what quantities do we want to compute ?

scattering amplitudes !

not obvious how, try something simpler: **energy spectrum**

two-point correlation functions and the spectrum

consider $\langle 0 | \, \mathcal{O}_\mathrm{f}(t) \, \mathcal{O}_\mathrm{i}^\dagger(0) \, | 0 \rangle$

Euclidean time-evolution $\mathcal{O}(t) = e^{Ht} \, \mathcal{O}(0) \, e^{-Ht}$

$$= \langle 0 | \, \mathcal{O}_\mathrm{f}(0) \, e^{-Ht} \, \mathcal{O}_\mathrm{i}^\dagger(0) \, | 0 \rangle$$

Hamiltonian has a complete set of eigenstates $H|\mathfrak{n}\rangle = E_\mathfrak{n}|\mathfrak{n}\rangle$

$$1 = \sum_\mathfrak{n} |\mathfrak{n}\rangle\langle\mathfrak{n}|$$

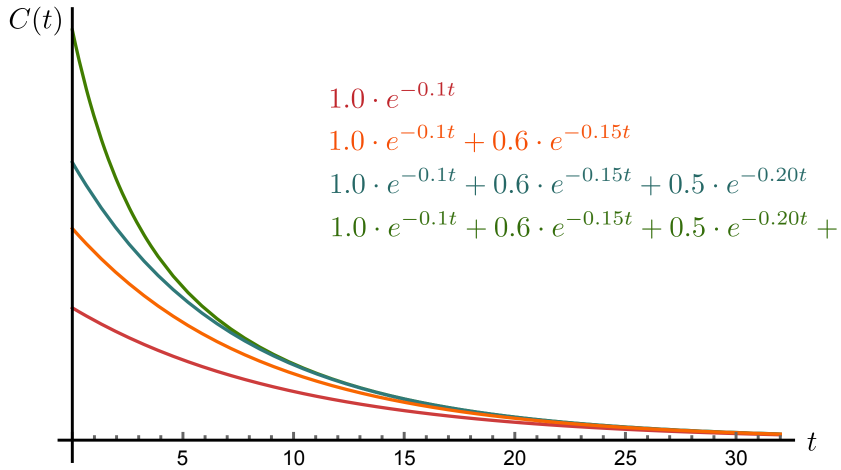
(only discrete eigenstates ?)

$$= \sum_\mathfrak{n} e^{-E_\mathfrak{n} t} \langle 0 | \mathcal{O}_\mathrm{f}(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_\mathrm{i}^\dagger(0) | 0 \rangle$$

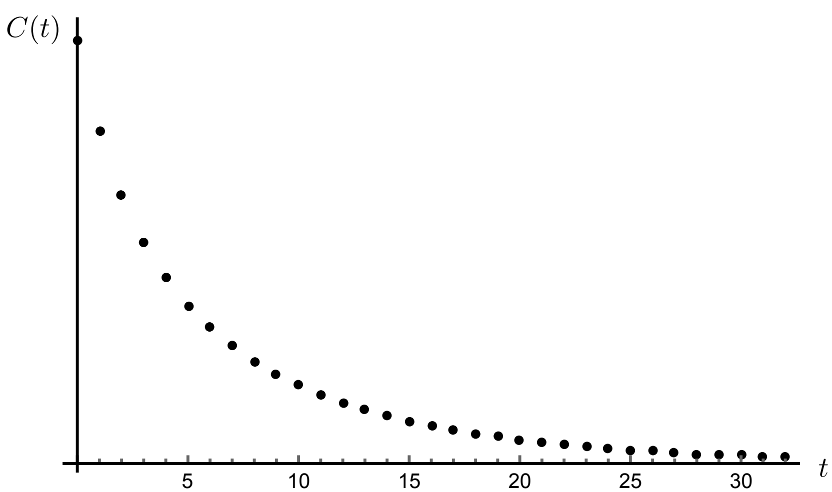
amplitude for $\mathcal{O}_\mathrm{i}^\dagger$ to ‘interpolate’ state $|\mathfrak{n}\rangle$ from the vacuum

time dependence of two point correlation functions

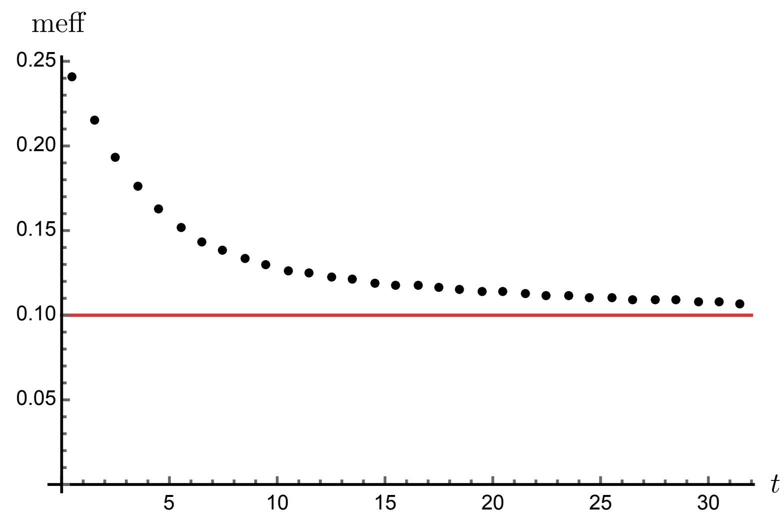
diagonal correlation function



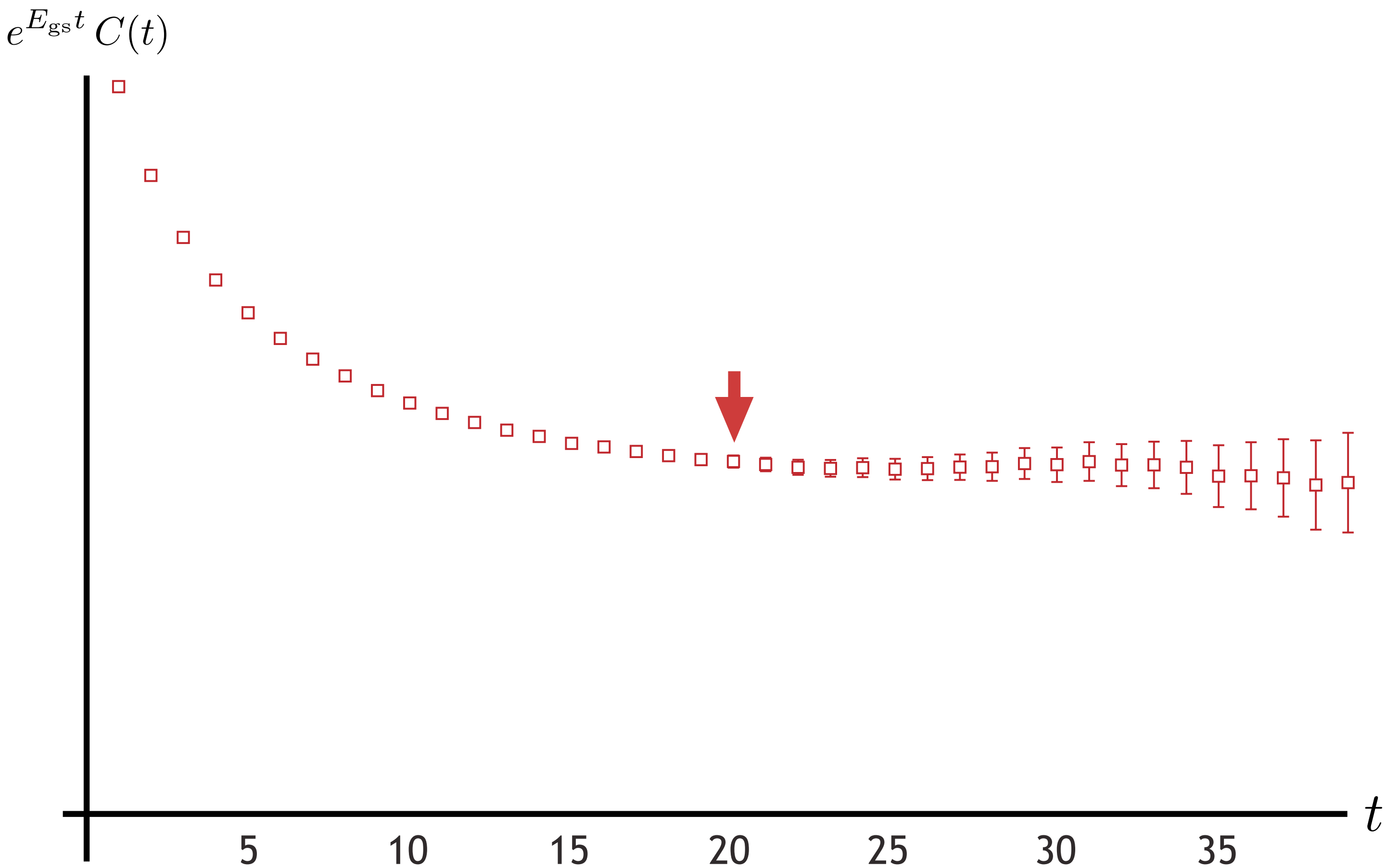
notice that as $t \to \infty$ $\qquad C(t) \to c \cdot e^{-E_{\mathrm{gs}} t}$



useful to define the 'effective mass' $\log \left[\dfrac{C(t)}{C(t+1)} \right]$

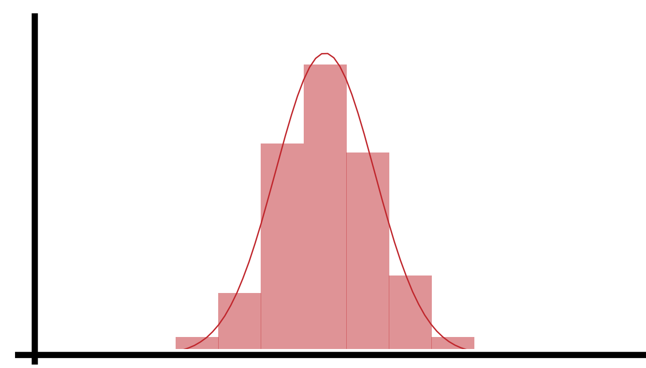


an actual lattice QCD two point correlation function



generally statistical noise increases with increasing t

ensemble dist<u>n</u>



noise growth — simplistic argument

at large times, the signal $\langle O(t)\, O(0) \rangle \sim e^{-Mt}$ for lightest state with mass *M*

the variance is the mean of the square $\big\langle [O(t)O(t)]\; [O(0)O(0)] \big\rangle$

and the operator *O* · *O* will have
a 0^+ component which overlaps with *ππ*

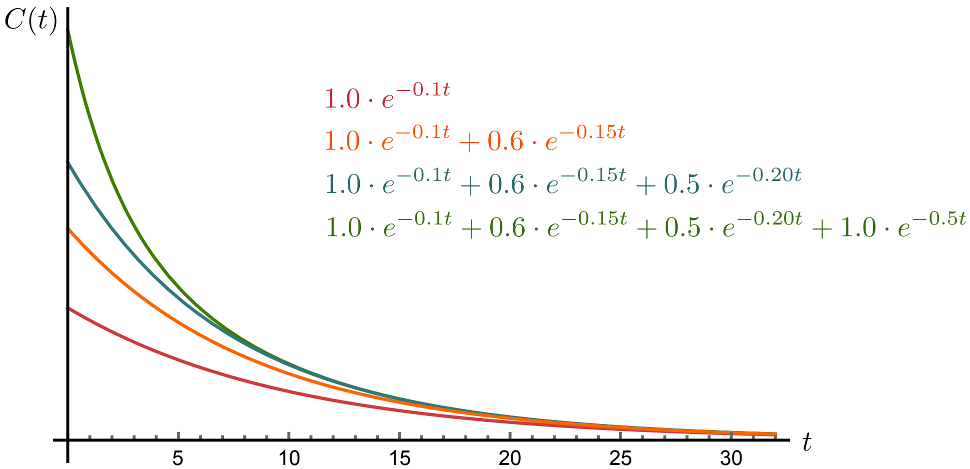
$$\big\langle [O(t)O(t)]\; [O(0)O(0)] \big\rangle \sim e^{-2m_\pi t}$$

$$\Rightarrow \quad \frac{\text{noise}}{\text{signal}} \sim e^{(M-m_\pi)t}$$

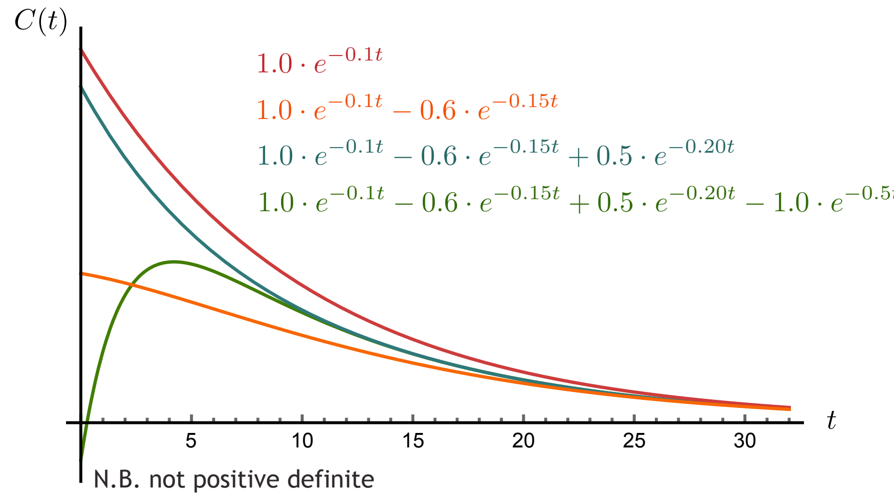
so for everything except pion correlators,
expect the noise to grow with *t*

time dependence of two point correlation functions

diagonal correlation function



off-diagonal correlation function



suggests non-linear fitting to a sum of exponentials …

? how many exponentials ?

? what if there are (near) degenerate states ?

… actually getting reliable results this way
for anything more than the ground state proves impractical …

extracting the excited spectrum

a more powerful approach makes use of a **basis of operators** $\{O_1, O_2, O_3, \ldots\}$

there should be a **linear combination** which **optimally produces the ground-state**
and **another** which **optimally produces the first-excited-state**
etc ...

$$\Omega_{\mathfrak{n}}^\dagger = \sum_i v_i^{(\mathfrak{n})} O_i^\dagger$$

how do we find these optimizing weights ?

