Gluons in QCD

Lecture 5

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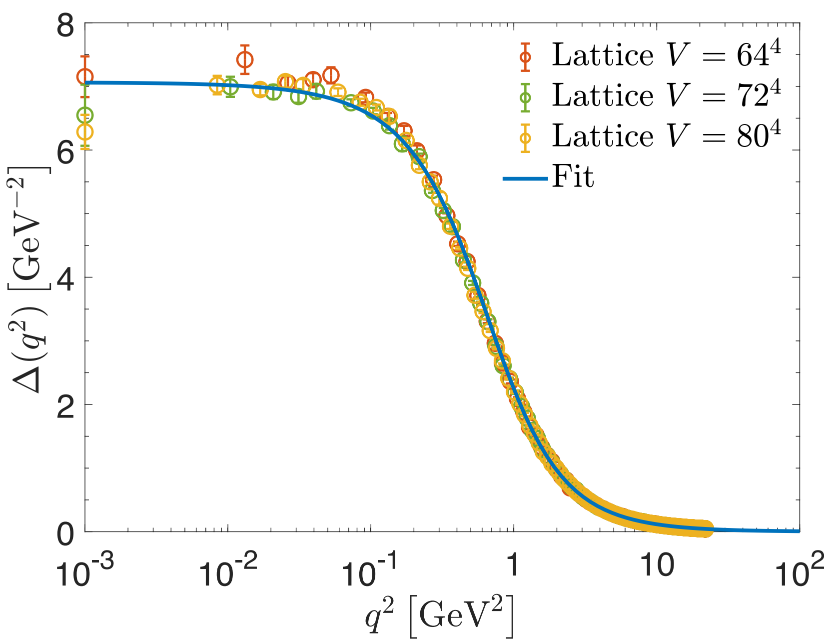
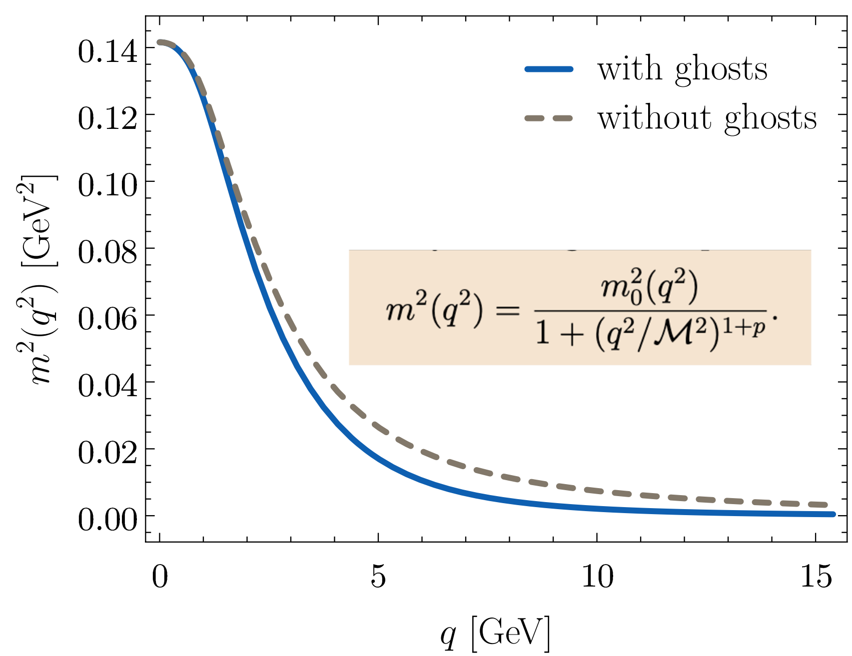
Dynamical gluon mass

- In the previous lecture, we have seen that the gluon propagator is infrared finite.

- Saturation of the gluon propagator can be explained by the generation of a *dynamical gluon mass.*

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$





Lattice data from:
I. L. Bogolubsky, et al, PoS LAT2007, 290 (2007).

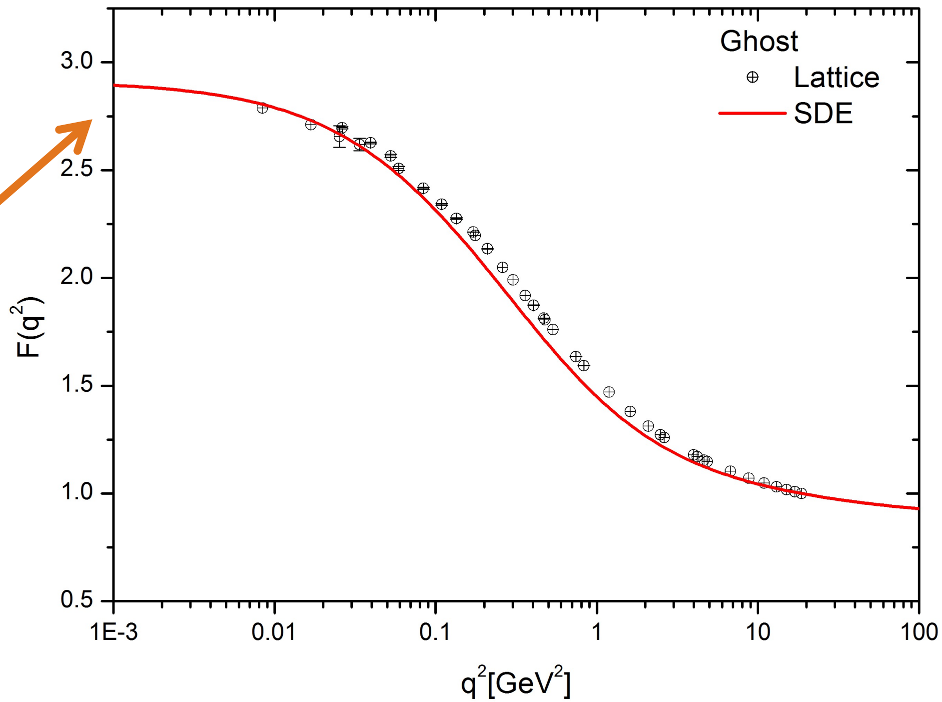
J. M. Cornwall, Phys. Rev. D26, 1453 (1982).
ACA., D. Binosi and J. Papavassiliou., Phys.Rev. D78, 025010 (2008)
ACA, D. Binosi, C. T. Figueiredo., and J. Papavassiliou, Eur. Phys. J. C78, no. 3, 181 (2018).

Ghost Sector

Ghost Sector

- The finiteness of the dressing of the gluon propagator, $F(p^2)$, is a consequence of the massiveness of the gluon propagator.

Finite result



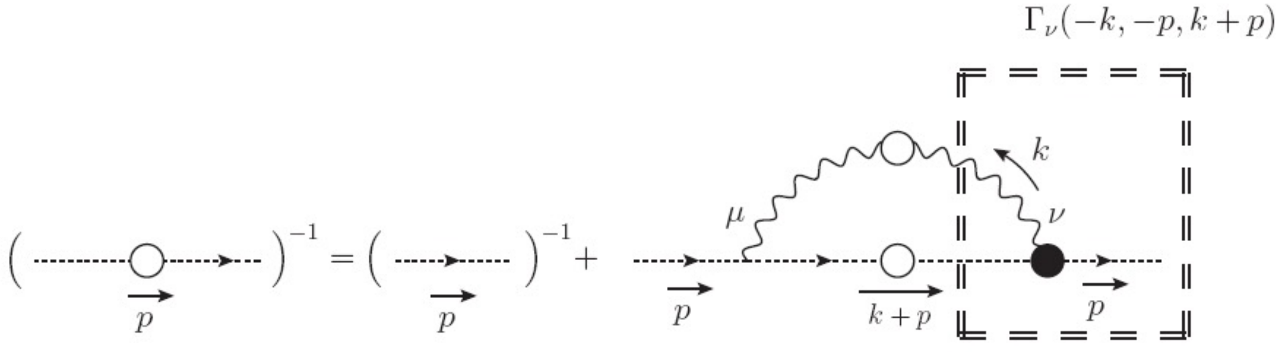
$$D(q^2) = \frac{F(q^2)}{q^2}$$

Gluon mass

$$F^{-1}(q^2) = 1 + \frac{C_A g^2}{24\pi^2} \ln\left(\frac{q^2 + \rho m^2(q^2)}{\mu^2}\right)$$

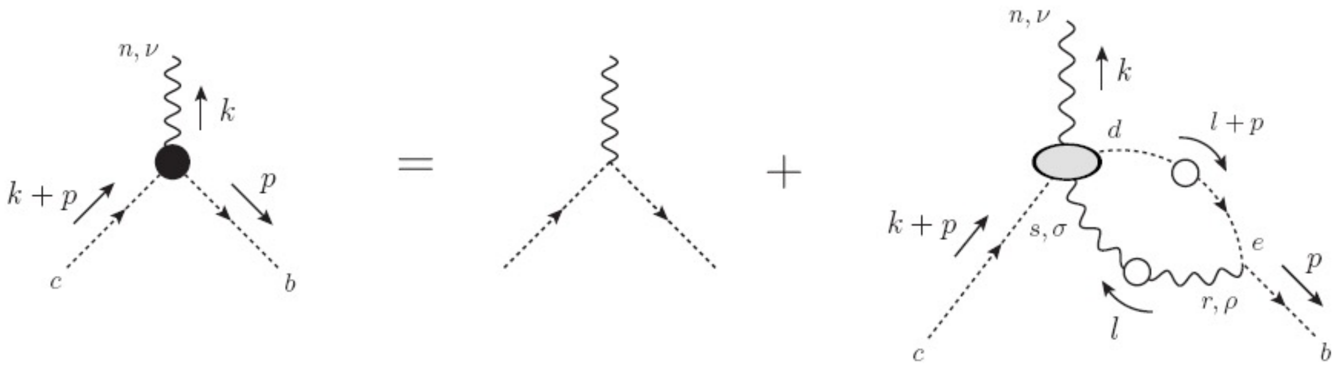
A. C. A, D. Binosi and J. Papavassiliou, Phys.Rev. D78 (2008) 025010
P. Boucaud, et al JHEP 0806, 099 (2008)

