The D-term: Last Item On The Checklist

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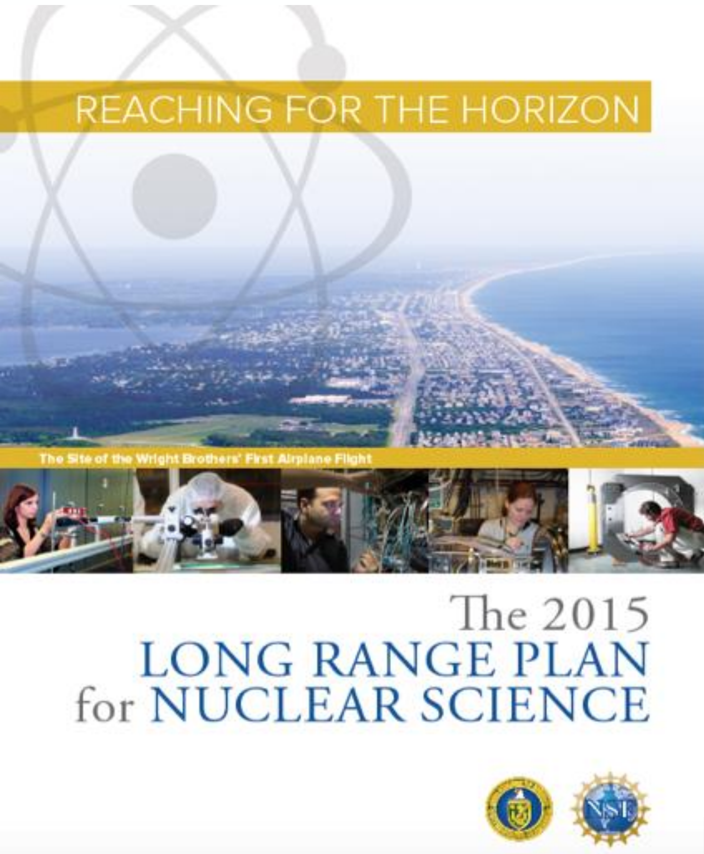
Outline

- Important Questions in Nuclear Physics
- What are Form Factors?
- GPD’s and the D-Term
- Scalar Diquark Model

Who Decides What's Important?

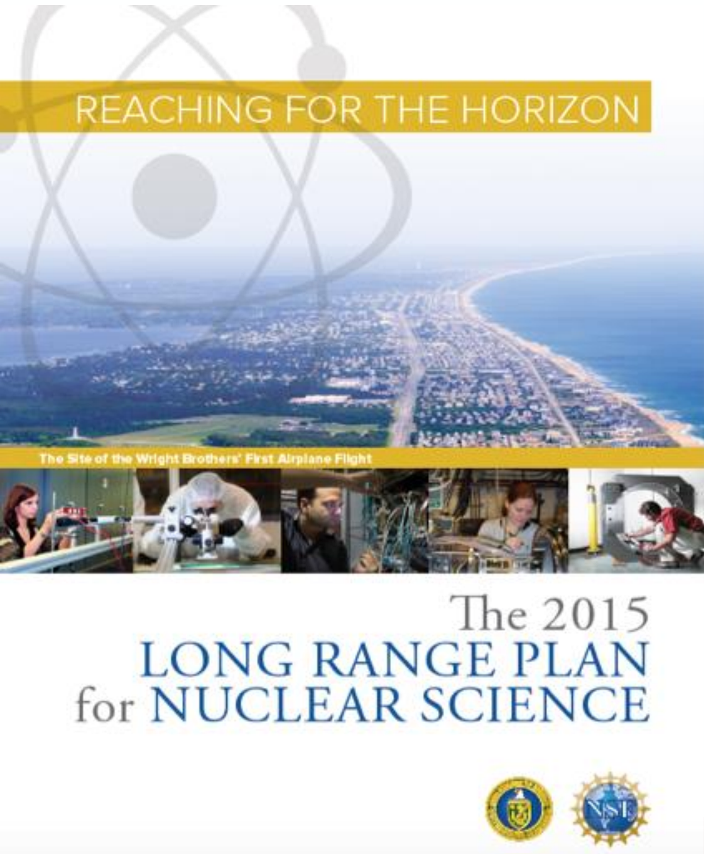


THE SCIENCE QUESTIONS

Nuclear science is a broad and diverse subject. The National Research Council Committee on the Assessment of and Outlook for Nuclear Physics 2013 report, *Nuclear Physics, Exploring the Heart of Matter*, (NP2010 Committee) framed the overarching questions "that are central to the field as a whole, that reach out to other areas of science, and that together animate nuclear physics today:

1. How did visible matter come into being and how does it evolve?
2. How does subatomic matter organize itself and what phenomena emerge?
3. Are the fundamental interactions that are basic to the structure of matter fully understood?
4. How can the knowledge and technical progress provided by nuclear physics best be used to benefit society?"

