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Manifestation of quantum correlation in the interpolating helicity amplitudes

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e-HUGS 2021

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Quantum Correlation due to orientation entanglement of spins

- The angle should be rotated to get the same initial configuration.
  - Spin- 0 → Any angle
  - Spin-1/2 → 720^0
  - Spin-1 → 360^0

- Spinors and polarization vectors
  - Spin orientation
  - Momentum of the particle

Quantum correlation in the interpolating helicity amplitudes ?

$$d^{(j)}_{m',m}(\beta) = < j, m'|exp(\frac{-ij_y\beta}{\hbar})|j, m >$$

(Wigner-d function)

Rotation by 180^0

Spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Spin-1/2 particles

$$|1/2,1/2> \longrightarrow |1/2,-1/2>$$
$$|1/2,-1/2> \longrightarrow -|1/2,1/2>$$

Spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$
$$|1,0> \longrightarrow -|1,0>$$
$$|1,-1> \longrightarrow |1,1>$$

Quantum –orientation entangled states

Interpolating Dynamic

Helicity is not invariant under the boost $\mathfrak{I}_3 \rightarrow \frac{\boldsymbol{P}.\boldsymbol{J}}{|\boldsymbol{P}|}$

IFD ($\delta = 0$)

δ_c

LFD ($\delta = \frac{\pi}{4}$)

Helicity is invariant under the boost

$\mathfrak{I}_3 \rightarrow J_3 + \frac{(P^1 E_2 - P^2 E_1)}{P^+}$

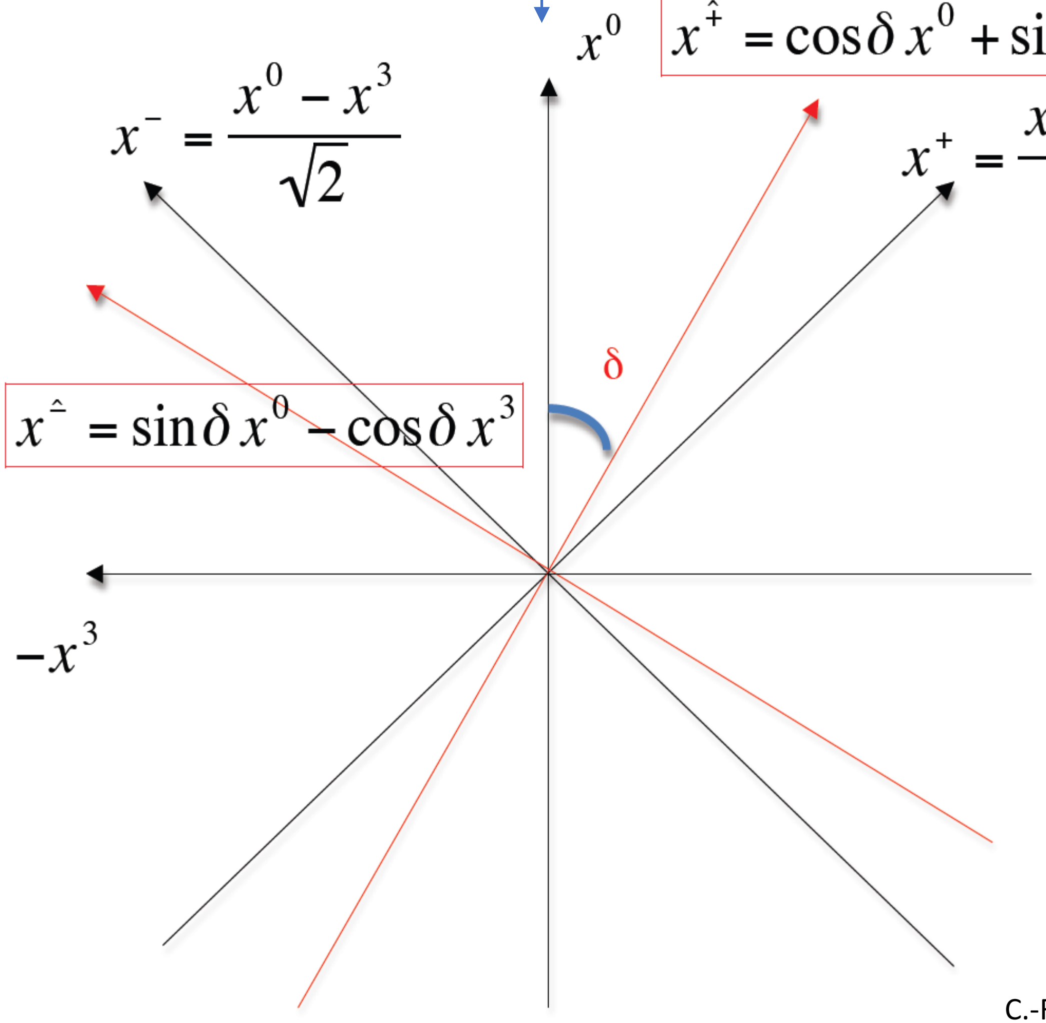
Interpolation space time transformation

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos(\delta) & 0 & 0 & \sin(\delta) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\delta) & 0 & 0 & -\cos(\delta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$x^{\hat{+}} = \cos\delta\, x^0 + \sin\delta\, x^3$$

$$x^- = \frac{x^0 - x^3}{\sqrt{2}} \qquad x^+ = \frac{x^0 + x^3}{\sqrt{2}}$$

$$x^{\hat{-}} = \sin\delta\, x^0 - \cos\delta\, x^3$$



- We connect two relativistic dynamics, proposed by Dirac

How quantum correlation manifest in different dynamic ?