‘variational’ approach

$$\Omega_{\mathfrak{n}}^\dagger = \sum_i v_i^{(\mathfrak{n})} O_i^\dagger \qquad \Omega_{\mathfrak{n}}^\dagger |0\rangle = |\mathfrak{n}\rangle + \sum_{\mathfrak{m}\neq\mathfrak{n}} \epsilon_{\mathfrak{m}} |\mathfrak{m}\rangle$$

with the $\epsilon_{\mathfrak{m}}$ as small as possible

‘optimal’ correlation function

$$\langle 0 | \Omega_{\mathfrak{n}}(t)\, \Omega_{\mathfrak{n}}^\dagger(0) | 0 \rangle = e^{-E_{\mathfrak{n}} t} + \sum_{\mathfrak{m}\neq\mathfrak{n}} |\epsilon_{\mathfrak{m}}|^2 e^{-E_{\mathfrak{m}} t}$$

minimize this

$$= \sum_{ij} v_i^* \langle 0 | O_i(t)\, O_j^\dagger(0) | 0 \rangle v_j = \sum_{ij} v_i^* C_{ij}(t)\, v_j$$

by varying the v_i

can avoid the trivial minimum ($v_i=0$) by fixing normalization $\sum_{ij} v_i^* C_{ij}(t_0)\, v_j = 1$

this choice will become clearer later

implement constraint via a Lagrange multiplier

minimize $\Lambda = \sum_{ij} v_i^* C_{ij}(t) v_j - \lambda\Big[\sum_{ij} v_i^* C_{ij}(t_0) v_j - 1\Big]$

⇒ generalized eigenvalue problem $\mathbf{C}(t)\mathbf{v} = \lambda(t)\mathbf{C}(t_0)\mathbf{v}$

generalized eigenvalue problem

$$\mathbf{C}(t)\mathbf{v} = \lambda(t)\mathbf{C}(t_0)\mathbf{v}$$

eigenvalues, *a.k.a* **principal correlators** $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

eigenvectors, *a.k.a* **optimized operator weights** $\Omega^\dagger_{\mathfrak{n}} = \sum_i v_i^{(\mathfrak{n})}\, O_i^\dagger$

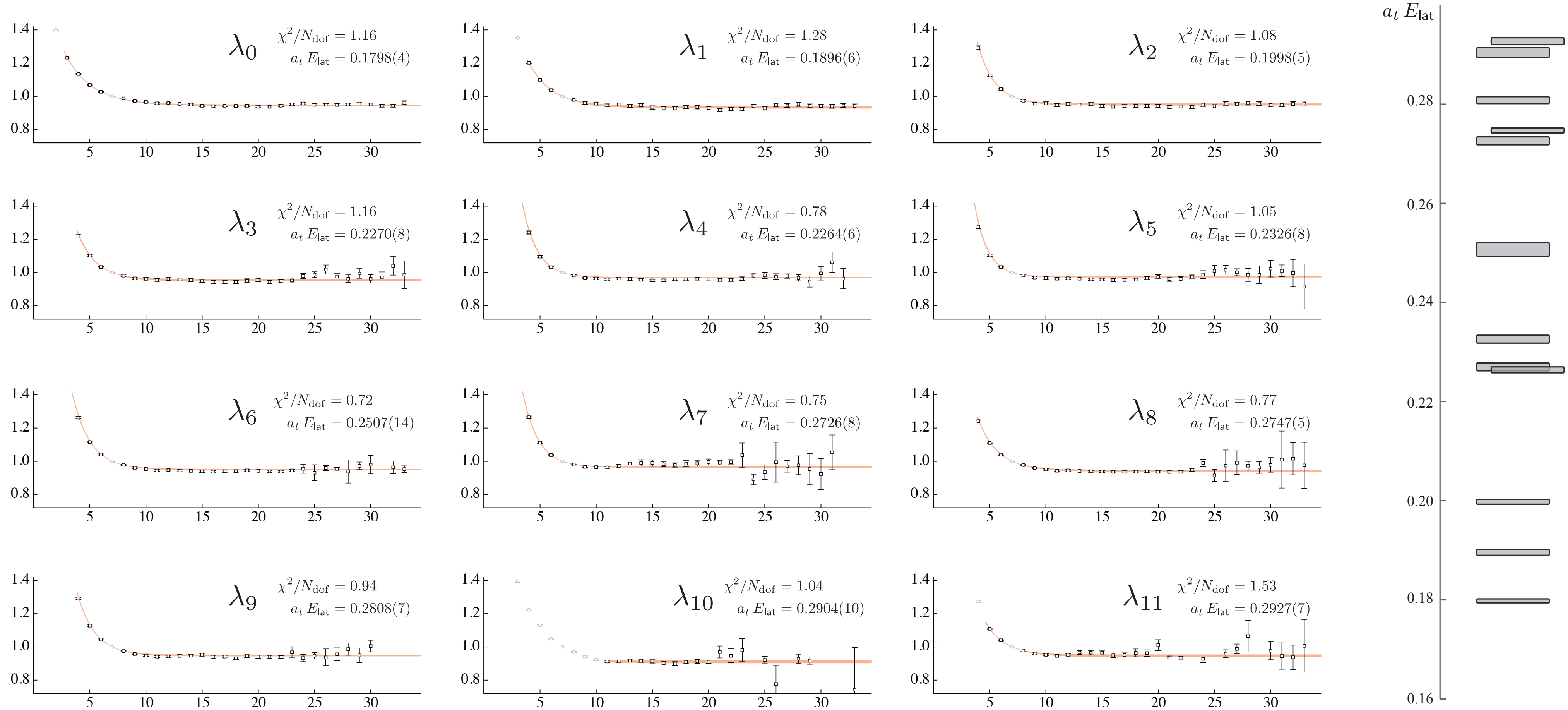
eigenvector orthogonality $\mathbf{v}^{(\mathfrak{n})} \cdot \mathbf{C}(t_0) \cdot \mathbf{v}^{(\mathfrak{m})} = \delta_{\mathfrak{n},\mathfrak{m}}$

things to think about:

? how do you select the value of t_0 ?

? is there some limit in which this gives the exact answer ?

principal correlators



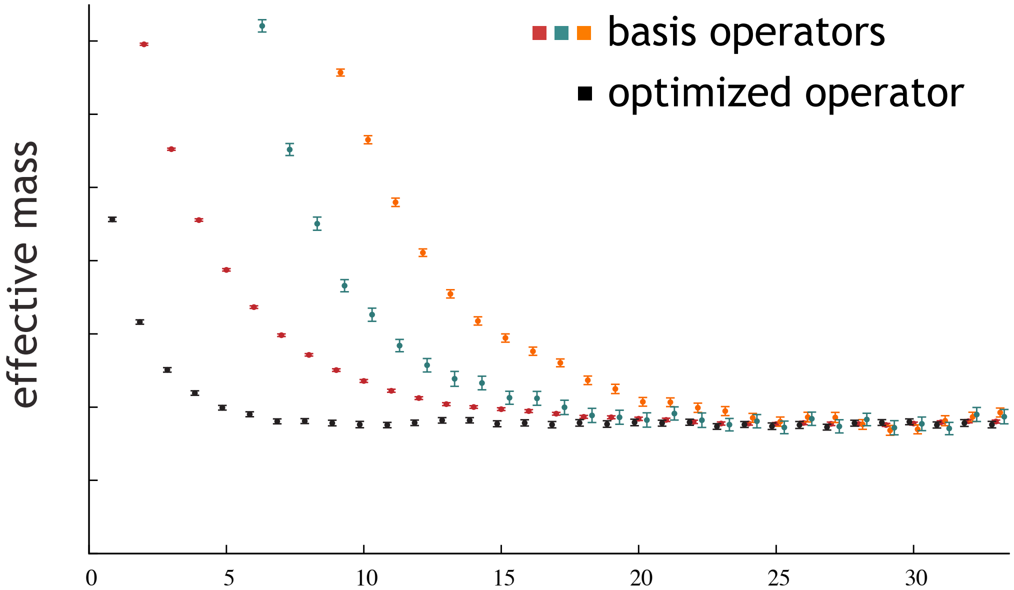
fitted with

$$\lambda_n(t) = (1 - A_n)\,e^{-E_n(t-t_0)} + A_n\,e^{-E'_n(t-t_0)}$$

with $A_n \ll 1$ and $E'_n \gg E_n$

optimized operator construction

[000] A_1 diagonal correlators



optimized operator saturated
by the pion by timeslice 7

ok, so it looks like we should be able to compute spectra, but we wanted **scattering amplitudes !**

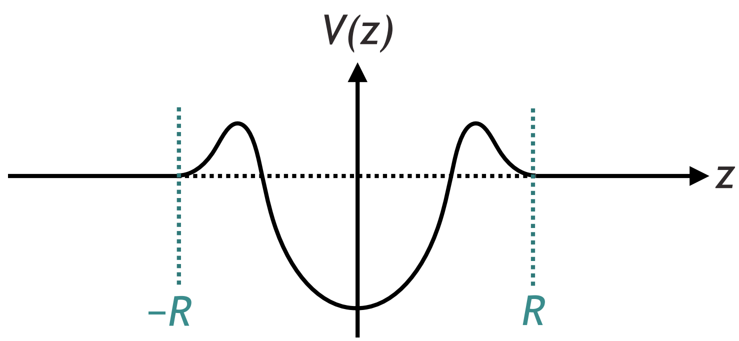
scattering in a finite volume ?

lattice defines a **(periodic) spatial volume** — usually a cube, side length a few fermi

spectra are discrete in a finite volume — no scattering continuum ?

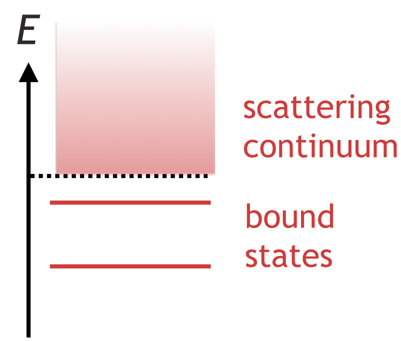
let's get a conceptual picture by returning to our one-dim quantum mechanics problem

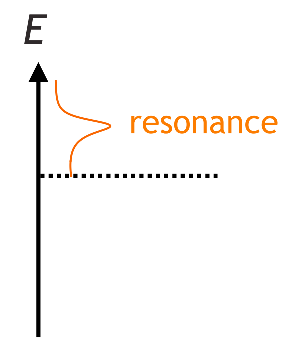
scattering



solve the Schrödinger equation

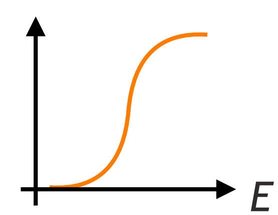
$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$





$$\psi(|z| > R) \sim \cos\left(p\,|z| + \delta(p)\right)$$

phase-shift



‘scattering’ in a finite-volume

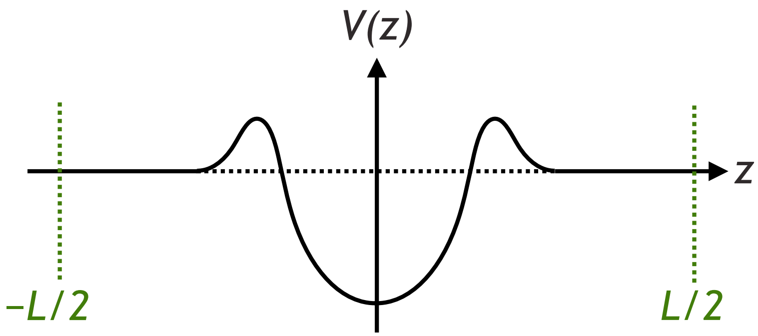
now put the system in a ‘box’ — periodic boundary condition at $z = \pm L/2$

$$\psi(|z| > R) \sim \cos\big(p\,|z| + \delta(p)\big)$$

$$\psi(L/2) = \psi(-L/2)$$
$$\frac{d\psi}{dz}(L/2) = \frac{d\psi}{dz}(-L/2)$$

momentum quantization condition

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$



for elastic scattering in a cube the corresponding relationship is $\cot\delta_\ell(E) = \mathcal{M}_\ell\big(E(L), L\big)$

in the simplest case of a single partial wave being non-zero

Lüscher 1986

⋮

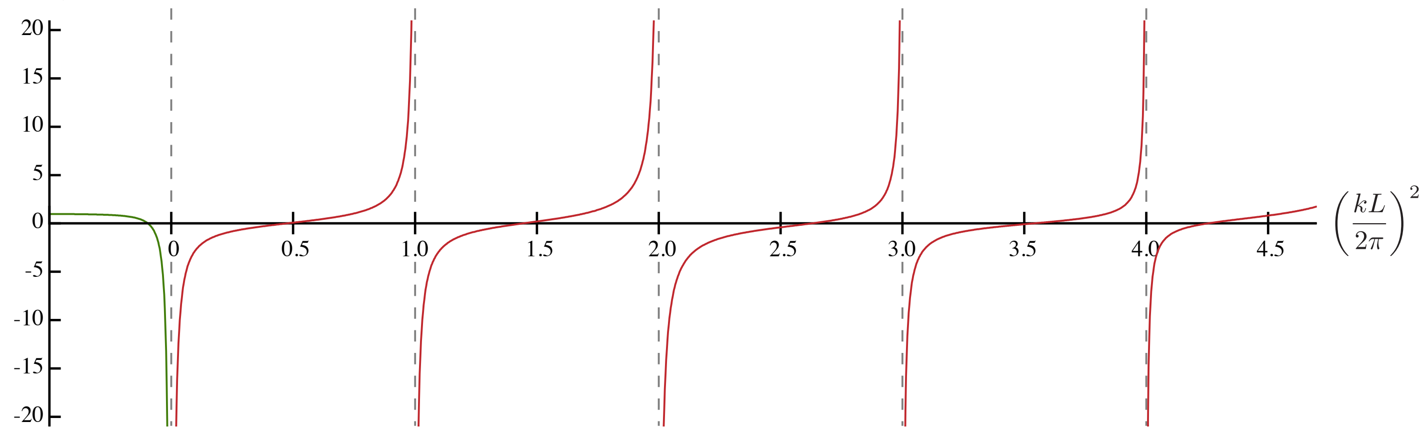
many subsequent works
see the RMP for a complete list

will present some complications later ...

$$\cot\delta_\ell(E) = \boxed{\mathcal{M}_\ell\big(E(L), L\big)}$$

known function expressing the 'kinematics' of the finite-volume

e.g. $\mathcal{M}_0$



$k = \frac{1}{2}\sqrt{E^2 - 4m^2}$

so find the intersections of this curve with $\cot\delta(E)$

finite-volume functions

X

$$\mathcal{M}_{\ell m,\ell' m'}(E,L) = \frac{1}{\pi^{3/2}} \sum_{\bar{\ell},\bar{m}} \sqrt{\frac{(2\ell+1)(2\bar{\ell}+1)}{(2\ell'+1)}} \langle \ell m; \bar{\ell}\bar{m} | \ell' m' \rangle \langle \ell 0; \bar{\ell} 0 | \ell' 0 \rangle \left(\tfrac{2\pi}{kL}\right)^{\bar{\ell}+1} Z_{\bar{\ell}\bar{m}}\left(s\to 1; \left(\tfrac{kL}{2\pi}\right)^2\right)$$

Lüscher zeta function

$$Z_{\ell m}\left(s; \left(\tfrac{kL}{2\pi}\right)^2\right) = \sum_{\mathbf{n}} \frac{|\mathbf{n}|^{\ell}\, Y_{\ell m}(\hat{\mathbf{n}})}{\left[|\mathbf{n}|^2 - \left(\tfrac{kL}{2\pi}\right)^2\right]^s}$$

$$\mathbf{n} = [n_x, n_y, n_z]$$

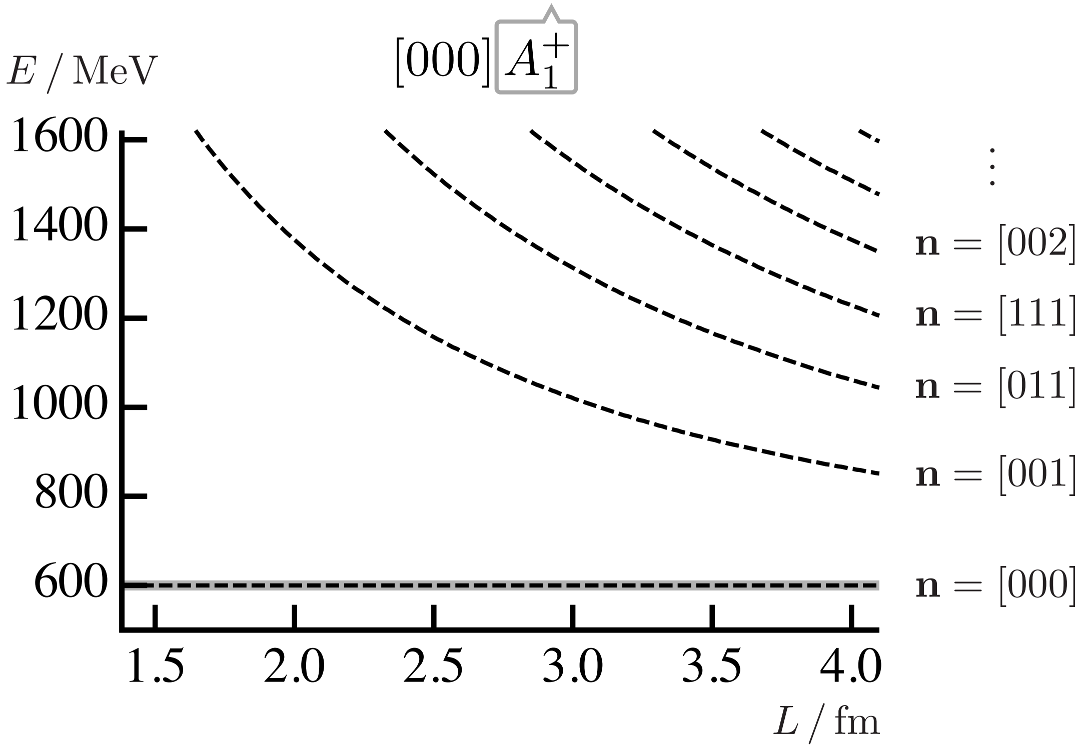
e.g. $\mathcal{M}_{00,00} = \dfrac{1}{\pi}\dfrac{1}{kL} \lim_{s\to 1^+} \sum_{\mathbf{n}} \dfrac{1}{\left[|\mathbf{n}|^2 - \left(\frac{kL}{2\pi}\right)^2\right]^s}$