The ghost SDE

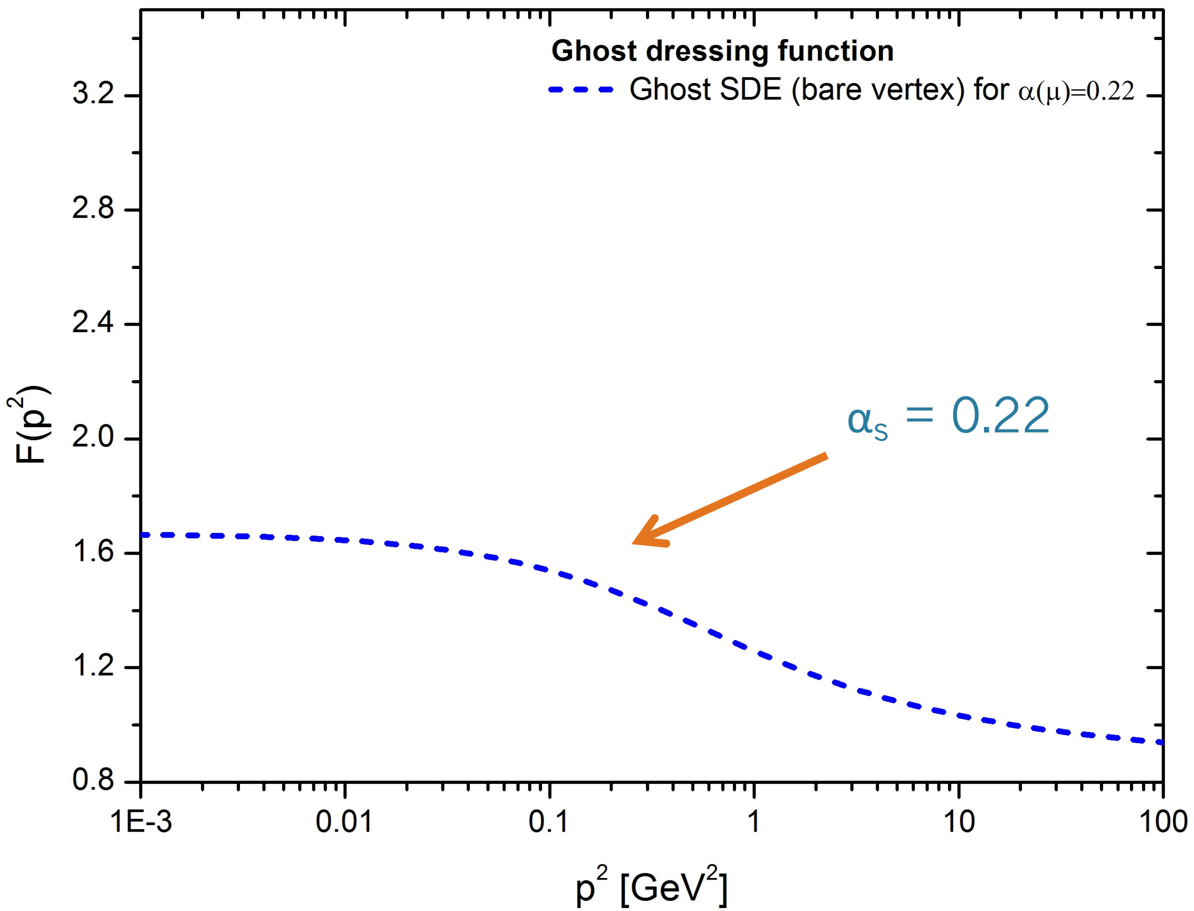


$$F^{-1}(p^2) = 1 + ig^2 C_{\mathrm{A}} \frac{1}{p^2} \int_k \Gamma_\mu^{[0]}(k,-k-p,p)\Delta^{\mu\nu}(k)\Gamma_\nu(-k,-p,k+p)D(k+p)\,,$$

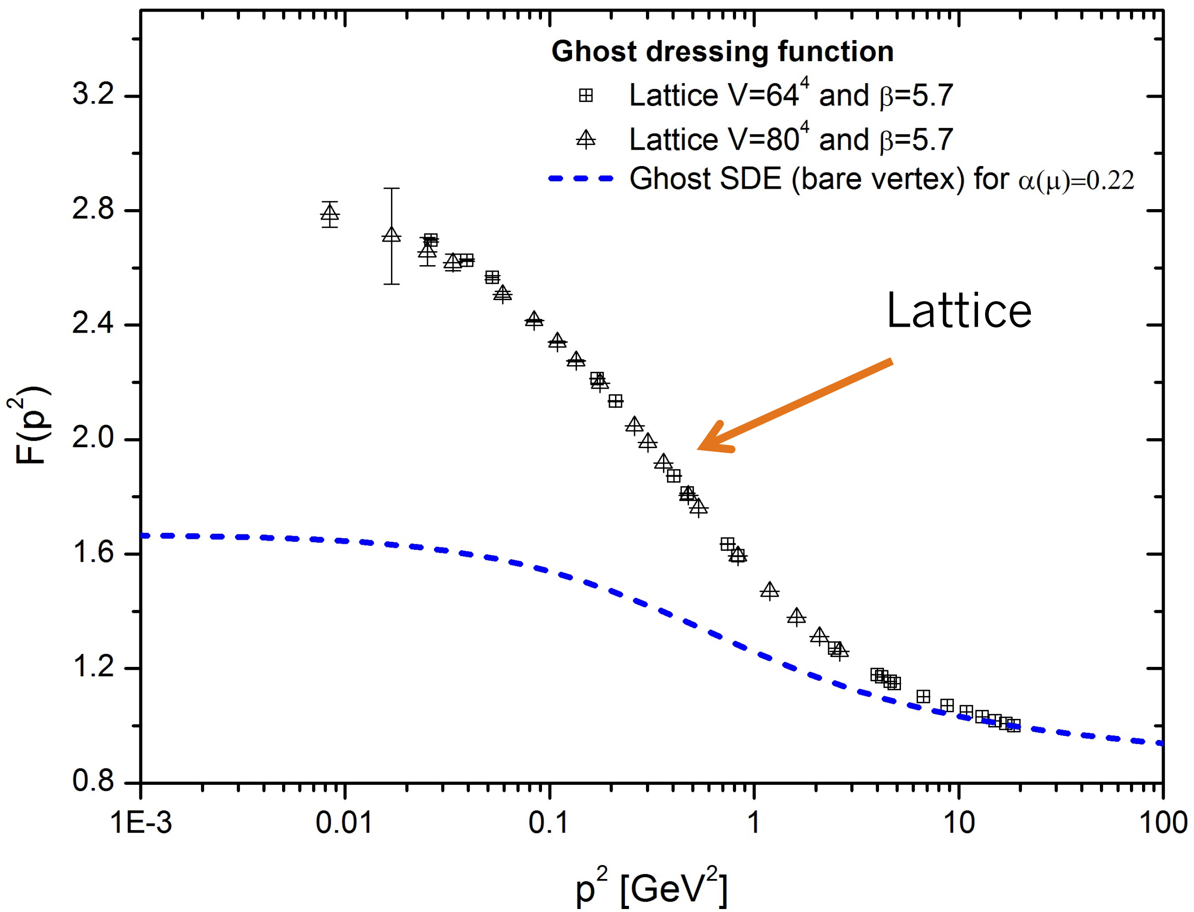
- The SDE for the ghost-gluon vertex is represented by



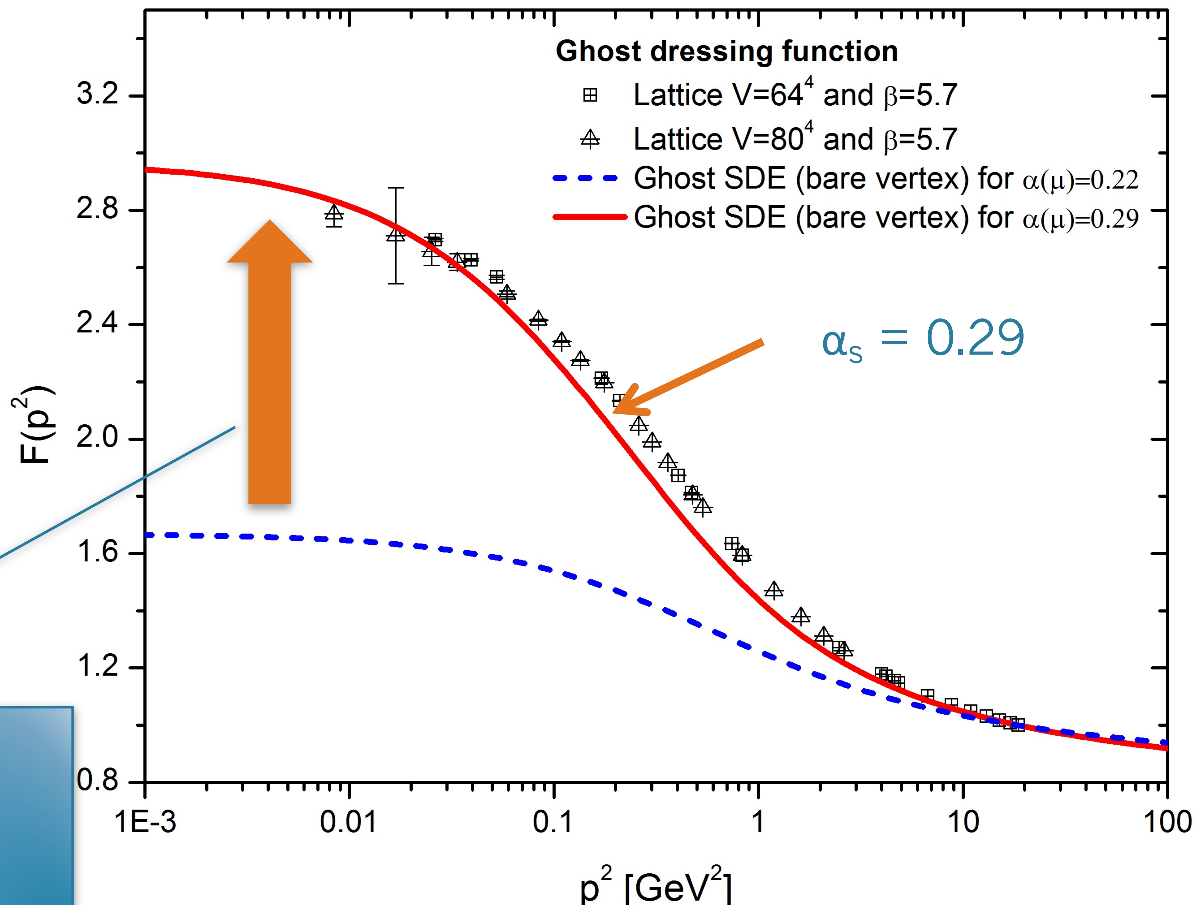
- When we use the bare gluon-ghost vertex the SDE result is



