



**Matteo Cerutti**

University of Pavia and INFN

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# GLOBAL FITS IN TMD FRAMEWORK

Results and Perspectives

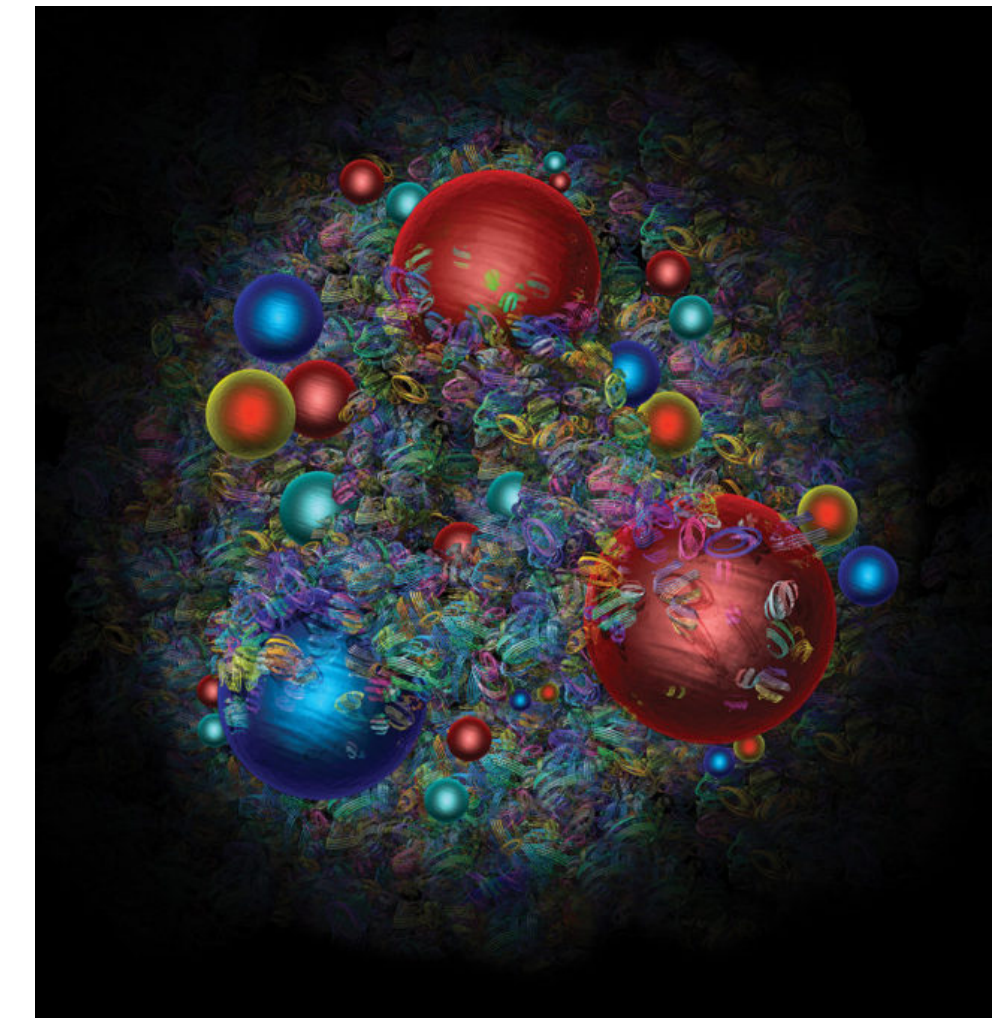
# Quantum ChromoDynamics (QCD)

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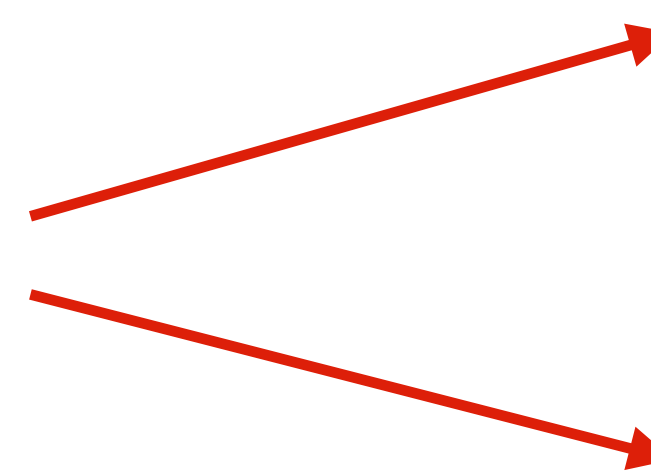
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a,\mu\nu} F_{a,\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$
$$D_\mu = \partial_\mu - ig T^a A_\mu^a$$

gluon field

quark field



But there are some features that cannot be proved from first principles!

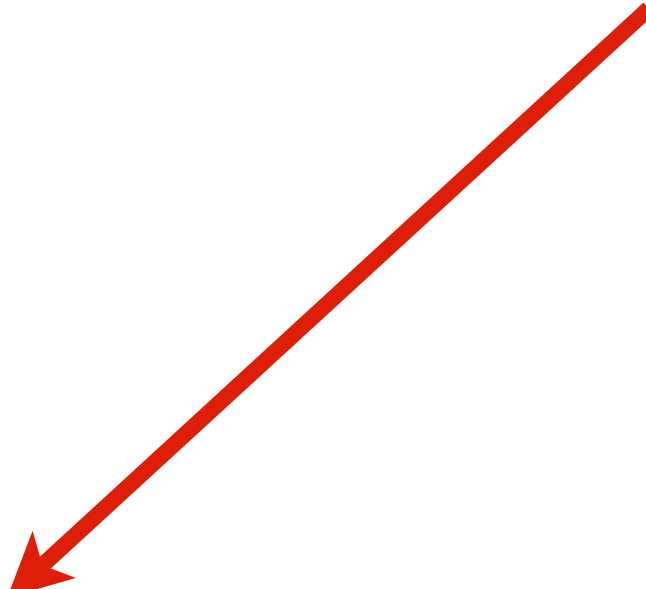


color confinement

chiral SSB

# COLOR CONFINEMENT

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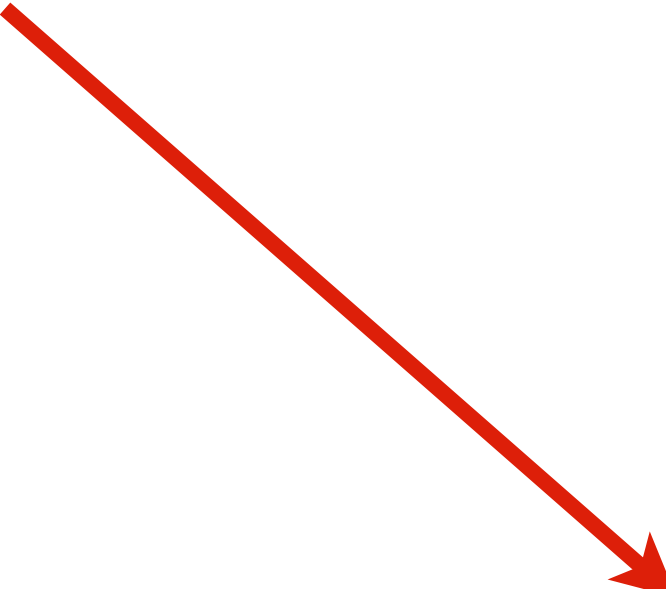
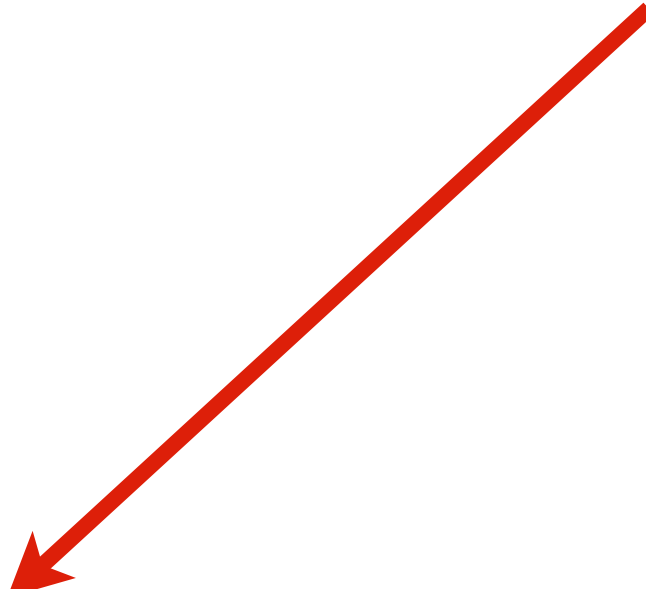
internal structure of hadrons



