TMD factorization for dijet and heavy meson pair production in DIS

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Rafael Fernández del Castillo, Universidad Complutense de Madrid

Outline

Soft Collinear Effective Theory

Dijet production

- Cross-section factorization
- TMD Soft Function
- Rapidity divergencies
- Consistency AD check

Heavy meson pair production

- Cross-section factorization
- Soft Function AD up to three loops

Based on the work published by

Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris & Ignazio Scimemi

https://arxiv.org/abs/2008.07531v4

Introduction

- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect.

- We consider two processes which are presently attracting increasing attention

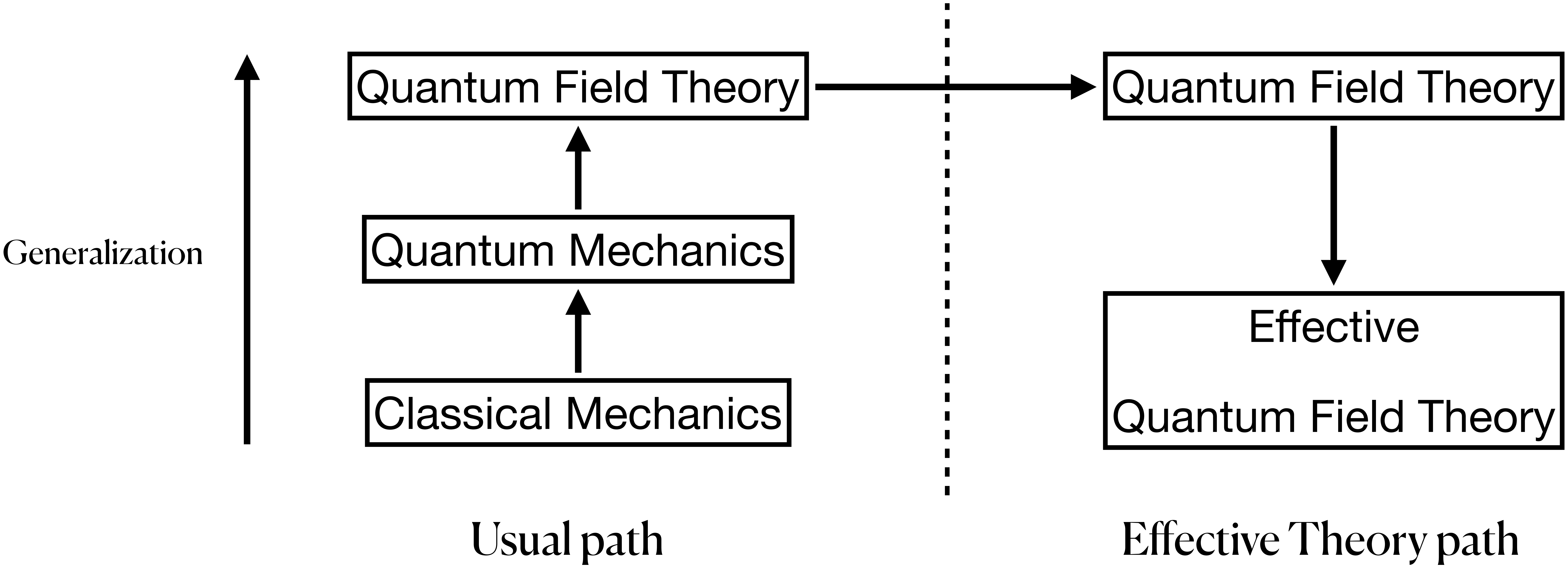
$$\ell + h \to \ell' + J_1 + J_2 + X \qquad\qquad \ell + h \to \ell' + H + \bar{H} + X$$

