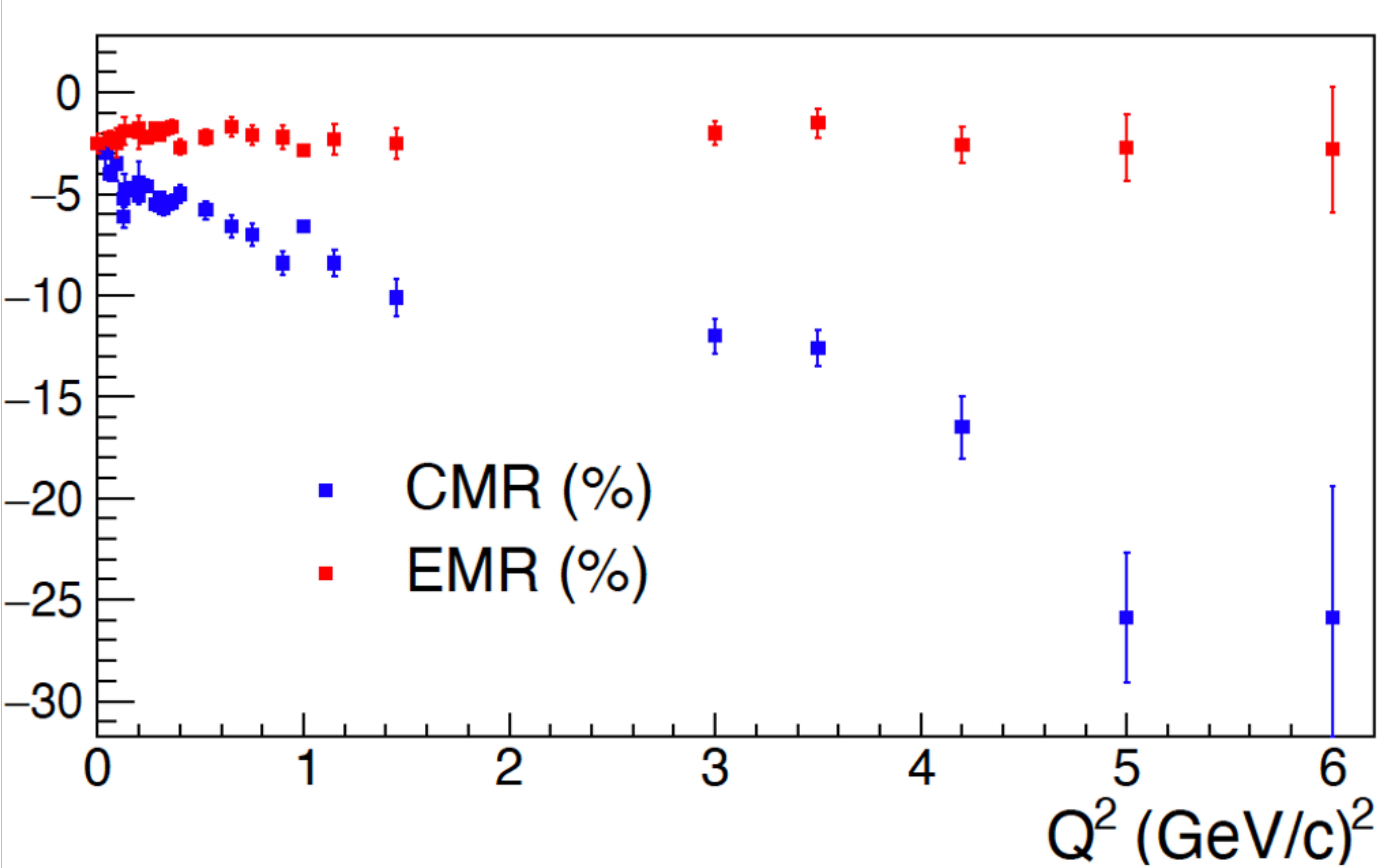
The dynamics of a meson cloud are important to describe the structure of the nucleon:
The nucleon structure directly relates to the nucleon radius.

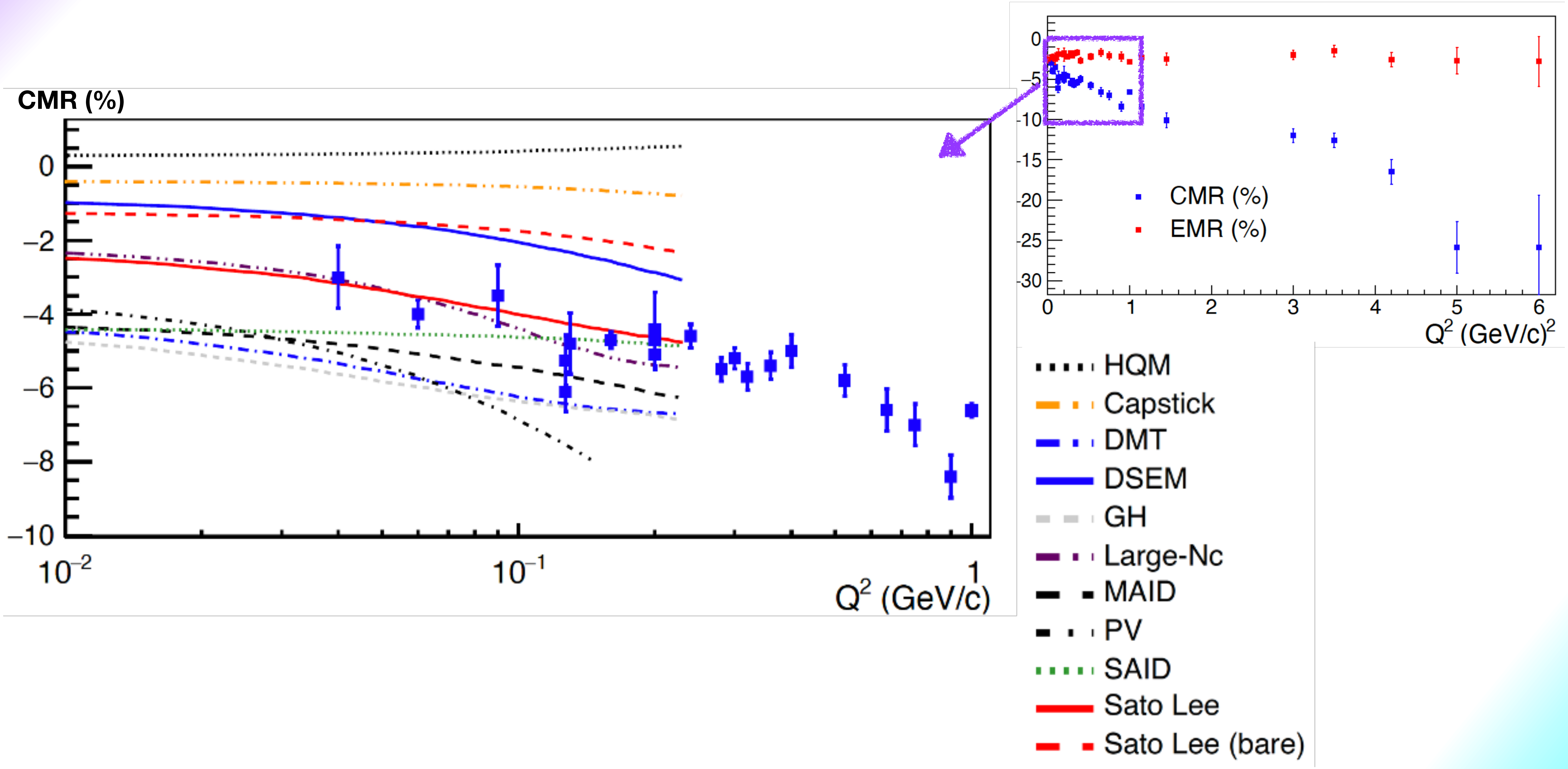
N-Δ transition as a pathway to nucleon structure

CMR & EMR converge at a small finite value as $Q^2 \to 0$

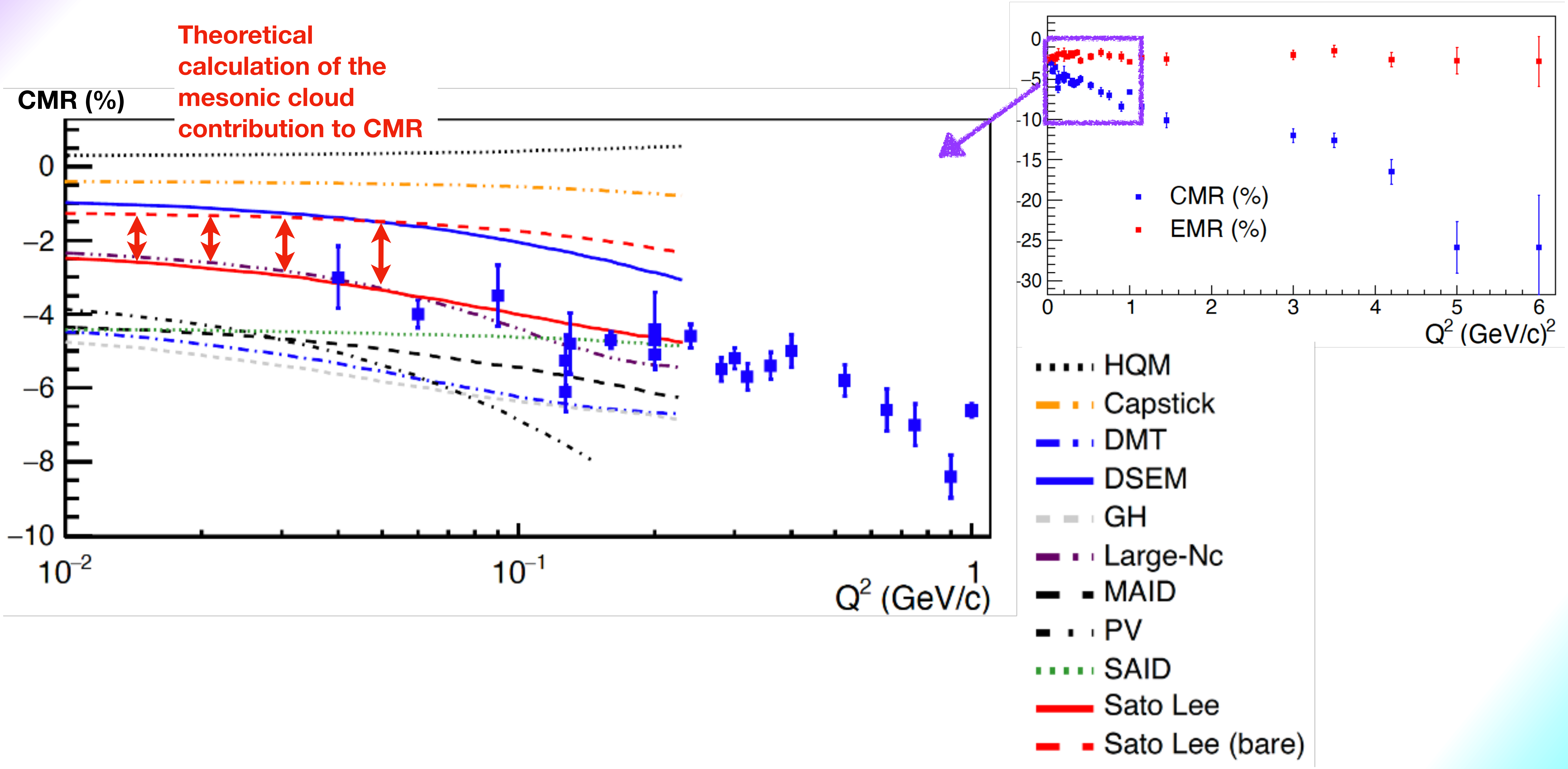


At large Q^2, no direct indication of EMR → 1 and CMR → constant (pQCD regime)

N-Δ transition as a pathway to nucleon structure



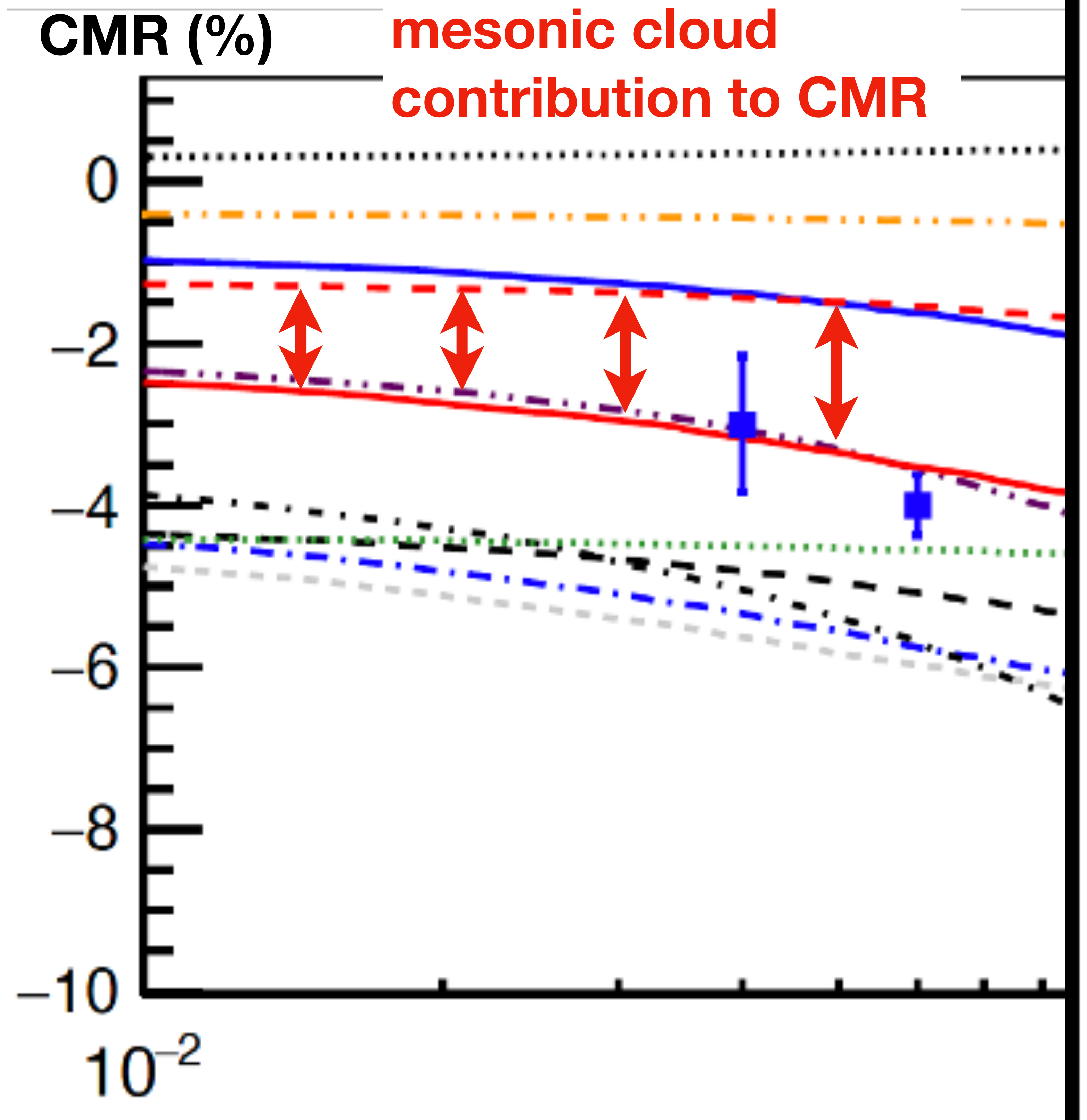
N-Δ transition as a pathway to nucleon structure



Theoretical calculation of the mesonic cloud contribution to CMR

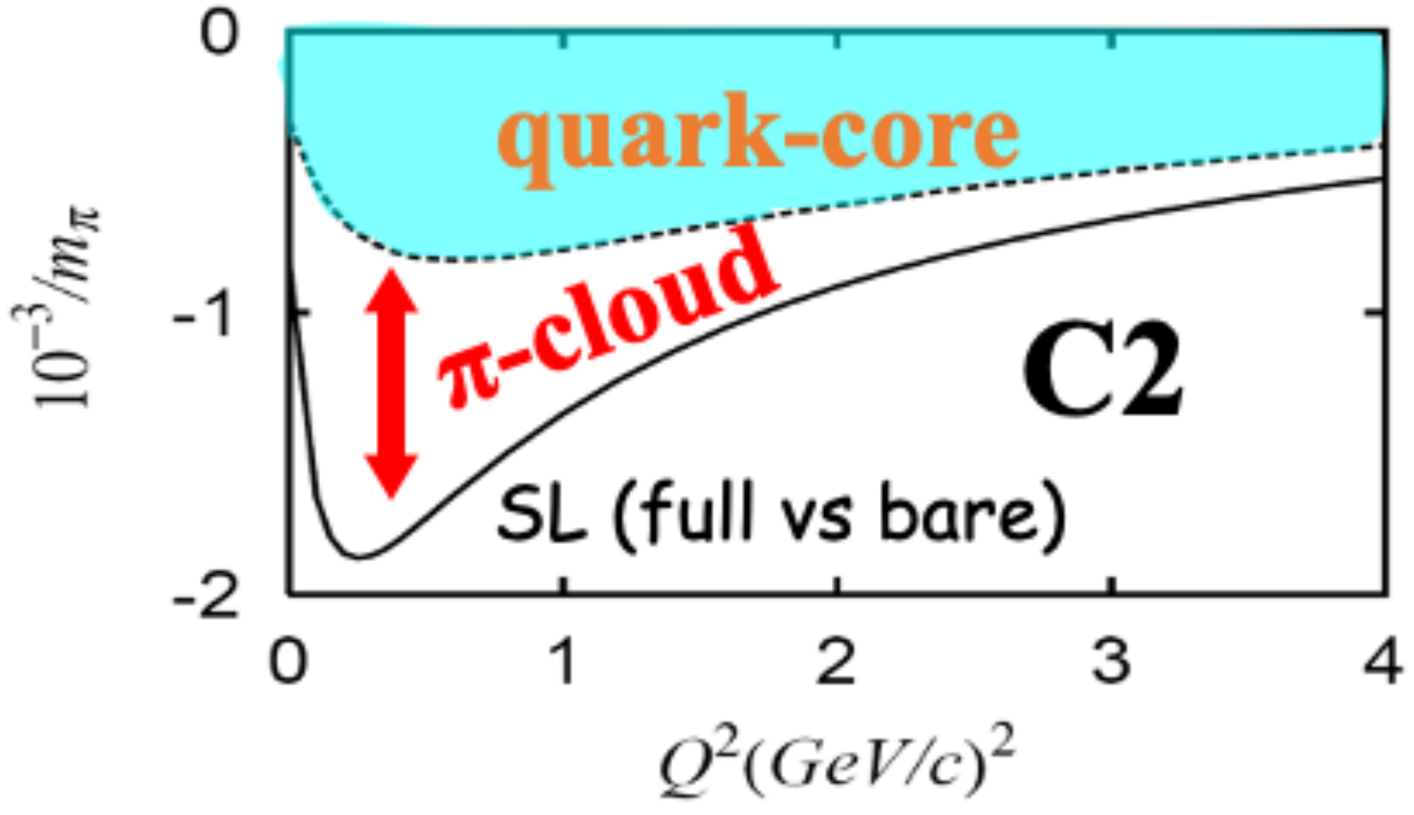
N-Δ transition as a pathway to nucleon structure

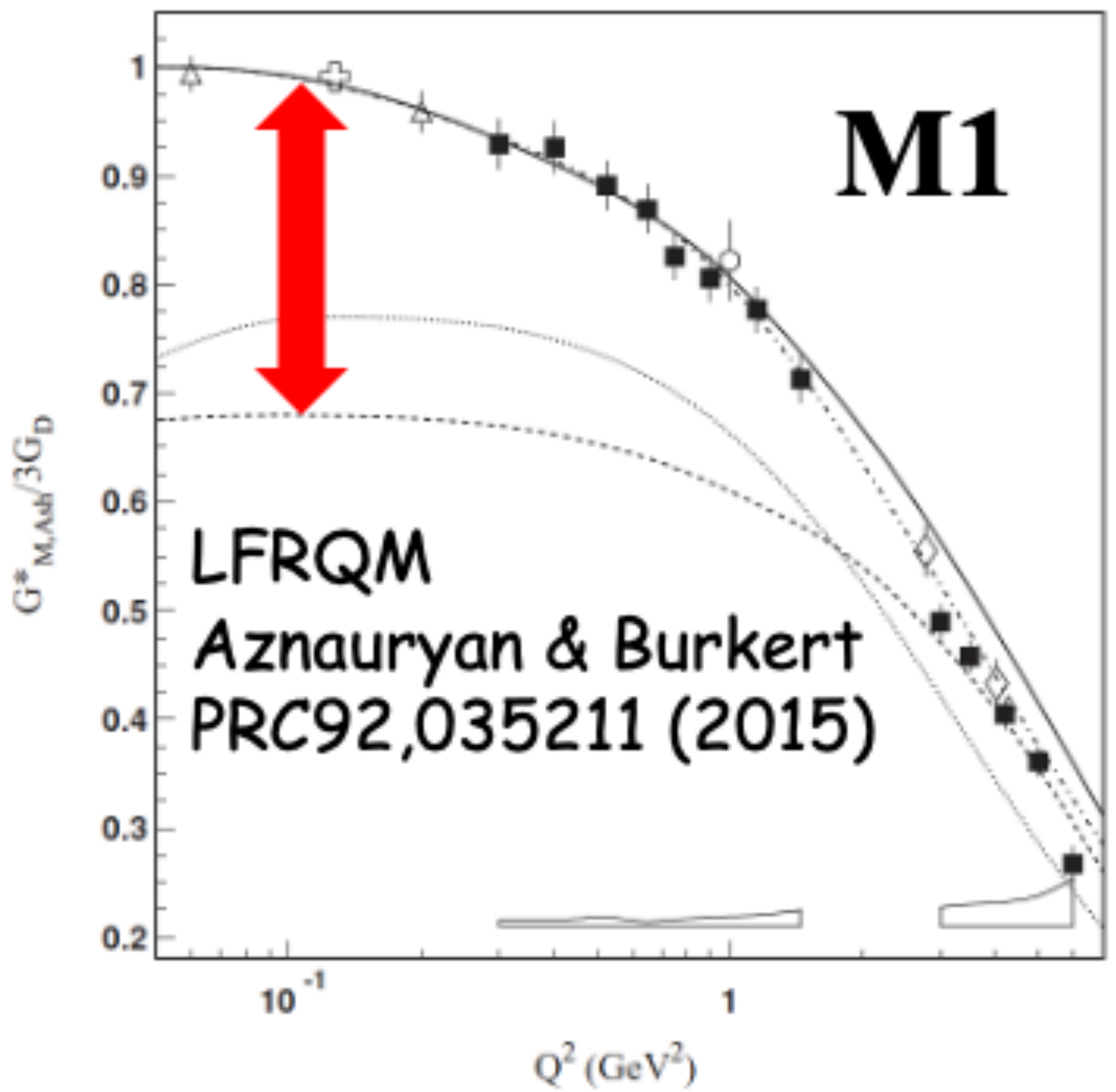
Theoretical calculation of the mesonic cloud contribution to CMR



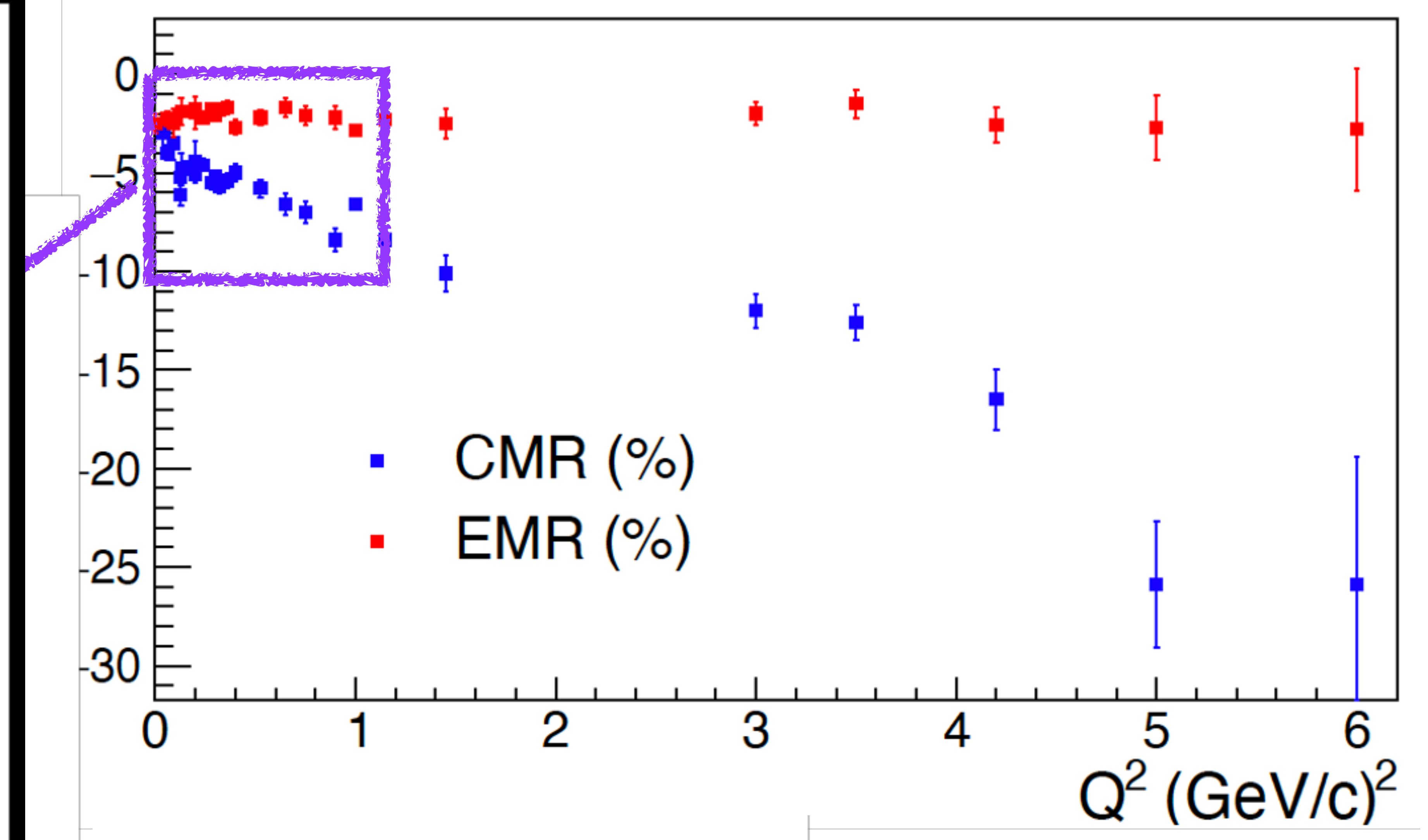
Dominant role of mesonic d.o.f. at large distance scale:

Mesonic cloud ~ 50% of the quadrupole amplitude magnitude & 1/3 of the magnetic dipole strength



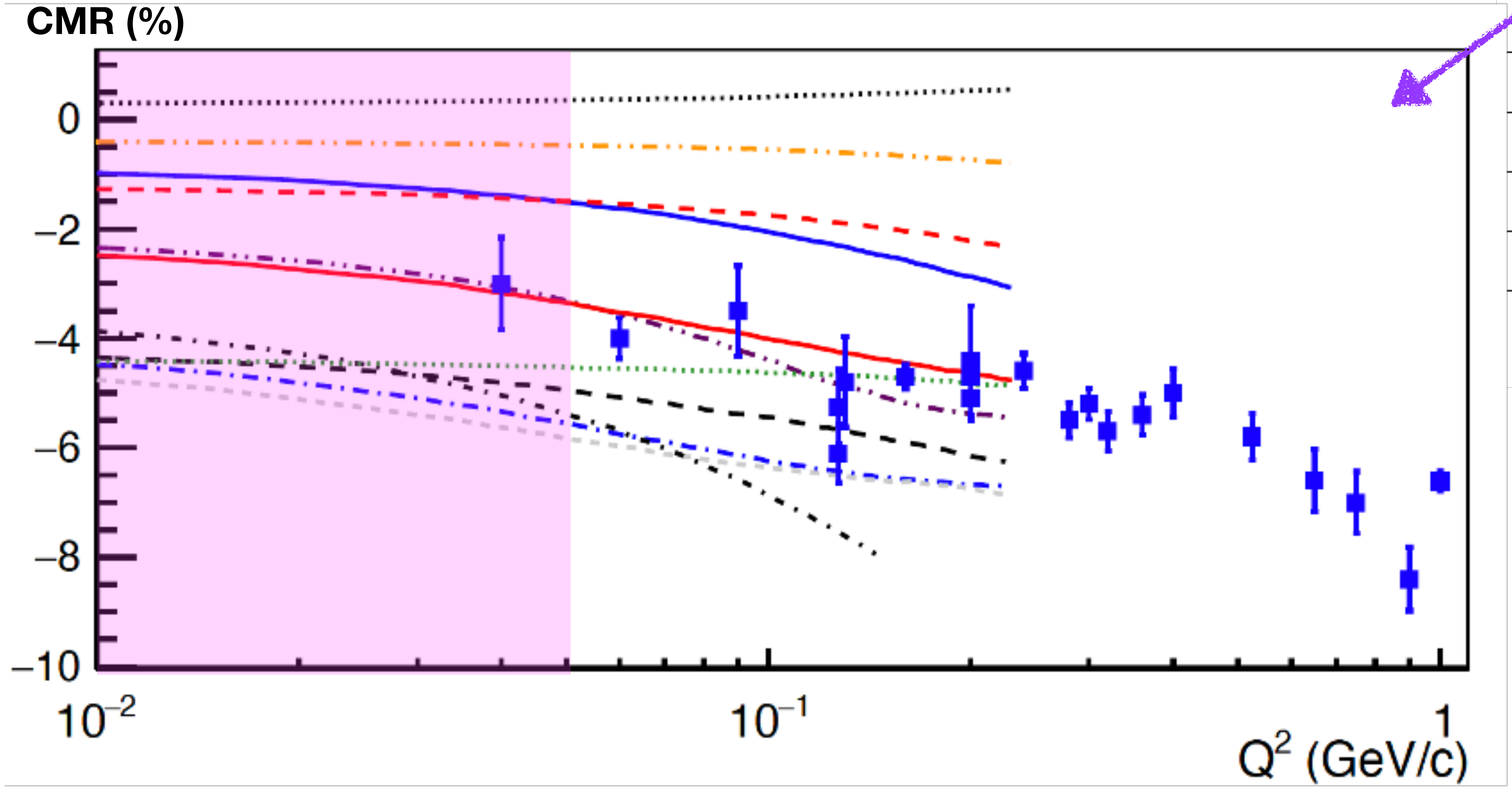


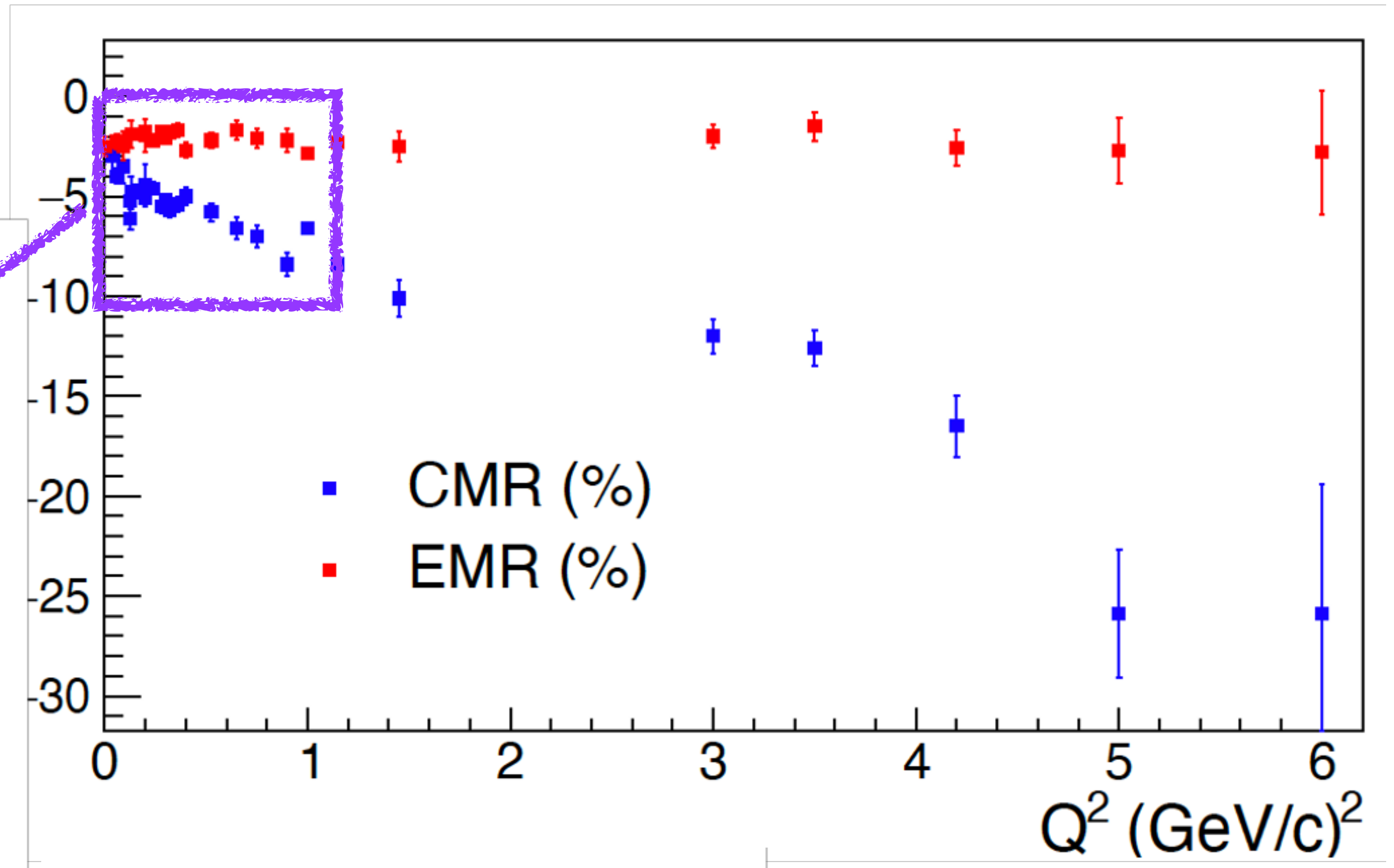
Signature of pion cloud



N-Δ transition as a pathway to nucleon structure

Low Q^2 region is poorly measured and can provide precision leverage for determining mesonic cloud contributions.