PDFs



# COLOR CONFINEMENT



**internal structure of hadrons**

**hadronization**



**PDFs**

**FFs**

**HOW TO STUDY THESE NON-PERTURBATIVE OBJECTS?**

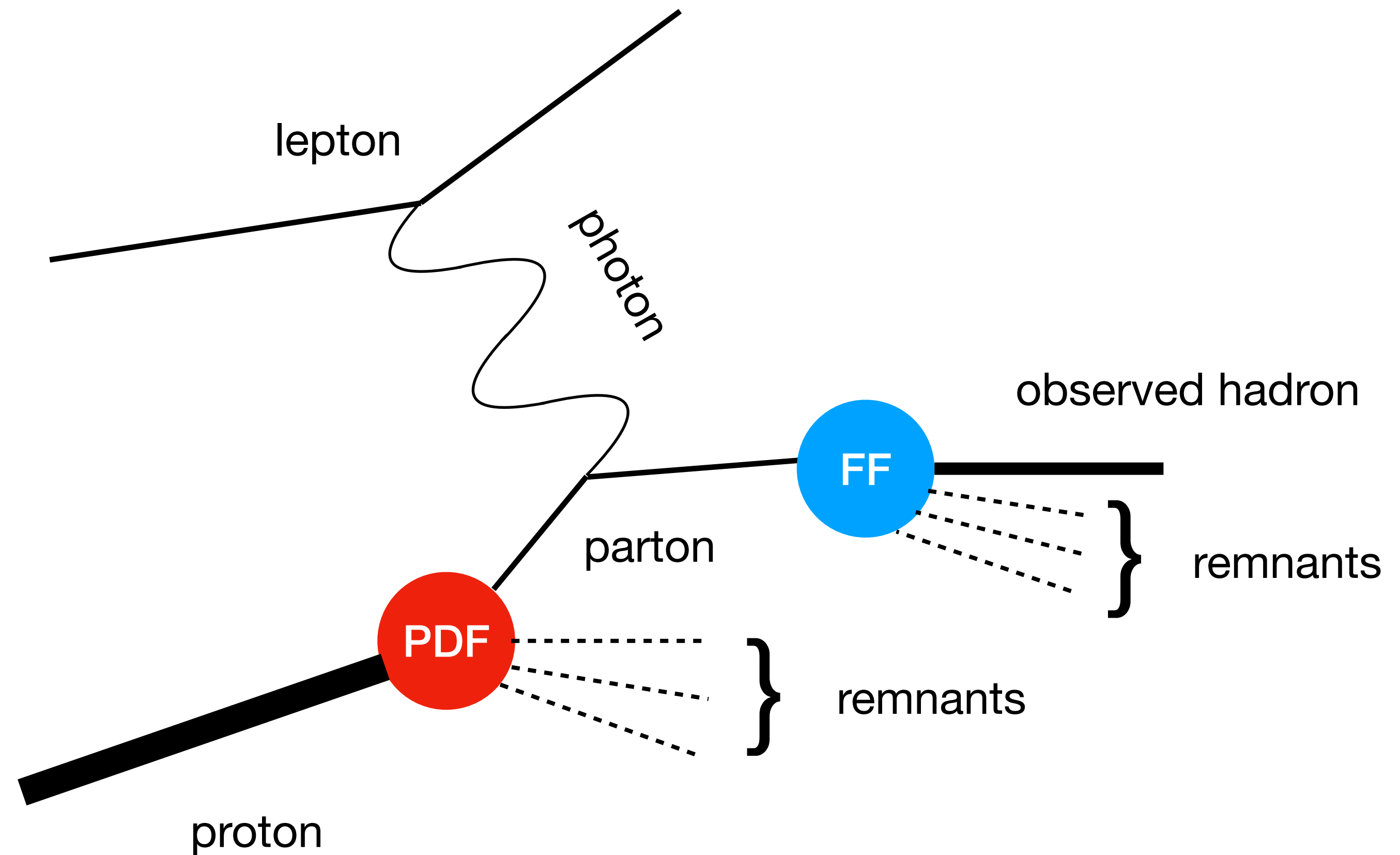
# Factorization of the cross section

Example: Semi-Inclusive Deep Inelastic Scattering (SIDIS)

$$l(\ell) + N(P) \rightarrow \gamma^*(q) \rightarrow l(\ell') + h(P_h) + X$$

If  $Q^2 \gg M^2$

$$d\sigma = \mathcal{H} \times \text{PDF} \otimes \text{FF}$$



# Factorization of the cross section

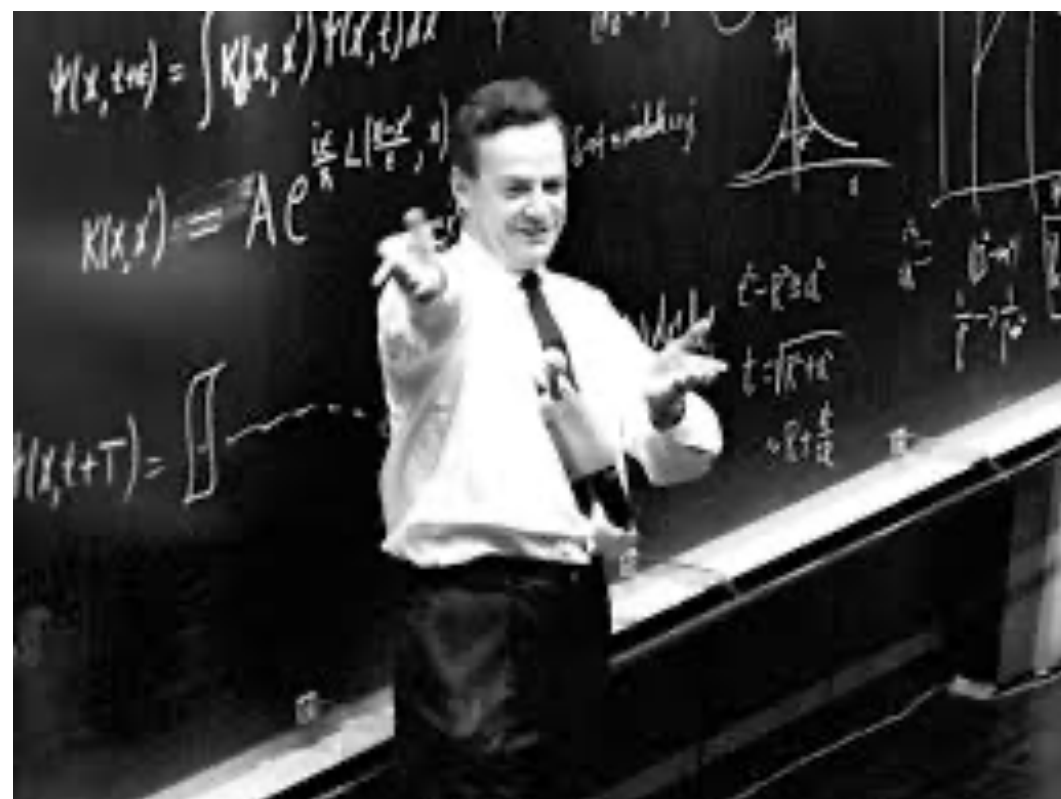
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partonic collision  
(perturbative)





# Factorization of the cross section

Example: Semi-Inclusive Deep Inelastic Scattering (SIDIS)

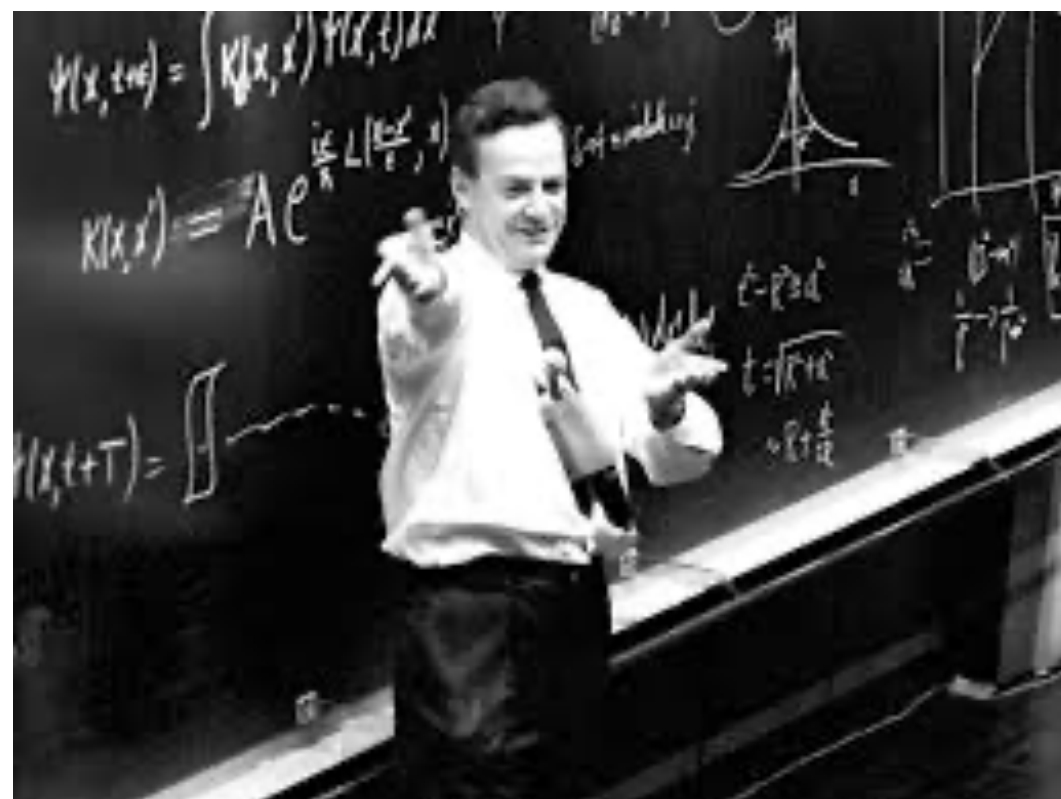
$$l(\ell) + N(P) \rightarrow \gamma^*(q) \rightarrow l(\ell') + h(P_h) + X$$

$$d\sigma = \mathcal{H} \times PDF \otimes FF$$

partonic collision  
(perturbative)

hadron structure  
(non-perturbative)

hadronization  
(non-perturbative)