Dijet — Heavy-meson

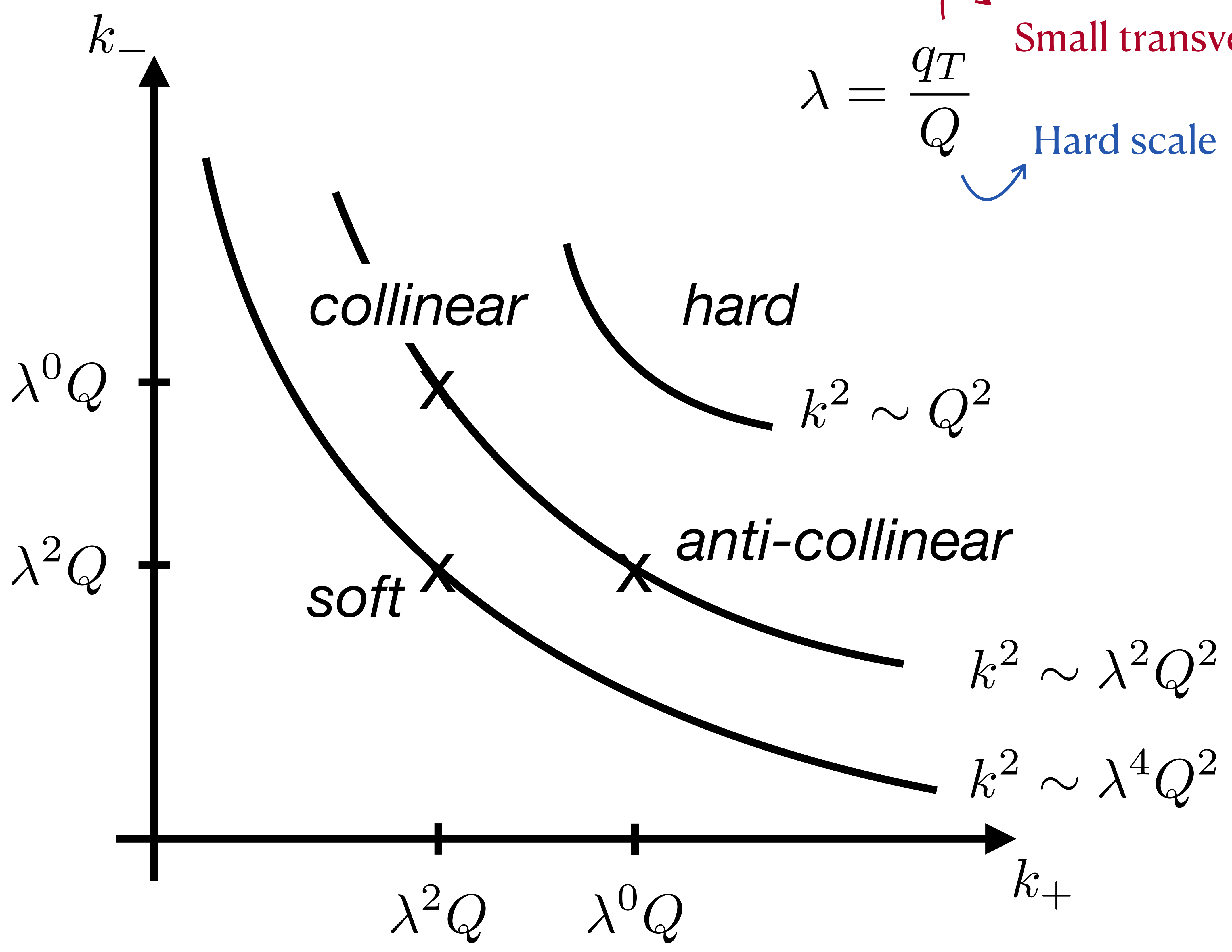
Working in the Breit frame

Dominguez, Xiao, Yuan, 2013
Boer, Brodsky, Mulders, Pisano, 2011
Zhang, 2017

What is an Effective Field Theory?



Soft Collinear Effective Theory



$$\lambda = \frac{q_T}{Q}$$

Small transverse momentum

Hard scale

- SCET is a QCD effective theory that separates soft and collinear modes
- We can build an effective lagrangian with its effective Feynman rules
- We achieve factorization through this effective theory

More on SCET

Iain Stewart Lecture Notes, 2013 (also on YouTube!)

Becher, Broggio, Ferroglia Lecture Notes, 2015

Dijet production

$r_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$

Transverse momentum imbalance (small)

