Probing nucleon structure at NLO

Víctor Martínez-Fernández
PhD student at the National Centre for Nuclear Research (NCBJ, Poland)
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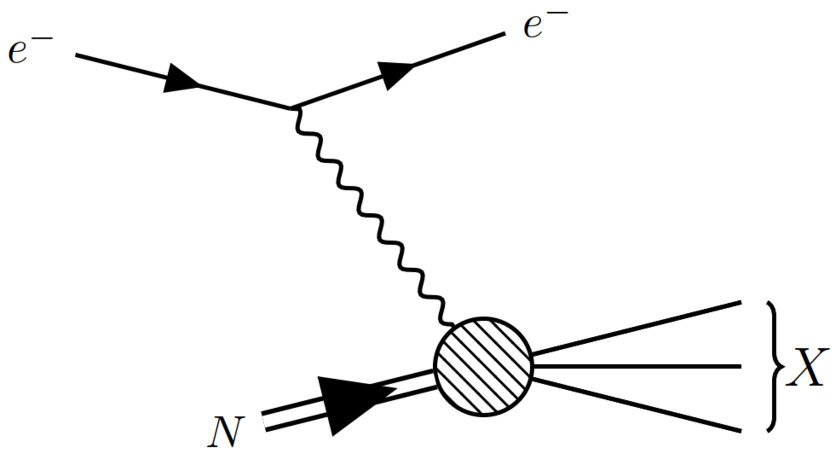
Why NLO corrections?

- Precise determination of GPDs (so nucleon tomography)
- Universality testing in process that at Born level look alike

Refreshing Deep Inelastic Scattering (DIS)



Factorization in cross-section

$$d\sigma_{\mathrm{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

pQED

Non-perturbative QCD

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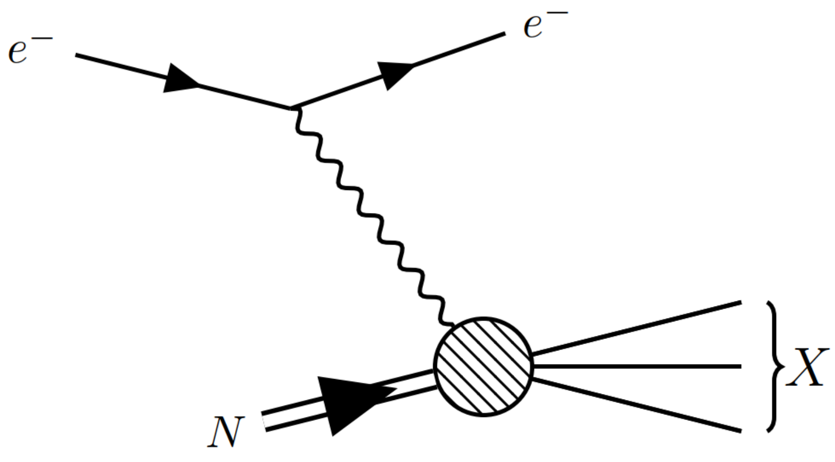

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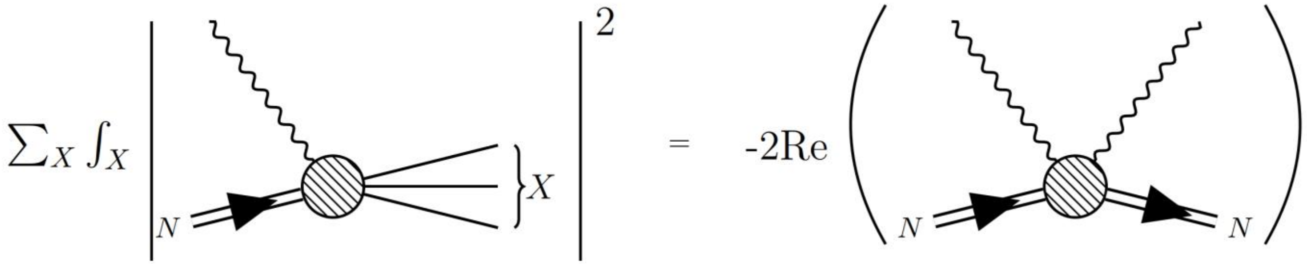
Optical theorem to DIS



