Semi-Exclusive Double Drell-Yan factorization and GTMDs

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Jefferson Lab

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STRONG 2020

Outline

- Goal of the study
- SCET Intro
- Factorization Theorem
  - SCET Current operator
  - GTMDs, Soft Factor
- Remove of overlapping and rapidity divergences
- Conclusions

Motivation

- Understanding multi dimensional inner structure of strongly interacting systems
- GTMDs absorb both GPDs and TMDs
- *How to obtain GTMDs in a cross-section using **SCET**?*

$$\pi(p_b) + N(p_a, \lambda_a) \to \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

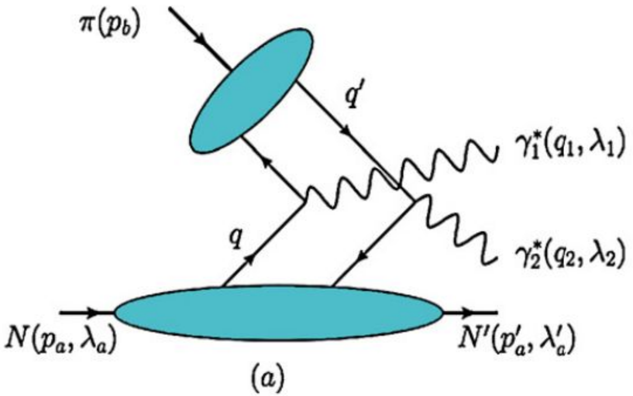


Figure 1: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400

Soft Collinear Effective Theory

- Light-cone coordinates:

$$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu \quad \text{with} \quad n_\mu = (1,0,0,1) \quad \text{and} \quad \bar{n}_\mu = (1,0,0,-1)\,.$$

- Dominant contributions from particles with collineal, anticollineal and soft momentum

$$\mathcal{L}(\phi) = \underbrace{\mathcal{L}(\phi_c)}_{\equiv\mathcal{L}_c} + \underbrace{\mathcal{L}(\phi_{\bar{c}})}_{\equiv\mathcal{L}_{\bar{c}}} + \underbrace{\mathcal{L}(\phi_s)}_{\equiv\mathcal{L}_s} + \mathcal{L}_{c+s}(\phi_c, \phi_{\bar{c}}, \phi_s)$$

- *Valid at small* $q_T << Q$

- Each of the dominant regions have a dedicated field

Soft Collinear Effective Theory

- Each field scales differently: **decoupling** of SCET modes

Hard	$q^\mu \sim Q(1,1,\lambda)$
Collinear	$k^\mu \sim Q(\lambda^2,1,\lambda)$
Anti-collinear	$k^\mu \sim Q(1,\lambda^2,\lambda)$
Soft	$k^\mu \sim Q(\lambda,\lambda,\lambda)$

$$\lambda = \frac{q_T}{p_a^+}$$

- After matching: only effective operator(s) joins the fields:

$$J_{\mu QCD} = \sum_{q_1} e_{q_1} \bar{\psi}\gamma^\mu \psi \longrightarrow J^\mu_{SCET} = \sum_q e_q C(Q^2/\mu^2) \bar{\chi}^q_{\bar{n}} S^{T\dagger}_{\bar{n}} \gamma^\mu S^T_n \chi^q_n$$

- Wilson Line (gauge invariance):

$$S^T_n = T_{sn} S_n$$

$$S_n(x) = P \exp\left[ig \int_{-\infty}^0 dsn \cdot A^a_s(x+sn)t^a\right]$$

- Cross-section factorization

SCET factorization

- Semi-Exclusive Double Drell-Yan cross-section:

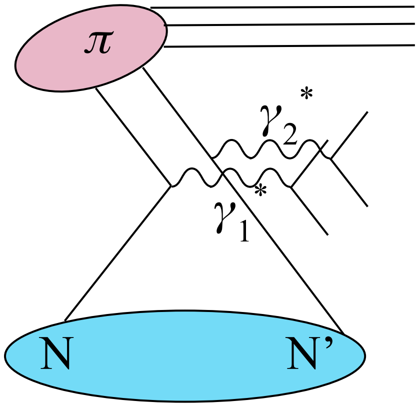
$$\frac{d\sigma}{d^4q_1 d^4q_2} \propto \sum_X \int dz_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \langle \Pi N | \overline{T}\{J^{\dagger\alpha}(z_1) J^{\dagger\beta}(z_2)\} | X N' \rangle \times \langle X N' | T\{J^{\mu}(z_3) J^{\nu}(0)\} | \Pi N \rangle$$