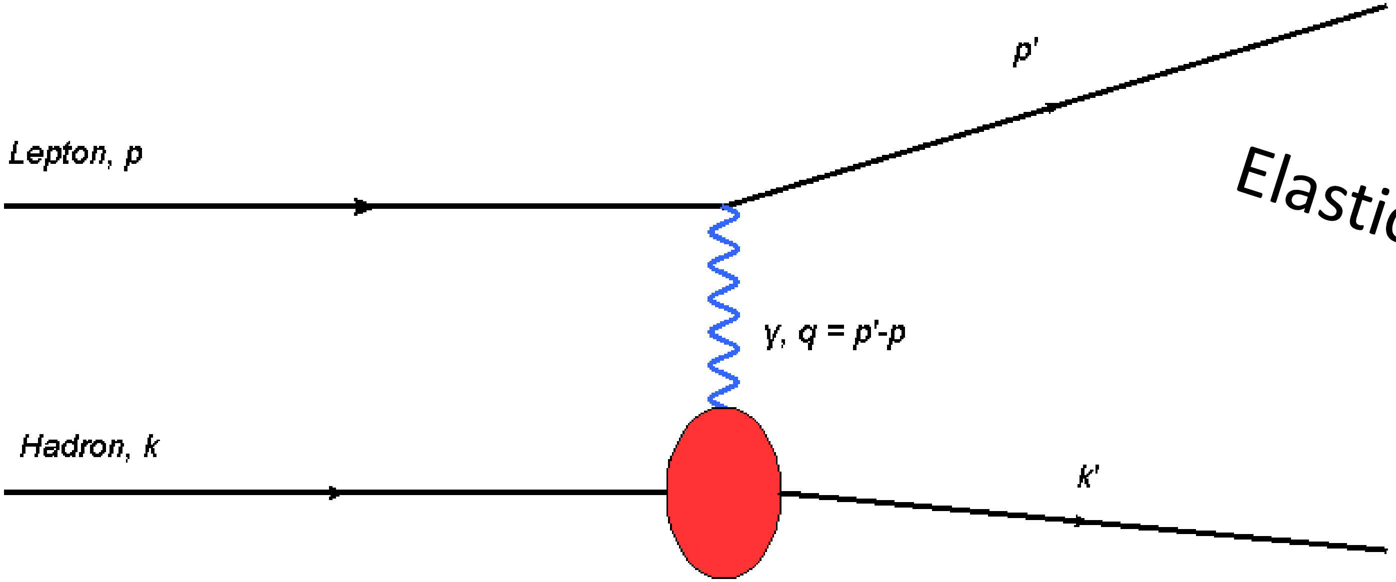
Who Decides What's Important?



Overarching Theme: Where Do Properties Of The Nucleon Come From, And How Are They Distributed?

That information is contained in **Form Factors**!

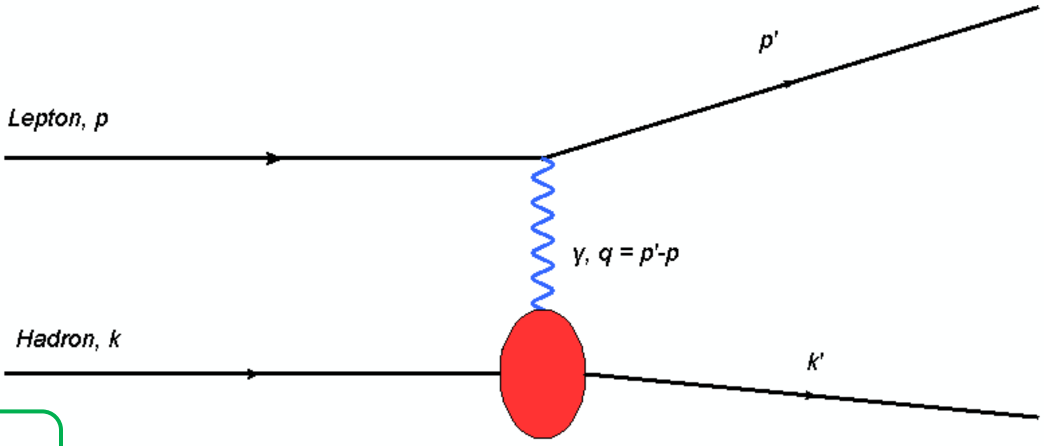
How Do Form Factors Arise in QFT?



Elastic Scattering

Question: **What must we include in our calculation to describe this process quantum mechanically?**

How Do Form Factors Arise in QFT?



Question: What must we include in our calculation to describe this process quantum mechanically?

We need:

- Operators to bring the lepton and hadron from their initial to their final state.
- A term characterizing how the exchange particle propagates.
- Strength of the interaction.

$$i\mathcal{M} = (-ie)^2 \left(\frac{-ig_{\mu\nu}}{q^2}\right) \langle p'|j_l^\mu(0)|p\rangle \langle k'|j_h^\nu(0)|k\rangle$$

Matrix Element

Conserved Current

How Do Form Factors Arise in QFT?

$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2$, and the conserved current for the lepton is given by

Conserved current: $j_l^\mu = \bar{\psi}\gamma^\mu\psi$ **What about j_h^μ?**

Because hadrons are composite particles, the conserved current is not so simple. However, we can still express the current as a linear combination of available tensors.