will have a divergence at each $k^2 = \left(\dfrac{2\pi}{L}|\mathbf{n}|\right)^2$

i.e. at the non-interacting energies

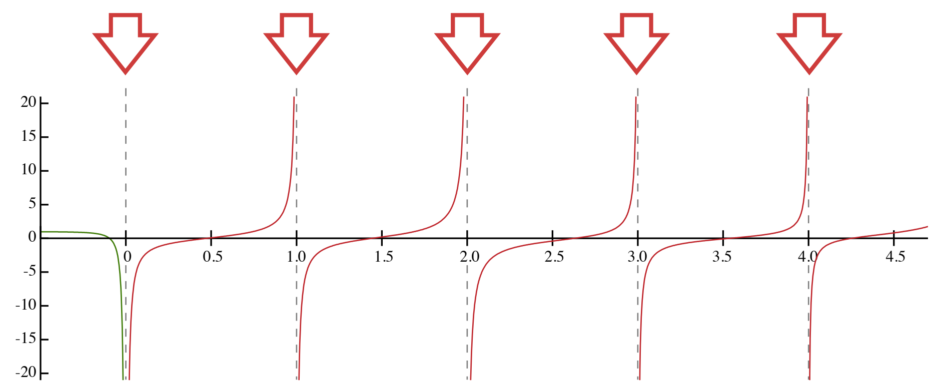
scattering particles m = 300 MeV

for now, think of this as S-wave



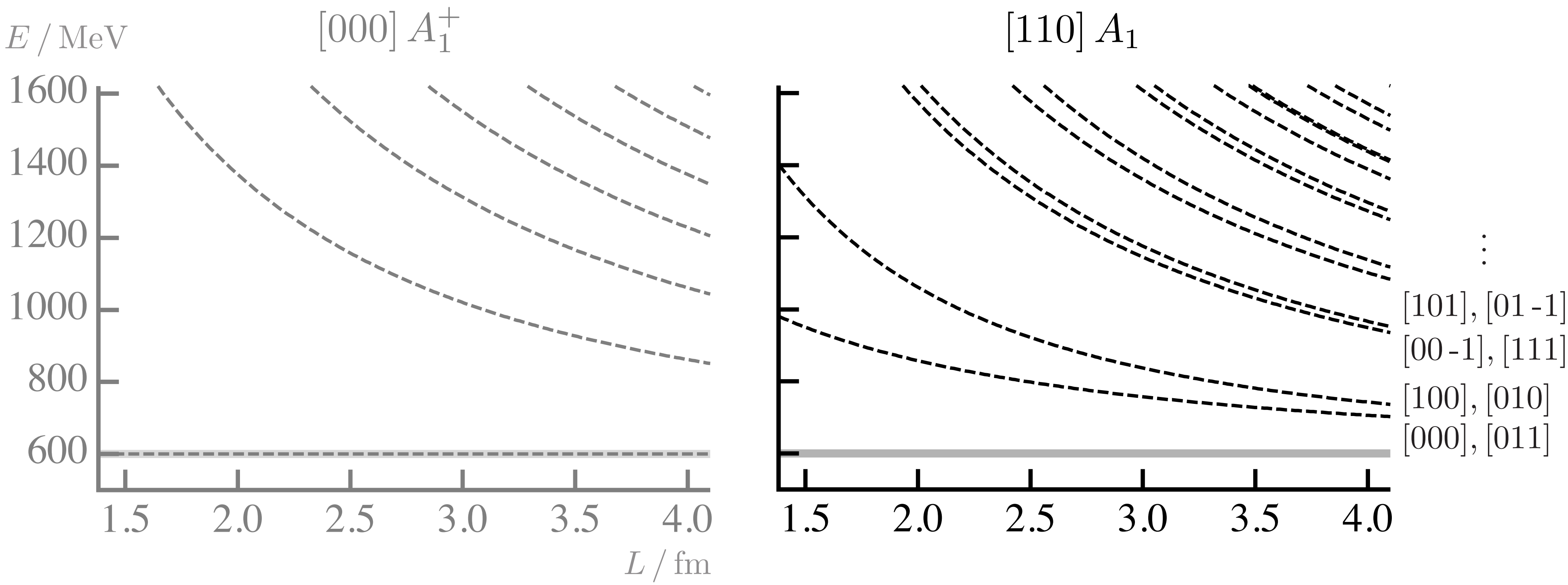
$$E_{\rm ni} = 2\sqrt{m^2 + \mathbf{p}^2} \qquad \mathbf{p} = \frac{2\pi}{L}\mathbf{n}$$

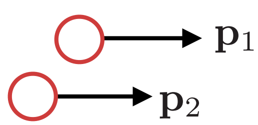
$$\cot\delta(E) = \mathcal{M}\big(E(L), L\big) \qquad \delta \to 0,\ \cot\delta \to \infty$$



divergences correspond to non-interacting energies

no interaction – in a moving frame





$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$$

$$E_{\text{ni}} = \sqrt{m^2 + \mathbf{p}_1^2} + \sqrt{m^2 + \mathbf{p}_2^2}$$

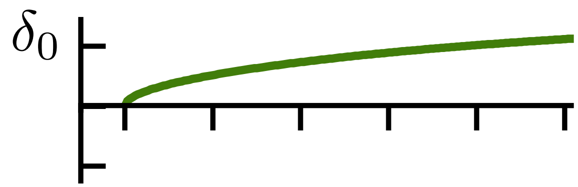
$$\mathbf{P} = \frac{2\pi}{L}\mathbf{n}$$

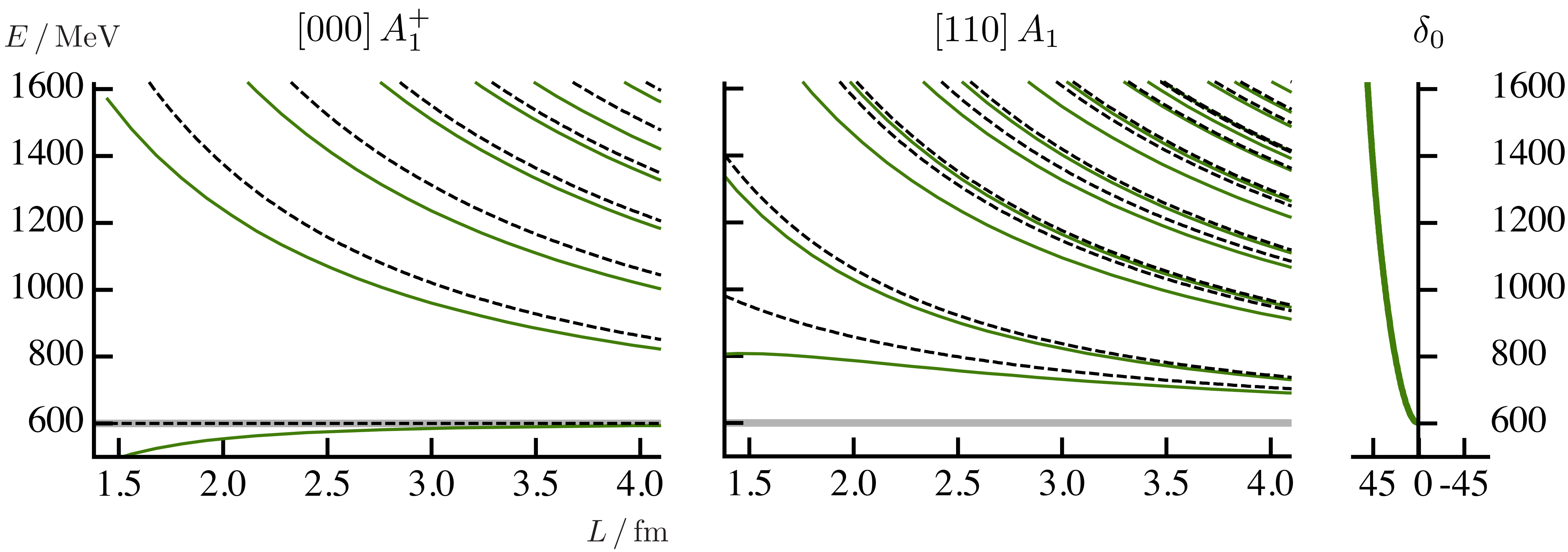
$$\mathbf{p}_{1,2} = \frac{2\pi}{L}\mathbf{n}_{1,2}$$

in the cm frame $E_{\text{cm}} = \sqrt{E^2 - \mathbf{P}^2}$

weak attraction

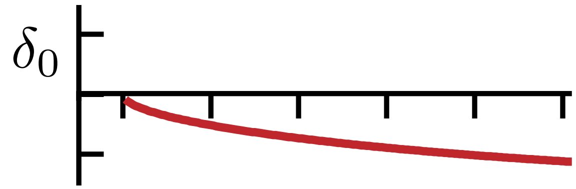
find the finite-volume spectrum for a **fixed phase-shift**

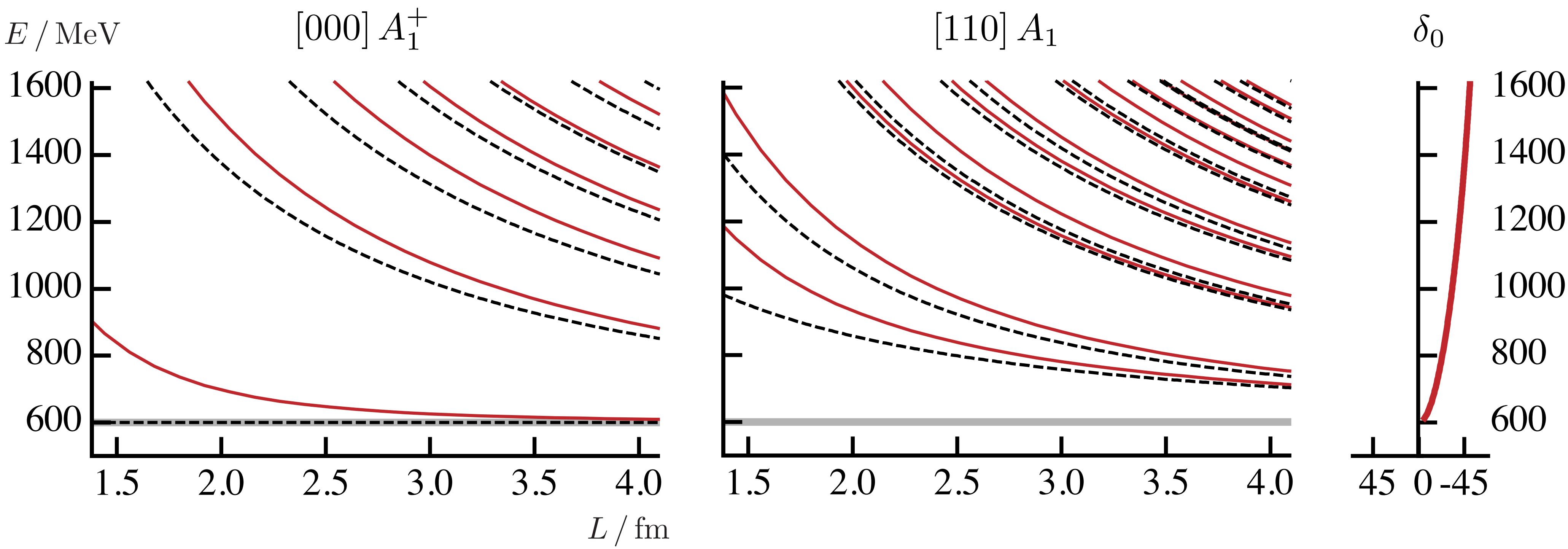




weak repulsion

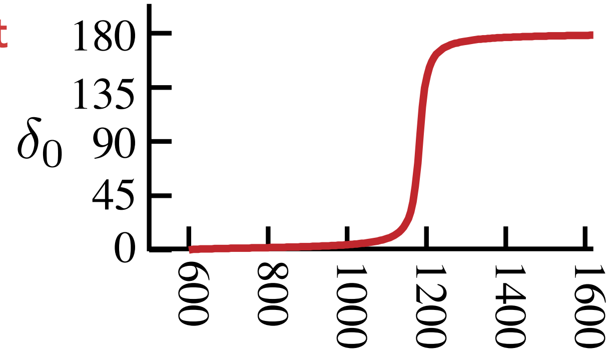
find the finite-volume spectrum for a **fixed phase-shift**





