The deconvolution problem of deeply virtual Compton scattering

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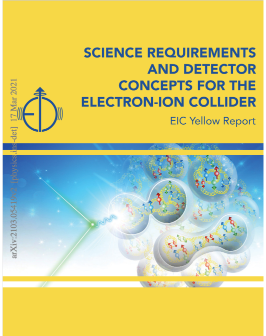
Exciting experimental promises



- With the **EIC yellow report** and **Chinese EIcC white paper**, deeply virtual Compton scattering (DVCS) will enter an era of more precise data over a much larger kinematic range.
- It is considered as a golden channel of extraction of generalised parton distributions (GPDs) and already provides many observables for fits. It is therefore necessary to re-examine the problem of unbiased extraction of GPDs from DVCS data.

Overview

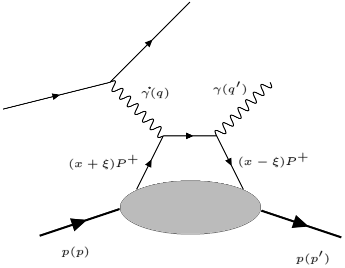
1. Deeply virtual Compton scattering and the structure of hadrons
2. Warming-up: extraction of gravitational form factors
3. Position of the problem: deconvoluting a Compton form factor
4. Shadow GPDs
5. Perspectives

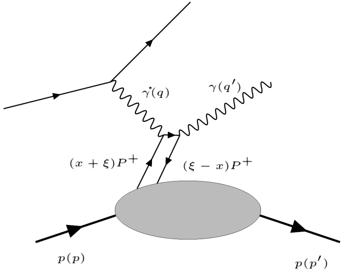
1. Deeply virtual Compton scattering and the structure of hadrons

Deeply virtual Compton scattering and the structure of hadrons

DVCS is the scattering of a lepton on a hadron via a photon of large virtuality, producing a real photon in the final state. It is an **exclusive process** with an intact recoil proton.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared





GPDs were introduced more than two decades ago in [Müller *et al*, 1994], [Radyushkin, 1996] and [Ji, 1997].

Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Deeply virtual Compton scattering and the structure of hadrons

Similarly to the introduction of **parton distribution functions** (PDFs) in the study of DIS,

- For a large photon virtuality $Q^2 = -q^2$, finite x_B and small total four-momentum transfer squared t, **factorisation theorems** describe DVCS in terms of a hard scattering part computable thanks to perturbative QCD, and a non-perturbative part described by **generalised parton distributions** (GPDs).
- The amplitude of DVCS is parametrised by **Compton form factors** (CFFs) $\mathcal{F}$, which write as convolutions of perturbative **coefficient functions** T_F^a and the **GPDs** F^a:

CFF convolution (leading twist) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} \, T_F^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) F^a(x, \xi, t, \mu^2) \tag{1}$$

$F^a(x, \xi, t, \mu^2) \to F^g(x, \xi, t, \mu^2)/x$ for the usual definition of gluon GPD

μ is the factorisation / renormalisation scale, α_s the strong coupling.

Deeply virtual Compton scattering and the structure of hadrons

Properties of GPDs

- For the proton, 4 GPDs without helicity transfer H^a, E^a, $\widetilde{H}^a$, $\widetilde{E}^a$ and 4 GPDs with helicity flip.
- GPDs are defined in terms of non-local matrix elements.
- They are real functions of (x, ξ, t, μ^2), with even parity in ξ.
- The forward limit $t \to 0$, $\xi \to 0$ gives back the usual **PDF**

$$H^q(x, \xi = 0, t = 0, \mu^2) = f^q(x, \mu^2) \tag{2}$$

- Their evolution with scale μ^2 generalizes the evolution kernels of the PDF (DGLAP) and the distribution amplitude (ERBL). [Müller, 1994]
- Because of the parity of the process, DVCS only involves the C-even – or singlet – GPDs, given *e.g.* for H^q by

$$H^{q(+)}(x, \xi, t, \mu^2) = H^q(x, \xi, t, \mu^2) - H^q(-x, \xi, t, \mu^2) \tag{3}$$