What tensors are available? $\gamma^\mu, \sigma^{\mu\nu}, q^\mu$. Can only use divergenceless combinations of these.

Our ignorance on the proportionality to those tensors is absorbed into *Form Factors (FF's):*

$$\langle p'|j_h^\mu|p\rangle = \bar{u}(p')_h\left[F_1(t)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}q_\nu F_2(t)\right]u(p)_h$$

How Do Form Factors Arise in QFT?

Conserved currents for composite particles → Form Factors → Related Densities. Relating FF's to densities is typically done with respect to a specific reference frame (i.e. Breit Frame)

Theory	Conserved Current	Related Densities
E&M	j^{μ}_{em}	Charge, Magnetic Moment
Gravity	$T^{\mu\nu}$	Mass, Angular Momentum, Pressure (?)
…		

Gravitational Form Factors of the EMT

For a spin-0 particle, we may decompose the EMT into linear combinations of available tensors. The common parameterization is in terms of the incoming and outgoing four-momentum $P^{\mu} = p'^{\mu} + p^{\mu}, \Delta^{\mu} = p'^{\mu} - p^{\mu}, t = \Delta^2$, and the metric $g^{\mu\nu}$.
The general form of the EMT is then:

$$< p'|T^{\mu\nu}(0)|p > = \frac{P^{\mu}P^{\nu}}{2}A(t) + \frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2}{2}D(t) \quad + \ldots \text{ J(t) for non spin-0}$$

$$\langle p'|\hat{T}^{Q,G}_{\mu\nu}(0)|p\rangle = \bar{N}(p')\Big[M_2^{Q,G}(t)\,\frac{\bar{P}_\mu \bar{P}_\nu}{m_N} + J^{Q,G}(t)\,\frac{i\bar{P}_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{m_N} + d^{Q,G}(t)\,\frac{1}{5m_N}\,\left(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2\right) \pm \bar{c}(t)g_{\mu\nu}\Big]N(p)$$

Mass distribution

Distribution of…D-term?

The D-term

The ij-components of the EMT define the *Stress-Tensor:*

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

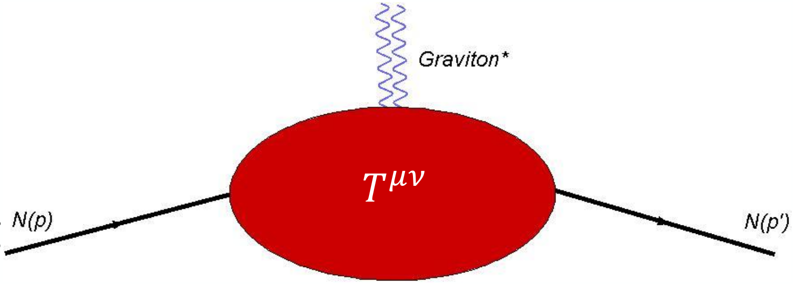
Where s(r) and p(r) are the shear forces and pressure distributions, respectively.

D-Term Describes Pressure and Shear Stress inside of Nucleon!

Can relate these densities to the D-term Schweitzer, Polyakov (2018)

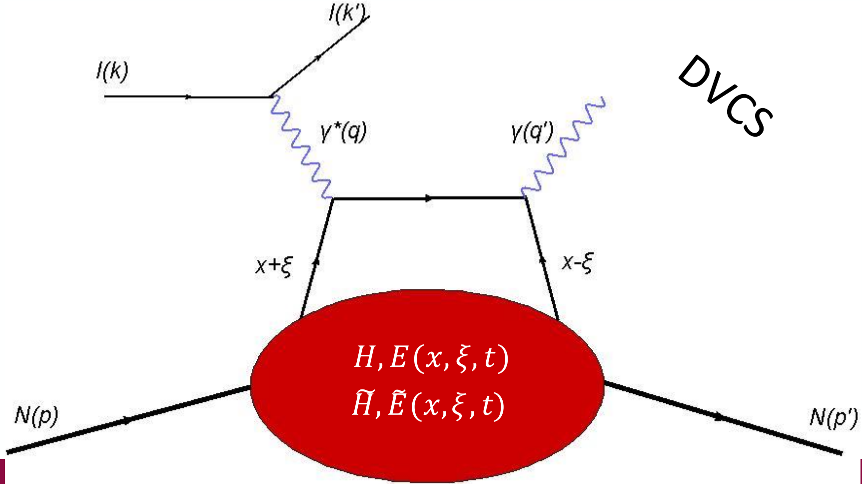
$$s(r) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \widetilde{D}(r), \qquad p(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \widetilde{D}(r), \quad \widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-\Delta r} D(-\Delta^2)$$

How These Form Factors May Be Extracted Experimentally



1) **Graviton scattering**

Direct access, and completely analogous to probing F_1 and F_2, but practically impossible…



2) **Generalized Parton Distributions (GPD's)**

Indirect access and non-trivial to extract, but more practical. Describes the transverse position of the partons and their longitudinal momentum.