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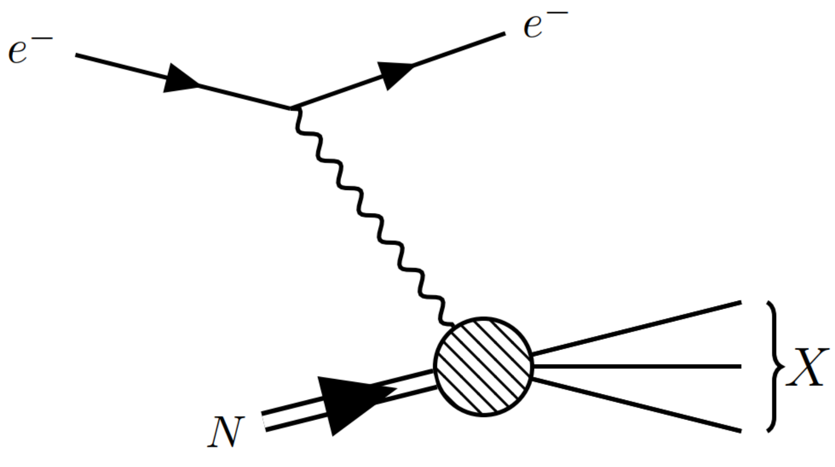

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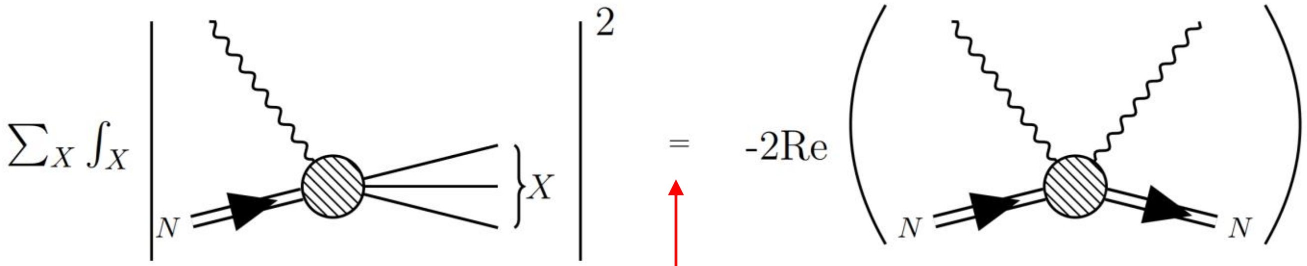

$$\sum_X \int_X |X\rangle\langle X| = 1$$ → DIS is inclusive process

Parton Distribution Functions (PDFs)

- Extracted from the hadronic tensor $W_{\mu\nu}$, they determine the internal nucleon structure

$$\mathrm{PDF}(x) = \frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixp^- z^+} \langle P|\bar{q}_f(-z/2)\gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2)|P\rangle\big|_{z_\perp = z^- = 0}$$

Wilson line definition

$$\mathcal{W}[z_1^+, z_2^+] = \mathbb{P}\exp\left[ig\int_{z_2^+}^{z_1^+} d\eta^+ A^-\right]$$

Samuel Wallon. Hard exclusive processes in perturbative QCD: from medium to asymptotical energies. Doctoral Schools ED 107 (Physique de la Région Parisienne) and ED 517 (Particules, Noyaux et Cosmos). 2014.

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$$\gamma^+ = \frac{\gamma^0 + \gamma^3}{\sqrt{2}}$$

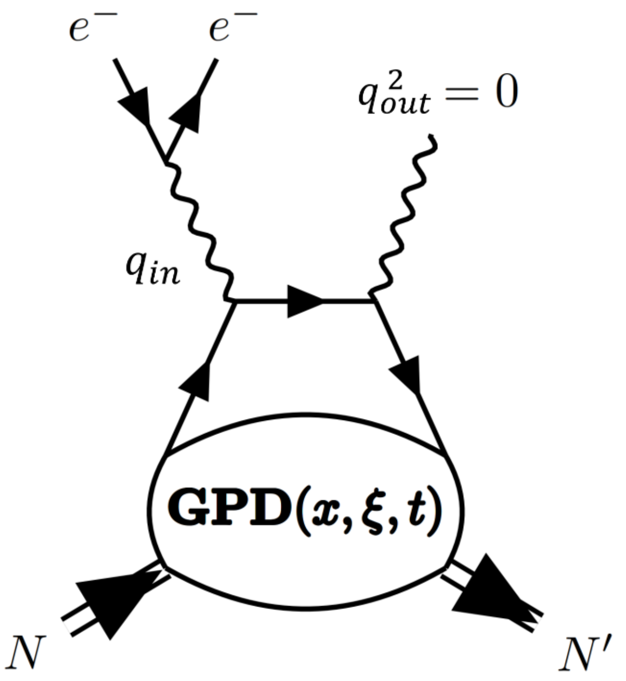
PDF is 1D

x = longitudinal parton momentum

Improving PDF's 1D picture

- In 1997, Ji introduces Generalized Parton Distributions (GPDs) through Deeply Virtual Compton Scattering (DVCS) process