P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949)

M. Jacob and G. Wick, Ann. Phys.**7**, 404 (1959)

C.-R. Ji, Z. Li, and B. Ma Phys. Rev. D **98**, 036017 (2018)

Spin orientation for generalize helicity spinors and vectors

Helicity transformation matrix

$$T = T_{12}T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$$

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta,$$

$$\mathcal{K}^{\hat{2}} = J^1 \cos\delta - K^2 \sin\delta,$$

$$(\delta \to \pi/4),\ \mathcal{K}^{\hat{1}} \to -E_1,\ \mathcal{K}^{\hat{2}} \to -E_2$$

$$(\delta \to 0),\ \mathcal{K}^{\hat{1}} \to -J^2,\ \mathcal{K}^{\hat{2}} \to J^1$$

- Depending on the spin and the representation Lorentz Group generators of rotation (J) and boost (K) change.

We consider this transformation for spin-up

$$T = B(\boldsymbol{\eta})\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}} e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s},$$

Boosts to momentum **P**
$\hat{n} = (\,sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$

Rotates the spin around the axis by a unit vector $\hat{m} = (\,-sin\varphi_s, cos\varphi_s, 0)$ by angle θ_s .

$$\mathrm{Cos}[\theta s] = \frac{\mathrm{Cos}[\alpha] + \mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\alpha]\mathrm{Cosh}[\beta 3] - \mathrm{Cosh}[\eta]}{1 + \mathrm{Cosh}[\eta]}$$

$$\mathrm{Cosh}[\eta] = \frac{\left((\mathrm{Cos}[\delta]\mathrm{Cosh}[\beta 3] + \mathrm{Sin}[\delta]\mathrm{Sinh}[\beta 3])\mathrm{Cos}[\delta]\right) - \left(\mathrm{Sin}[\delta]\mathrm{Cos}[\alpha](\mathrm{Sin}[\delta]\mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\delta]\mathrm{Sinh}[\beta 3])\right)}{\mathrm{Cos}[2\delta]}$$

$$\mathrm{Cos}[\phi s] = \frac{\beta 1}{\beta 1^2 + \beta 2^2} = \mathrm{Cos}[\phi] \qquad \mathrm{Sin}[\phi s] = \frac{\beta 2}{\beta 1^2 + \beta 2^2} = \mathrm{Sin}[\phi]$$

Without loss and generality, we can make $\phi = \phi s = 0$

$$e^{-\beta_3} = \frac{P^{\hat{+}} - \mathbb{P}}{M(\cos\delta - \sin\delta)}$$

$$e^{\beta_3} = \frac{P^{\hat{+}} + \mathbb{P}}{M(\sin\delta + \cos\delta)}$$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}}$$

$$\alpha = \sqrt{\mathbb{C}(\beta_1^2 + \beta_2^2)}$$

$$\frac{\beta_j}{\alpha} = \frac{P^j}{\sqrt{\mathbf{P}_{\perp}^2 \mathbb{C}}} \qquad (j = 1, 2)$$

When, we fix the particle's initial momentum direction as +z ($\theta = 0$) ,

We can simplify

$$\theta_s = 0 \ , \cos\alpha = P_{\hat{-}}/\mathbb{P} \to 1 \quad P_{\hat{-}} > 0$$

$$\theta_s \to \pi \ , \cos\alpha = P_{\hat{-}}/\mathbb{P} \to -1 \quad P_{\hat{-}} < 0$$

$$\mathbb{P} = \sqrt{P_{\hat{-}}^2 + \mathbf{P}_\perp^2 \mathbb{C}}$$

$\Rrightarrow$ Indicates sign change of $P_{\hat{-}}$

QC in Spin-1/2 spinors

$$U^{+1/2}(P_{\hat{-}} > 0) \Rrightarrow U^{-1/2}(P_{\hat{-}} > 0)$$
$$U^{-1/2}(P_{\hat{-}} > 0) \Rrightarrow -U^{+1/2}(P_{\hat{-}} > 0)$$

QC in spin-1 spinors

$$U^{+1}(P_{\hat{-}} > 0) \Rrightarrow U^{-1}(P_{\hat{-}} > 0)$$
$$U^{0}(P_{\hat{-}} > 0) \Rrightarrow -U^{0}(P_{\hat{-}} > 0)$$
$$U^{-1}(P_{\hat{-}} > 0) \Rrightarrow U^{+1}(P_{\hat{-}} > 0)$$

QC in polarization vectors

$$\epsilon^{+1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$
$$\epsilon^{0}(P_{\hat{-}} > 0) \Rrightarrow -\epsilon^{0}(P_{\hat{-}} > 0)$$
$$\epsilon^{-1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$

$P_{\hat{-}}$ ← Interpolating longitudinal momentum

- Initial direction of particles' momentum
- Boost of the frame
- Interpolation angle

Interpolating Longitudinal Momentum

$$P_1 = \{E_0, 0,0, P_v\}$$
$$P_2 = \{E_0, 0,0, -P_v\}$$

$$P_{1\hat{-}} = [(P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2})\sin\delta + (E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2})\cos\delta]/\bar{E}$$

$$P_{2\hat{-}} = [(-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2})\sin\delta + (E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2})\cos\delta]/\bar{E}$$

We see $0 \le \delta < \frac{\pi}{4}$ range $P_{\hat{-}}$ can get any real value, but exactly at the LF we do not see $P_{\hat{-}} < 0$ values , since $P_{\hat{-}} \to P^+$ at $\delta = \frac{\pi}{4}$.