Relating GPD's to the Form Factors

GPD's are not directly observable, but are related to many measurable quantities:

$$H_q(x,0,0) = q(x)$$
$$\widetilde{H}(x,0,0) = \Delta q(x)$$

Momentum and Angular Momentum Distributions

We also have

$$\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boxed{A^q(t) + \xi^2 D^q(t)}$$
$$\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boxed{B^q(t) - \xi^2 D^q(t)}$$

EMT Form Factors

Ji's Sum Rule: $\int dx x\big(H(x,\xi,t) + E(x,\xi,t)\big) = 2J(t)$

Relating GPD's to the Form Factors

1. Calculate the Compton Form Factor (CFF) from GPD H

$$\mathcal{H}(\xi, t) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] H(x, \xi, t)$$

2. Relate real and imaginary parts of CFF through dispersion relation:

$$\mathrm{Re}\mathcal{H}(\xi, t) = \frac{1}{\pi} \mathcal{P} \int_{0}^{1} d\xi' \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right] \mathrm{Im}\mathcal{H}(\xi', t) + 4D(t)$$

3. Subtraction "constant" proportional to D-term

Free Field Case

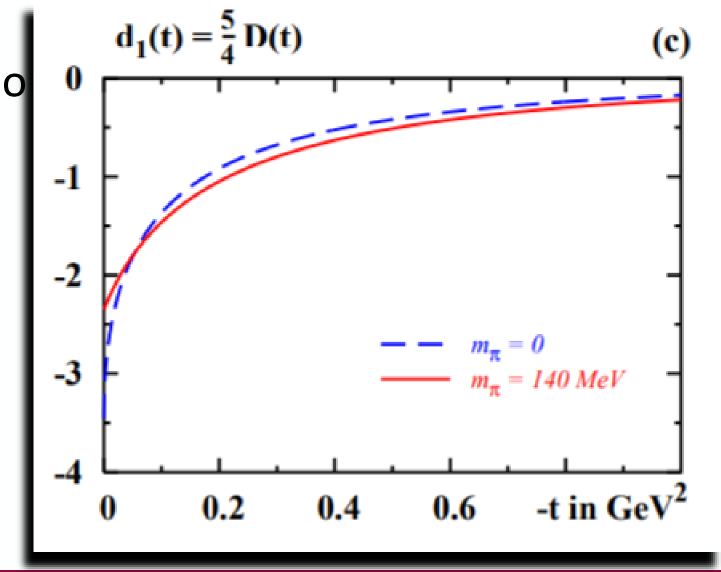
For a real Klein-Gordan Field: $A(t) = 1$, $D(t) = -1$

Question: Is the D-term sensitive to interactions?

D-term

Goeke et al, PRD75 (2007) 094021

Only o



or some

ϕ^3-Theory and Scalar Diquark Model

$$\mathcal{L} = \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3$$

Nucleon

Quark/Scalar Diquark

How they interact

$$\mathcal{L} = \frac{1}{2}\bar{N}(i\overrightarrow{\not\partial} - i\overleftarrow{\not\partial})N + \frac{1}{2}\bar{q}(i\overleftrightarrow{\not\partial} - i\overleftrightarrow{\not\partial})q - m_q\bar{q}q + (\partial_\mu\phi\partial^\mu\phi^* - m_s^2|\phi|^2) - i\lambda_s(\bar{N}q\phi^* - \bar{q}N\phi)$$

ϕ^3 is the training wheels version of the scalar-diquark Lagrangian.

Still insightful because of how little we know about the D-term.

Scalar Diquark Model

GPD's already calculated to lowest order (Bhattacharya et al. 2018):

$$H(x,\xi,t)=\begin{cases}0 & -1\le x\le -\xi\\ \dfrac{g^2(x+\xi)(1+\xi)(1-\xi^2)}{4(2\pi)^3}\displaystyle\int d^2\vec{k}_\perp \frac{N_H}{D_1\,D_2^{-\xi\le x\le\xi}} & -\xi\le x\le\xi\,,\\ \dfrac{g^2(1-x)(1-\xi^2)}{2(2\pi)^3}\displaystyle\int d^2\vec{k}_\perp \frac{N_H}{D_1\,D_2^{x\ge\xi}} & x\ge\xi\,,\end{cases}$$

$$N_H = \vec{k}_\perp^2+(m_q+xM)^2+(1-x)^2\frac{t}{4}-(1-x)\xi t\,\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}$$

$$\begin{aligned}D_1 &= (1+\xi)^2\vec{k}_\perp^2+\frac{1}{4}(1-x)^2\vec{\Delta}_\perp^2-(1-x)(1+\xi)\vec{k}_\perp\cdot\vec{\Delta}_\perp+(1-x)(1+\xi)m_q^2+(x+\xi)(1+\xi)m_s^2\\ &\quad-(1-x)(x+\xi)M^2\,,\\ D_2^{-\xi\le x\le\xi} &= \xi(1-\xi^2)\vec{k}_\perp^2+\frac{1}{4}(1-x^2)\xi\vec{\Delta}_\perp^2+x(1-\xi^2)\vec{k}_\perp\cdot\vec{\Delta}_\perp+\xi(1-\xi^2)m_q^2-\xi(x^2-\xi^2)M^2\,,\\ D_2^{x\ge\xi} &= (1-\xi)^2\vec{k}_\perp^2+\frac{1}{4}(1-x)^2\vec{\Delta}_\perp^2+(1-x)(1-\xi)\vec{k}_\perp\cdot\vec{\Delta}_\perp+(1-x)(1-\xi)m_q^2+(x-\xi)(1-\xi)m_s^2\\ &\quad-(1-x)(x-\xi)M^2\,.\end{aligned}$$