Deeply virtual Compton scattering and the structure of hadrons

Polynomiality of Mellin moments: [Ji, 1998], [Radyushkin, 1999]
Due to Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2)\xi^k \tag{4}$$

This property implies that the GPD is the Radon transform of a **double distribution** F^q (DD) with an added ***D*-term** on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| < 1\}$:

Double distribution formalism [Radyushkin, 1997], [Polyakov, Weiss, 1999]

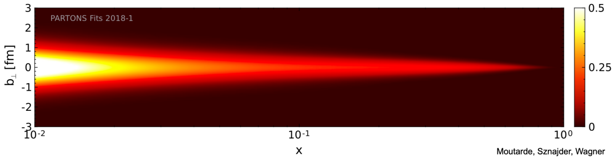
$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi) \left[F^q(\beta, \alpha, t, \mu^2) + \xi\delta(\beta) D^q(\alpha, t, \mu^2)\right] \tag{5}$$

Deeply virtual Compton scattering and the structure of hadrons

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \tag{6}$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks (valence GPD) in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework [Moutarde *et al*, 2018].

Deeply virtual Compton scattering and the structure of hadrons

- Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

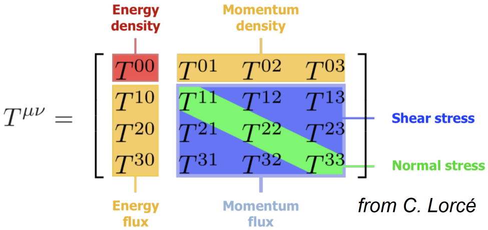
Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s\rangle = \bar{u}(p', s')\left\{ \frac{P^\mu P^\nu}{M} A_a(t,\mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t,\mu^2) + M\eta^{\mu\nu}\bar{C}_a(t,\mu^2) \right.$$
$$\left. + \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t,\mu^2) + B_a(t,\mu^2)\right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a(t,\mu^2) \right\} u(p,s) \tag{7}$$

where

$$\Delta = p' - p,\ \ t = \Delta^2,\ \ P = \frac{p+p'}{2} \tag{8}$$

Deeply virtual Compton scattering and the structure of hadrons



from C. Lorcé

In the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the **GFFs** w.r.t. variable $\vec{\Delta}$ [Polyakov, 2003].

- Example of such distribution: radial pressure anisotropy profile

$$s_a(r, \mu^2) = -\frac{4M}{r^2} \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^2} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left[t^{5/2}\, C_a(t, \mu^2) \right] \tag{9}$$

- This pressure profile can be extracted from **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{10}$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \tag{11}$$

Extraction of GFFs

2. Warming-up: extraction of gravitational form factors from experimental data

Extraction of GFFs

- At this stage, we don't need to fully extract the GPDs H or E to conveniently access the GFF $C_q(t, \mu^2)$. The **polynomiality property** gives that the GFF $C_q(t, \mu^2)$ only depends on the D-term via

$$\int_{-1}^{1} \mathrm{d}z \, z D^q(z, t, \mu^2) = 4 C_q(t, \mu^2) \tag{12}$$

- The experimental data is sensitive to the D-term through the **subtraction constant** defined by the **dispersion relation** (see *e.g.* [Diehl, Ivanov, 2007])

LO dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\, \mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 \mathrm{d}\xi' \, \mathrm{Im}\, \mathcal{H}(\xi', t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \tag{13}$$

The subtraction constant $\mathcal{C}_H(t, Q^2)$ is a function of the D-term given at LO by

$$\mathcal{C}_H(t, Q^2) = 2 \sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z, t, Q^2)}{1 - z} \tag{14}$$

Extraction of GFFs

- How do we get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \text{ ?} \tag{15}$$

- This is a prototype of the more complicated GPD extraction problem we will face later on. The known solution is through evolution.
- Let's expand the D-term on a basis of Gegenbauer polynomials

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{16}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{17}$$

Extraction of GFFs

- Because Gegenbauer polynomials diagonalize the LO ERBL [Lepage, Brodsky, 1979], [Efremov, Radyushkin, 1979] evolution kernel, each term $d_n^q(t,\mu^2)$ actually $d_n^{\pm}$ but that does not change the argument evolves multiplicatively with a different anomalous dimension. Since exponentials are a free family on any non-vanishing interval, the decomposition

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{18}$$

  is **unique, non-ambiguous and theoretically allows to entirely retrieve the** D**-term from the knowledge of the subtraction constant on any non-vanishing interval in** $Q^2 = \mu^2$.
- All is well on paper, but what about in real life?