- Change of K^3 from "dynamic" operator to the "kinematic " operator at LF

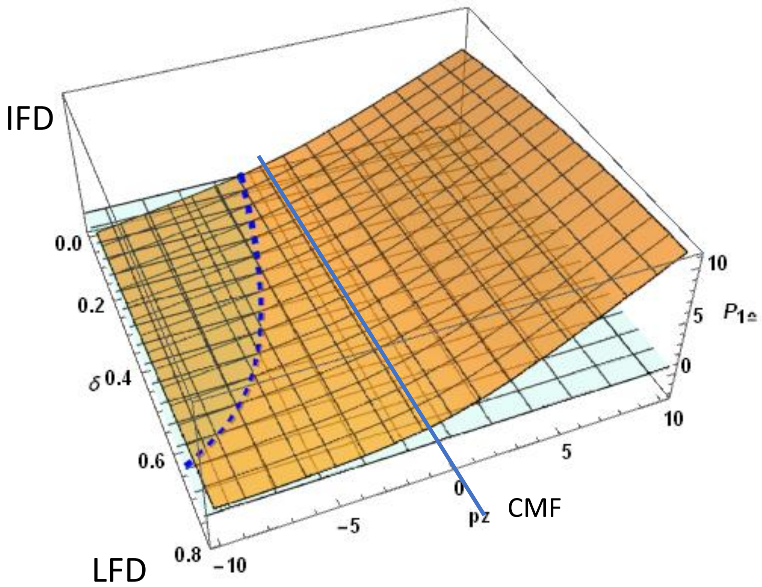
$\Longrightarrow$

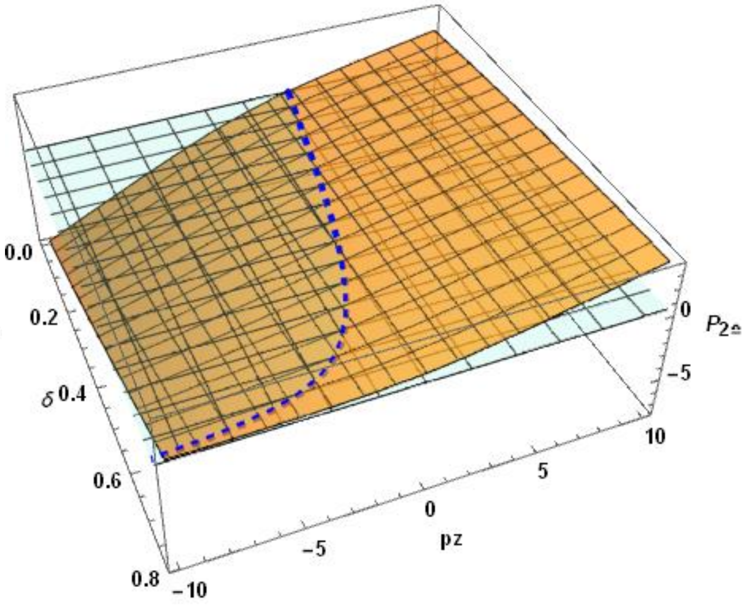
$P_{1\hat{-}} = 0$, and $P_{2\hat{-}} = 0$, $P_1^+ \to 0$ and $P_2^+ \to 0$

$$P^z \to -\infty$$

Helicity Boundaries

$\bar{E} = 2E_0$





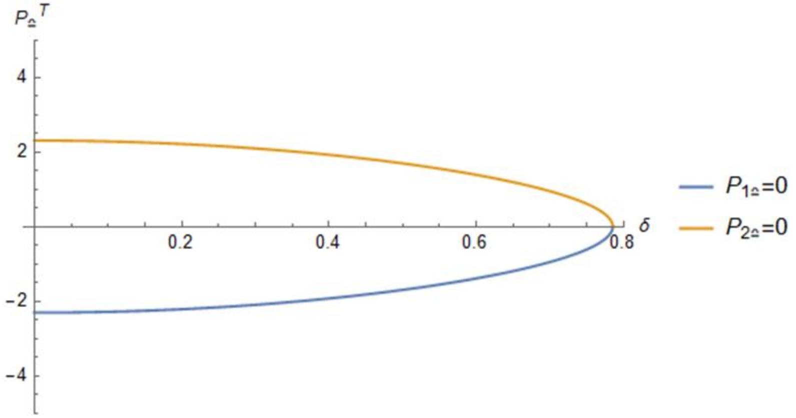
$$\tan(\delta_{c1}) = -\frac{E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2}}{P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}} \qquad \tan(\delta_{c2}) = -\frac{E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2}}{-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}}$$

It seems that the QC of LF is accumulated in the zero-mode

To see the QC in the zero-mode we consider total longitudinal momentum of the system

$$(P_{\hat{-}})^T = \sqrt{\bar{E}^2 + (P^z)^2}\,\sin\delta + P^z\cos\delta$$

$$\Rightarrow P^z = \frac{P_{\hat{-}}^T\cos\delta - \sin\delta\sqrt{(P_{\hat{-}}^T)^2 + \bar{E}^2\cos 2\delta}}{\cos 2\delta}$$