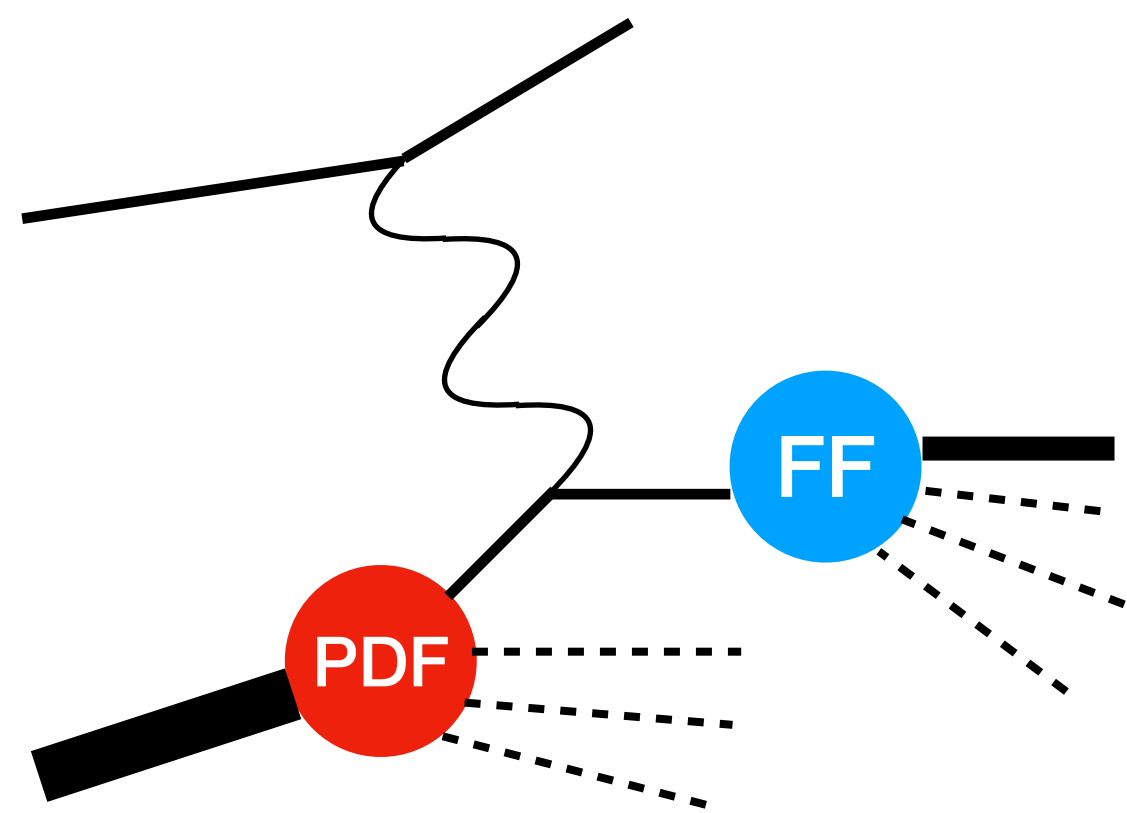


# Factorization of the cross section

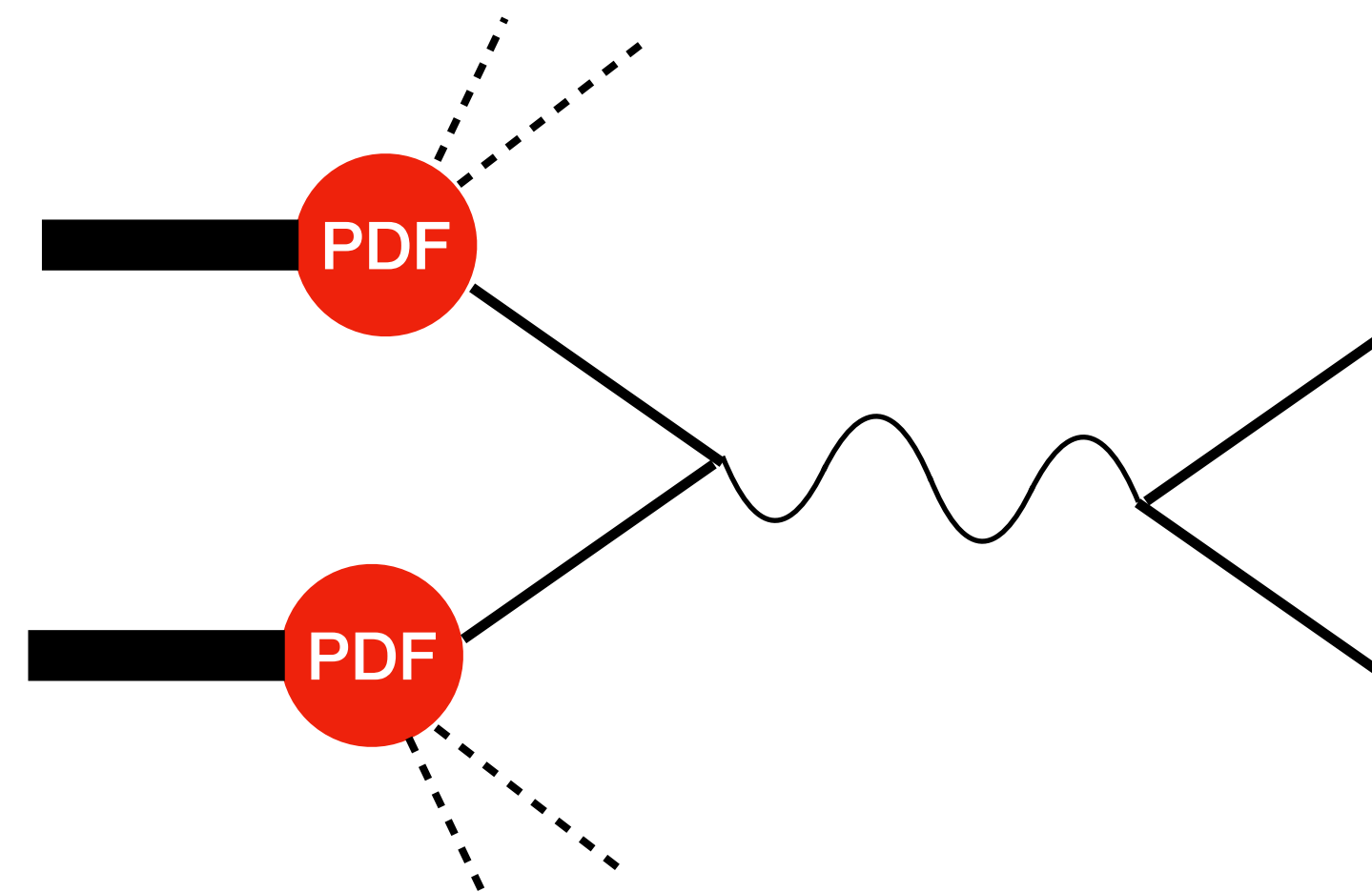
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## UNIVERSALITY:

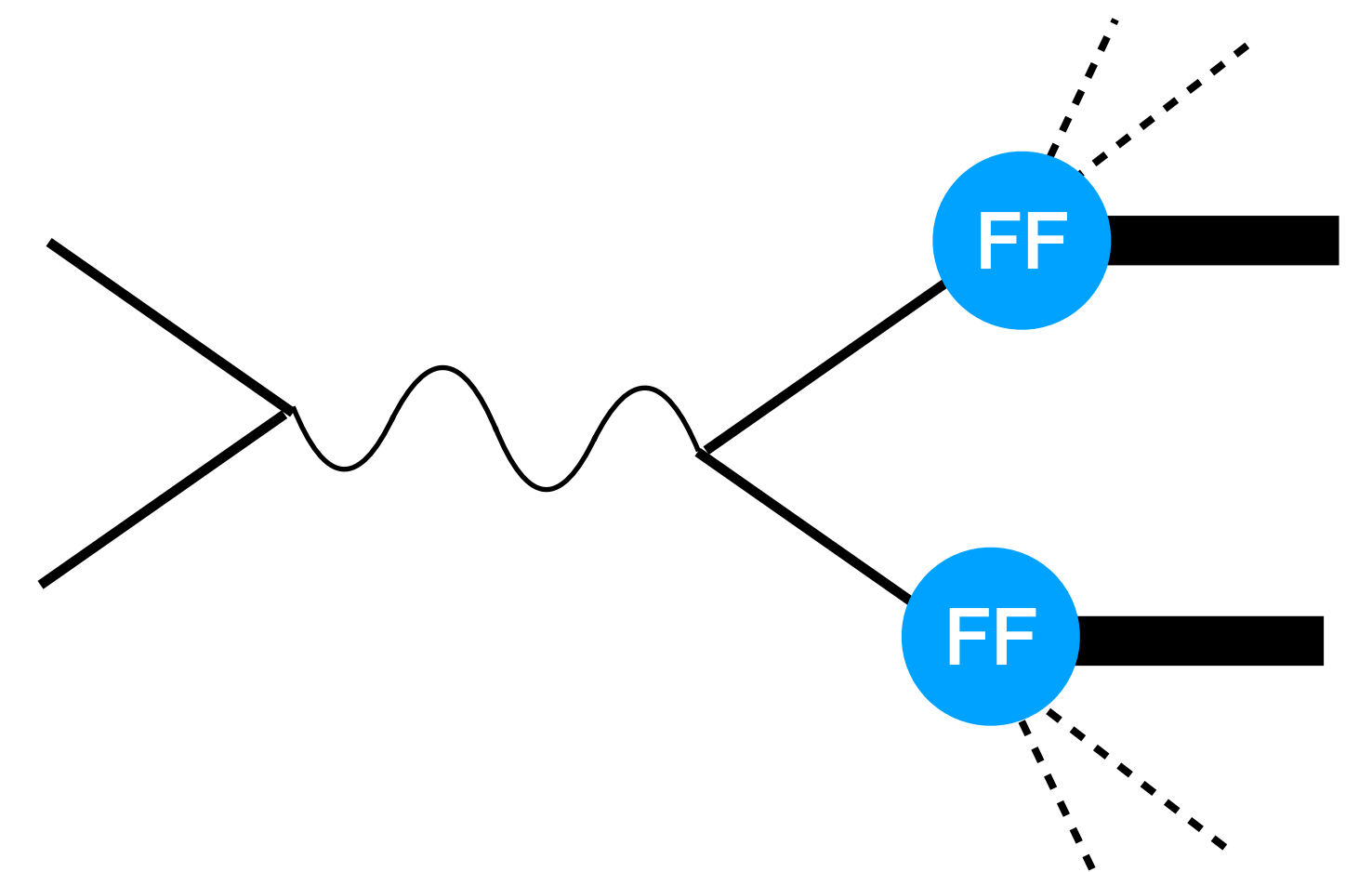
Same PDFs and FFs for a given hadron in different processes!