- The point now is to study the conversión of a virtual photon into a real one



Xiang-Dong Ji. Gauge-Invariant Decomposition of Nucleon Spin. *Phys. Rev. Lett.*, 78:610–613, 1997

DVCS = exclusive process = factorization in amplitude

Sketch of DVCS amplitude

$$\begin{aligned}\mathcal{A}_{\mathrm{DVCS}} &\sim \int_{-1}^{1} dx\, \frac{1}{x-\xi+i0}\mathrm{GPD}(x,\xi,t)+\cdots \\ &= \mathrm{PV}\left(\frac{1}{x-\xi}\right)(\mathrm{GPD}(x,\xi,t)) - \int_{-1}^{1} dx\; i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t)+\cdots\end{aligned}$$

So we can measure GPDs at $x=\pm\xi$ only, i.e., we can access $\mathrm{GPD}(\pm\xi,\xi,t)$

GPD definition: 3D distribution

$$\mathrm{GPD}(x, \xi, t) = \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixp^- z^+} \langle P' | \bar{q}_f(-z/2) \gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2) | P \rangle \Big|_{z_\perp = z^- = 0}$$

Measure the difference between P and P'

$$\xi = -\frac{(q_{in} - q_{out})(q_{in} + q_{out})}{(P + P')(q_{in} + q_{out})}$$

$$t = (P - P')^2$$

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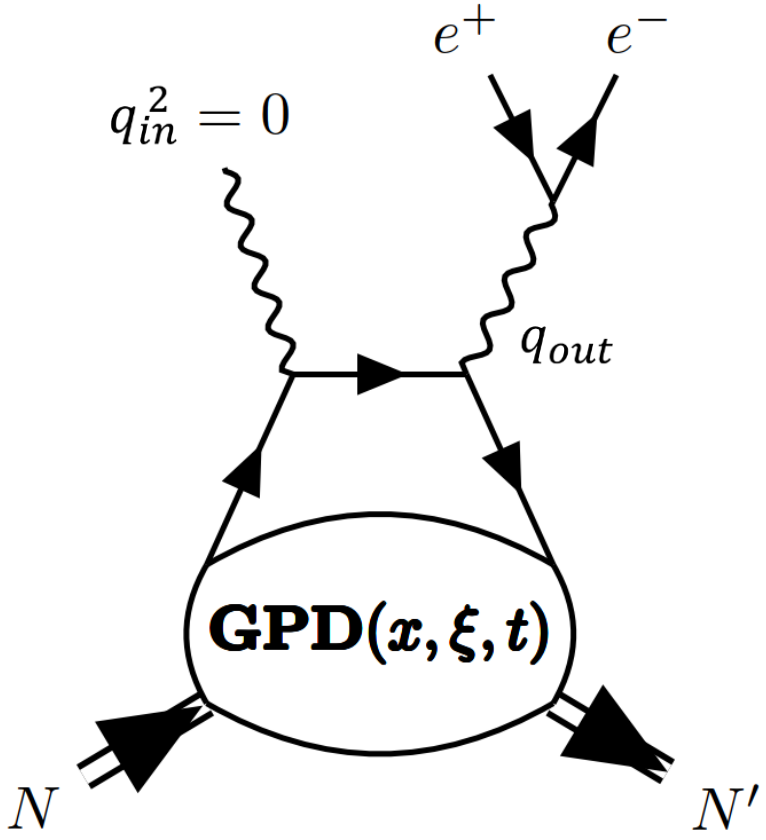
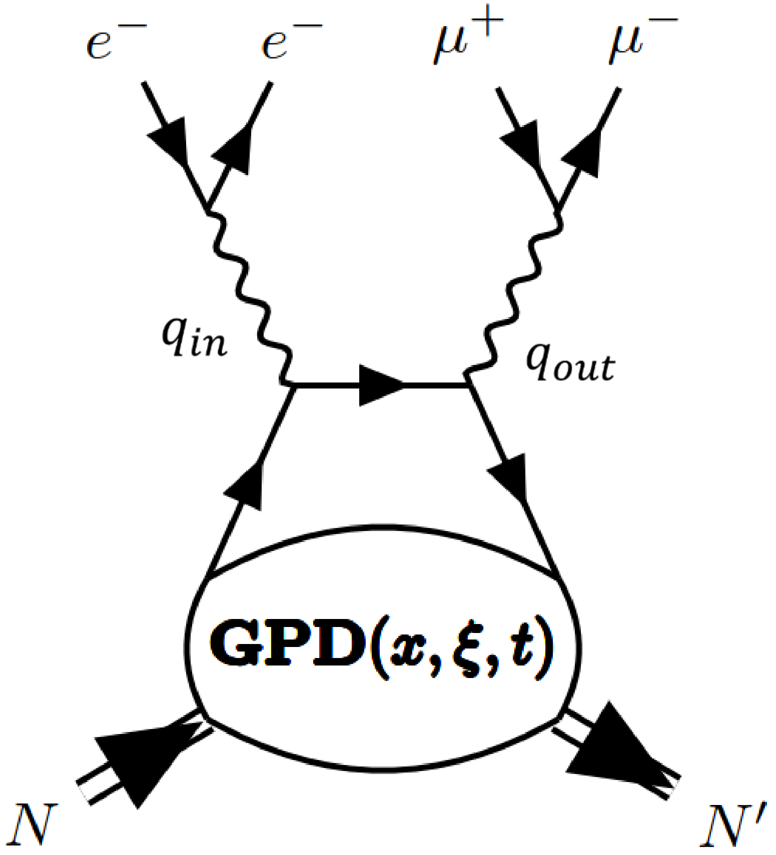
Nucleon tomography via Fourier transform in the plane transverse to proton motion

