NLO Corrections to Di-Hadron Production in DIS Using the Color Glass Condensate Formalism

Filip Bergabo and Jamal Jalilian-Marian

fbergabo@gradcenter.cuny.edu
CUNY Graduate Center & Baruch College

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CITY UNIVERSITY OF NEW YORK

Abstract

The Color Glass Condensate (CGC) is an effective theory of QCD where a nuclear target can be treated as a *classical* background field. The CGC has been successful in modelling a range of phenomena in pA collisions, and we expect similar success for eA scattering. The Leading Order (LO) di-hadron production cross section in DIS using the CGC is well known, but corrections will be needed for the increased sensitivity of the electron ion collider (EIC). In this project we are calculating the Next-to-Leading Order (NLO) corrections to di-hadron production in DIS where the target nucleus is a CGC.

Outline

- CGC Introduction
- The LO Result
- NLO Corrections

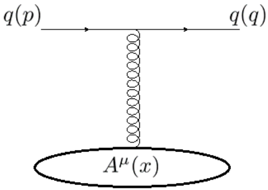
CGC Background

Intuitive picture of the color glass condensate: Nucleus is a distribution of quarks and gluons. At high speed, it becomes "saturated" with gluons. Due to length contraction and time dilation, it becomes a frozen gluon pancake.

In a high energy scattering process, we can therefore treat the nucleus as a *classical* backgroud field $A^\mu(x)$.

Quark Scattering on a Background Field

How does a quark interact a background field?



$$\bar{u}(q) ig\gamma^{\mu} t^{a} \Big[\ldots?\Big] u(p)$$

We need something with a spacetime index μ and an adjoint color index a. We also want to include the possibility of scattering at any momentum k and at any location x.

$$\Big[\ldots?\Big] \to \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{d}^4 x A^a_\mu(x) e^{-ik\cdot x}$$

The Background Field

The background field $A^\mu(x)$ should satisfy the *classical* equation of motion.

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^2$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{A}^\mu} = \frac{\partial \mathcal{L}}{\partial A^\mu} \longrightarrow D_\mu F^{\mu\nu} = J^\nu$$

Simplify for the case of a fast nucleus moving in the $-z$ direction and choose the light cone gauge $A^+ = 0$:

$$\boldsymbol{\nabla}_\perp^2 A^-(x^+, \mathbf{x}) = -\rho(x^+, \mathbf{x})$$

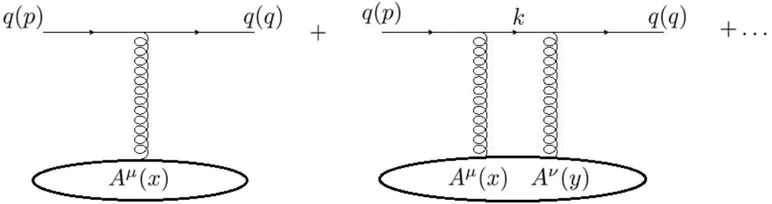
Solving this equation, one finds

$$A^-(x^+, \mathbf{x}) \sim \frac{1}{g}$$

Disaster! The amplitude is now independent of g. A diagrammatic expansion using perturbation theory is bound to fail... Or is it?

Multiple Scattering

We can fix the problem by summing up every possible number of interactions with the background field.



$$\bar{u}(q) \quad 2\pi\delta(p^+ - q^+)\gamma^+ \int \mathrm{d}^2\mathbf{x}(V(\mathbf{x}) - 1)e^{-i(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}} \quad u(p).$$

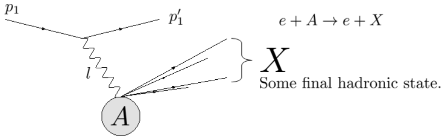
$$V(x) = \mathcal{P}\exp\left(ig\int \mathrm{d}x^+ A^-(x^+, \mathbf{x})\right)$$

.

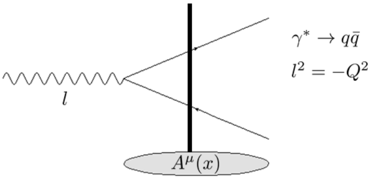
A *Wilson Line* has appeared. We can now insert this multiple scattering contribution into more complicated diagrams.