Scalar Diquark Model

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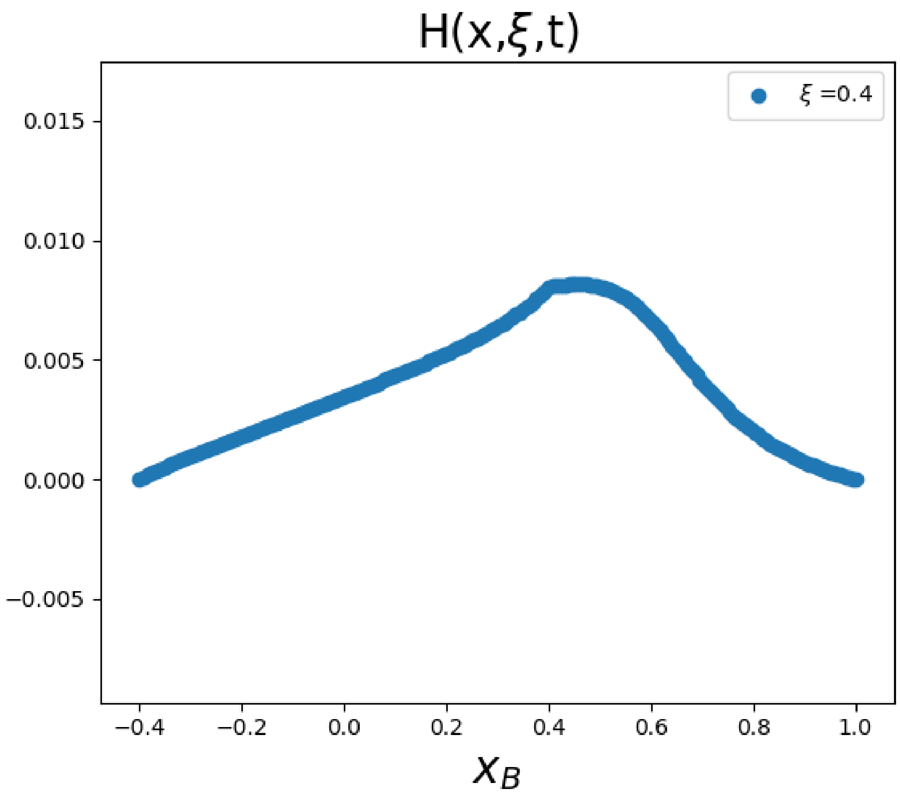
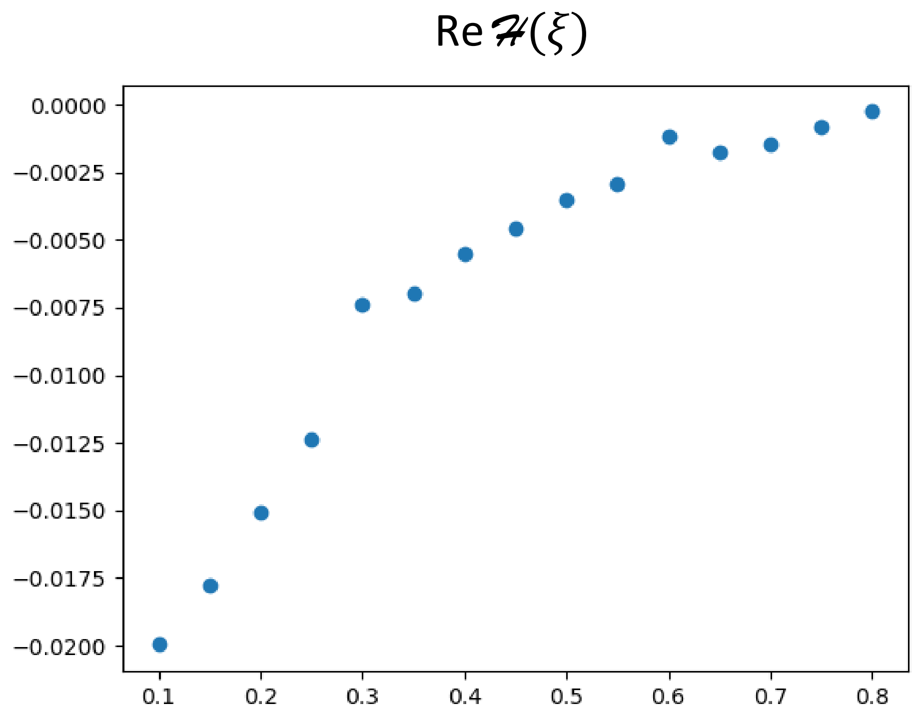
Project is front heavy, reproducing GPD's calculated by Bhattacharya et al. has been challenging, particularly at $|\Delta_\perp| = 0$.