- However the lattice simulation tell us



- One way of correcting → increase the value of the coupling

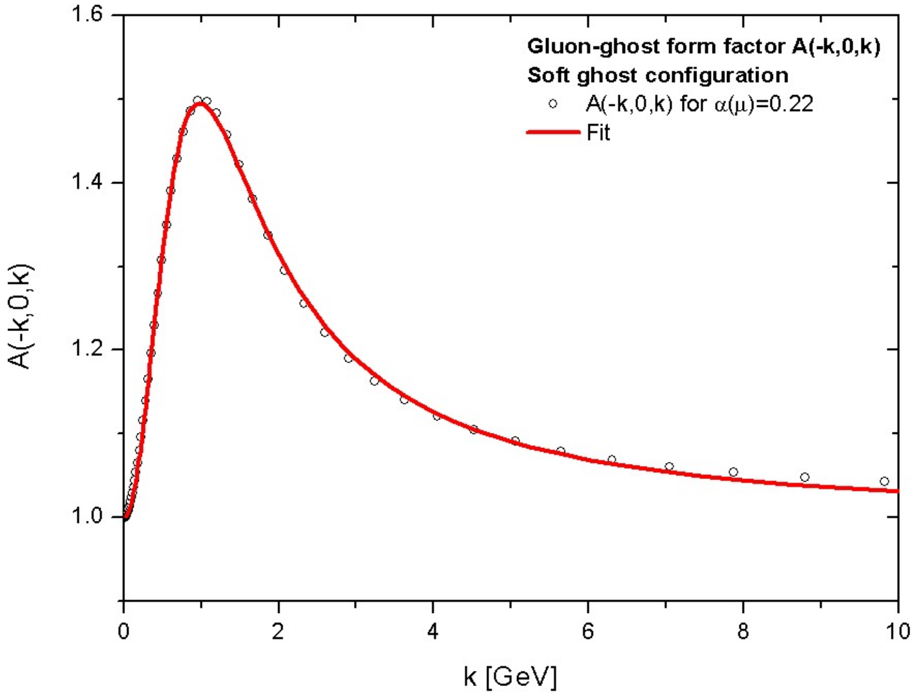


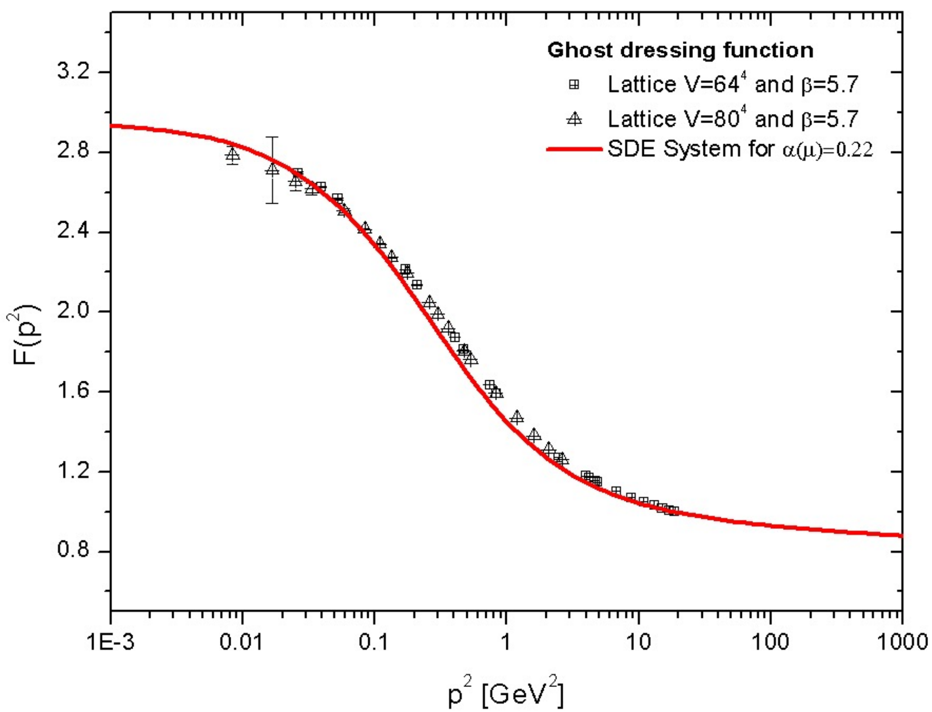
artificial increase of the gauge coupling

However the correct procedure is …

to add the contribution of the vertex, and solve the system for the

ghost gluon vertex + ghost SDEs

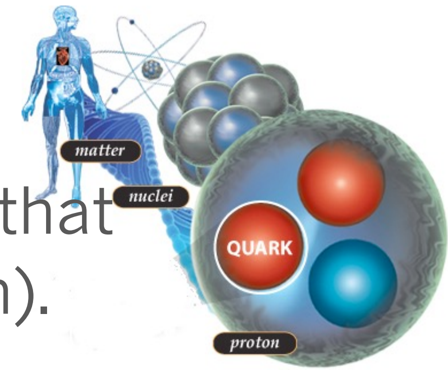




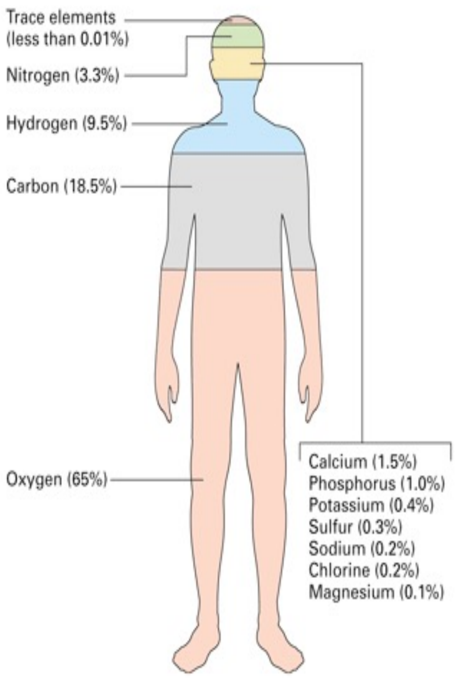
A.C. A., D. Ibáñez and J. Papavassiliou, Phys. Rev. D87, 114020 (2013)
D. Dudal, O. Oliveira and J. Rodriguez-Quintero, Phys. Rev. D86, 105005 (2012)
I. L. Bogolubsky, et al. PoS LATTICE, 290 (2007).

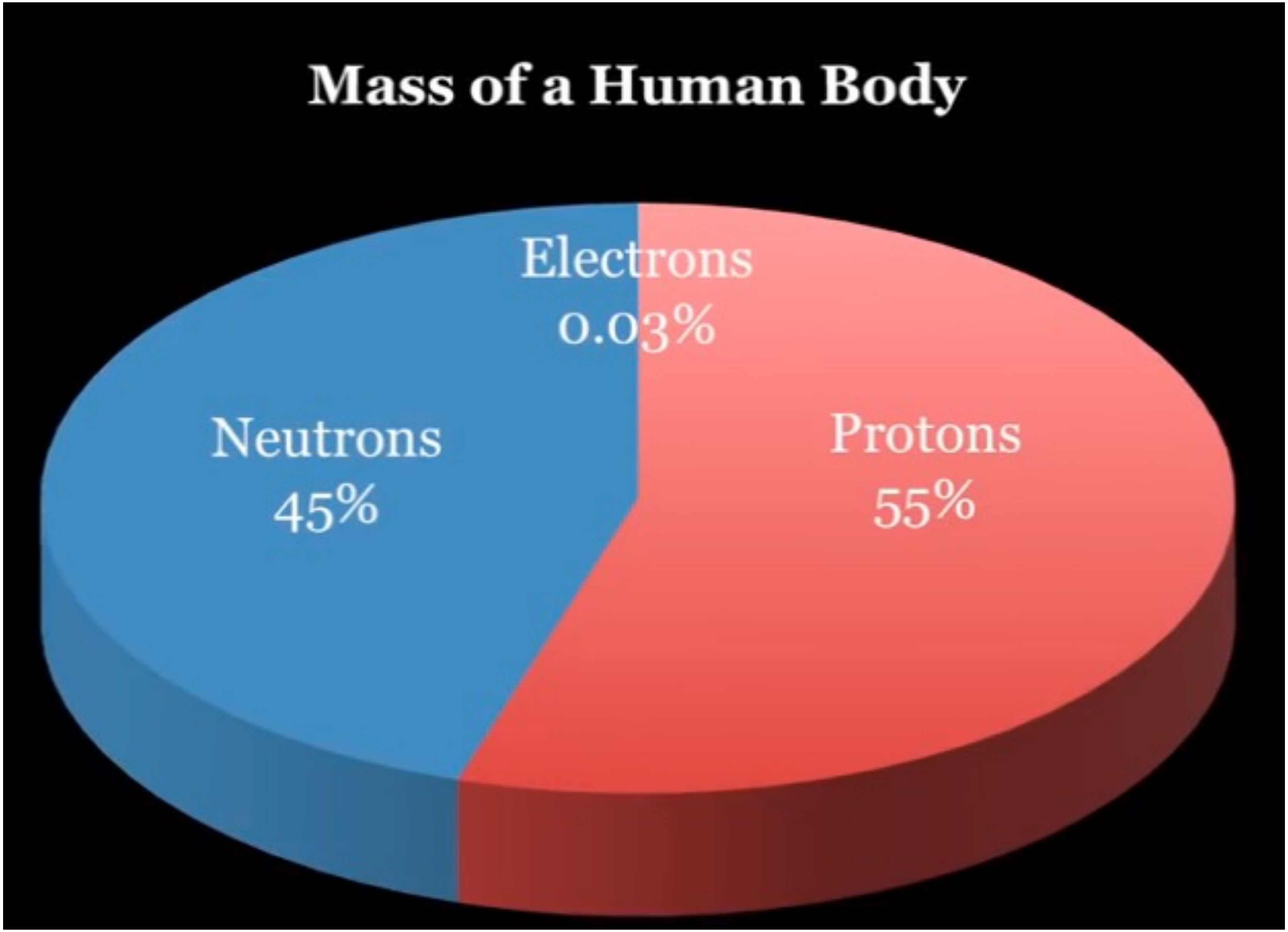
Quark Sector

Did you know that



- The ***sum of the masses of all electrons*** that form your body is only ~ 21 g (70 kg person).
- All the ***electron mass comes from the Higgs mechanism*** .
- However, most of our masses does not come from the Higgs mechanism!
- Our masses are generated through ***another mechanism which is much more powerful and efficient.***
- That's because ***the vast majority*** of our body mass and everything around us ***are made up of protons and neutrons.***



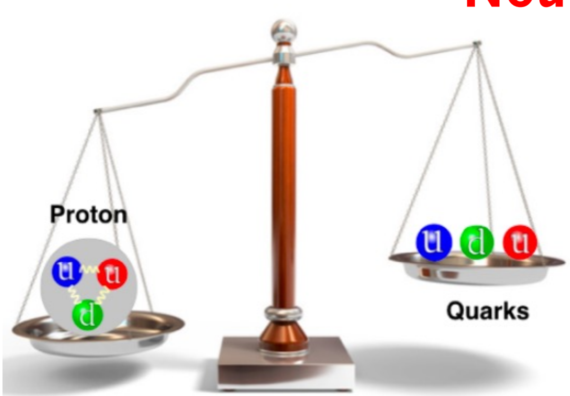

Mass of a Human Body
Electrons
0.03%
Neutrons
45%
Protons
55%

The QCD mass problem

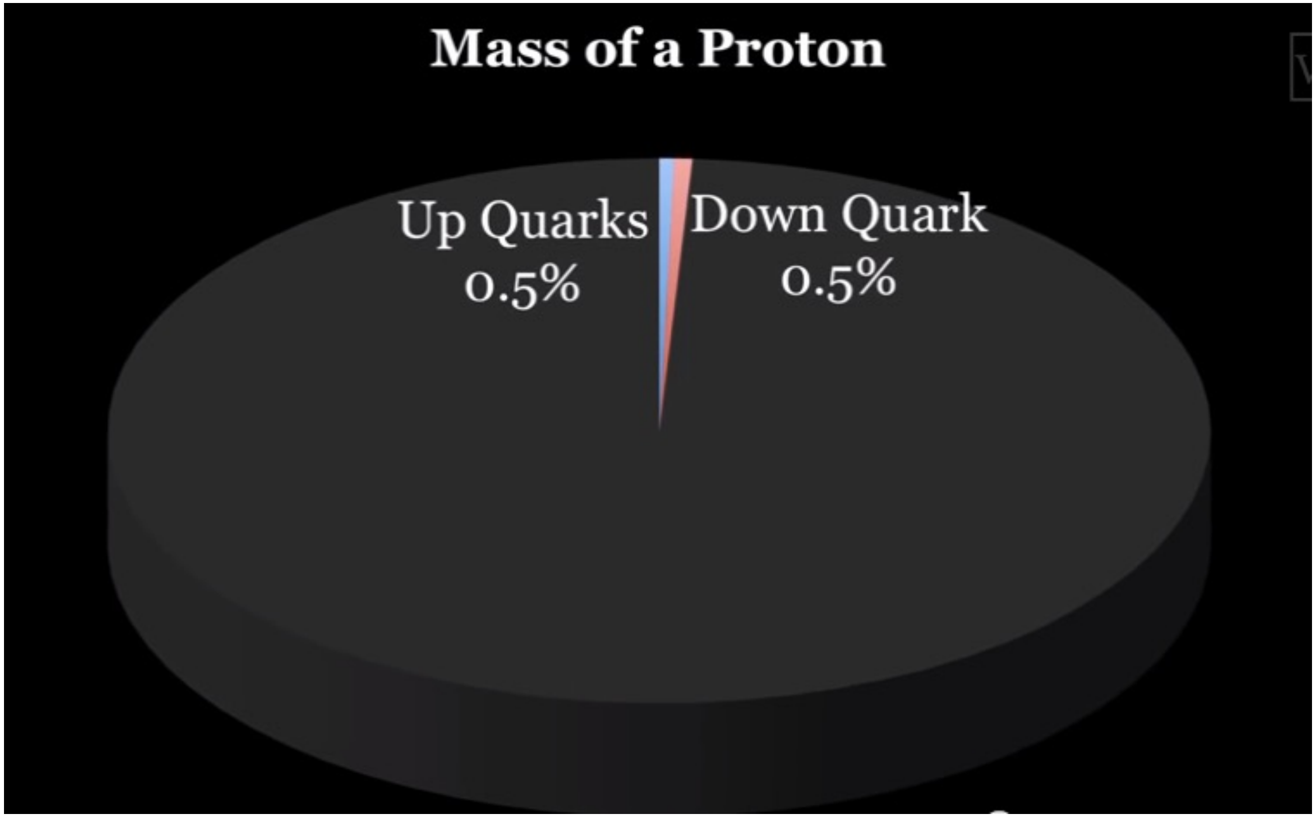
- Higgs generates masses for the up and down of the orders of

- A naive sum of the individual quark masses composing the proton/neutron gives us

Proton → 2.3 + 2.3 + 4.8 = 9.4 MeV
Neutron → 2.3 + 4.8 + 4.8 = 11.9 MeV



But, the proton/neutron masses are ~ 1 GeV (~100 times bigger)!