Low Q^2 region measurements can also provide high precision benchmarks for LQCD and ChEFT calculations.

N-Δ transition as a pathway to nucleon structure

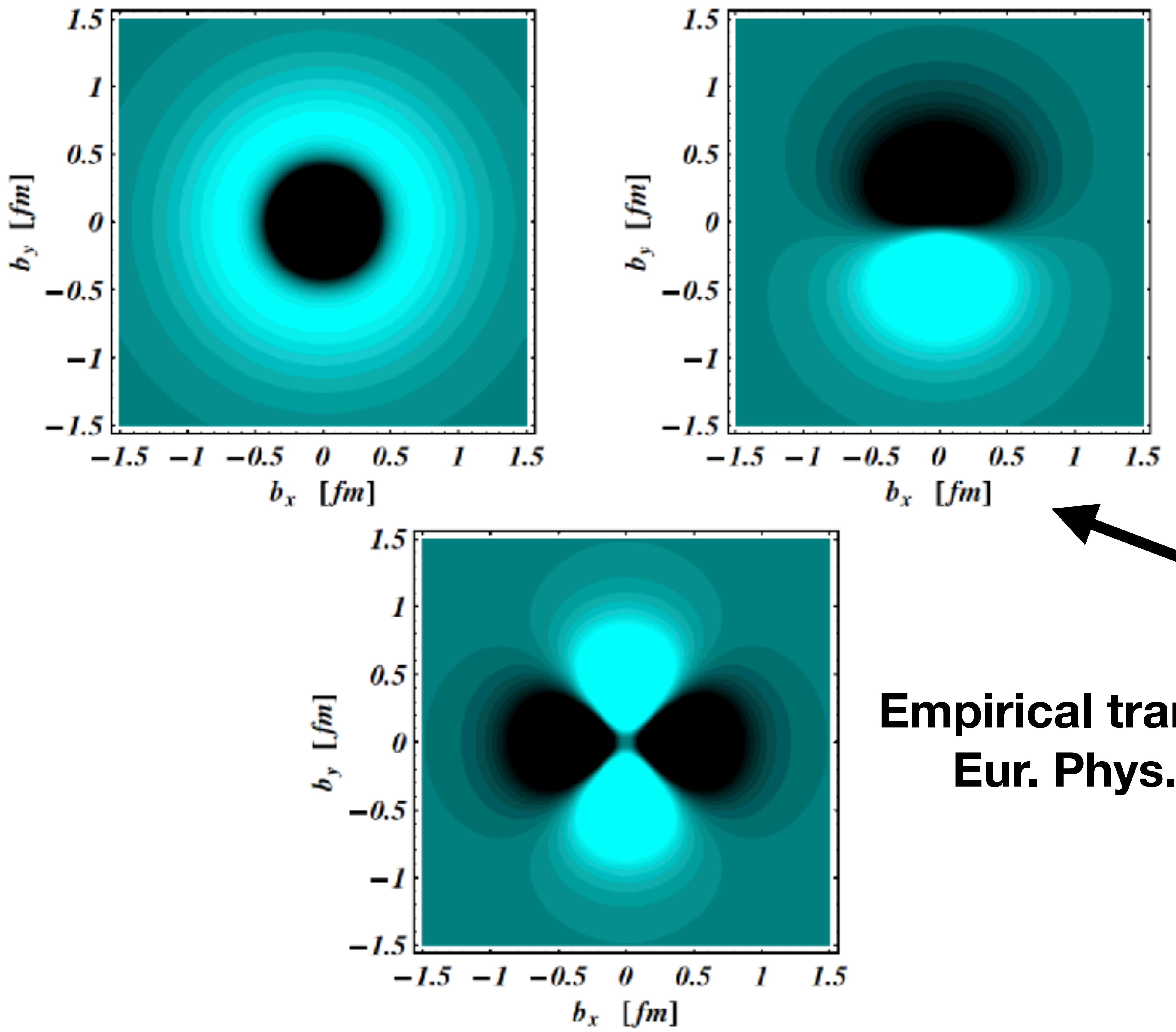


Fig. 18. Quark transverse charge density corresponding to the $p \to \Delta(1232)P_{33}$ e.m. transition. Upper left panel: p and Δ are in a light-front helicity +1/2 state ($\rho_0^{pP_{33}}$). Upper right panel: p and Δ are polarized along the x-axis ($\rho_T^{pP_{33}}$) as in Fig. 14. The lower panel shows the quadrupole pattern, whose contribution to the polarized transition density is very small due to the weak $E2/C2$ admixtures in the $N\Delta$ transition and practically invisible in the upper right panel. The light (dark) regions correspond to positive (negative) densities. For the $p \to P_{33}(1232)$ e.m. transition FFs, we use the MAID2007 parametrization.

Empirical transverse charge transition densities,
Eur. Phys. J. Special Topics 198, 141 (2011)

Latice QCD: Quark transverse charge density in Δ+(1232)
Phys. Rev. D. 79, 014507 (2009)

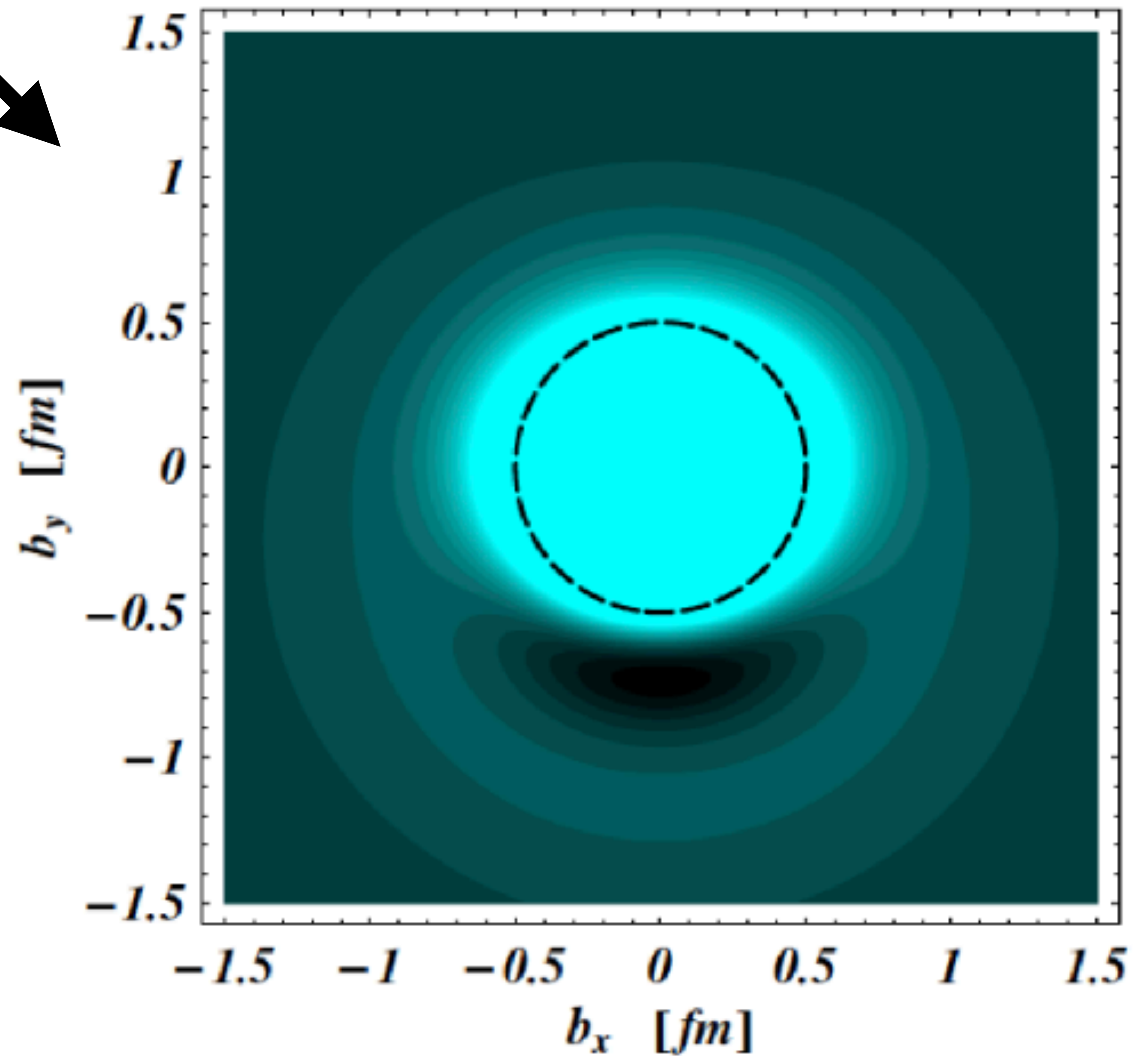


FIG. 10: Lattice QCD results for the quark transverse charge density $\rho^{\Delta}_{T\,\frac{3}{2}}$ in a $\Delta^+(1232)$ which is polarized along the positive x-axis. The light (dark) regions correspond to the largest (smallest) values of the density. In order to see the deformation more clearly, a circle of radius 0.5 fm is drawn for comparison. The density is obtained from quenched lattice QCD results at $m_\pi = 410$ MeV for the Δ e.m. FFs [48].

CMR and EMR measurements at low Q^2

- **On the need for low Q^2 Proton N-Δ Transition Form Factor measurements:**
  - Low Q^2 landscape (< 0.1 GeV^2/c^2) is an important region to measure:
    - Essentially unmeasured region
    - Mesonic cloud effects are predicted to be:
      - dominant in explaining the magnitude of the TFFs
      - changing most rapidly over all Q^2
  - Provides an excellent low-Q^2 test bed for ChEFT and LQCD calculations
  - Can inform spatial extractions of the TFFs and Delta charge density.
  - Can be used, in conjunction with existing world data, to explore nucleon structure (more on that in the following slides).

Neutron Considerations

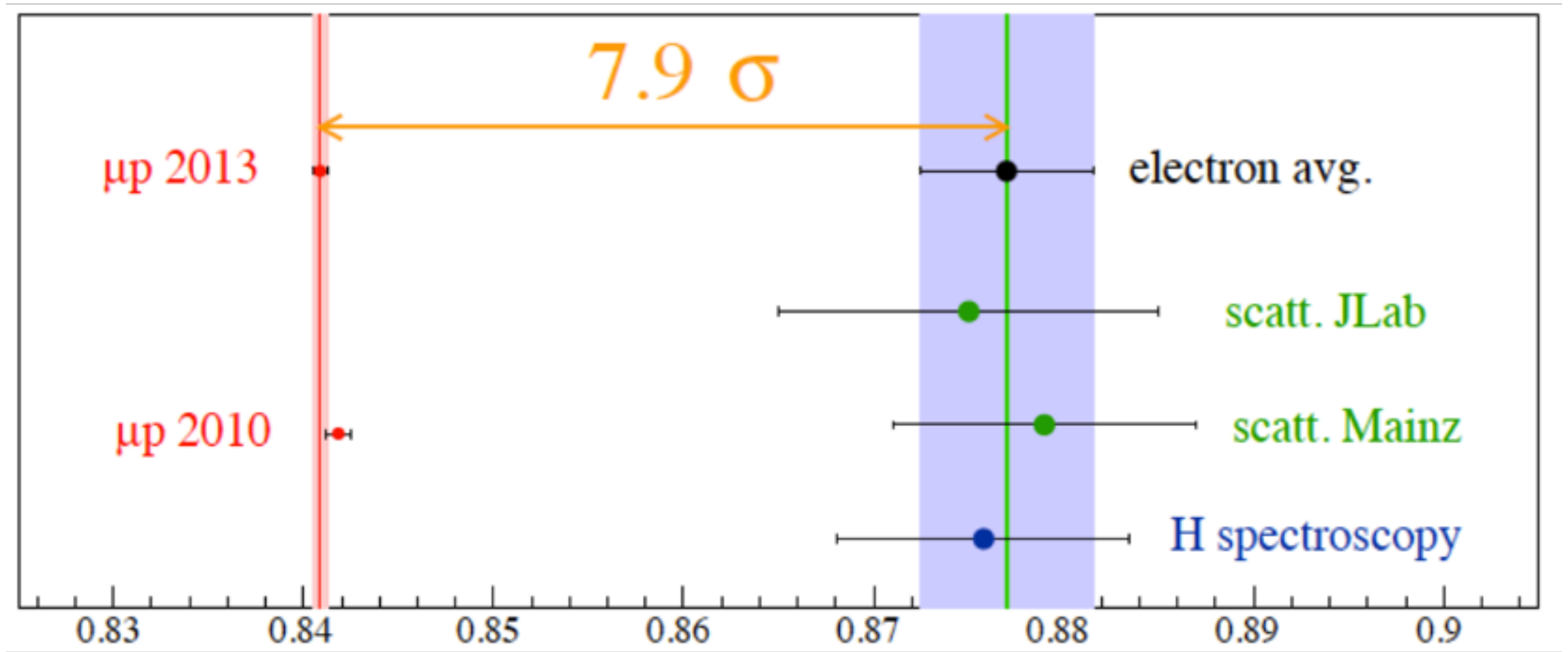
The fundamental properties of the neutron play a significant role in our understanding of nature. Compared to the proton, those properties have been notoriously more difficult to measure.

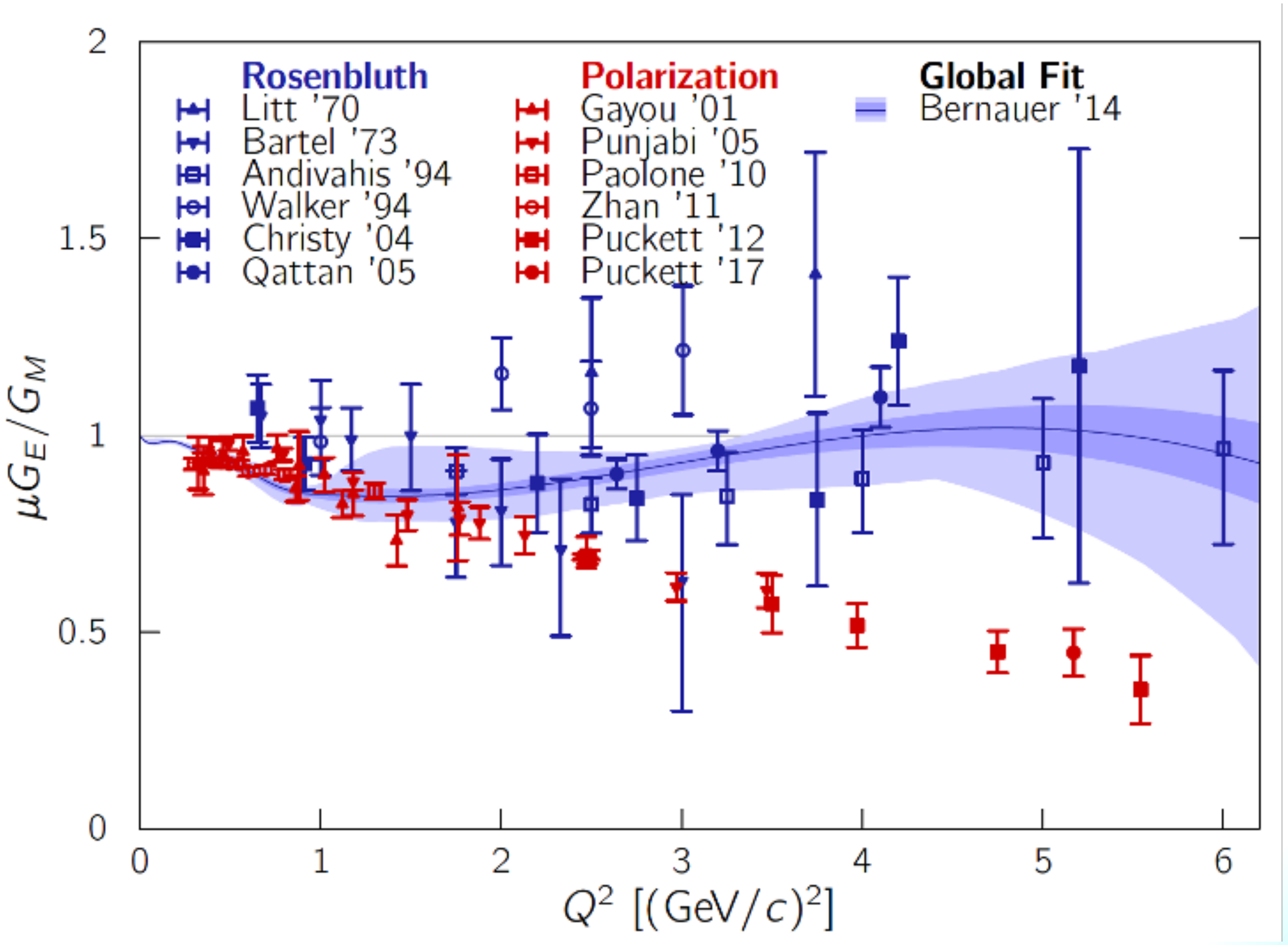
- **The significance of understanding the neutron cannot be overstated:**
  - A cornerstone in the understanding of the hadronic structure.
  - Plays a central role in cosmological theories: it’s properties offer valuable constraints in searches for new physics.
- **Precision is key:**
  - It is required in the determination of its properties in order to achieve the required level of understanding – consequence of the system dynamics & the interactions of the constituents
- **What if...**
  - ... the proton-neutron mass difference (~0.1%) were swapped?
    - There would be no hydrogen, water, stable long-lived stars which use hydrogen as a nuclear fuel... The universe would be drastically different.

Bottom line: A precise understanding of the neutron's basic properties is critical.
The charge radius is one of those properties.

Surprises with the proton

- **We have been startled twice concerning the fundamental properties of the proton over the last 20 years!**
  - First concerning the electro-magnetic structure…
  - And more recently concerning the charge radius!



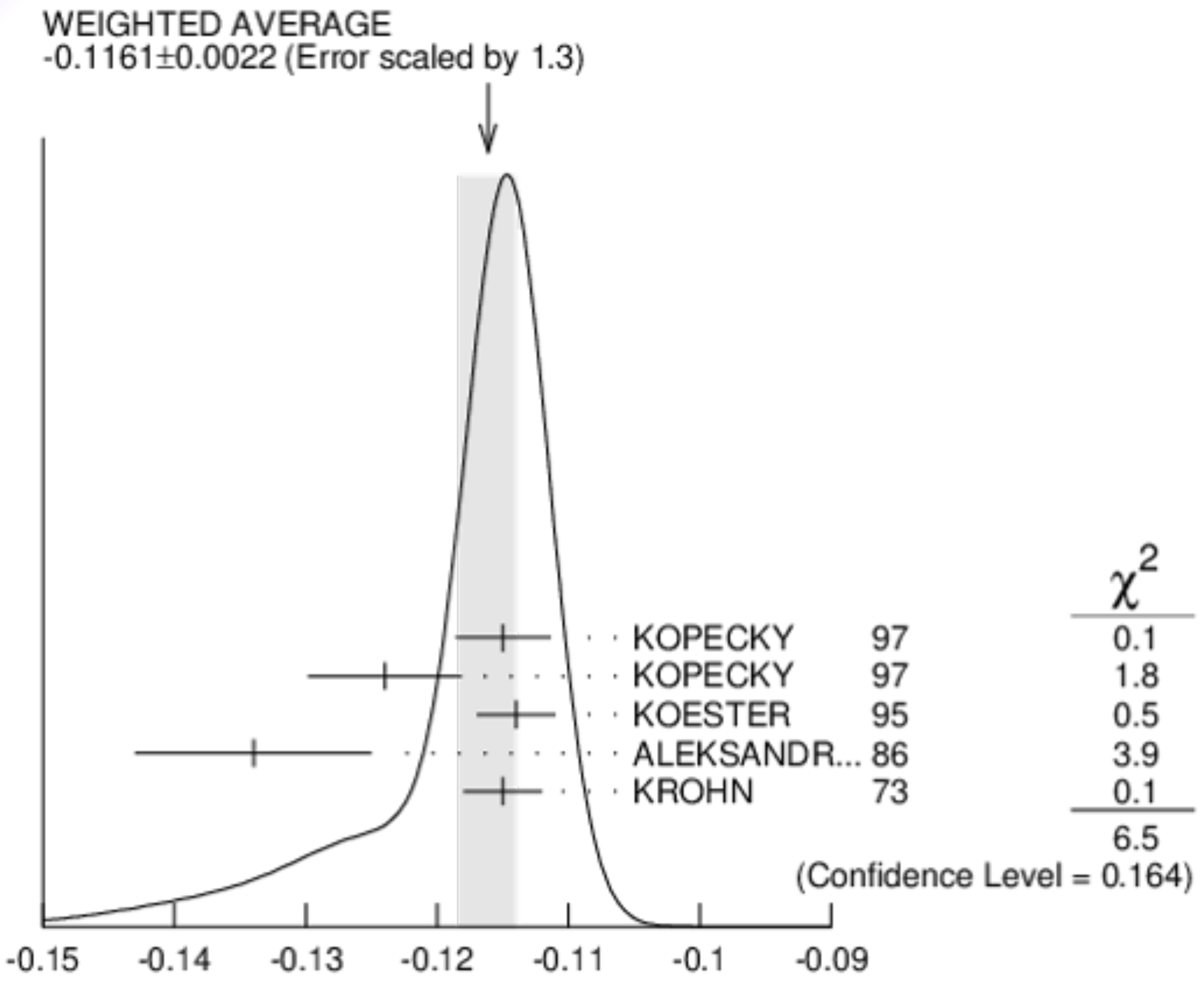


These issues concerning our understanding of the basic proton properties would have not have come to light when they did unless alternative measurement methods were considered and employed!!!!

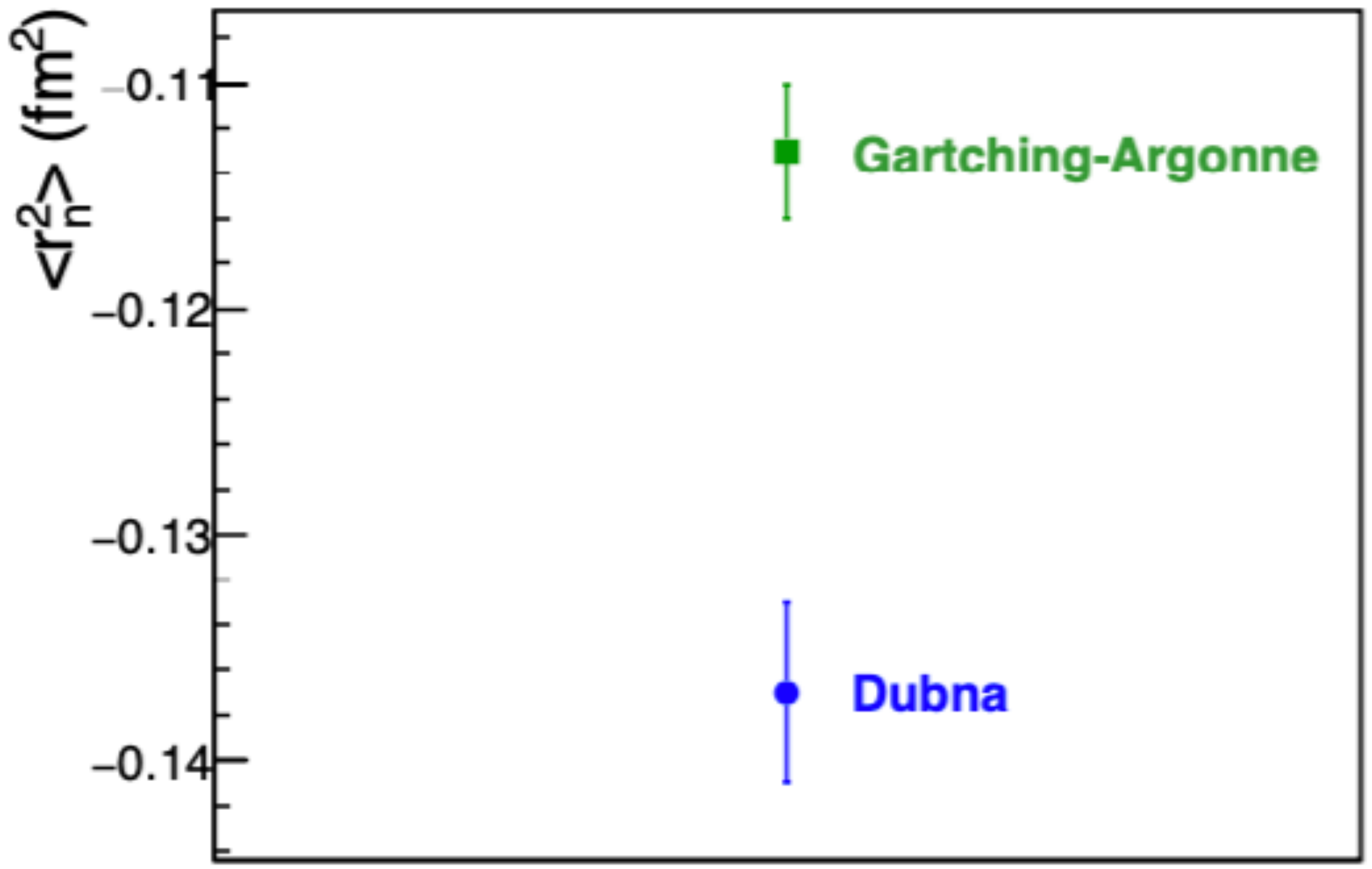
Alternative measurement methodologies are crucially important!

Our current understanding of the neutron charge radius

The value of $< r_n^2 >$ is based on one method of extraction → measurement of b_{ne} using Pb, Bi, …(very indirect method)



- **Some details on the PDG compiled neutron radius:**
  - Most recent measurements over 2 decades old.
  - Some world data is omitted.
  - Input data shows significant tension
    - Simply averaging data with significant discrepancies can be misleading.



The world data results essentially come from two research groups:
Gartching-Argonne and Dubna
With a 5σ tension between them!!!