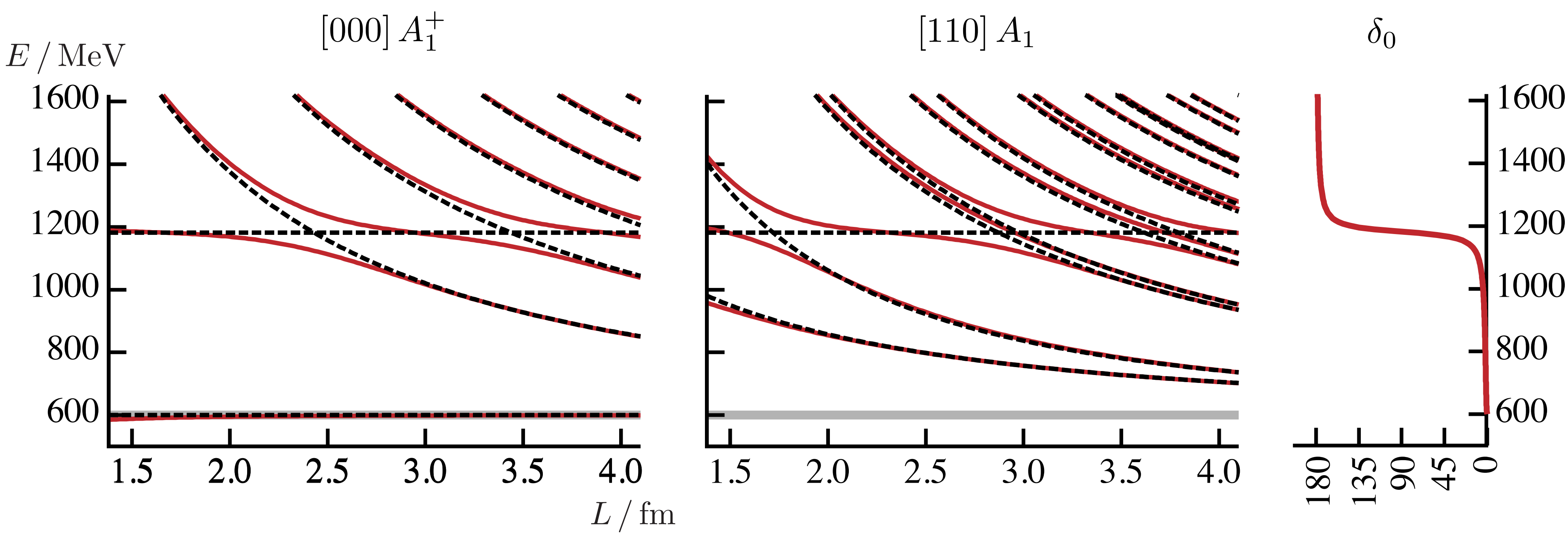
an elastic resonance

find the finite-volume spectrum for a **fixed phase-shift**

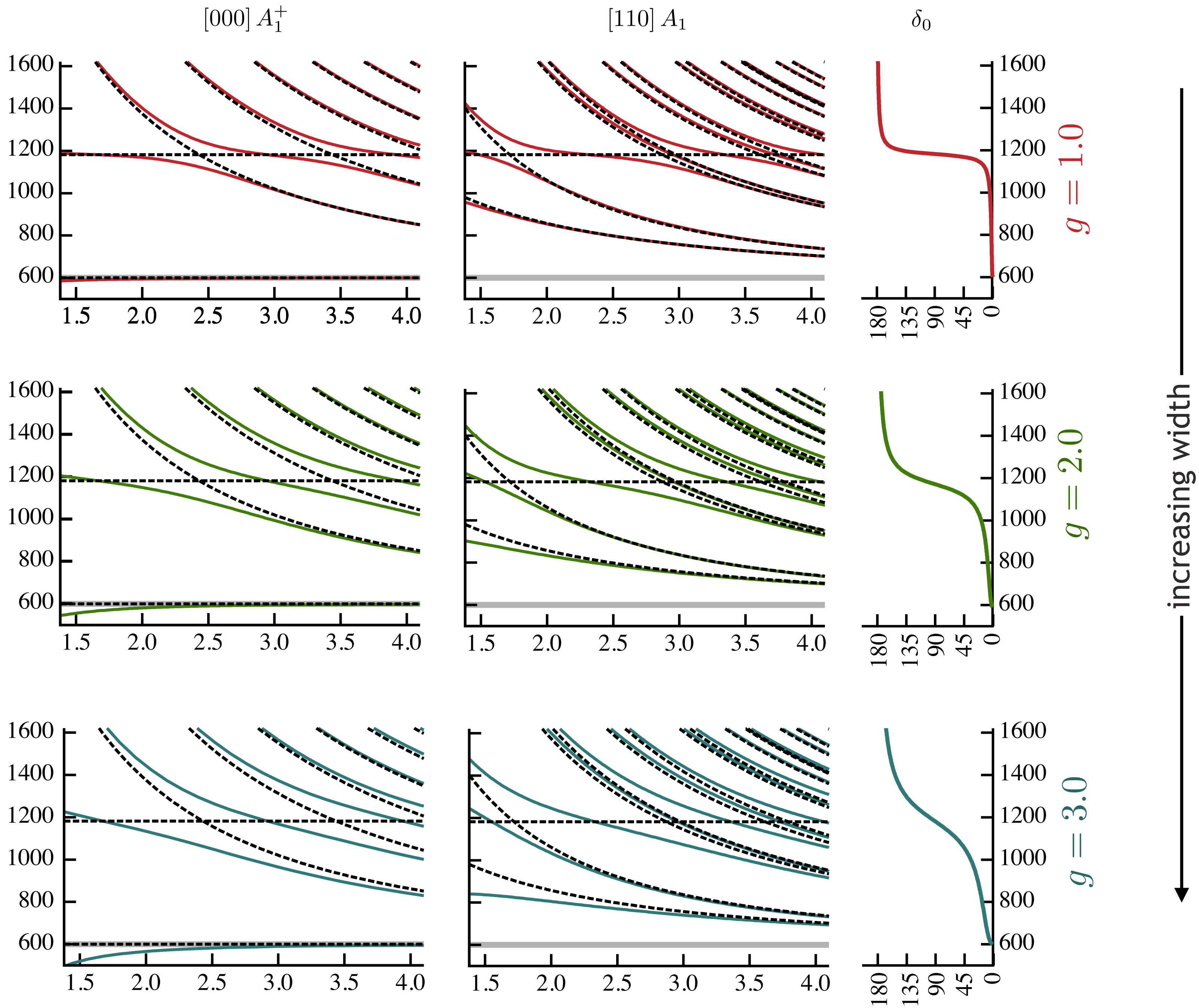


$$\tan\delta = \frac{E\,\Gamma(E)}{m_R^2 - E^2}$$

$$\Gamma(E) = \frac{g^2}{6\pi}\frac{m_R^2}{E^2}k(E)$$

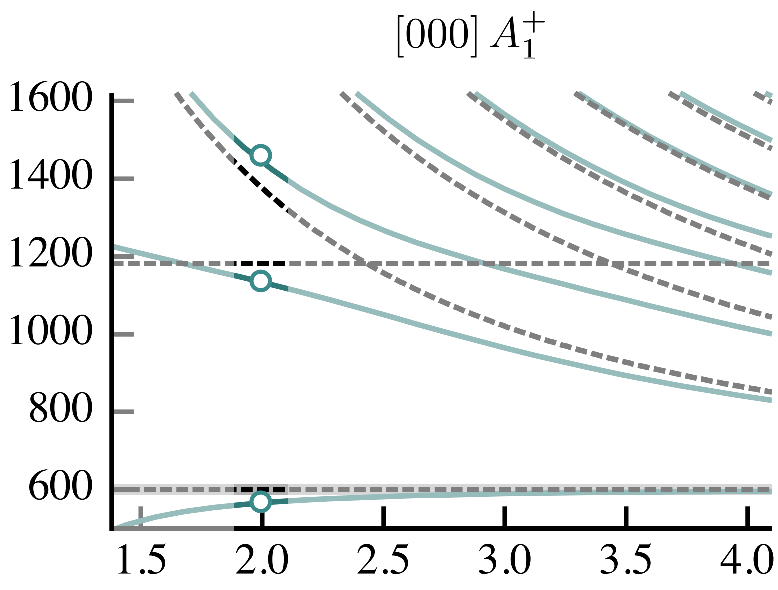


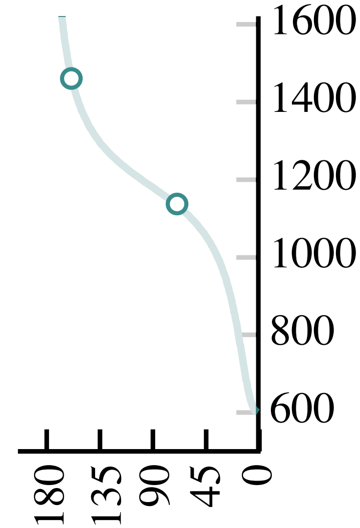
note the avoided level crossings



$$\Gamma(E) = \frac{g^2}{6\pi}\frac{m_R^2}{E^2}k(E)$$

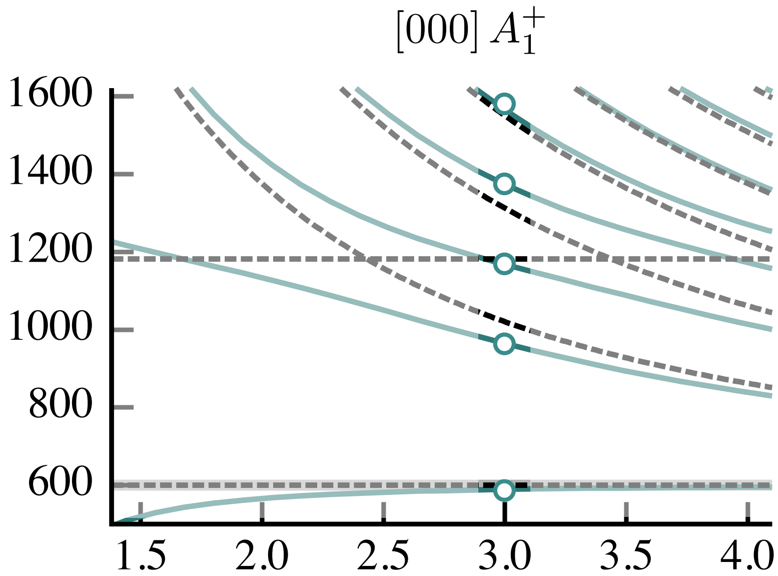
what about the reverse process — obtain the phase-shift from the finite-volume spectrum ?

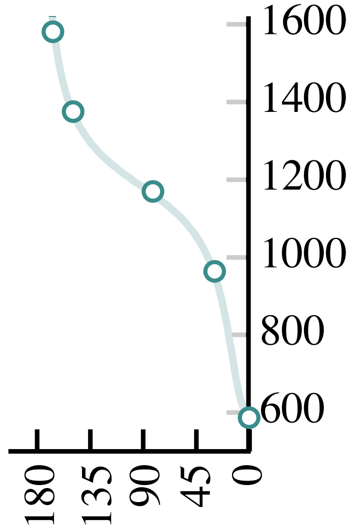




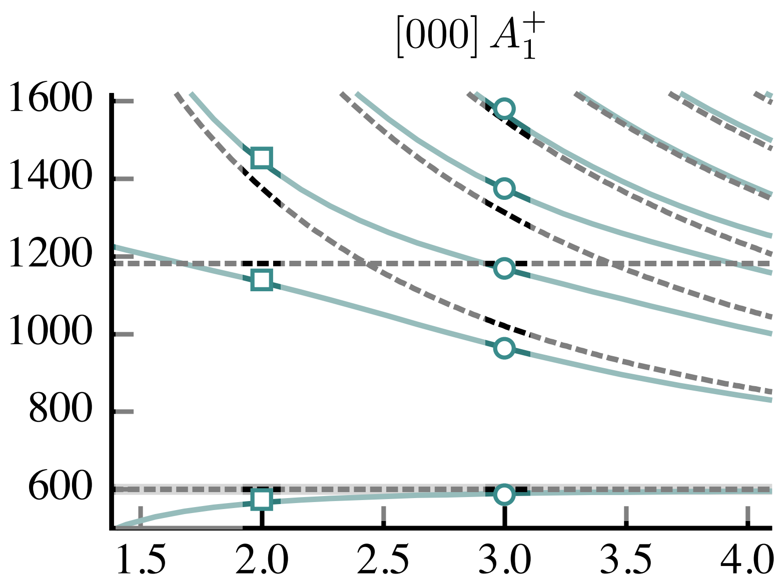
an elastic resonance — finite-volume mapping

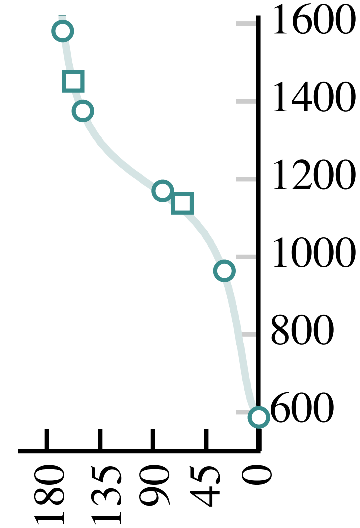
what about the reverse process — obtain the phase-shift from the finite-volume spectrum ?





what about the reverse process — obtain the phase-shift from the finite-volume spectrum ?

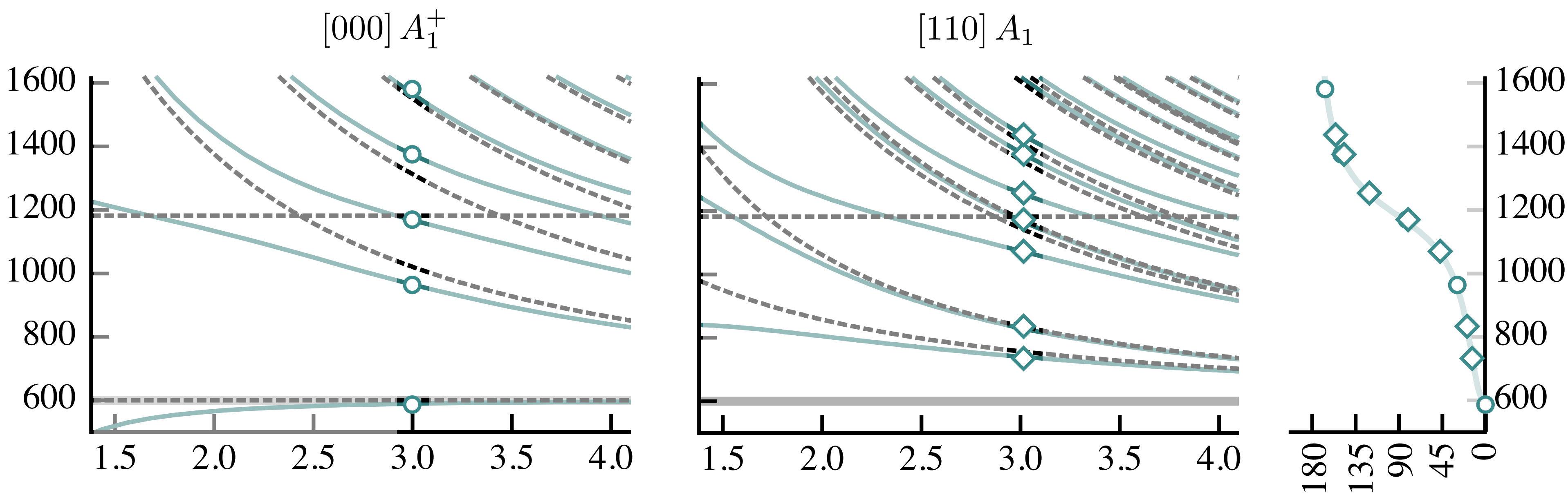




more volumes give more information,

but each new volume is a completely new lattice calculation
and hence very computationally costly

an elastic resonance — finite-volume mapping



determining the **moving-frame spectrum** provides much more information

it looks like given enough finite volume energies,
we can reconstruct the elastic scattering phase-shift …

some annoying technical stuff — breaking of rotational symmetry

a **finite cubic lattice** has a **smaller rotational symmetry group** than an infinite continuum

simpler example of the problem: a rotationally symmetric two-dim system $\psi(r,\theta) = R_m(r)\, e^{im\theta}$

now considered on a square grid — minimum rotation is by $\pi/2$

m and $m+4n$ transform the same !

back in 3D — **irreducible representations** of the reduced symmetry group contain multiple spins

cubic symmetry

$\Lambda(\text{dim})$	$A_1(1)$	$T_1(3)$	$T_2(3)$	$E(2)$	$A_2(1)$
J	$\mathbf{0}, 4\ldots$	$\mathbf{1}, 3, 4\ldots$	$\mathbf{2}, 3, 4\ldots$	$\mathbf{2}, 4\ldots$	$\mathbf{3}\ldots$

subduction $|\Lambda, \rho\rangle = \sum_m S^{\Lambda,\rho}_{J,m}\, |J, m\rangle$

for non-zero momentum it's even worse
— in continuum have **little group**, those rotations which don't change ***p***

⇒ label by **helicity**

can subduce helicity states into irreps of the reduced cubic symmetry

PRD85 014507 (2012)

some annoying technical stuff — breaking of rotational symmetry

reduction of rotational symmetry is an important feature of the quantization condition too

for elastic scattering, what we previously presented as $\cot\delta_\ell(E) = \mathcal{M}_\ell\big(E(L), L\big)$

should actually be $0 = \det\Big[\cot\delta_\ell\,\delta_{\ell,\ell'}\,\delta_{m,m'} - \mathcal{M}_{\ell m;\ell' m'}\Big]$

which when subduced becomes $0 = \det\Big[\cot\delta_\ell\,\delta_{\ell,\ell'}\,\delta_{n,n'} - \mathcal{M}^{\Lambda}_{\ell n;\ell' n}\Big]$

features **all ℓ** subduced into irrep Λ

n = embedding of ℓ into Λ

e.g. [000] A_1

$$0 = \det\left[\begin{pmatrix}\cot\delta_0(E) & 0 & \cdots \\ 0 & \cot\delta_4(E) & \cdots \\ \vdots & \vdots & \ddots\end{pmatrix} - \begin{pmatrix}\mathcal{M}^{A_1}_{01;01}(E,L) & \mathcal{M}^{A_1}_{01;41}(E,L) & \cdots \\ \mathcal{M}^{A_1}_{41;01}(E,L) & \mathcal{M}^{A_1}_{41;41}(E,L) & \cdots \\ \vdots & \vdots & \ddots\end{pmatrix}\right]$$

what allows us to make progress is that $\delta_\ell(E) \sim k^{2\ell+1}$ at energies not too far from threshold

so higher angular momenta are naturally suppressed

in practice, truncate at some ℓ_{max} …

where are we ... ?

matrix of correlation functions → finite volume spectra → elastic scattering amplitudes

$$\langle 0 | O_i(t)\, O_j^\dagger(0) | 0 \rangle$$

but what operator basis $\{O_i\}_{i=1\ldots N}$ should we use ?