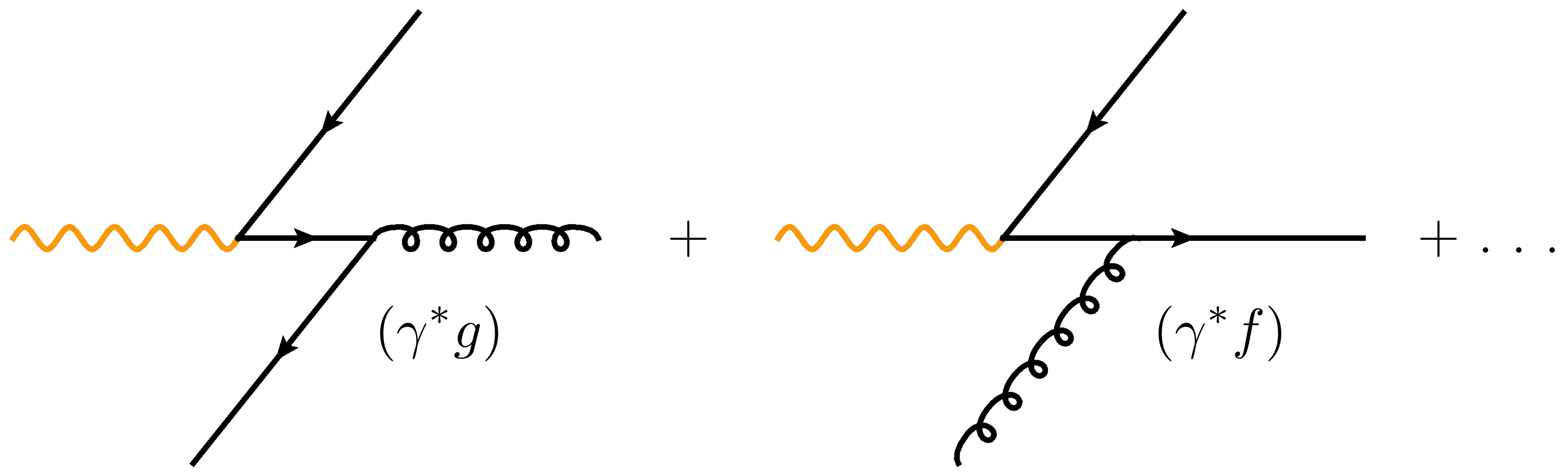
$p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$

Total transverse momentum (hard scale)

$$\lambda = \frac{r_T}{p_T}$$

$$\ell + h \to \ell' + J_1 + J_2 + X$$

dijet LO process:



- Sensitive of polarized and unpolarized TMDPDFs
- Experimental observation should be possible in the future EIC Page, Chu, Aschenauer, 2020
- Jets here described have $p_T \in [2, 20]$ GeV and are found in the central rapidity region
- Factorization within SCET

Cross-section factorization

Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_i jet pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

Hard Function — Gluon TMDPDF

$$(\gamma^* g) \qquad \frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g \to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_{g,\mu\nu}(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma g}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2) \Big(\mathcal{C}_f(\boldsymbol{b},R,\mu) J_f(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu) J_{\bar{f}}(p_T,R,\mu)\Big)$$

Soft Function — Collinear-Soft & Jet function

$$(\gamma^* f) \qquad \frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{fU} \sum_f H^U_{\gamma^* f \to g\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_f(\xi,\boldsymbol{b},\mu,\zeta_1)$$
$$\times S_{\gamma f}(\boldsymbol{b},\zeta_2,\mu)\Big(\mathcal{C}_g(\boldsymbol{b},R,\mu) J_g(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu) J_{\bar{f}}(p_T,R,\mu)\Big)$$

TMD soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

The Soft Function describes the interaction through soft gluons between initial and final hadronic states

Soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\boldsymbol{b}, -\infty)_{ca'} \mathrm{Tr}\Big[S_{v_2}(+\infty, \boldsymbol{b}) T^{a'} S_{v_1}^\dagger(+\infty, \boldsymbol{b}) \times S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \Big] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

Wilson lines

$$S_v(+\infty, \xi) = P\exp\Big[-ig \int_0^{+\infty} d\lambda \; v \cdot A(\lambda v + \xi) \Big] \qquad S_{\bar{v}}^\dagger(+\infty, \xi) = P\exp\Big[ig \int_0^{+\infty} d\lambda \; \bar{v} \cdot A(\lambda \bar{v} + \xi) \Big]$$

$$S_n(+\infty, \xi) = \lim_{\delta^+ \to 0} P\exp\Big[-ig \int_0^{+\infty} d\lambda \; n \cdot A(\lambda n + \xi) e^{-\delta^+ \lambda} \Big]$$

δ - regulator for rapidity div.

