Probing Linearly Polarized Gluon Distribution In J/ψ electroproduction at EIC

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Plan of Talk

Gluon TMDs

Linearly Polarized Gluon Distribution

Probing Linearly Polarized Gluon Distribution In J/ψ Production at EIC

Azimuthal Asymmetry : $\cos 2\phi$

Numerical Results

Conclusions

Gluon TMDs

- TMD-PDFs (Transverse Momentum Dependent Parton Distribution Functions): $f(x, k_\perp, Q^2)$ gives the number density of partons, with their intrinsic transverse motion and spin, inside a nucleon.

- Gluon TMD correlator are defined as Fourier transform of forward matrix elements of bilocal products of the gluon field strength

Gluon field tensor

$$\Gamma^{+i;+j}(x, \boldsymbol{k}_T; P,S) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ik\cdot\xi} \langle P,S|\mathrm{Tr}[F^{+i}(0)W_{[0,\xi]}F^{+j}(\xi)W_{[\xi,0]}]|P,S\rangle_{|\xi^+=0}$$

Gauge links

- In the literature, in small-x region, we have Weizsacker-Williams (WW) type and dipole distribution with simple gauge link configurations $++\ or\ --$ and $+-\ or\ -+$ respectively.

Hadron Pol.		Gluon Polarization: U	Circular	Linear
	U	f_1^g		$h_1^{\perp g}$
	L		g_{1L}^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

At leading twist

Linearly Polarized Gluon TMDs

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

It affects unpolarised cross section and cause azimuthal asymmetries, $\cos 2\phi, \cos 4\phi$.

It's a time-reversal even function and in small-x region, it can be WW type or Dipole distribution depending on gauge link

It can be probed in Drell-Yan process and SIDIS process. Though it has not been extracted from the data yet, but lot of theoretical studies has been done.

Initial state interactions and final state interactions may affect the generalized factorization. Such effects are less complicated in ep compared with pp and pA.

$h_1^{\perp g}$ in J/ψ Production

The Leading order process contributing to the $\cos(2\phi)$ asymmetry is $\gamma^* + g \to c + \bar{c}$

A. Mukherjee and S. Rajesh, EPJC 77, 854 (2017)

Contributes at $z = 1$, where z is energy fraction of γ^* carried by J/ψ in proton rest frame

We extended it to the kinematical region $z < 1$ in the CS model.

RK and A. Mukherjee; *Phys.Rev.D* 99 (2019) 5, 054012

With the heavy quark pair produced in the hard process: $\gamma^* + g \to c + \bar{c} + g$.

We further extend it and incorporates the CO contributions to J/ψ production mechanism.

Quarkonium Production

- Quarkonium is a bound state of heavy quark and anti-quark ($Q\bar{Q}$)

Describes conversion of $Q\bar{Q}[n]$ states into final quarkonium state.

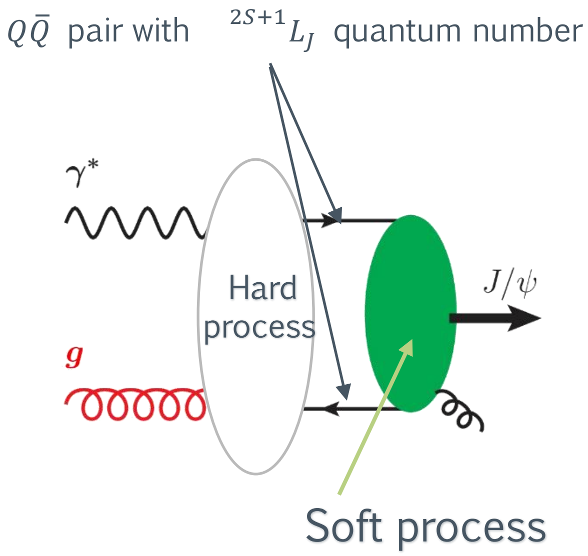
Non-perturbative, long distance matrix elements LDMEs

NRQCD factorization

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

Perturbative short distance coefficient

Cross section in particular color, angular momentum and spin state “n” : ${}^{2S+1}L_J$, calculated by perturbative QCD