- Matching to SCET current

- Particles assigned to sectors

$$N \to n, \Pi \to \bar{n}, X \to s, \bar{n}$$

$$\sum_X |X\rangle\langle X| = \sum_{X_{\bar{n}}} |X_{\bar{n}}\rangle\langle X_{\bar{n}}| \times \sum_{X_s} |X_s\rangle\langle X_s| = 1$$



- Decoupling of SCET sectors after fierzing:

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \int d^4 z_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \sum_{\Gamma} \sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \Phi_{DDY}(z) f_{pion}(z) f_{NN'}(0, z_3) f_{N'N}(z_1, z_2)$$

SCET factorization: GTMDs

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y,\vec{b}_\perp\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

where:

$$\tilde{f}(x;\vec{b}_\perp) = \int d^2\vec{k}_\perp e^{i\vec{k}_\perp\cdot\vec{b}_\perp} f(x;\vec{k}_\perp)$$

- Two pure GTMDs evaluated in different positions (unsubtracted, with rapidity divergences)

One-loop calculation in Echevarría Et. al. Physics Letters B, 759, 336-341

$$f_{pp',\lambda\lambda'}^{[\Gamma],0}(P,\Delta,x,\vec{k}_\perp) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i(z^- k^+/2 - \vec{z}\cdot\vec{k}_\perp)} \langle p',\lambda'|\bar{q}W_n(-z/2)\Gamma W_n^\dagger q(z/2)|p,\lambda\rangle|_{z^+=0}$$

$P = (p + p')/2$ $\quad \Delta = p - p'$ $\quad k^+ = xp^+$

SCET factorization: Soft factor

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y,\vec{b}_\perp\}) \tilde{f}_{NN'}\tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

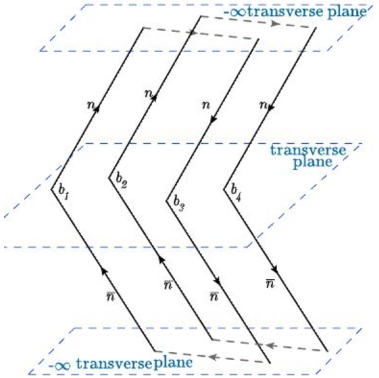


Figure 2: Double Wilson Loop Soft Factor.
Vladimirov, A. (2018) Journal of High Energy Physics, 2018(4), 1-46

- Soft factor depends on b_T:

$$\tilde{\Phi}_{DDY}(\vec{b}_{1\perp},\vec{b}_{2\perp},\vec{b}_{3\perp}) = \left\langle 0|S_n^{T\dagger}(\vec{b}_{1\perp})S_{\bar{n}}^{T}(\vec{b}_{1\perp})S_{\bar{n}}^{T\dagger}(\vec{b}_{2\perp})S_n^{T}(\vec{b}_{2\perp})S_{\bar{n}}^{T\dagger}(\vec{b}_{3\perp})S_n^{T}(\vec{b}_{3\perp})S_n^{T\dagger}(0)S_{\bar{n}}^{T}(0)|0\right\rangle$$

Overlapping regions and divergences

$$d\sigma^{[1,11,1]} \propto f^{[1]}_{pion}(\{y, \vec{b}_\perp\})\Phi_{11}(\{b\}) f^{[1]}_{NN'} f^{[1]}_{N'N}(\{x, \vec{b}_\perp\})$$

- Rapidity divergences
- When doing loop calculations: remove overlapping between regions
- Subtracted cross-section (without rapidity divergences):

$$d\sigma^{[1,11,1]}_{DDY\ sub}(\{b\}) \propto H(Q^2/\mu^2)\frac{\hat{f}^{[1]}_{pion}}{\Phi_{11}}\Phi_{11}(\{b\})\frac{\hat{f}^{[1]}_{NN'}\hat{f}^{[1]}_{N'N}(\{x, \vec{b}_\perp\})}{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})\Phi_{DY}(0, \vec{b}_{3\perp})} =$$

$$= H(Q^2/\mu^2)\frac{\hat{f}^{[1]}_{pion}}{\sqrt{\Phi_{11}}}\frac{\sqrt{\Phi_{11}}}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})}\sqrt{\Phi_{DY}(0, \vec{b}_{3\perp})}}\frac{\hat{f}^{[1]}_{NN'}\hat{f}^{[1]}_{N'N}(\{x, \vec{b}_\perp\})}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})}\sqrt{\Phi_{DY}(0, \vec{b}_{3\perp})}} =$$