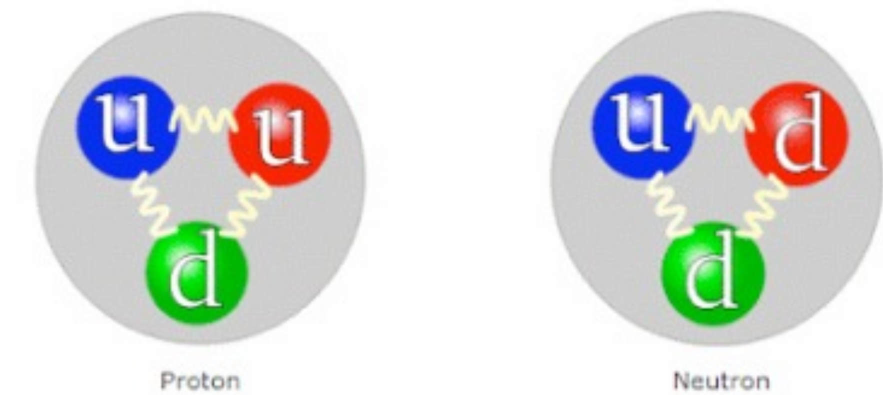


- Therefore only **1.0% - 1.5 % of the nucleon masses** are generated by the Higgs field…almost irrelevant.
- QCD dynamics should generate almost all mass.
- We need *a very efficient mechanism* which will be able to generate **98.5% of the proton/neutron mass.**

The weight of the world is QCD

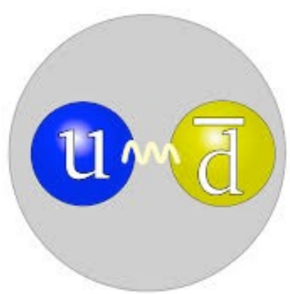
Andreas S. Kronfeld, Science 322, 1198 (2008)

- The proton/neutron has a mass ~1 GeV, it suggests that the quarks should **have an effective mass of the order ~300-350 MeV**



938 MeV 940 MeV

- But, what about the pion?



pion
140 MeV

It is too light!

$$\mathcal{L}_{\text{quarks}} = i\bar{q}\gamma^{\mu}\partial_{\mu}q$$

- We can decompose the quark fields in left and right hands components

$$q_L = \frac{1}{2}(1-\gamma_5)q \qquad q_R = \frac{1}{2}(1+\gamma_5)q$$

- *The chiral fields decouple one from each other!*

$$\mathcal{L}_{\text{quarks}} = i\bar{q}_L\gamma^{\mu}\partial_{\mu}q_L + i\bar{q}_R\gamma^{\mu}\partial_{\mu}q_R$$

- Therefore, the Lagrangian is invariant under the chiral transformation

$$q_L \to e^{i\alpha_L}q_L\,; \qquad q_R \to e^{i\alpha_R}q_R$$

Chiral Symmetry

- *Mass term breaks the chiral symmetry → it couples the chiral fields*

$$m_0\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

Chiral symmetry breaking

⇕

Generation of a quark mass

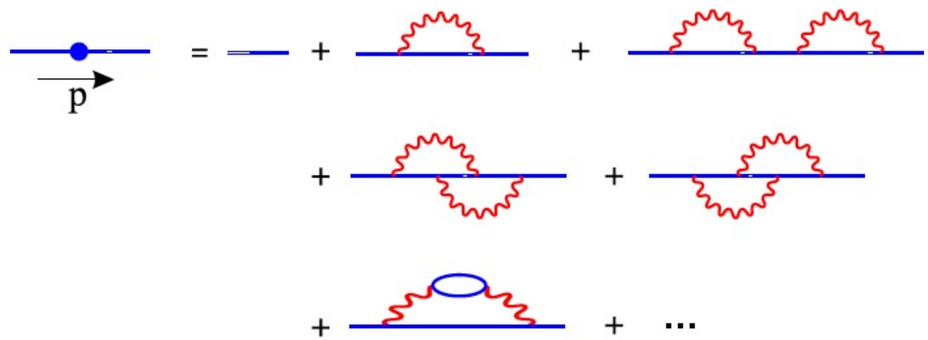
- The quarks u and d which form the visible matter have very small masses.

- *Chiral symmetry is an approximate symmetry* for our Lagrangian → in the chiral limit the pion is the Nambu-Goldstone boson – zero mass.

- Pion does not acquire a higher mass because it is protected by this approximate symmetry.

- Is it possible to generate masses of the order of 300-350 MeV *using perturbative methods*?

- In quantum field theory nothing is constant.
- Let us compute the corrections to the quark mass perturbatively:



$$\mathcal{M}(p) = \underline{m_0}\left(1 + c_1\,\alpha\,\ln\left(\frac{p^2}{\mu^2}\right) + c_2\,\alpha^2\,\ln^2\left(\frac{p^2}{\mu^2}\right) + \cdots\right)$$

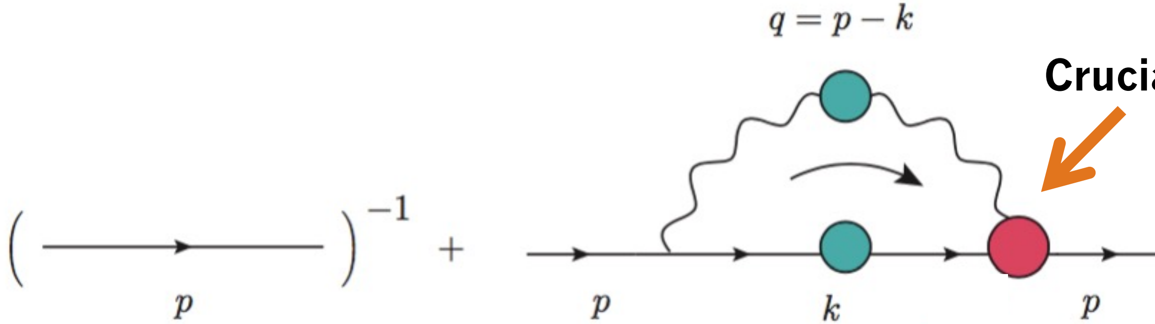
- Perturbatively, the dynamical mass at all orders are proportional to the bare mass m_0 (the mass that Higgs generates).

- **When m_0=0, all perturbative corrections vanish!**

- Therefore, the dynamical quark mass should be generated by *purely nonperturbative effects.*

Dynamical quark masses

$S^{-1}(p) =$



$$\underbrace{S^{-1}(p) = \not{p} - m_0 - iC_{\rm r} g^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S(k) \Gamma_\nu(k,p) \Delta^{\mu\nu}(p-k)}_{\textbf{Gap Equation}}$$

Full quark propagator

$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

Dynamical quark mass ⇨

$$\mathcal{M}(p) = \frac{B(p)}{A(p)}$$

Chiral Symmetry breaking occurs when $B \neq 0$

Simple Ansatz for Γ_μ

- The quark dynamical mass equation is given by

$$\mathcal{M}(p^2) = 4\int_k \mathcal{K}(p,k)\,\frac{\mathcal{M}(k^2)}{k^2 + \mathcal{M}^2(k^2)}$$

- The kernel $\mathcal{K}(p,k)$ depends on the approximation used for the quark gluon vertex.

- A simple Ansatz is the Abelian approximation for Γ_μ (satisfies the Ward identity).

$$q^\mu \Gamma_\mu(p,k) = S^{-1}(p) - S^{-1}(k)$$

- In this case

$$\mathcal{K}(p,k) \propto g^2 \Delta(p-k)$$

- However, the kernel does not have enough strength for generating the quark mass!

We have to "inflate" the kernel

Inflating the kernel

↓

means better knowledge of the quark-gluon vertex

↓

- **Use an improved quark-gluon vertex (abelianization not good)**

  ✓ **Slavnov-Taylor identity instead of Ward identity**

$$q^\mu \Gamma_\mu^{\rm STI}(q,p_2,-p_1) = F(q)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)S^{-1}(p_2)]\,.$$

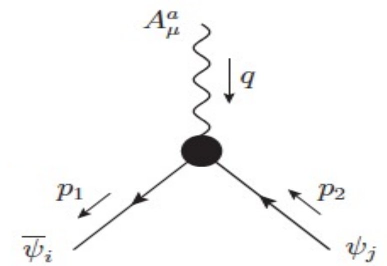
$$q^\mu \Gamma_\mu(p,k) = S^{-1}(p) - S^{-1}(k)$$

  ✓ **Include quark-ghost scattering kernel H is numerically crucial!**

$$D(q) = \frac{F(q)}{q^2}$$

A. C. A. and J. Papavassiliou, Phys. Rev. D83, 014013 (2011).
A. C. A., J. C. Cardona, M. N. Ferreira and J. Papavassiliou, Phys. Rev. D96, no. 1, 014029 (2017).

The quark-gluon vertex



- The vertex has 12 tensorial structures. It can be decomposed in a "longitudinal" and a transverse parts

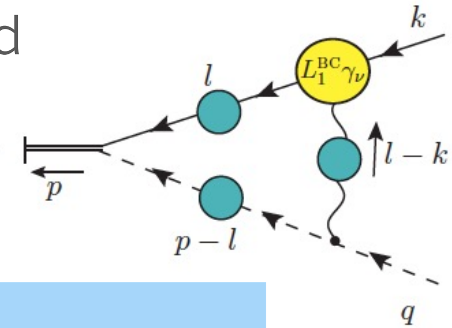
$$\Gamma_\mu(q,p_2,-p_1) = \Gamma^{\rm STI}_\mu(q,p_2,-p_1) + \Gamma^{\rm T}_\mu(q,p_2,-p_1)\,.$$

$$q^\mu \Gamma^{\rm T}_\mu(q,p_2,-p_1) = 0\,,$$

- The transverse part has 8 tensorial structures and it satisfies
- In this work we will not study/consider the contributions of the transverse pieces.