must be constructed
out of quark/gluon fields

easiest constructions with meson quantum numbers — **fermion bilinears** $\bar{\psi}\boldsymbol{\Gamma}\psi$

well motivated by
success of quark model

'looks' like a $q\bar{q}$ system

Γ = Dirac gamma
+ gauge-covariant derivatives

meson operators

easiest constructions with meson quantum numbers — **fermion bilinears** $\bar{\psi}\mathbf{\Gamma}\psi$

well motivated by
success of quark model

'looks' like a $q\bar{q}$ system

Wick
contractions

'annihilation'
required for isospin=0

quark propagation from t to t
⇒ matrix inversions on many t

easiest constructions with meson quantum numbers — **fermion bilinears** $\bar{\psi}\mathbf{\Gamma}\psi$

well motivated by success of quark model

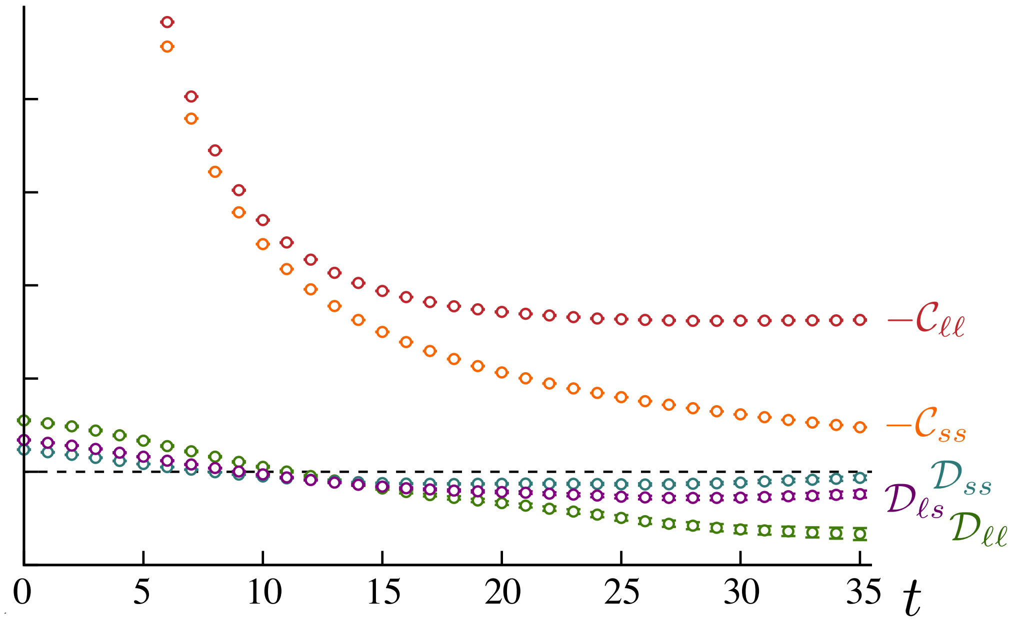
'looks' like a $q\bar{q}$ system

Wick contractions

'annihilation' required for isospin=0

quark propagation from *t* to *t*
⇒ matrix inversions on many *t*

an isospin=0 correlation function



turns out this is not enough …

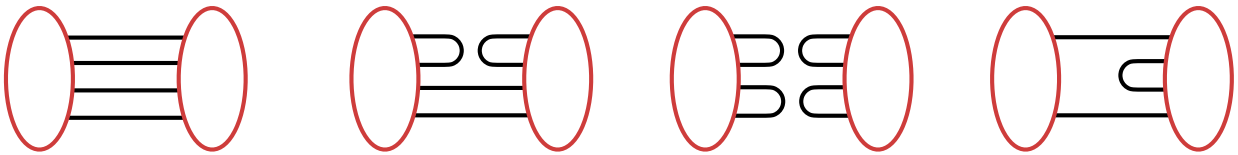
easiest constructions with meson quantum numbers – **fermion bilinears** $\bar{\psi}\Gamma\psi$

but can also construct operators with **more quark fields**

e.g. ‘local’ tetraquark operators $\bar{\psi}_{\mathbf{x}}\bar{\psi}_{\mathbf{x}}\psi_{\mathbf{x}}\psi_{\mathbf{x}}$

e.g. ‘meson-meson’-like operators $\sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\,\bar{\psi}_{\mathbf{x}}\Gamma\psi_{\mathbf{x}}\,\sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}}\,\bar{\psi}_{\mathbf{y}}\Gamma'\psi_{\mathbf{y}}$

schematic
Wick
contractions



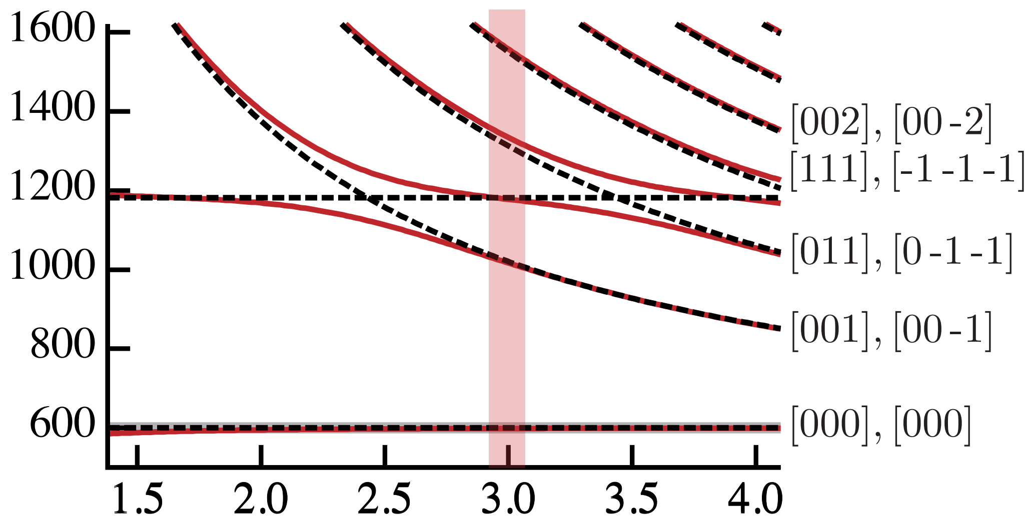
‘annihilation’
generally
required

and can clearly include still more quark fields ad infinitum …

… is there some organizing principle
which suggests what operator basis we should use ?

the non-interacting spectrum as an operator basis guide

e.g. narrow resonance (in rest frame)



suppose we want to determine all states up to 1500 MeV on a 3 fm lattice

we might try an operator basis featuring 'meson-meson'-like operators with back-to-back momentum up to [111]

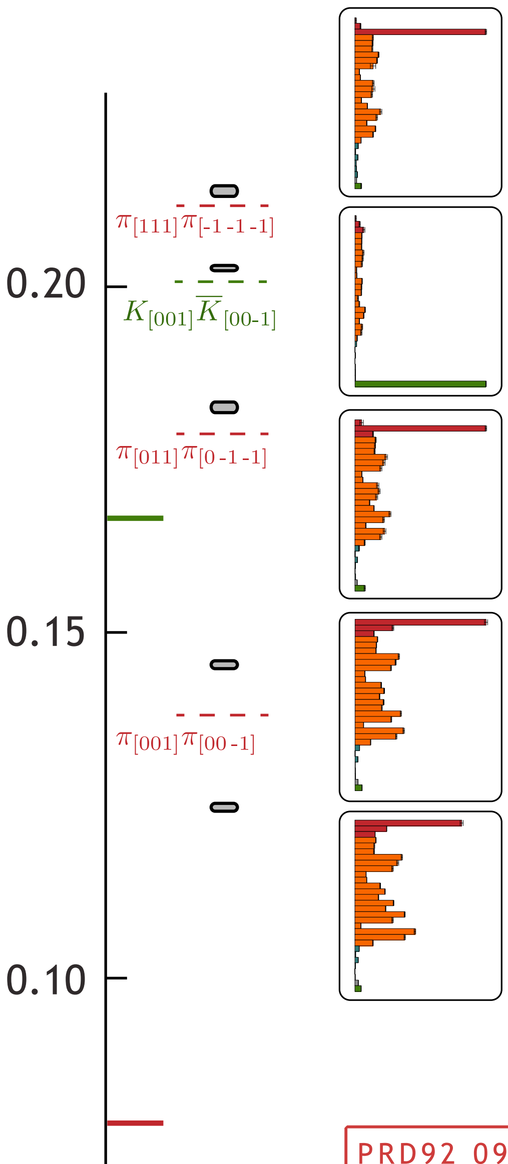
'look like' the expected meson-meson basis states

plus a set of $\bar{\psi}\Gamma\psi$ operators

'look like' a bound $q\bar{q}$-like basis state

isospin=1 T_1^- irrep spectrum

variational analysis of 30×30 correlation matrix: $3\times\pi\pi$, $26\times\bar{\psi}\Gamma\psi$, $1\times K\bar{K}$

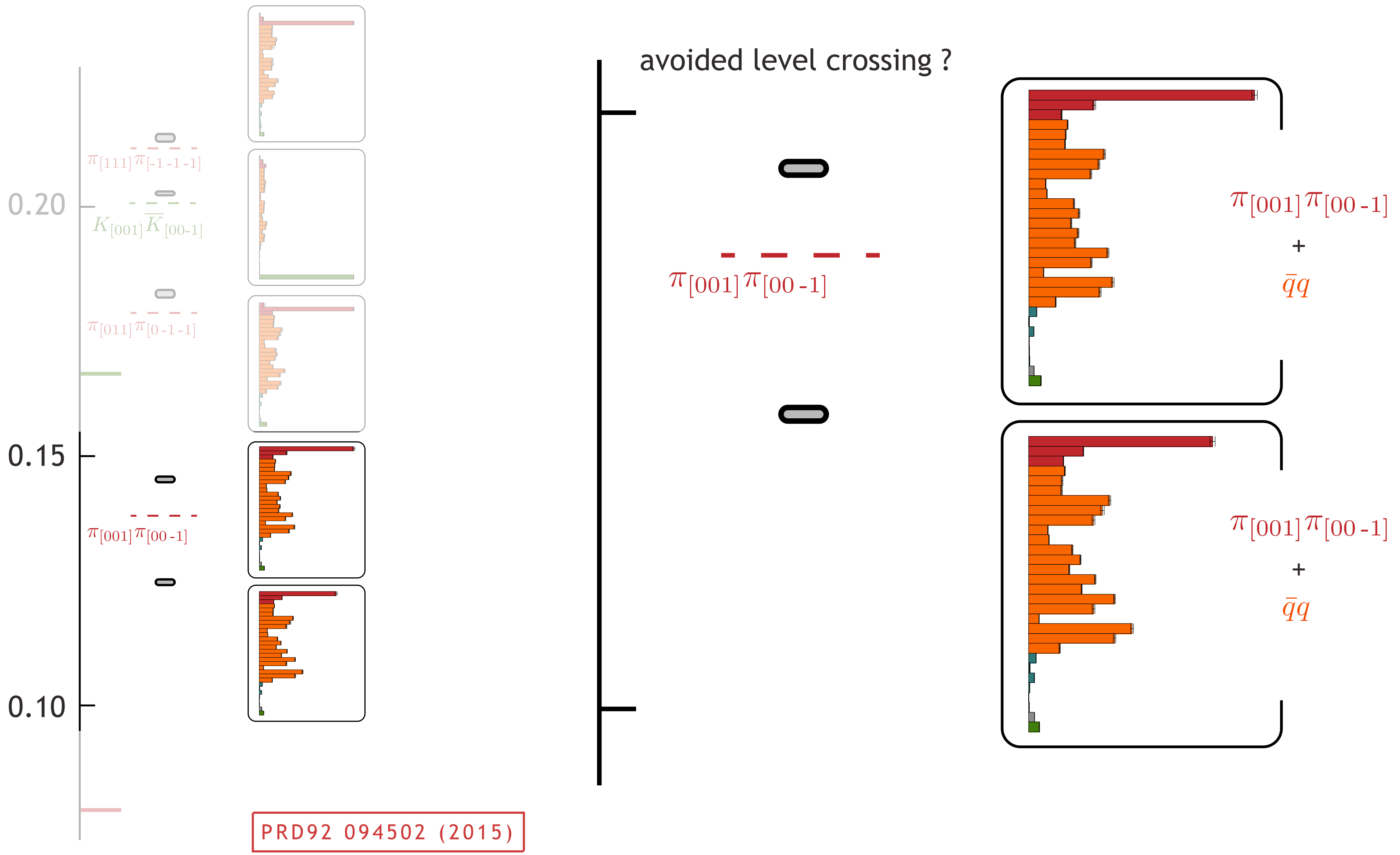


PRD92 094502 (2015)

$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

isospin=1 T_1^- irrep spectrum

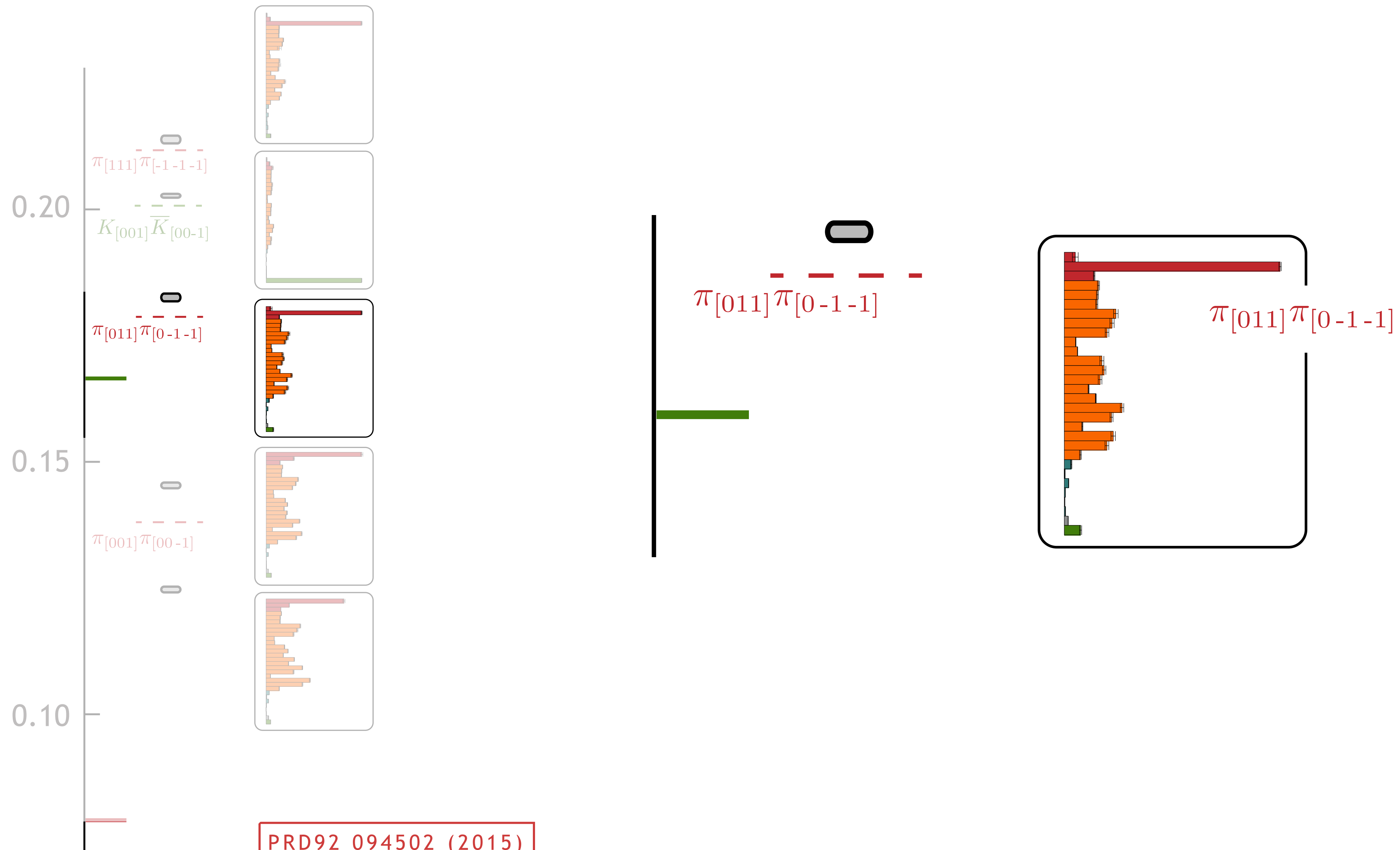
variational analysis of 30×30 correlation matrix: $3\times\pi\pi$, $26\times\bar{\psi}\Gamma\psi$, $1\times K\bar{K}$



$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

isospin=1 T_1^- irrep spectrum

variational analysis of 30×30 correlation matrix: $3\times\pi\pi$, $26\times\bar{\psi}\Gamma\psi$, $1\times K\bar{K}$



