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University of Pavia and INFN

GLOBAL FITS IN TMD FRAMEWORK

Results and Perspectives

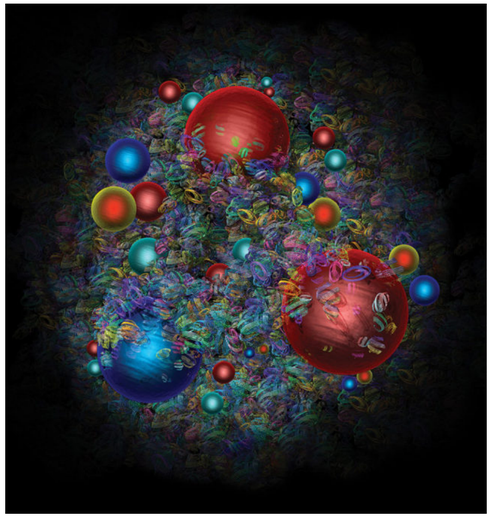
Quantum CromoDynamics (QCD)

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a,\mu\nu} F_{a,\mu\nu} + \overline{\psi}(i\not{D} - m)\psi$$

quark field

$$D_\mu = \partial_\mu - ig T^a A^a_\mu$$

gluon field

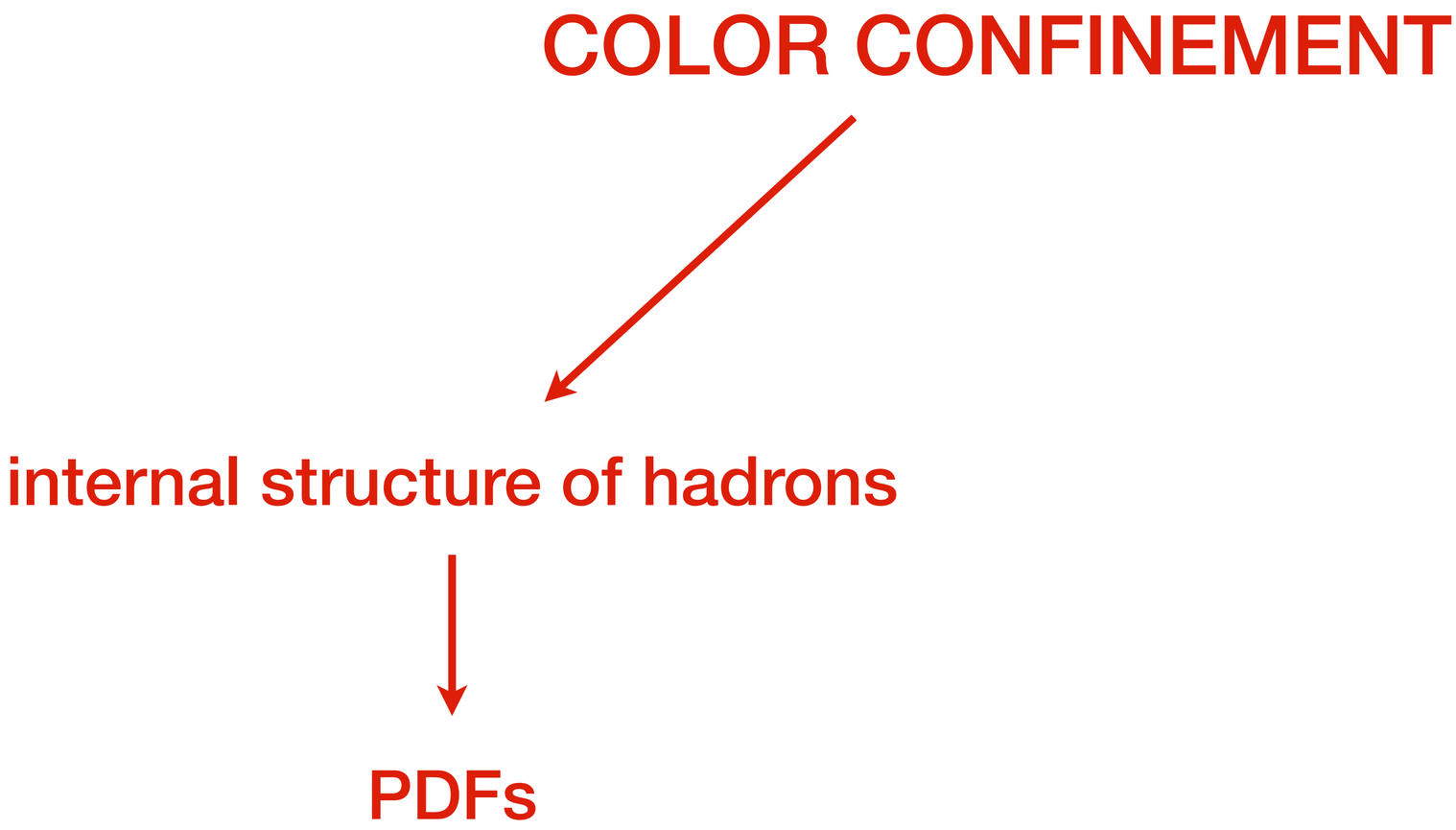


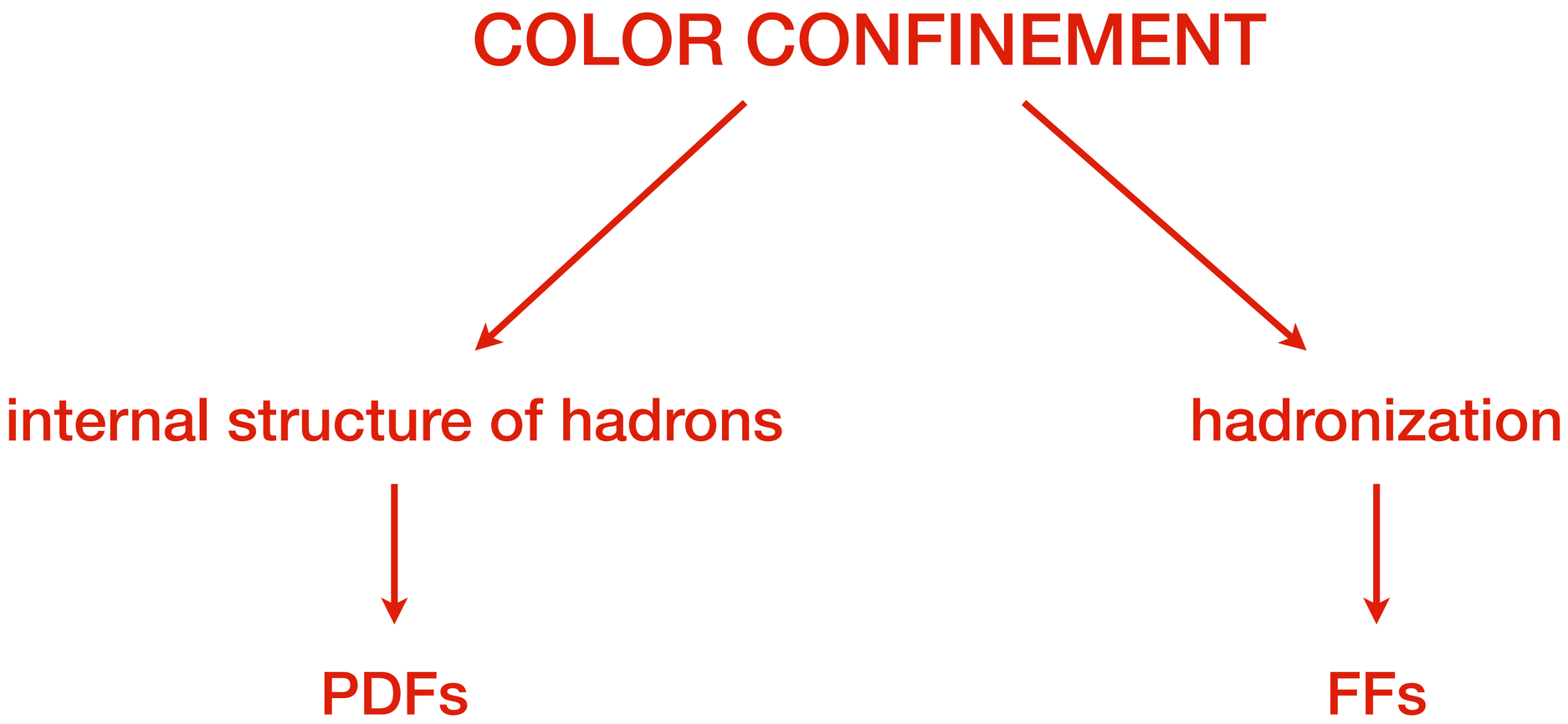
But there are some features that cannot be proved from first principles!