Other 2 “golden channels”: TCS & DDVCS

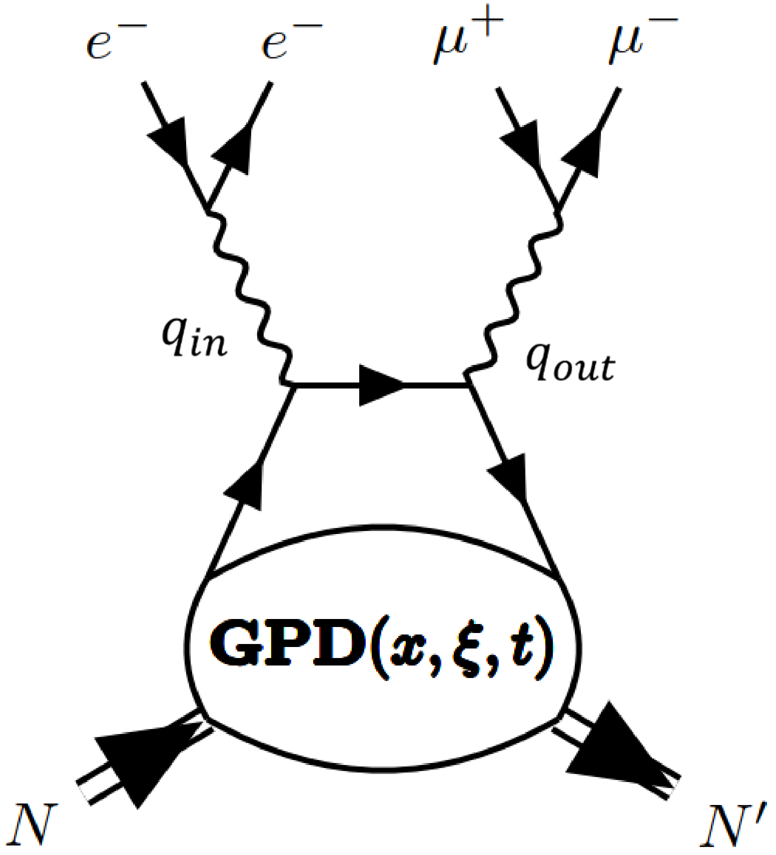
- TCS or timelike Compton scattering
- Counterpart of DVCS
- A real photon transforms into a virtual one (lepton photo-production)



- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)



- DDVCS or double DVCS
- 2 virtual photons: spacelike (incoming) and timelike (outgoing)
- **Allows to measure GPDs outside $x = \pm\xi$**