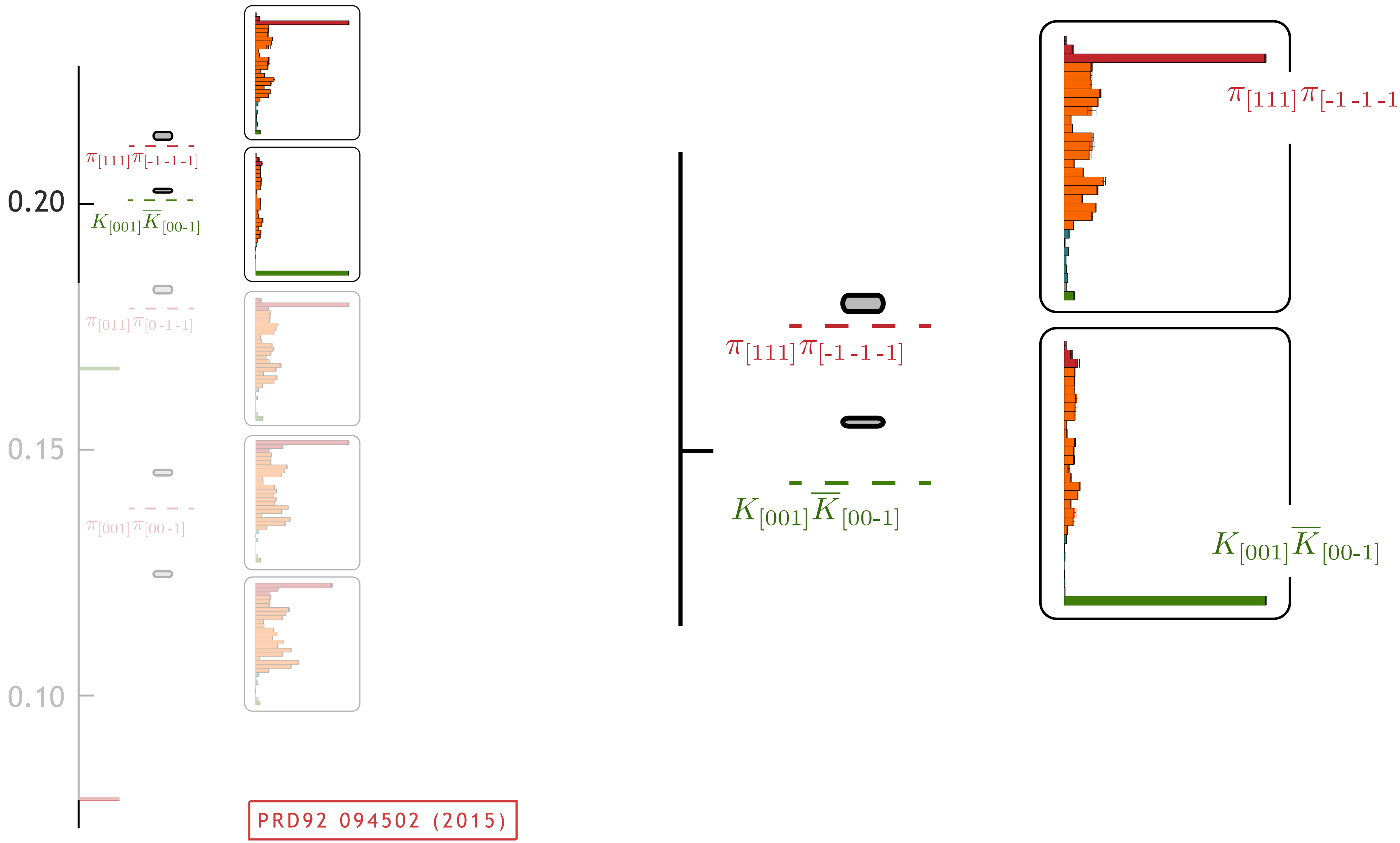
PRD92 094502 (2015)

$m_\pi = 0.039$
$m_K = 0.083$

$L \sim 3.8$ fm

isospin=1 T_1^- irrep spectrum

variational analysis of 30×30 correlation matrix: $3\times\pi\pi$, $26\times\bar{\psi}\Gamma\psi$, $1\times K\bar{K}$

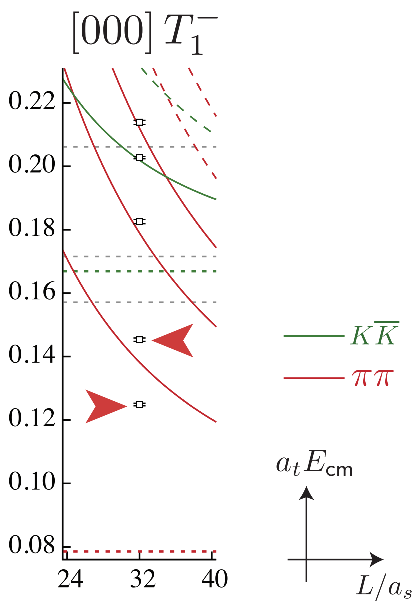


$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

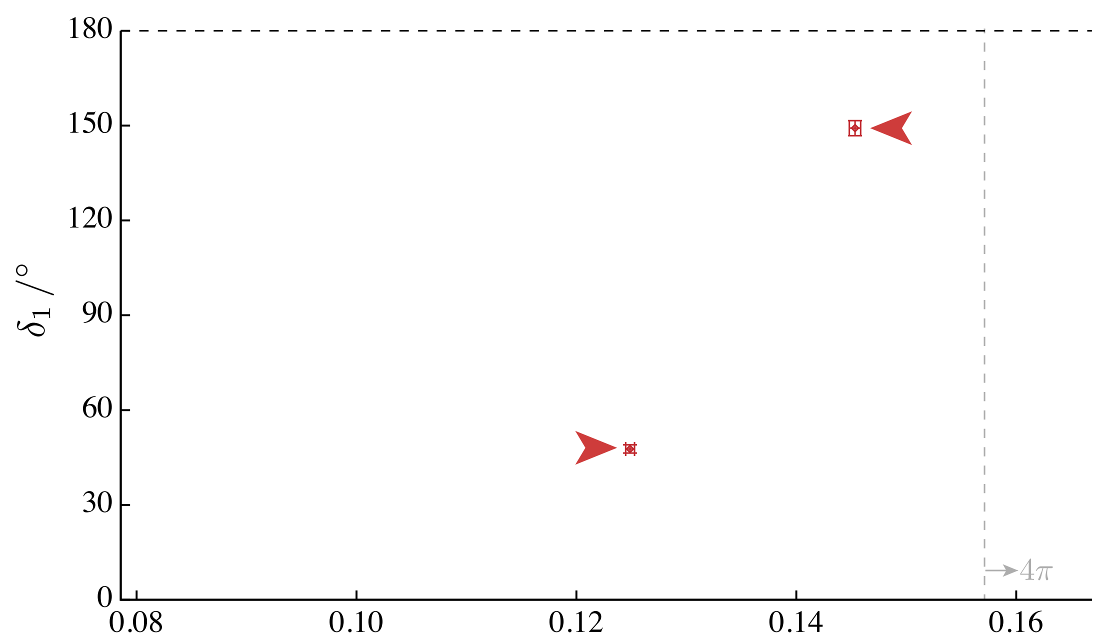
$\pi\pi$ I=1 J^P=1^- scattering

m_π ~ 236 MeV

PRD 92 094502 (2015)



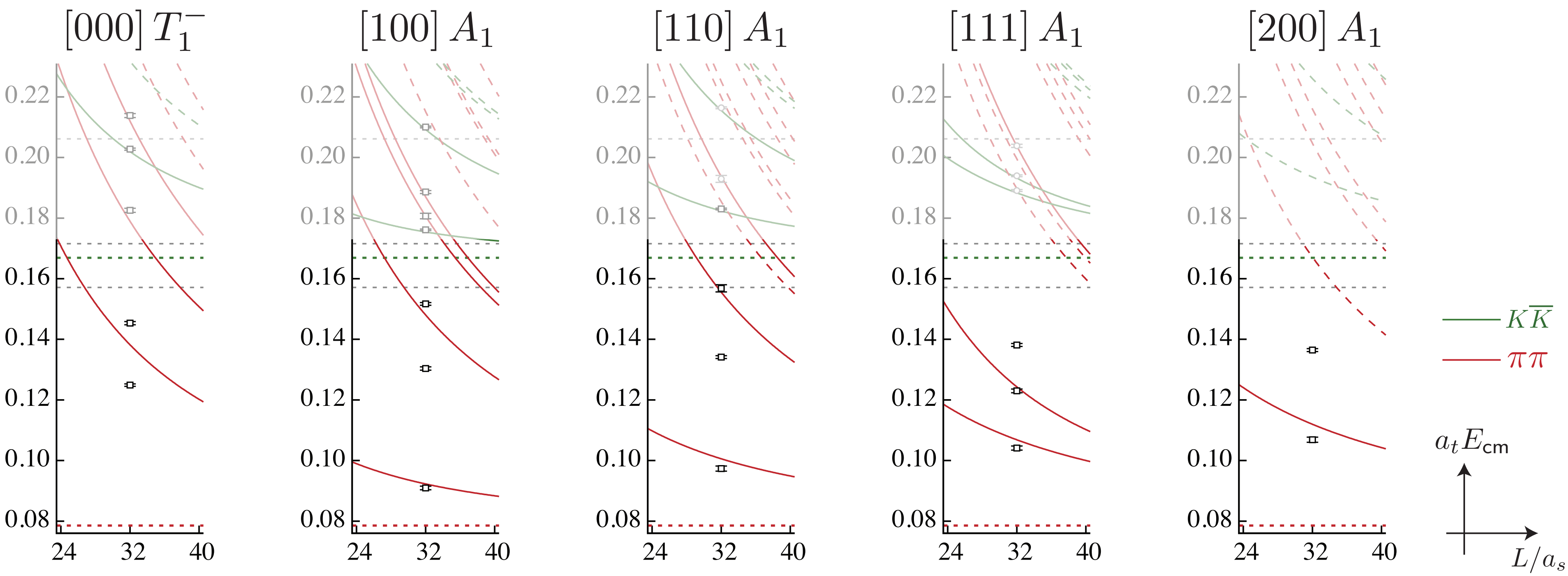
$$t(E) = \frac{1}{\rho(E)} e^{i\delta(E)} \sin\delta(E)$$



m_π = 0.039

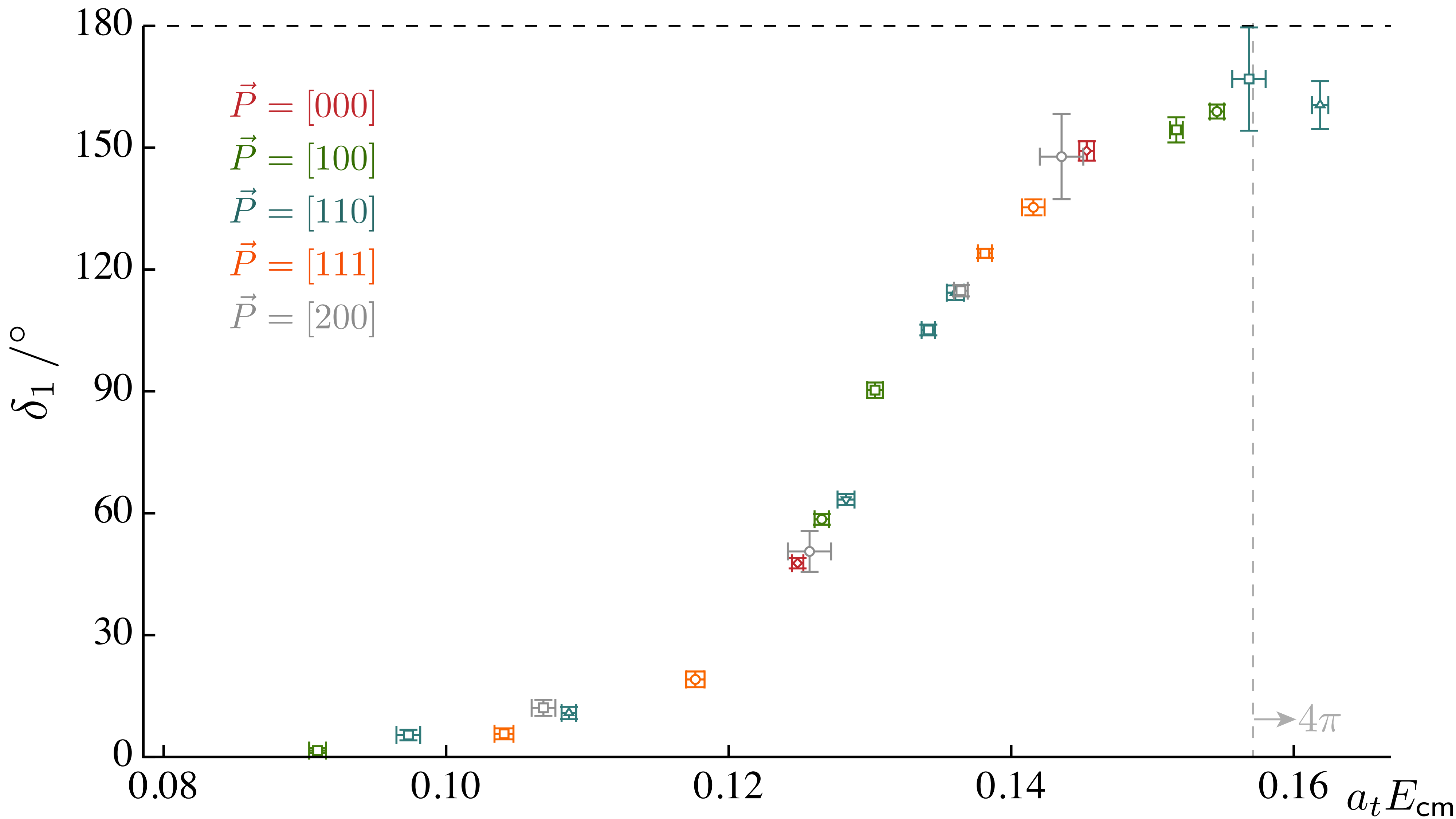
m_K = 0.083

L ~ 3.8 fm



$m_\pi = 0.039$
$m_K = 0.083$
L ~ 3.8 fm

… giving a phase-shift



$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

… looks like a classic resonance signal …

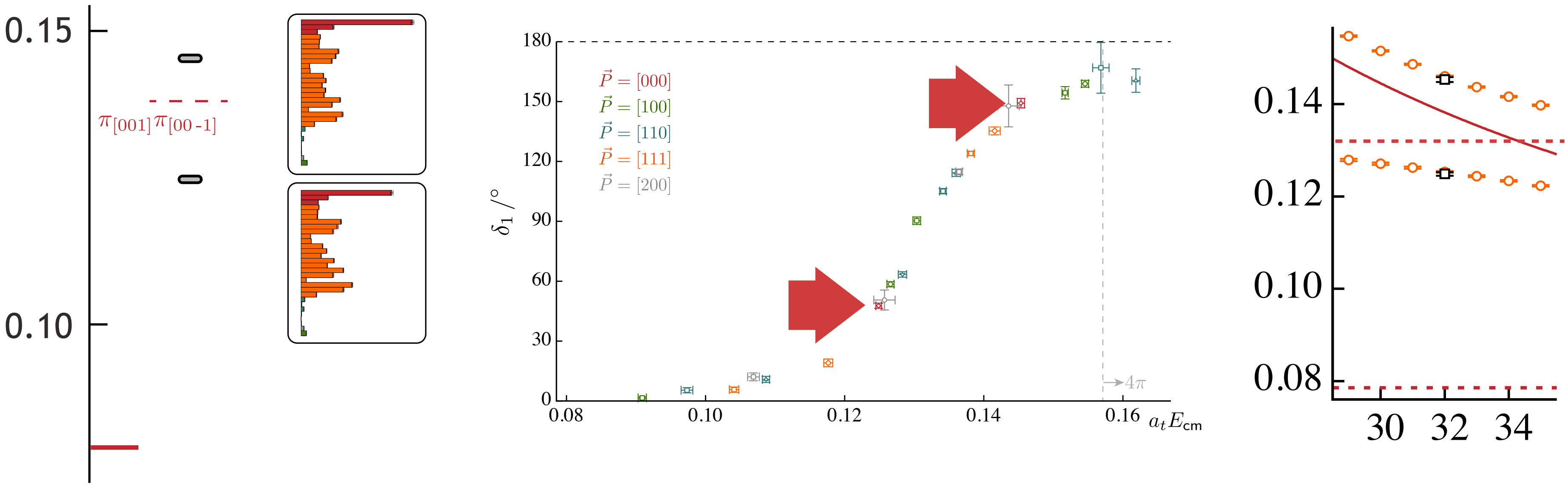
what happens if we vary the operator basis ?

$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

what's happening here ?