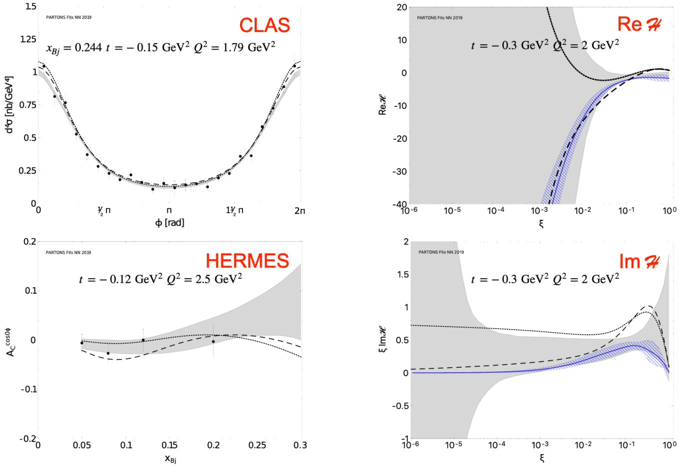
Extraction of GFFs

- We performed an analysis of the subtraction constant using most of the world DVCS dataset obtained over 17 years of experiments.
- The CFFs are fitted using a neural network (NN) to assess realistic uncertainties in [Moutarde *et al*, 2019]. Replicas of the NN are freely accessible on PARTONS (https://partons.cea.fr).
- The resulting uncertainty is considerably larger than in constrained parametrization fits.
- Complete details, notably about evolution, are found in **[Dutrieux et al, Eur.Phys.J.C 81 (2021) 4, 300]**.

Extraction of GFFs

Neural Networks

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614



PARTONS ANN

PARTONS 2018
EPJC 78 (2018) 11, 890

VGG
GK
LO evaluation

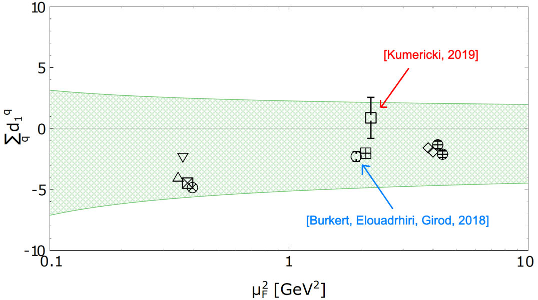
$$\xi \approx x_{Bj}/(2 - x_{Bj})$$

Extraction of GFFs

We then assume that the D-term takes the following form (multipole Ansatz for t-dependence and neglecting all terms in the Gegenbauer expansion except the first)

$$D^q(z,t,\mu^2) = 3(1-z^2)z\left(1-\frac{t}{M_D^2}\right)^{-\alpha} d_1^q(\mu^2) \tag{19}$$

Choosing beforehand the t dependence fixes qualitatively the obtained pressure profile, obtained by Fourier transforming with respect to $\vec{\Delta}$.



In green, 68% confidence interval found for $\sum_q d_1^q(t=0,\mu^2)$, a critical parameter to evaluate pressure profiles and results obtained by other studies (black markers). The parameter is compatible with 0 with current experimental data.

Extraction of GFFs

- What about we had not assumed only d_1^q was non-zero? Then as the space of functional dependence of $D^q(z)$ increases, so does the possibility of stumbling on **a D-term with negligible contributions to the subtraction constant, but considerable contributions to the GFF** (for instance, a D-term which is an eigenvector for a negligible eigenvalue of the linear operator represented by the subtraction constant). Since

$$\mathcal{C}_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu^2) \tag{20}$$