Sketch of DDVCS amplitude

$$
\begin{aligned}
\mathcal{A}_{\text{DDVCS}} &\sim \int_{-1}^{1} dx \, \frac{1}{x - x_B + i0} \text{GPD}(x,\xi,t) + \cdots \\
&= \text{PV}\left(\frac{1}{x - x_B}\right)(\text{GPD}(x,\xi,t)) - \int_{-1}^{1} dx \; i\pi\delta(x - x_B)\text{GPD}(x,\xi,t) + \cdots
\end{aligned}
$$

So now we can access $\text{GPD}(\pm x_B, \xi, t)$

Details in DDVCS

Here, x_B is the *generalized* Bjorken variable,

$$x_B = \frac{-q^2}{2pq}, \quad q = \frac{q_{in} + q_{out}}{2}, \quad p = \frac{P + P'}{2}$$

q_{in}, q_{out} are the 4-momenta of incoming & outgoing photon
P, P' are the 4-momentum of incoming & outgoing proton

Experimentally, DDVCS is very demanding: x-sec smaller than DVCS' → EIC will have enough luminosity to accurate measurements

B. Pire, L. Szymanowski, and J. Wagner. NLO corrections to timelike, spacelike and double deeply virtual Compton scattering. *Phys. Rev. D*, 83:034009, 2011.

Renormalization

- Need to renormalize both GPDs and hard part. Amplitude:

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} \tilde{T}^q(x)\tilde{F}^q(x) + \tilde{T}^g(x)\tilde{F}^g(x) \right]$$

Bare GPD

Bare hard-part coefficients

- We can work in DDVCS and use

$$\begin{aligned} \mathrm{DDVCS}|_{x_B=\xi} &\rightarrow \mathrm{DVCS} \\ \mathrm{DDVCS}|_{x_B=-\xi} &\rightarrow \mathrm{TCS} \end{aligned}$$

GPDs at NLO

Renormalize the associated Green function

$$G^{qq'} = \int d^4x_1 d^4x_2\, e^{-i(p_1x_1+p_2x_2)} \langle\Omega|\mathbb{T}\left\{\bar{\psi}_q(z_1)\mathcal{W}[z_1,z_2]\gamma^+\psi_q(z_2)\bar{\psi}_{q'}(x_1)\psi_{q'}(x_2)\right\}|\Omega\rangle\big|_{z_{1,2}=z^+_{1,2}}\,, \quad \gamma^+ = \frac{\gamma^0+\gamma^3}{\sqrt{2}}$$

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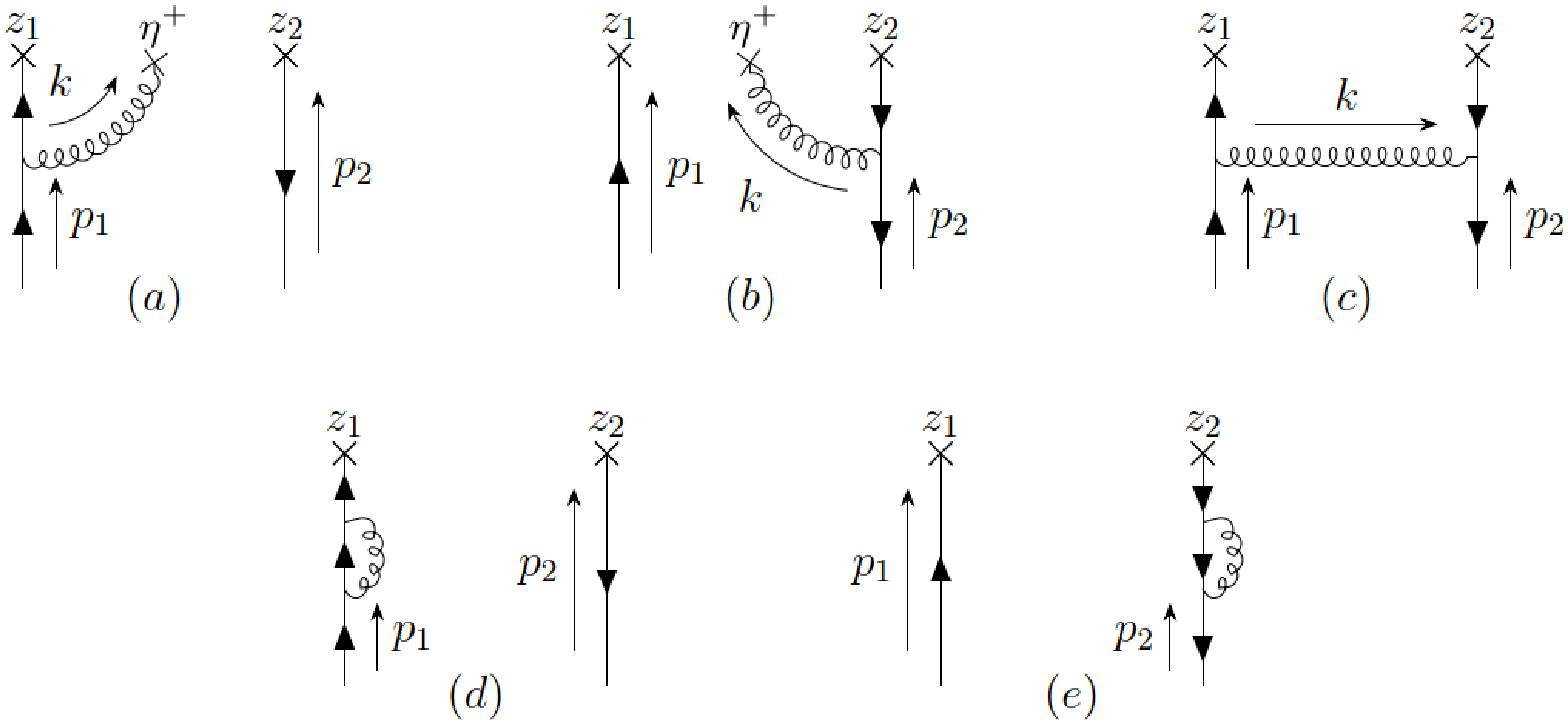