→ color confinement

→ chiral SSB

COLOR CONFINEMENT


COLOR CONFINEMENT
internal structure of hadrons
PDFs


COLOR CONFINEMENT
internal structure of hadrons
hadronization
PDFs
FFs

HOW TO STUDY THESE NON-PERTURBATIVE OBJECTS?

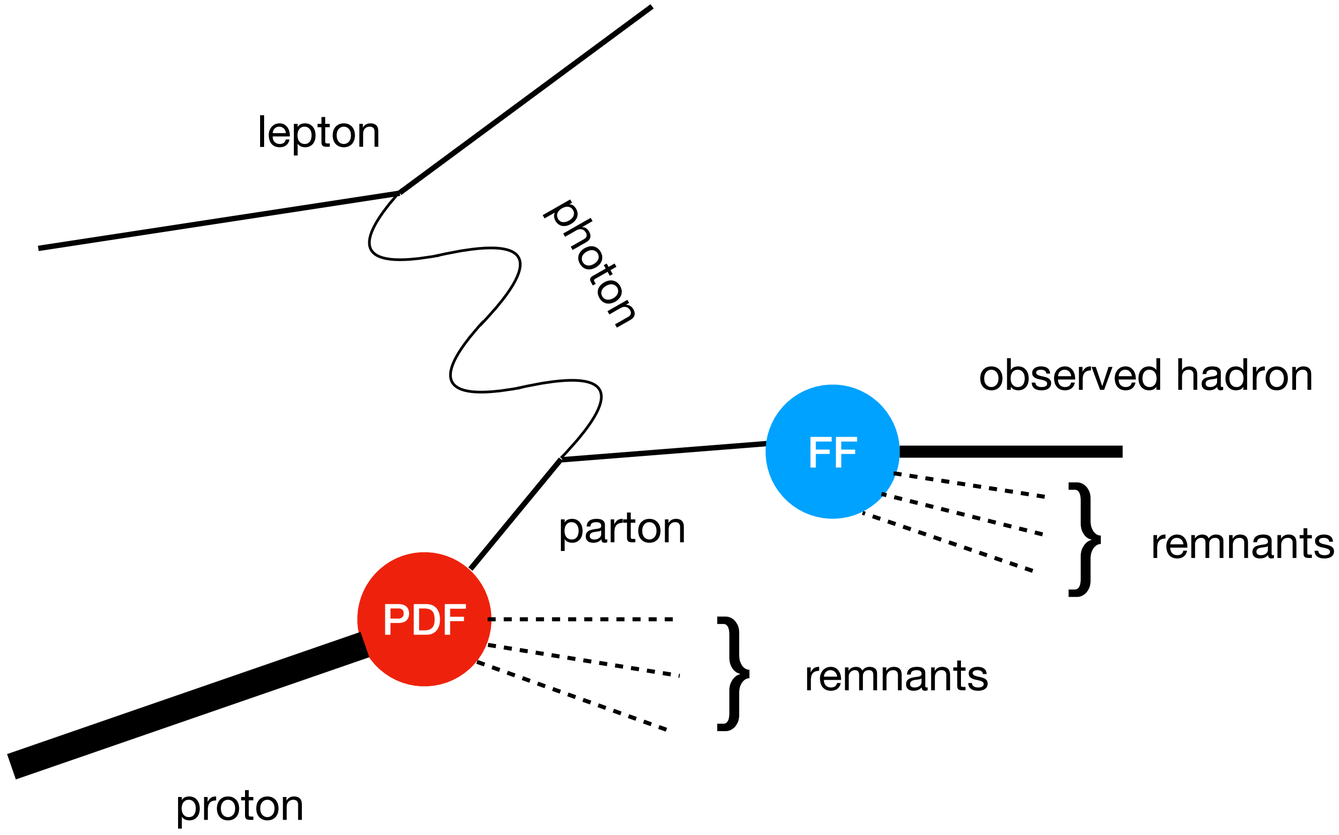
Factorization of the cross section

Example: Semi-Inclusive Deep Inelastic Scattering (SIDIS)

$$l(\ell) + N(P) \to \gamma^*(q) \to l(\ell') + h(P_h) + X$$

If $Q^2 \gg M^2$

$$d\sigma = \mathcal{H} \times PDF \otimes FF$$



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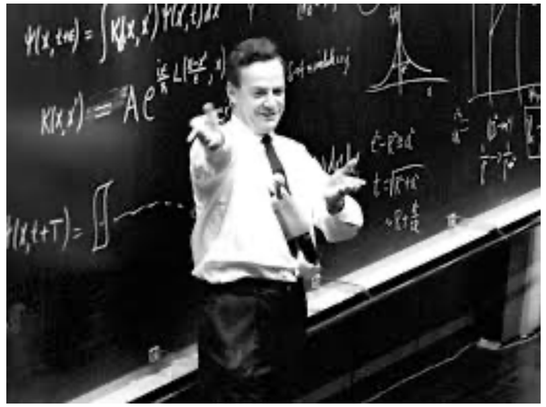
partonic collision
(perturbative)



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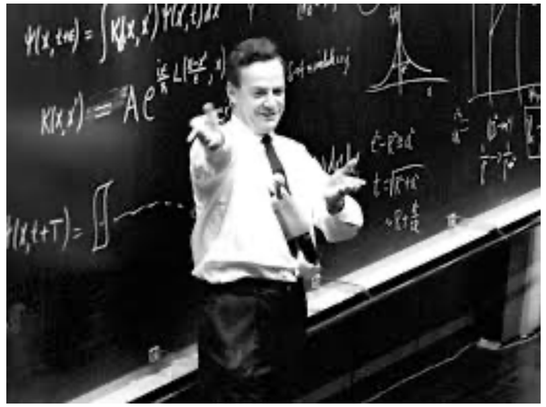
partonic collision
(perturbative)

hadron structure
(non-perturbative)

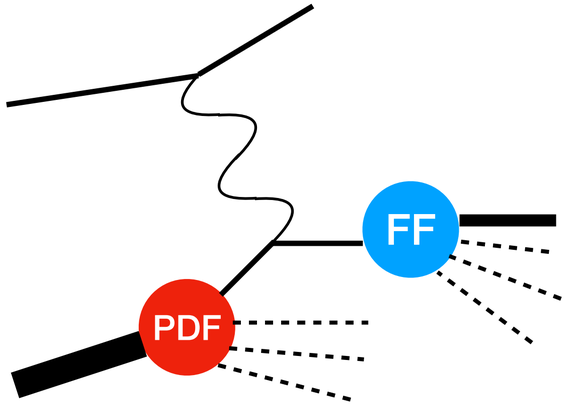
hadronization
(non-perturbative)