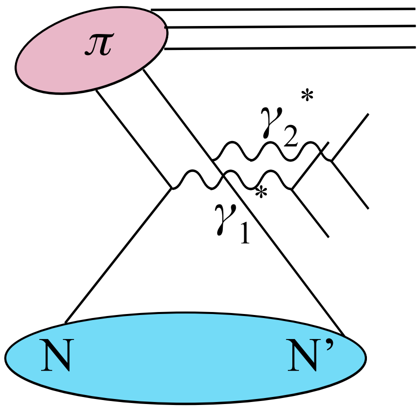
$$= H(Q^2/\mu^2) \times 2TMD \times \Phi_{New} \times GTMD * GTMD'$$

- New ratio of soft factors term

Conclusions

- First factorization of Semi-Exclusive DDY cross-section into functions with different scales
- Semi-Exclusive DDY factorization gives access to GTMDs (and 2TMD)
- New ratio of soft factors not present before: important for pheno!
- Future work:
  - More complicated structures (color, polarizations)
  - Evolution: *Resummation via RG equations*

THANK YOU FOR YOUR ATTENTION!



Extra slides

SCET factorization

$$\frac{d\sigma}{d^4q_1d^4q_2} = \int d^4z_{1,2,3}e^{-iq_1z_1-iq_2z_2+iq_1z_3}\sum_{\Gamma}\sum_{\Gamma'}C_\Gamma C_{\Gamma'}H(Q^2/\mu^2)\Phi_{DDY}(z)f_{pion}(z)f_{NN'}(0,z_3)f_{N'N}(z_1,z_2)$$

- Multipole expansion + FTs properties lead to:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'}C_\Gamma C_{\Gamma'}H(Q^2/\mu^2)\int\frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6}e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}}\tilde{\Phi}_{DDY}(\{\vec{b}_\perp\})\tilde{f}_{pion}(\{y,\vec{b}_\perp\})\tilde{f}_{NN'}\tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

Factorized cross-section in impact parameter space with:

$$\tilde{f}(x;\vec{b}_\perp) = \int d^2\vec{k}_\perp e^{i\vec{k}_\perp\cdot\vec{b}_\perp}f(x;\vec{k}_\perp)$$

and

Φ_{DDY} - Soft factor with 8 Wilson lines

f_{pion} - Naive Double TMD

$f_{NN'}f_{N'N}$ - Naive Two GTMDs

GTMDs

- General correlator for GTMDs:

$$W^{[\Gamma],0}_{\lambda\lambda'}(P,\Delta,x,\vec{k}_\perp) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i(z^-k^+/2-\vec{z}\cdot\vec{k}_\perp)} \underbrace{\langle p',\lambda'|\bar{q}W_n(-z/2)\Gamma W^\dagger_n q(z/2)|p,\lambda\rangle|_{z^+=0}}_{\phi^{[\Gamma],q}_{\lambda\lambda'}}$$

- Subtracted correlator: Echevarría Et. al. Physics Letters B, 759, 336-341 [1]

  Free from rapidity divergences

$$W^{[\Gamma],q}_{\lambda\lambda'} = \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{+i\left(\frac{1}{2}z^-\bar{k}^+ - \boldsymbol{z}_\perp\cdot\bar{\boldsymbol{k}}_\perp\right)} \phi^{[\Gamma],q}_{\lambda\lambda'}(0,z^-,\boldsymbol{z}_\perp)\, S^{\frac{1}{2}}(z_T)$$

- Soft factor:

$$S(z_T) = \frac{\mathrm{Tr}_c}{N_c}\langle 0|\mathcal{S}^\dagger_n\left(-\frac{z}{2}\right)\mathcal{S}_{\bar{n}}\left(-\frac{z}{2}\right)\mathcal{S}^\dagger_{\bar{n}}\left(\frac{z}{2}\right)\mathcal{S}_n\left(\frac{z}{2}\right)|0\rangle\Big|_{z^\pm=0}$$

Exclusive DDY

- GTMDs: Exclusive Double Drell-Yan

$$\pi(p_b) + N(p_a, \lambda_a) \rightarrow \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

- Amplitude calculation at LO by Bhattacharya et.al.