Preliminary Results





Starting to extract CFF from GPD's. Still need to reproduce GPD H at $|\Delta_\perp| = 0$.

Summary

The D-term is an important property of matter related to how pressure is distributed inside of particles.

Because gravity is so much weaker than the other interactions, must extract D-term experimentally though other methods such as DVCS.

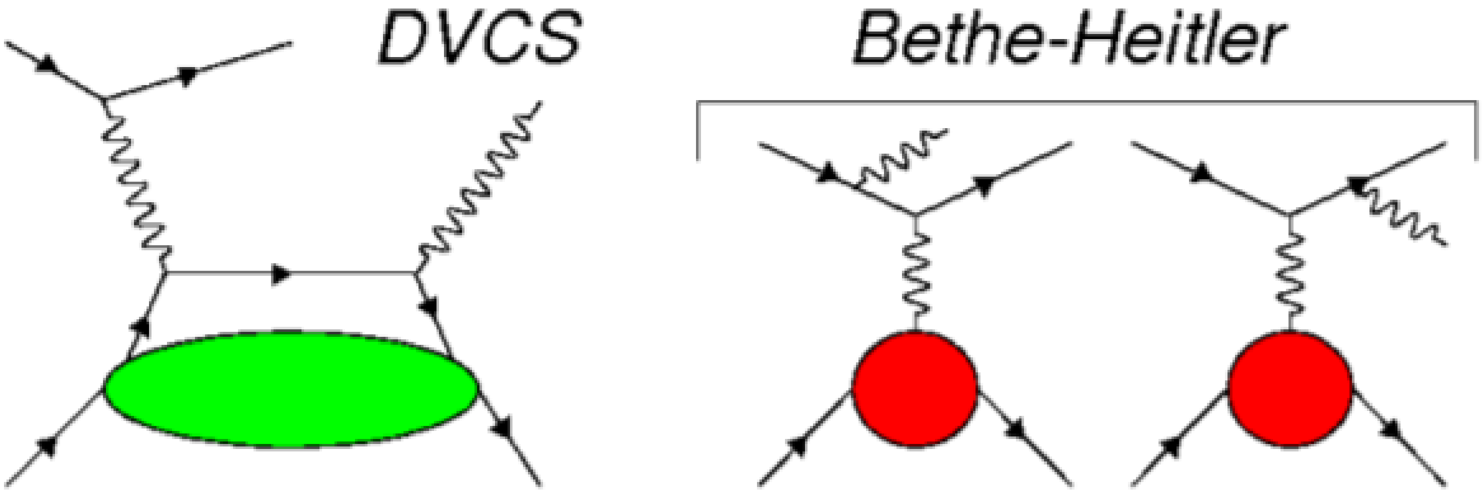
The D-term can be calculated exactly for a free field, but even in simple models such as ϕ^3 and Scalar Diquark it has proven to be difficult to extract.

Future: Once the D-term has been calculated from the CFF, we can verify its value by calculating the relevant Feynman Diagrams for the matrix element of the EMT directly. Then we can relate it to pressure distributions, perhaps in IMF for 2-D Interpretation. Additional insight might be gained by calculating the form factor classically as well (Filip Bergabo).

References

- Reaching for the Horizon: The 2015 Long Range Plan for Nuclear Science. United States: N. p., 2015. Web.
- M. V. Polyakov, P. Schweitzer arXiv:1805.06596 [hep-ph] 14 Sep 2018
- L. S. Brown and J. C. Collins, Annals Phys. 130, 215 (1980).
- S. Bhattacharya et al. arXiv:1808.01437v2 [hep-ph] 25 Sep 2018

Back Up Slides


DVCS
Bethe-Heitler

The Energy-Momentum Tensor (EMT)