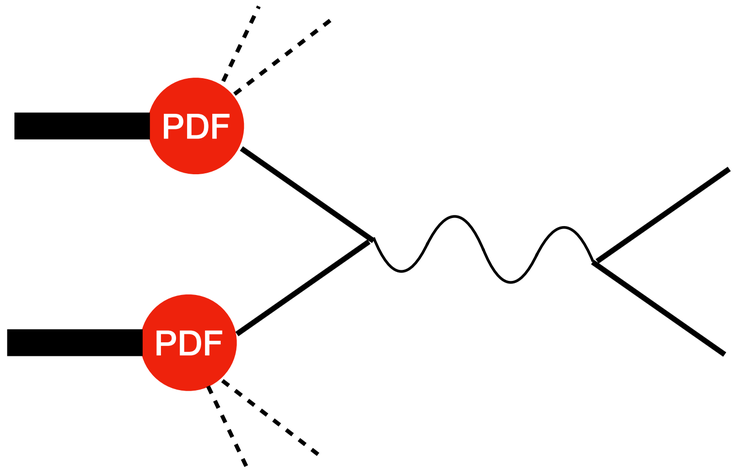
Factorization of the cross section

UNIVERSALITY:

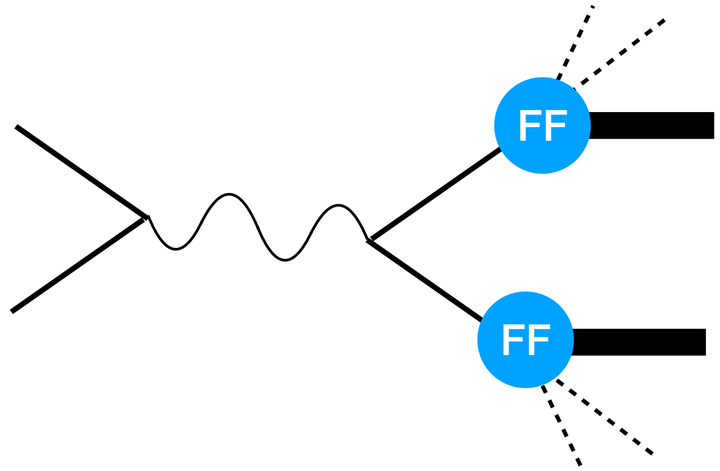
Same PDFs and FFs for a given hadron in different processes!



SIDIS



Drell-Yan

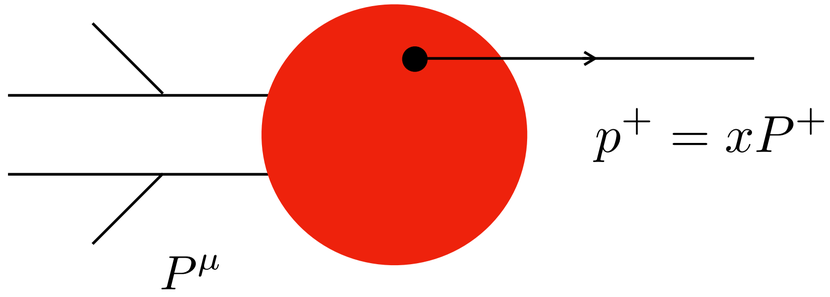


electron-positron annihilation

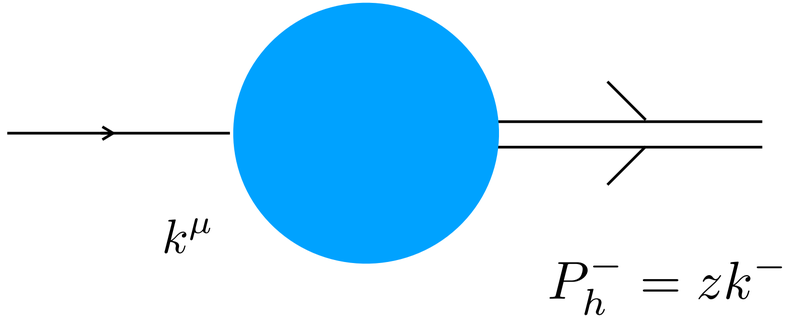
HOW TO PROVE THAT THE FORMALISM IS VALID?


GLOBAL FITS

Collinear case - factorization

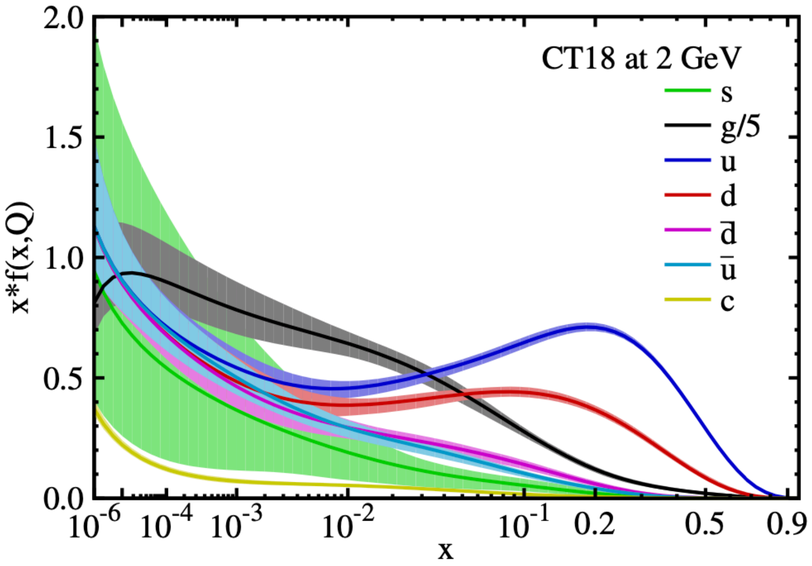


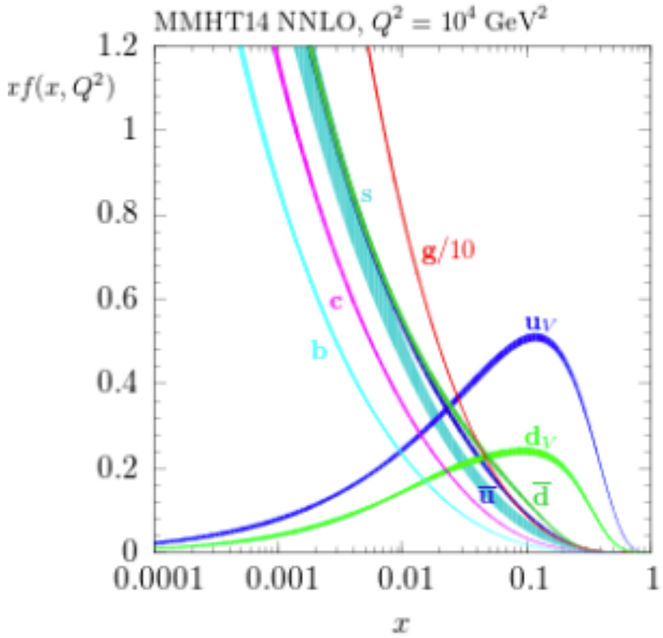
$$\Phi_{ij}(x,S) = \int \frac{d\xi^-}{2\pi} e^{ip\cdot\xi} \langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle_{\xi^+=\xi_T=0}$$