Final GPDs at NLO

$$\tilde{F}^q(x) = F^q(x) - \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qq}(x,x') \otimes F^q(x')$$
$$- \left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{e^\gamma \mu_F^2}{4\pi\mu_R^2}\right) K^{qg}(x,x') \otimes F^g(x')$$

Kernels can be read from

M. Diehl, Phys. Rept. **388** (2003) 41;
A. V. Belitsky and A. V. Radyushkin, Phys. Rept. **418**, 1 (2005)

Hard part at NLO

$$\tilde{T}^q = C_0^q + \left(\frac{|Q^2|\, e^{\gamma}}{4\pi\mu_R^2}\right)^{\epsilon/2} \left(\frac{1}{\epsilon}\, C_{coll}^q + C_1^q\right),$$

- The divergent part of the whole amplitude at NLO (1st order in α_s) is

$$\mathcal{A}_{div} = -\frac{2}{\varepsilon}\sum_j \int_{-1}^{1} dx\, \mathrm{GPD}_R^j(x)\mathcal{C}_{coll.}^j(x) +$$

$$+\frac{2}{\varepsilon}\sum_{i,j}\int_{-1}^{1} dx\, dx'\, K^{ij}(x,x')\mathrm{GPD}_R^j(x')\mathcal{C}_0^i(x')$$

Cancelling divergences, a.k.a., renormalizing

$$\mathcal{A}_{div} = -\frac{2}{\varepsilon}\sum_j \int_{-1}^{1} dx\, \mathrm{GPD}^j_R(x)\mathcal{C}^j_{coll.}(x) +$$

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The cancelation of divergences occurs if

$$\boxed{\mathcal{C}^j_{coll.}(x) = \sum_i \int_{-1}^{1} K^{ij}(y,x)\mathcal{C}_0^i(y)dy}$$