- The longitudinal part is composed by 4 structures and satisfies

$H^{[1]}(q,k,-p) \;=\; 1 \;-$



$$q^\mu\Gamma_\mu^{\rm STI}(q,p_2,-p_1) = F(q)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)S^{-1}(p_2)]\,.$$

whose decompositions are given by

$$H(q,p_2,-p_1) = X_0\mathbb{I} + X_1\not{p}_1 + X_2\not{p}_2 + X_3\tilde{\sigma}_{\mu\nu}p_1^\mu p_2^\nu\,,$$
$$\overline{H}(-q,p_1,-p_2) = \overline{X}_0\mathbb{I} + \overline{X}_2\not{p}_1 + \overline{X}_1\not{p}_2 + \overline{X}_3\tilde{\sigma}_{\mu\nu}p_1^\mu p_2^\nu\,,$$

$$\Gamma_\mu^{\rm STI}(q,p_2,-p_1) = L_1\gamma_\mu + L_2(\not{p}_1 - \not{p}_2)(p_1-p_2)_\mu + L_3(p_1-p_2)_\mu + L_4\tilde{\sigma}_{\mu\nu}(p_1-p_2)^\nu\,,$$

- Substituting the decompositions in the STI

We obtain…

- The quark-gluon form factors are given by

$$L_1 = \frac{F(q)}{2}\left\{A(p_1)[X_0 - (p_1^2 + p_1 \cdot p_2)X_3] + A(p_2)[\overline{X}_0 - (p_2^2 + p_1 \cdot p_2)\overline{X}_3]\right\} + \frac{F(q)}{2}\left\{B(p_1)(X_2 - X_1) + B(p_2)(\overline{X}_2 - \overline{X}_1)\right\};$$

functions of two momenta and the angle between them

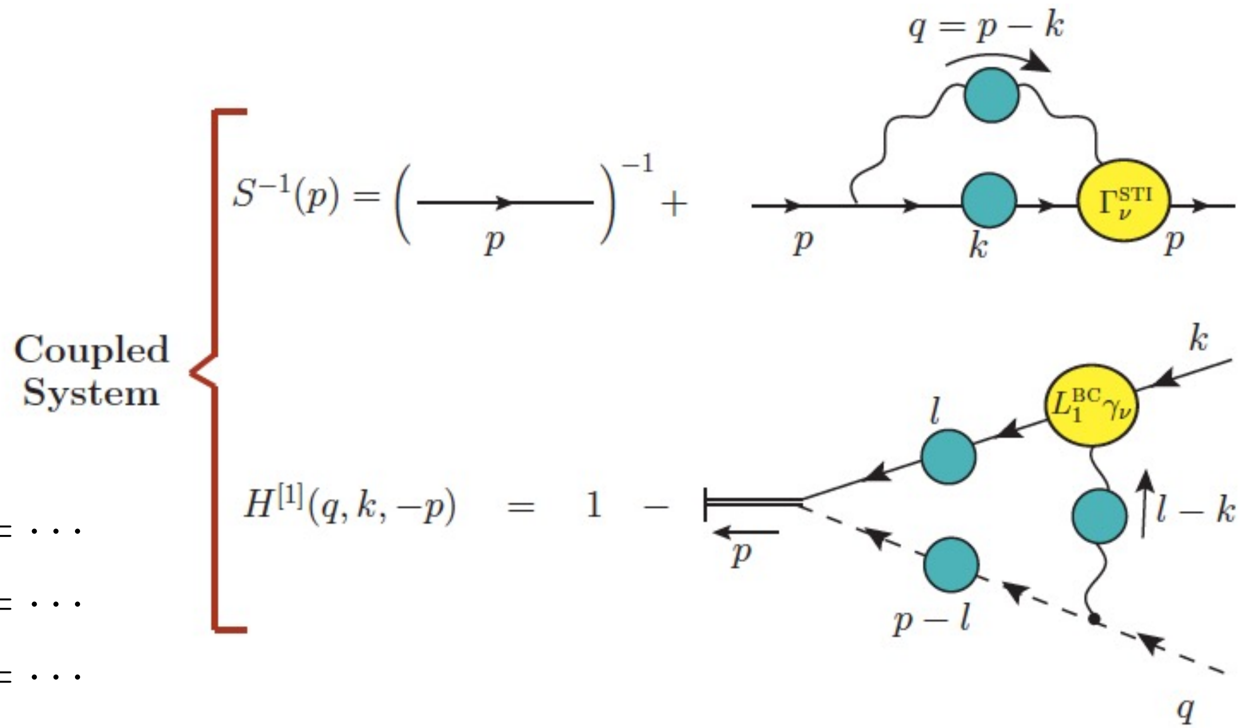
- Similar expressions will be obtained for

$$L_2 = \cdots$$
$$L_3 = \cdots$$
$$L_4 = \cdots$$

A. C. A. and J. Papavassiliou , Phys. Rev. D83, 014013 (2011).

Coupled system

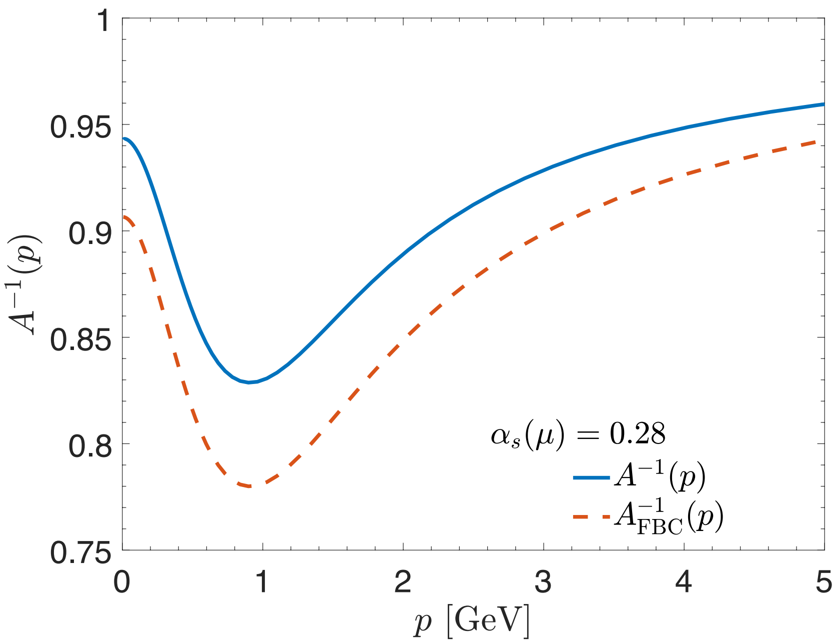
- We solve numerically a coupled system of *six nonlinear integral equations*

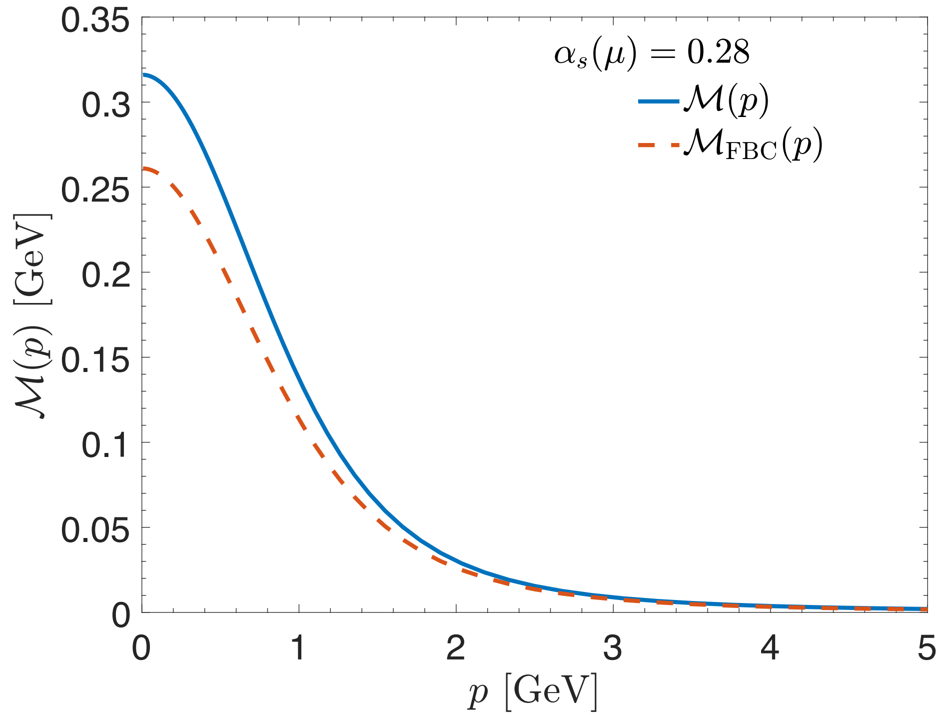


$$A(p) = \cdots$$
$$B(p) = \cdots$$
$$X_1(p, k, \theta) = \cdots$$
$$X_2(p, k, \theta) = \cdots$$
$$X_3(p, k, \theta) = \cdots$$
$$X_4(p, k, \theta) = \cdots$$

Numerical Results

- The quark propagator results





- Generates a dynamical mass of $\mathcal{M}(0) = 316\,\mathrm{MeV}$
- The effect of H increases ~20% of the value of the dynamical mass!

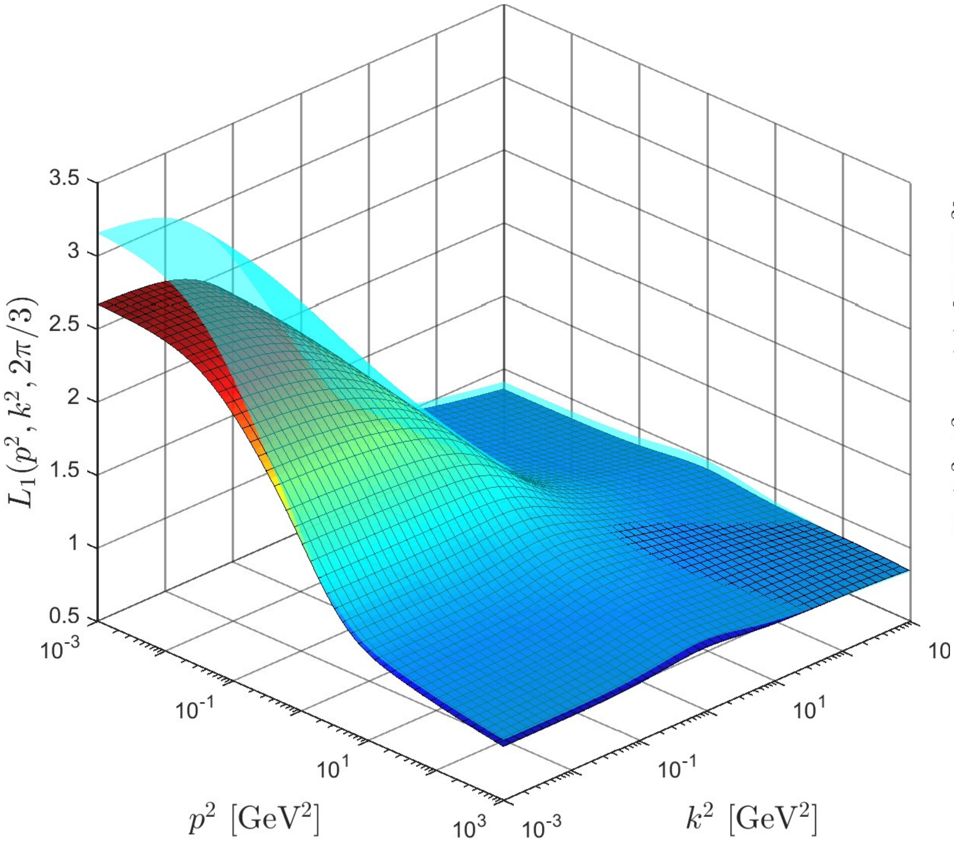
ACA, J. C. Cardona, M. N. Ferreira and J.Papavassiliou, Phys.Rev.D 98, no. 1, 014002 (2018)