G. T. Bodwin et al, PRD51 (1995), Lepage 95

J/ψ Production in ep collision

- We present a $\cos(2\phi_h)$ asymmetry in Process: $e(l) + p(P) \to e(l') + J/\psi(P_h) + X$
- J/ψ is produced using NRQCD based CS and CO models.
- In the kinematic region $z < 1$.
- The corresponding hard process is : $\gamma^* + g \to c\bar{c} + g$ final state gluon is not detected
- The heavy quark pair then hadronizes to form J/ψ, through a soft process.
- $z = P \cdot P_h / P \cdot q$, z is energy fraction of J/ψ in proton rest frame.

$$q = l - l' \qquad Q^2 = s x_B y \qquad s = (l + P)^2 \qquad x_B = \frac{Q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot l}$$

- The incoming and outgoing electron forms the leptonic plane. The azimuthal angles are measured w.r.t. this plane.

J/ψ Production in ep collision

Assuming TMD factorization hold, the differential cross-section is given by

$$d\sigma = \frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_h}{(2\pi)^3 2E_{P_h}}\int\frac{d^3p_g}{(2\pi)^3 2E_g}\int dx d^2\boldsymbol{k}_\perp (2\pi)^4\delta(q+k-P_h-p_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi^{\nu\nu'}(x,\boldsymbol{k}_\perp)\mathcal{M}_{\mu\nu}^{\gamma^*+g\to J/\psi+g}\mathcal{M}_{\mu'\nu'}^{*\gamma^*+g\to J/\psi+g}$$

Lepton tensor: $L^{\mu\mu'}(l,q) = e^2(-g^{\mu\mu'}Q^2 + 2(l^\mu l'^{\mu'} + l^{\mu'} l^\mu))$

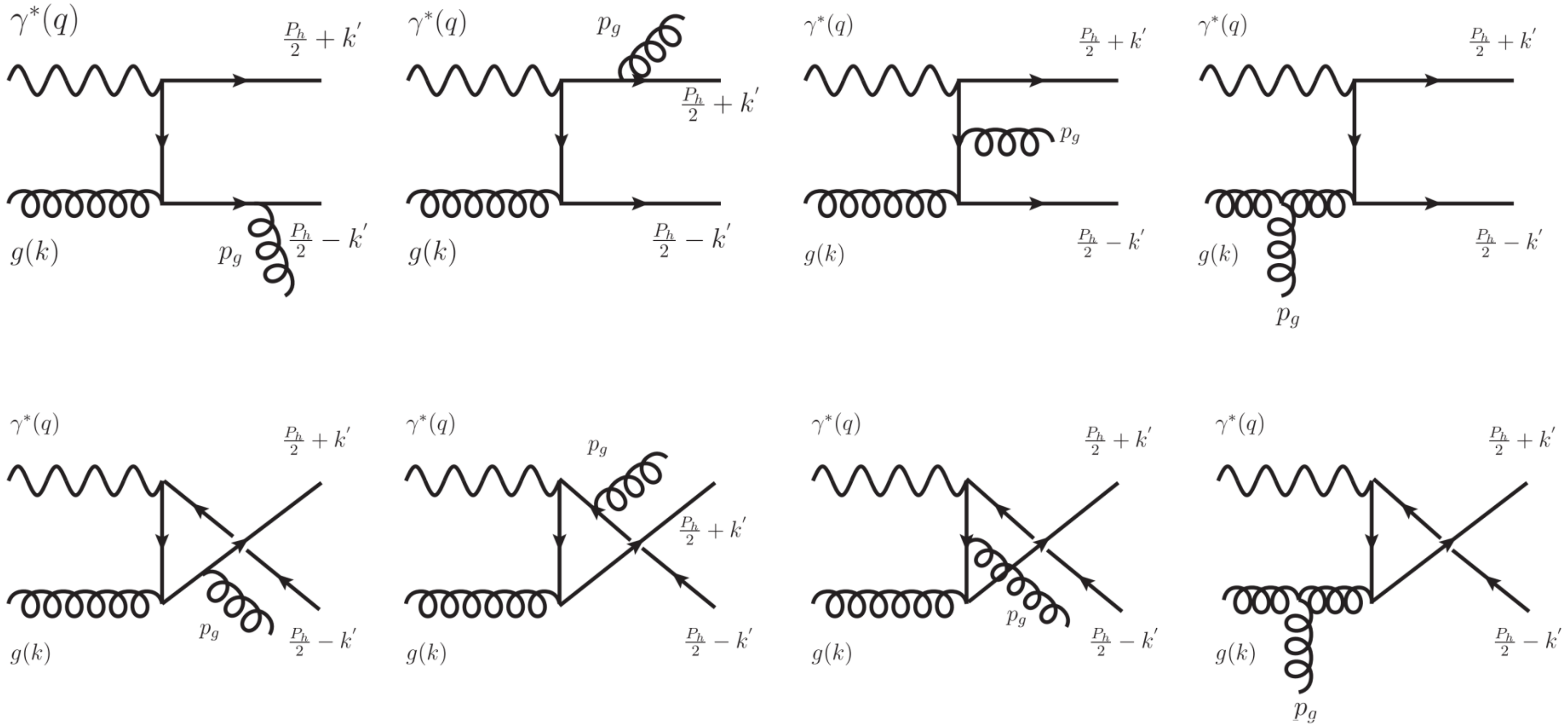
Gluon correlator for unpolarized proton at 'Leading Twist'