Leading Order Di-Hadron Production in DIS

We are interested in the DIS process $\gamma^* A \to q\bar{q}X$.



The LO diagram has the photon split into a $q\bar{q}$ pair which then interacts with the CGC. (Solid line represents the multiple scattering)



This LO result is well understood [Gelis and Jalilian-Marian, 2003].

Modified Propagators

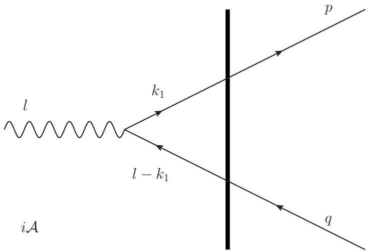
We form "modified" propagators in the presence of the background field by inserting the multiple scattering contribution τ_F between two free propagators.

$$S(q,p) = S_0(q)\tau_F(q,p)S_0(p)$$

Since gluons can also interact with the backgroud field, we'll need a modified gluon propagator.

$$G(q,p)^{ab}_{\mu\nu} = G^0_{\mu\sigma}(q)\tau^{ab}_G(q,p)G^{0\,\sigma}_{\nu}(p)$$

The LO Result



$$i\mathcal{A} = \int \frac{d^4 k_1}{(2\pi)^4}\, \bar{u}(p) \left[S^0(p)\right]^{-1} S(p, k_1)\, ie\not{\epsilon}(l)\, S(k_1 - l, -q) \left[S^0(-q)\right]^{-1} v(q)$$

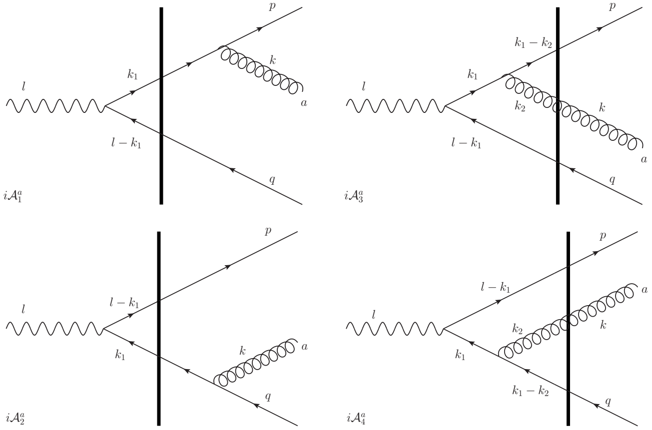
$$i\mathcal{M} = -4el^{+} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} d^2\mathbf{x}_1\, d^2\mathbf{x}_2 \frac{N V_1 V_2^{\dagger}}{(\mathbf{k}_1^2 + Q_1^2)} e^{-i(\mathbf{p}-\mathbf{k}_1)\cdot\mathbf{x}_1} e^{-i(\mathbf{k}_1+\mathbf{q})\cdot\mathbf{x}_2}$$

$$N = \frac{1}{8(l^{+})^2} \bar{u}(p)\not{n}\not{k}_1\not{\epsilon}(l)(\not{k}_1 - \not{l})\not{n} v(q).$$

Example: $N^{L;+} = -Q(z_1 z_2)^{3/2}$

Real Corrections

There are four real diagrams to consider in the eikonal limit using the shockwave approximation. [Ayala et al., 2017]



We need to integrate over the phase space of the outgoing gluon.

Real Corrections

Amplitudes:

$$i\mathcal{M}_1^a = -\,ieg16(l^+)^2 \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\mathrm{d}^4 x \frac{N_1 e^{i\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{k_1^2(k_1-l)^2} t^a V_1 V_2^\dagger e^{-i\mathbf{q}\cdot\mathbf{x}_2} e^{-i(\mathbf{p}+\mathbf{k})\cdot\mathbf{x}_1}, \quad \text{where } k_1^+ = p^+ + k^+.$$