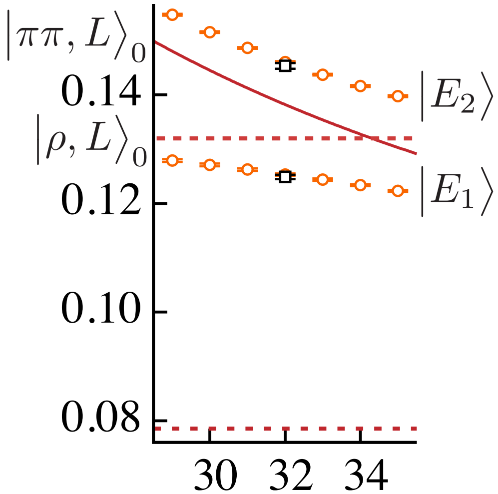
X

focus on the lowest two states



an avoided level crossing

what's happening here ?



think about this as a **two-state problem**

imagine we could turn off the coupling so
a 'bound-state' and a 'meson-meson' state were eigenstates

$|\rho, L\rangle_0$ $\qquad$ $|\pi\pi, L\rangle_0$

with the coupling turned on, the eigenstates are admixtures

$$|E_1\rangle = \cos\theta\, |\rho, L\rangle_0 + \sin\theta\, |\pi\pi, L\rangle_0$$
$$|E_2\rangle = -\sin\theta\, |\rho, L\rangle_0 + \cos\theta\, |\pi\pi, L\rangle_0$$

with operators that 'look-like' $|\rho, L\rangle_0$ and $|\pi\pi, L\rangle_0$ in the basis, the variational method separates $|E_1\rangle$,$|E_2\rangle$

$$\begin{pmatrix} C_{\rho,\rho}(t) & C_{\rho,\pi\pi}(t) \\ C_{\pi\pi,\rho}(t) & C_{\pi\pi,\pi\pi}(t) \end{pmatrix} = \begin{pmatrix} Z_\rho & 0 \\ 0 & Z_{\pi\pi} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} e^{-E_1 t} & 0 \\ 0 & e^{-E_2 t} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} Z_\rho & 0 \\ 0 & Z_{\pi\pi} \end{pmatrix}$$

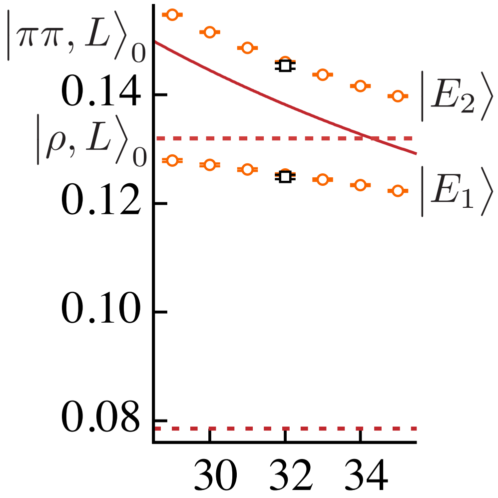
$$\mathcal{O}_\rho|0\rangle = Z_\rho|\rho, L\rangle_0 + \epsilon|\pi\pi, L\rangle_0$$
$$\mathcal{O}_{\pi\pi}|0\rangle = Z_{\pi\pi}|\pi\pi, L\rangle_0 + \epsilon|\rho, L\rangle_0$$

GEVP eigenvectors will find the rotation

and the principal correlators $\quad \lambda_1(t) \sim e^{-E_1 t}$

$$\lambda_2(t) \sim e^{-E_2 t}$$

what's happening here ?



think about this as a **two-state problem**

imagine we could turn off the coupling so
a 'bound-state' and a 'meson-meson' state were eigenstates

$|\rho, L\rangle_0$ $\qquad$ $|\pi\pi, L\rangle_0$

with the coupling turned on, the eigenstates are admixtures

$$|E_1\rangle = \cos\theta\,|\rho, L\rangle_0 + \sin\theta\,|\pi\pi, L\rangle_0$$
$$|E_2\rangle = -\sin\theta\,|\rho, L\rangle_0 + \cos\theta\,|\pi\pi, L\rangle_0$$

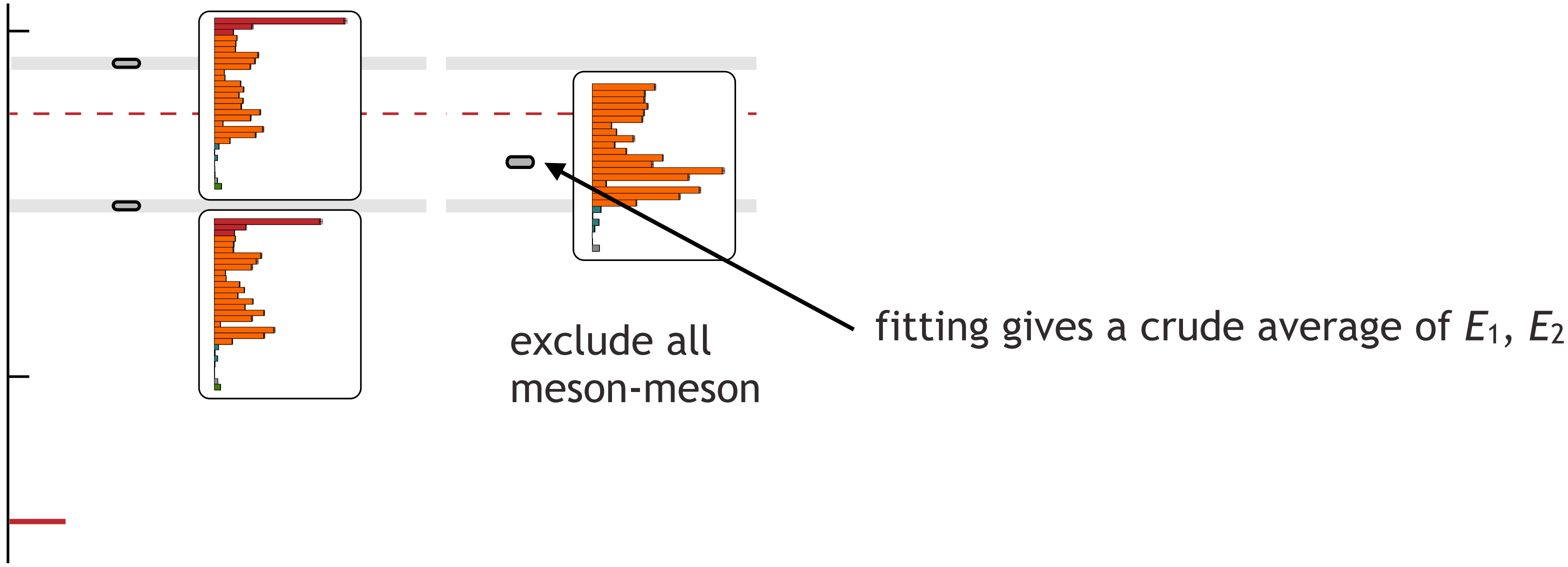
now suppose we used only the $\mathcal{O}_\rho$ operators

then $C(t) \propto \cos^2\theta\, e^{-E_1 t} + \sin^2\theta\, e^{-E_2 t}$ and there'll be two energies present …

… and they're very hard to separate

what’s happening here ?