PRC 56, 2229 (1997) ; Annu. Rev. Nucl. Part. Sci. 55, 27 (2005) ; PRD 77 034020 (2008) …

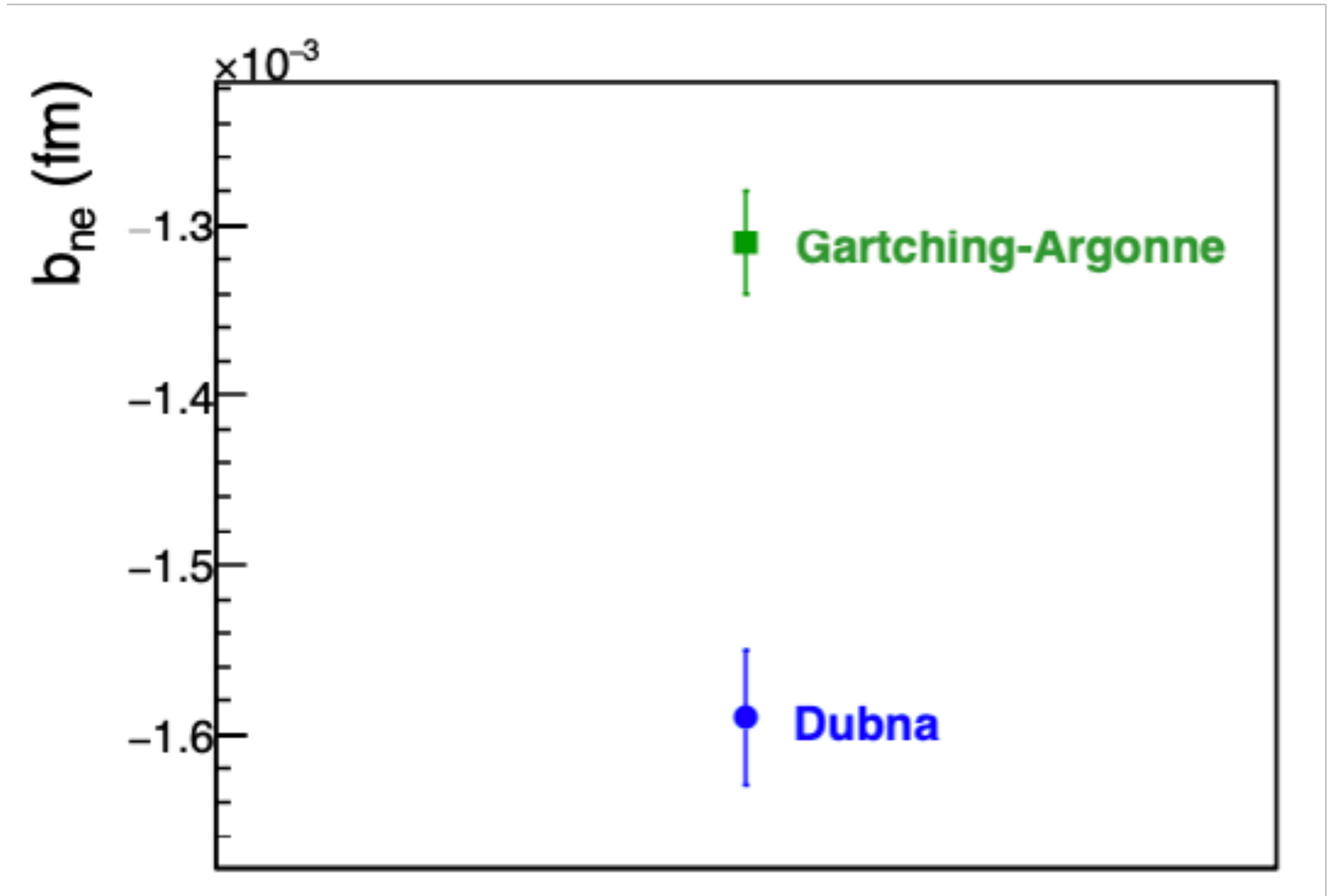
Our current understanding of the neutron charge radius

The value of $< r_n^2 >$ is based on one method of extraction → measurement of b_{ne} using Pb, Bi, …(very indirect method)

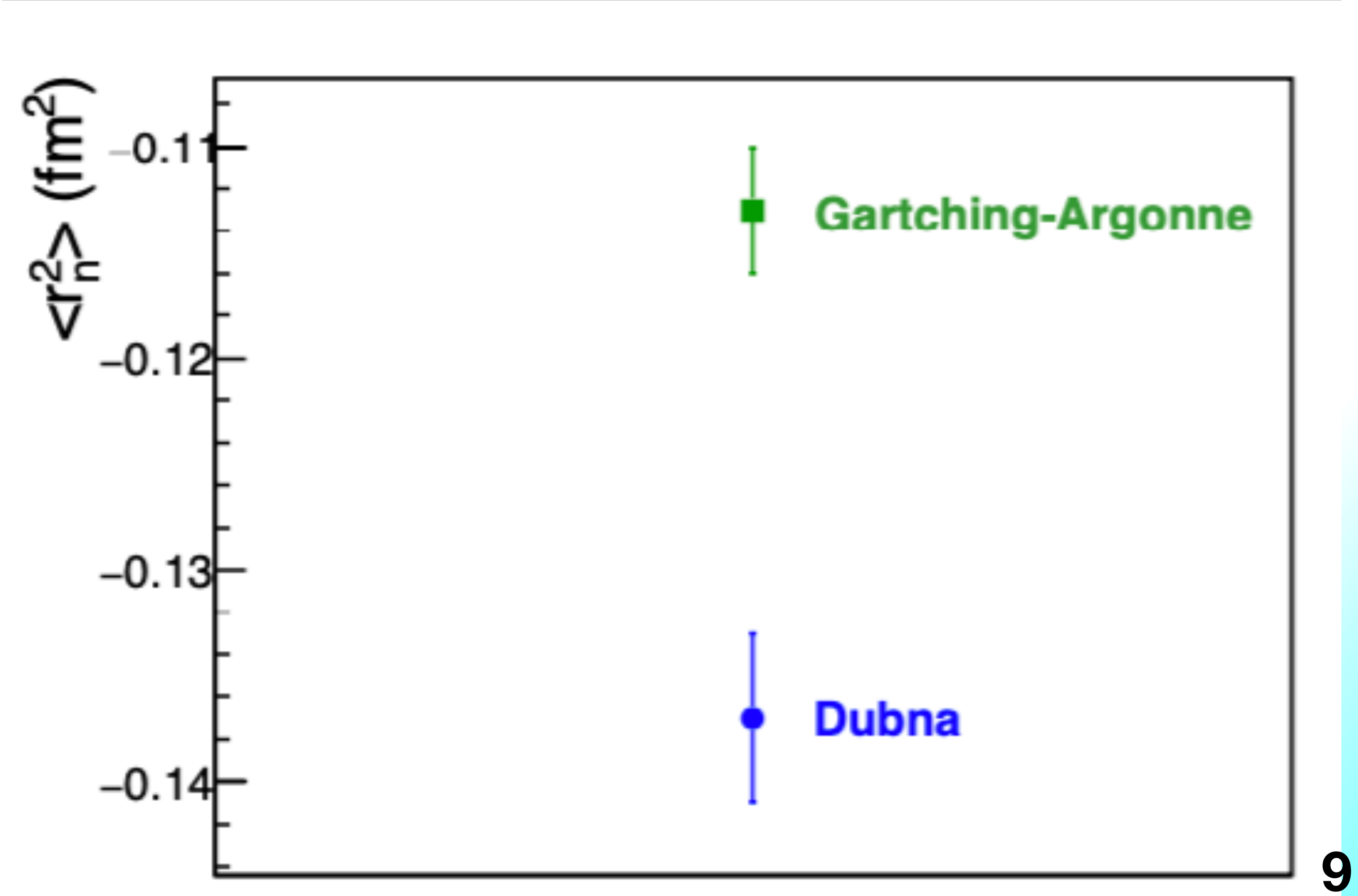
The same methodology is used in each group's radius extraction: a measurement of b_{ne}

A 5σ discrepancy most likely implies an underestimation of systematic uncertainty associated with the methodology

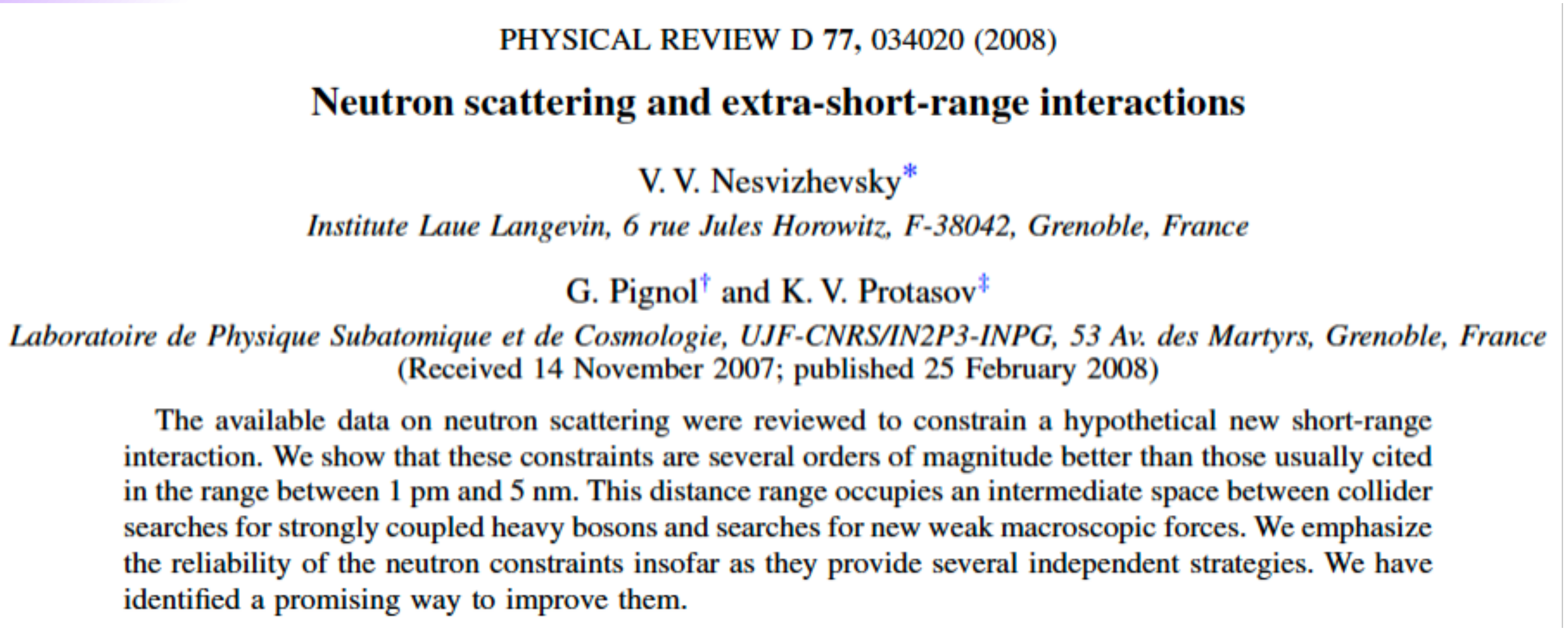
This is a long standing discrepancy and there is NO obvious path using neutron scattering alone that can resolve this.



$$< r_n^2 > = 3(m_e a_0/m_n)\, b_{ne}$$



Some consequences of the current precision



PHYSICAL REVIEW D **77**, 034020 (2008)

Neutron scattering and extra-short-range interactions

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(Received 14 November 2007; published 25 February 2008)

The available data on neutron scattering were reviewed to constrain a hypothetical new short-range interaction. We show that these constraints are several orders of magnitude better than those usually cited in the range between 1 pm and 5 nm. This distance range occupies an intermediate space between collider searches for strongly coupled heavy bosons and searches for new weak macroscopic forces. We emphasize the reliability of the neutron constraints insofar as they provide several independent strategies. We have identified a promising way to improve them.

PHYSICAL REVIEW D 77, 034020 (2008)

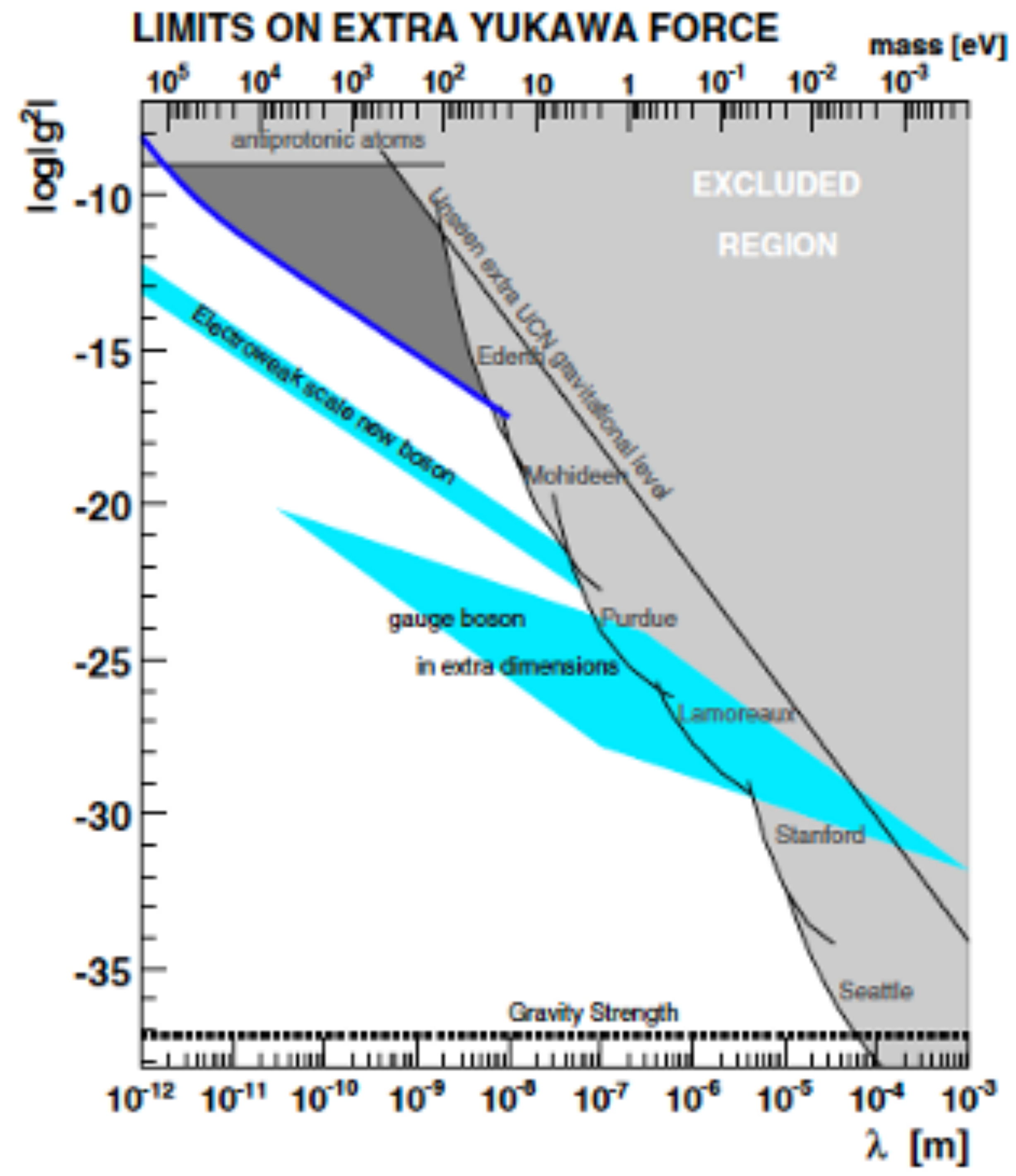
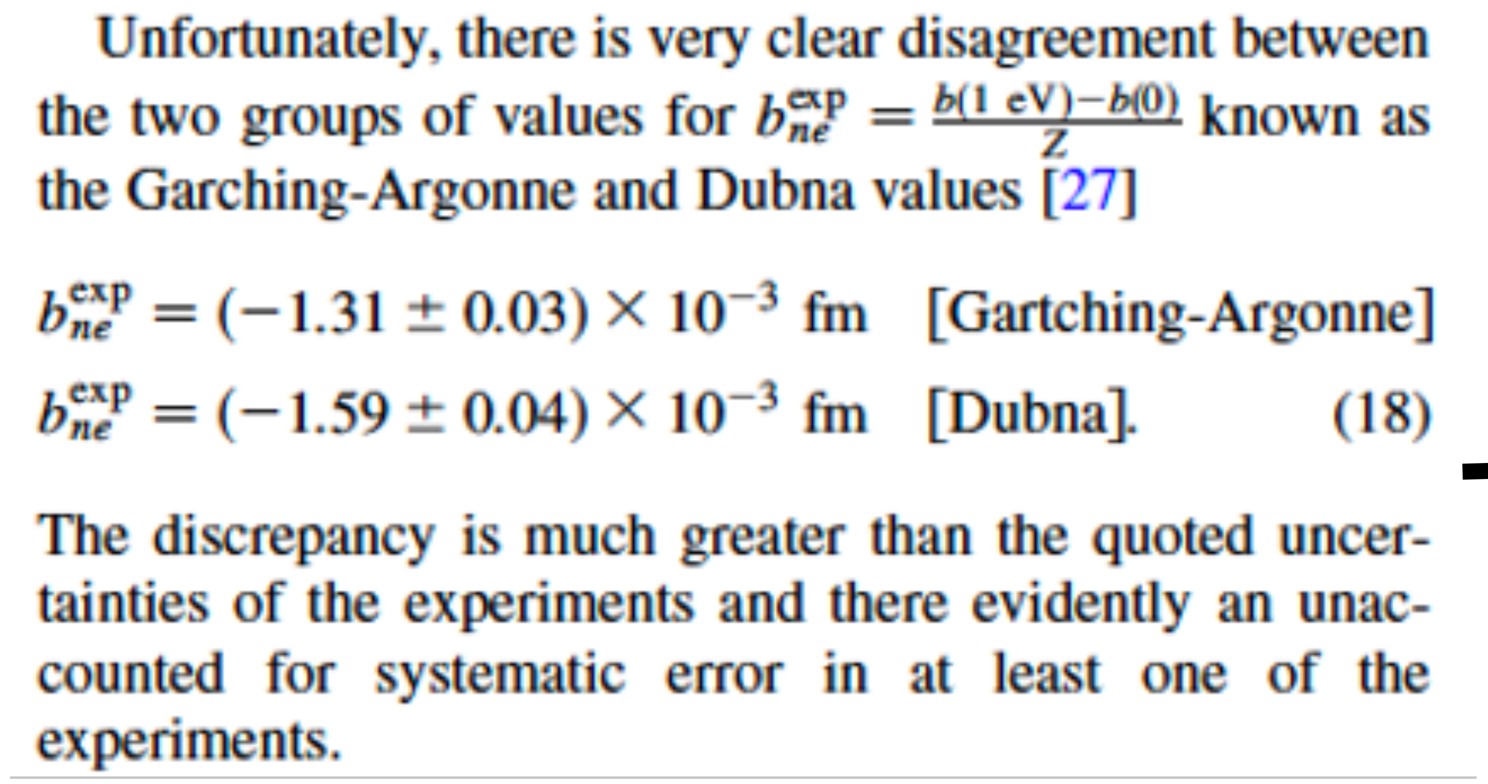


FIG. 8 (color online). Experimental limits on extra interactions including the best neutron constraint obtained in this article (bold line). Two theoretical regions of interest are shown: a new boson with mass induced by electroweak symmetry breaking [10], and a new boson in extra large dimensions [4].

BSM physics: constrains on forces due to new bosons modeled by a Yukawa-type scattering potential: $f(q) = f_{\text{nucl}}(q) + \boxed{f_{ne}(q)} + f_{\text{new}}(q)$

Depends on b_{ne} , limited by precision

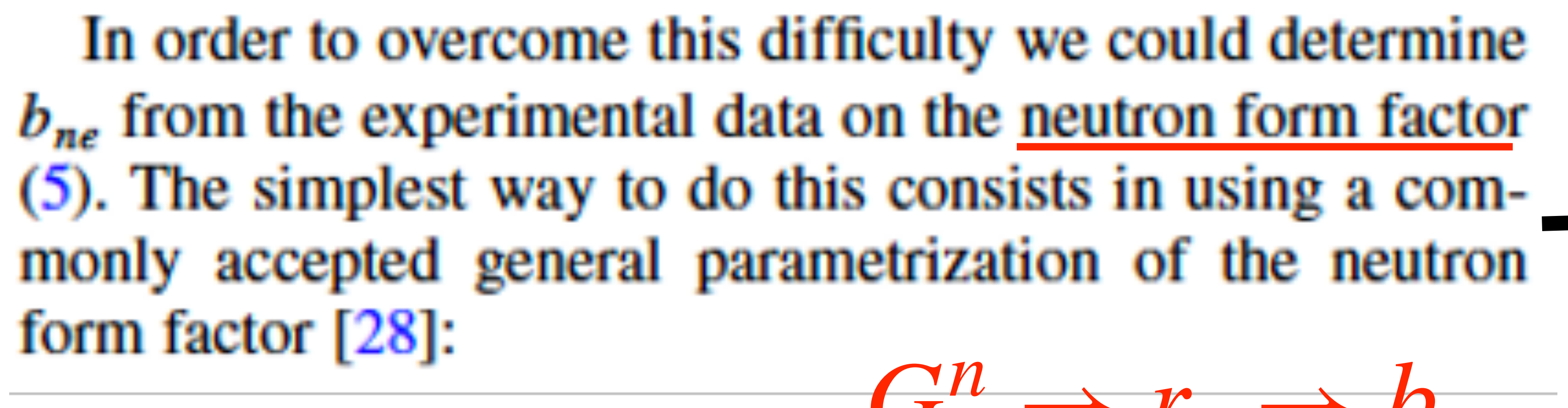


Unfortunately, there is very clear disagreement between the two groups of values for $b_{ne}^{\exp} = \frac{b(1\ \text{eV}) - b(0)}{Z}$ known as the Garching-Argonne and Dubna values [27]

$$b_{ne}^{\exp} = (-1.31 \pm 0.03) \times 10^{-3}\ \text{fm} \quad [\text{Gartching-Argonne}]$$
$$b_{ne}^{\exp} = (-1.59 \pm 0.04) \times 10^{-3}\ \text{fm} \quad [\text{Dubna}]. \qquad (18)$$

The discrepancy is much greater than the quoted uncertainties of the experiments and there evidently an unaccounted for systematic error in at least one of the experiments.

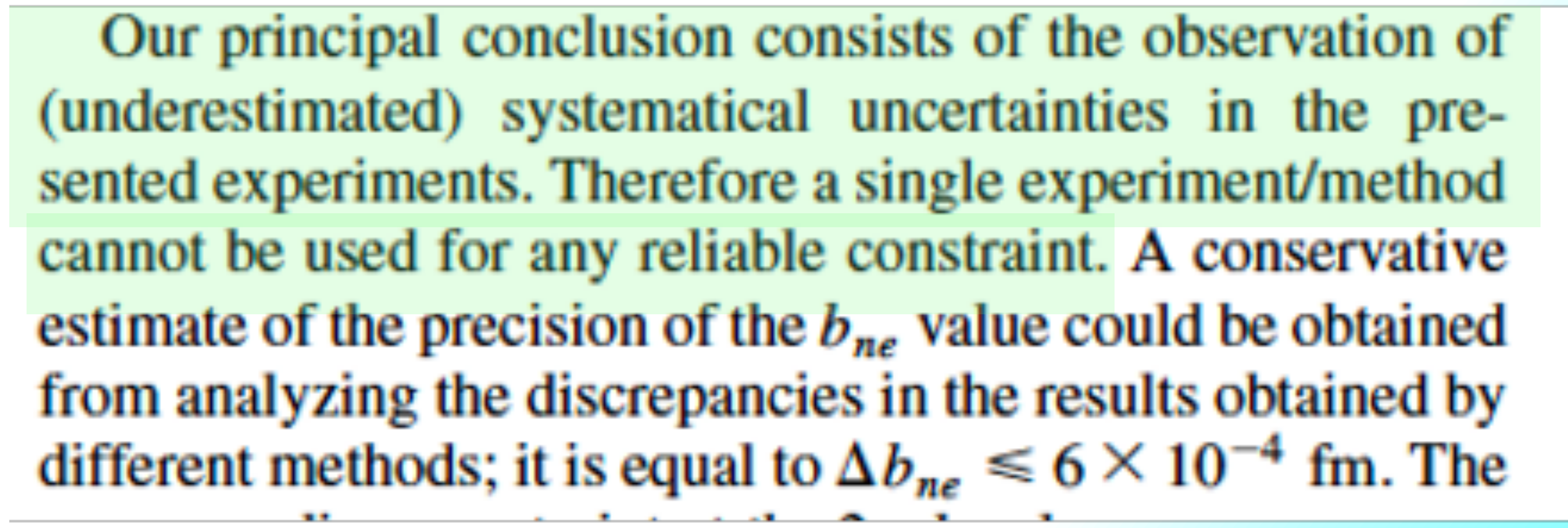
→



In order to overcome this difficulty we could determine b_{ne} from the experimental data on the neutron form factor (5). The simplest way to do this consists in using a commonly accepted general parametrization of the neutron form factor [28]:

$G^n \rightarrow r_n \rightarrow b_{ne}$

→

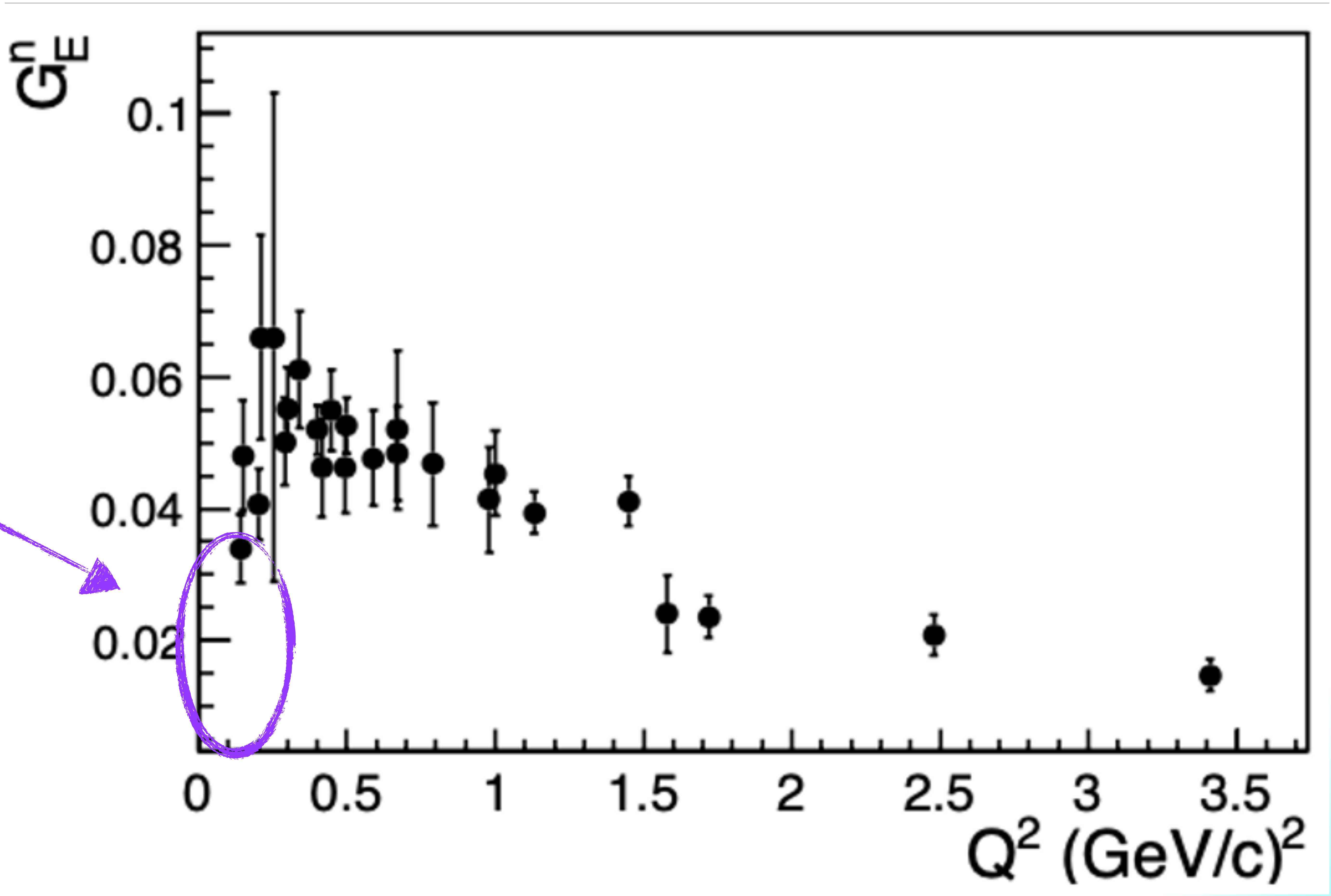


Our principal conclusion consists of the observation of (underestimated) systematical uncertainties in the presented experiments. Therefore a single experiment/method cannot be used for any reliable constraint. A conservative estimate of the precision of the b_{ne} value could be obtained from analyzing the discrepancies in the results obtained by different methods; it is equal to $\Delta b_{ne} \lesssim 6 \times 10^{-4}$ fm. The

An alternative method to measure the neutron charge radius

$$\langle r_n^2 \rangle = -6 \frac{dG_E^n(Q^2)}{dQ^2}\bigg|_{Q^2 \to 0}$$

If one can measure with precision $G_E^n(Q^2 \to 0)$, one can determine the neutron charge radius.



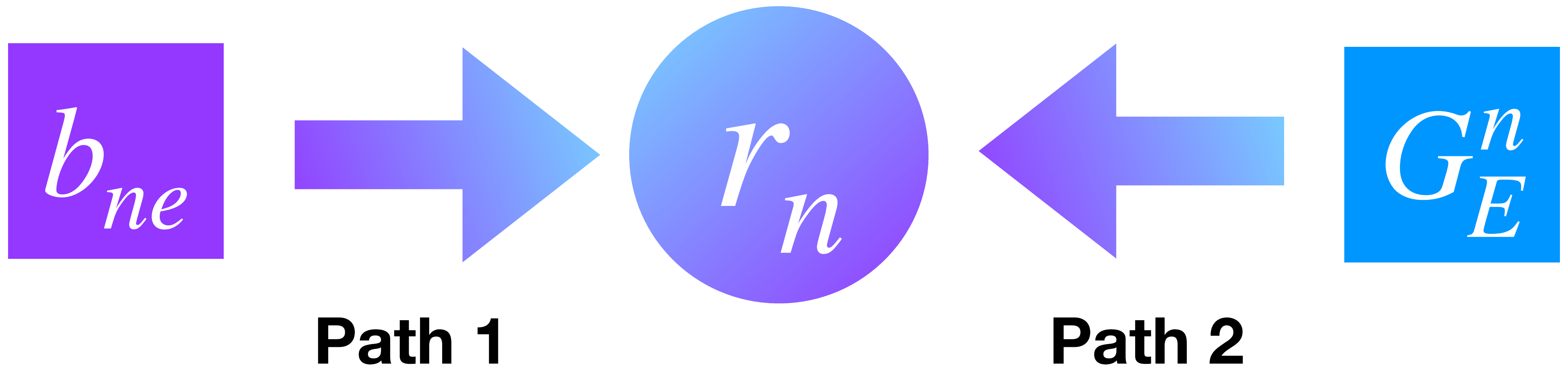
Doing such would provide an alternative path to the charge radius, and provide an important cross-check to the existing measurements. (And could reveal surprises!)