$$\phi_g^{\nu\nu'}(x,\boldsymbol{k}_\perp^2) = \frac{1}{2x}\left[-g_\perp^{\nu\nu'} f_1^g(x,\boldsymbol{k}_\perp^2) + \left(\frac{k_\perp^\nu k_\perp^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\boldsymbol{k}_\perp^2}{2M_p^2}\right) h_1^{\perp g}(x,\boldsymbol{k}_\perp^2)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

Feynman Diagrams



Feynman diagrams for process: $\gamma^* + g \to c\bar{c} \ +g$

Amplitude Calculations

The amplitude can be written as

$$\mathcal{M}\left(\gamma^* g \to Q\bar{Q}\left[{}^{2S+1}L_J^{(1,8)}\right](P_h) + g\right) = \sum_{L_z S_z} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}') \langle LL_z; SS_z | JJ_z \rangle$$

$$Tr[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')].$$

The operator $O(q,k,P_h,k')$ is calculated from the Feynman diagrams

$$O(q,k,P_h,k') = \sum_{m=1}^{8} C_m O_m(q,k,P_h,k')$$

The spin projection operator that projects the spin triplet and spin singlet states

$$\mathcal{P}_{SS_z}(P_h,k') = \sum_{s_1,s2} \left\langle \frac{1}{2}s_1; \frac{1}{2}s_2 | SS_z \right\rangle v\left(\frac{P_h}{2} - k', s_1\right) \bar{u}\left(\frac{P_h}{2} + k', s_2\right),$$

$$= \frac{1}{4M^{3/2}}(-\not{P}_h + 2\not{k}' + M)\Pi_{SS_z}(\not{P}_h + 2\not{k}' + M) + \mathcal{O}(k'^2)$$

Amplitude Calculations

Since, $k' \ll P_h$, amplitude expanded in Taylor series about $k' = 0$

First term in the expansion gives the S-states (L=0, J=0,1). The linear term in k' gives the P-states (L=1, J=0,1,2).

The S-states amplitude :

$$\mathcal{M}\left[{}^{2S+1}S_J^{(8)}\right](P_h,k) = \frac{1}{\sqrt{4\pi}} R_0(0) Tr\left[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')\right]|_{k'=0}$$
$$= \frac{1}{\sqrt{4\pi}} R_0(0) Tr\left[O(0)\mathcal{P}_{SS_z}(0)\right],$$

The P-states amplitude :

$$\mathcal{M}\left[{}^{2S+1}P_J^{(8)}\right] = -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(P_h)\langle LL_z; SS_z|JJ_z\rangle \frac{\partial}{\partial k'^{\alpha}} Tr[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')]|_{k'=0}$$
$$= -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(P_h)\langle LL_z; SS_z|JJ_z\rangle Tr[O_\alpha(0)\mathcal{P}_{SS_z}(0) + O(0)\mathcal{P}_{SS_z\alpha}(0)]$$

Where,

$$O(0) = O(q,k,P_h,k')\Big|_{k'=0} \quad \mathcal{P}_{SS_z}(0) = \mathcal{P}_{SS_z}(P_h,k')\Big|_{k'=0} \quad O_\alpha(0) = \frac{\partial}{\partial k'^{\alpha}} O(q,k,P_h,k')\Big|_{k'=0} \quad \mathcal{P}_{SS_z\alpha}(0) = \frac{\partial}{\partial k'^{\alpha}} \mathcal{P}_{SS_z}(P_h,k')\Big|_{k'=0}$$