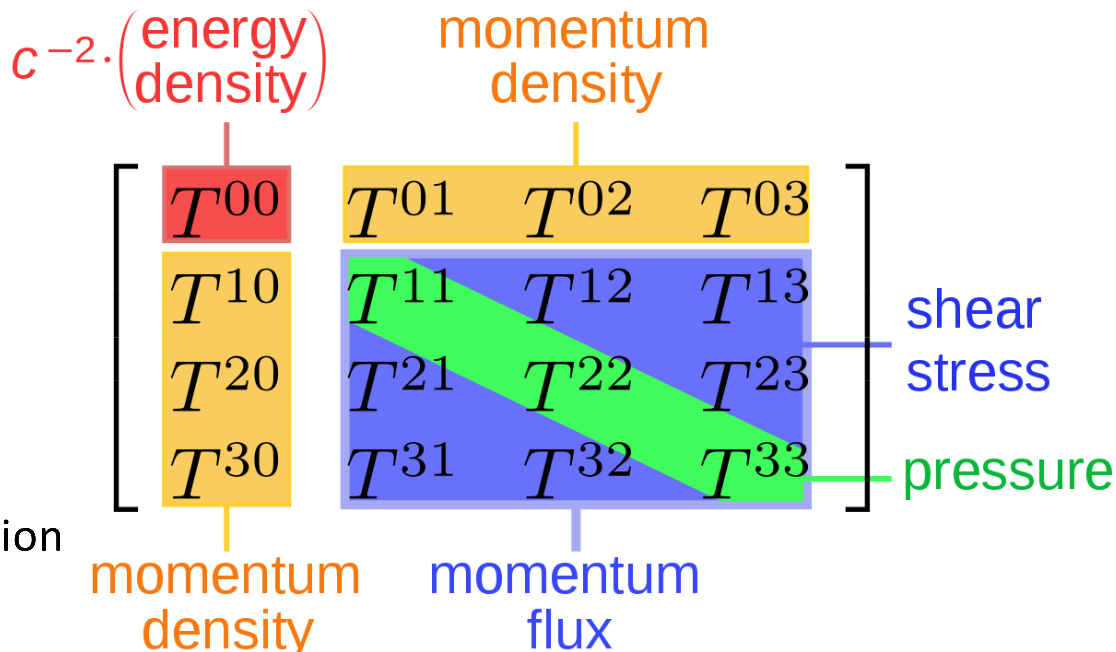
A Lagrangian that exhibits global space-time translational invariance (i.e. $x^\mu \to x^\mu + \alpha^\mu$) yields the EMT as it's conserved current:

$$T^{\mu\nu} = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \partial^\nu \phi_n - g^{\mu\nu}\mathcal{L}$$

Which must satisfy $\partial_\mu T^{\mu\nu} = 0$

Example: $\partial_\mu T^{\mu 0} = \frac{\partial E}{\partial t} + \nabla \cdot \vec{p} = 0$

This is a statement of the conservation of energy and momentum!



Free Field Case

The EMT for a free real scalar field is:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\, \partial^\nu \phi - g^{\mu\nu}\mathcal{L}, \text{ with } \mathcal{L} = \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2} m^2 \phi^2$$

$$= \partial^\mu\, \phi \partial^\nu \phi - \frac{1}{2} g^{\mu\nu}\left(\partial_\alpha \phi \partial^\alpha \phi - m^2 \phi^2\right)$$

Its matrix elements (Hudson, Schweitzer (2017) and reproduced in our work) are:

$$< p'|T^{\mu\nu}(0)|p > = \frac{P^\mu P^\nu}{2} - \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{2}$$

And since $< p'|T^{\mu\nu}(0)|p > = \frac{P^\mu P^\nu}{2} A(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D(t)$,

$$\boxed{A(t) = 1, D(t) = -1}$$

EMT for ϕ^3-Theory

$$\mathcal{L} = \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3$$

Free-Field EMT

Contribution due to Interaction

The corresponding EMT for this theory is:

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}(m_{ph}^2 + \delta m^2)\phi^2 - \frac{\lambda}{3!}\phi^3\right)$$

Where $m^2 = m_{ph}^2 + \delta m^2$,and δm^2 is a *mass counterterm* to subtract the divergence of the self energy diagram.

EMT for ϕ^3-Theory

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}(m_{ph}^2 + \delta m^2)\phi^2 - \frac{\lambda}{3!}\phi^3\right)$$

The matrix element to arbitrary order in perturbation theory calculated by expanding:

$$< p'\left|T^{\mu\nu}(x,y)e^{i\int d^4 z L_{int}}\right|p > = < p'\left|T^{\mu\nu}_{free}(x,y)\right|p >$$

$$+\int d^4 z\left(-\frac{i\lambda}{3!}\right) < p'|T^{\mu\nu}(x,y)\phi^3(z)|p > \ + \cdots$$

For this theory, we obtain no contributions to the free field EMT at $\mathcal{O}(\lambda)$. That is,

$$< p'\left|T^{\mu\nu}_{\mathcal{O}(\lambda)}(x,y)\right|p > = 0$$

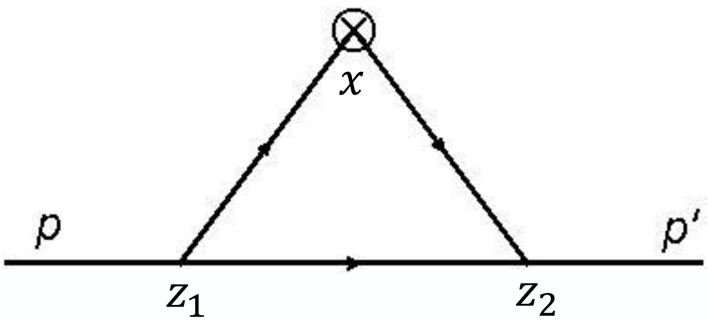
Must go to $\mathcal{O}(\lambda^2)$!