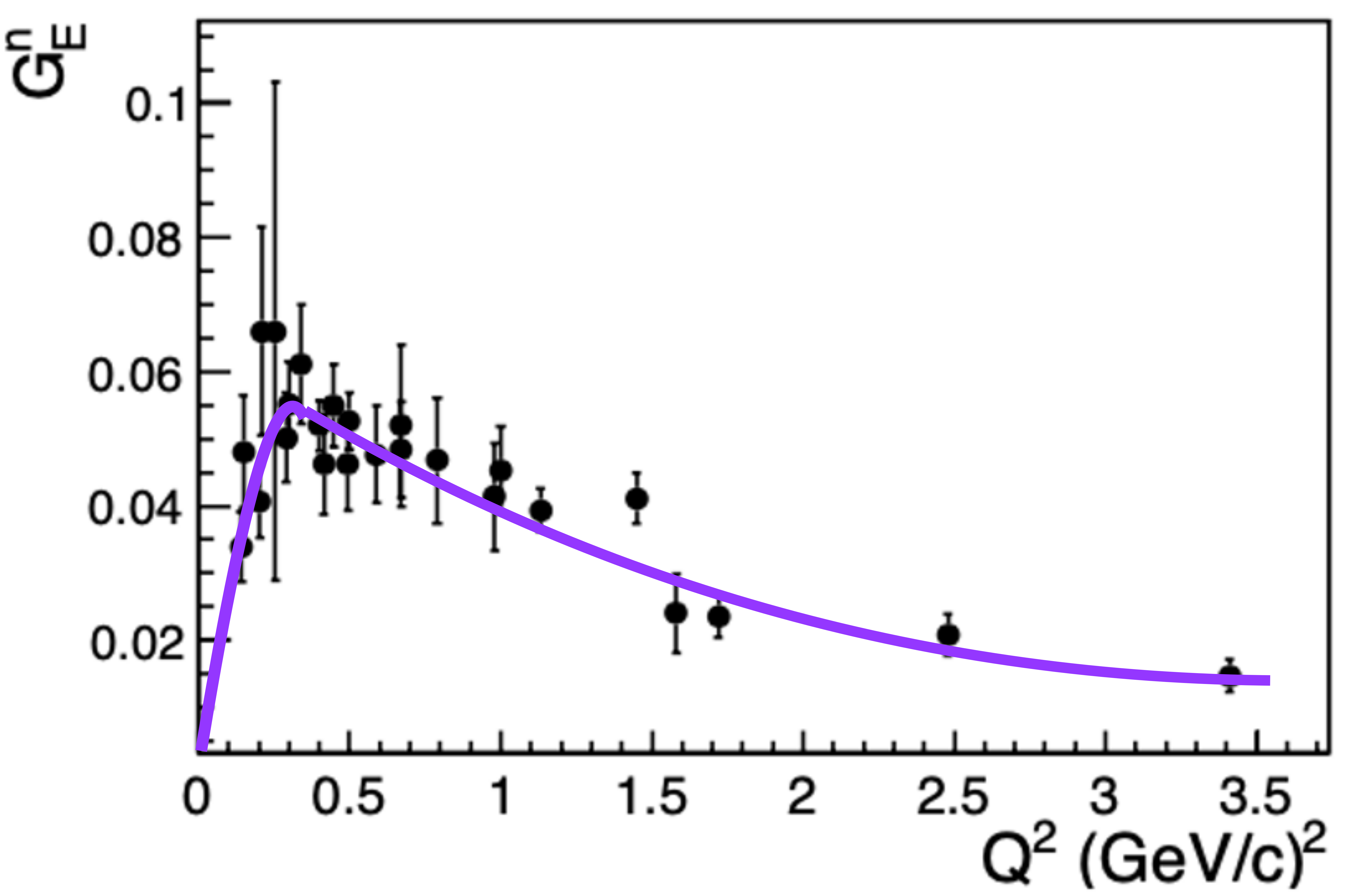
b_{ne} $\langle r_n^2 \rangle$ G_E^n

Path 1 **Path 2**

An alternative method to measure the neutron charge radius

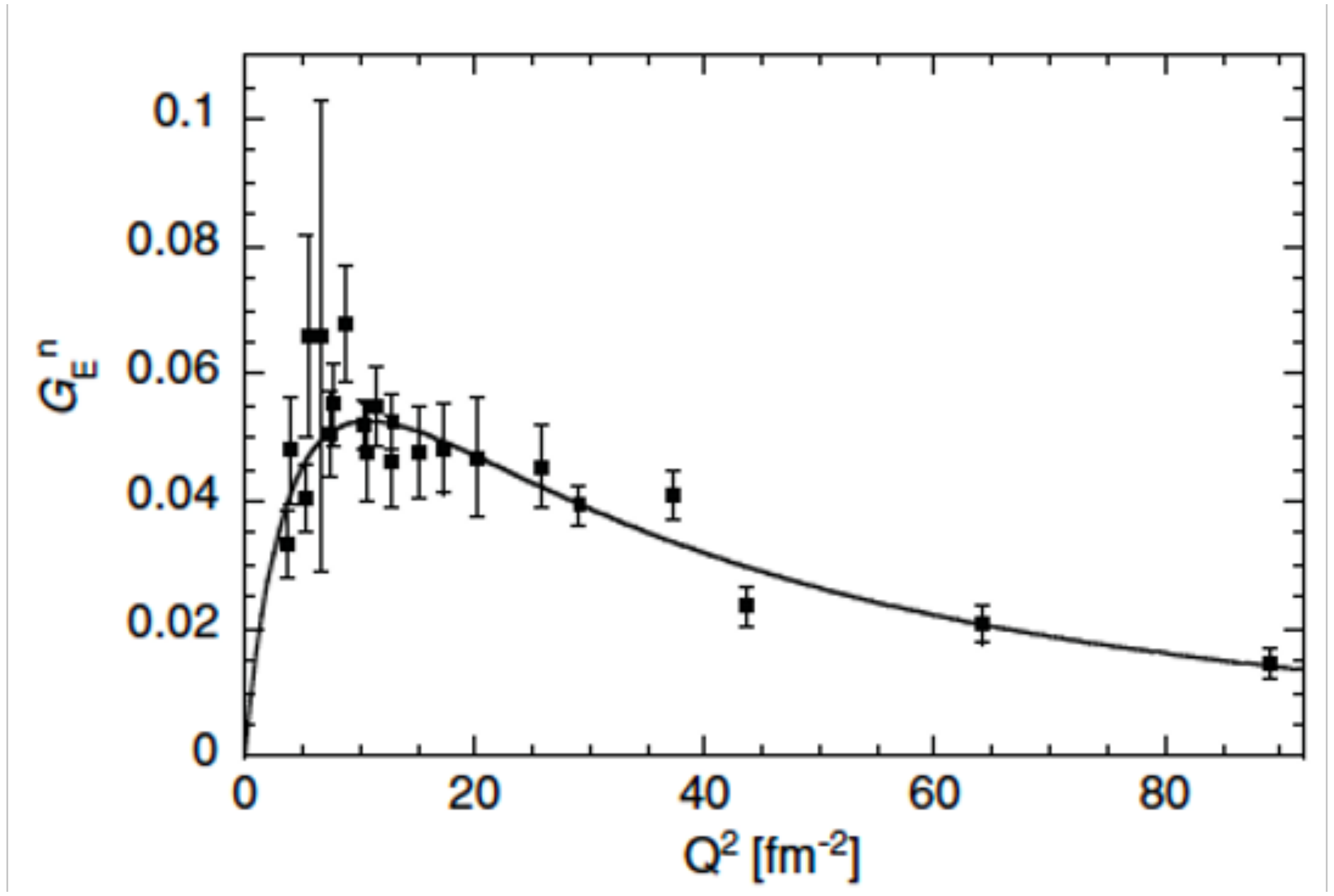
- **Historical G_E^n measurements:**
    - No truly "free" neutron target
    - Polarized ^{2}H, ^{3}He targets & polarized electron beam
    - Quasi-elastic electron scattering
    - Double polarization observables
- **A fit is needed for $Q^2 \to 0$**
    - Relies on precision of measurements
    - ... and on how close measurements are to $Q^2 = 0$





An alternative method to measure the neutron charge radius

T.R. Gentile & C.B. Crawford
PRC 83, 055203 (2011)



- **Parameterizations of the fit forms are not well constrained as $Q^2 \to 0$**
  - Recent attempts using quasi-free neutron target measurements of G_E^n have yielded radii ~33% from pdg values.

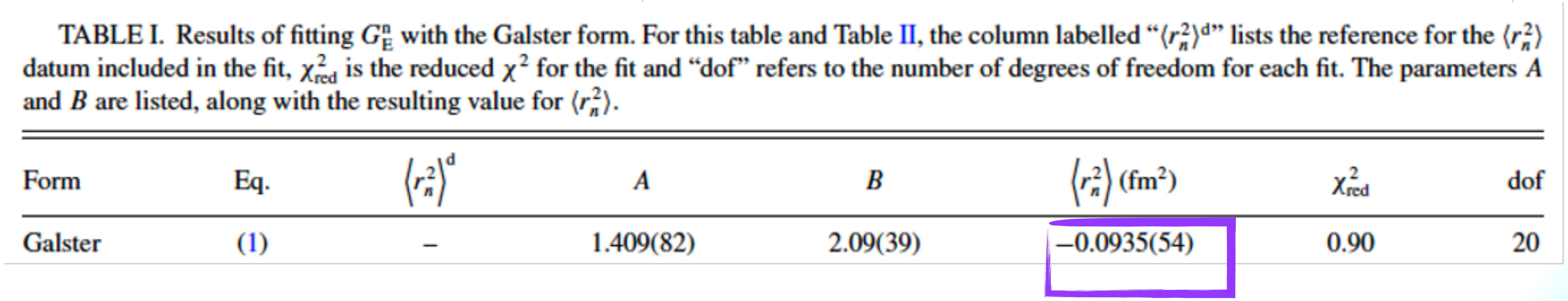


TABLE I. Results of fitting G_E^n with the Galster form. For this table and Table II, the column labelled "$\langle r_n^2\rangle^d$" lists the reference for the $\langle r_n^2\rangle$ datum included in the fit, χ^2_{red} is the reduced χ^2 for the fit and "dof" refers to the number of degrees of freedom for each fit. The parameters A and B are listed, along with the resulting value for $\langle r_n^2\rangle$.

Form	Eq.	$\langle r_n^2\rangle^d$	A	B	$\langle r_n^2\rangle$ (fm^2)	χ^2_{red}	dof
Galster	(1)	–	1.409(82)	2.09(39)	–0.0935(54)	0.90	20

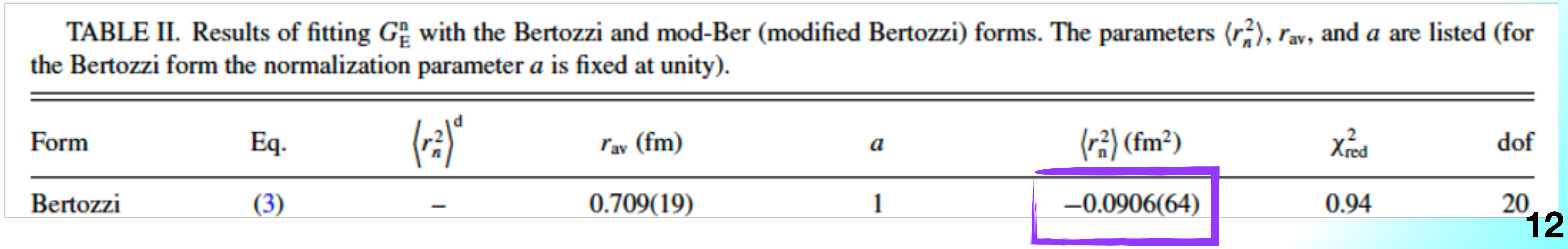
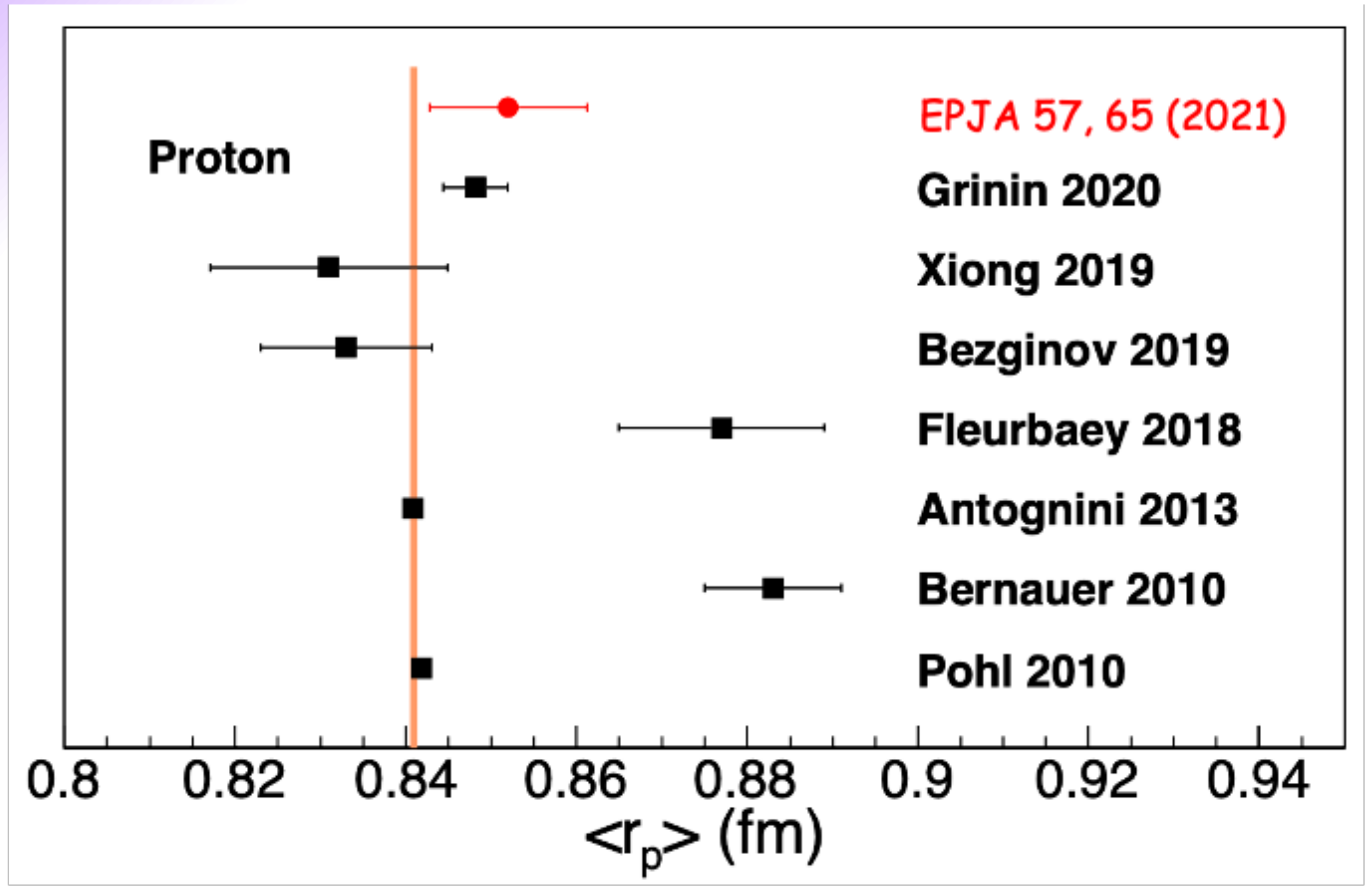
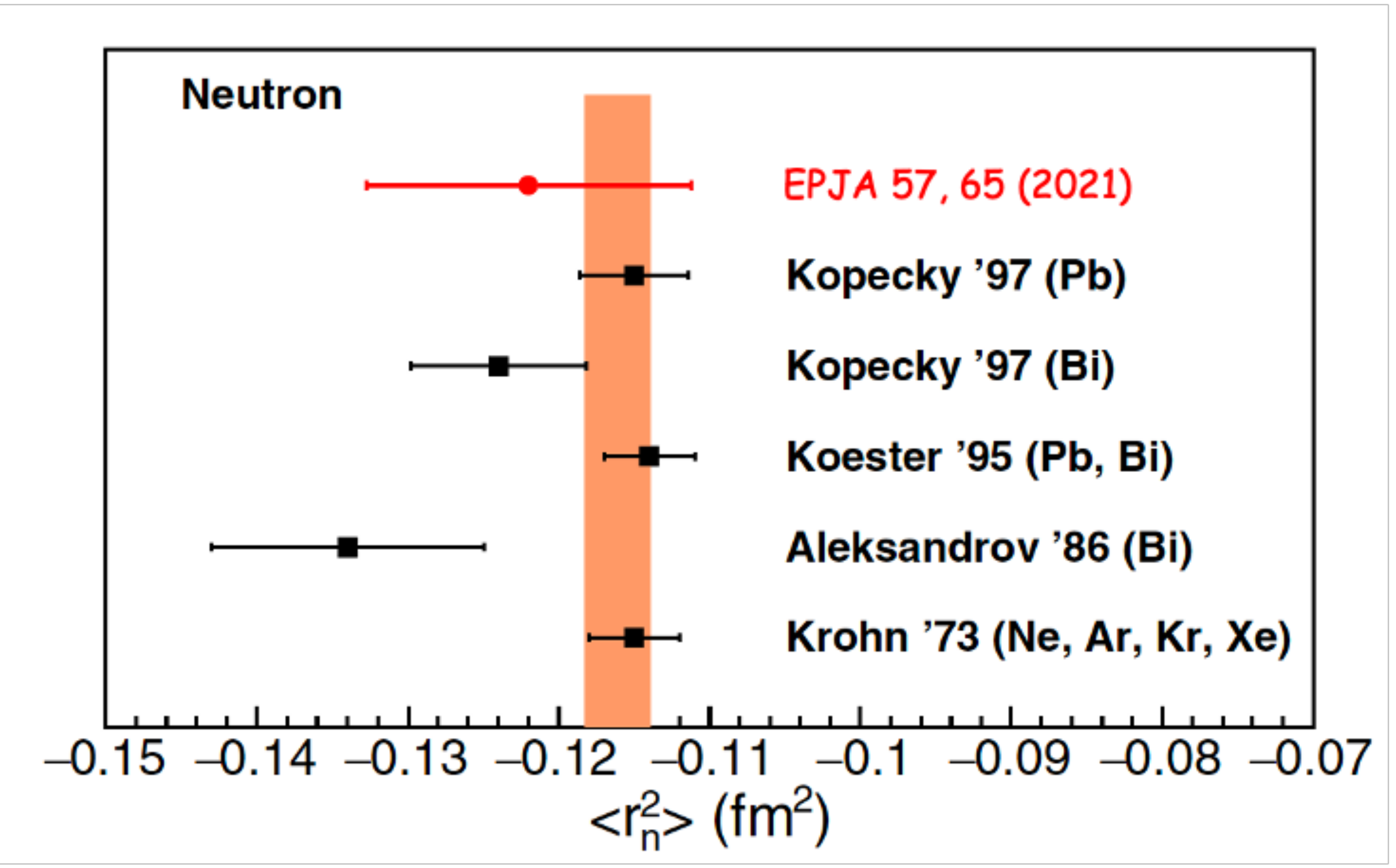


TABLE II. Results of fitting G_E^n with the Bertozzi and mod-Ber (modified Bertozzi) forms. The parameters $\langle r_n^2\rangle$, r_{av}, and a are listed (for the Bertozzi form the normalization parameter a is fixed at unity).

Form	Eq.	$\langle r_n^2\rangle^d$	r_{av} (fm)	a	$\langle r_n^2\rangle$ (fm^2)	χ^2_{red}	dof
Bertozzi	(3)	–	0.709(19)	1	–0.0906(64)	0.94	20

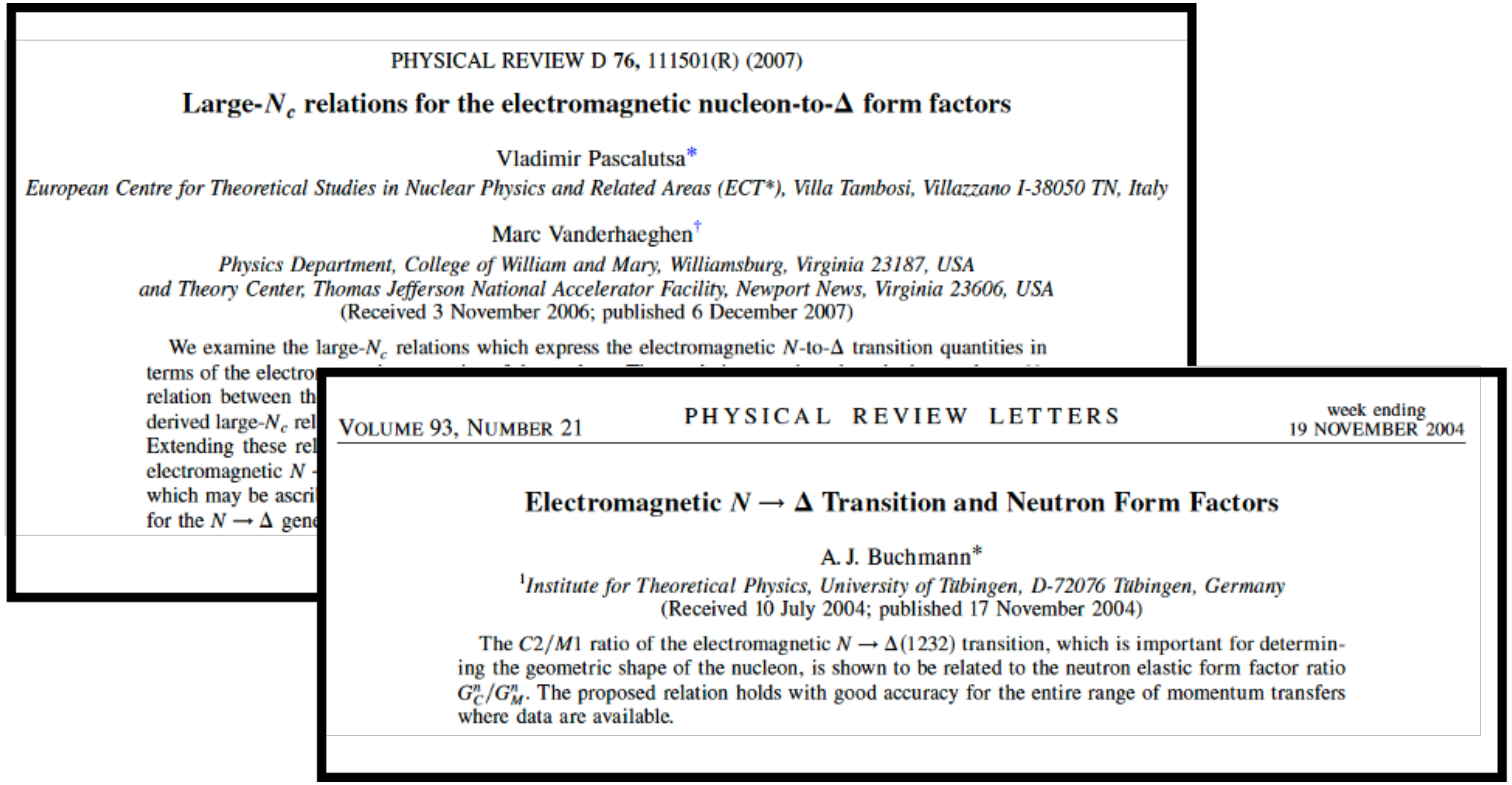
Radius extraction through flavor decomposition





- **By using the neutron and proton FF data together, a flavor decomposition can be performed.**
  - Exploiting isospin symmetry, both proton and neutron radii can be extracted simultaneously.
    - Eur. Phys. J. A 57, 65 (2021), H. Atac, M. Constantinou, Z.E. Meziani, M. Paolone, N. Sparveris:
      - $\langle r_p \rangle = 0.852 \pm 0.002_{\text{(stat.)}} \pm 0.009_{\text{(syst.)}}$ (fm)
      - $\langle r_n^2 \rangle = -0.122 \pm 0.004_{\text{(stat.)}} \pm 0.010_{\text{(syst.)}}$ (fm^2)
  - Provides new nucleon radii points:
    - Neutron precision (~9%) remains inadequate to reconcile discrepancies.

A path to extend our low Q^2 reach for G_E^n



PHYSICAL REVIEW D **76,** 111501(R) (2007)

Large-N_c relations for the electromagnetic nucleon-to-Δ form factors

Vladimir Pascalutsa*

European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT), Villa Tambosi, Villazzano I-38050 TN, Italy*

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Physics Department, College of William and Mary, Williamsburg, Virginia 23187, USA
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(Received 3 November 2006; published 6 December 2007)

We examine the large-N_c relations which express the electromagnetic N-to-Δ transition quantities in
terms of the electro
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Extending these rel
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VOLUME 93, NUMBER 21 PHYSICAL REVIEW LETTERS week ending 19 NOVEMBER 2004

Electromagnetic $N \to \Delta$ Transition and Neutron Form Factors

A. J. Buchmann*

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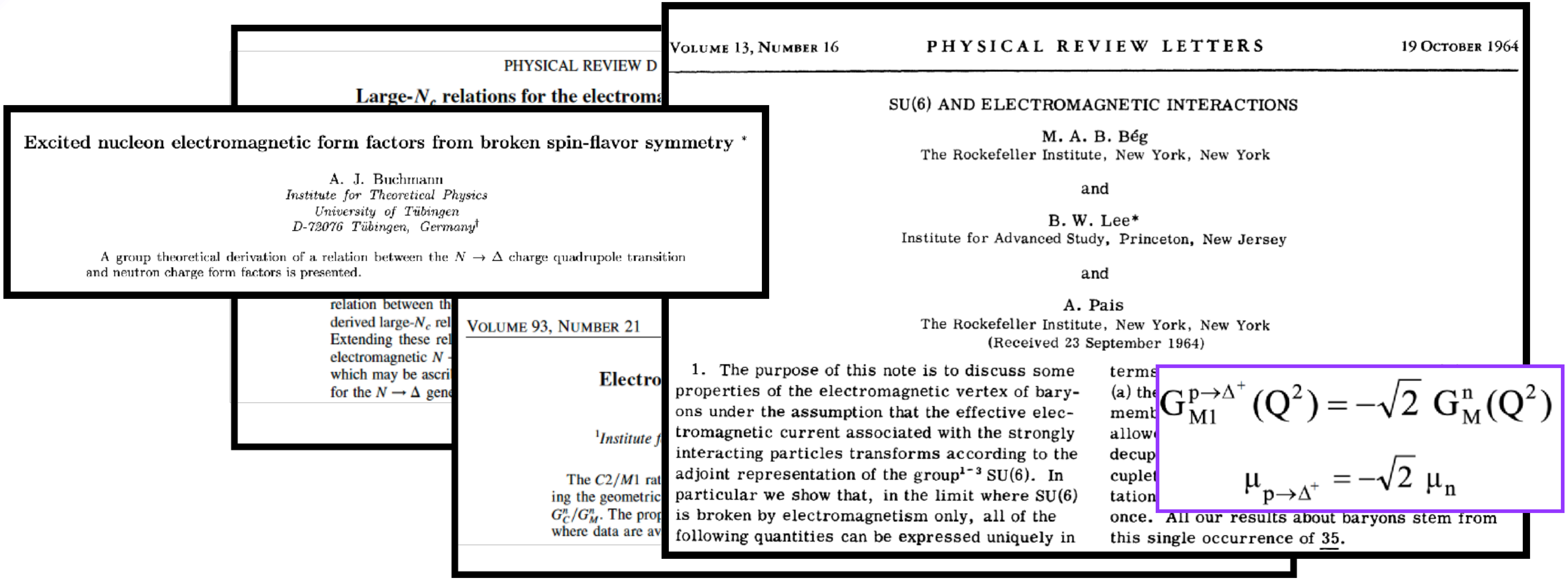
(Received 10 July 2004; published 17 November 2004)

The $C2/M1$ ratio of the electromagnetic $N \to \Delta(1232)$ transition, which is important for determining the geometric shape of the nucleon, is shown to be related to the neutron elastic form factor ratio G_C^n/G_M^n. The proposed relation holds with good accuracy for the entire range of momentum transfers where data are available.

- **It has been long known that there is a correlation between the N-Δ TFFs and G_E^n**
  - Initially exploited in reverse to infer information for the N-Δ TFFs, while they were not yet very well measured.
  - 15 years later: the N-Δ TFFs can be accessed at lower Q^2 and with higher precision, compared to the current G_E^n measurements

A path to extend our low Q^2 reach for G_E^n



PHYSICAL REVIEW D

Large-N_c relations for the electroma

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Excited nucleon electromagnetic form factors from broken spin-flavor symmetry *

A. J. Buchmann
Institute for Theoretical Physics
University of Tübingen
D-72076 Tübingen, Germany†

A group theoretical derivation of a relation between the $N \to \Delta$ charge quadrupole transition and neutron charge form factors is presented.