Zero-mode of the light front

$$P^z \to -\infty\,,\ (P_{\hat{-}})^T \to 0$$

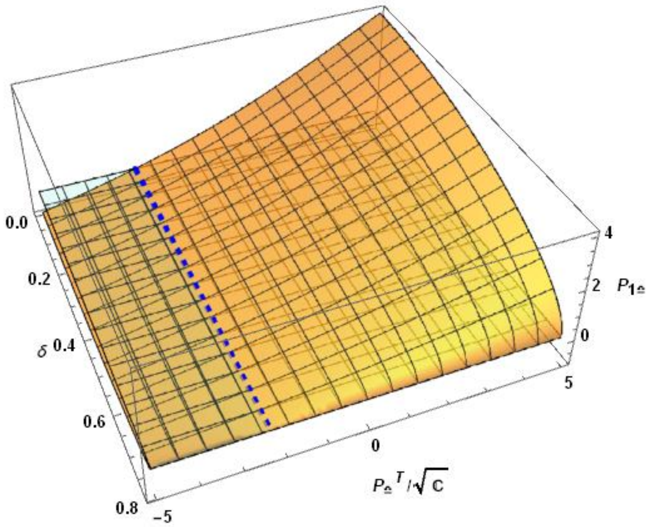
This further confirm that the QC accumulated in the zero-mode

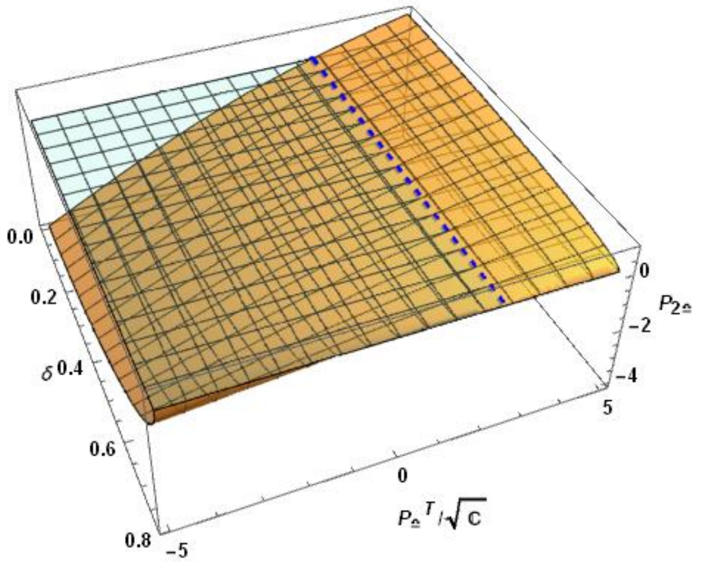
Scanning the zero-mode

At $\delta = \frac{\pi}{4}$ $\quad \frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}$ ⟶ Finite Value

$$\mathbb{C} = \cos 2\delta$$

QC Boundaries





$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c1} = -\frac{\bar{E}P_v}{M} = -2.3094$$

$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c2} = \frac{\bar{E}P_v}{M} = 2.3094$$

$$\bar{E} = 2E_0,\, E_0 = 2,\, P_v = 1,\, M = \sqrt{3}$$

Scalar particle and its anti-particle production by two neutral massive spin-1 particles

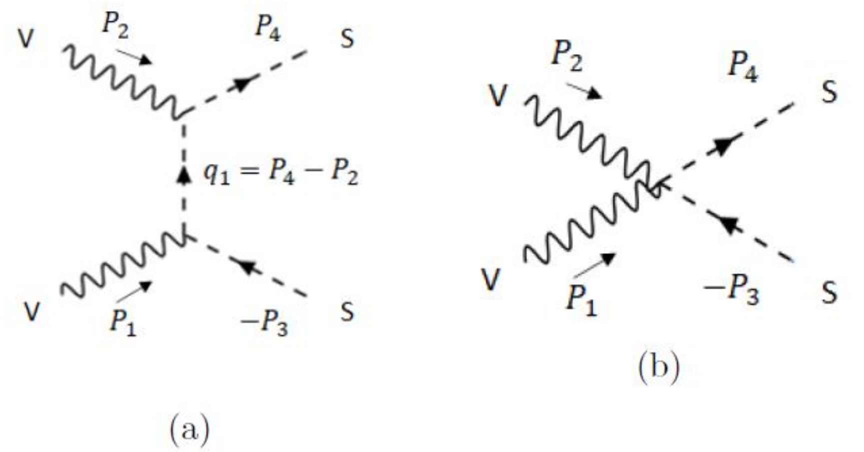


Fig. (a) t-channel Feynman diagram, the cross channel (u-channel) can be drawn by crossing the two final states' particles. Fig. (b) is drawn for the seagull channel.

v -> Vector particle (Spin-1)
s -> Scalar particle (Spin-0)

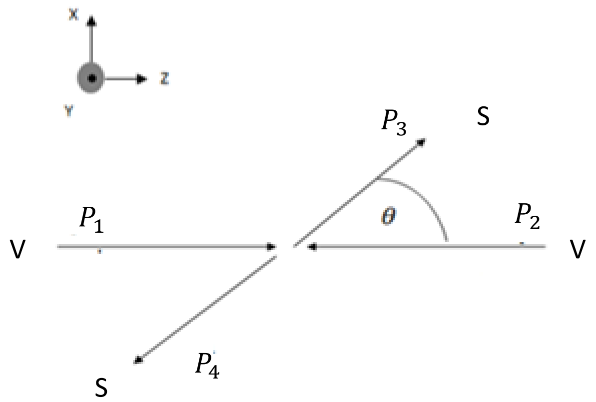
Interpolating helicity amplitudes

$$M_t^{\lambda_1\lambda_2} = (-p_3 + q_1)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1, \lambda_1) \frac{1}{q_1^2 - m_s{}^2} (p_4 + q_1)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2, \lambda_2)$$

$$M_u^{\lambda_1\lambda_2} = (-p_3 + q_2)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2, \lambda_2) \frac{1}{q_2^2 - m_s{}^2} (-p_4 + q_2)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1, \lambda_1)$$

$$M_{se}^{\lambda_1\lambda_2} = -2 g_{\hat{\mu}\hat{\nu}} \varepsilon^{\hat{\mu}} (p_1, \lambda_1)\, \varepsilon^{\hat{\nu}} (p_2, \lambda_2)$$