$$i\mathcal{M}_2^a = -\,ieg16(l^+)^2 \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\mathrm{d}^4 x \frac{N_2 e^{i\mathbf{k}_1\cdot(\mathbf{x}_2-\mathbf{x}_1)}}{k_1^2(k_1-l)^2} V_1 V_2^\dagger t^a e^{-i\mathbf{p}\cdot\mathbf{x}_1} e^{-i(\mathbf{q}+\mathbf{k})\cdot\mathbf{x}_2}, \quad \text{where } k_1^+ = q^+ + k^+.$$

$$i\mathcal{M}_3^a = 2eg16(l^+)^2 k^+ \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\frac{\mathrm{d}^3 k_2}{(2\pi)^3}\mathrm{d}^6 x \frac{N_3 e^{i\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{x}_2)} e^{i\mathbf{k}_2\cdot(\mathbf{x}_3-\mathbf{x}_1)}}{k_1^2(k_1-l)^2 k_2^2(k_2-k_1)^2} V_1 t^b V_2^\dagger U_3^{ab} e^{-i(\mathbf{p}\cdot\mathbf{x}_1+\mathbf{q}\cdot\mathbf{x}_2+\mathbf{k}\cdot\mathbf{x}_3)},$$

where $k_2^+ = k^+, \quad k_1^+ = p^+ + k^+$.

$$i\mathcal{M}_4^a = 2eg16(l^+)^2 k^+ \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\frac{\mathrm{d}^3 k_2}{(2\pi)^3}\mathrm{d}^6 x \frac{N_4 e^{i\mathbf{k}_1\cdot(\mathbf{x}_2-\mathbf{x}_1)} e^{i\mathbf{k}_2\cdot(\mathbf{x}_3-\mathbf{x}_2)}}{k_1^2(k_1-l)^2 k_2^2(k_2-k_1)^2} V_1 t^b V_2^\dagger U_3^{ab} e^{-i(\mathbf{p}\cdot\mathbf{x}_1+\mathbf{q}\cdot\mathbf{x}_2+\mathbf{k}\cdot\mathbf{x}_3)},$$

where $k_2^+ = k^+, \quad k_1^+ = q^+ + k^+$.

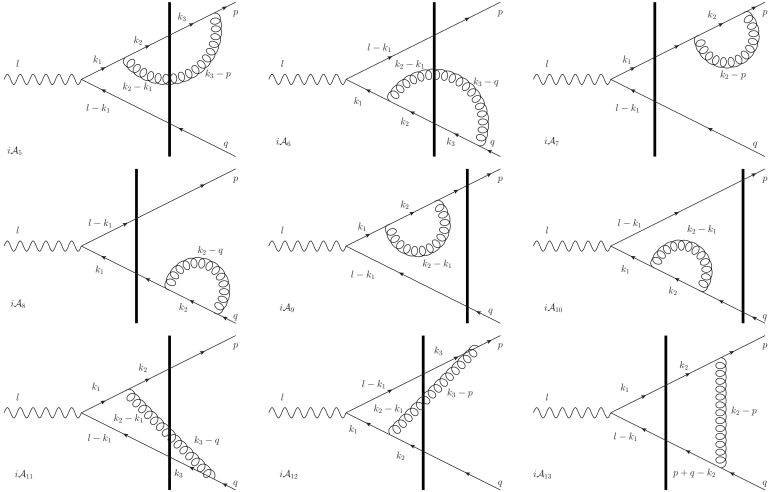
$$N_1 = \frac{\bar{u}(p)\not\epsilon^*(k)(\not p + \not k)\not n \not k_1 \not\epsilon(l)(\not k_1 - \not l)\not n v(q)}{16(l^+)^2(p+k)^2}, \qquad N_2 = \frac{\bar{u}(p)\not n(\not l - \not k_1)\not\epsilon(l)\not k_1 \not n(\not k + \not q)\not\epsilon^*(k)v(q)}{16(l^+)^2(q+k)^2},$$

$$N_3 = \frac{\bar{u}(p)\not n(\not k_1 - \not k_2)\gamma^\sigma \not k_1 \not\epsilon(l)(\not l - \not k_1)\not n v(q) d_{\mu\sigma}(k_2)\epsilon^{\mu *}(k)}{16(l^+)^2},$$

$$N_4 = \frac{\bar{u}(p)\not n(\not l - \not k_1)\not\epsilon(l)\not k_1 \gamma^\sigma(\not k_2 - \not k_1)\not n v(q) d_{\mu\sigma}(k_2)\epsilon^{\mu *}(k)}{16(l^+)^2}.$$