Diagrams That Contribute at $\mathcal{O}(\lambda^2)$

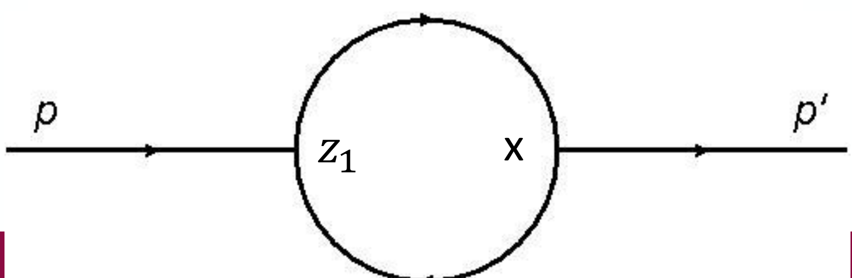
The diagrams which contribute at $\mathcal{O}(\lambda^2)$:

$$\langle p'|\phi(x)\phi(y)\phi(z_1)\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)\phi(z_2)|p\rangle$$

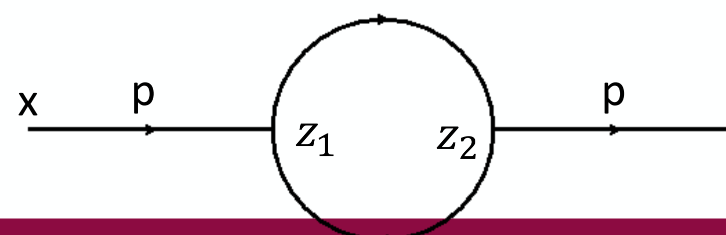
Let x = y after derivatives are taken



$$\langle p'|\phi(x)\phi(x)\phi(x)\phi(z_1)\phi(z_1)\phi(z_1)|p\rangle$$



$$\text{Self} = \langle 0|\phi(x)\phi(y)\phi(z_1)\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)\phi(z_2)|0\rangle$$



Not Done Yet

$$\langle p'|\phi(x)\phi(y)\phi(z_1)\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)\phi(z_2)|p\rangle$$

$$\langle p'|\phi(x)\phi(x)\phi(x)\phi(z_1)\phi(z_1)\phi(z_1)|p\rangle$$

$$\mathrm{Self} = \langle 0|\phi(x)\phi(y)\phi(z_1)\phi(z_1)\phi(z_1)\phi(z_2)\phi(z_2)\phi(z_2)|0\rangle$$

These are the important diagrams, but still need to take various derivatives of them and/or multiply by appropriate factors according to the EMT definition:

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}(m_{ph}^2 + \delta m^2)\phi^2 - \frac{\lambda}{3!}\phi^3\right)$$

Putting It All Together

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \frac{P^\mu P^\nu}{2}A(t) - \frac{\Delta^\mu\Delta^\nu}{2}\left(1+\frac{\lambda^2\mu^2}{(4\pi)^2}\int dx_i^2\delta(x_i^2-1)\frac{1+x_2(1-x_2)+2x_2x_3-x_2-x_3}{\Delta_2}\right)$$
$$-g^{\mu\nu}\left[-\frac{1}{2}t+\frac{\lambda^2\mu^2}{(4\pi)^2}\left(\ln m_{ph}-\int dx_i^1\delta(x_i^1-1)\ln\Delta_1+\int dx_i^2\delta(x_i^2-1)\left(\ln\Delta_2+\frac{1}{\Delta_2}(-\frac{1}{2}t-2C(x_i^2))\right)\right)\right]$$

with

$$A(t)\equiv 1+\frac{\lambda^2\mu^2}{(4\pi)^2}\int dx_i^2\delta(x_i-1)\frac{1+2x_2(x_2-1)+2x_2x_3-x_3-x_2}{\Delta_2}$$

$$\Delta_1 = (x_2p)^2 + x_1m^2$$
$$\Delta_2 = (x_2p+x_3p')^2 + x_1m^2$$

Should have $< p'|T^{\mu\nu}(0)|p > = \frac{P^\mu P^\nu}{2}A(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{2}D(t),$

Key Issues:

- No mutual factor to pull out of $\Delta^\mu\Delta^\nu$ and $g^{\mu\nu}$ terms -> No D-term yet
- A(t) factored out, however doesn't satisfy A(0)=1
- Uniqueness?

Improvement Term

One may modify the definition of the EMT so long as it remains symmetric (GR) and divergenceless (NT) (and, if this were a gauge theory, not contribute to the Ward Identities)

Define $T^{\mu\nu}_{imp} = T^{\mu\nu} + \Theta^{\mu\nu}$

Where $\Theta^{\mu\nu} = -h(\partial^\mu \partial^\nu - g^{\mu\nu} \,\Box)\phi^2$

Introducing this improvement term will affect the D-term by a constant, but not the A-term Polyakov, Schweitzer (2018)

Interesting because D-term is an observable and should be uniquely defined.
Has been shown that this is the correct improvement term even when studying ϕ^4 theory in a weakly curved gravitational background Brown, Collins (1980)