Where $q_2 = p_3 - p_2$



$$P_1 = \{E_0, 0,0, P_v\}$$

$$P_2 = \{E_0, 0,0, -P_v\}$$

$$P_3 = \{E_0, P_s Sin(\theta), 0, P_s Cos(\theta)\}$$

$$P_4 = \{E_0, -P_s Sin(\theta), 0, -P_s Cos(\theta)\}$$

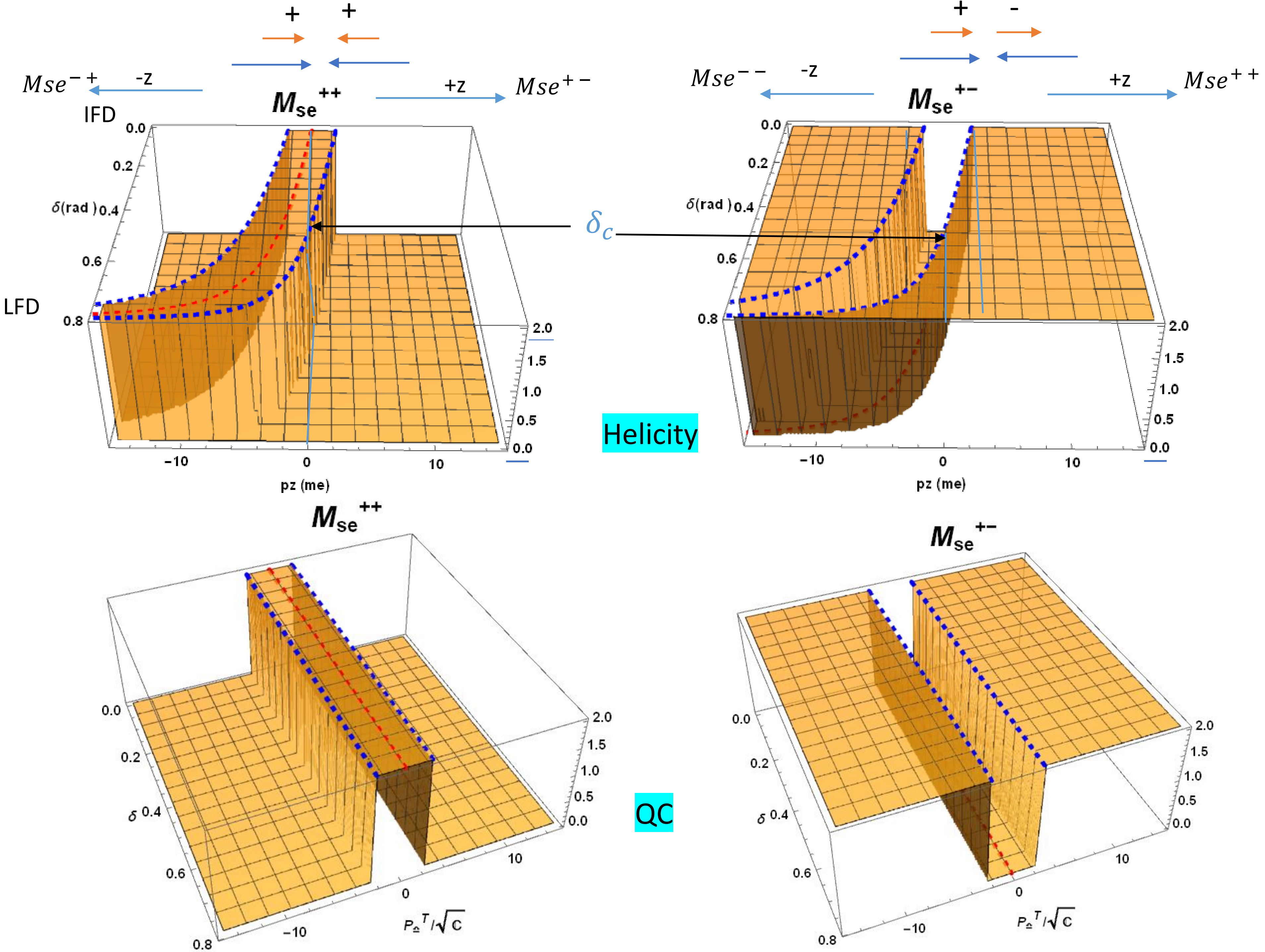
- Center of mass frame and the four momenta of the particles

Seagull Channel

- Contact interaction - Angular momentum is conserved without involving orbital angular momentum

$$\epsilon^{+1}(P_{\hat{-}} > 0) \nRightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$

$$\epsilon^{-1}(P_{\hat{-}} > 0) \nRightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$