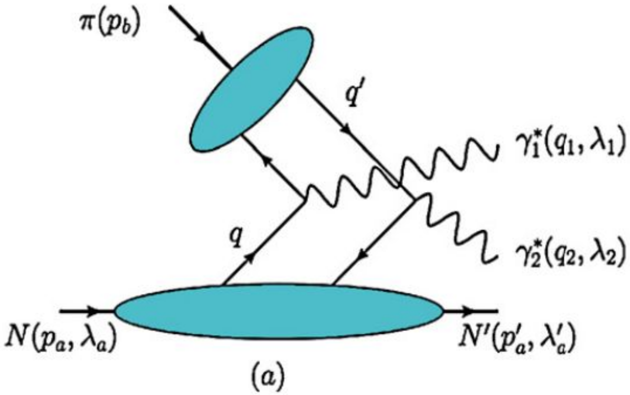


Figure 2: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400 [2]

$$d\sigma^{\lambda_1,\lambda_2}_{\lambda_a,\lambda'_a} = \frac{\pi}{2s^{3/2}} \frac{1+\xi_a}{1-\xi_a} |\mathcal{T}^{\lambda_1,\lambda_2}_{\lambda_a,\lambda'_a}|^2 \delta(p'^0_a + q_1^0 + q_2^0 - \sqrt{s}) \times \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4},$$

Exclusive DDY

- GTMDs: Exclusive Double Drell-Yan

$$\pi(p_b) + N(p_a, \lambda_a) \to \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

- Amplitude calculation at LO by Bhattacharya et.al.

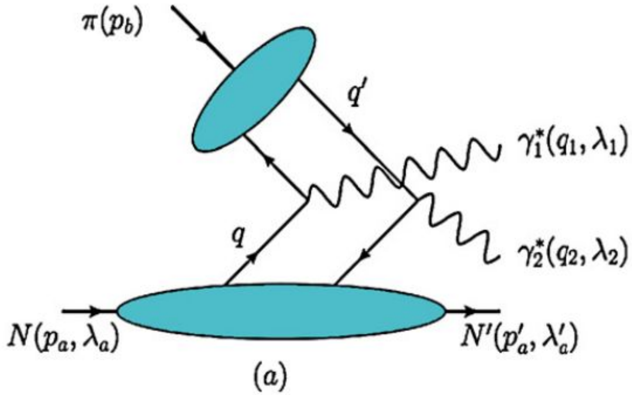


Figure 2: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400 [2]

$$\begin{aligned}\mathcal{T}^{\mu\nu}_{\lambda_a,\lambda'_a} \;=\; & i\sum_{q,q'} e_q e'_q e^2 \frac{(2\pi)^4}{N_c}\int d^2\vec{k}_{a\perp}\int d^2\vec{k}_{b\perp}\,\delta^{(2)}\!\left(\frac{\Delta\vec{q}_\perp}{2}-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right)\Phi^{q'q}_\pi(x_b,\vec{k}^2_{b\perp}) \\ & \Big[-i\varepsilon^{\mu\nu}_\perp\Big(W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp}) - W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\Big) \\ & \quad - g^{\mu\nu}_\perp\Big(W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(x_a,\vec{k}_{a\perp}) + W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda'_a}(-x_a,-\vec{k}_{a\perp})\Big)\Big],\end{aligned}$$

SCET factorization: 2TMD

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y,\vec{b}_\perp\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

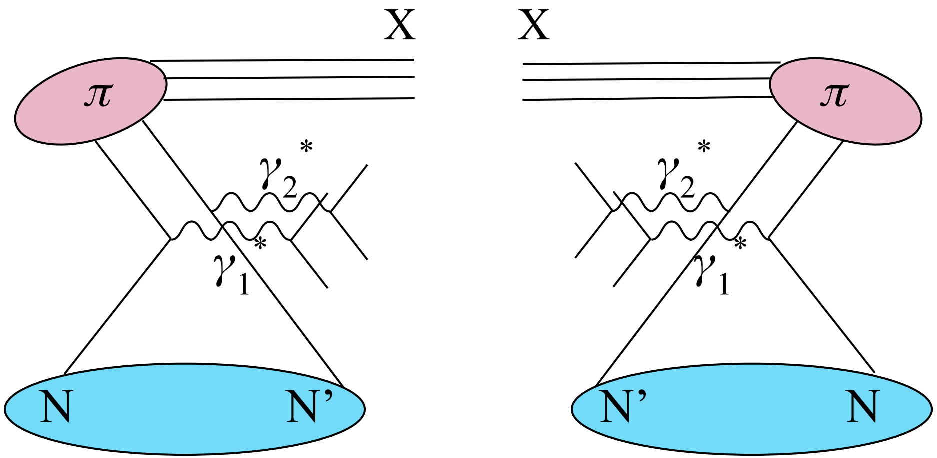
- Pure Double TMDPDF (unsubtracted, with rapidity divergences):