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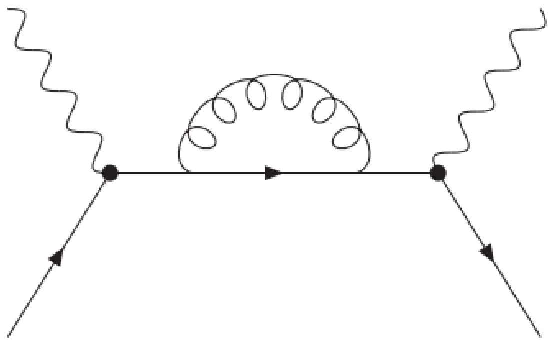
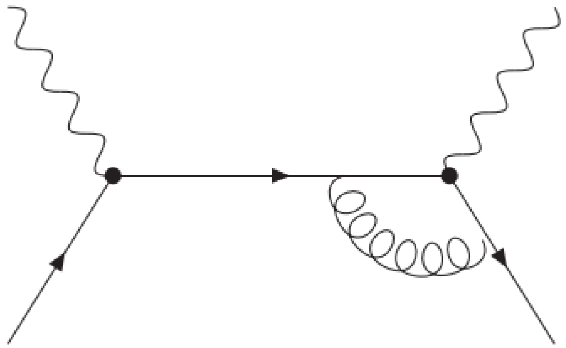
$$T^q = C_0^q + C_1^q + \frac{1}{2}\ln\left(\frac{|Q^2|}{\mu_F^2}\right)\cdot C^q_{coll}$$

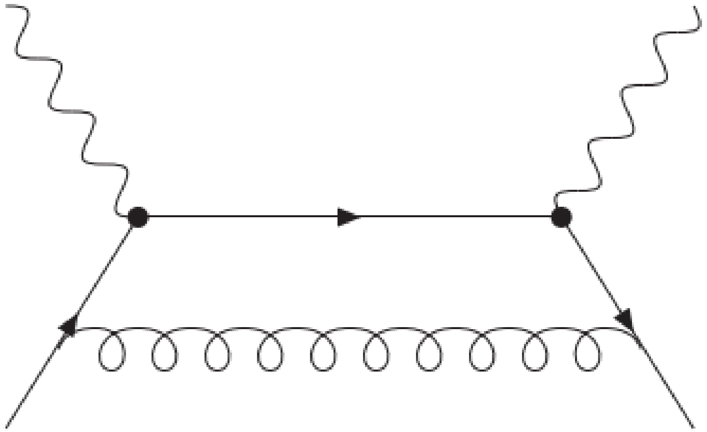
Renormalized hard part

Oskar Grocholski's Master thesis (still in preparation)