Parity conservation

$$Mse^{--} = Mse^{++}$$

$$Mse^{-+} = Mse^{+-}$$

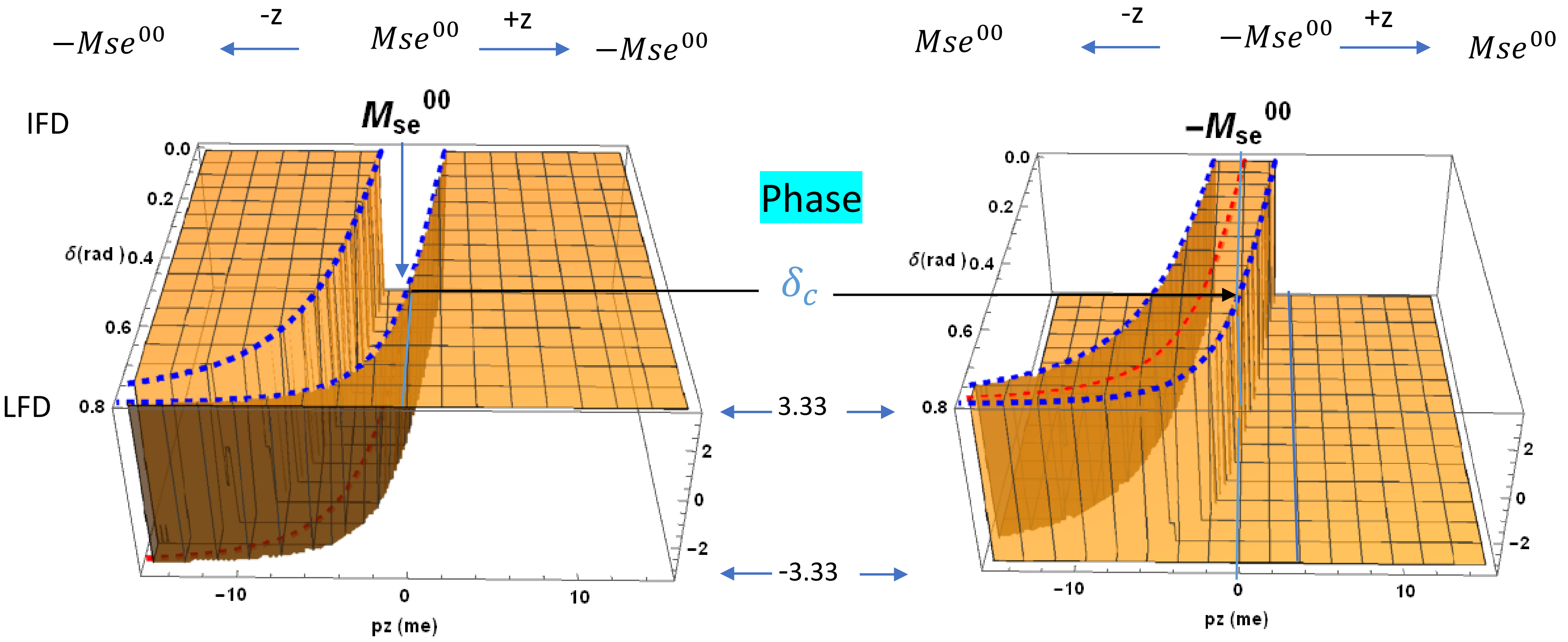
[C-R . Ji, B.L.G.Bakker , International Journal of Modern Physics,(2013)]