it is easy to see that at a given μ_0^2, if

$$d_1^q(t, \mu_0^2) = -d_3^q(t, \mu_0^2) \tag{21}$$

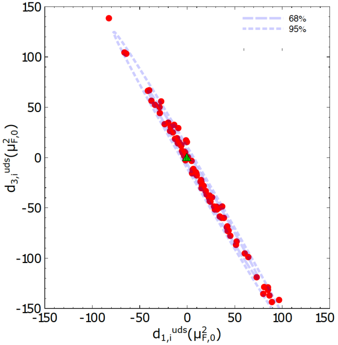
the subtraction constant vanishes, but not the GFF

$$C_q(t, \mu_0^2) = \frac{1}{5} d_1^q(t, \mu_0^2) \tag{22}$$

Extraction of GFFs

- If the effect of evolution is not significant enough, when allowing d_3^q to be non-zero, the result is polluted by large configurations where $d_1^q = -d_3^q$. Since the initial result was compatible with 0, these configurations become dominant.

$$d_1^{uds}(\mu_F^2) \quad -0.5 \pm 1.2 \qquad \longrightarrow \qquad \begin{array}{ll} d_1^{uds}(\mu_F^2) & 11 \pm 25 \\ d_3^{uds}(\mu_F^2) & -11 \pm 26 \end{array}$$



The correlation coefficient between d_1^q and d_3^q is of -0.997.

Conclusion: We have to find a way to evaluate the conditioning of our inverse problem given a functional liberty on the function of interest and a range of evolution in μ^2.

3. Position of the problem:
deconvoluting a Compton form factor

Deconvoluting a Compton form factor

We remind that DVCS experimental data are parametrized in terms of CFFs, which write as the convolution (given for GPD H^a)

$$\mathcal{H}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T_F^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) H^a(x, \xi, t, \mu^2) \tag{23}$$

Position of the problem

Assuming a CFF has been extracted from experimental data with excellent precision – and the different gluon and flavour contributions have been separated, through a global analysis with various targets and processes – we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{24}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" eq. (24) to recover $H^q(x, \xi, t, \mu^2)$ from $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$?**

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question remains essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of a LO shadow GPD

For a given scale μ_0^2,

$$\forall\xi, \forall t,\ T^q_{LO}(Q^2,\mu_0^2)\otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x,\xi=0,t=0,\mu_0^2) = 0 \tag{25}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{LO}(Q^2,\mu^2)\otimes H^q(\mu^2) = \mathcal{O}(\alpha_s(\mu^2)) \tag{26}$$

- Let H^q be a LO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same LO CFF up to a numerically small and theoretically subleading contribution.

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question remains essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t, T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{25}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, \; T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}({\alpha_s}^2(\mu^2)) \tag{26}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

4. Shadow GPDs

arXiv:2104.03836, to appear in Phys. Rev. D

Shadow GPDs at leading order

- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We also omit t since it is untouched by the convolution.
- **Leading order** It is well-known that the LO CFF only probes the GPD on the $x = \xi$ line and the D-term, so a LO shadow GPD is simply given by:

$$\mathrm{Im}\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) \propto H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \tag{27}$$

$$H^q(x, \xi = 0, \mu_0^2) = 0 \tag{28}$$

where $H^{q(+)}$ denotes the singlet GPD (x-odd part of the GPD).

Shadow GPDs at leading order

- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{29}$$

- The associated GPD is obtained by the linear Radon transform, given by the matrix R for $x > |\xi|$ *(not diverging for $|\xi| \to 1$ thanks to the cancellation of poles when $x \to 1$)*:

$$H^{q(+)}(x, \xi, \mu_0^2) = \sum_{u=1}^{N+1} \frac{1}{(1+\xi)^u} + \frac{1}{(1-\xi)^u} \sum_{v=0}^{N+1} q_{uv}\, x^v \quad \text{where} \quad q_{uv} = \sum_{m,n} R_{uv}^{mn} c_{mn} \tag{30}$$

$$R_{uv}^{mn} = \sum_{j=0}^{n} \frac{(-1)^{u+v+j}}{m+j+1} \binom{n}{j} \binom{j}{m-u+j+1} \binom{m+j+1}{v-n+j} \tag{31}$$