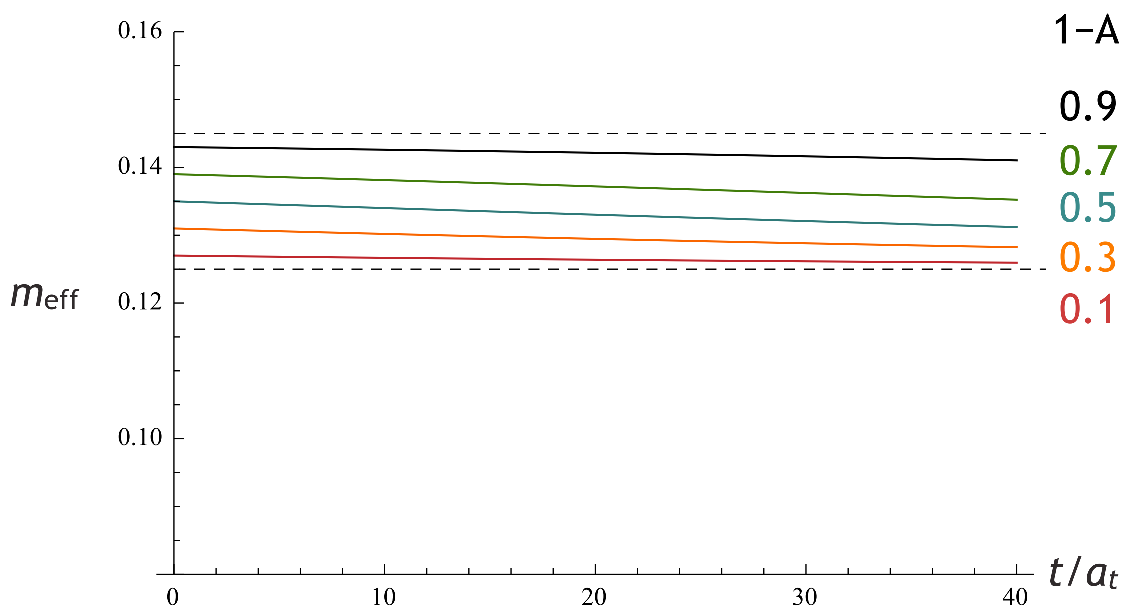
it looks like this is what’s happening

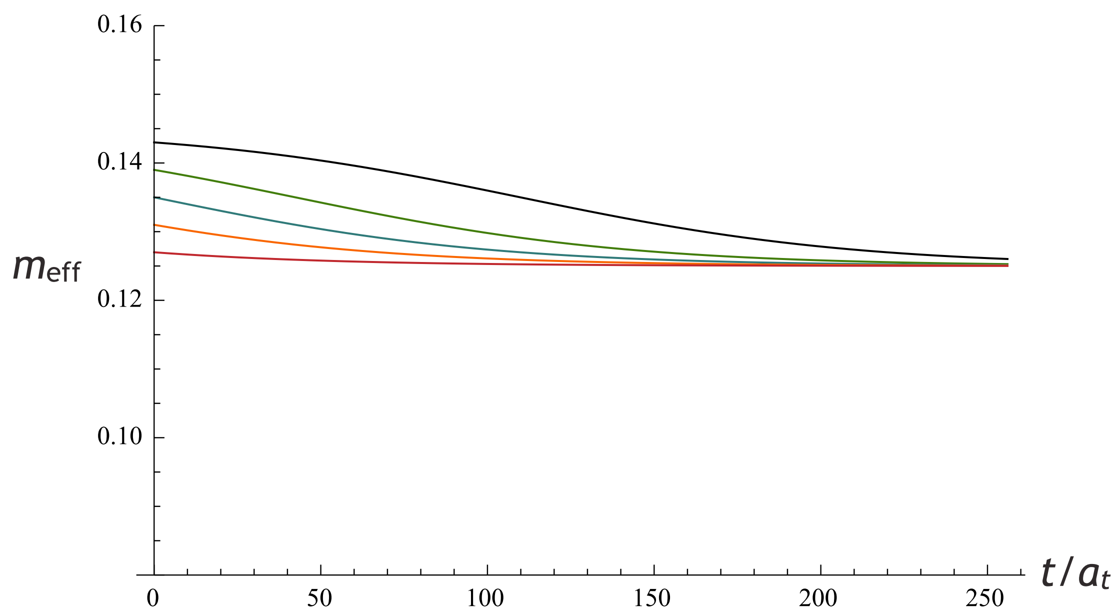


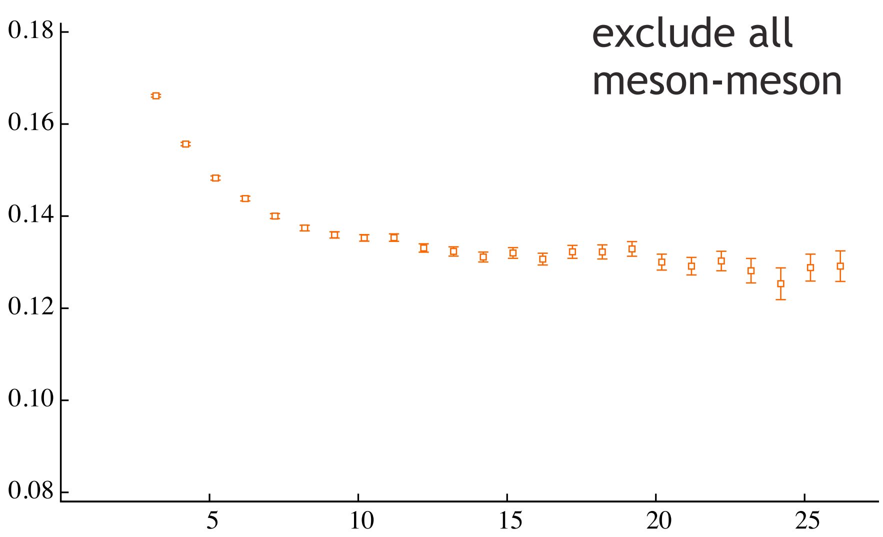
two-state admixture

$$C(t) = A\, e^{-0.125\, t/a_t} + (1-A)\, e^{-0.145\, t/a_t}$$

X



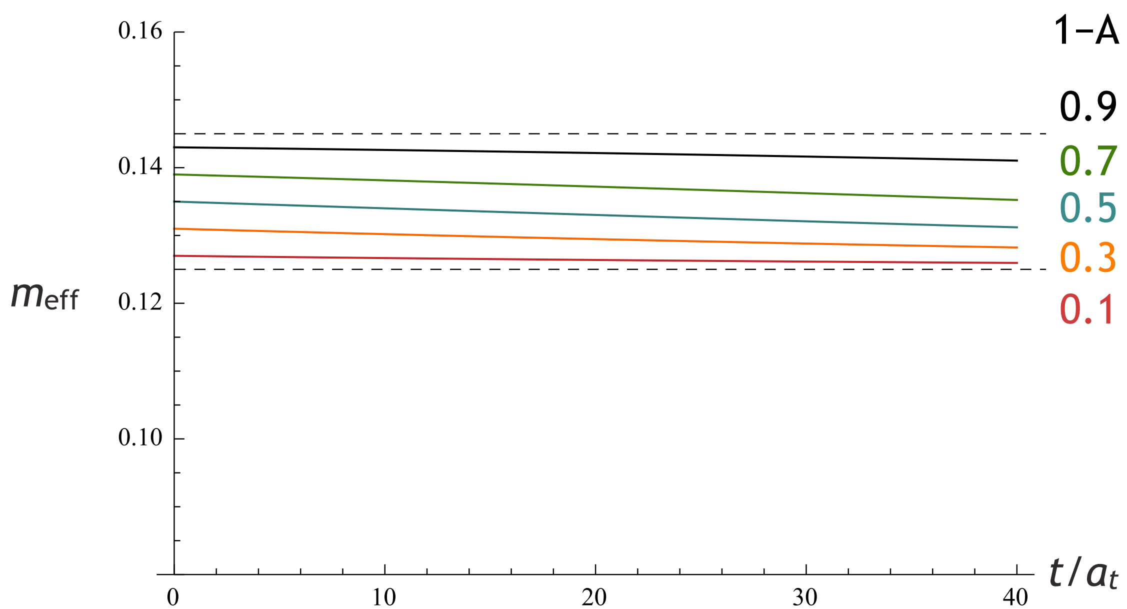


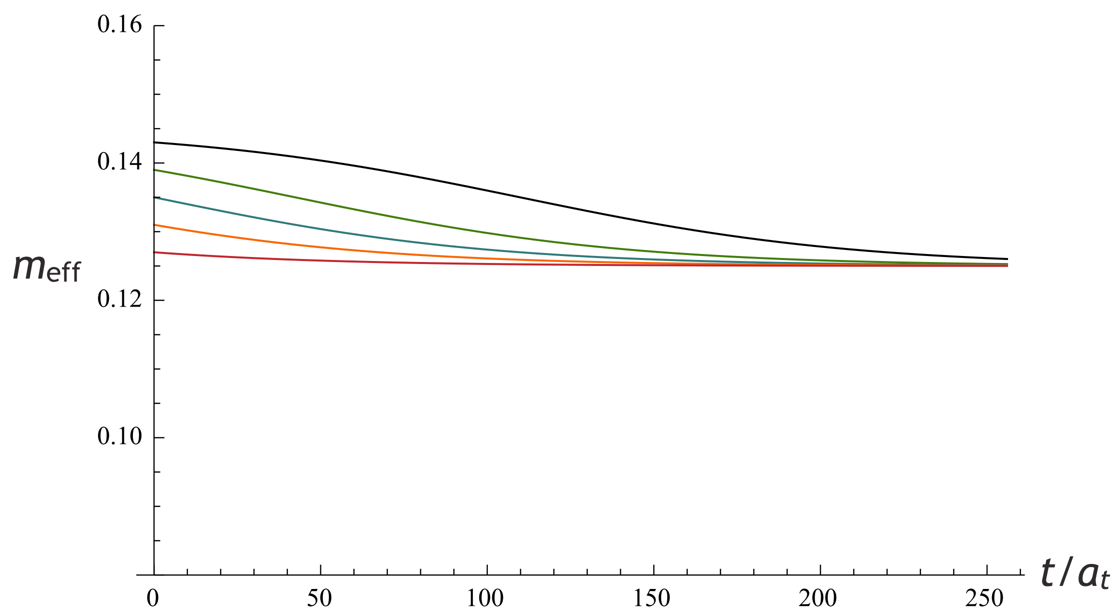


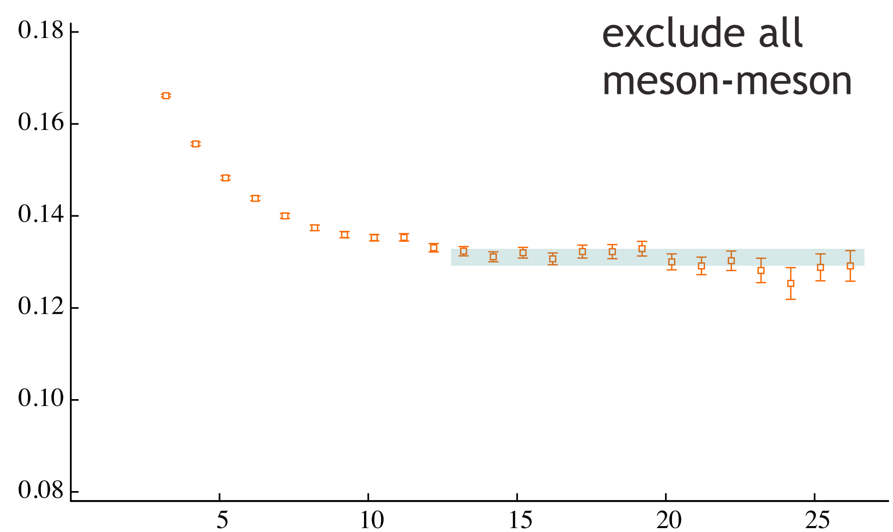
two-state admixture

$$C(t) = A\,e^{-0.125\,t/a_t} + (1-A)\,e^{-0.145\,t/a_t}$$

X



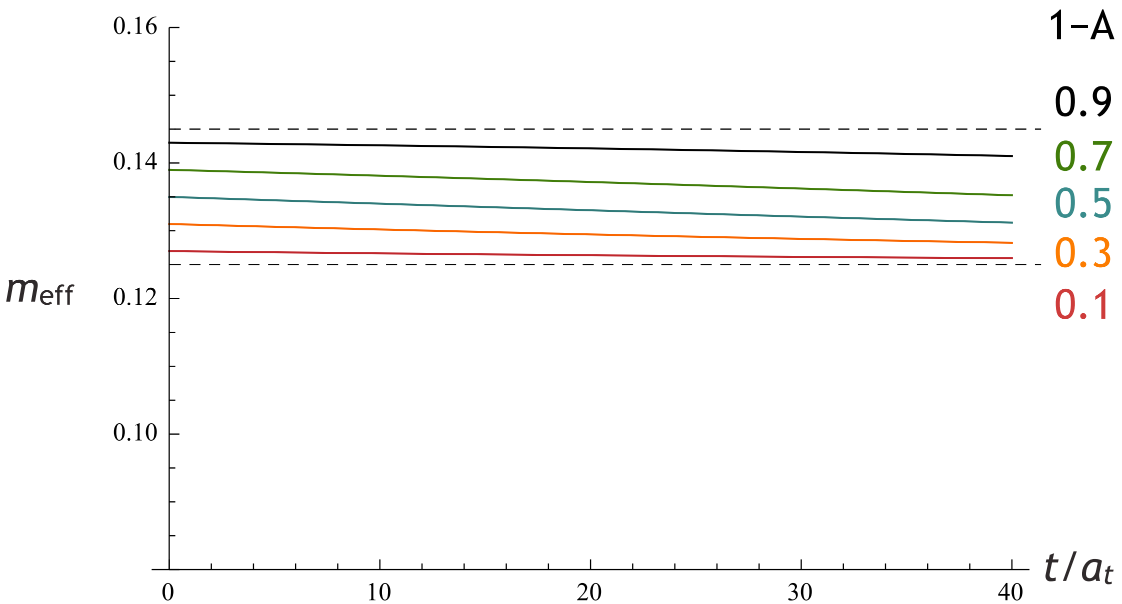


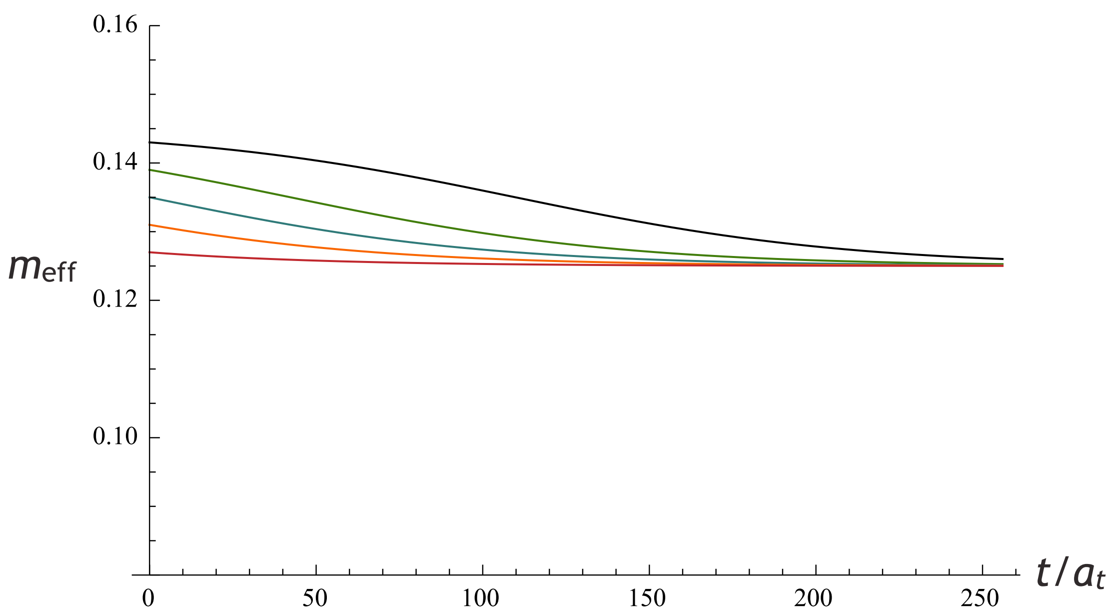


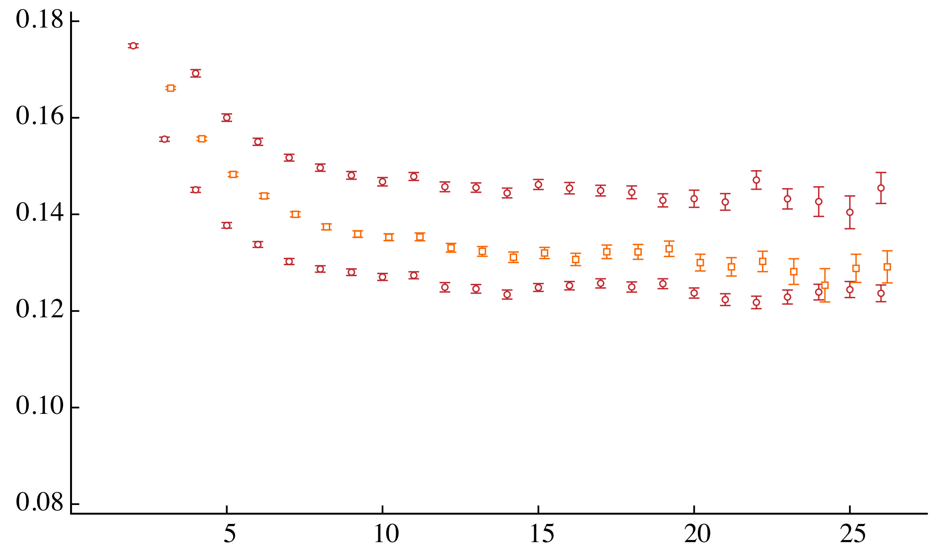
two-state admixture

$$C(t) = A\, e^{-0.125\, t/a_t} + (1 - A)\, e^{-0.145\, t/a_t}$$

X







‘single-meson’-like versus ‘meson-meson’-like ?

volume dependence !

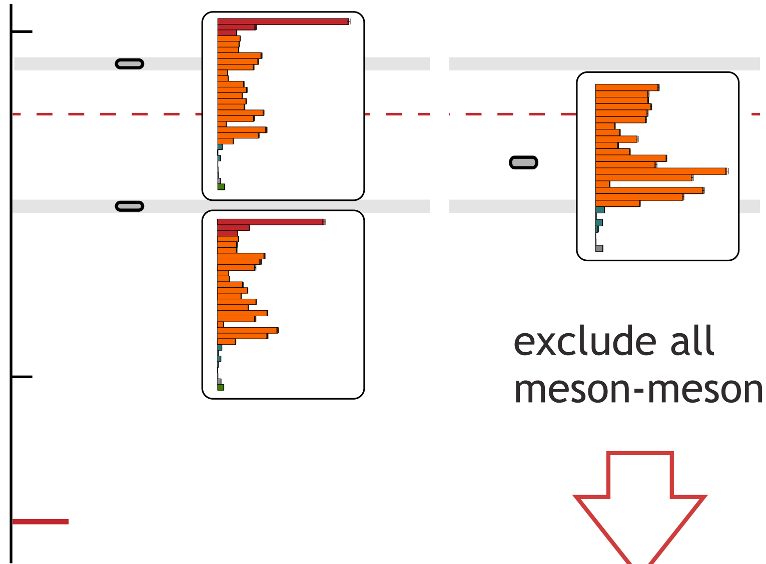
‘meson-meson’-like $\sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, \bar{\psi}_{\mathbf{x}}\mathbf{\Gamma}\psi_{\mathbf{x}} \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}}\, \bar{\psi}_{\mathbf{y}}\mathbf{\Gamma}'\psi_{\mathbf{y}}$ samples the whole volume of the lattice

‘single-meson’-like $\sum_{\mathbf{x}} e^{i\mathbf{P}\cdot\mathbf{x}}\, \bar{\psi}_{\mathbf{x}}\mathbf{\Gamma}\psi_{\mathbf{x}}$ samples a single point (translated)

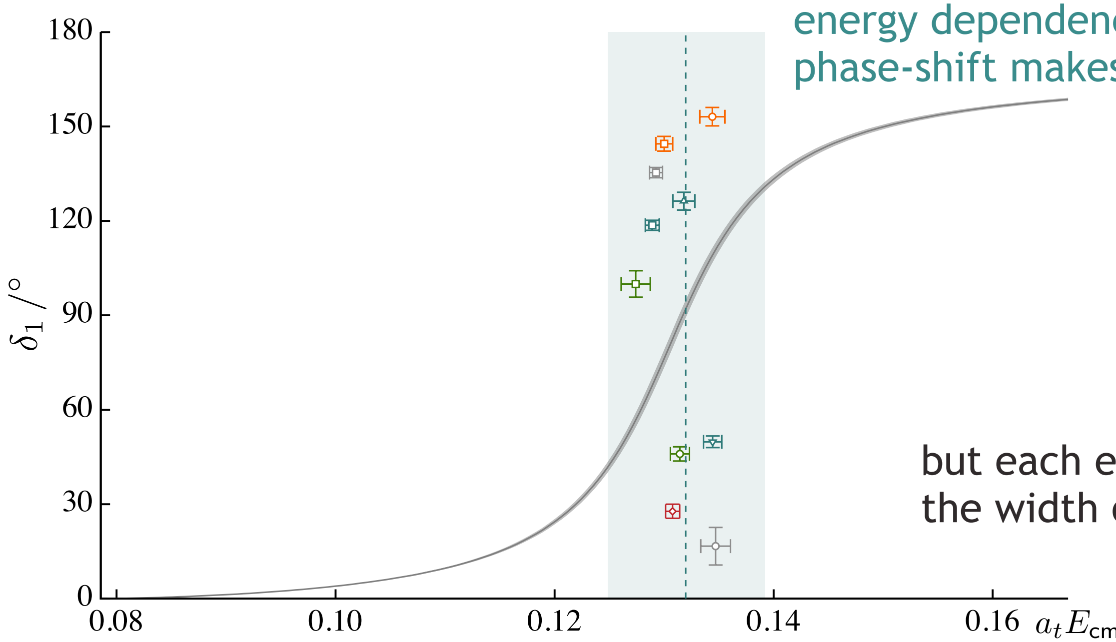
so: ‘looks-like’ = ‘has the same volume sampling as’

interesting side note:
tetraquark operators won’t work well for interpolating meson-meson components — wrong volume sampling

how bad is it really to get the wrong energies ?



exclude all
meson-meson



energy dependence of the
phase-shift makes no sense

but each energy within
the width of the resonance … ?

some technical stuff — 'meson-meson'-like operators

what actually goes into a '$\pi\pi$'-like operator ?

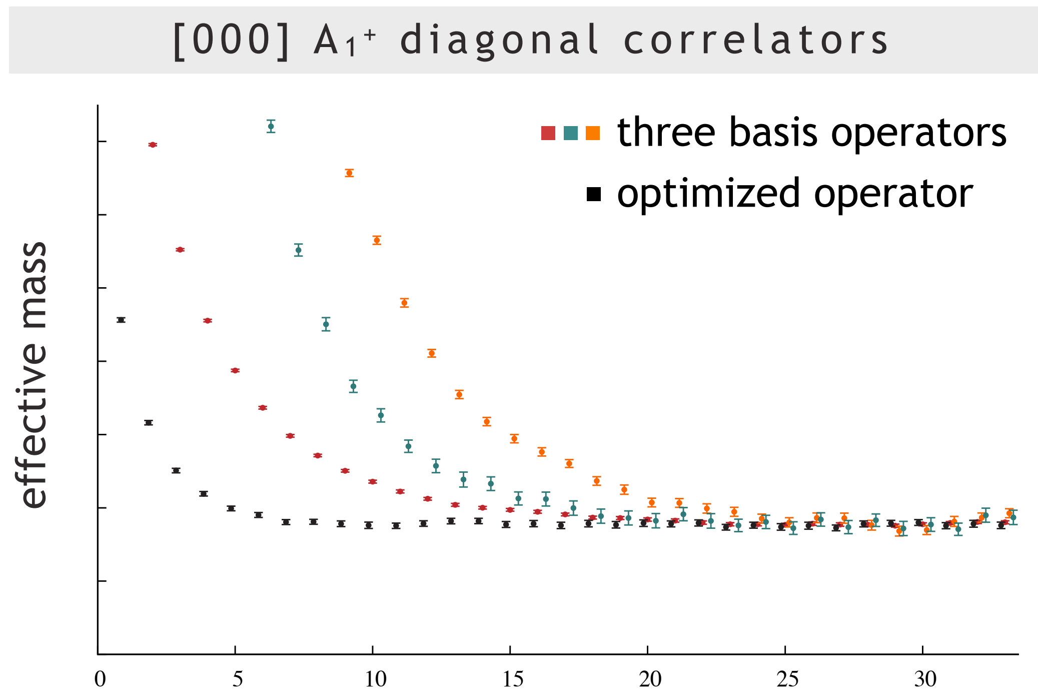
one option for construction is to use products of single-meson operators in lattice irreps

$$\sum_{\substack{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2 \\ \mathbf{p}_1+\mathbf{p}_2=\mathbf{P}}} C_{\Lambda_1\otimes\Lambda_2\to\Lambda}(\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2)\,\pi(\mathbf{p}_1;\Lambda_1)\,\pi(\mathbf{p}_2;\Lambda_2)$$

'lattice' Clebsch-Gordan coefficients

some group theory to work them out

then each single-meson operator can be the **variationally optimized** one for that ***p***, *Λ*



optimized operator saturated by the pion by timeslice 7