SIDIS



Drell-Yan



electron-positron  
annihilation



**HOW TO PROVE THAT THE FORMALISM IS VALID?**





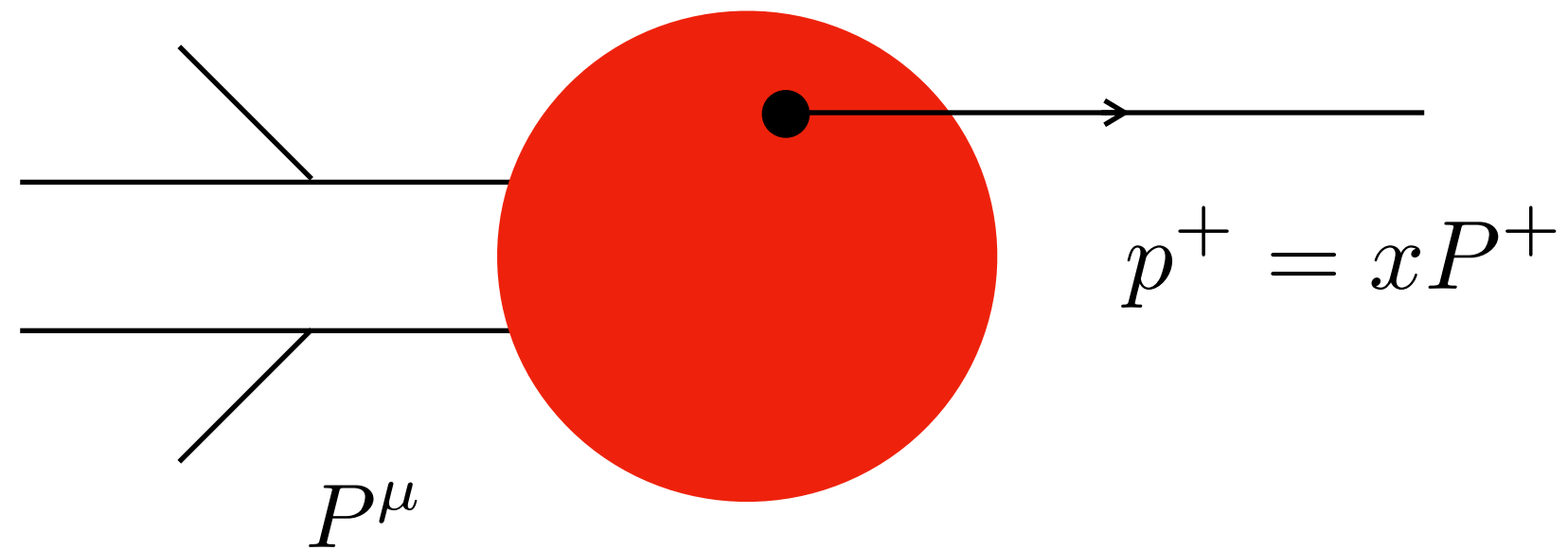


A conceptual image featuring a hand holding a globe. The globe is rendered with a grid of white dots and is overlaid with a network of white lines and glowing nodes. The background is a blurred laptop keyboard. A red rectangular box is centered over the globe, containing the text "GLOBAL FITS" in white, bold, uppercase letters.