$P_s = \sqrt{3}$ $\quad P_v = 1$ $\quad E_0 = 2$

Seagull Channel

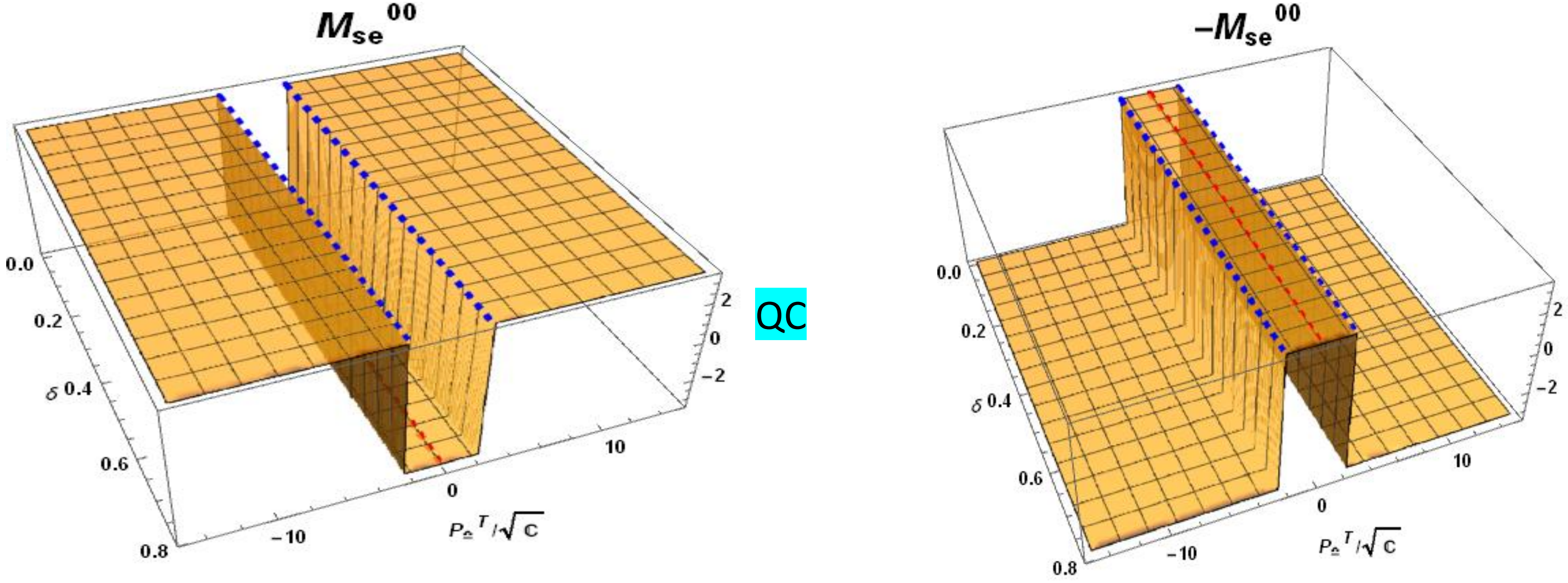
$|1,0> \longrightarrow -|1,0>$

$\epsilon^0(P_{\hat{-}} > 0) \Rightarrow -\epsilon^0(P_{\hat{-}} > 0)$



- $Mse^{+0} = Mse^{-0} = Mse^{0+} = Mse^{0-} = 0$

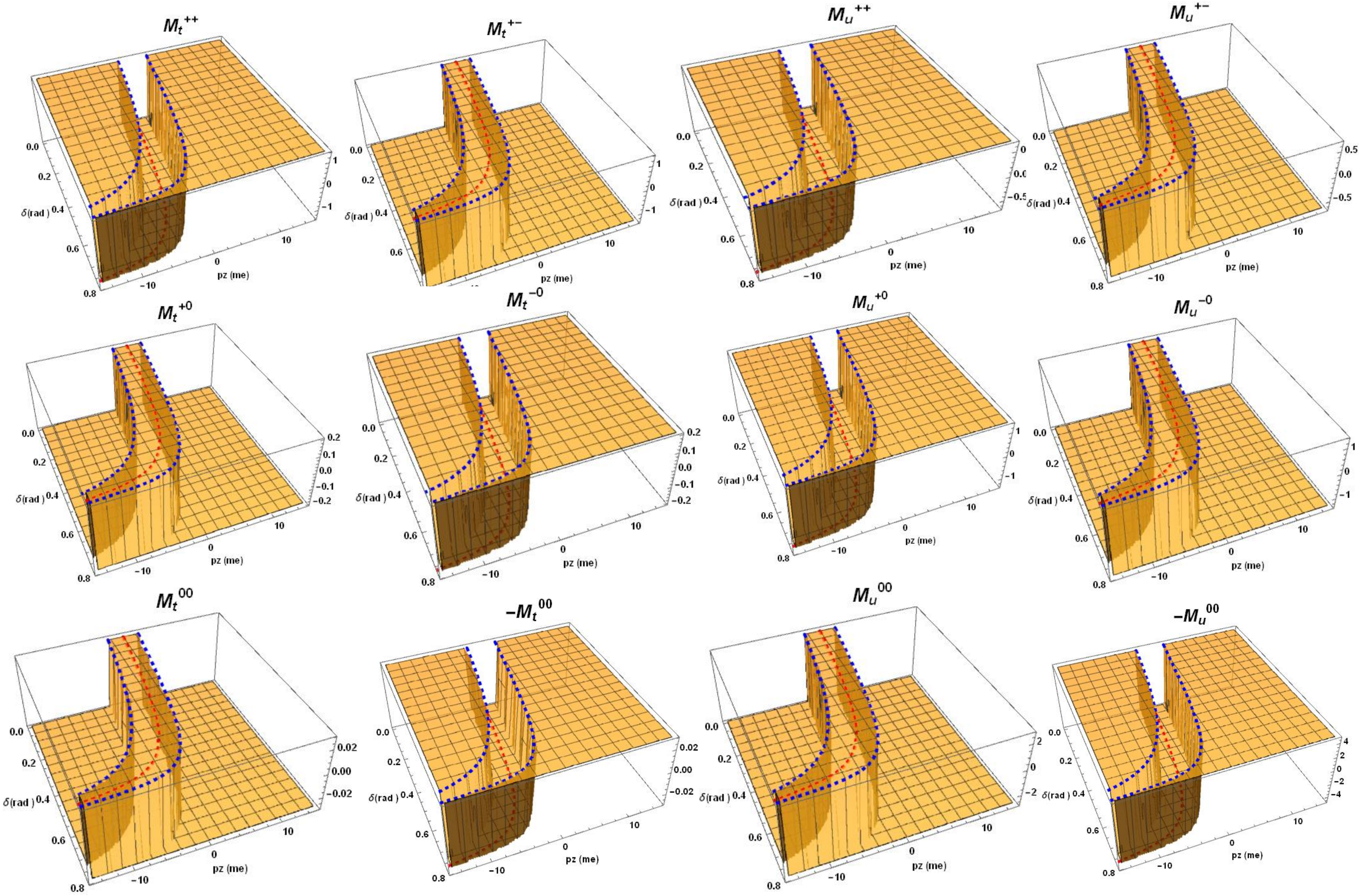
Do not satisfy conservation of total angular momentum in any frame