Shadow GPDs at leading order

- The Radon transform is expressed in terms of the rectangular matrix R between the appropriately chosen bases.
- R is a block-diagonal, triangular inferior matrix with a correct ordering of both bases.
- The inverse Radon transform is obtained by inverting a submatrix of R. We find

$$c_{m,n} = -\binom{n+m}{m}(n+m+1)\sum_{k=0\,\mathrm{even}}^{m}\binom{m}{k}E_{m-k}\;q_{k+1,n+m+1} \tag{32}$$

where the E_{2i} are Euler numbers, notably defined by

$$\frac{1}{\cosh(t)} = \sum_{k=0\,\mathrm{even}}^{\infty} E_k\,\frac{t^k}{k!}, \quad \mathrm{or} \quad \sum_{k=0\,\mathrm{even}}^{n}\binom{n}{k}E_k = 0 \text{ for } n \text{ even} \geq 1 \tag{33}$$

Shadow GPDs at leading order

- For our LO shadow GPD, we first want $H^{q(+)}(\xi,\xi,\mu_0^2) = 0$, so we notice that

$$H^{q(+)}(\xi,\xi,\mu_0^2) = \sum_{w=1}^{N+1} \frac{k_w}{(1+\xi)^w} \quad \text{where} \quad k_w = \sum_{u,v} C_w^{uv}\, q_{uv}\,, \quad C_w^{uv} = (-1)^{u+v+w} \binom{v}{u-w}$$

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{34}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{34}$$

- We then want $H^{q(+)}(x, \xi = 0, \mu_0^2) = 0$, so we notice that

$$H^{q(+)}(x, 0, \mu_0^2) = \sum_{w=0}^{N+1} q_w x^w \quad \text{where} \quad q_w = \sum_{u,v} Q_w^{uv}\, q_{uv}\,, \quad Q_w^{uv} = 2\delta_w^v$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{35}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{34}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{35}$$

- Both linear systems $C.R$ and $Q.R$ are systems of $\sim N$ equations for $\sim N^2$ variables, so the number of solutions grows quadratically with N, order of the polynomial DD.

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2)=0 \implies (c_{mn})_{m,n}\in \ker(C.R) \tag{34}$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2)=0 \implies (c_{mn})_{m,n}\in \ker(Q.R) \tag{35}$$

LO shadow GPDs

Here is an example of an infinite family of LO shadow DDs, each being of degree $N\geq 9$ odd

$$F_N(\beta,\alpha,\mu_0^2)=\beta^{N-8}\Bigg[\alpha^8-\frac{28}{9}\alpha^6\left(\frac{N^2-3N+20}{(N+1)N}+\beta^2\right)+\frac{10}{3}\alpha^4\left(\frac{N^2-7N+40}{(N+1)N}+\frac{2(N^2-3N+44)}{3(N+1)N}\beta^2+\beta^4\right)$$
$$-\frac{4}{3}\alpha^2\left(\frac{N^2-11N+60}{(N+1)N}-\frac{N-8}{N}\beta^2-\frac{N^2-3N-28}{(N+1)N}\beta^4+\beta^6\right)+\frac{1}{9}(1-\beta^2)^2\left(\frac{N^2-15N+80}{(N+1)N}-\frac{2(N-8)}{N}\beta^2+\beta^4\right)\Bigg] \tag{36}$$

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \tag{37}$$

An explicit calculation of each term for our polynomial double distribution gives that

$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right) \left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w} \right] \tag{38}$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2) \left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w} \right]$$