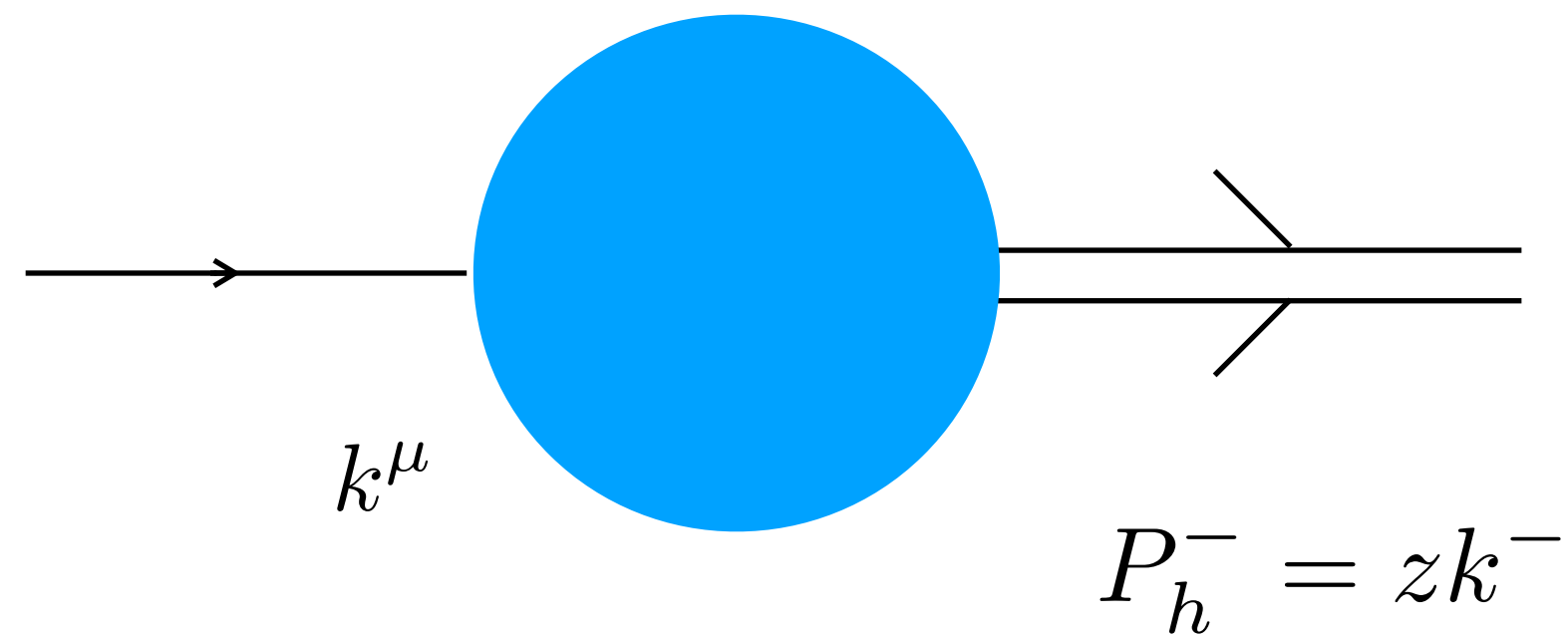
**GLOBAL FITS**



# Collinear case - factorization

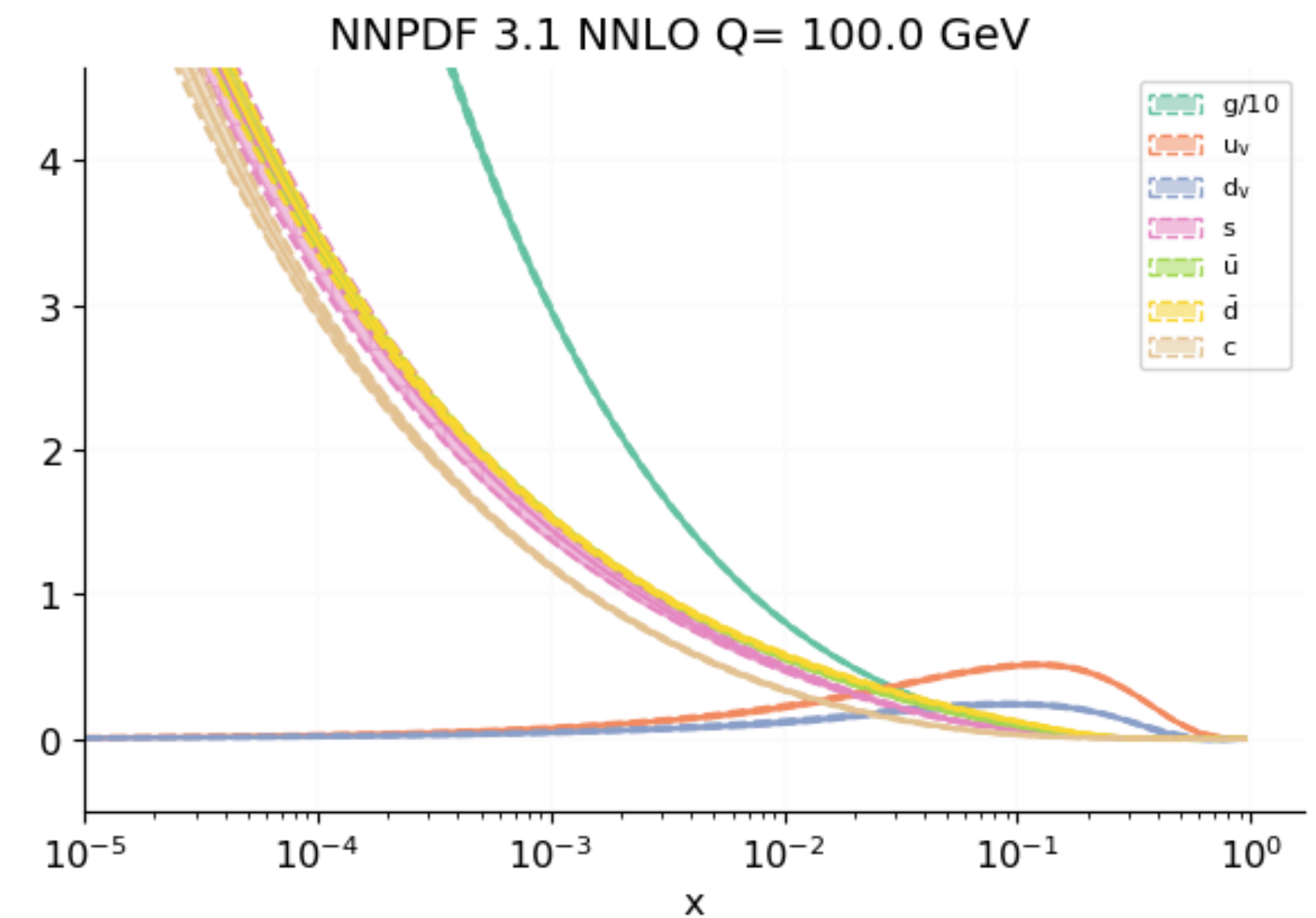
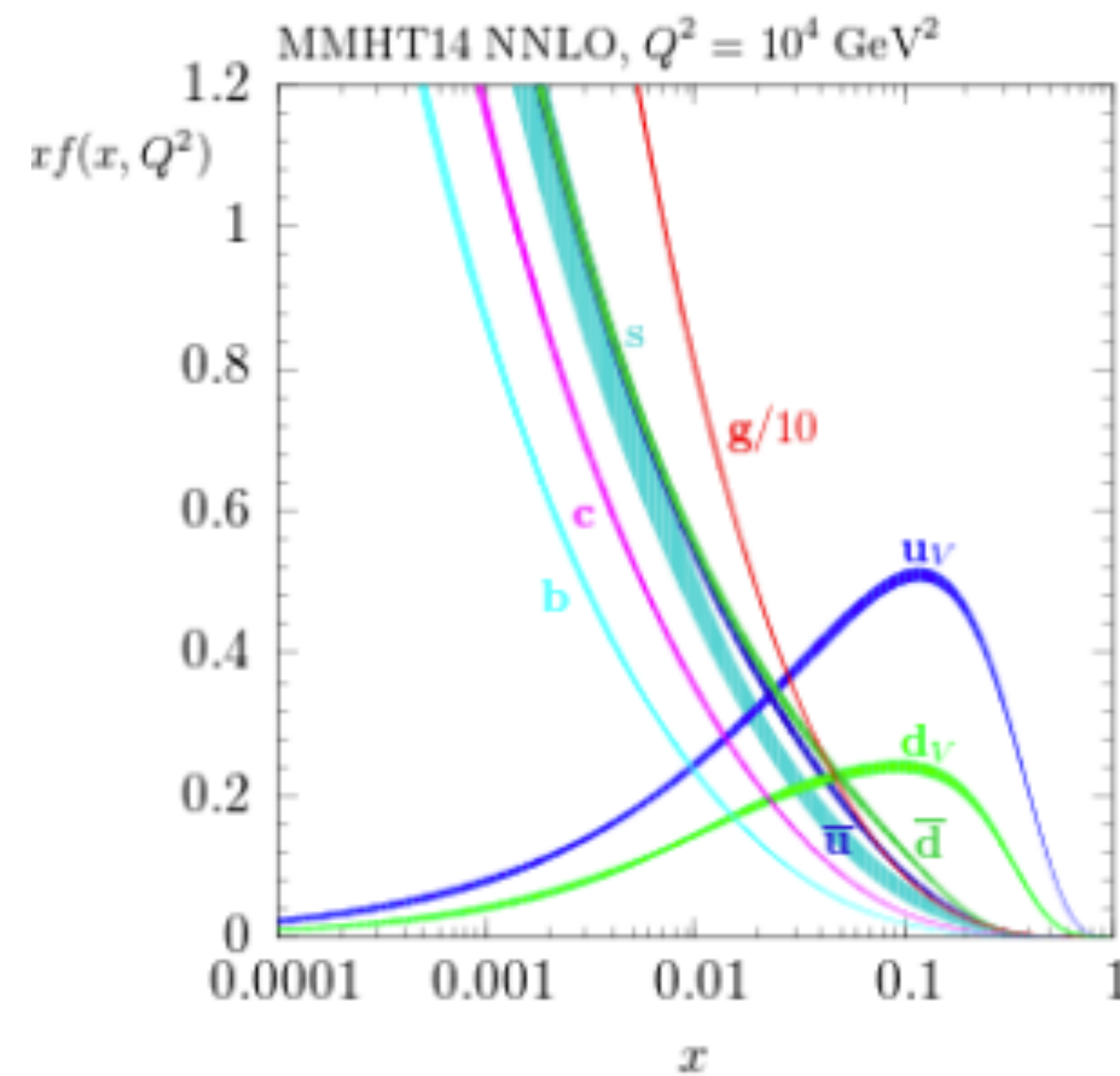
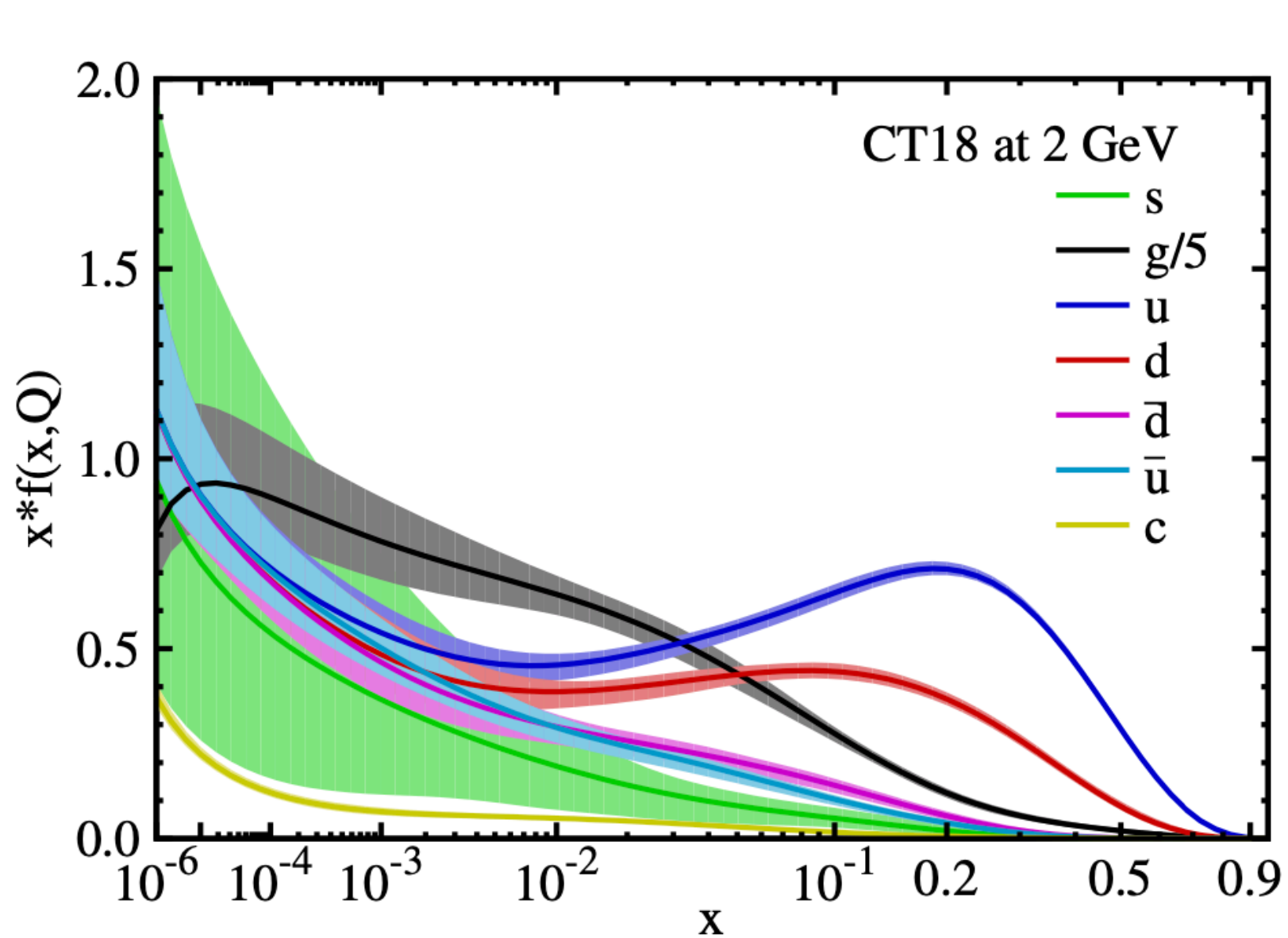


$$\Phi_{ij}(x, S) = \int \frac{d\xi^-}{2\pi} e^{ip \cdot \xi} \langle PS | \bar{\psi}_j(0) \psi_i(\xi) | PS \rangle_{\xi^+ = \xi_T = 0}$$



$$\Delta_{ij}(z, S) = \frac{1}{2z} \sum_X \int \frac{d\xi^+}{2\pi} e^{ip \cdot \xi} \langle 0 | \bar{\psi}_i(\xi) | h, X \rangle \langle h, X | \psi_j(0) | 0 \rangle_{\xi^- = \xi_T = 0}$$