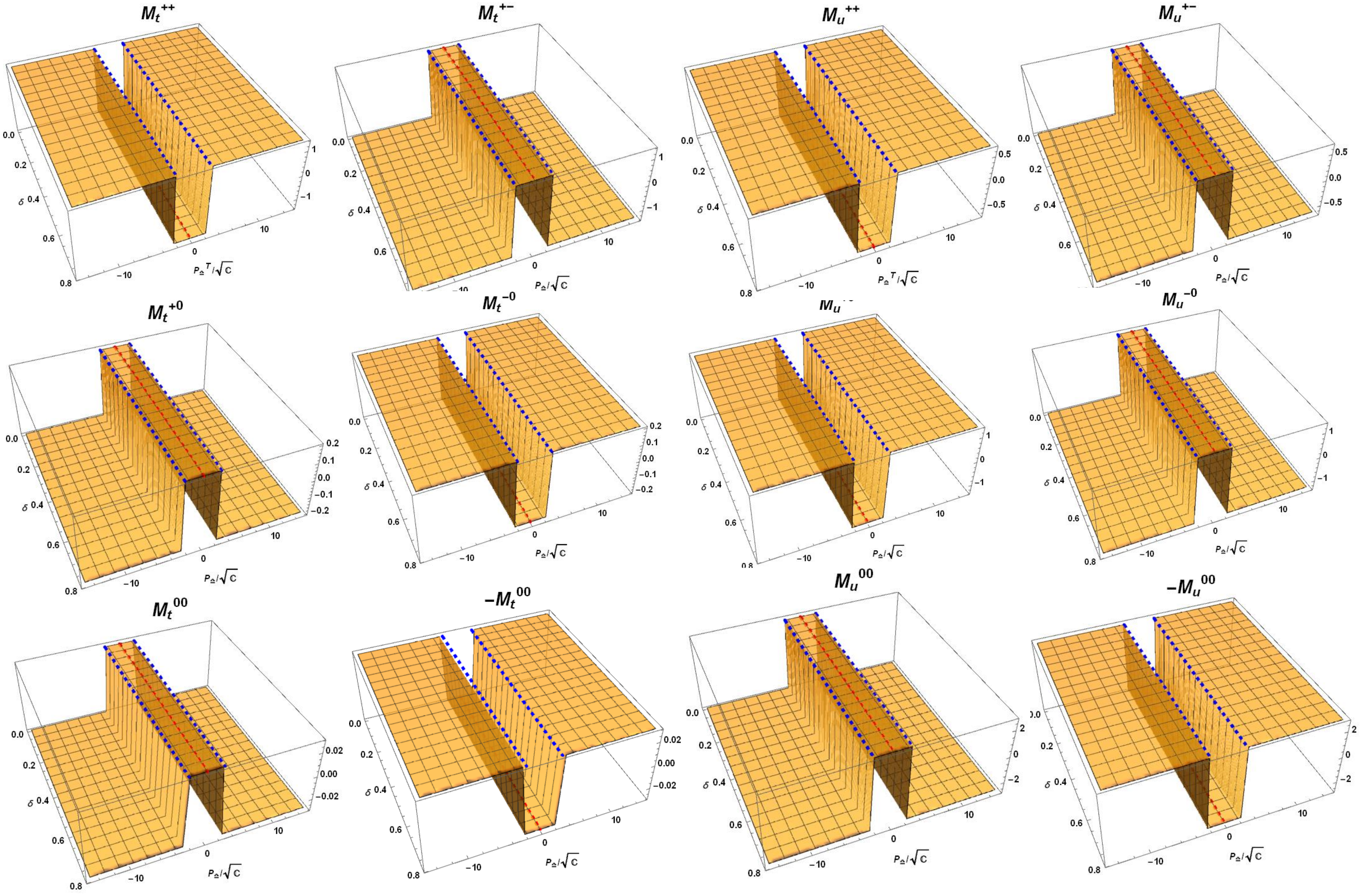
$P_s = \sqrt{3}$ $P_v = 1$ $E_0 = 2$

T and U channels helicity amplitudes

- Depend on orbital angular momentum involving impact parameter
- Same Features can be observed



QC in T and U channels helicity amplitudes



CONCLUSION

- We confirm QC in interpolating spin-1/2 spinors , interpolating spin-1 spinors and polarization vectors.
- Quantum correlation manifest itself as district boundaries in the landscape of helicity amplitudes when we change the interpolation angle and normalized total longitudinal momentum of the system.
- We discuss the conditions which enable us to see the QC for all reference frame and for all interpolation angle
- Specially we show that LF QC appears in the zero-mode . (Quantum entanglement in the LF)
- This hints the ‘a la Einstein’s “spooky action at a distance “ even in the LFD.

THANK YOU

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Interpolating spin- ½ spinors

$$u_H^{(-1/2)}(P) = \begin{pmatrix} -P^L\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ -P^L\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \end{pmatrix} \quad u_H^{(1/2)}(P) = \begin{pmatrix} \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \\ P^R\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ P^R\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \end{pmatrix}$$

Interpolating Spin-1 polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}P^R, \frac{(P_1P_{\hat{-}}-iP_2\mathbb{P})}{P^L}, \frac{(P_2P_{\hat{-}}+iP_1\mathbb{P})}{P^L}, -\mathbb{C}P^R\right)$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}P^L, \frac{(P_1P_{\hat{-}}+iP_2\mathbb{P})}{P^R}, \frac{(P_2P_{\hat{-}}-iP_1\mathbb{P})}{P^R}, -\mathbb{C}P^L\right)$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{M\mathbb{P}}\left(P_{\hat{+}}-\frac{M^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right)$$

Interpolating Spin-1 Helicity spinors

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}}\begin{pmatrix} \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \\ \sqrt{2}P^R(P^{\hat{+}}+\mathbb{P}) \\ \frac{(A-B)(P^R)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \sqrt{2}P^R(P^{\hat{+}}-\mathbb{P}) \\ \frac{(A+B)(P^R)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \end{pmatrix}, \quad u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}}\begin{pmatrix} \frac{(A+B)(P^L)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}-\mathbb{P}) \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \frac{(A-B)(P^L)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}+\mathbb{P}) \\ \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \end{pmatrix} \quad u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}}\begin{pmatrix} -(A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A-B)P^R \\ (-A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A+B)P^R \end{pmatrix}$$

$P^L = P^1 - iP^2$, $\quad P^R = P^1 + iP^2 \quad A = \cos\delta,\ B = -\sin\delta \quad \mathbb{X} \equiv \frac{P^{\hat{+}}-\mathbb{P}}{\mathbb{C}} = \frac{P^{\hat{+}}-\sqrt{(P^{\hat{+}})^2-M^2\mathbb{C}}}{\mathbb{C}}$