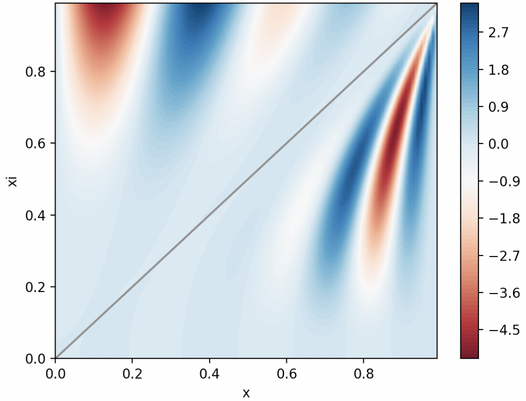
Shadow GPDs at next-to-leading order

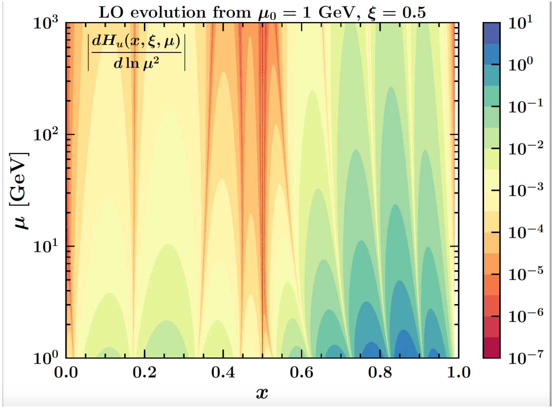
- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found with a polynomial DD of order $N = 21$.
- Furthermore, we add the condition that the DD vanishes at the edges of its support to ensure continuity at the $(x, \xi) = (1, 1)$ point. Indeed,

$$\lim_{\varepsilon \to 0} H^{q(+)}\left(1 - \frac{\varepsilon}{\lambda}, 1 - \varepsilon\right) = \int_0^{1/\lambda} \mathrm{d}\alpha\, F^{q(+)}(1 - \alpha, \alpha) \tag{40}$$

  so unless $F^{q(+)}(1 - \alpha, \alpha) = 0$, the limit would be different depending on the path taken to the limit coming from the $x > |\xi|$ region.
- Adding this condition, a first solution is found with a polynomial DD of order $N = 25$ (see below).

Shadow GPDs at next-to-leading order





Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ [Bertone, 2018] and PARTONS. Notice that the diagonal $x = \xi$ barely evolves.

Shadow GPDs at next-to-leading order

- Under evolution, the quark component of the NLO CFF writes

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right)$$

$$+\alpha_s(\mu^2)\, C_0^q \otimes K_{qq}^{(0)} \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{\mu_0^2}\right) + \mathcal{O}(\alpha_s^2(\mu^2)) \tag{41}$$

- By construction of an NLO shadow GPD, we specifically cancelled all terms on the first line. Since

$$C_{coll}^q + C_0^q \otimes K_{qq}^{(0)} = 0 \tag{42}$$

by requirement that the CFF does not exhibit a scale dependence other than the residual dependence resulting from the perturbative truncation, the first term of the second line vanishes as well.

- The evolution of the diagonal corresponds to the evolution of the LO CFF, and is also of order $\mathcal{O}(\alpha_s^2(\mu^2))$, explaining so specifically small.

Shadow GPDs at next-to-leading order

- Under evolution, the quark component of the NLO CFF writes

$$\mathcal{H}^q(\xi, Q^2) = \cancel{C_0^q \otimes H^{q(+)}(\mu_0^2)} + \alpha_s(\mu^2)\, \cancel{C_1^q \otimes H^{q(+)}(\mu_0^2)} + \alpha_s(\mu^2)\, \cancel{C_{coll}^q \otimes H^{q(+)}(\mu_0^2)} \log\left(\frac{\mu^2}{Q^2}\right)$$

$$+\alpha_s(\mu^2)\, C_0^q \otimes K_{qq}^{(0)} \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{\mu_0^2}\right) + \mathcal{O}(\alpha_s^2(\mu^2)) \qquad (43)$$

- By construction of an NLO shadow GPD, we specifically cancelled all terms on the first line. Since

$$C_{coll}^q + C_0^q \otimes K_{qq}^{(0)} = 0 \qquad (44)$$

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$$+\alpha_s(\mu^2)\, \cancel{C_0^q \otimes K_{qq}^{(0)} \otimes H^{q(+)}(\mu_0^2)} \log\left(\frac{\mu^2}{\mu_0^2}\right) + \mathcal{O}(\alpha_s^2(\mu^2)) \tag{45}$$

- By construction of an NLO shadow GPD, we specifically cancelled all terms on the first line. Since

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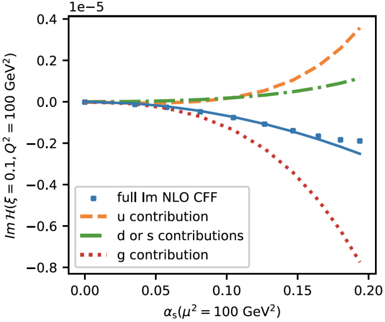
- The evolution of the diagonal corresponds to the evolution of the LO CFF, and is also of order $\mathcal{O}(\alpha_s^2(\mu^2))$, explaining so specifically small.