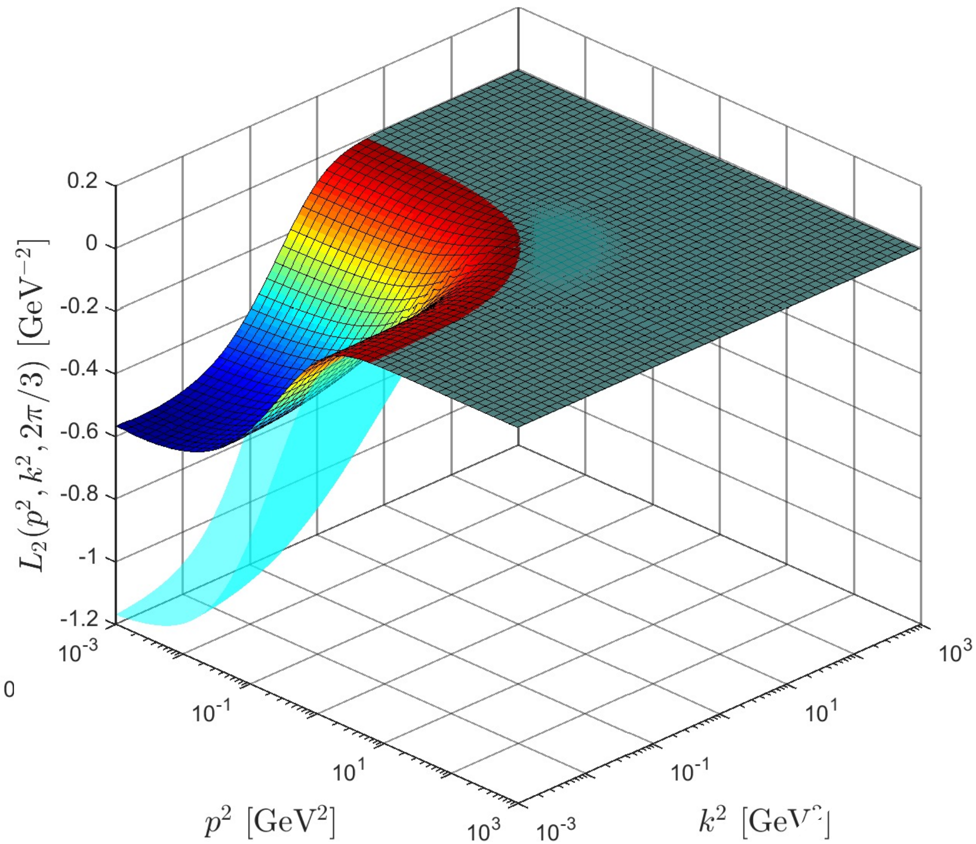
Quark-gluon form factors

$$\Gamma_\mu^{\rm STI}(q,p_2,-p_1) = \underline{L_1}\gamma_\mu + \underline{L_2}(\not{p}_1 - \not{p}_2)(p_1-p_2)_\mu + L_3(p_1-p_2)_\mu + L_4\tilde{\sigma}_{\mu\nu}(p_1-p_2)^\nu\,,$$

- Functions of three variables: 2 momenta p_1=p and p_2=k and the angle between them.

- Sizable effect of H

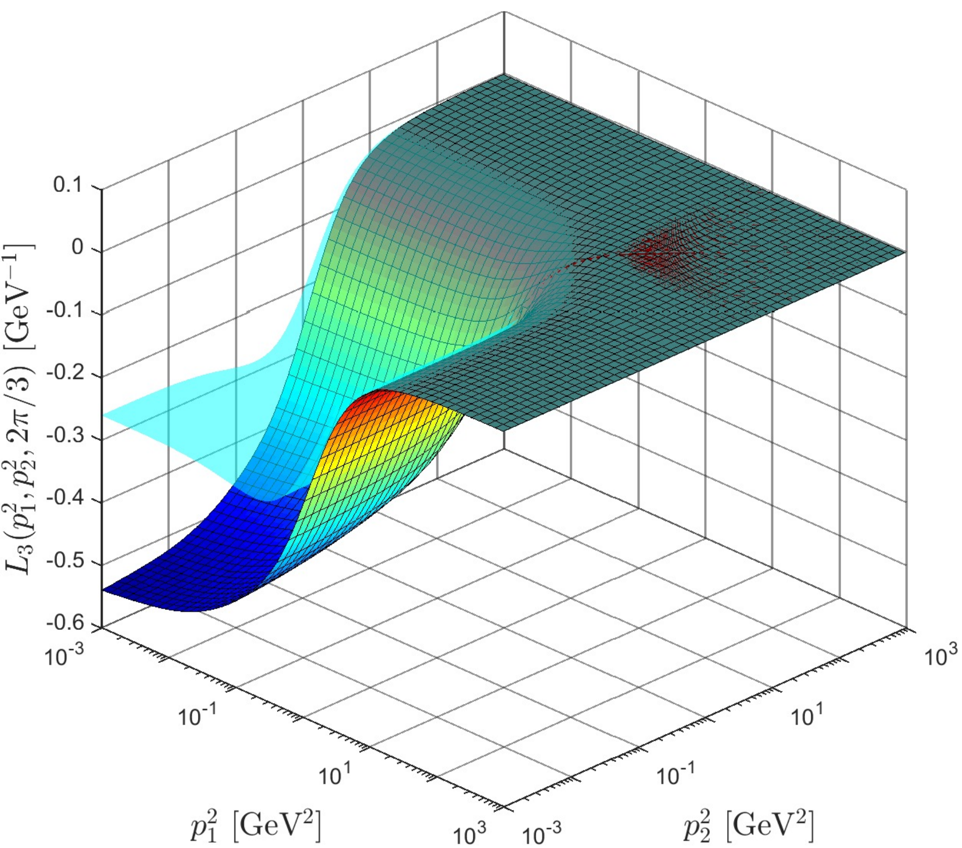


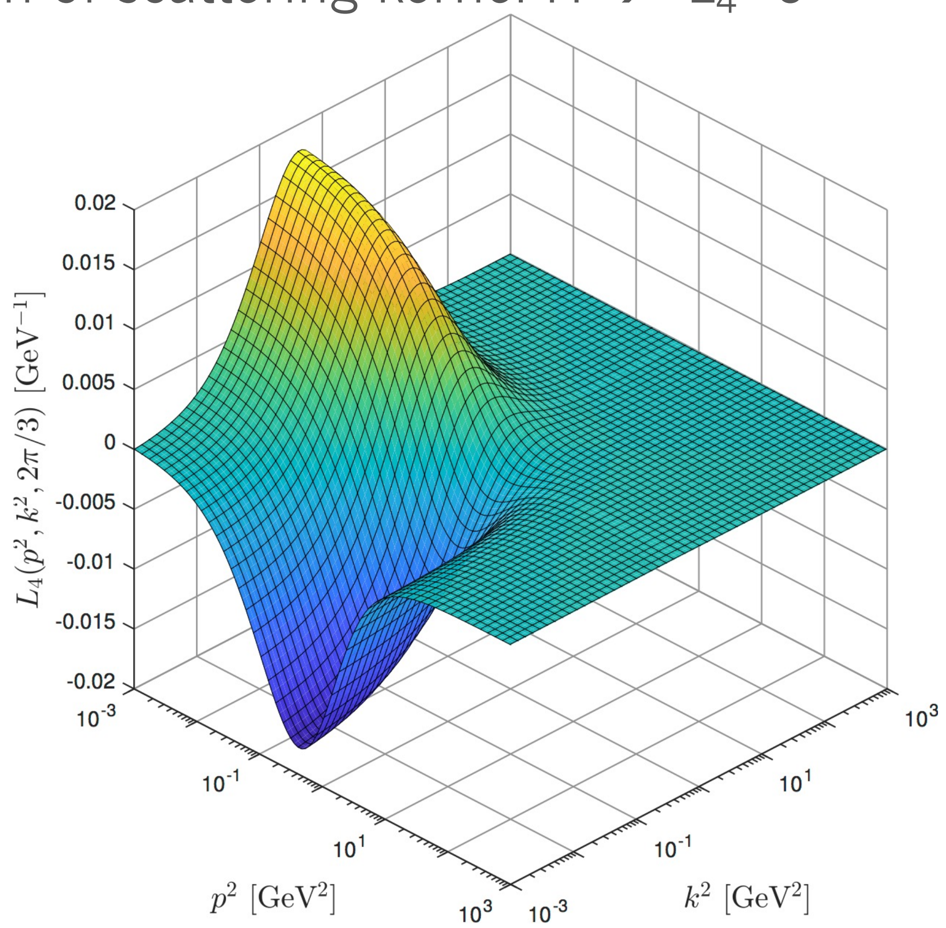


Quark-gluon form factors

$$\Gamma_\mu^{\rm STI}(q,p_2,-p_1) = L_1\gamma_\mu + L_2(\not{p}_1 - \not{p}_2)(p_1-p_2)_\mu + L_3(p_1-p_2)_\mu + L_4\tilde{\sigma}_{\mu\nu}(p_1-p_2)^\nu\,,$$

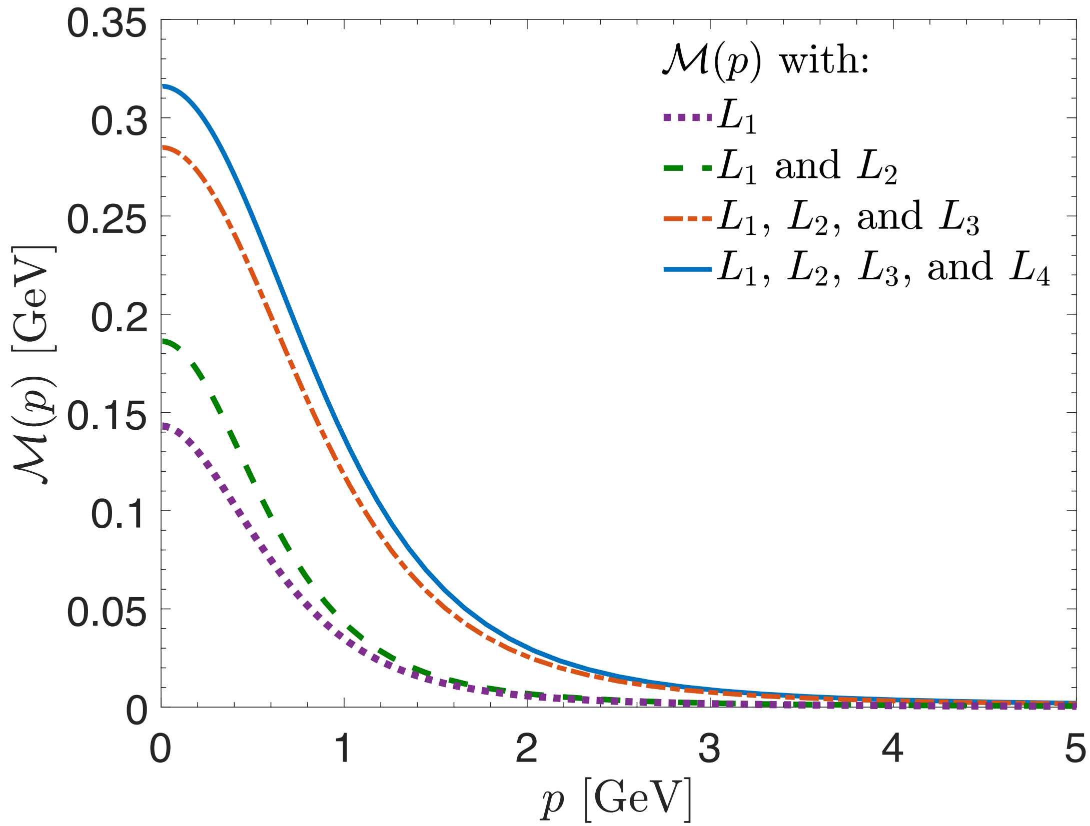
- L_4 has a suppressed structure but nonvanishing!
- When we neglected the contribution of scattering kernel H → L_4=0