Buffing, M. G., Diehl, M., & Kasemets, T. (2018). Journal of High Energy Physics, 2018(1), 1-112 [4]

$$\tilde{f}_{pion}(\{y,\vec{b}_\perp\}) = \prod_j \int \frac{dr_j^+}{2\pi} e^{-ir_j^+ y_j p_b^-} \left\langle \Pi | \bar{\chi}_{\bar{n}}(r_2^+,0^-,\vec{b}_{2\perp})\Gamma^\mu \chi_{\bar{n}}(r_1^+,0^-,\vec{b}_{1\perp})\bar{\chi}_{\bar{n}}(r_3^+,0^-,\vec{b}_{3\perp})\Gamma'^\beta \chi_{\bar{n}}(0)|\Pi \right\rangle$$

where $j= 1,...,3$ and $y_3 = - y_1$

Color



- Consider color structure

$$f_{pion}^{[d_4,\ldots,d_1]} \propto \left\langle \Pi|\bar{\chi}_{\bar{n}}^{d_2}(r_2^+,0^-,\vec{b}_{2\perp})\Gamma^\mu \chi_{\bar{n}}^{d_1}(r_1^+,0^-,\vec{b}_{1\perp})\bar{\chi}_{\bar{n}}^{d_3}(r_3^+,0^-,\vec{b}_{3\perp})\Gamma'^\beta \chi_{\bar{n}}^{d_4}(0)|\Pi\right\rangle$$

$$f_{NN'}f_{N'N}^{[a_4,\ldots,a_1]} \propto \left\langle N|\bar{\chi}_n^{a_1}(0^+,r_1^-,\vec{b}_{1\perp})\Gamma^\alpha \chi_n^{a_2}(0^+,r_2^-,\vec{b}_{2\perp})|N'\right\rangle \left\langle N'|\bar{\chi}_n^{a_4}(0)\Gamma'^\nu \chi_n^{a_3}(0^+,r_3^-,\vec{b}_{3\perp})|N\right\rangle$$

$$\Phi_{DDY}^{a_1,\ldots,a_4,b_4,\ldots,b_1}(\{b\}) = \left\langle 0|S_n^{T\dagger a_1}S_{\bar{n}}^{Td_1}(\vec{b}_{1\perp})S_{\bar{n}}^{T\dagger d_2}S_n^{Ta_2}(\vec{b}_{2\perp})S_{\bar{n}}^{T\dagger d_3}S_n^{Ta_3}(\vec{b}_{3\perp})S_n^{T\dagger a_4}S_{\bar{n}}^{Td_4}(0)|0\right\rangle$$

Color

- Obtain valid color structures using projectors

- Cross-section:

$$d\sigma \propto \left(f^{[1]}_{pion}(\{y,\vec{b}_\perp\}), f^{[8]}_{pion}(\{y,\vec{b}_\perp\})\right) \times \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix} \times \begin{pmatrix} f^{[1]}_{NN'} f^{[1]}_{N'N}(\{x,\vec{b}_\perp\}) \\ 0 \end{pmatrix}$$

- Singlet term:

$$d\sigma^{[1,11,1]} \propto f^{[1]}_{pion}(\{y,\vec{b}_\perp\})\Phi_{11}(\{b\}) f^{[1]}_{NN'} f^{[1]}_{N'N}(\{x,\vec{b}_\perp\})$$

- Up until now: un-subtracted terms

Color

- Consider DPS color structure

- DPS projectors to get singlet states:

$$I_1 = \frac{\delta_{a_1 a_4}\delta_{a_2 a_3}}{N_c^2} \qquad I_8 = \frac{2t^A_{a_1 a_4} t^A_{a_2 a_3}}{N_c\sqrt{N_c^2-1}}$$

- Soft factor:

$$\Phi_{MN}(\{b\}) = I_M \times \Phi_{DDY}^{[a_1,\ldots,a,4][b_1,\ldots,b_4]}(\{b\}) \times I_N$$

with:

$$\Phi_{DDY}^{a_1,\ldots,a_4,b_4,\ldots,b_1}(\{b\}) = \left\langle 0 \middle| S_n^{T\dagger a_1} S_{\bar{n}}^{Td_1}(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger d_2} S_n^{Ta_2}(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger d_3} S_n^{Ta_3}(\vec{b}_{3\perp}) S_n^{T\dagger a_4} S_{\bar{n}}^{Td_4}(0) \middle| 0 \right\rangle$$

$$\Rightarrow \qquad \Phi_{DDY}(\{b\}) = \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix}$$