Echevarría, Scimemi, Vladimirov, 2016

Rapidity divergencies

Collinear matrix element and SF are plagued with unregularized and uncanceled divergences. They show up perturbatively through integrals of the form

$$\int_0^1 dt\ t^{-1}$$

These divergencies are also known as light-cone singularities and appear due to the fact that Wilson lines are defined along light-like trajectories

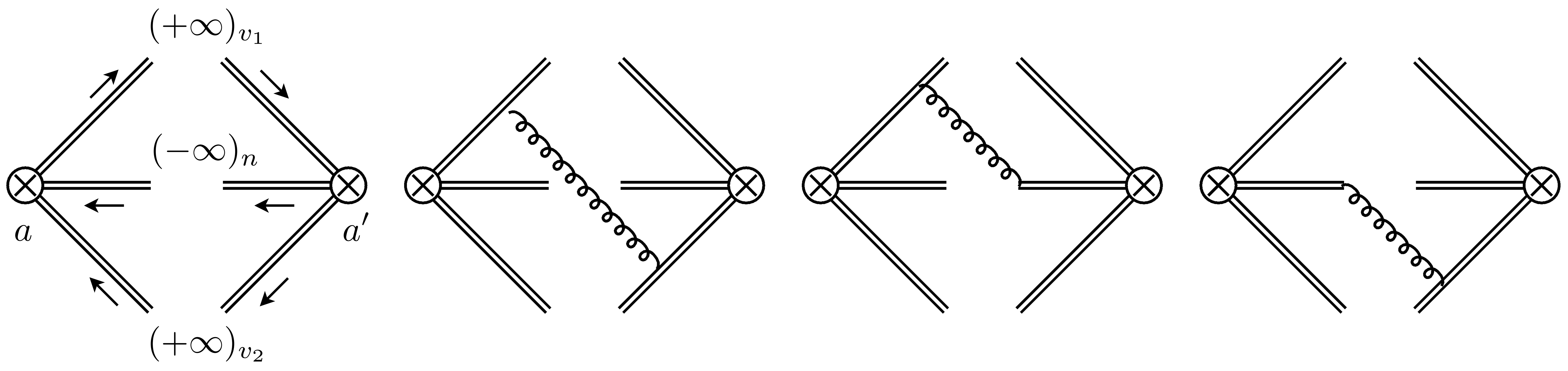
Rapidity $$y = \frac{1}{2}\ln\frac{k_+}{k_-}$$

δ-regulator regulates both IR and rapidity divergencies

Rapidity divergencies should cancel between the TMDs and SF

TMD soft function

Finite result



+ virtual diagrams

at one-loop order...

$$\hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) = 1 + a_s\Big\{C_A\Big[\ln\big(B\,\mu^2 e^{2\gamma_E}\big)\Big(\ln\big(B\mu^2 e^{2\gamma_E}\big) + 4\ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + 2\ln(2A_n)\Big) - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{6} - 2\text{Li}_2(1+A_{\boldsymbol{b}})\Big] + C_F\Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\text{Li}_2(1+A_{\boldsymbol{b}})\Big]\Big\},$$

with... $A_{\boldsymbol{b}} = \dfrac{(v_1\cdot v_2)}{2\,(v_1\cdot\hat{b})\,(v_2\cdot\hat{b})} = -\dfrac{\hat{s}}{4\,p_T^2\,c_b^2}$ $\qquad A_n = \dfrac{(v_1\cdot v_2)}{2\,(v_1\cdot n)(v_2\cdot n)}$

$(\gamma^* g)$ - channel

Consistency check

Dijet-production

$\epsilon \longrightarrow \mu$

$\delta \longrightarrow \zeta$

$$\frac{d}{d\ln\mu} G(\mu) = \gamma_G(\mu)\, G(\mu)$$

$(\gamma^* g)$-channel $\quad \gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{\mathcal{C}_1} + \gamma_{\mathcal{C}_2} + \gamma_\alpha = 0$

$(\gamma^* f)$-channel $\quad \gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{\mathcal{C}_f} + \gamma_{\mathcal{C}_g} + \gamma_\alpha = 0$

The sum of all anomalous dimensions should cancel for each channel

$$\gamma^{[1]}_{S_{\gamma g}} = 4\Big\{ -C_A \ln\zeta_2 + 2C_F\Big[\ln\big(B\mu^2\, e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big]\Big\}\,,$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\Big\{(C_F + C_A)\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big] + (C_F - C_A)\Big[\ln\left(\frac{\hat{t}}{\hat{u}}\right) - \kappa(v_f)\Big] - C_F \ln\zeta_2\Big\}$$

$$\gamma^{[1]}_{F_i} = 4C_i\Big[-\ln\left(\frac{\zeta_1}{\mu^2}\right) + \gamma_i\Big]\,,$$

$$\gamma^{[1]}_{\mathcal{C}_g} = 4C_A\Big[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_g)\Big]$$

$$\gamma^{[1]}_{\mathcal{C}_i} = 4C_F\Big[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_i)\Big]$$