Shadow GPDs at next-to-leading order

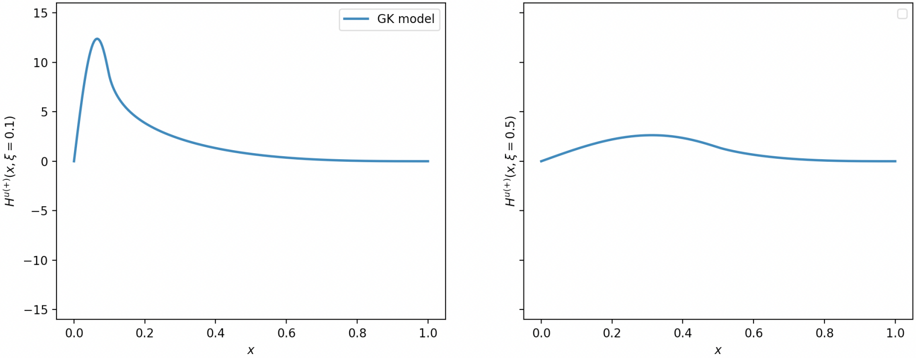
- By linearity of both the CFF convolution and the evolution equation, we can evaluate separately the contribution to the CFF of a quark shadow NLO GPD under evolution.
- We probe the prediction of evolution as $\mathcal{O}(\alpha_s^2(\mu^2))$ with our previous NLO shadow GPD on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.



- The fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

Shadow GPDs at next-to-leading order

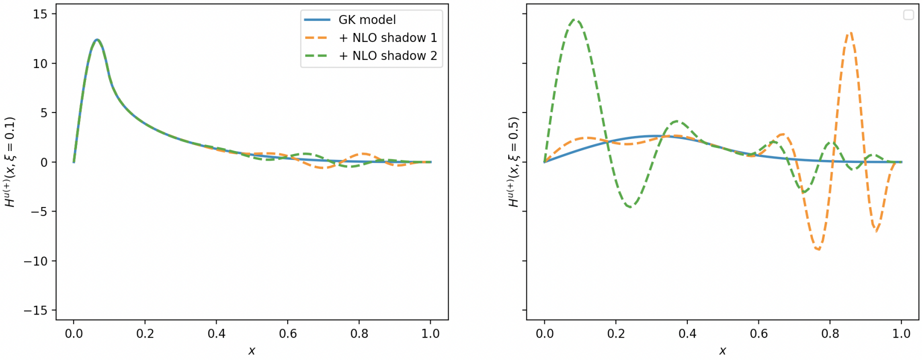
In practice, this is the Goloskov-Kroll (GK) GPD model at scale 1 GeV^2



$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

Shadow GPDs at next-to-leading order

The orange and brown models are **GK + NLO shadow GPDs**. For ξ close to 0 and x close to ξ, by design, they are very close, but vastly different otherwise. They give rise to NLO CFFs which are exactly identical at this scale, and different by a negligible amount for expected Q^2 lever arm.

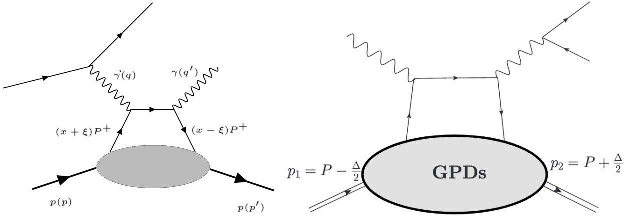


$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

5. Perspectives

Perspectives

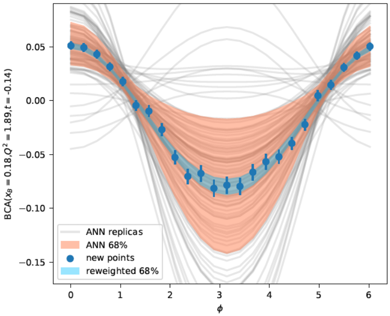
- We have explicitly demonstrated the difficulties of extracting GPDs with a pure DVCS + DIS approach even at NLO. It is foreseeable this discussion extends to higher orders of DVCS.
- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger *et al*, 2002]. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller *et al*, 2012]) makes it subject to the same shadow GPDs.