# Collinear case - Global Fits of PDFs

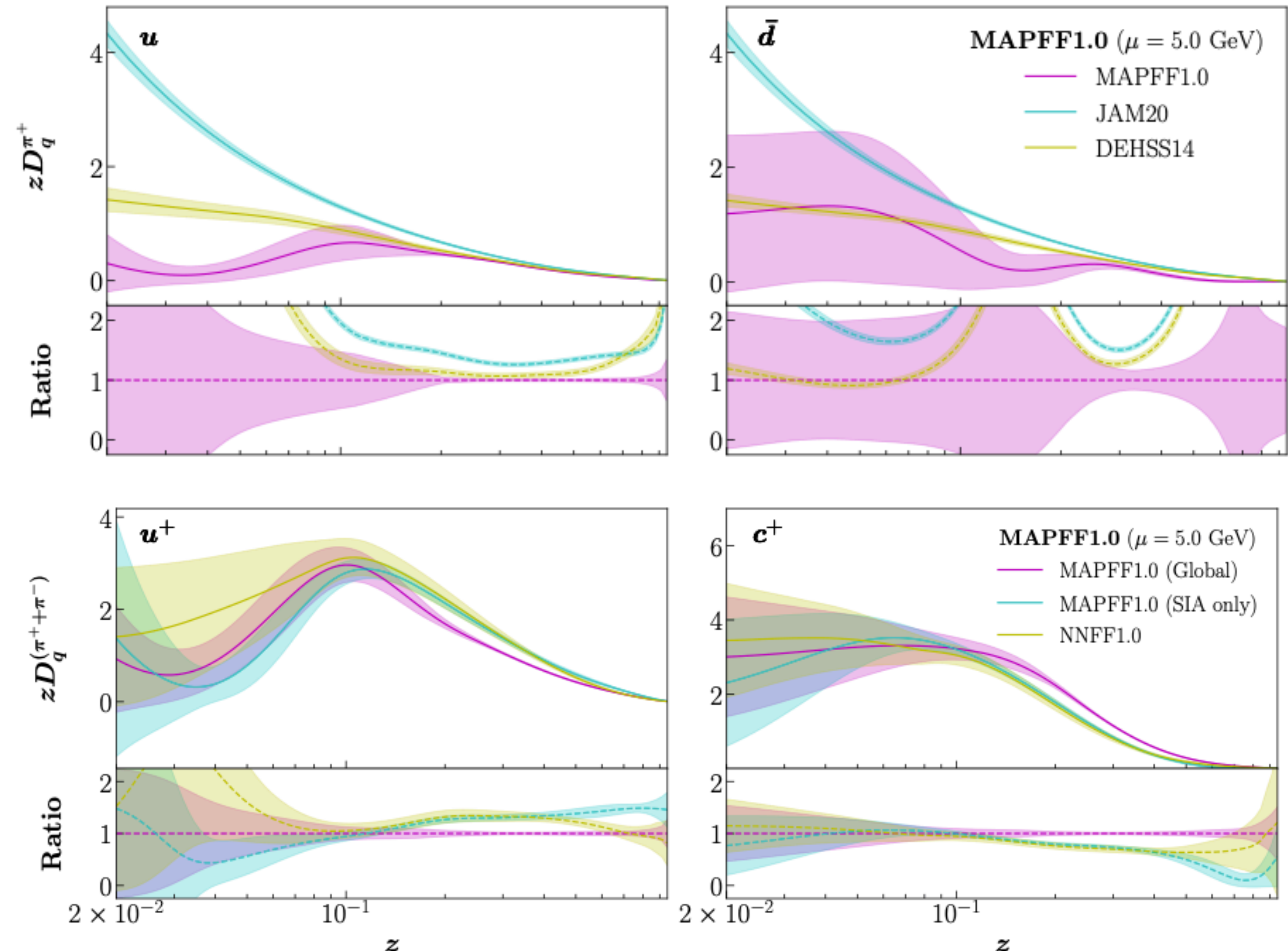


Quite a lot determination available  
See e.g. the LHAPDF library  
<https://lhapdf.hepforge.org>

# Collinear case - Global Fits of FFs

## Extractions:

- MAP Collaboration
- JAM Collaboration
- DEHSS
- Neural Network FF Collaboration





# Collinear case - What does not work

---

## Evidences:

- Single Spin Asymmetries (SSA)  
J. Adams et al., P.R.L. 92 (2004) 171801, . . .
- Violation of the Lam-Tung rule  
J. S. Conway et al. P.R. D39 (1989) 92-122, . . .
- Results of the EMC, SMC, . . ., experiments  
J. Ashman et al., P.L. B206 (1988) 364, . . .

# Collinear case - What does not work

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# TMD case - factorization

Transverse Momentum Dependent parton densities

$$\Phi_{ij}(x, \mathbf{p}_T, S) = \int \frac{d\xi^- d^2\xi_T}{2\pi} e^{ip \cdot \xi} \langle PS | \bar{\psi}_j(0) \psi_i(\xi) | PS \rangle_{\xi^+ = 0}$$

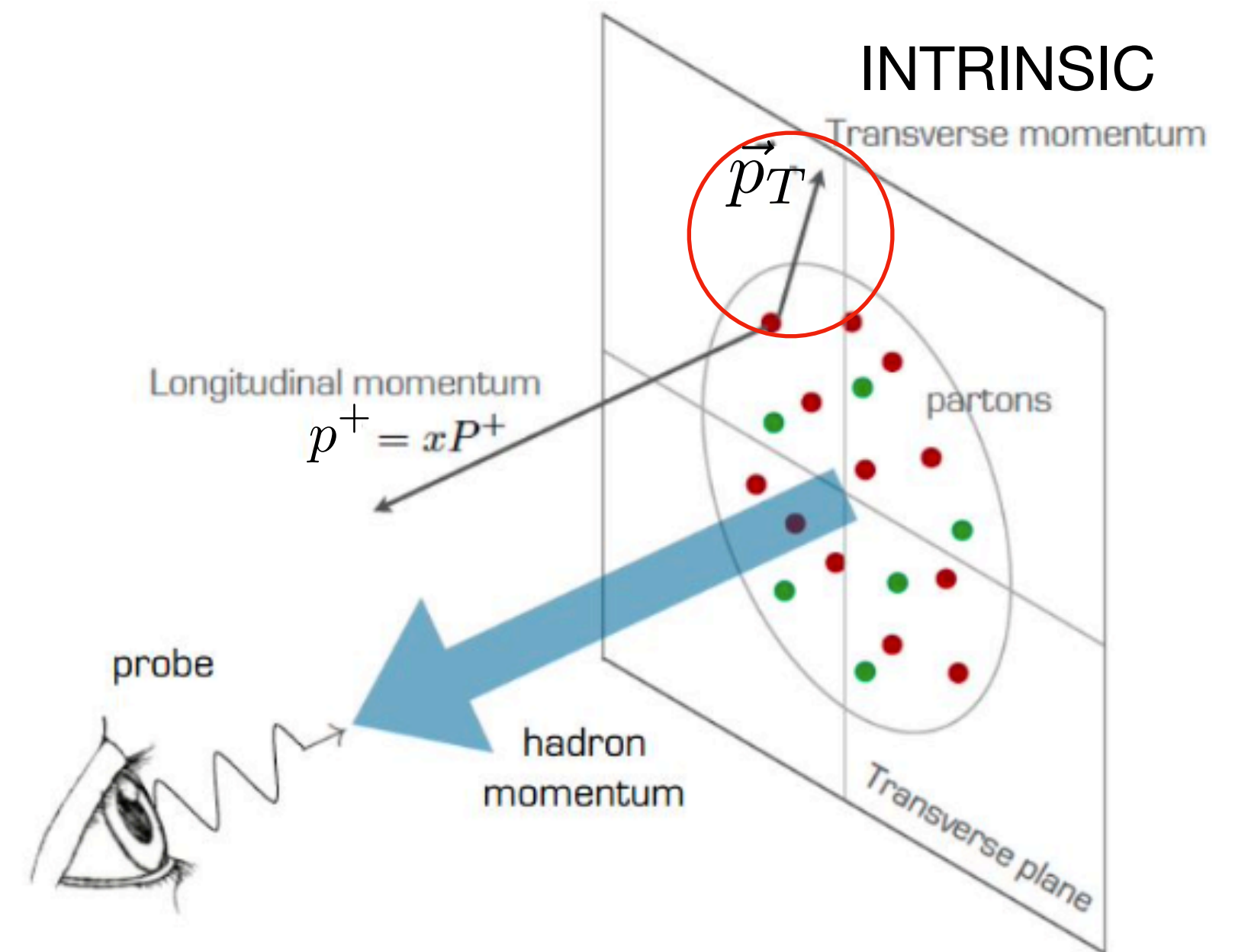
$$\Downarrow$$

$$F_a(x, p_T^2, \mu, \zeta)$$

$$\Delta_{ij}(z, \mathbf{k}_T, S) = \sum_X \int \frac{d\xi^+ d^2\xi_T}{2\pi} e^{ip \cdot \xi} \langle 0 | \bar{\psi}_i(\xi) | X P_h S_h \rangle \langle X P_h S_h | \psi_j(0) | 0 \rangle_{\xi^- = 0}$$