Asymmetry Calculations

Final expression of the diff. cross section

$$\frac{d\sigma}{dy\,dx_B\,dz\,d^2\mathbf{P}_{hT}} = d\sigma^U(\phi_h) + d\sigma^T(\phi_h),$$

$$d\sigma^U(\phi_h) = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int k_\perp dk_\perp \{(A_0 + A_1 cos(\phi_h) + A_2 cos(2\phi_h))f_1^g(x,\mathbf{k}_\perp^2)\}$$

$$d\sigma^T(\phi_h) = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int dk_\perp \frac{k_\perp^3}{M_p^2}\{(B_0 + B_1 cos(\phi_h) + B_2 cos(2\phi_h))h_1^{\perp g}(x,\mathbf{k}_\perp^2)\}$$

We are interested in small-x region, we neglected higher terms in x_B.

- We consider the region : $P_{h\perp} < M$, M is mass of J/ψ
- We expanded in $P_{h\perp}/M$ and keep the terms up to $\mathcal{O}(P_{h\perp}^2/M^2)$

$$\langle cos(2\phi_h)\rangle = \frac{\int d\phi_h cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma}$$

- $\cos(2\phi_h)$ asymmetry as function of $P_{h\perp}, y, x_B, and\ z$

$$\langle cos(2\phi_h)\rangle \propto \frac{\int k_\perp dk_\perp \left(A_2\ f_1^g(x,\mathbf{k}_\perp^2) + \frac{k_\perp^2}{M_p^2}\ B_2\ h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right)}{\int k_\perp dk_\perp \left(A_0\ f_1^g(x,\mathbf{k}_\perp^2) + \frac{k_\perp^2}{M_p^2}\ B_0\ h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right)}$$

Kinematical regions for asymmetry

We use a framework based on generalized parton model with the inclusion of intrinsic transverse momentum.

- We consider the region : $P_{h\perp} < M$. This upper limit on $P_{h\perp}$, reduces the fragmentation contributions from heavy quark.

- We impose cutoff on z : $0.1 < z < 0.9$

- Cutoff $z < 0.9$ made outgoing gluon hard and no contributions from virtual diagrams.
- Cutoff $0.1 < z$ to eliminate fragmentation of hard gluon to J/ψ.

Parameterization of TMDs

- Gaussian parametrization

$$f_1^g(x,\mathbf{k}_\perp^2) = f_1^g(x,\mu)\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$h_1^{\perp g}(x,\mathbf{k}_\perp^2) = \frac{M_p^2 f_1^g(x,\mu)}{\pi\langle k_\perp^2\rangle^2}\frac{2(1-r)}{r}e^{1-\frac{k_\perp^2}{r\langle k_\perp^2\rangle}}$$

$r\ (0<r<1)$ *and* $\langle k_\perp^2\rangle$ *are parameters.*

Satisfy the positivity bound but do not saturate it.

$$\frac{\mathbf{k}_\perp^2}{2M_p^2}\left|h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right| \le f_1^g(x,\mathbf{k}_\perp^2)$$

- In small-x, region the WW gluon distributions are calculated in McLerran-Venugopalan (MV) model.

$$f_1^g(x,\mathbf{k}_\perp^2) = \frac{S_\perp C_F}{\alpha_s\pi^3}\int d\rho\frac{J_0(k_\perp\rho)}{\rho}\left(1-\exp^{\frac{-\rho^2}{4}Q_{sg}^2(\rho)}\right)$$

$$h_1^{\perp g}(x,\mathbf{k}_\perp^2) = \frac{2S_\perp C_F}{\alpha_s\pi^3}\frac{M_P^2}{k_\perp^2}\int d\rho\frac{J_2(k_\perp\rho)}{\rho\log\left(\frac{1}{\rho^2\lambda_{QCD}^2}\right)}\left(1-\exp^{\frac{-\rho^2}{4}Q_{sg}^2(\rho)}\right)$$