VOLUME 93, NUMBER 21

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VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

SU(6) AND ELECTROMAGNETIC INTERACTIONS

M. A. B. Bég
The Rockefeller Institute, New York, New York

and

B. W. Lee*
Institute for Advanced Study, Princeton, New Jersey

and

A. Pais
The Rockefeller Institute, New York, New York
(Received 23 September 1964)

1. The purpose of this note is to discuss some properties of the electromagnetic vertex of baryons under the assumption that the effective electromagnetic current associated with the strongly interacting particles transforms according to the adjoint representation of the group¹⁻³ SU(6). In particular we show that, in the limit where SU(6) is broken by electromagnetism only, all of the following quantities can be expressed uniquely in

terms
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once. All our results about baryons stem from this single occurrence of 35.

$$G_{M1}^{p\to\Delta^+}(Q^2) = -\sqrt{2}\, G_M^n(Q^2)$$

$$\mu_{p\to\Delta^+} = -\sqrt{2}\,\mu_n$$

- **It has been long known that there is a correlation between the N-Δ TFFs and G_E^n**
  - Initially exploited in reverse to infer information for the N-Δ TFFs, while they were not yet very well measured.
  - 15 years later: the N-Δ TFFs can be accessed at lower Q^2 and with higher precision, compared to the current G_E^n measurements

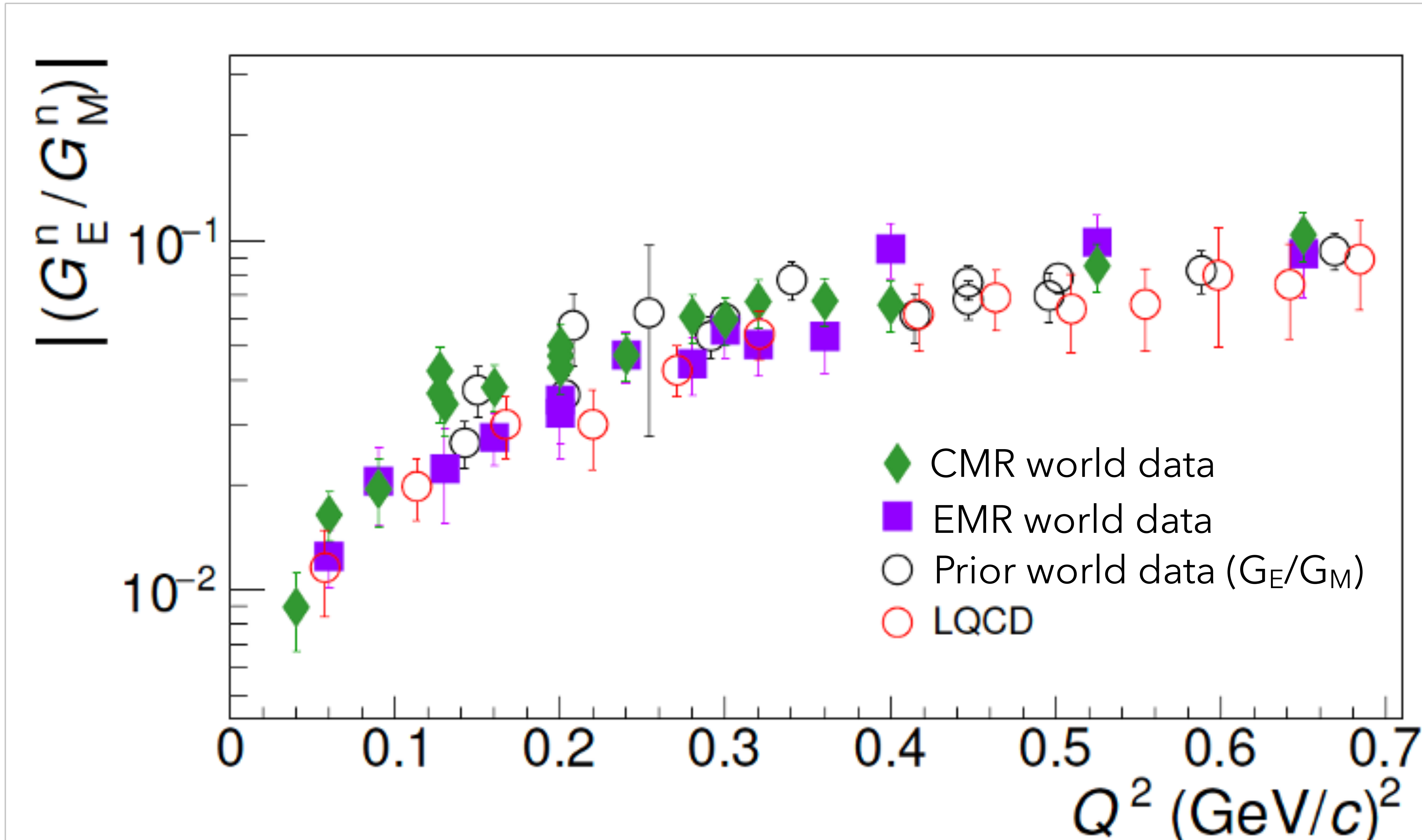
A path to extend our low Q^2 reach for G_E^n

Large-N$_c$ Relations (Pascalutsa & Vanderhaeghen)
Phys. Rev. D76. 93, 111501(R) (2007)

$$\frac{E2}{M1}\left(Q^2\right) = \left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M_N^2}{2Q^2} \frac{G_E^n\left(Q^2\right)}{F_2^p\left(Q^2\right) - F_2^n\left(Q^2\right)}$$

$$\frac{C2}{M1}\left(Q^2\right) = \left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{Q_+Q_-}{2Q^2} \frac{G_E^n\left(Q^2\right)}{F_2^p\left(Q^2\right) - F_2^n\left(Q^2\right)}$$

- **Large-Nc relations:**
  - Carry about 15% theoretical uncertainty.
  - Two relations (CMR and EMR) can be used to cross-check validity.



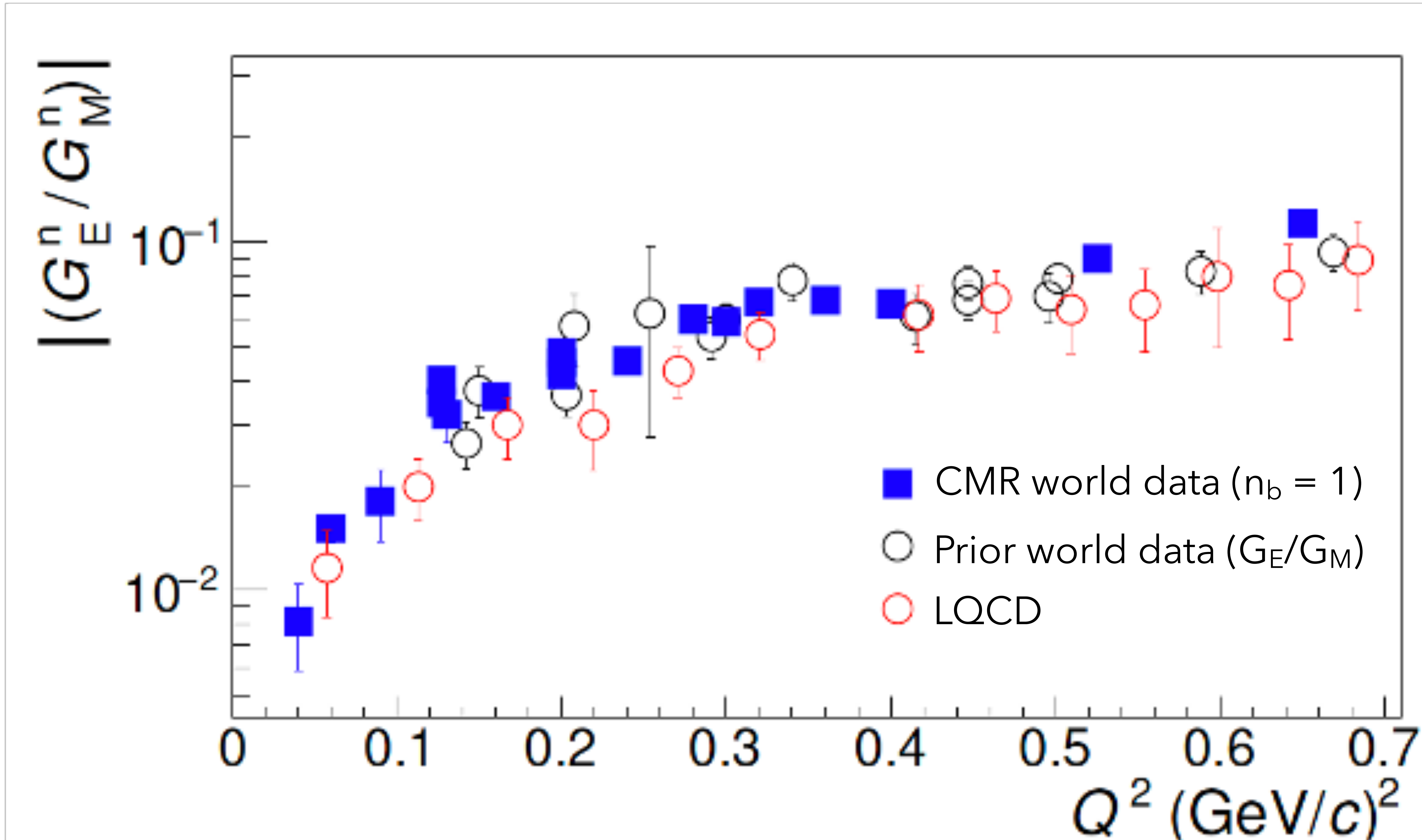
A path to extend our low Q^2 reach for G_E^n

A. J. Buchmann

Phys. Rev. Lett. 93, 212301 (2004)

$$\frac{G_E^n(Q^2)}{G_M^n(Q^2)} = \frac{Q}{|\mathbf{q}|}\frac{2Q}{M_N}\frac{1}{n_b(Q^2)}\frac{C2}{M1}(Q^2)$$

- **Buchmann SU(6) form:**
  - Ratios are related due to the underlying spin-flavor symmetry and its breaking by spin-dependent two- and three-quark currents
  - Theoretical correction (n_b) is ~10% (i.e. it reduces the G_E^n/G_M^n ratio by n_b~1.1) mainly due to third order SU(6) breaking terms (three-quark currents) omitted in the relation between G_M^n and $G_{M1}^{N\rightarrow\Delta}$



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This uncertainty can be parameterized from world CMR and ratio data

$$n_b(Q^2) = \frac{\frac{Q}{|\mathbf{q}|}\frac{2Q}{M_N}\frac{C2}{M1}(Q^2)}{\frac{G_E^n(Q^2)}{G_M^n(Q^2)}}$$

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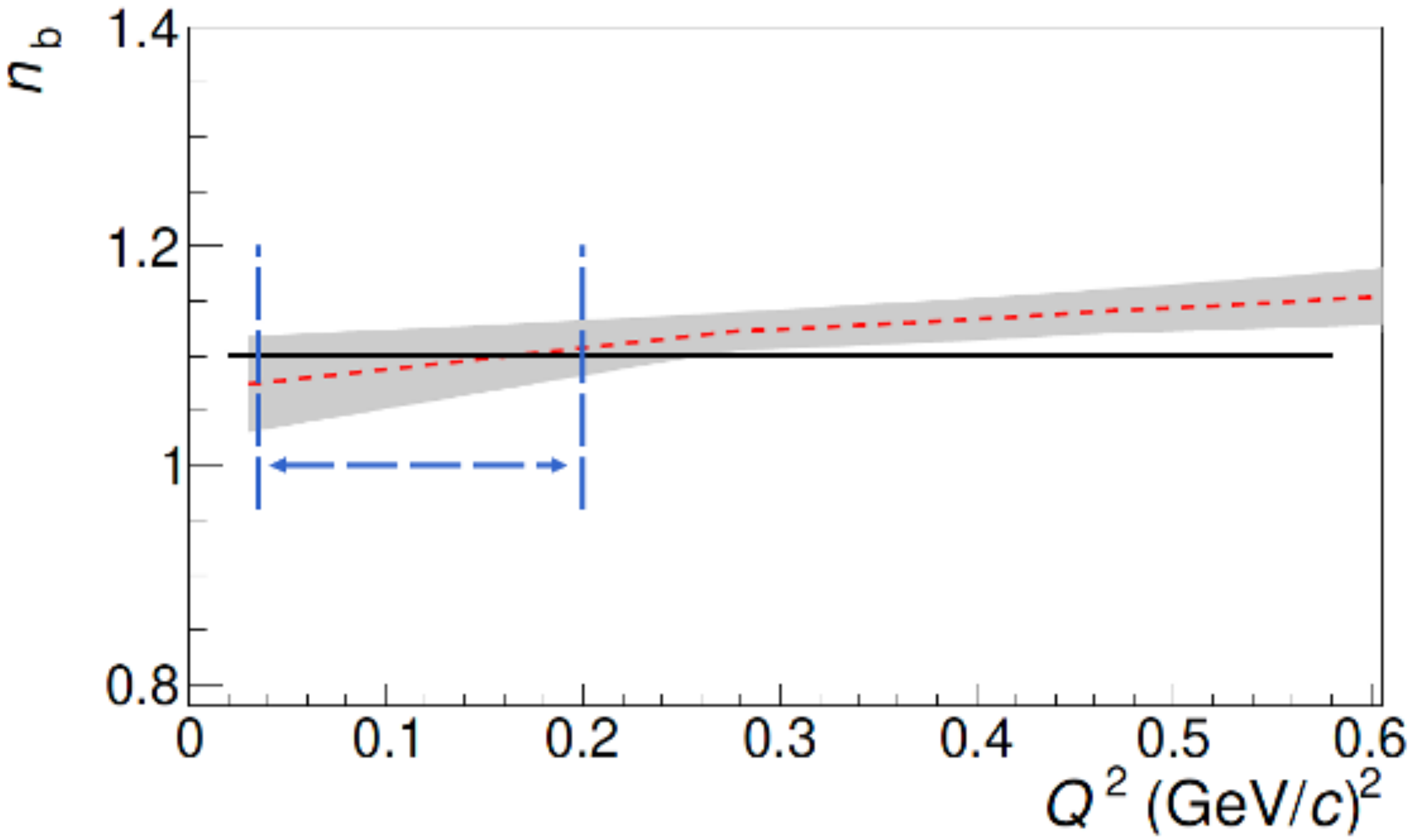

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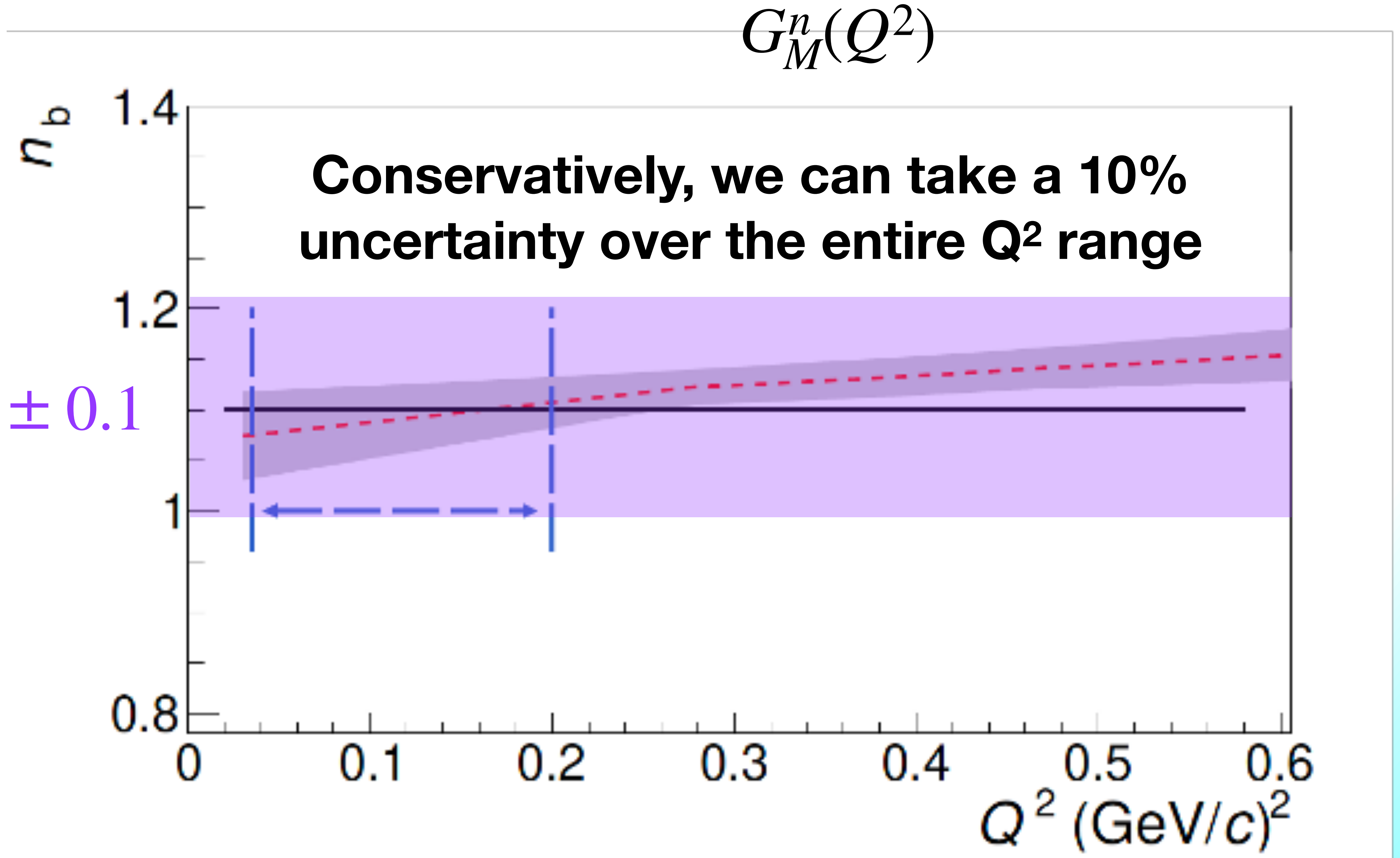
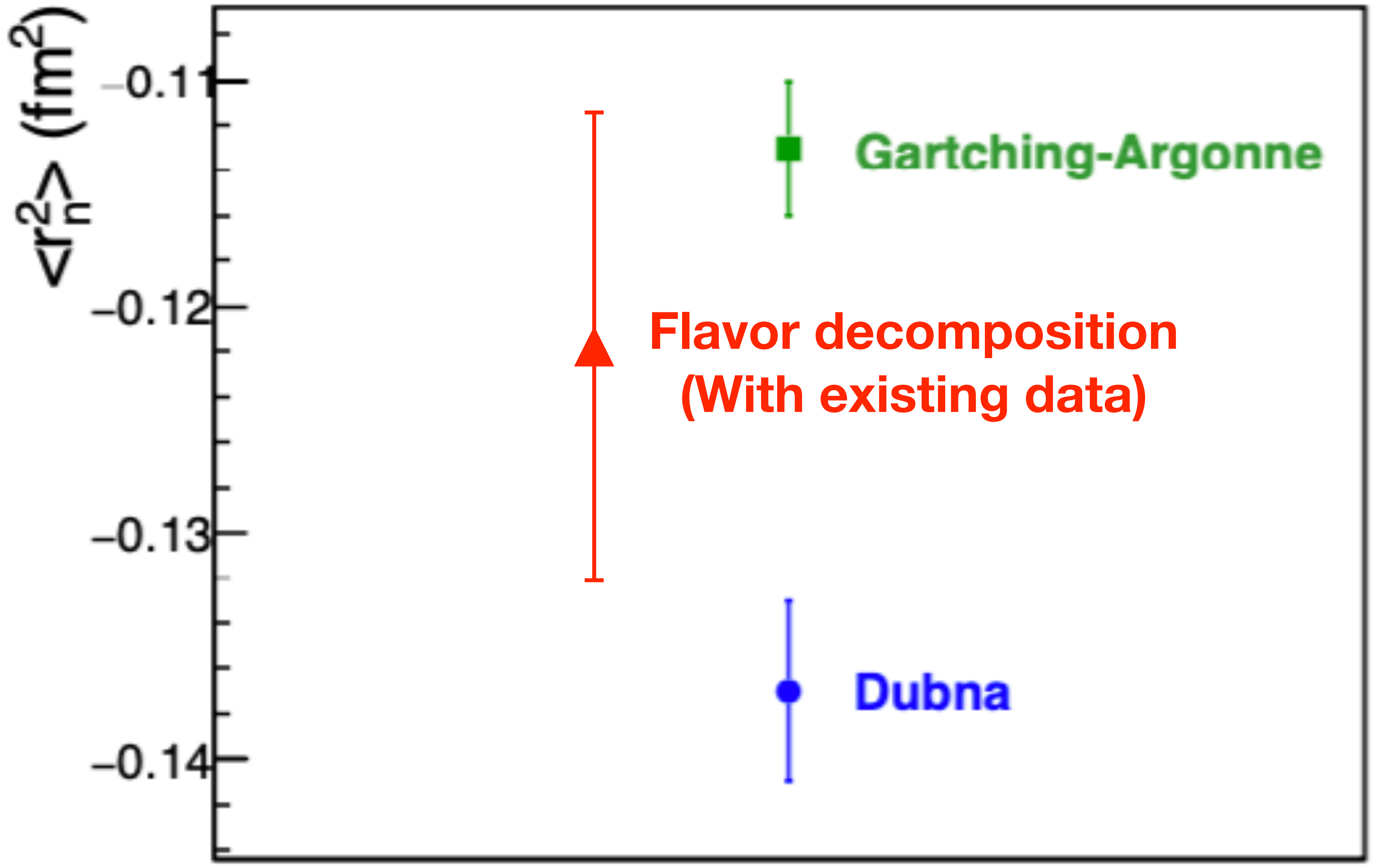


Conservatively, we can take a 10% uncertainty over the entire Q^2 range

$n_b = 1.1 \pm 0.1$

Radius extraction options are limited



- **From theory report:**
  - [The neutron scattering method] "… is certainly the most direct, and should also be the most reliable, notwithstanding the fact that there is discrepancy between the Dubna and Gartching-Argonne experiments, **which should be explored further**."
- This is a long standing issue, and there is no clear path via direct scattering that can resolve these discrepancies.
- There is also no clear path via electron - quasi-free neutron scattering that can provide the precision at low Q^2 to improve on $\langle r_n^2 \rangle$ extraction.
- **There is no currently known alternative path except to add high precision data points via CMR or EMR measurements.**
  - Underlying model dependence should not, on its own, invalidate such calculations.

On uncertainties in the G_E^n calculation or $\langle r_n^2 \rangle$ extraction

- **From the theory report or correspondence:**
  - "The quoted relation between GEn and the ... transition form factors is not based on any symmetries of strong interactions that could provide a reference point and enable one to theoretically estimate corrections due to deviations from the idealized symmetric situation, e.g. in a parametric expansion."
    - Some comments:
      - The NRCQM is expanded to include two-body terms at LO and NLO to enforce current conservation. One result is the relation: $\langle r_\Delta^2 \rangle = \langle r_p^2 \rangle - \langle r_n^2 \rangle$
        - This relation has been tested via "general parametrization" of QCD where 3rd order terms and loops contribute to a ~10-20% deviation. (Dillon, Mortugo, PRB 448)
          - The same procedure cannot be done for the TFF and G_E^n relation because the quadrupole relations connect certain matrix elements or expectation values of a quantity that transforms as a tensor under space rotations to the expectation value of a scalar.
        - ***We claim using such relations do not intrinsically invalidate the calculation.***

On uncertainties in the G_E^n calculation or $\langle r_n^2 \rangle$ extraction

- **From the theory report or correspondence:**
  - "The quoted 15-20% accuracy of the relation is a purely empirical statement based on the comparison between the measured GEn and N -> Delta transition FFs in the region where data are available. "
    - We humbly disagree:
      - The uncertainties are determined from analysis of corrections or neglected terms in the NRCQM calculation via various studies (including relativistic corrections, D-state admixtures, etc..) and for large-Nc, the level of uncertainty can be constrained on the order of (1/Nc) or the mass-splitting.
        - We've had repeated communication with authors of the derivations and they stand by their theoretical uncertainties.
      - The authors do occasionally compare to existing data and express that as a percentage, but this is not the basis for the uncertainty of the calculation.

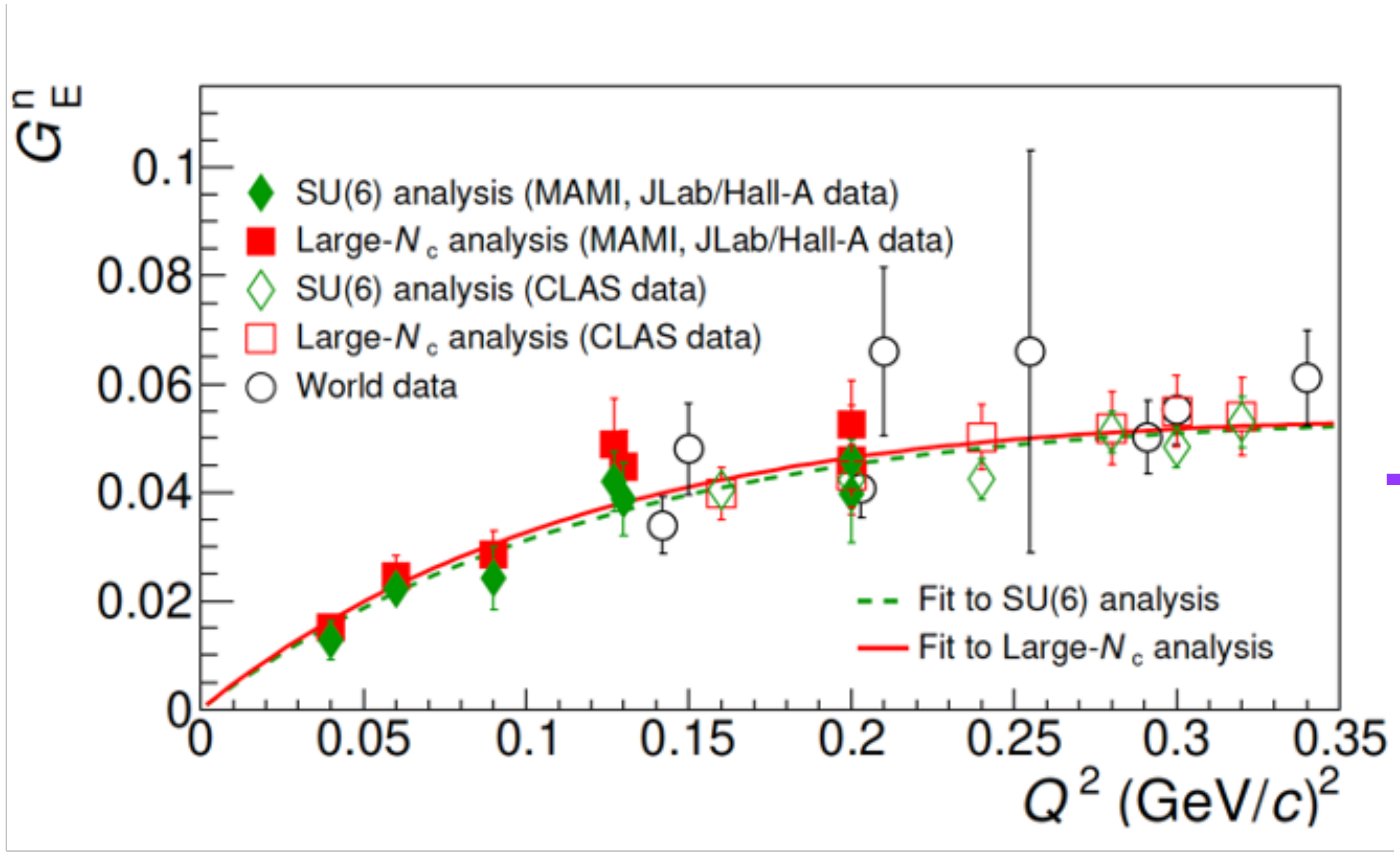
On uncertainties in the G_E^n calculation or $\langle r_n^2 \rangle$ extraction

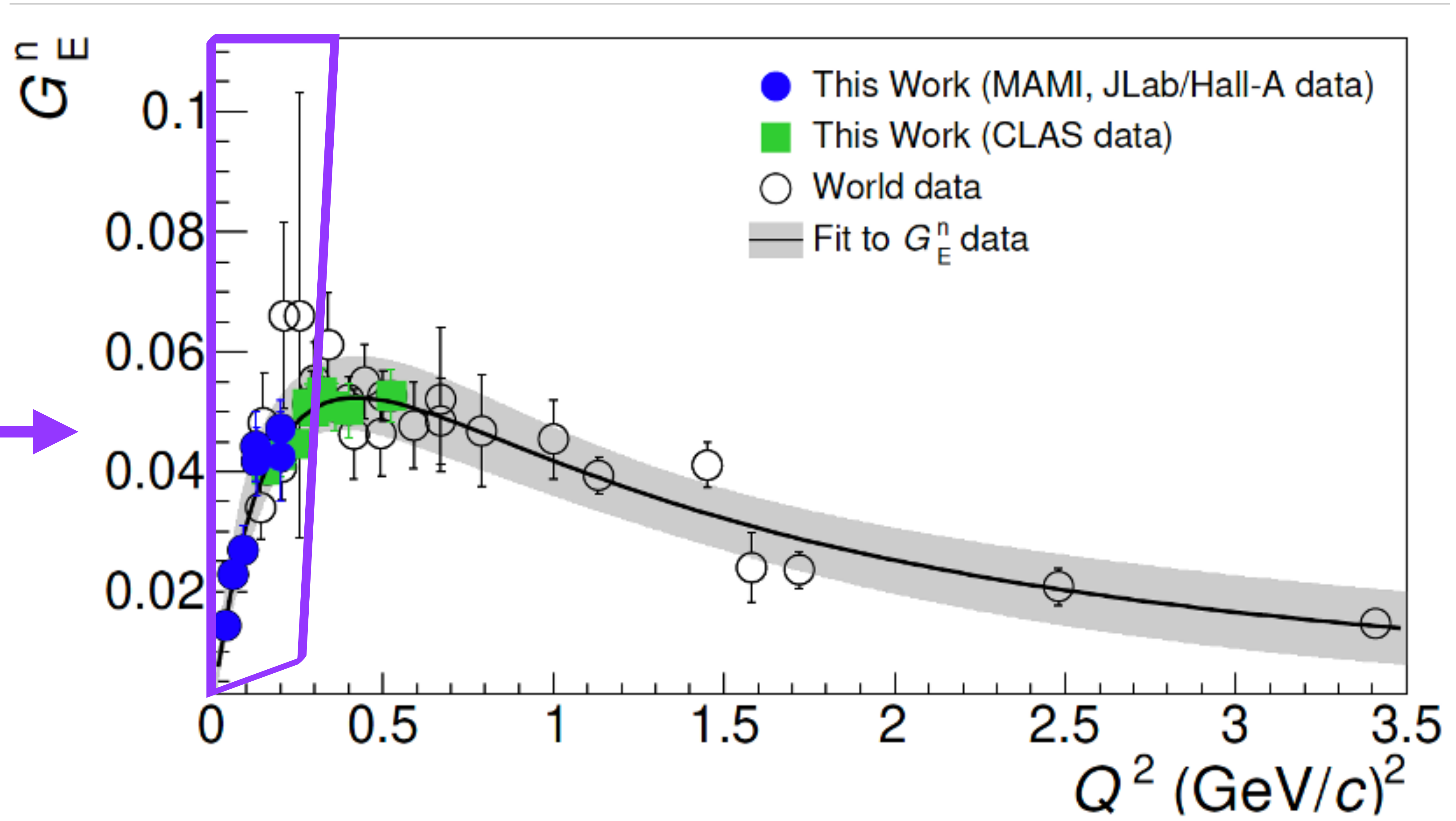
- **From the theory report or correspondence:**
  - "..differences between the [isoscalar and isovector] channels are not expressed in the quark model, and one cannot expect the relation to remain valid at lower Q2 in any meaningful sense."
    - Again, we humbly disagree:
      - An analysis of the soundness of the G_E^n calculation specifically in the low Q2 region to the highest measured values was performed by Bachmann (arxiv.org/pdf/hep-ph/0412421).
      - The isovector and isoscalar components are not ignored in the CQM, but instead the careful relation of the charge radii of the proton, neutron, and Delta include cancelations of terms.
      - The Q2 dependence of GEn and the cancelling relation between the isoscalar and isovector components that culminate in an exact cancelation at Q2 = 0 implies there must be an intrinsic dynamical relation between them.
      - To the level of existing measurements at the lowest Q2 points (0.3 to 0.5 GeV2/c2), the calculations seem to agree with data reasonably well.