Getting the coefficients C_1^q, C_{coll}^q

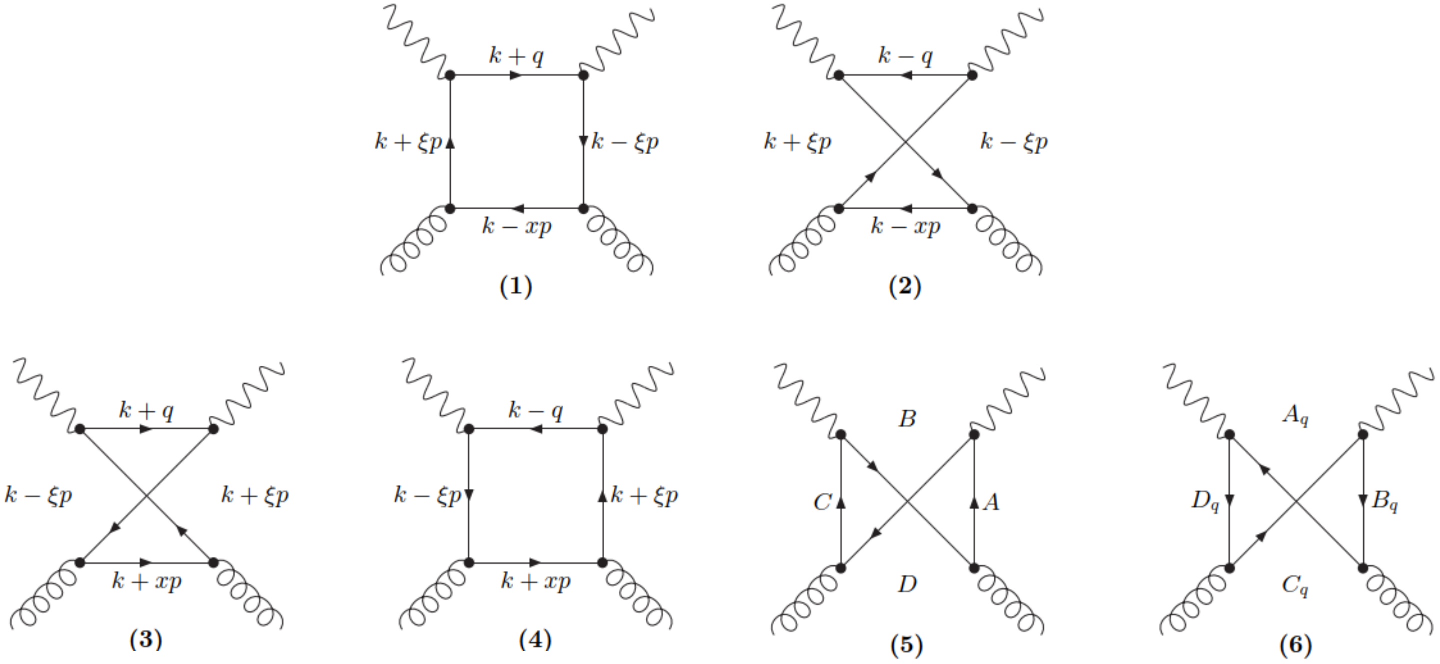






Gluon sector

- In the gluon sector there are also coefficients of that type that require more work:



Conclusions

$$C^q_{0(DVCS)} = {C^q_{0(TCS)}}^*$$

$$C^q_{coll(DVCS)} = {C^q_{coll(TCS)}}^*$$

$$\frac{{C^q_{1(TCS)}}^* - C^q_{1(DVCS)}}{\frac{e^2\alpha_S C_F}{4\pi}} = \frac{1}{x-\xi+i\varepsilon}\left[\left(3-2\log 2+2\log|1-\frac{x}{\xi}|\right)(i\pi)+\pi^2\left(1+\theta(x-\xi)-\theta(-x+\xi)\right)\right]$$
$$+\frac{1}{x+\xi-i\varepsilon}\left[\left(3-2\log 2+2\log|1+\frac{x}{\xi}|\right)(i\pi)+\pi^2\left(1+\theta(-x-\xi)-\theta(x+\xi)\right)\right]$$

sizable corrections, be careful with universality

B. Pire, L. Szymanowski, and J. Wagner. NLO corrections to timelike, spacelike and double deeply virtual Compton scattering. *Phys. Rev. D*, 83:034009, 2011.

Conclusions

$$R^q_{T-S} = \frac{C^q_{1(TCS)} - C^{q\,*}_{1(DVCS)}}{C^q_0}$$

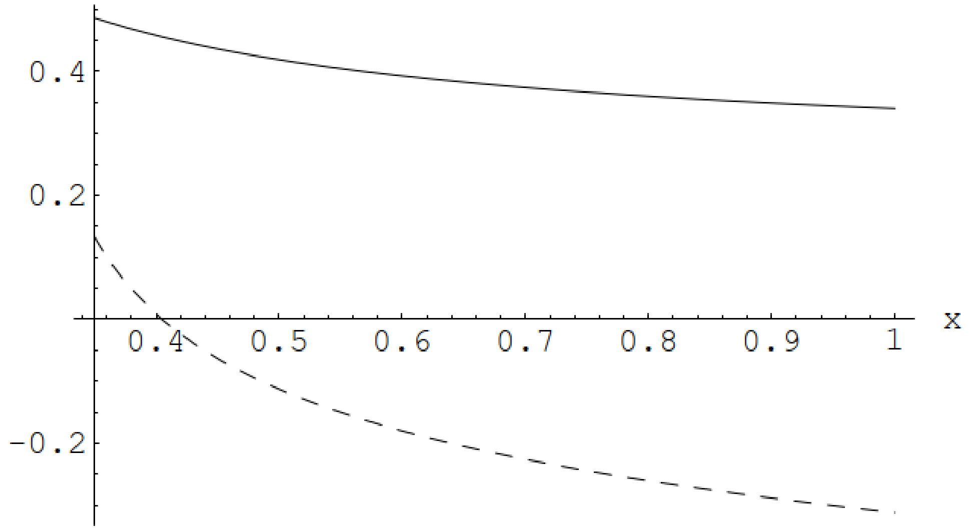


Figure 8: Real (solid line) and imaginary (dashed line) part of the ratio R^q_{T-S} of difference of NLO quark coefficient functions to the LO coefficient functions in the TCS and DVCS as a function of x

Future perspectives

- Phenomenology study of these NLO corrections

- PARTONS platform: open-source C++ program
  - Contains several GPD models
  - Leading twist… but higher twists will be included in near future
  - Can be used by theorists and experimentalists
  - Provides x-secs, Compton form factors, etc

PARTONS

PARtonic Tomography Of Nucleon Software

To download and for tutorials
http://partons.cea.fr

For detail description of architecture see:
Eur. Phys. J. C78 (2018), 478