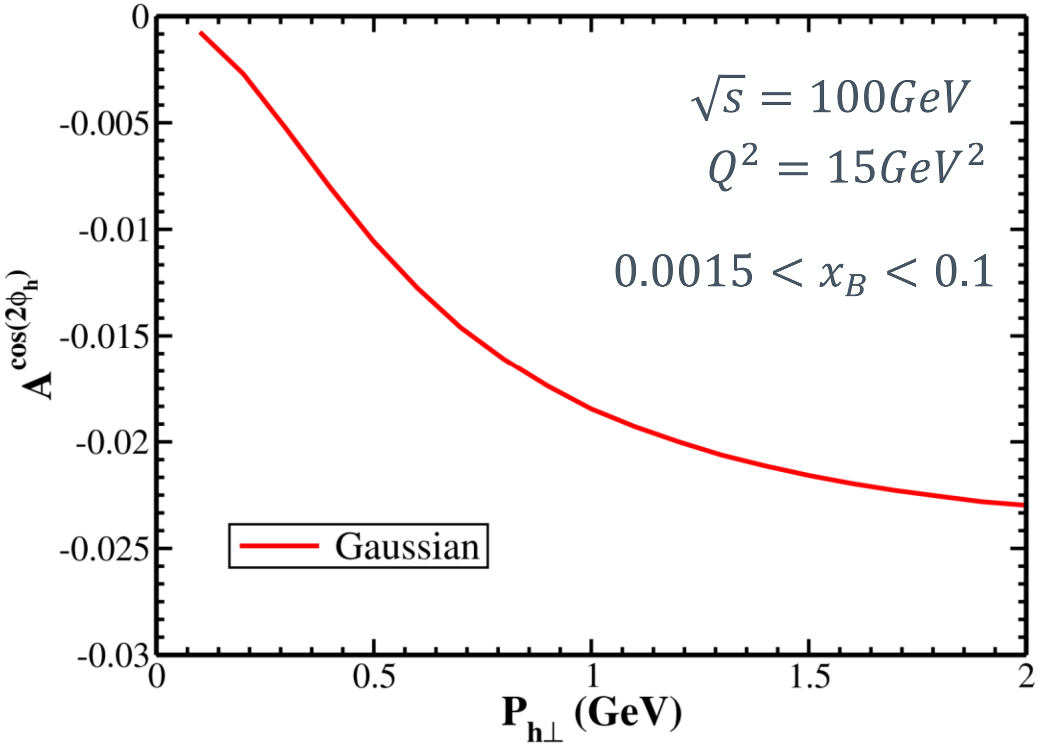
$S_\perp$ is the transverse size of the proton

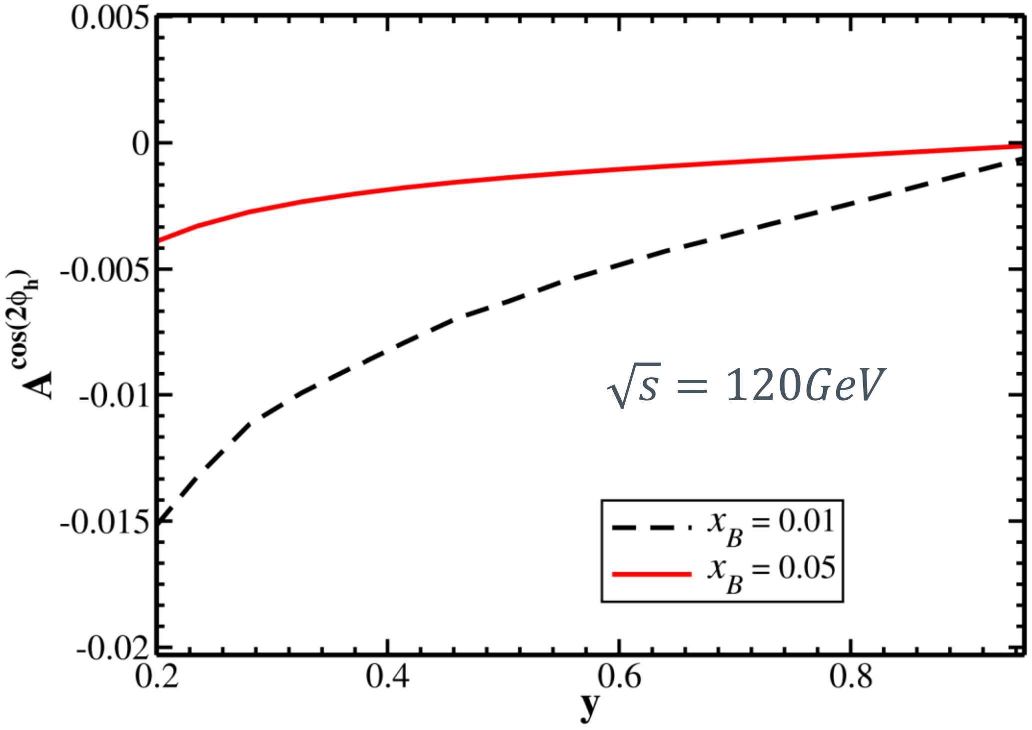
McLerran and Venugopalan, PRD (1994)