Impact of the form factors on the quark mass

- When we turn on one by one the form factors



Form Factor	% of the mass generated
L_1	54%
L_2	13%
L_3	23%
L_4	10%

$$f_\pi = 90\,\mathrm{MeV}$$

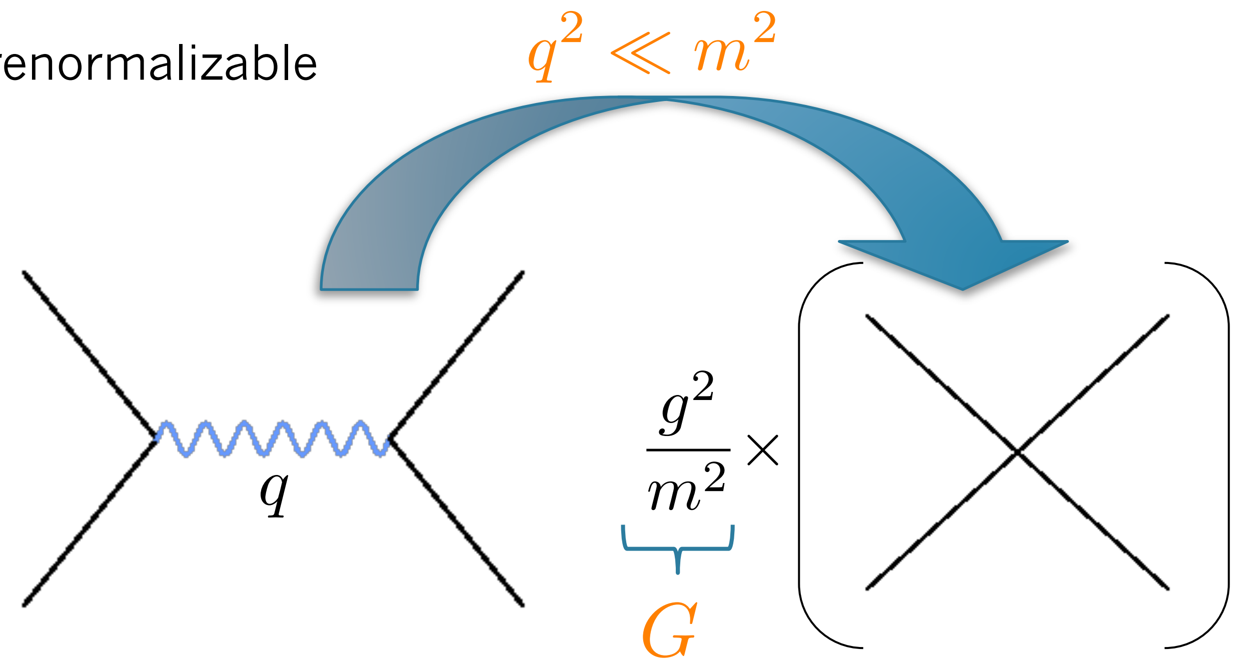
$$f_\pi^{exp} = 93\,\mathrm{MeV}$$

◎ L_4 is usually neglected, but its impact is of the order of the L_2

Bound States

Nambu-Jona Lasinio model from first principles

- A phenomenological successful model
- Not renormalizable



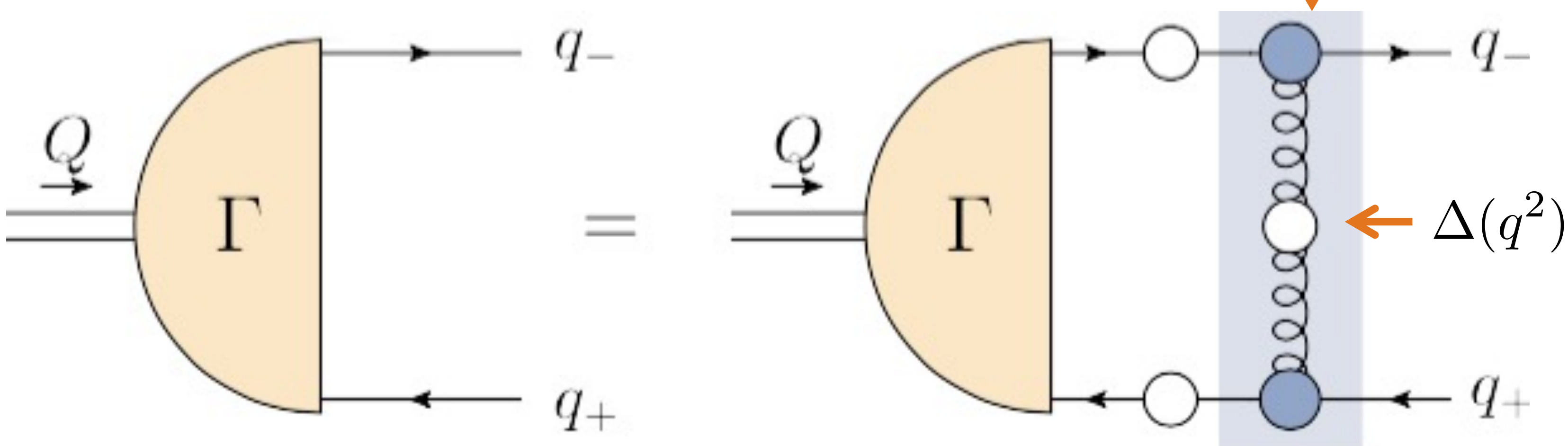
- Reproduced from QCD → limit of low momentum exchange

S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts and D. J. Wilson, Phys. Rev. C 84, 042202 (2011).

Bethe-Salpeter equation

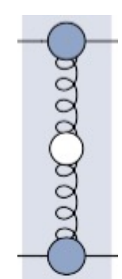
◎ Describes the formation of bound states (mesons) using the fundamental Green's function

$$\frac{g}{[1+G(q^2)]} \approx gF(q^2)$$



P. Maris and C. D. Roberts, Int. J. Mod. Phys. E 12, 297 (2003)
P. Maris and P. C. Tandy, Phys. Rev. C60, 055214 (1999)

✓ Fundamental ingredient of the BS equation



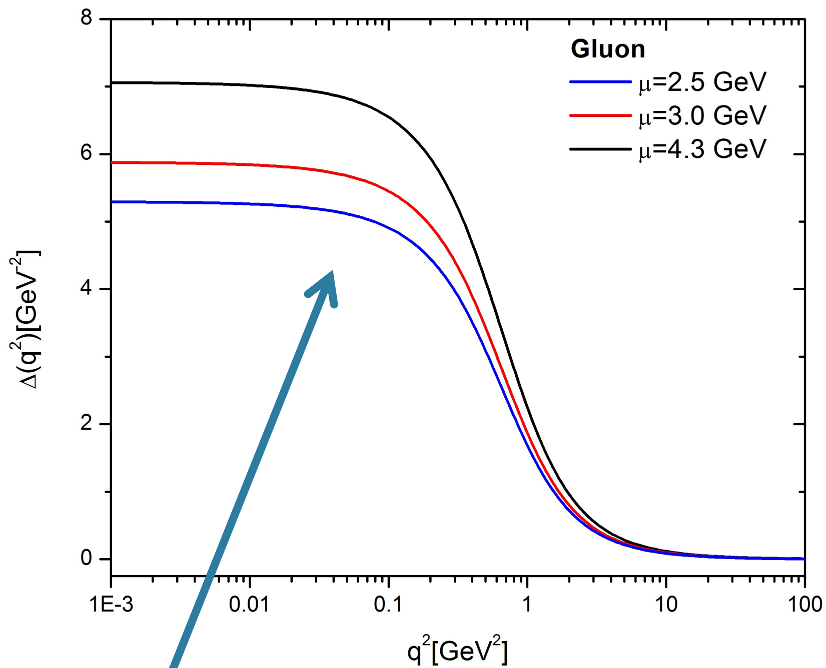
$$\frac{g}{[1+G(q^2)]} \approx gF(q^2)$$

✓ RGI quantity → μ–independent

$$\widehat{d}(p^2) = \alpha_s \widehat{\Delta}(p^2) = \frac{\alpha_s \Delta(p^2)}{[1+G(p^2)]^2},$$