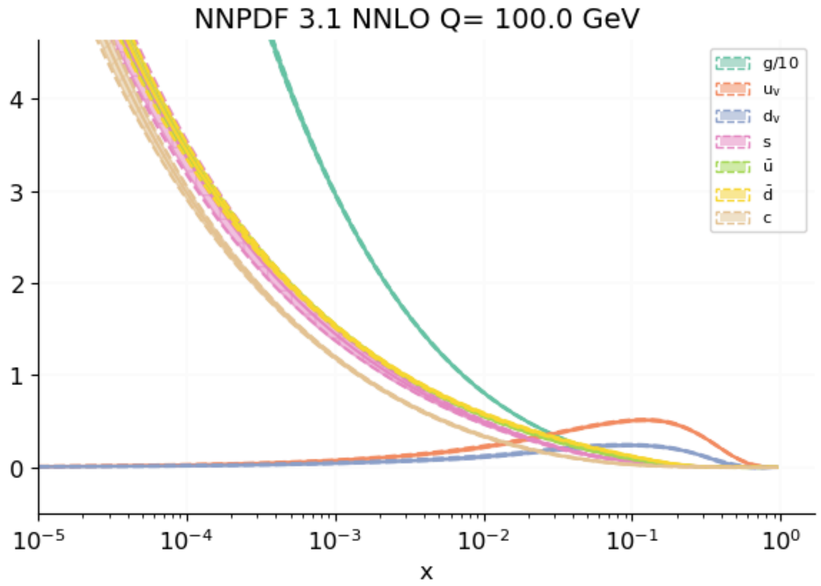


$$\Delta_{ij}(z,S) = \frac{1}{2z}\sum_X \int \frac{d\xi^+}{2\pi} e^{ip\cdot\xi} \langle 0|\bar{\psi}_i(\xi)|h,X\rangle\langle h,X|\psi_j(0)|0\rangle_{\xi^-=\xi_T=0}$$

Collinear case - Global Fits of PDFs





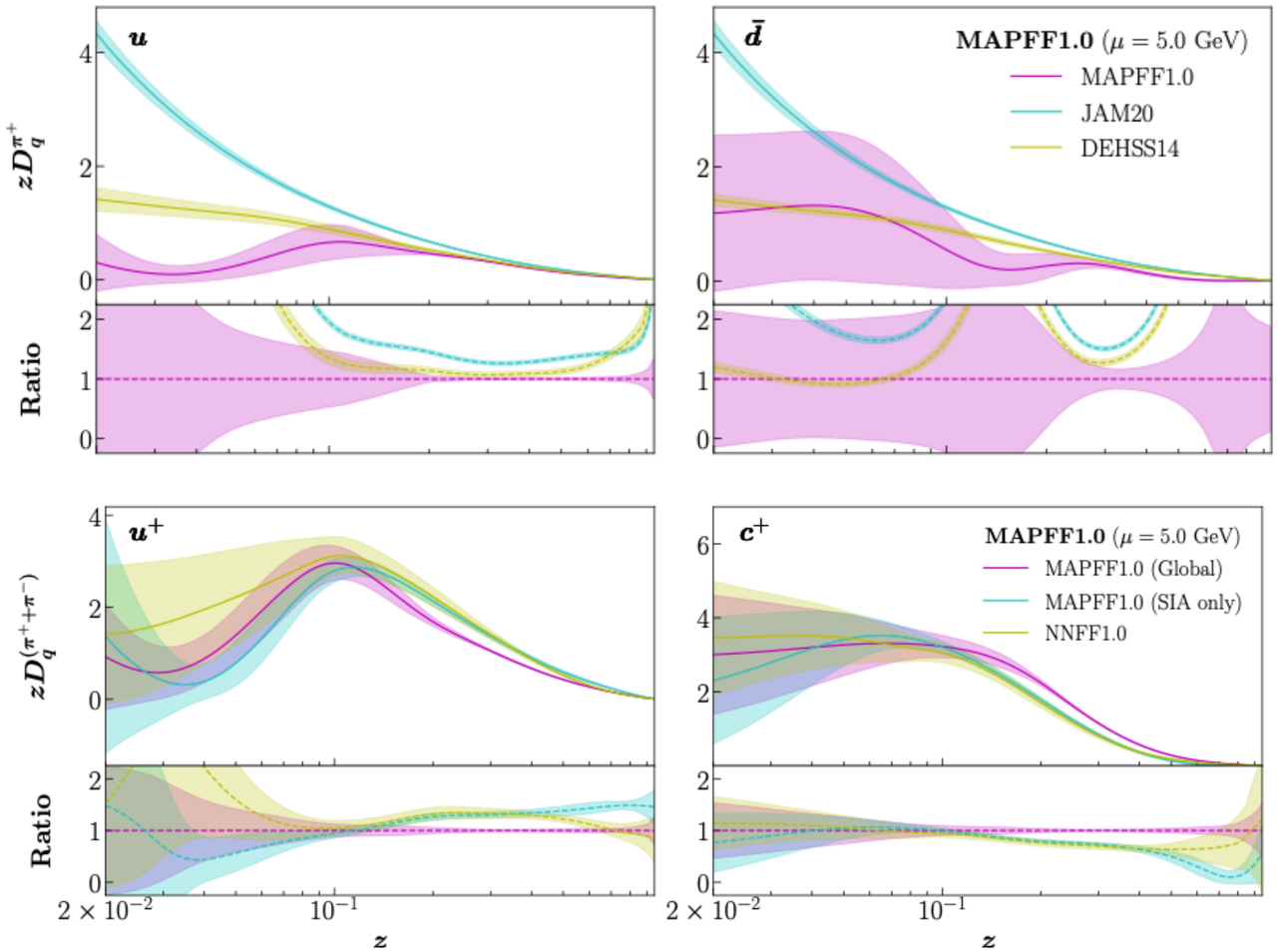


Quite a lot determination available
See e.g. the LHAPDF library
https://lhapdf.hepforge.org

Collinear case - Global Fits of FFs

Extractions:

- MAP Collaboration
- JAM Collaboration
- DEHSS
- Neural Network FF Collaboration



R. A. Khalek, V. Bertone and E. R. Nocera, [arXiv:2105.08725 [hep-ph]]

Collinear case - What does not work

Evidences:

- Single Spin Asymmetries (SSA)

  J. Adams et al., P.R.L. 92 (2004) 171801, . . .

- Violation of the Lam-Tung rule

  J. S. Conway et al. P.R. D39 (1989) 92-122, . . .

- Results of the EMC, SMC, …, experiments

  J. Ashman et al., P.L. B206 (1988) 364, . . .

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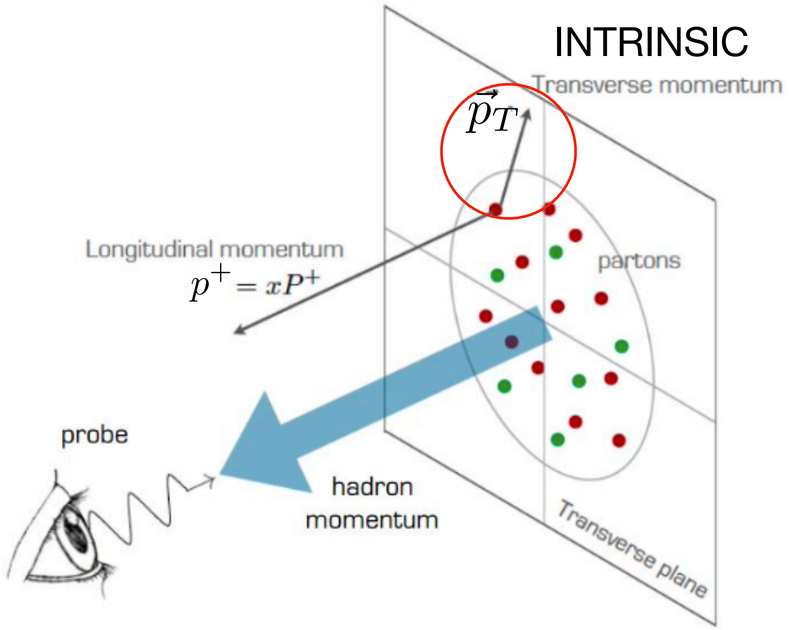
TMD case - factorization