Q_{sg} Is the saturation scale

$$Q_{sg}^2(\rho) = Q_{sg0}^2\ln\left(1/\rho^2\lambda^2+\epsilon\right) \qquad Q_{sg0}^2 = (N_c/C_F)Q_{s0}^2$$

Numerical Estimates for EIC





Asymmetry plotted using Gaussian parameterizaion of TMDs

We have used MSTW2008 pdfs, TMD evolution not used.

The European Physical Journal C 63, 189 (2009)

y and Q^2 are related through the relation $Q^2 = yx_Bs$

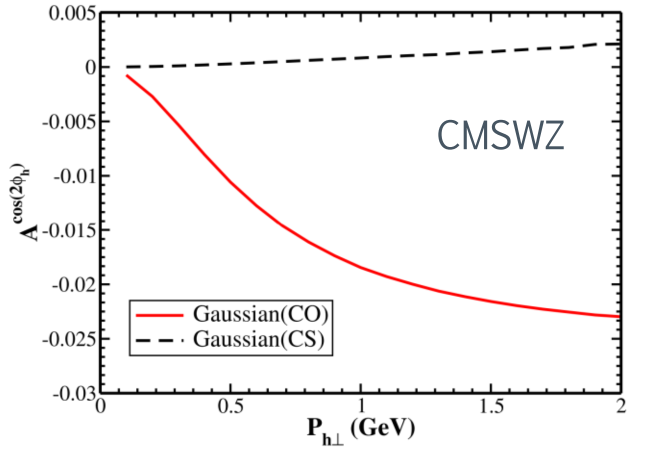
We used CMSWZ set of LDMEs : CMSWZ: Phys Rev letters 108, 242004(2012)