$$\Downarrow$$

$$D^{a \rightarrow h}(z, k_T^2, \mu, \zeta)$$

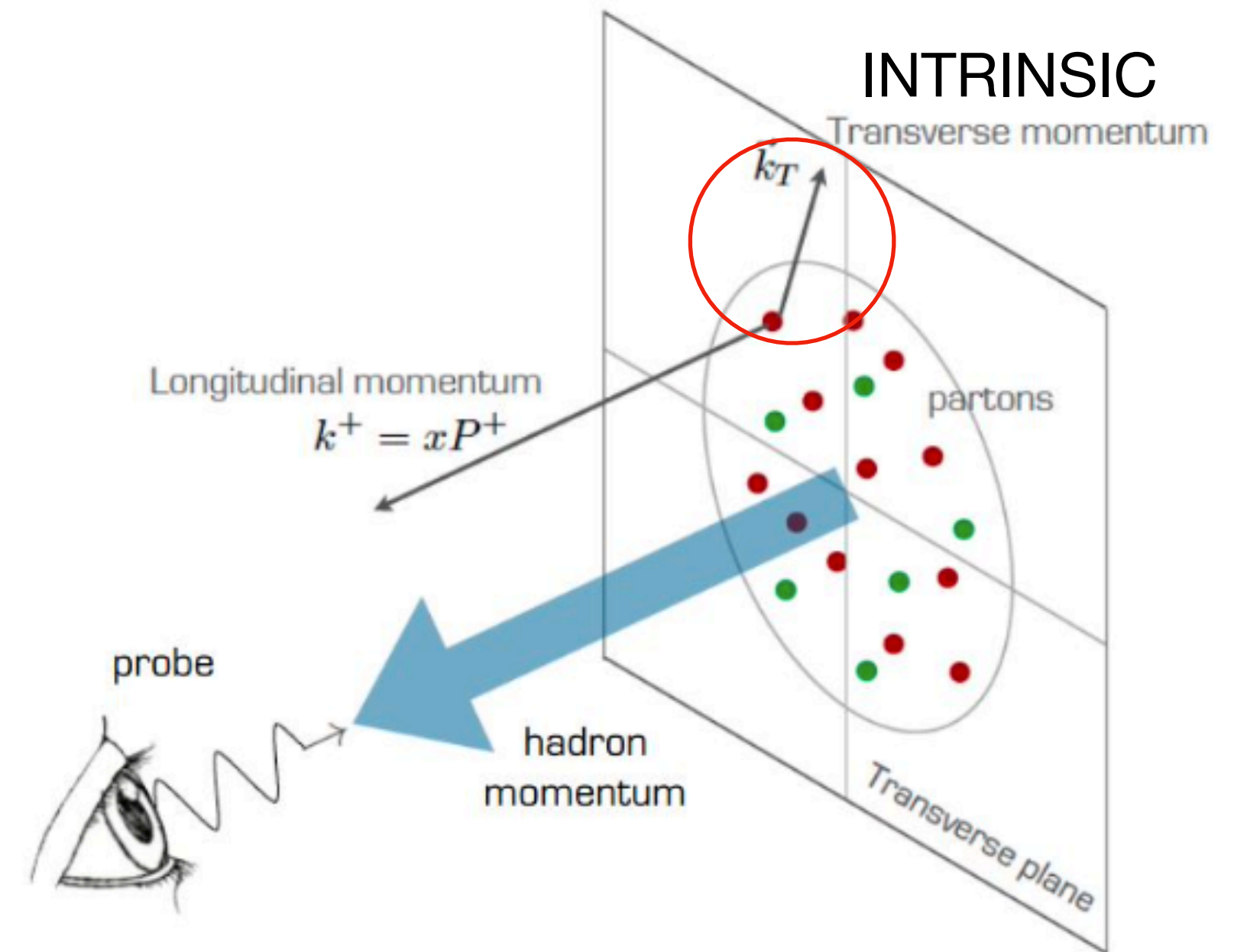


**HOW TO PARAMETRIZE PDFS AND FFS?**

# TMD case - Parametrization

Fourier Transform in  $b_T$  space

$$\tilde{F}_a(x, b_T^2; \mu, \zeta) = \int \frac{d^2 k_T}{(2\pi)^2} e^{i b_T \cdot k_T} F_a(x, k_T^2; \mu, \zeta)$$



How to model a TMD distribution?

$$\tilde{F}_a(x, b_T^2; Q, Q^2) = [C_{a/b}(x, b_T^2, \mu_b) \otimes F_a(x; \mu_b)] e^{S_{\text{pert}}(\mu_b^2, Q^2)} e^{S_{\text{NP}}(b_T, Q^2; \lambda)} \tilde{F}_{a, NP}(x, b_T^2; \lambda')$$

Matching coefficient  
(Perturbative calc.)

Collinear PDF  
(previous fit)

Evolution  
(perturbative calc.)

Evolution  
(model-dependent)

TMD part of the parton density  
(model-dependent)

# PAVIA RESULTS



**PV17 global fit**

# Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

## Datasets:

## Number of data:

SIDIS	COMPASS experiment	6252
	HERMES experiment	1514
Drell-Yan	E288 experiment	168
	E605 experiment	35
Z production	CDF experiment	68
	D0 experiment	22
		<hr/>
		8059



# Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

---

TMD parametrization:

$$f_{1\text{NP}}^a(x, \mathbf{k}_\perp^2) = \frac{1}{\pi} \frac{(1 + \lambda \mathbf{k}_\perp^2)}{g_{1a} + \lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}},$$
$$D_{1\text{NP}}^{a \rightarrow h}(z, \mathbf{P}_\perp^2) = \frac{1}{\pi} \frac{1}{g_{3a \rightarrow h} + (\lambda_F/z^2)g_{4a \rightarrow h}^2} \left( e^{-\frac{\mathbf{P}_\perp^2}{g_{3a \rightarrow h}}} + \lambda_F \frac{\mathbf{P}_\perp^2}{z^2} e^{-\frac{\mathbf{P}_\perp^2}{g_{4a \rightarrow h}}} \right)$$

Gaussian + weighted Gaussian

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Gaussian + weighted Gaussian

Gaussian widths:

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$g_{3,4}(x) = N_{3,4} \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$



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11 parameters to be fixed

4 -> TMD PDF

6 -> TMD FF

1 -> NP Evolution

# Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

---

Accuracy of the simulation: Next to Leading Log (NLL)

resummation of  $\log^{2n-1} \frac{Q^2}{q_T^2}$



# Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]

Accuracy of the simulation: Next to Leading Log (NLL)

resummation of  $\log^{2n-1} \frac{Q^2}{q_T^2}$

Results of the fit:

	HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Points	190	190	189	187
$\chi^2/\text{points}$	$4.83 \pm 0.42$	$2.47 \pm 0.28$	$0.91 \pm 0.14$	$0.82 \pm 0.17$