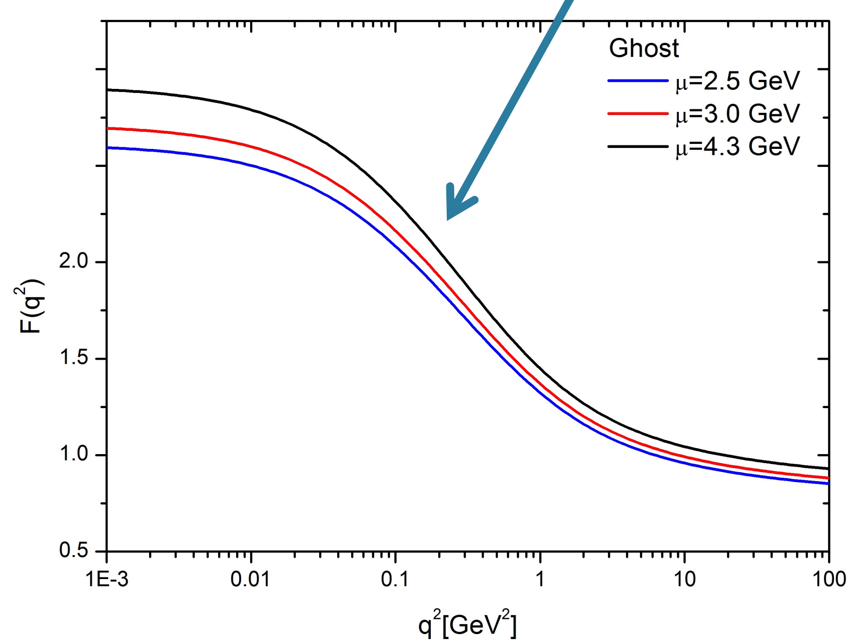
Dimensionful effective charge

The μ-dependent ingredients



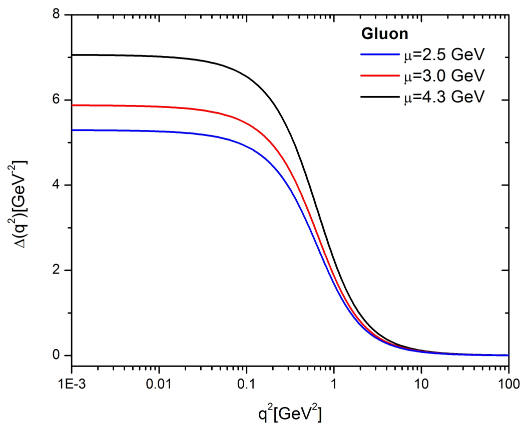
μ-dependence

μ-dependence

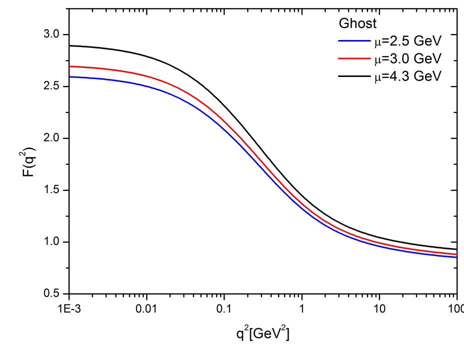


Forming the μ-independent product

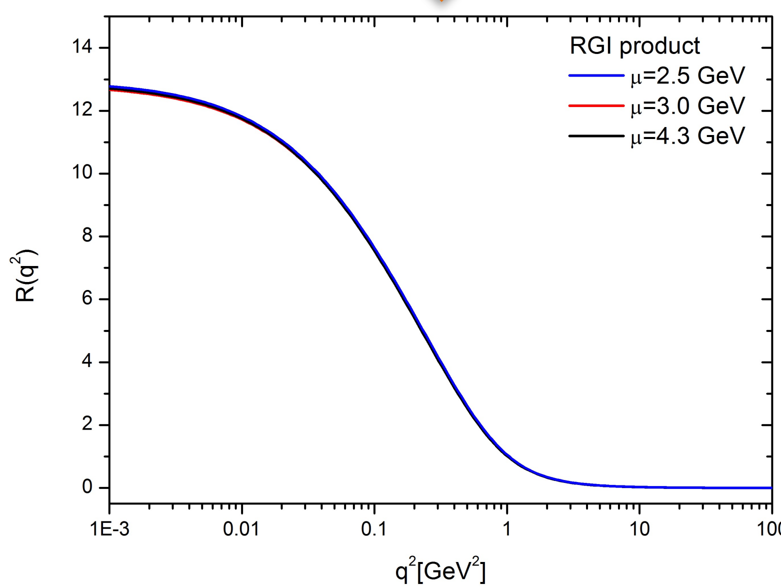
$\alpha_s(\mu)\times$



$\times \Big\{$



$\Big\}^2$



$\alpha_s(\mu)$ fixed from ghost SDE → reproduce the lattice data

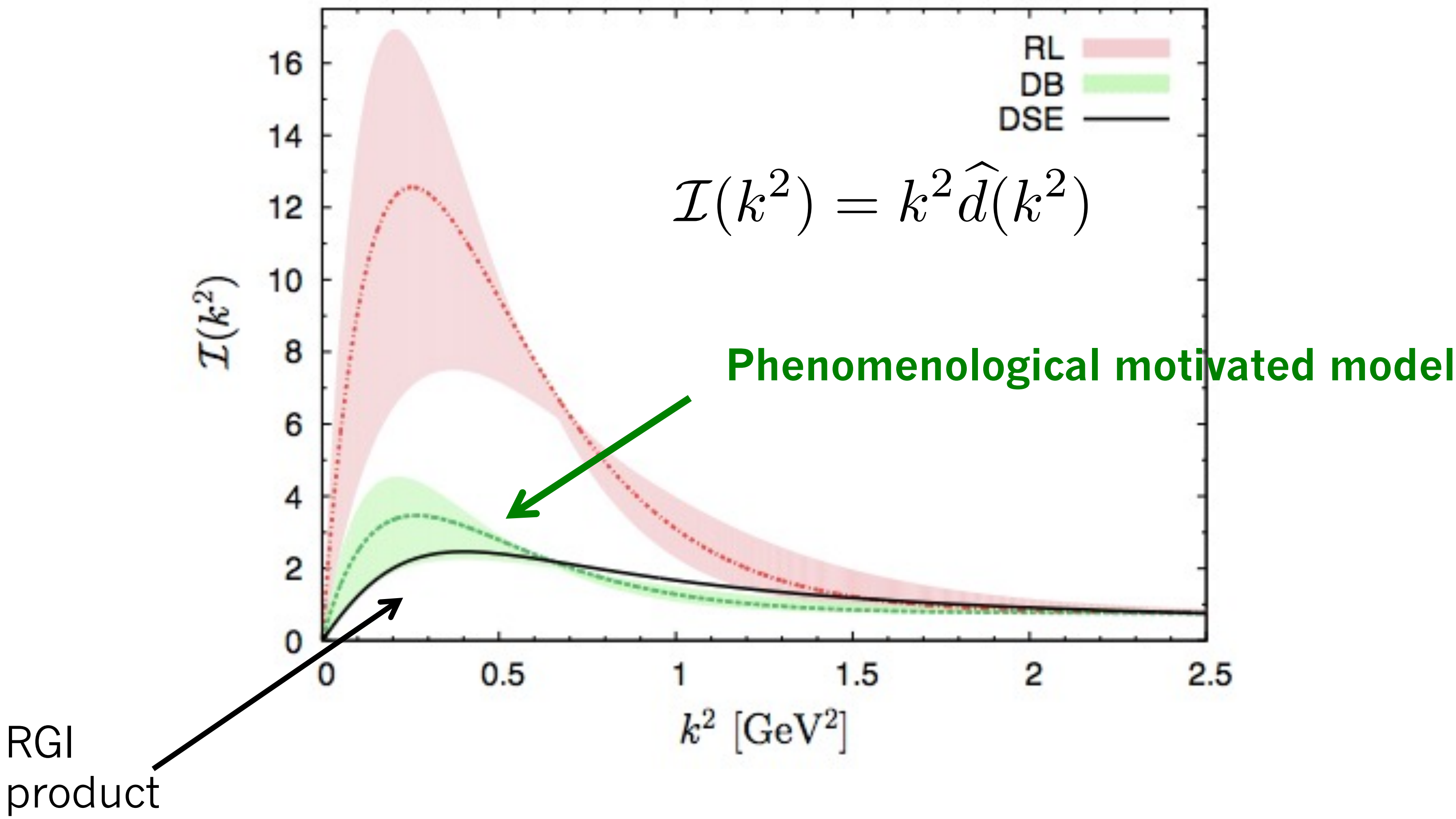
$\alpha_s(\mu) = 0.22$

$\alpha_s(\mu) = 0.30$

$\alpha_s(\mu) = 0.36$

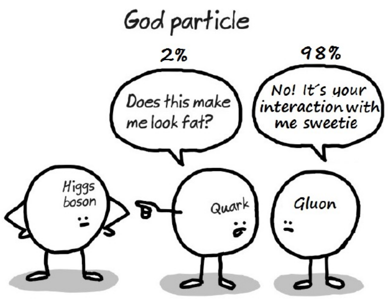
A. C. A., D. Binosi, J. Papavassiliou. and J. Rodriguez-Quintero, Phys. Rev. D80, 085018 (2009)

- The RGI product is a good match to the phenomenological motivated model behaviour required to describe a wide range of hadron observables using the most sophisticated BSE analysis.



D. Binosi, L. Chang. J. Papavassiliou, C. D. Roberts, Phys.Lett. B742 (2015) 183-188

Conclusions



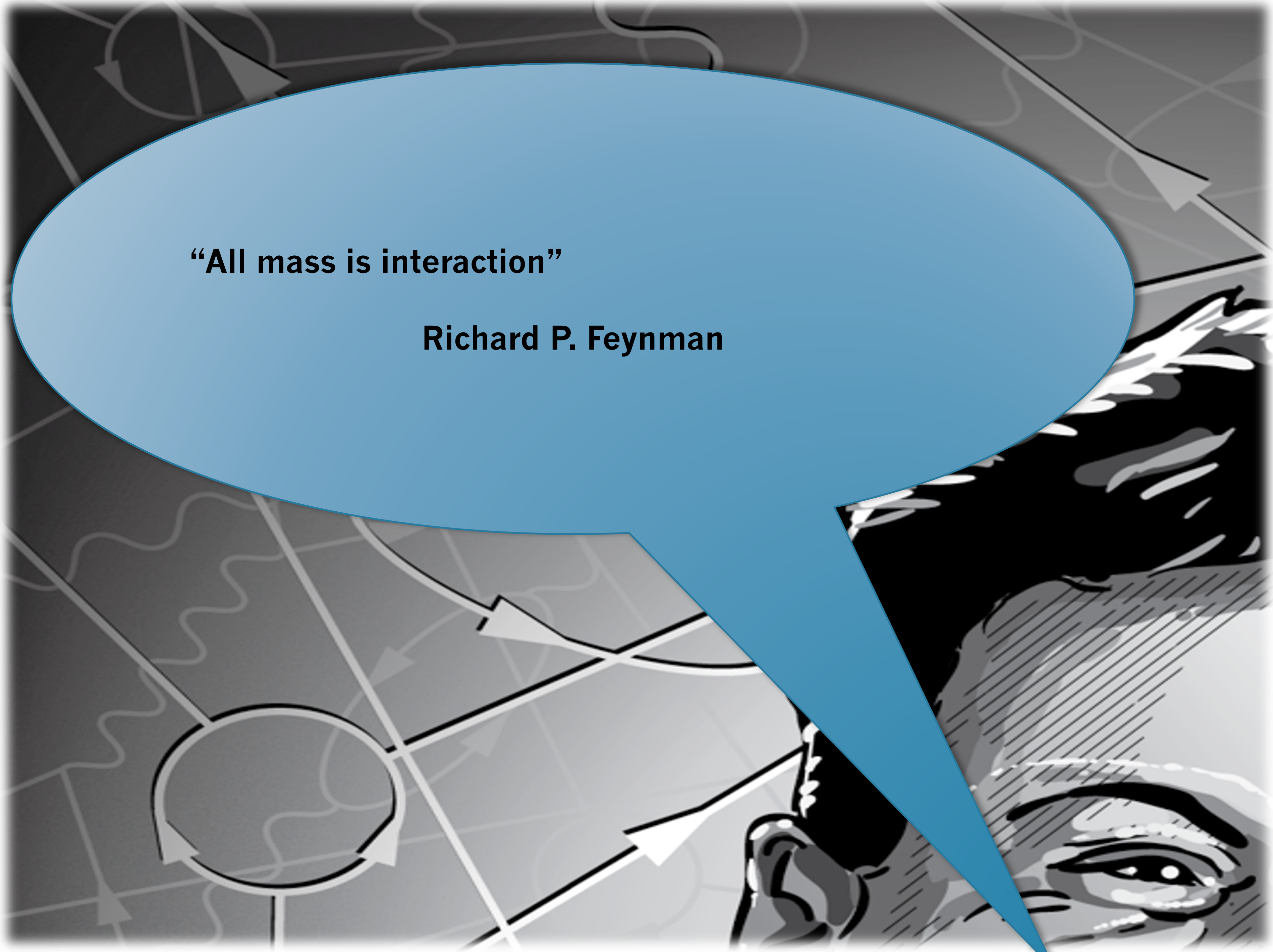
- The level of reliability of the SDEs has been improving steadily over the years.

- The synergy with the lattice opens new unexplored avenues. It must be nurtured and strengthened.

- CSB with realistic results can be obtained from the study of the gap equation, supplemented by:
  - ✓ Complete non-Abelian quark-gluon vertex
  - ✓ Non-perturbative ingredients from the lattice.

- The resulting effective charge is in good agreement with the interaction strengthen model employed on the BSE analysis.

- The fundamental issue of mass generation can be addressed in a self-consistent framework.


"All mass is interaction"
Richard P. Feynman