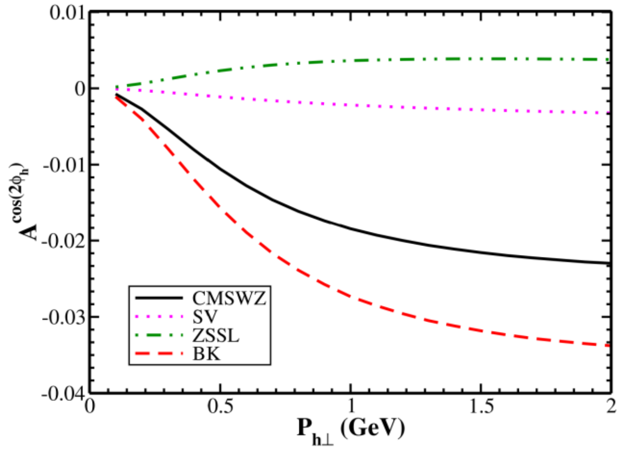
Integration ranges

$0.1 < z < 0.9$

$0 < P_{h\perp} < 2\ GeV$

Numerical Estimates for EIC



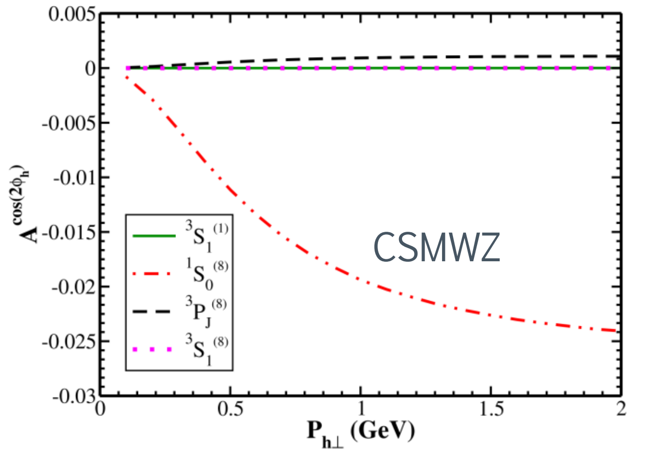


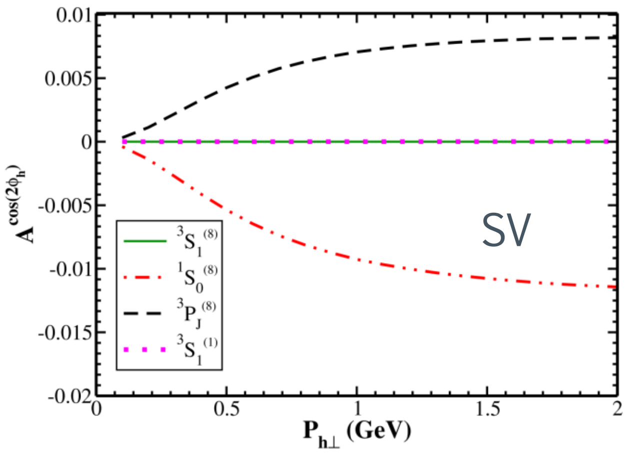
CMSWZ: Phys Rev letters 108, 242004(2012)

BK: Phys Rev D 84, 051501 (2011)

SV: Phys Rev C 87, 044905 (2013)

ZSSL: Phys Rev letters 114, 092006 (2015)

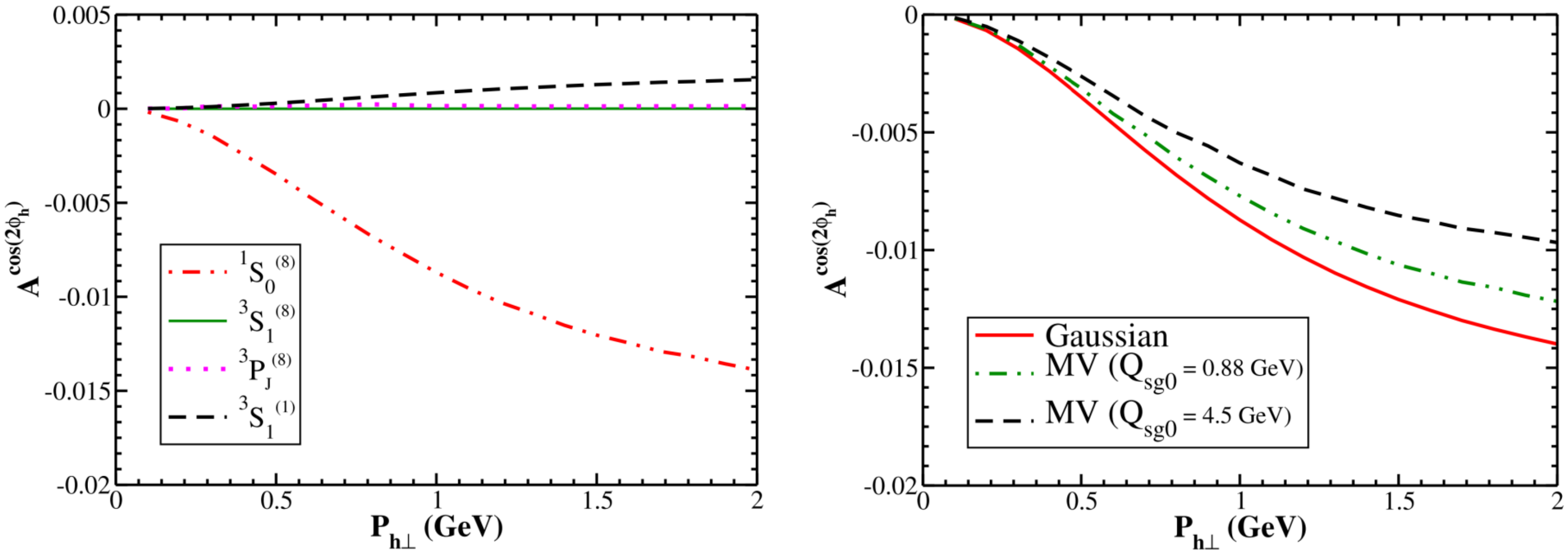




At EIC	Integration ranges
$\sqrt{s} = 100GeV$	$0.1 < z < 0.9$
$Q^2 = 15GeV^2$	$0.0015 < x_B < 0.1$

- CO states giving the significant contributions to the asymmetry as compared with asymmetry in CS model.
- Both the magnitude and sign of asymmetry depends on the choice of LDMEs sets.
- For CMSWZ, there is a dominance contribution of one state, $^1S_0^{(8)}$. Whereas for SV, we have major contribution from two states $^1S_0^{(8)}$ and $^3P_J^{(8)}$.