Transverse Momentum Dependent parton densities

$$\Phi_{ij}(x, \boldsymbol{p}_T, S) = \int \frac{d\xi^- d^2\xi_T}{2\pi} e^{ip\cdot\xi} \langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle_{\xi^+=0}$$

$$\Downarrow$$

$$F_a(x, p_T^2, \mu, \zeta)$$



$$\Delta_{ij}(z, \boldsymbol{k}_T, S) = \sum_X \int \frac{d\xi^+ d^2\xi_T}{2\pi} e^{ip\cdot\xi} \langle 0|\bar{\psi}_i(\xi)|X P_h S_h\rangle\langle X P_h S_h|\psi_j(0)|0\rangle_{\xi^-=0}$$

$$\Downarrow$$

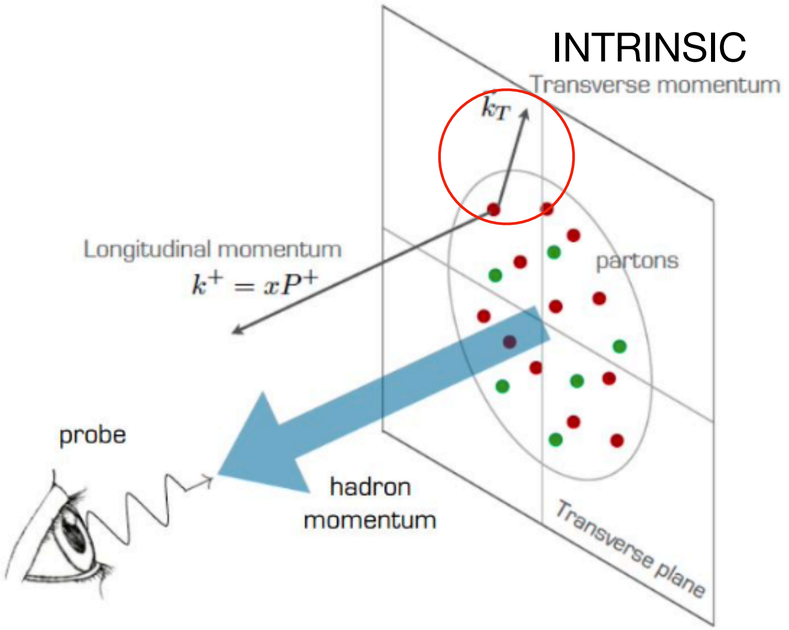
$$D^{a\to h}(z, k_T^2, \mu, \zeta)$$

HOW TO PARAMETRIZE PDFS AND FFS?

TMD case - Parametrization

Fourier Transorm in b_T space

$$\tilde{F}_a(x, b_T^2; \mu, \zeta) = \int \frac{d^2 k_T}{(2\pi)^2} e^{i b_T \cdot k_T} F_a(x, k_T^2; \mu, \zeta)$$



How to model a TMD distribution?

$$\tilde{F}_a(x, b_T^2; Q, Q^2) = [C_{a/b}(x, b_T^2, \mu_b) \otimes F_a(x; \mu_b)] e^{S_{\rm pert}(\mu_b^2, Q^2)} e^{S_{\rm NP}(b_T, Q^2; \lambda)} \tilde{F}_{a,NP}(x, b_T^2; \lambda')$$

Matching coefficient (Pertubative calc.)

Collinear PDF (previous fit)

Evolution (perturbative calc.)

Evolution (model-dependent)

TMD part of the parton density (model-dependent)

PAVIA RESULTS

PV17 global fit

Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

Datasets:		Number of data:
SIDIS	COMPASS experiment	6252
	HERMES experiment	1514
Drell-Yan	E288 experiment	168
	E605 experiment	35
Z production	CDF experiment	68
	D0 experiment	22
		8059

Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

TMD parametrization:

$$f^a_{1\mathrm{NP}}(x,\boldsymbol{k}_\perp^2) = \frac{1}{\pi}\,\frac{\left(1+\lambda \boldsymbol{k}_\perp^2\right)}{g_{1a}+\lambda\, g_{1a}^2}\; e^{-\frac{\boldsymbol{k}_\perp^2}{g_{1a}}}\,,$$

$$D^{a\to h}_{1\mathrm{NP}}(z,\boldsymbol{P}_\perp^2) = \frac{1}{\pi}\,\frac{1}{g_{3a\to h}+\left(\lambda_F/z^2\right) g_{4a\to h}^2}\left(e^{-\frac{\boldsymbol{P}_\perp^2}{g_{3a\to h}}}+\lambda_F\frac{\boldsymbol{P}_\perp^2}{z^2}\; e^{-\frac{\boldsymbol{P}_\perp^2}{g_{4a\to h}}}\right)$$

Gaussian + weighted Gaussian

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Gaussian + weighted Gaussian

Gaussian widths:

$$g_1(x) = N_1\frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$g_{3,4}(x) = N_{3,4}\frac{(z^\beta+\delta)(1-z)^\gamma}{(\hat{z}^\beta+\delta)(1-\hat{z})^\gamma}$$

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11 parameters to be fixed

- 4 -> TMD PDF
- 6 -> TMD FF
- 1 -> NP Evolution

Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

Accuracy of the simulation: Next to Leading Log (NLL)

resummation of $\log^{2n-1}\frac{Q^2}{q_T^2}$

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Accuracy of the simulation: Next to Leading Log (NLL)

resummation of $\log^{2n-1}\frac{Q^2}{q_T^2}$

Results of the fit:

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Points	190	190	189	187
χ^2/points	4.83 ± 0.42	2.47 ± 0.28	0.91 ± 0.14	0.82 ± 0.17

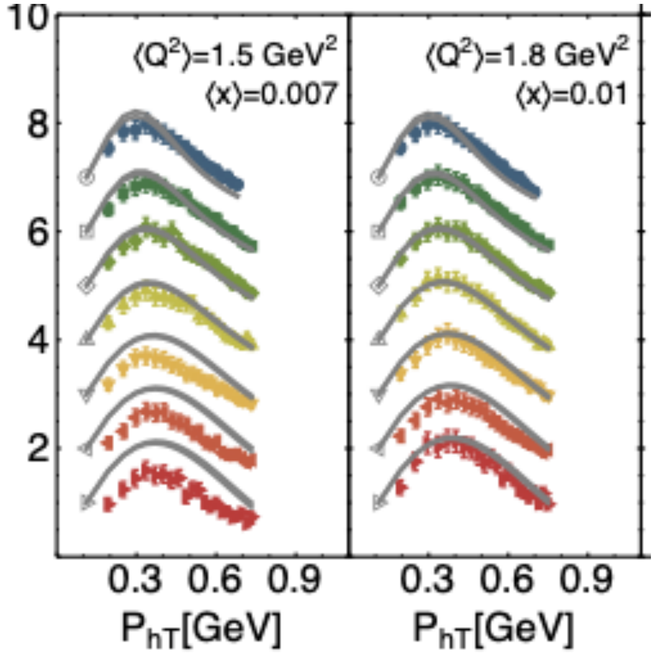
	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36 ± 0.00	1.11 ± 0.02	2.00 ± 0.02	1.73 ± 0.01

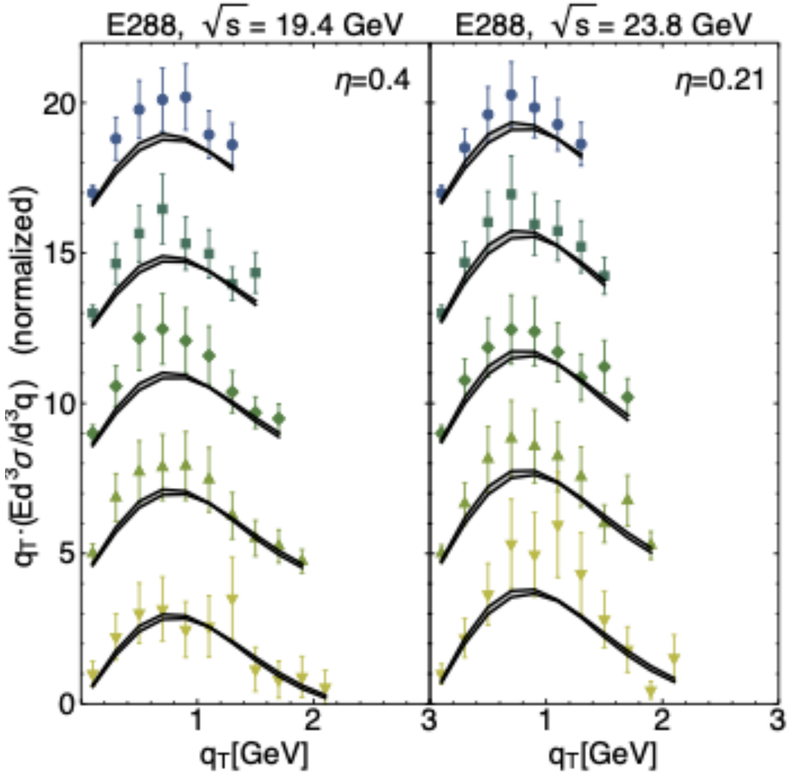
	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
χ^2/points	0.99 ± 0.09	0.84 ± 0.10	0.32 ± 0.01	1.12 ± 0.08