On uncertainties in the G_E^n calculation or $\langle r_n^2 \rangle$ extraction

- **From the theory report or correspondence:**
  - "An extraction of the the neutron charge radius from GEn data assuming this relation would not be conclusive, since the accuracy of the indirectly obtained GEn data could not be quantified. "
    - While the theoretical calculations cannot be directly tested via methods like power counting, they are still analytically determined and can be defended.
    - This relation is not a new concept:
      - The relationship between proton properties and TFFs was established in the 60's.
      - The relationship between G_E^n and the TFFs has been established for over 25 years and the calculation has been employed in many peer-reviewed journals by many different authors.
        - Where appropriate, the authors were able to quote and use the relevant theoretical uncertainty with our issue.
      - The relations have been redefined in the frameworks of SU(6) breaking and large-Nc relations.

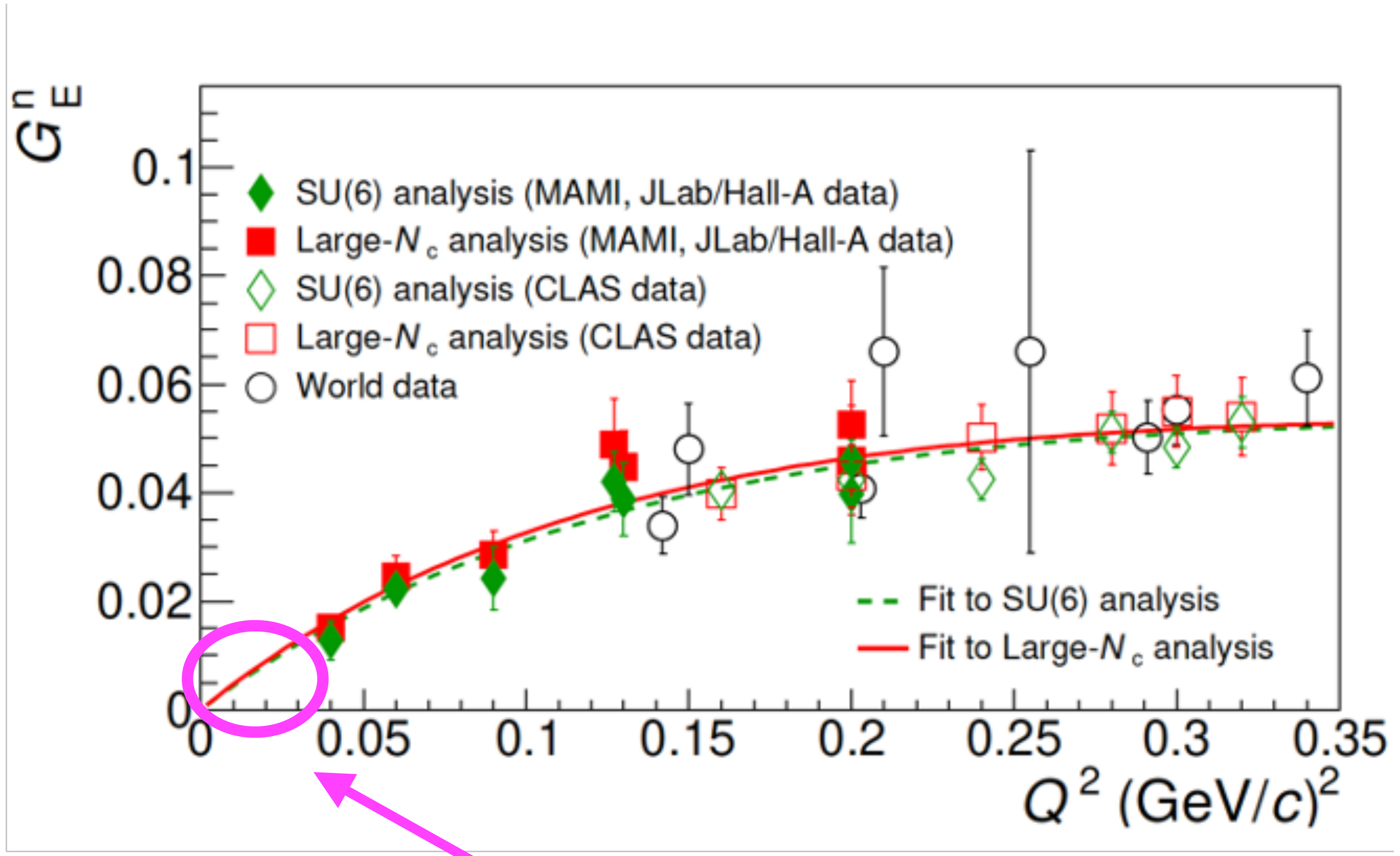
A path to extend our low Q^2 reach for G_E^n

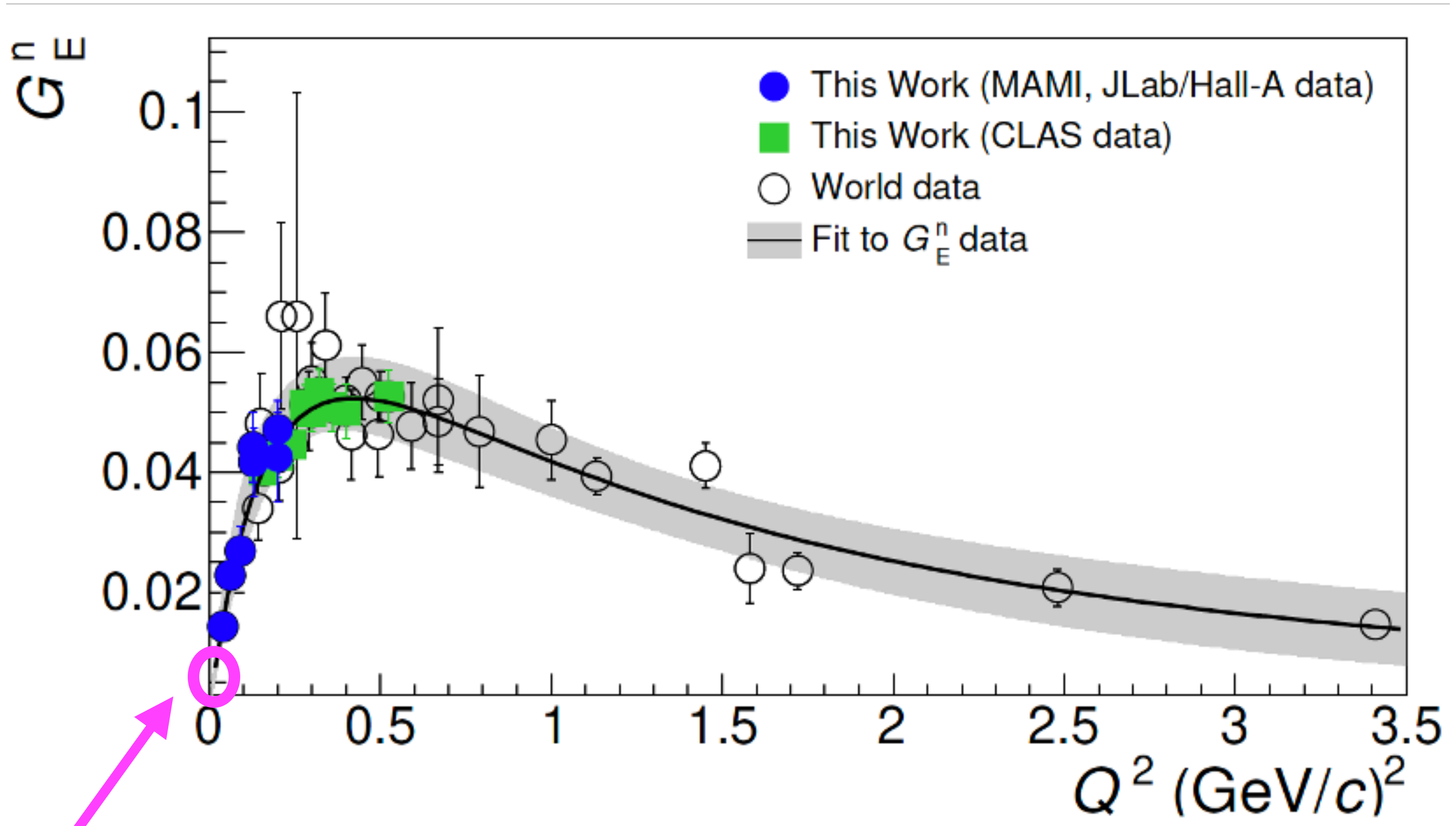


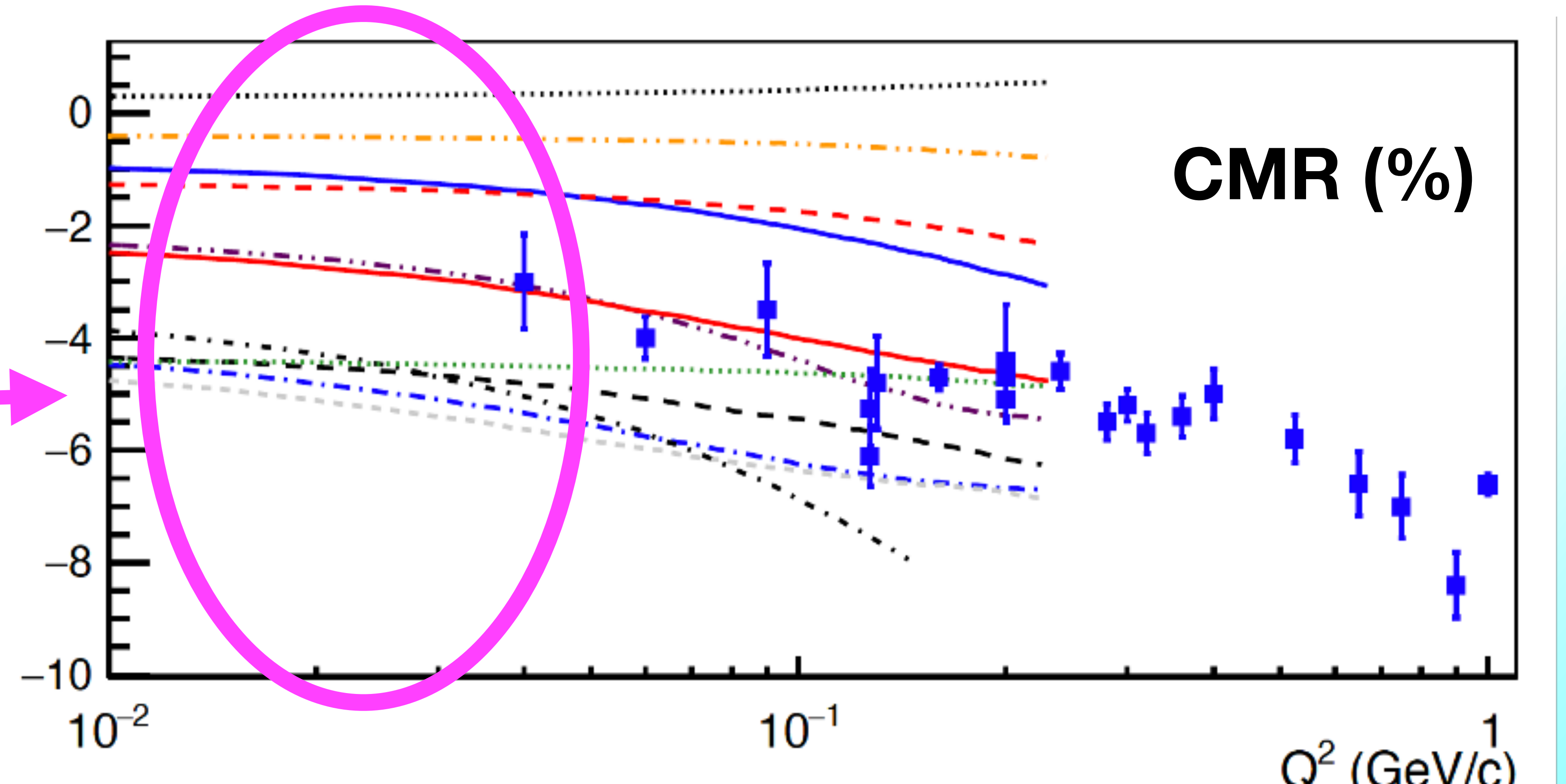


Global analysis using this method has been published in Nature Comm. 12, 1759 (2021)

A path to extend our low Q^2 reach for G_E^n

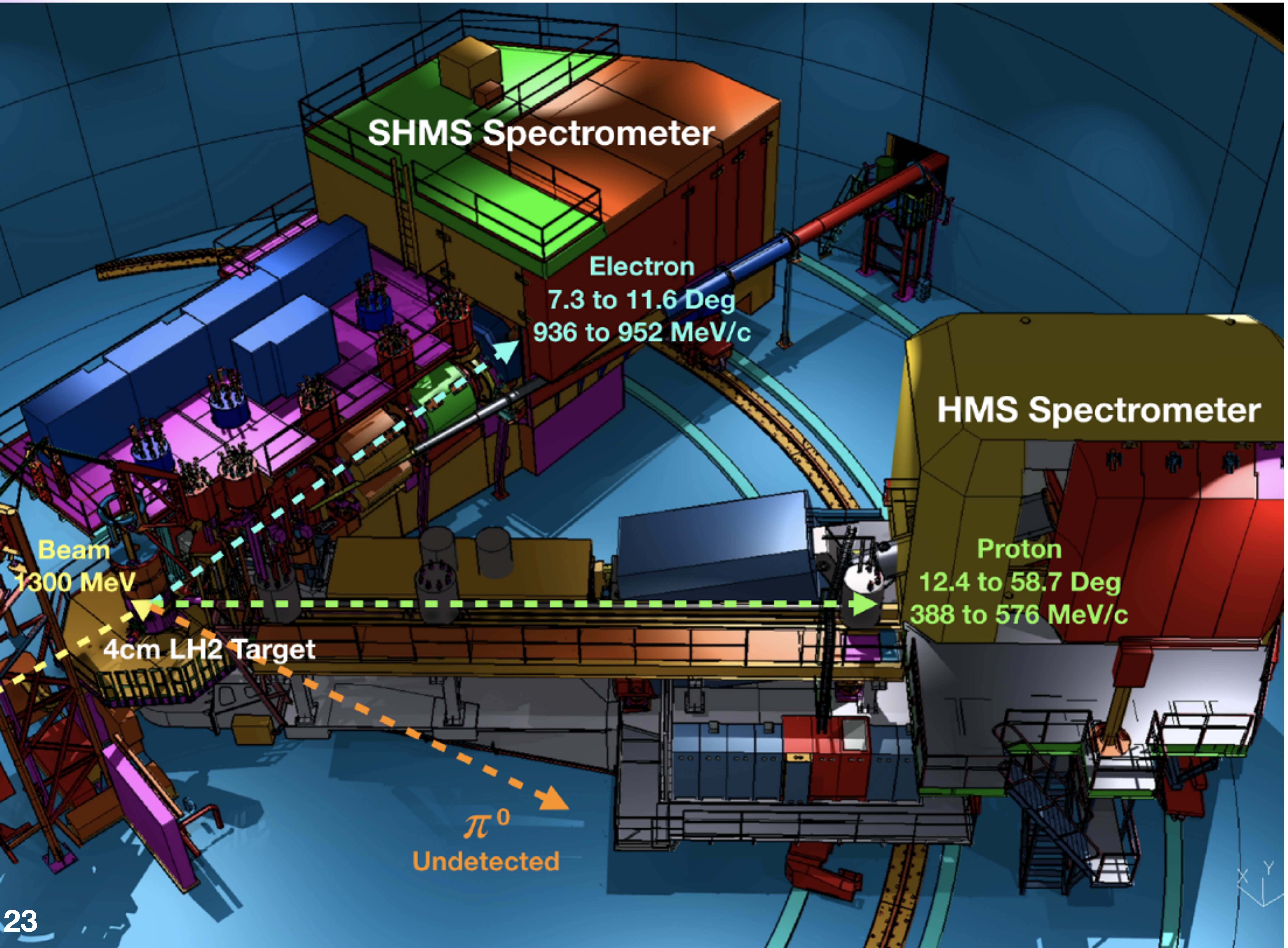




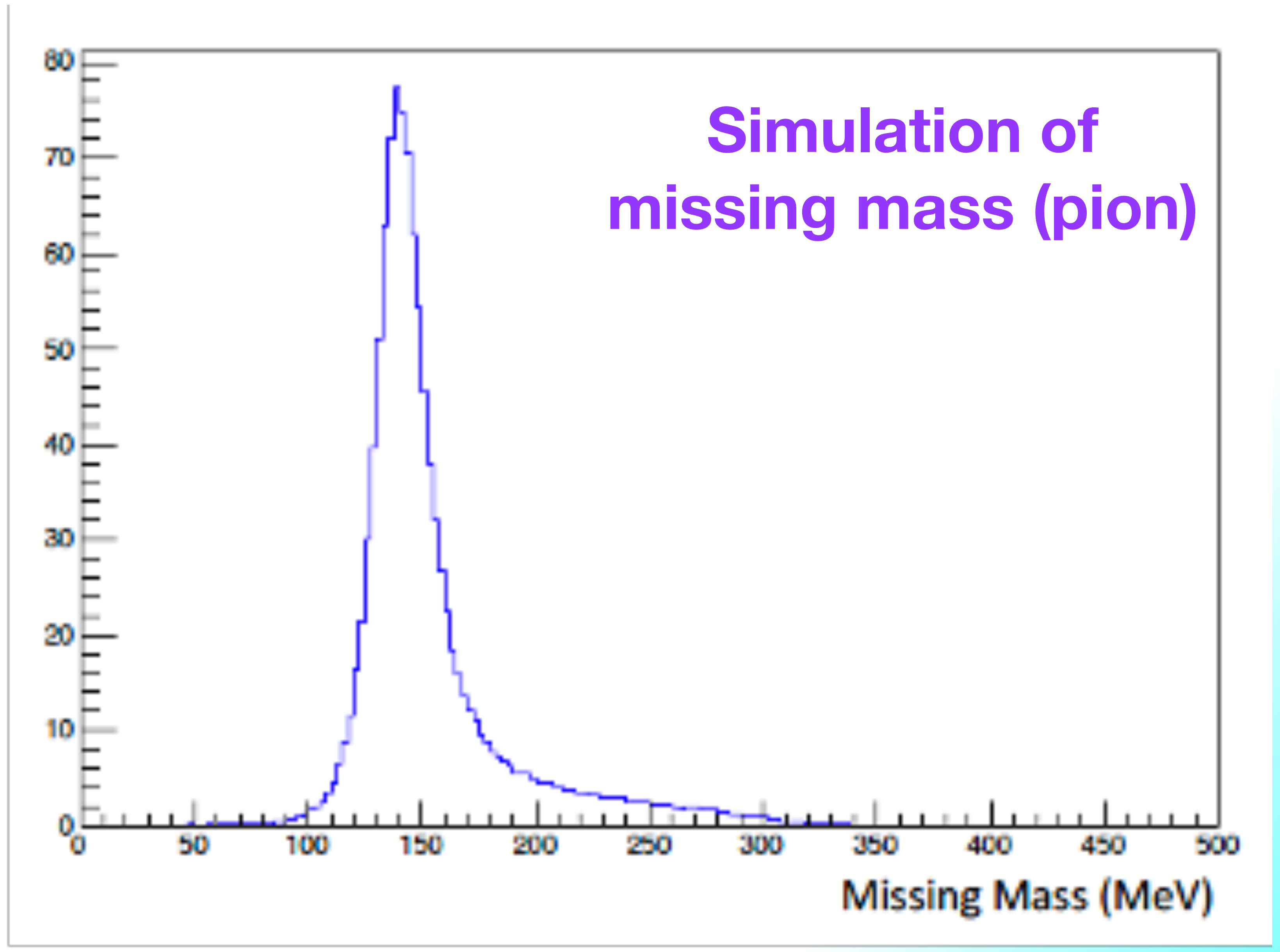


This proposal

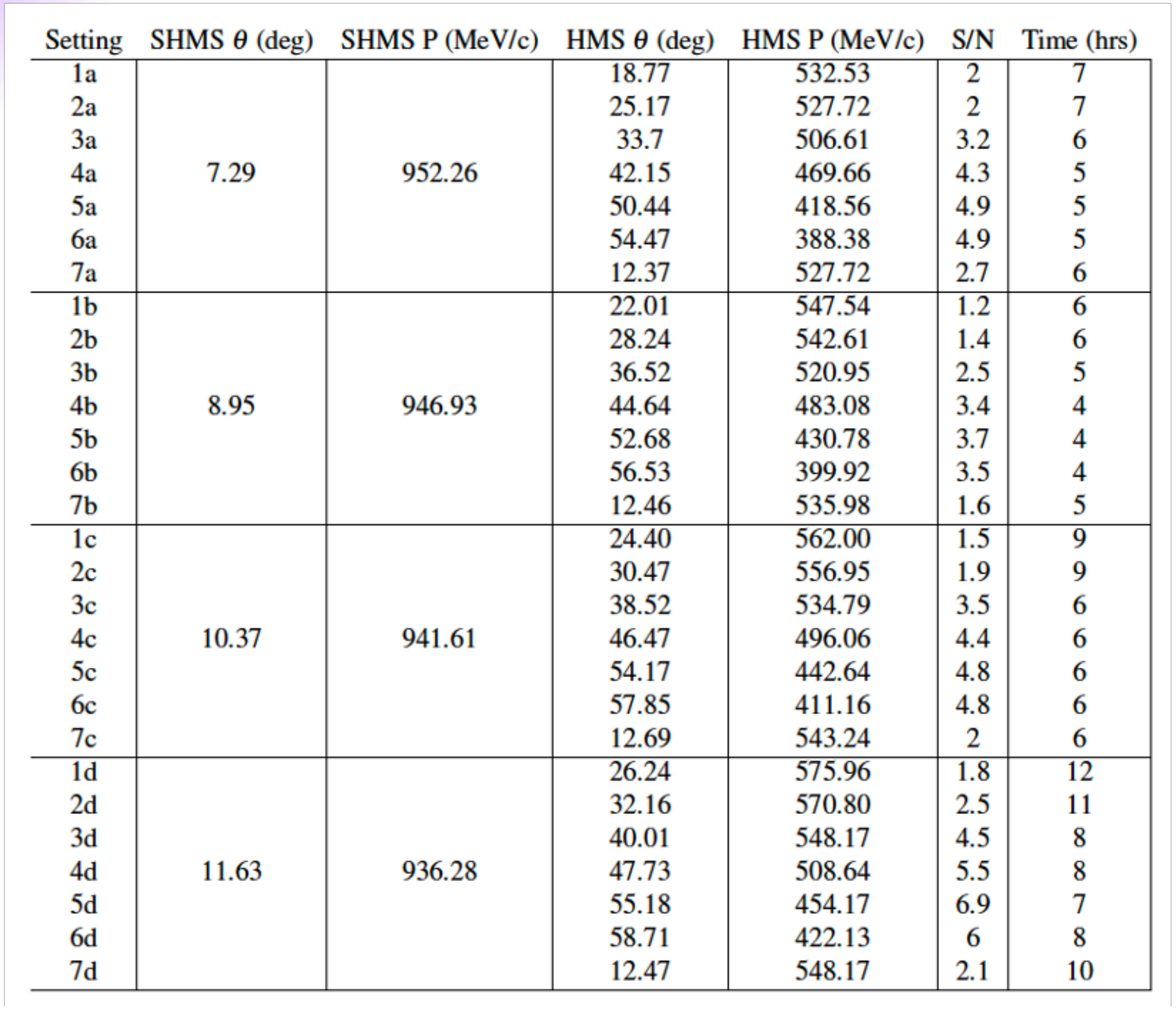
Experimental Setup



- **Standard Hall-C equipment**
  - 1300 MeV electron beam
  - Detect proton and electron in coincidence
  - Reconstruct pion from missing mass.