Numerical Estimates for EIC



$\sqrt{s} = 150GeV$

$x = 0.01$

$z = 0.7$

Integration ranges

$0.2 < y < 0.9$

$0.005 < x_B < 0.009$

The asymmetry in MV model depends on the saturation scale Q_{sg}.

We used CSMWZ set of LDMEs for the above plots.

Gaussian parameterization of the TMDs gives larger $\cos(2\phi_h)$ asymmetry than the asymmetry in MV model.

Conclusion

- Presented a calculation of $\cos(2\phi_h)$ asymmetry in the electroproduction of J/ψ at the EIC
- In the kinematical region $z < 1$ and small-x, where contribution from the process $\gamma^* + g \to c + \bar{c} + \mathrm{g}$ dominates
- We calculated the asymmetry in NRQCD based CO model.
- The asymmetry depends on the parameterization of the gluon TMDs used. We used both the Gaussian as well as the MV model for the parameterization. The magnitude of the asymmetry was found to be larger for Gaussian parameterization.
- The asymmetry depends on the LDMEs used, in particular, contributions from individual states were found to depend substantially on the set of LDMEs used.
- We obtain a small but sizable $\cos(2\phi_h)$ asymmetry. It could be a useful channel to probe the ratio of linearly polarized gluon TMD to unpolarized gluon TMD at EIC.

Thanks for your attention

Backup…

In the CS model, as J/ψ is $^3S_1$ state, we calculate contribution only from $^3S_1^{(1)}$.

$$\mathcal{M}[^3S_1^{(1)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right]$$

$$\begin{aligned}\sum_{m=1}^{3} O_m(0) =& g_s^2(ee_c)\varepsilon^{\rho*}_{\lambda_g}(p_g)\left[\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\mu(-\not{P}_h - 2\not{p}_g + M)\gamma_\rho}{(\hat{s}-M^2)(\hat{u}-M^2+q^2)}\right.\\ &+\frac{\gamma_\rho(\not{P}_h + 2\not{p}_g + M)\gamma_\nu(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{s}-M^2)(\hat{t}-M^2)}\\ &\left.+\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\rho(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{t}-M^2)(\hat{u}-M^2+q^2)}\right].\end{aligned}$$

CO contributions coming from: $^3S_1^{(8)}$, $^1S_0^{(8)}$, $^3P^{(8)}_{J(=0,1,2)}$

$^3S_1^{(8)}$ Amplitude

$$\mathcal{M}[^3S_1^{(8)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\sqrt{2}}{2} d_{abc} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right]$$

$^1S_0^{(8)}$ Amplitude

$$\mathcal{M}[^1S_0^{(8)}](P_h,k) = \frac{R_0(0)}{4\sqrt{\pi M}} \frac{\sqrt{2}}{2} i f_{abc} \mathrm{Tr}\left[(O_1(0) - O_2(0) - O_3(0) + 2O_4(0))(-\not{P}_h + M)\gamma^5\right]$$

$^3P^{(8)}_{J(=0,1,2)}$ Amplitude

$$\begin{aligned}\mathcal{M}[^3P_J^{(8)}](P_h,k) =& \frac{\sqrt{2}}{2} f_{abc} \sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(P_h)\langle 1L_z; 1S_z | JJ_z\rangle\\ &\mathrm{Tr}\Big[(O_{1\alpha}(0) - O_{2\alpha}(0) - O_{3\alpha}(0) + 2O_{4\alpha}(0))\mathcal{P}_{SS_z}(0)\\ &+ (O_1(0) - O_2(0) - O_3(0) + 2O_4(0))\mathcal{P}_{SS_z\alpha}(0)\Big].\end{aligned}$$

$$O_4 = 2g_s^2(ee_c)\varepsilon^{\rho*}_{\lambda_g}(p_g)\gamma_\nu \frac{\not{P}_h + 2\not{k}' - 2\not{q} + M}{(P_h + 2k' - 2q)^2 - M^2}\gamma_\sigma \frac{1}{(k-p_g)^2} \mathcal{T}_{\mu\rho\sigma}(k,p_g)$$

Three gluon vertex