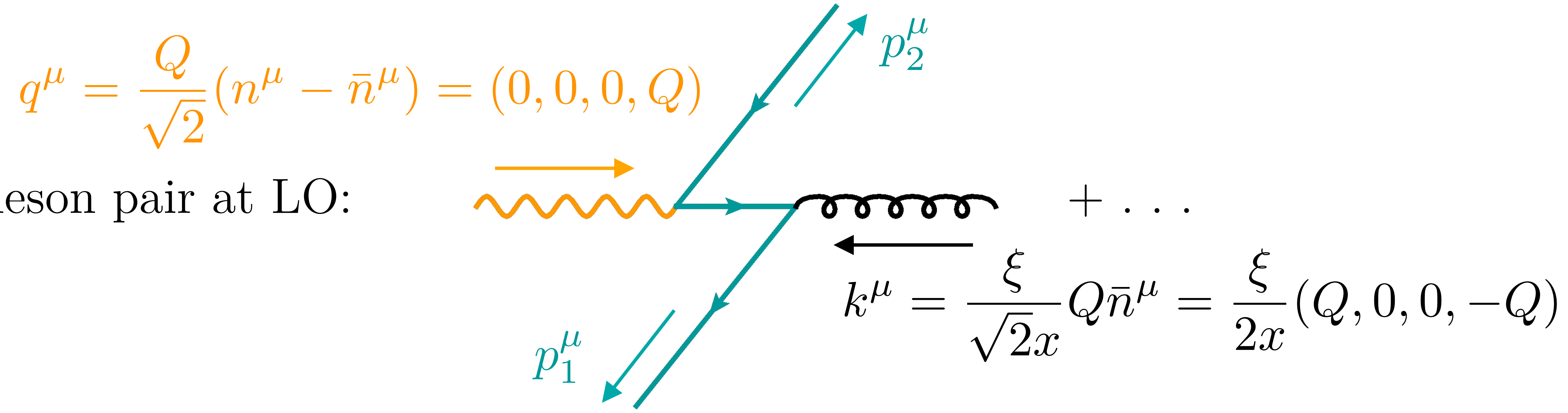
$$\kappa(v_f) = -\kappa(v_{\bar{f}}) = -\kappa(v_g) = i\pi\,\mathrm{sign}(c_{\boldsymbol{b}})$$

They cancel !!!

Heavy-meson pair production

$$\ell + h \to \ell' + H + \bar{H} + X$$

heavy meson pair at LO:



- Experimentally more challenging
- Observation of charmed mesons could be possible

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

Li, Liu, Vitev, 2020

Chudakov, Higinbotham, Hyde, Furletov, Furletova, Nguyen, 2016

Cross-section factorization

Heavy meson pair production

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T}$$

We measure over

- x Bjorken variable
- η_H, $\eta_{\bar{H}}$ heavy meson pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$$\frac{d\sigma(\gamma^* g)}{dx\, d\eta_H\, d\eta_{\bar{H}}\, dp_T\, d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1) \times S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Heavy Quark Jet Function

Fickinger, Fleming, Kim, Mereghetti, 2016

Region sensitive to TMD $|\boldsymbol{r}_T| \ll p_T^{H,\bar{H}}$

We have a new scale m_Q

Consistency check

Heavy meson pair production

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

$= 2\gamma_J$

They cancel !!!

This sum is known up to three-loops...

We can use this consistency relation to extend the SF AD up to three loops

Soft function

anomalous dimension

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\left[2C_F \ln\left(\frac{B\mu^2 e^{2\gamma_E}}{-A_b}\right) - C_A \ln\zeta_2\right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma^{[1]}_{S_{\gamma g}} = 0$$

$$\delta\gamma^{[2]}_{S_{\gamma g}} = C_F\left[C_A\left(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\right) + n_f T_F\left(-\frac{448}{27} + \frac{8}{9}\pi^2\right)\right]$$

$$\delta\gamma^{[3]}_{S_{\gamma g}} = \dots$$

This let us $\longrightarrow$

- Check future perturbative calculations of the SF
- Compute its evolution kernel up to three loops

Conclusion

- We give a general idea of how factorization works and the role of universal functions
- We have established factorization for the dijet and heavy meson production
- Both cases can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- Future: Phenomenology (SOON!)

Thank you for listening!