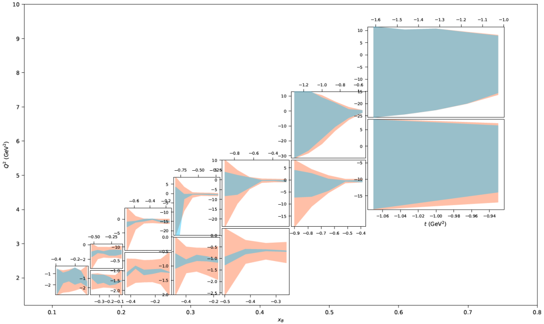


DVCS (left) and TCS (right)

Perspectives

- Reducing uncertainties on CFFs itself, even if not a solution to the deconvolution problem presented here, is a very useful task. *e.g.* hadron matter properties were compatible with 0 largely because of the uncertainty on $\mathrm{Re}\,\mathcal{H}$ in [Dutrieux *et al*, Eur.Phys.J.C 81 (2021) 4, 300].
- The proposal to install a positron beam at JLab [Afanasev *et al*, 2019] can help on this task. We have performed in **[Dutrieux et al, arXiv:2105.09245]** a **reweighting** of our neural network replicas of CFFs against simulated new experimental points.





Perspectives

- **Deeply virtual meson production** (DVMP) [Collins *et al*, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{47}$$

  with $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Perspectives

- **New experimental channels**: more experimentally challenging processes offer a richer access to GPDs thanks to more handles with kinematic variables.
  - Double deeply virtual Compton scattering (DDVCS) – proposed at JLab with SOLID (LOI12-15-005) and CLAS12 (LOI12-16-004) – which gives access directly to the (x, ξ) value of GPDs in the ERBL region at LO.
  - Multiparticle production: diphoton [Pedrak *et al*, 2017], photon-rho [Boussarie *et al*, 2017]
- **Lattice QCD**: low order Mellin moments of GPDs will not change significantly the previously exposed picture. Where a new order of DVCS put N constraints on a DD of polynomial order N, a new Mellin moment only brings a finite number of constraints.
- Extractions of the x-dependence of parton distributions are an interesting prospects, which we start to consider.

Perspectives

Positivity constraints [Radyushkin, 1999], [Pire *et al*, 1999], [Diehl *et al*, 2001], [Pobylitsa, 2002]

- Stemming from the representation of GPDs as overlap between light-front wave functions, positivity constraints are a Cauchy-Schwart like inequality relating GPDs to the PDFs, *e.g.* for $x \geq |\xi|$

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{1}{1-\xi^2} f^q\left(\frac{x+\xi}{1+\xi}\right) f^q\left(\frac{x-\xi}{1-\xi}\right)} \qquad (48)$$

- This inequality puts a maximal bound on the size of shadow GPDs in the DGLAP region, and is especially constraining for large x.
- Since shadow GPDs are maximally violating positivity (their forward limit is 0), they are a tool to correct a model giving satisfactory experimental agreement, but violating positivity. (Work in progress)

6. Conclusion

Conclusion

- Explicit demonstration of LO and NLO shadow GPDs of considerable size with a very small and subleading contribution to CFFs. **Such shadow GPDs will be hidden in typical statistical and systematic uncertainties of DVCS**. TCS or LO DVMP face similar issues. We foresee that our discussion can be extended to higher order DVCS. Other exclusive processes will help discriminate the DVCS shadow GPDs. Especially DDVCS or Lattice QCD for instance should escape the dimensionality of data problem.
- Potential impact on **hadron tomography** due to the $\xi \to 0$ extrapolation, determination of **OAM** and mechanical properties to study.
- An extraction of GPDs with lesser systematic uncertainty requires a **multi-channel analysis**, and the development of integrated analysis tools, like **PARTONS**
- More precise data over a much larger Q^2 range promised by future colliders will be very welcomed here and for the extraction of mechanical properties as well.
- More theoretical constraints, like **positivity** could play a significant role in reducing the uncertainty.