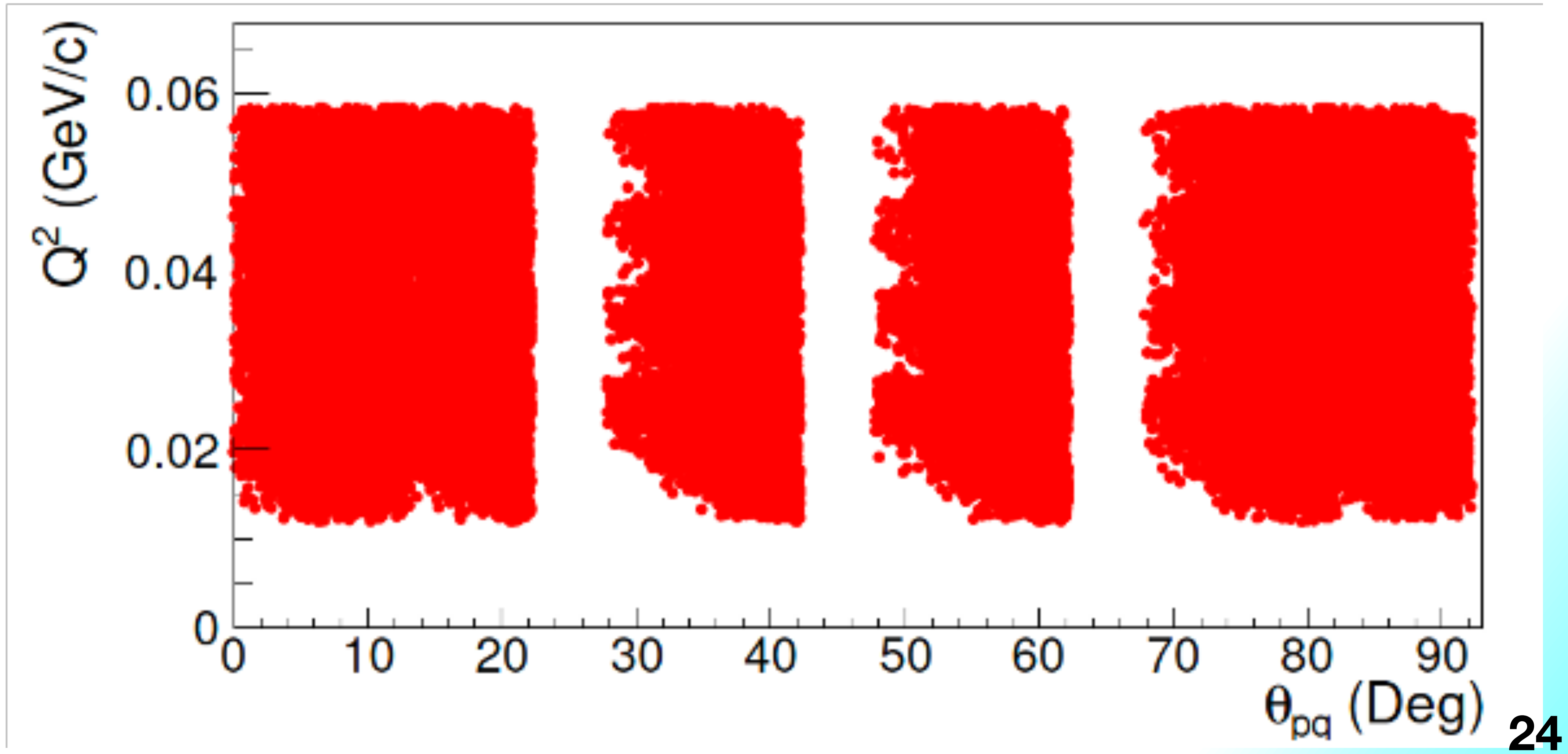


Measurement Settings

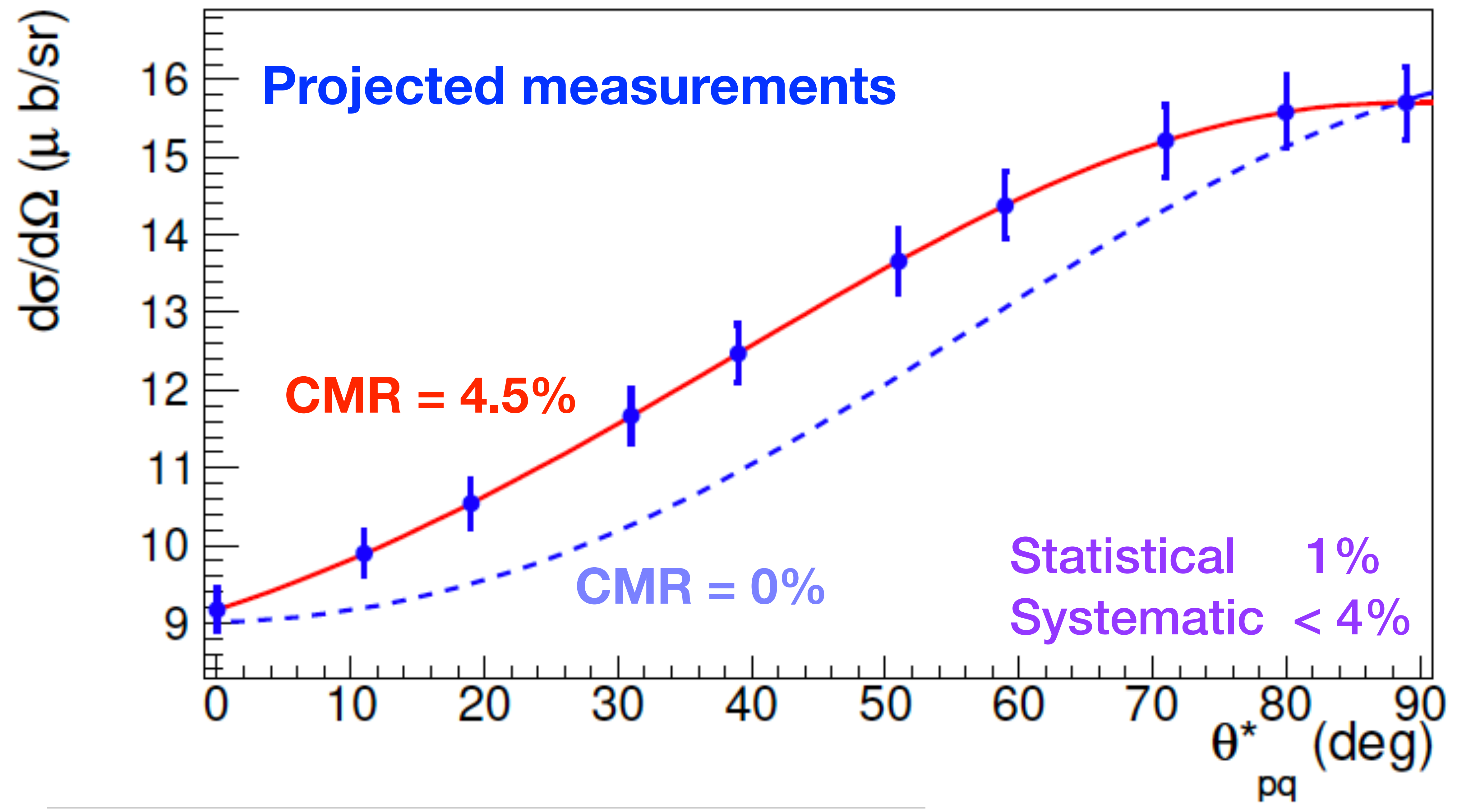


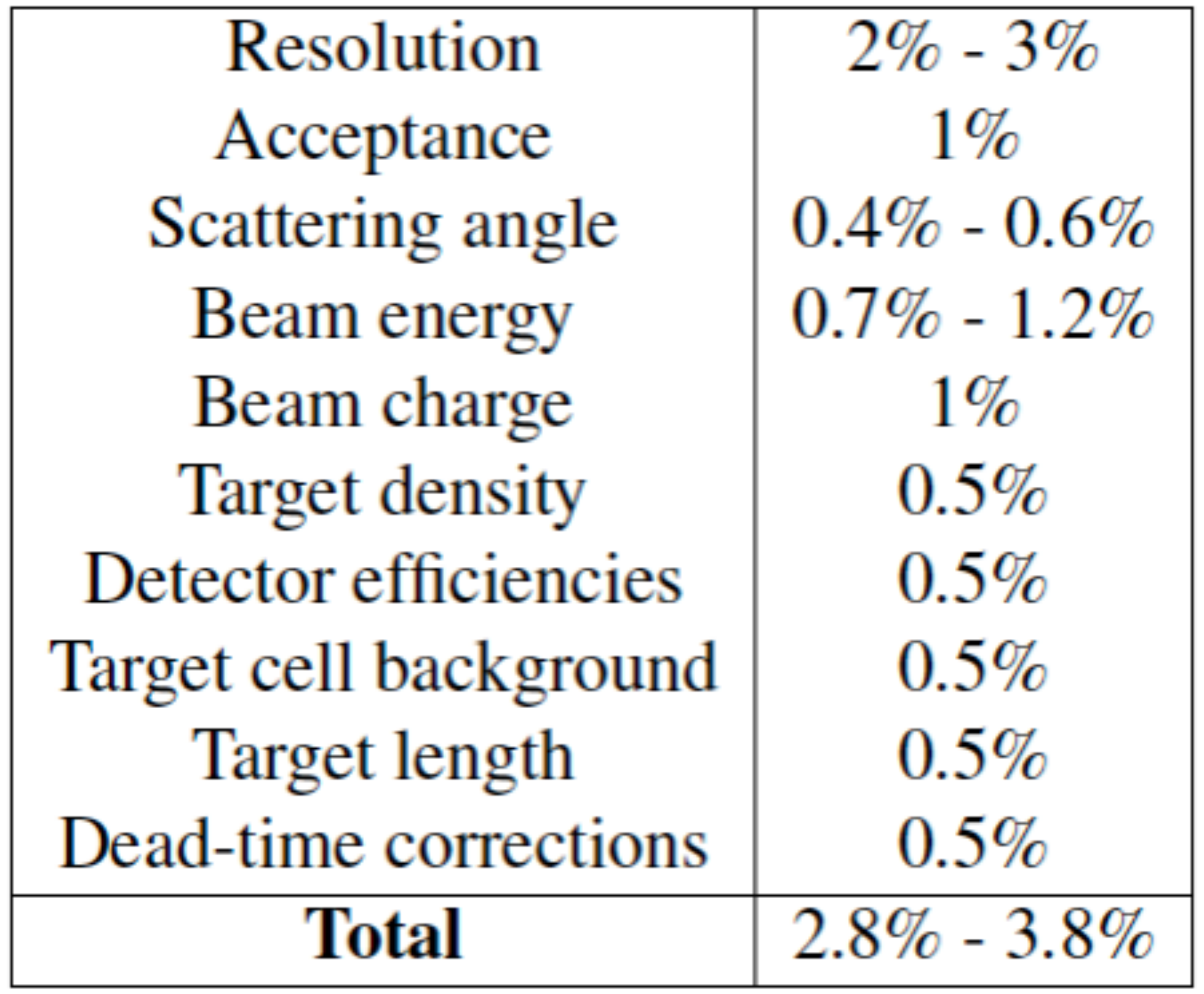
Setting	SHMS θ (deg)	SHMS P (MeV/c)	HMS θ (deg)	HMS P (MeV/c)	S/N	Time (hrs)
1a	7.29	952.26	18.77	532.53	2	7
2a			25.17	527.72	2	7
3a			33.7	506.61	3.2	6
4a			42.15	469.66	4.3	5
5a			50.44	418.56	4.9	5
6a			54.47	388.38	4.9	5
7a			12.37	527.72	2.7	6
1b	8.95	946.93	22.01	547.54	1.2	6
2b			28.24	542.61	1.4	6
3b			36.52	520.95	2.5	5
4b			44.64	483.08	3.4	4
5b			52.68	430.78	3.7	4
6b			56.53	399.92	3.5	4
7b			12.46	535.98	1.6	5
1c	10.37	941.61	24.40	562.00	1.5	9
2c			30.47	556.95	1.9	9
3c			38.52	534.79	3.5	6
4c			46.47	496.06	4.4	6
5c			54.17	442.64	4.8	6
6c			57.85	411.16	4.8	6
7c			12.69	543.24	2	6
1d	11.63	936.28	26.24	575.96	1.8	12
2d			32.16	570.80	2.5	11
3d			40.01	548.17	4.5	8
4d			47.73	508.64	5.5	8
5d			55.18	454.17	6.9	7
6d			58.71	422.13	6	8
7d			12.47	548.17	2.1	10

- **Cover a Q^2 range of 0.015 to 0.055 $(\mathrm{GeV/c})^2$**
  - 28 arm configurations
  - Coverage for 9 Q2 bins.
  - 7.8 days production
  - 1.7 days other (dummy, calibration, etc..)



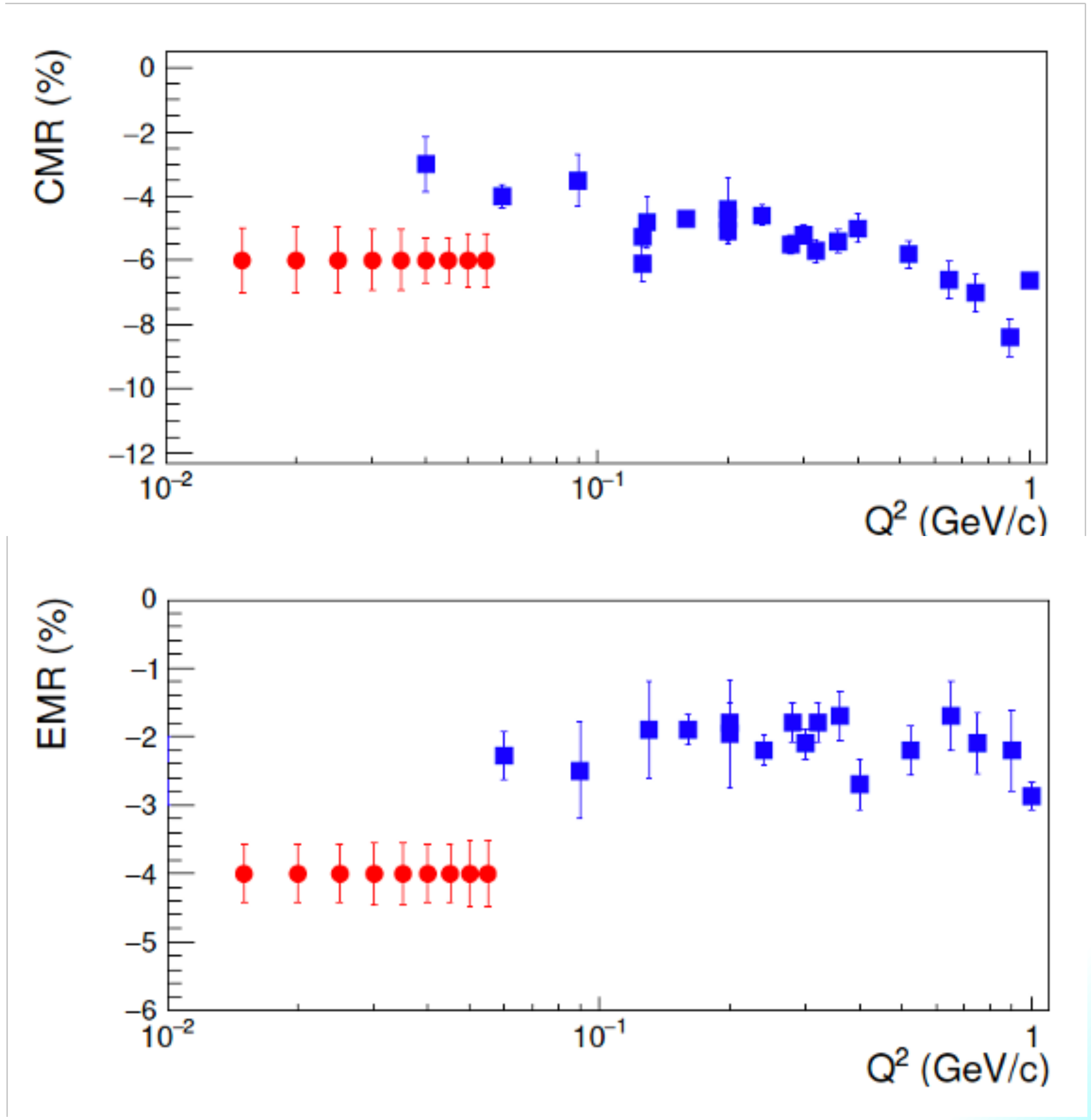
Projected CMR and EMR measurements





Resolution	2% - 3%
Acceptance	1%
Scattering angle	0.4% - 0.6%
Beam energy	0.7% - 1.2%
Beam charge	1%
Target density	0.5%
Detector efficiencies	0.5%
Target cell background	0.5%
Target length	0.5%
Dead-time corrections	0.5%
Total	2.8% - 3.8%

- High precision in very low Q^2 region that is sparsely populated
  - Region where pion-cloud effects are expected to be prominent



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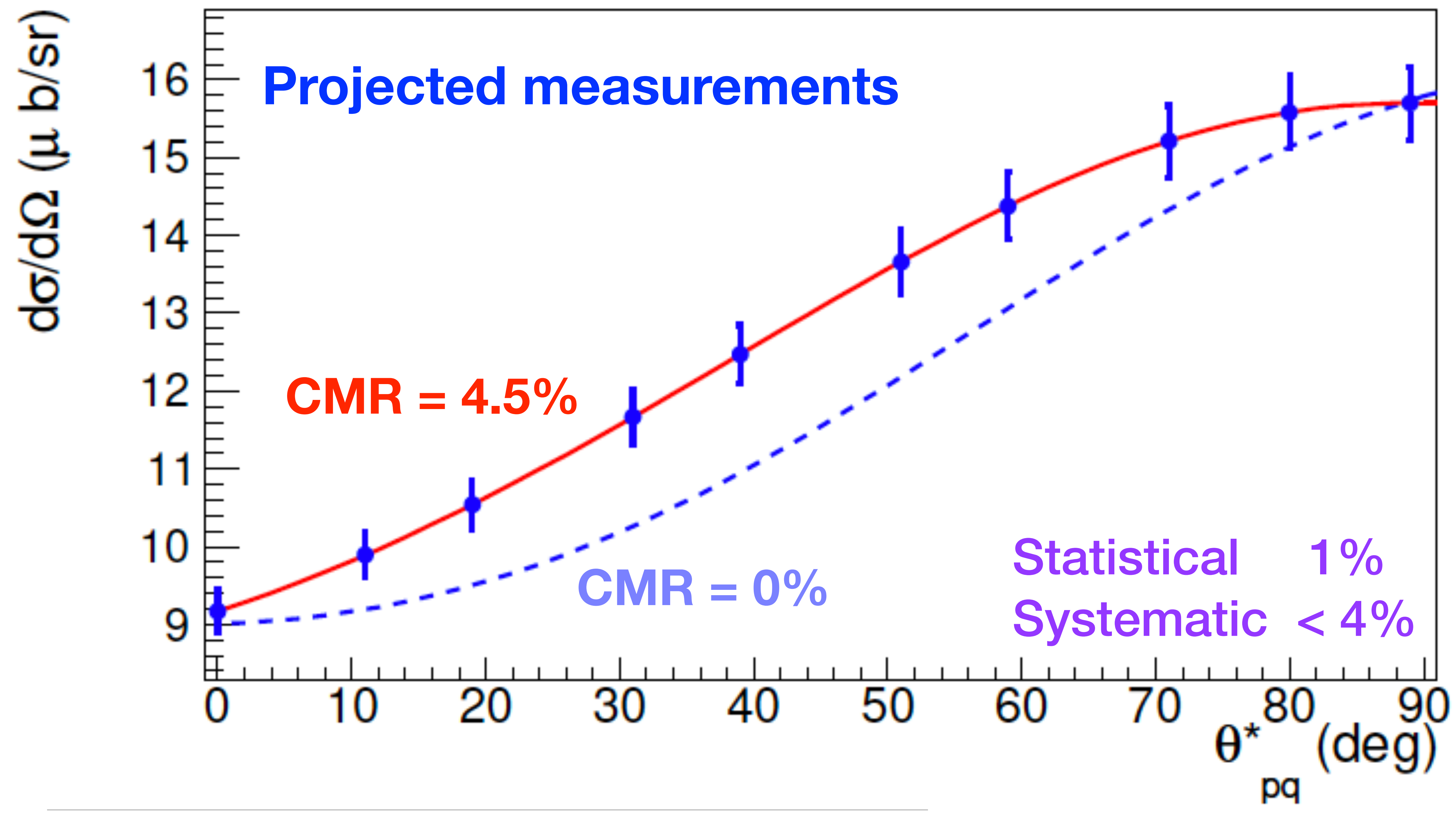

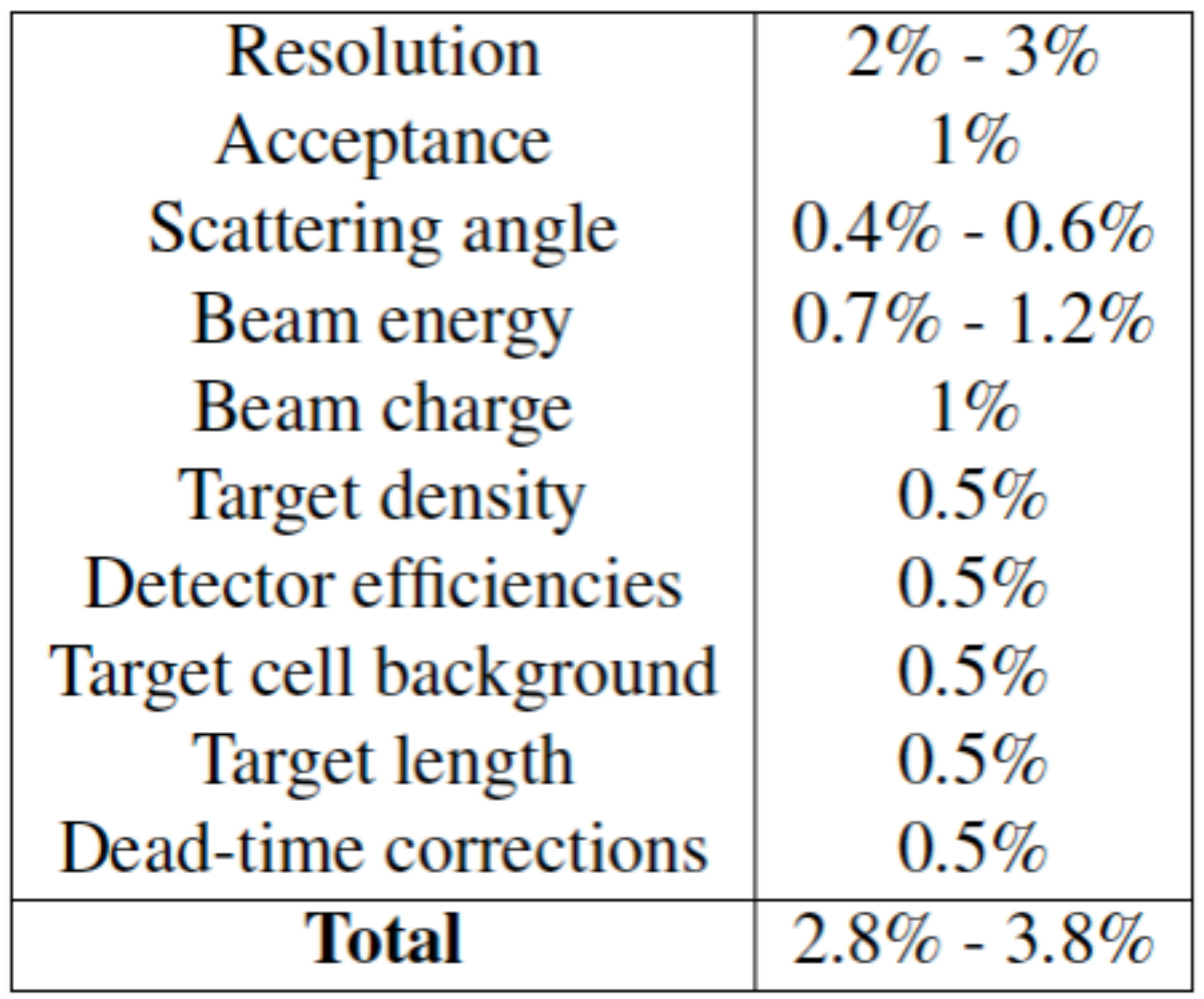

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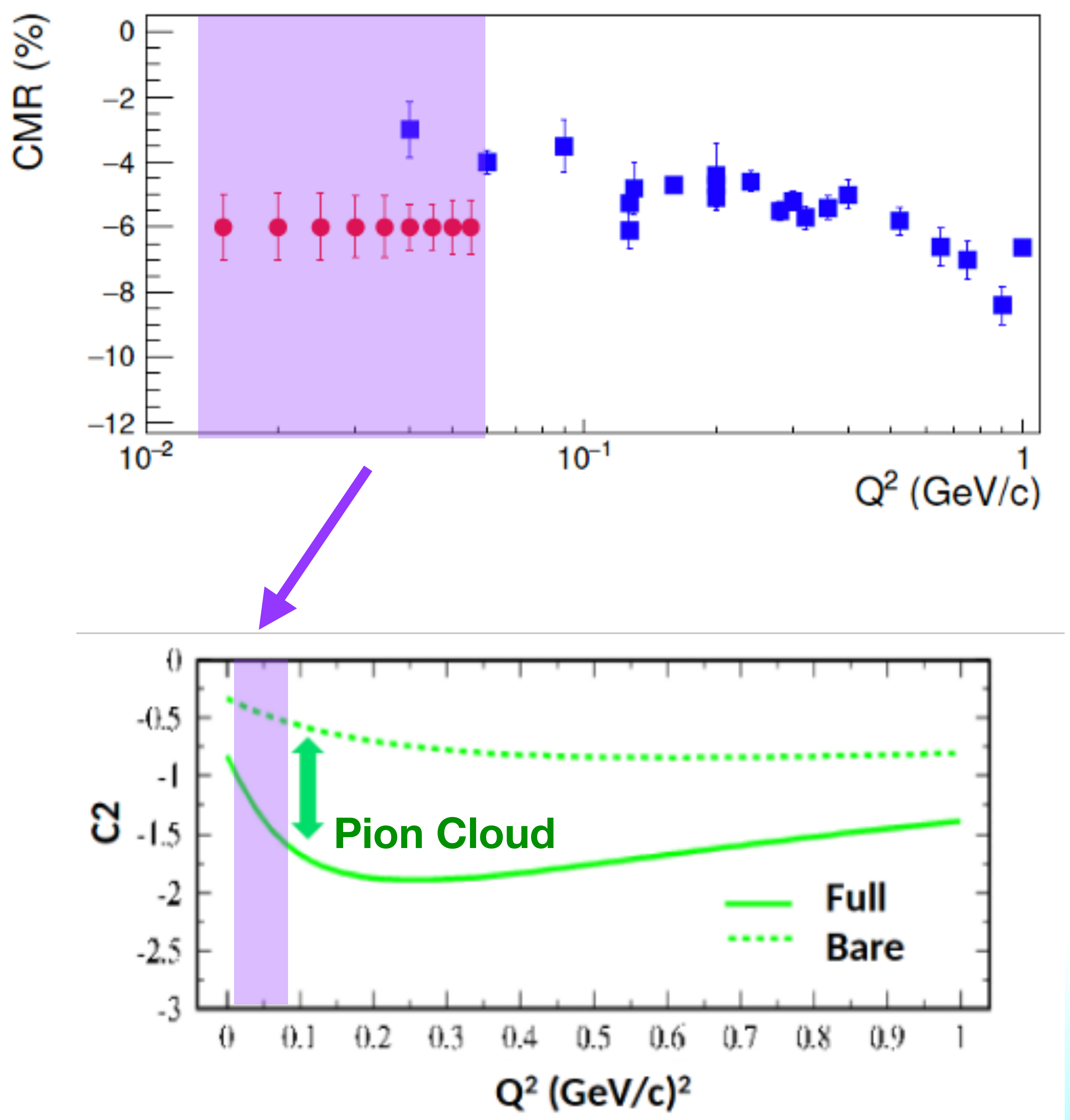
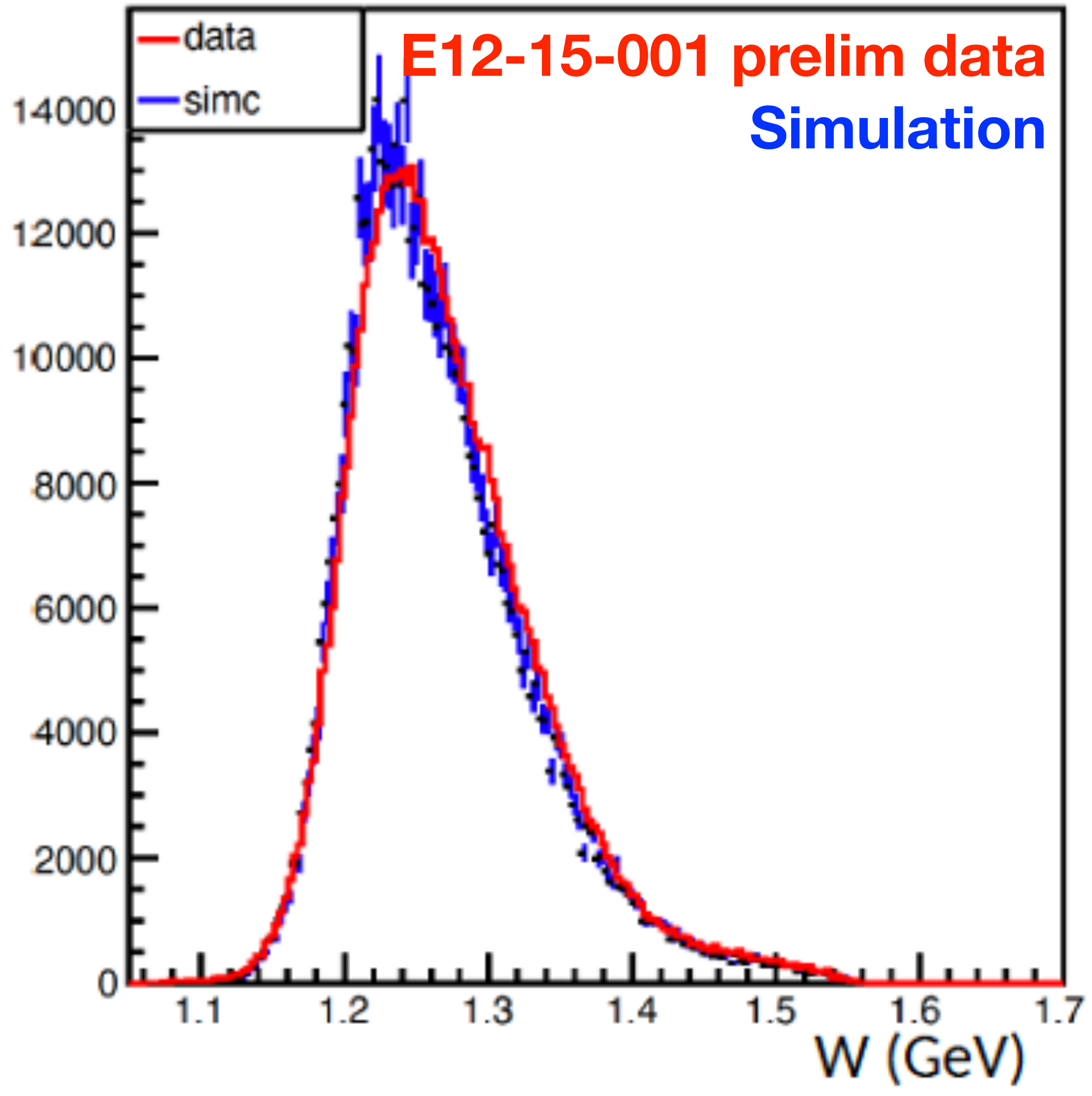
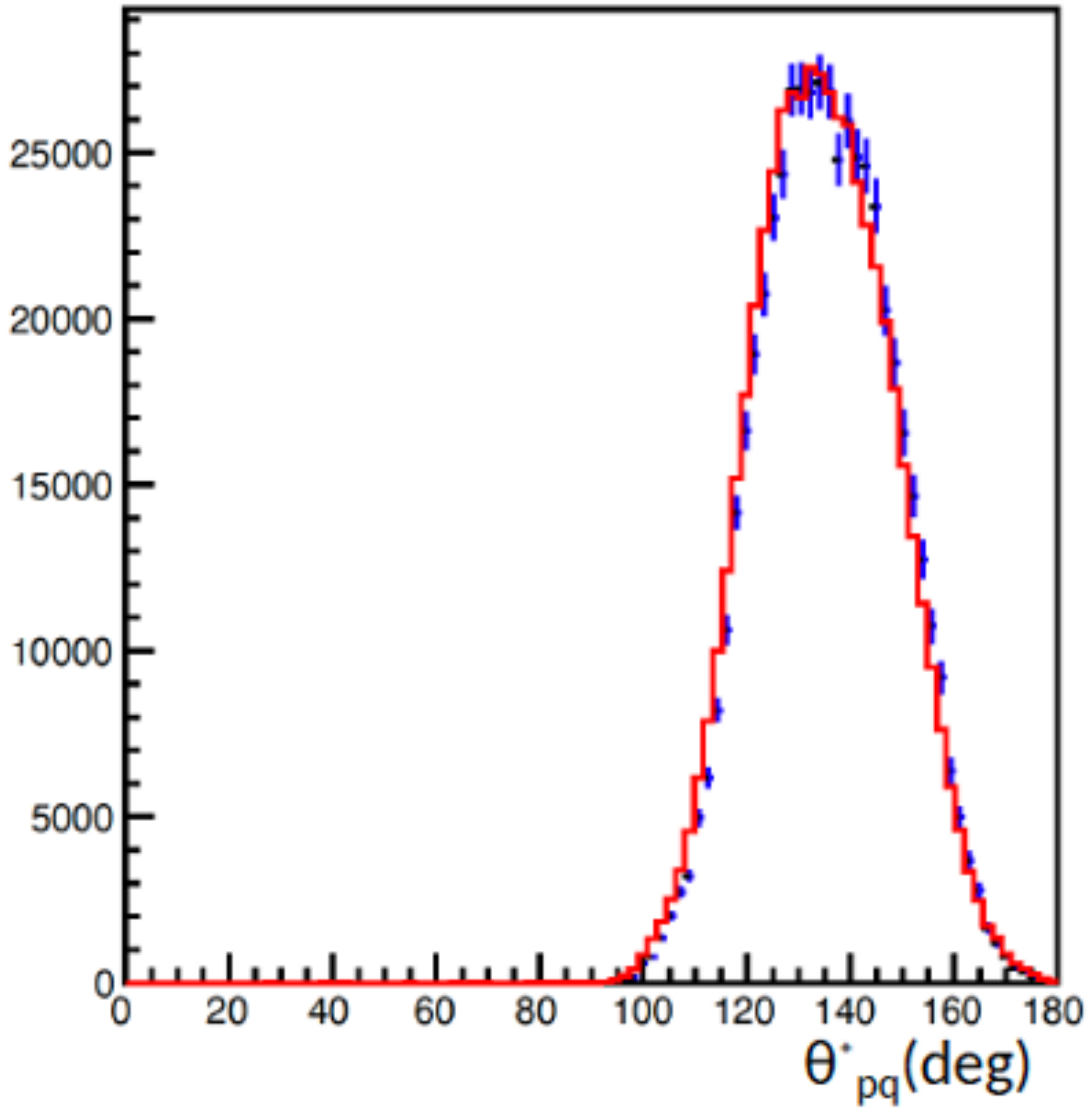