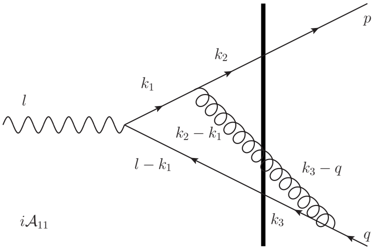
[Ayala et al., 2017]

There are several virtual diagrams to consider in the eikonal limit using the shockwave approximation.



Virtual Corrections: an Example

One of the virtual diagrams:



$$i\mathcal{M}_{11} = \frac{eg^2}{4l^+ z_1 z_2} \int \frac{\mathrm{d}^6\mathbf{k}}{(2\pi)^6} \frac{\mathrm{d}z_3}{2\pi} \mathrm{d}^6x \frac{N_{11} V_1 t^a V_2^\dagger t^b U_3^{\dagger ab} e^{i\mathbf{k}_1\cdot(\mathbf{x}_3-\mathbf{x}_2)} e^{i\mathbf{k}_2\cdot(\mathbf{x}_1-\mathbf{x}_3)} e^{i\mathbf{k}_3\cdot(\mathbf{x}_3-\mathbf{x}_2)} e^{-i(\mathbf{p}\cdot\mathbf{x}_1+\mathbf{q}\cdot\mathbf{x}_3)}}{\left[\mathbf{k}_1^2 + z_3(1-z_3)Q^2\right]\left[Q^2 + \frac{\mathbf{k}_1^2}{z_3} + \frac{\mathbf{k}_2^2}{z_1} + \frac{(\mathbf{k}_1-\mathbf{k}_2)^2}{z_2-z_3}\right]\left(\mathbf{k}_3 - \frac{z_3}{z_2}\mathbf{q}\right)^2}.$$

$$N_{11} = \bar{u}(p)\not{n}\not{k}_2\gamma^\mu\not{k}_1\not{\epsilon}(l)(\not{k}_1 - \not{l})\not{n}\not{k}_3\gamma^\nu v(q) d_{\mu\sigma}(k_2-k_1) d^\sigma_\nu(k_3-q).$$

Numerators: an Example

Numerators are more complicated than in the LO case! Below is an example for a longitudinally polarized photon and transverse polarized quark.

$$N_{11}^{L;+} = \frac{-2^5 Q(l^+)^2 z_3^2 (1-z_3) z_1^{3/2} \sqrt{z_2}}{(z_3 - z_2)^2} \Bigg[\frac{z_1 z_2}{z_3(1-z_3)} (\mathbf{k}_1 \cdot \epsilon)(\mathbf{k}_3 \cdot \epsilon^*) + \frac{z_3(1-z_3)}{z_1 z_2} (\mathbf{q} \cdot \epsilon)(\mathbf{k}_2 \cdot \epsilon^*)$$
$$- \frac{z_1}{(1-z_3)} (\mathbf{k}_1 \cdot \epsilon)(\mathbf{q} \cdot \epsilon^*) - \frac{(1-z_3)}{z_1} (\mathbf{k}_3 \cdot \epsilon)(\mathbf{k}_2 \cdot \epsilon^*) - \frac{z_2}{z_3} (\mathbf{k}_2 \cdot \epsilon)(\mathbf{k}_3 \cdot \epsilon^*) - \frac{z_3}{z_2} (\mathbf{q} \cdot \epsilon)(\mathbf{k}_1 \cdot \epsilon^*)$$
$$+ (\mathbf{k}_3 \cdot \epsilon)(\mathbf{k}_1 \cdot \epsilon^*) + (\mathbf{k}_2 \cdot \epsilon)(\mathbf{q} \cdot \epsilon^*) \Bigg].$$

Ongoing Work

- Derive all NLO contributions to the di-hadron cross section.
- Establish factorization (or lack thereof) in NLO corrections to di-hadron production in DIS.

Thank You

- Thank you for listening!
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