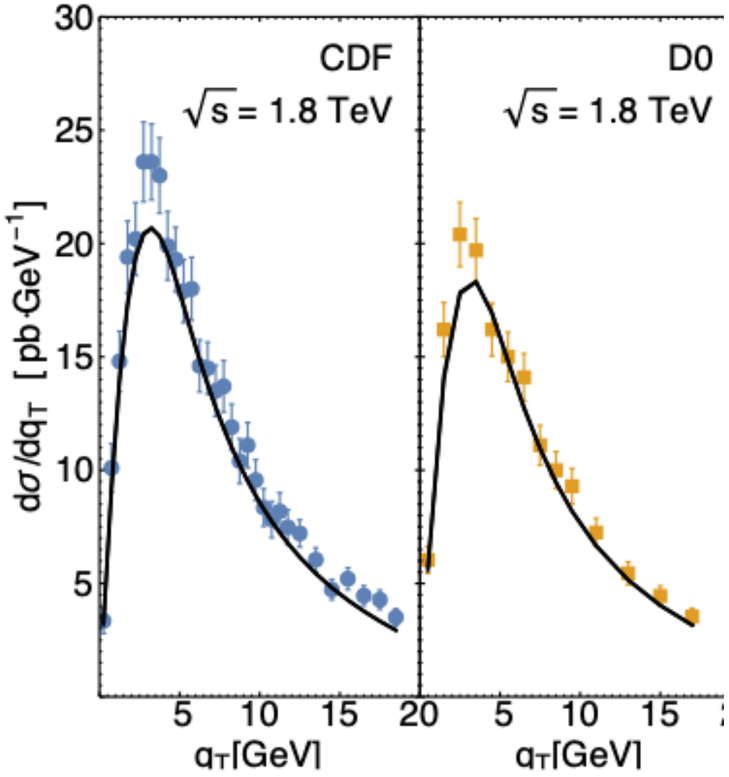
Points	Parameters	χ^2	χ^2/d.o.f.
8059	11	12629 ± 363	1.55 ± 0.05

Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]







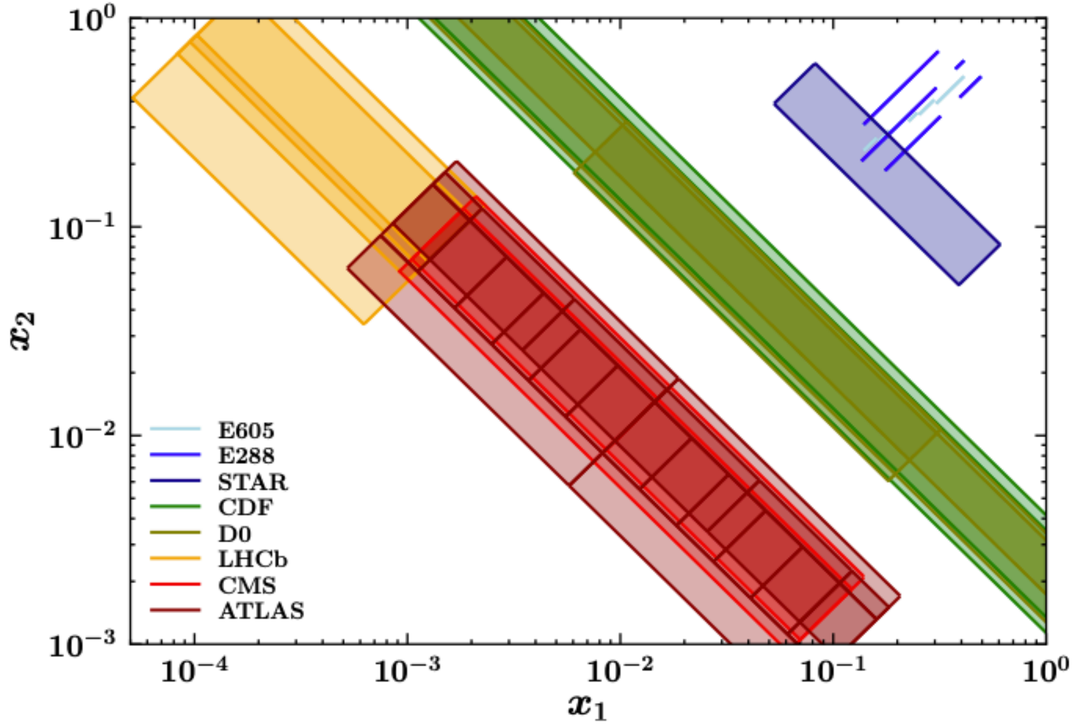
PV19 global fit

Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

Datasets:

only Drell-Yan



Total number of data:

353

Not so “global” but useful to know if we are in the right direction!

Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

TMD parametrization:

$$f_{\rm NP}(x,b_T,\zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right] \times \exp\left[-\left(g_2 + g_{2B}b_T^2\right)\ln\left(\frac{\zeta}{Q_0^2}\right)\frac{b_T^2}{4}\right]$$

b*Gaussian + Gaussian

quadratic + quartic terms in b

Global Fit: PV19

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b*Gaussian + Gaussian

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Gaussian widths:

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

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9 parameters to be fixed

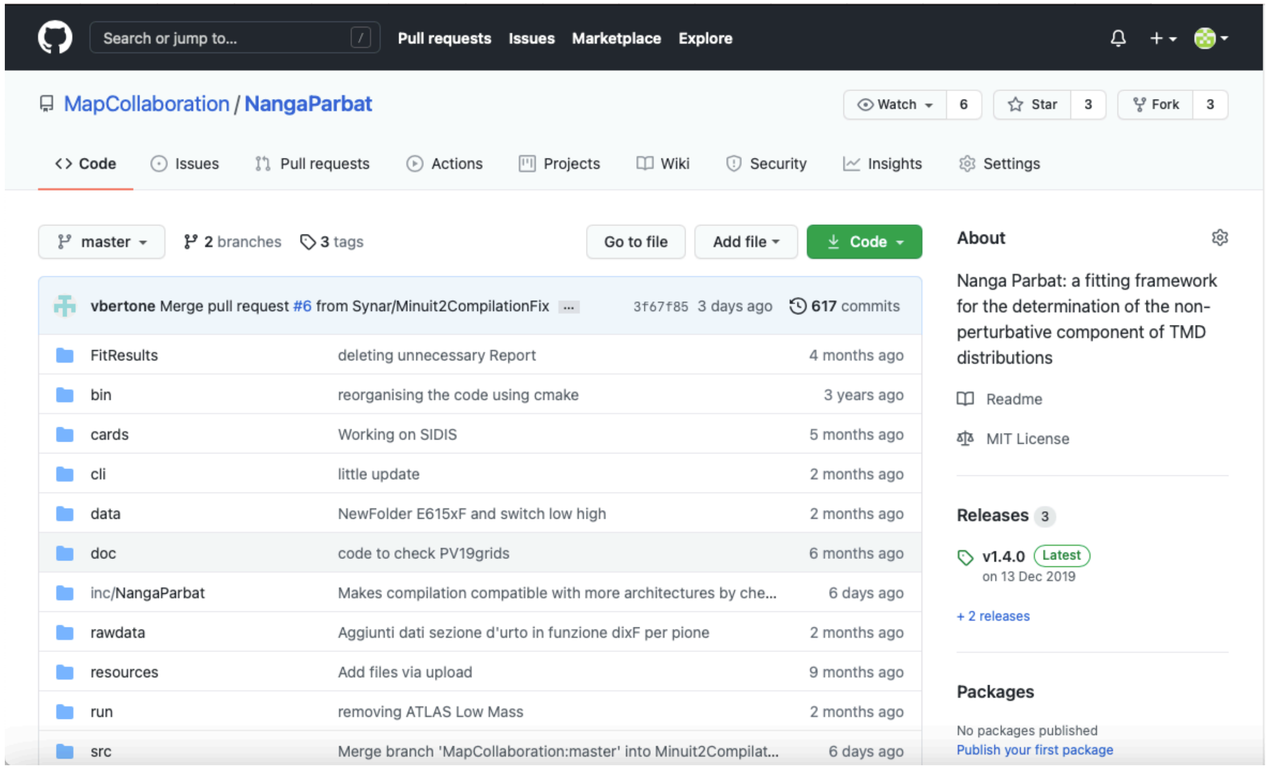
7 -> TMD PDF

2 -> NP Evolution

Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

Computational part: Nanga-Parbat fitting framework



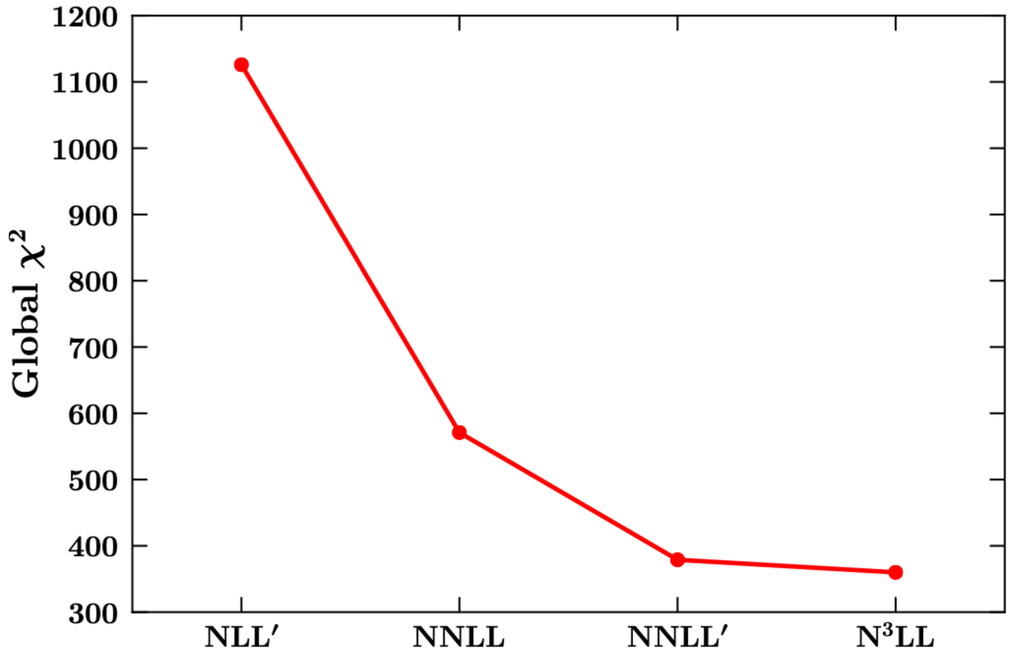
Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

Accuracy of the simulation: Next to Next to Next to Leading Log (N3LL)

$$\text{resummation of } \log^{2n-3}\frac{Q^2}{q_T^2}$$

Results:



The higher the accuracy is, the better the fit is performed

$$\chi^2_R = 1.02$$

Global Fit: PV19

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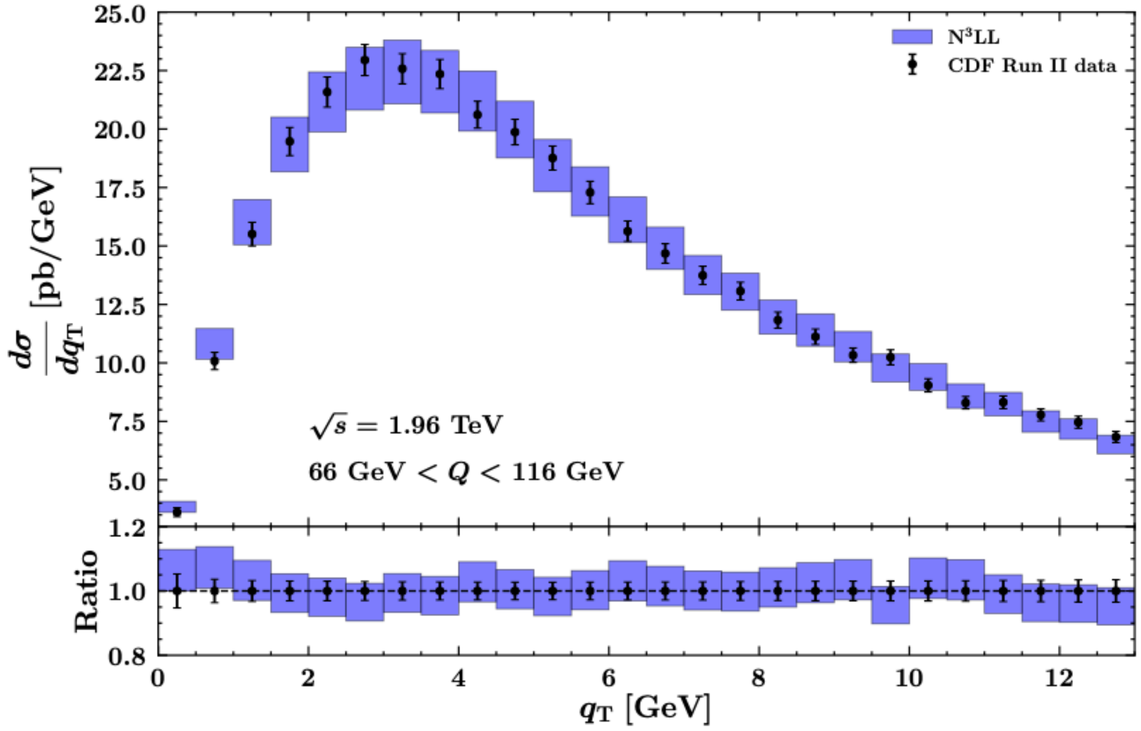
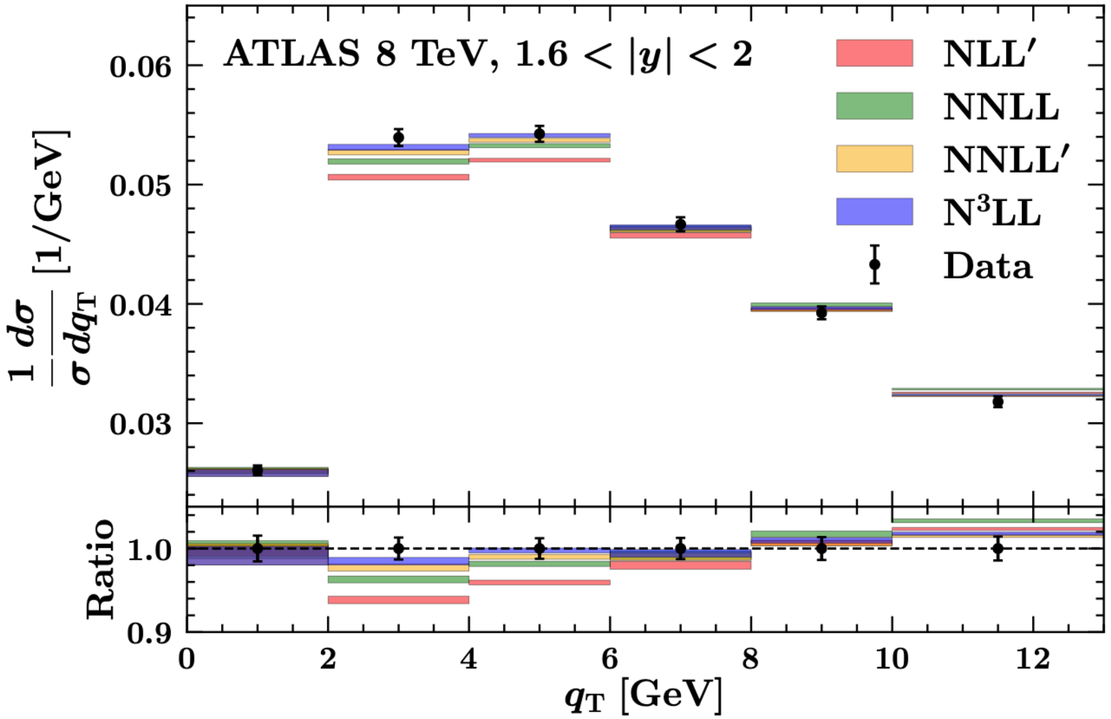
Results:

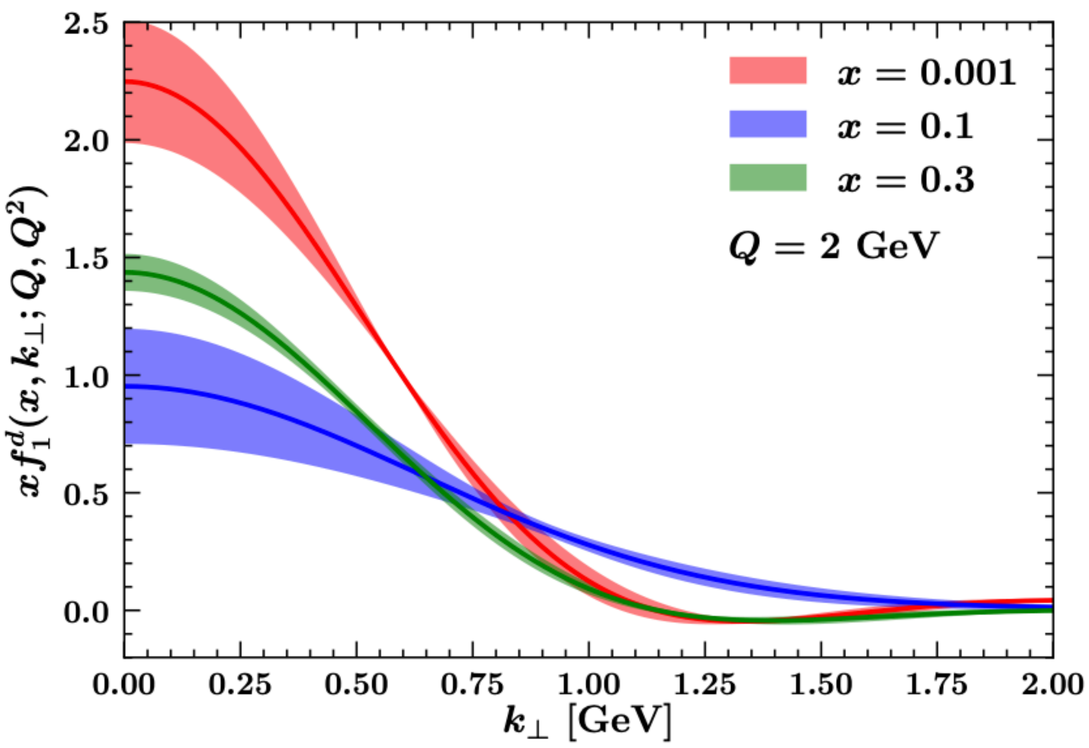


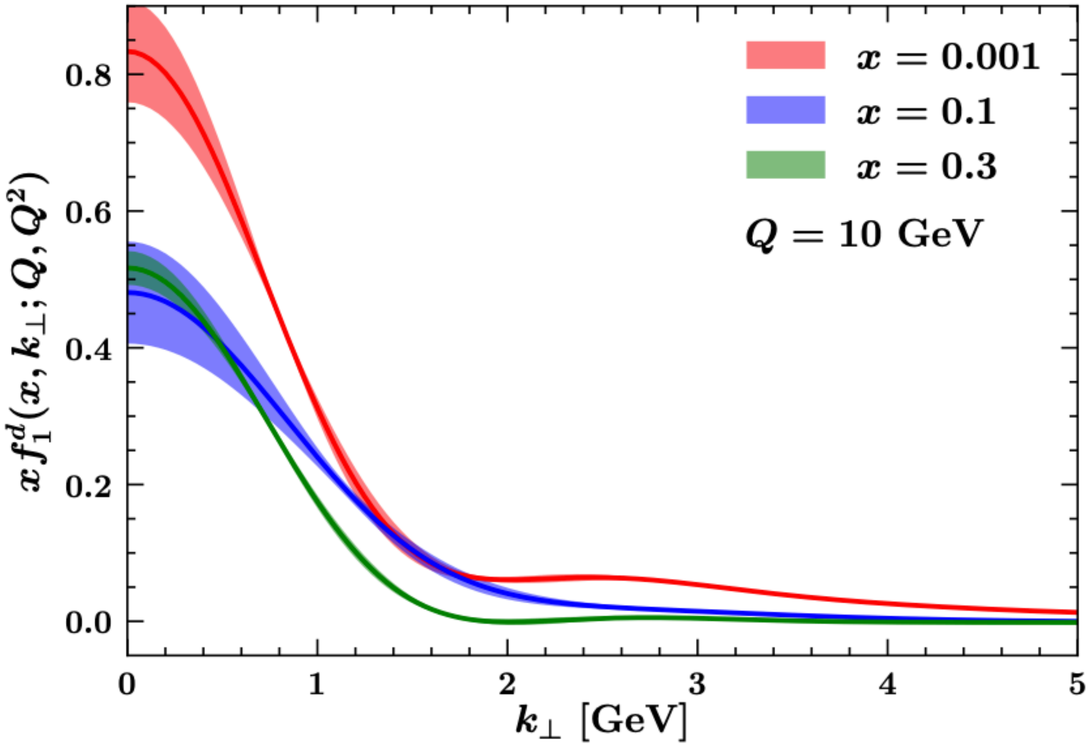


Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

TMD PDF extraction:





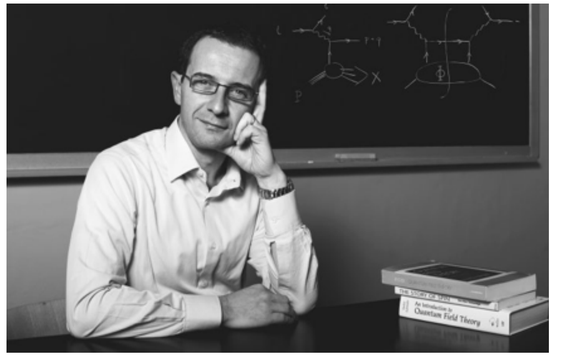
New global fit?

Global Fit: PV202?

Work still in progress!

http://www.hadronicphysics.it/hasqcd/

Alessandro Bacchetta



Valerio Bertone

Chiara Bissolotti

Giuseppe Bozzi

Fulvio Piacenza

Andrea Signori

Marco Radici


STAY TUNED!