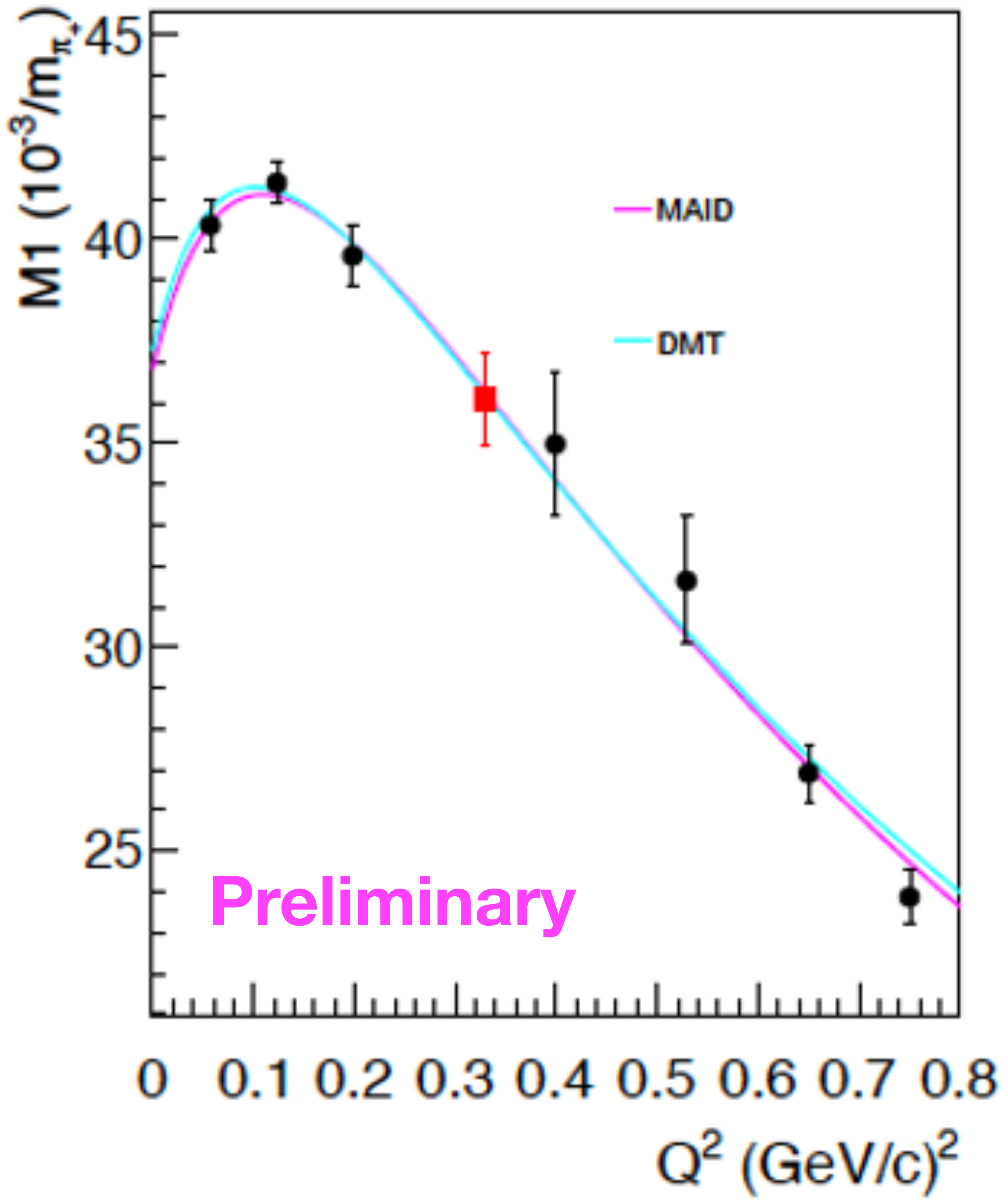
Confidence in methods and projections

VCS Experiment E12-15-001 ran in Hall-C (2019) with a similar set-up at Q2 = 0.33 (GeV/c)2

Main difference with proposed experiment: Lower Q2 -> lower beam energy and lower central momentum settings

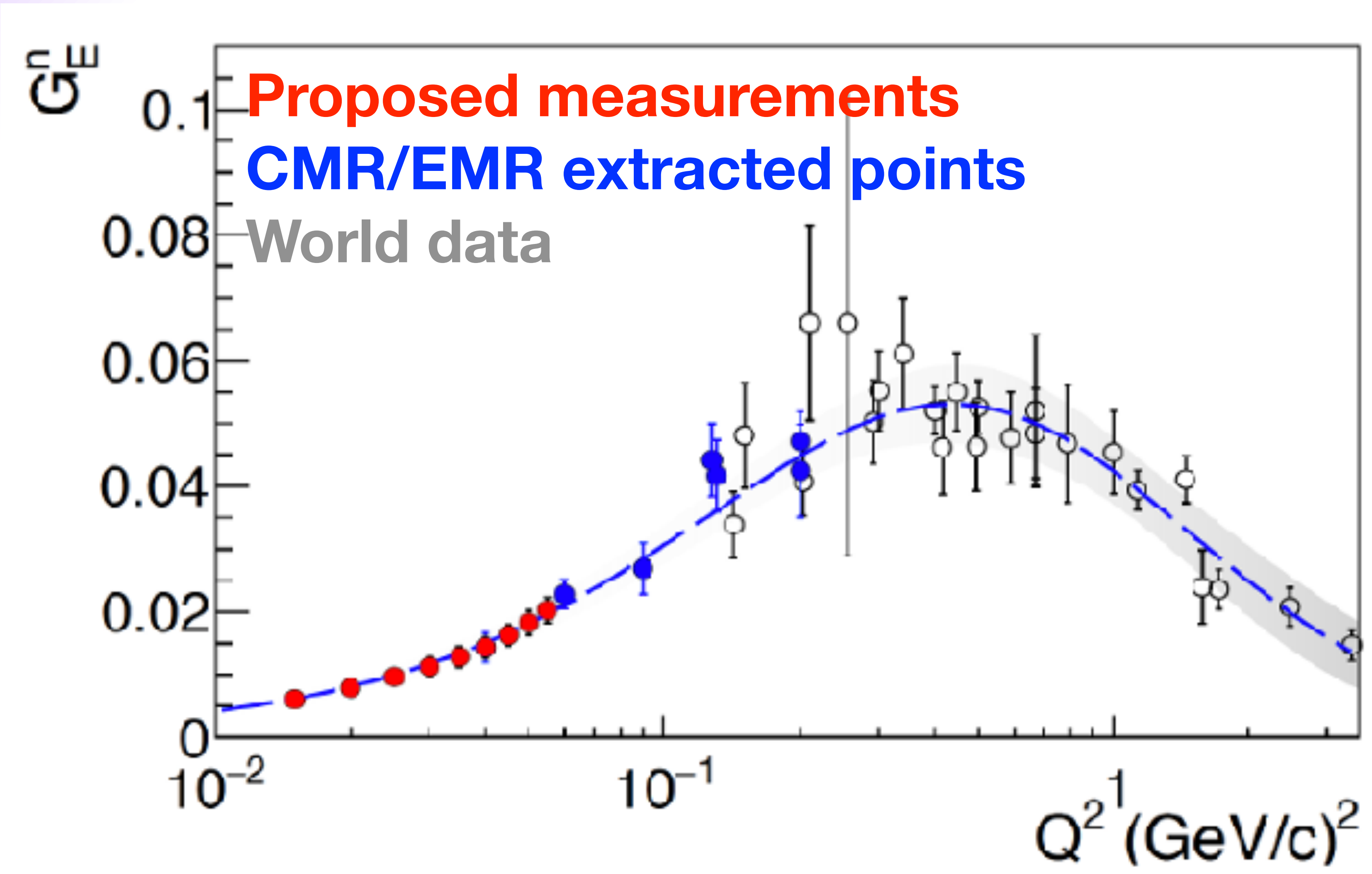


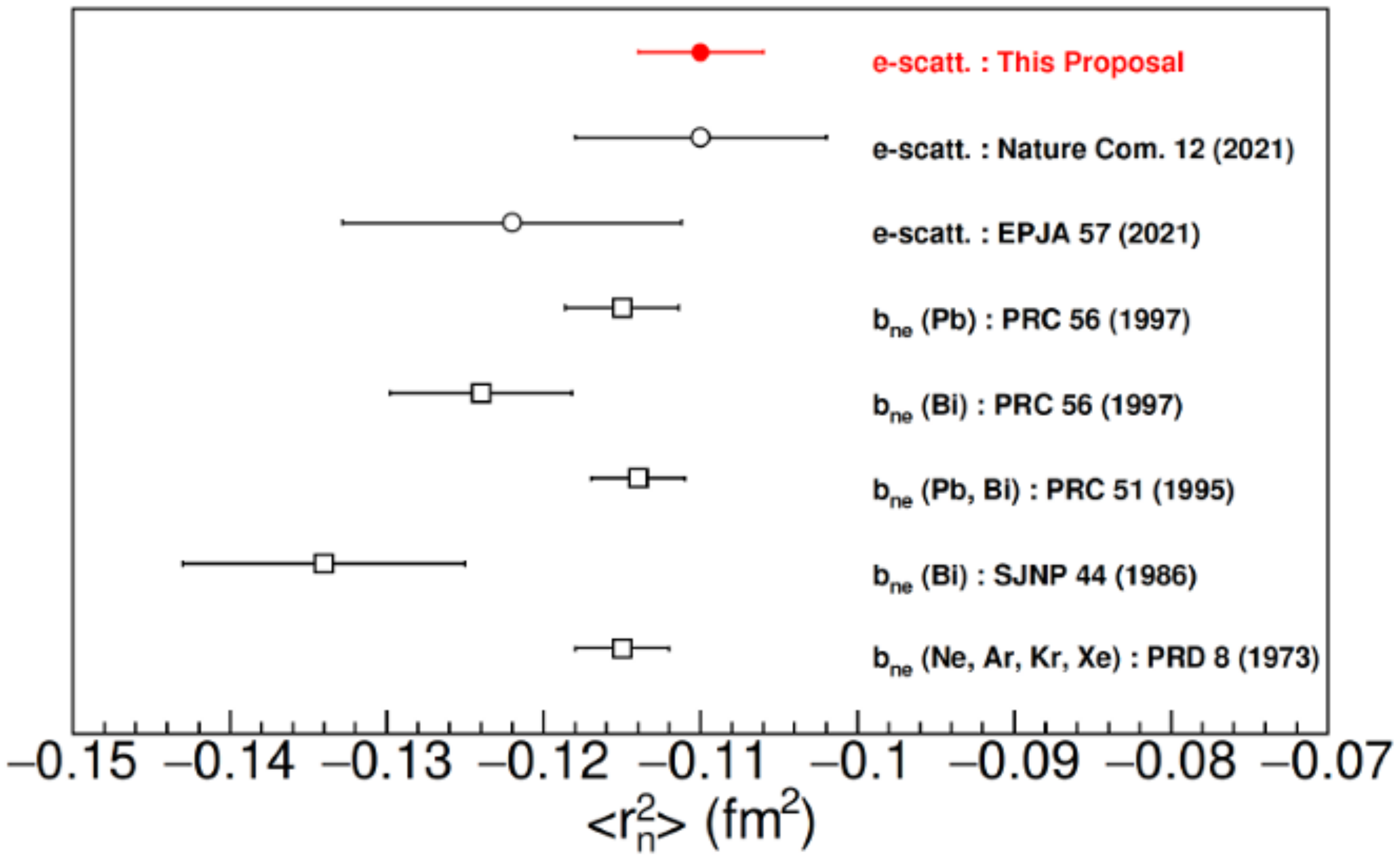


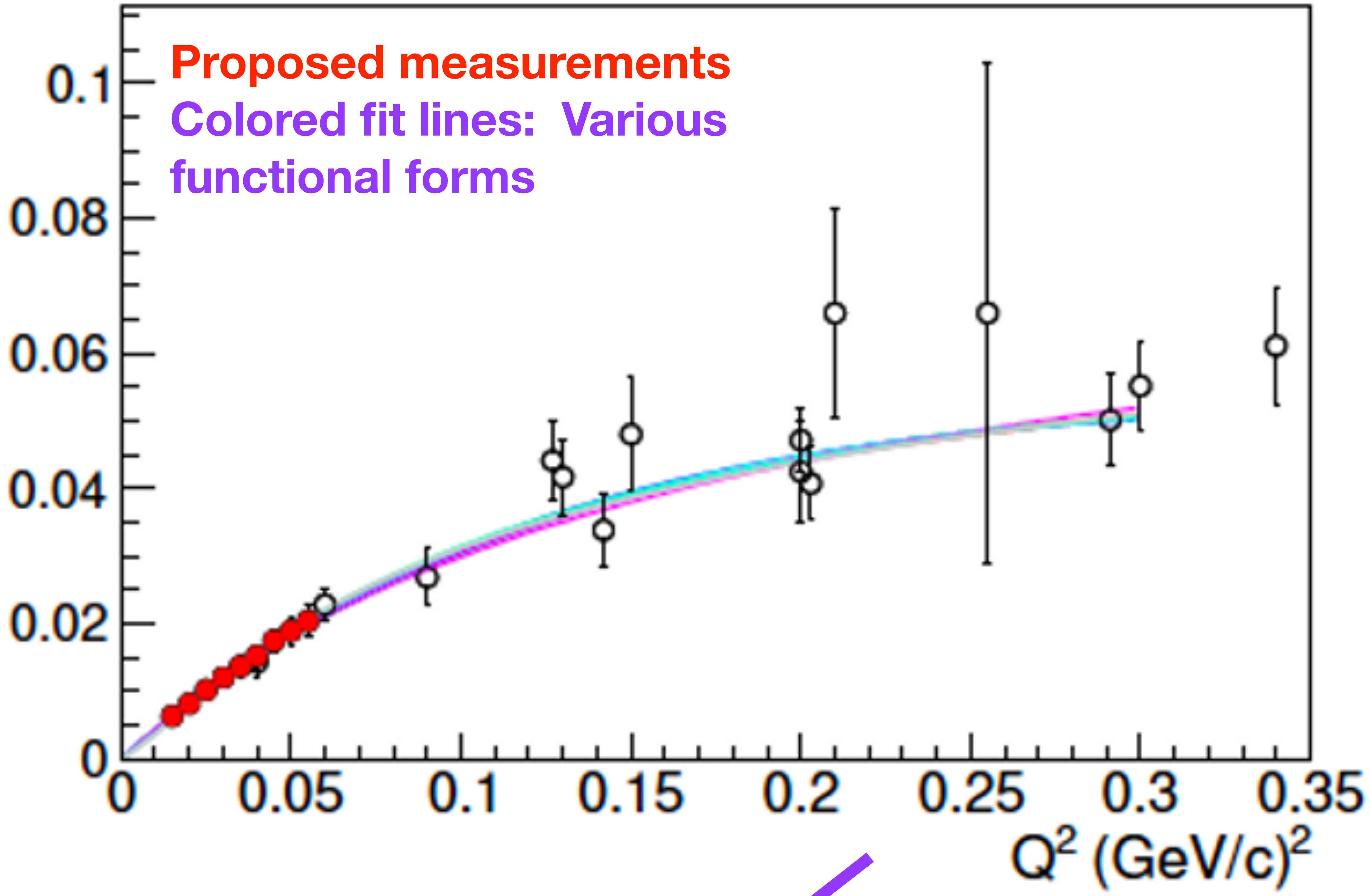


Collaboration has experience extracting N-Δ transition measurements from Hall-C data.

$\langle r_n^2 \rangle$ extraction through direct G_n^E fitting







Proposed measurements

Colored fit lines: Various functional forms

$$\langle r_n^2 \rangle = -6 \frac{dG_E^n(Q^2)}{dQ^2}\Bigg|_{Q^2 \to 0}$$

Extract radius through fit of slope as $Q^2 \to 0$

Projected precision: ~ 3.7% !!!

$\langle r_n^2 \rangle$ extraction through direct G_n^E fitting

$\langle r_n^2 \rangle$ depends on the derivative

$$\langle r_{\mathrm{n}}^2 \rangle = -6 \frac{dG_{\mathrm{E}}^{\mathrm{n}}(Q^2)}{dQ^2}\bigg|_{Q^2 \to 0}$$

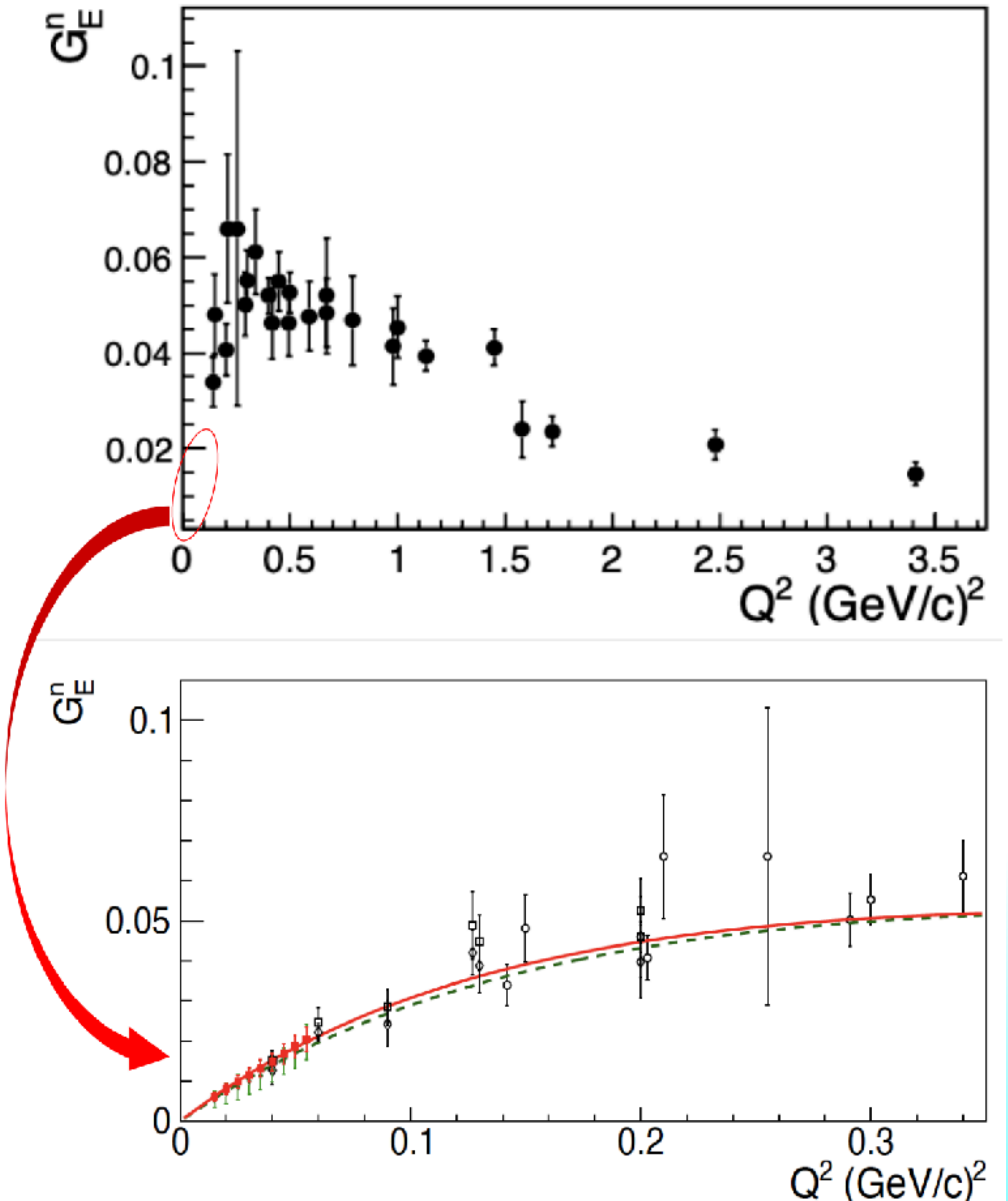
GEn rapidly changing
varies ~400% within
the measured Q² range

$\delta_{exp.}$ ~ 20%-25%

$\delta_{theoretical}$ ~15% → $\delta \langle r_n^2 \rangle$ =3.7%

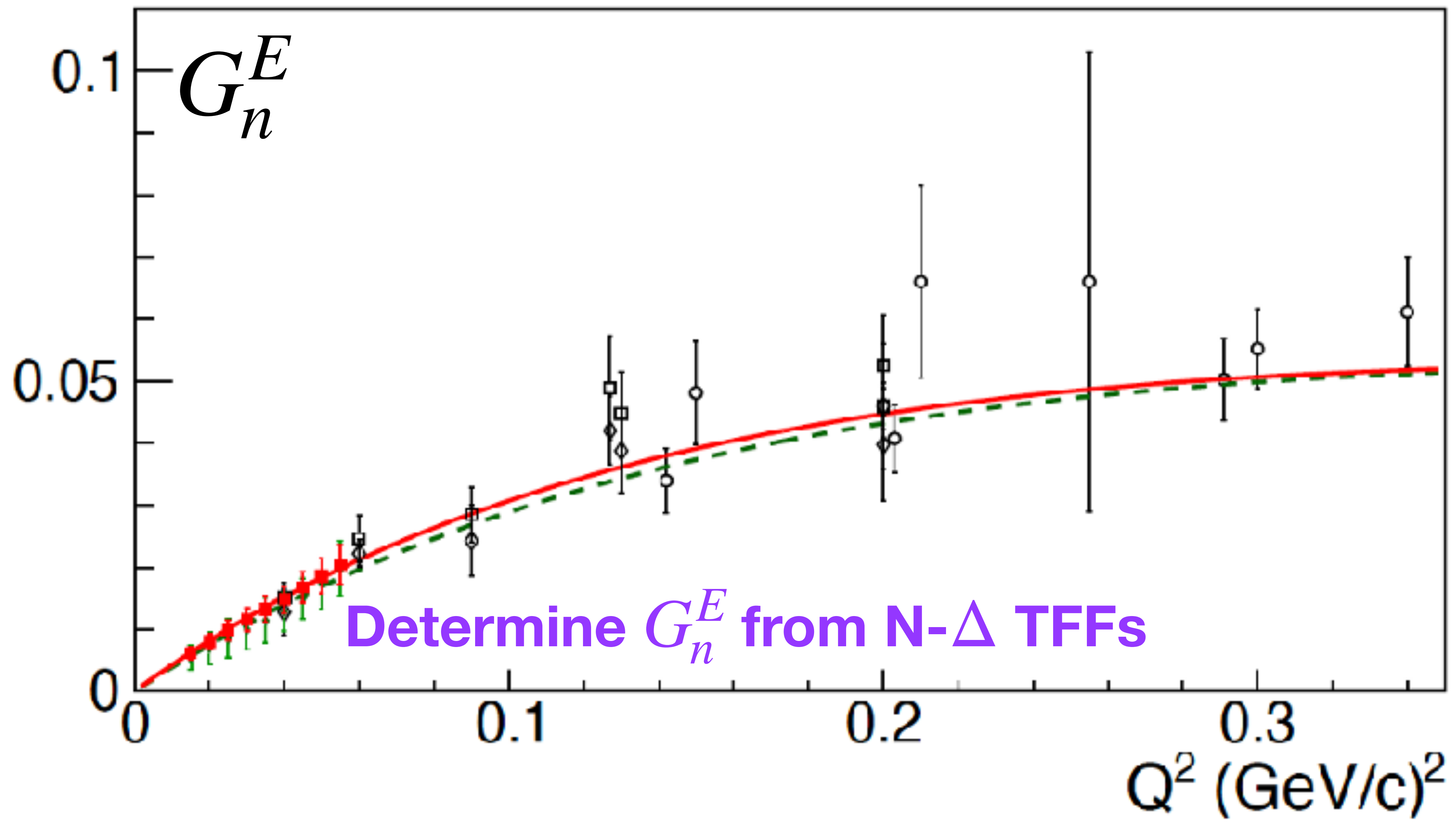
Inflating the
theoretical uncertainty:

$\delta_{theoretical}$ ~25% → $\delta \langle r_n^2 \rangle$ =4.3%



Increasing the theoretical uncertainty from 15 to 25% only increases the extraction uncertainty by 0.6%

$\langle r^2_{n,p}\rangle$ extraction and flavor decomposition



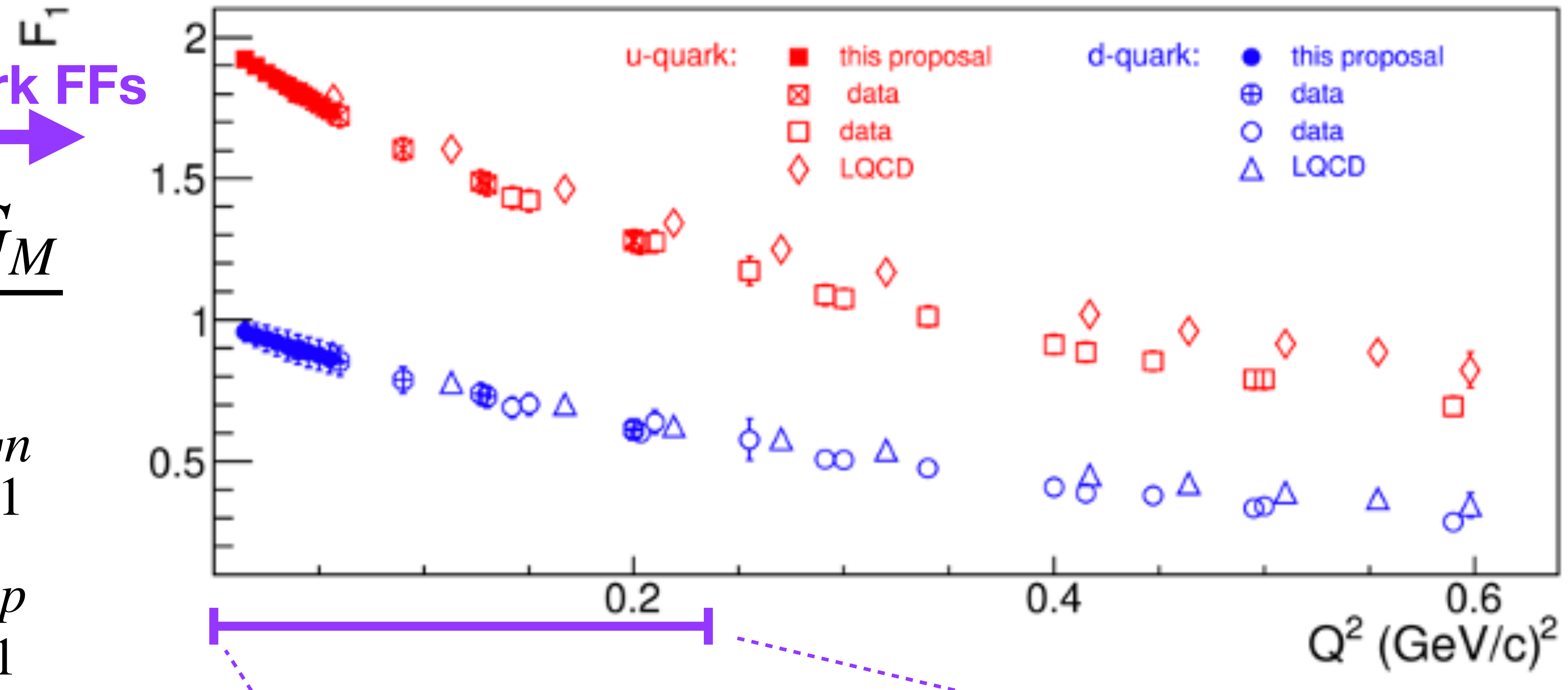
Determine G_n^E from N-Δ TFFs

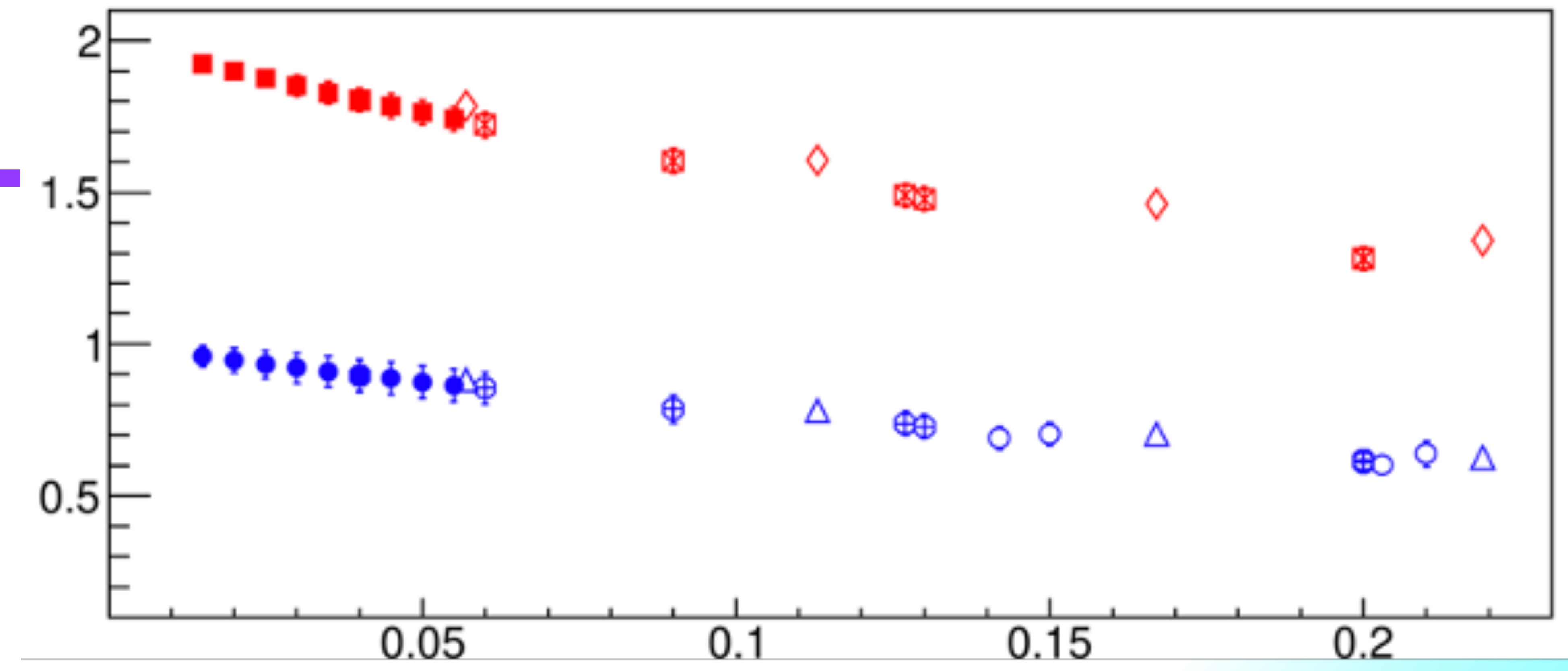
Decompose quark FFs

$$F_1 = \frac{G_E + \tau G_M}{1+\tau}$$

$$F_1^u = 2F_1^p + F_1^n$$

$$F_1^d = 2F_1^n + F_1^p$$





Determine transverse flavor radii

$$\langle b^2_{u(d)}\rangle = \frac{-4}{F_1^{u(d)}(0)} \frac{dF_1^{u(d)}(Q^2)}{dQ^2}\bigg|_{Q^2\to 0}$$

Recover nucleon radii

$$\langle r_p^2\rangle = 2\langle b_u^2\rangle - \frac{1}{2}\langle b_d^2\rangle + \frac{3}{2}\frac{\kappa_N}{M_N^2}$$

$$\langle r_n^2\rangle = \langle b_d^2\rangle - \langle b_u^2\rangle + \frac{3}{2}\frac{\kappa_N}{M_N^2}$$

Summary

- **Proposed: Measurement of the N-Δ TFFs in a mostly unmeasured region where the mesonic cloud dynamics is predicted to be dominant and rapidly changing**
  - Offers a test-bed for ChEFT and LQCD calculations
- **Proposed: A precise measurement (~3.7%) of the neutron charge radius.**
  - A very basic system property; sensitive to the internal structure & dynamics of the nucleon
  - Traditional method of extraction shows discrepancies which indicates unaccounted / underestimated systematics
    - PDG world data average value is elusive
    - Cross check with a different method ensures the honesty of the measurement and is a scientific obligation, whenever possible.
- **Resolve the long-standing neutron-electron scattering length discrepancies**
  - Important in setting constraints for the existence of new forces in nature
- **Direct extraction of the u- and d-quark distributions TMSR**
- **Request:**
  - 9.5 days
  - Beam energy: 1.3 GeV (flexible within +/- 0.1 GeV)
  - Hall C standard setup

Thank you!