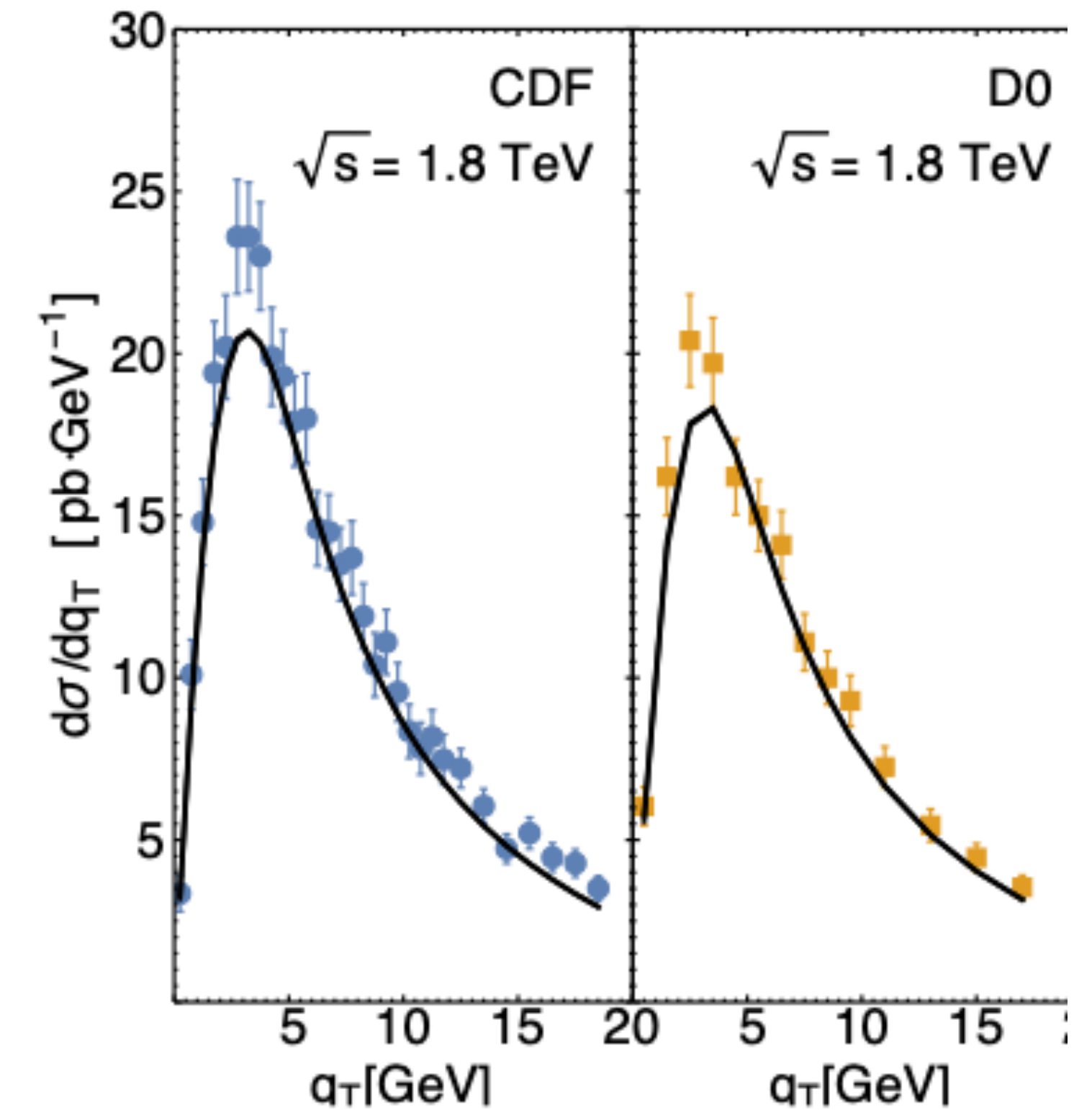
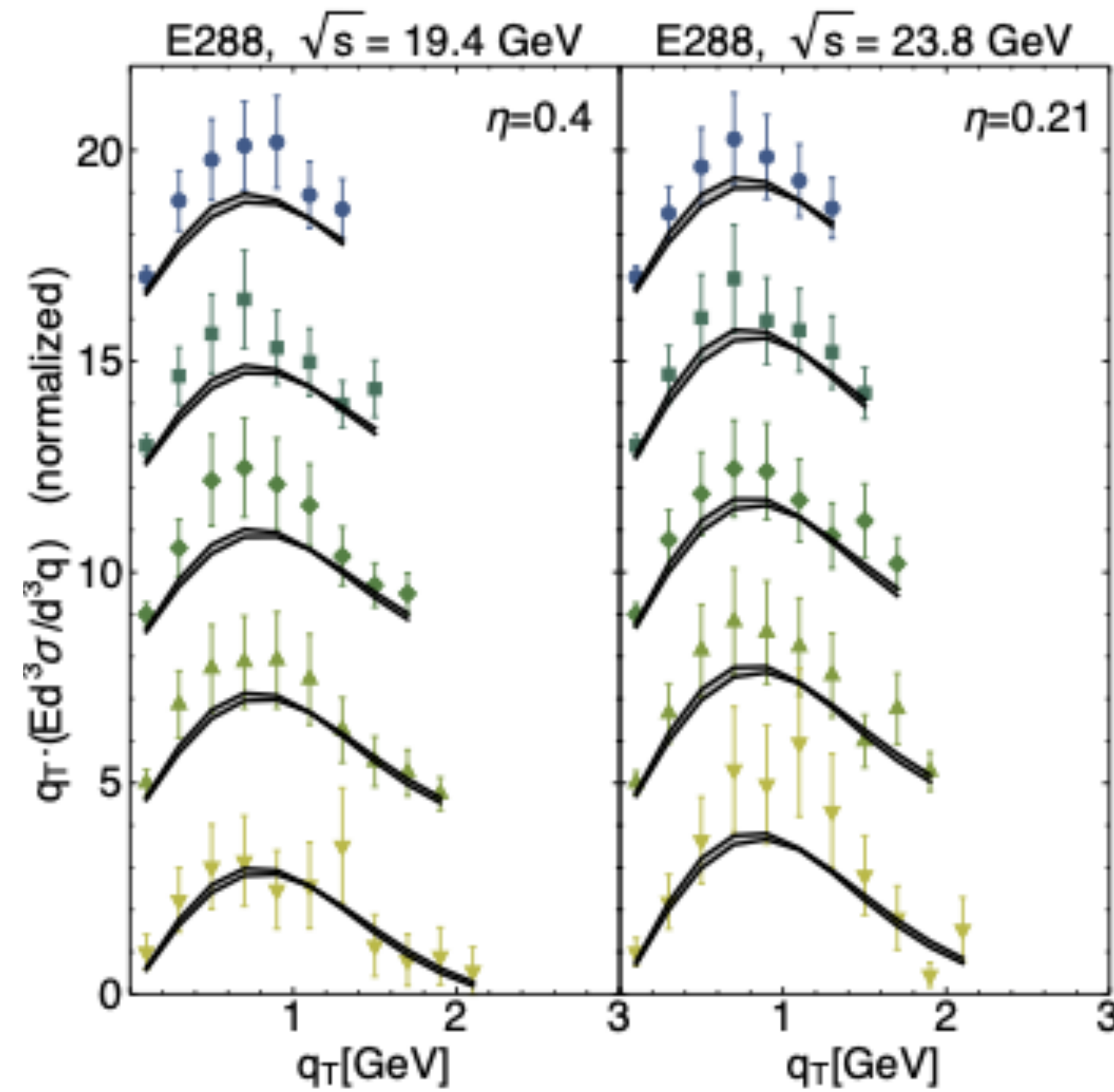
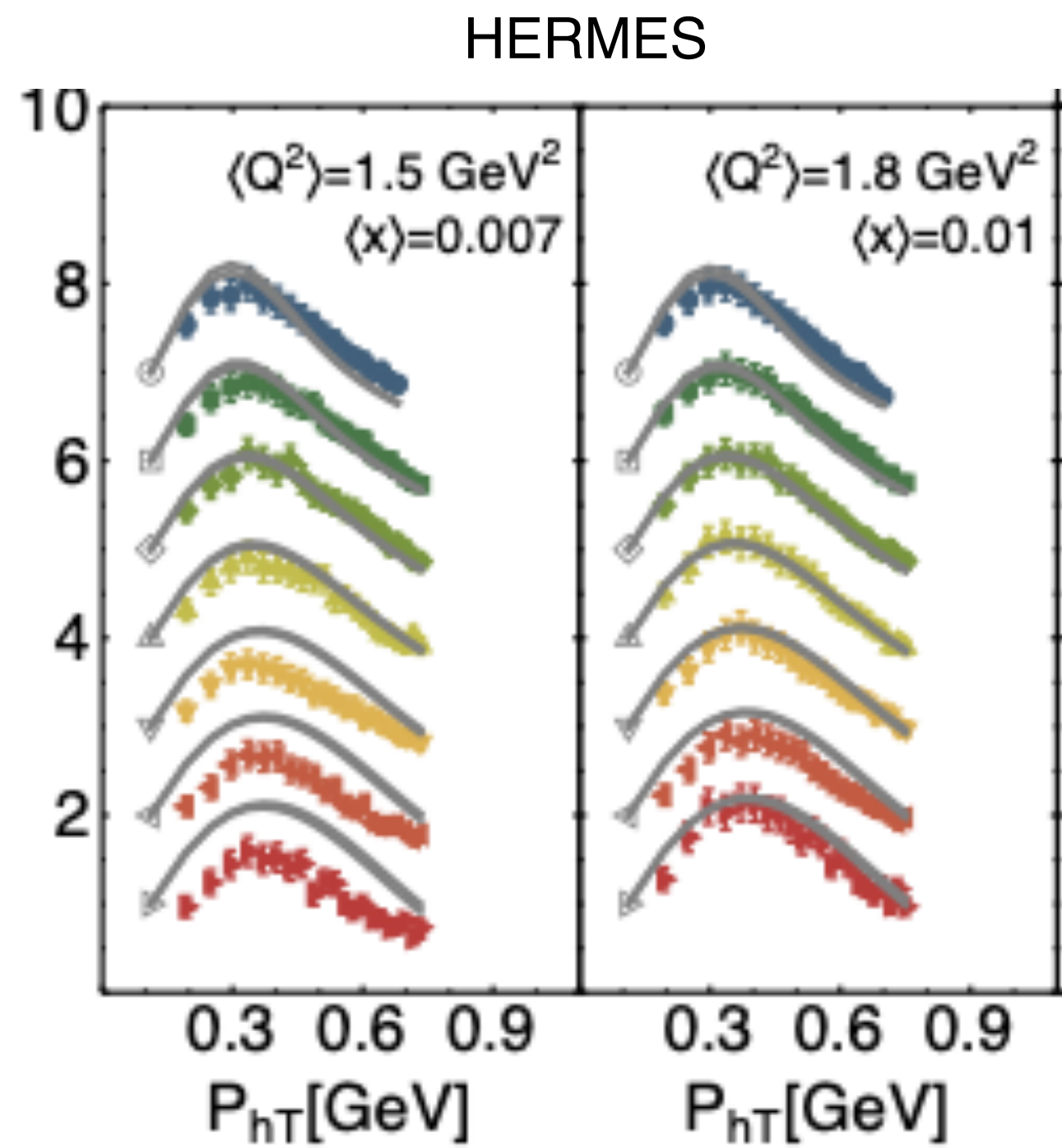
	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
$\chi^2/\text{points}$	$1.36 \pm 0.00$	$1.11 \pm 0.02$	$2.00 \pm 0.02$	$1.73 \pm 0.01$

	E288 [200]	E288 [300]	E288 [400]	E605
Points	45	45	78	35
$\chi^2/\text{points}$	$0.99 \pm 0.09$	$0.84 \pm 0.10$	$0.32 \pm 0.01$	$1.12 \pm 0.08$

Points	Parameters	$\chi^2$	$\chi^2/\text{d.o.f.}$
8059	11	$12629 \pm 363$	$1.55 \pm 0.05$

# Global Fit: PV17

Bacchetta, Delcarro, Pisano, Radici, Signori [arXiv:1703.10157]



**PV19 global fit**



# Global Fit: PV19

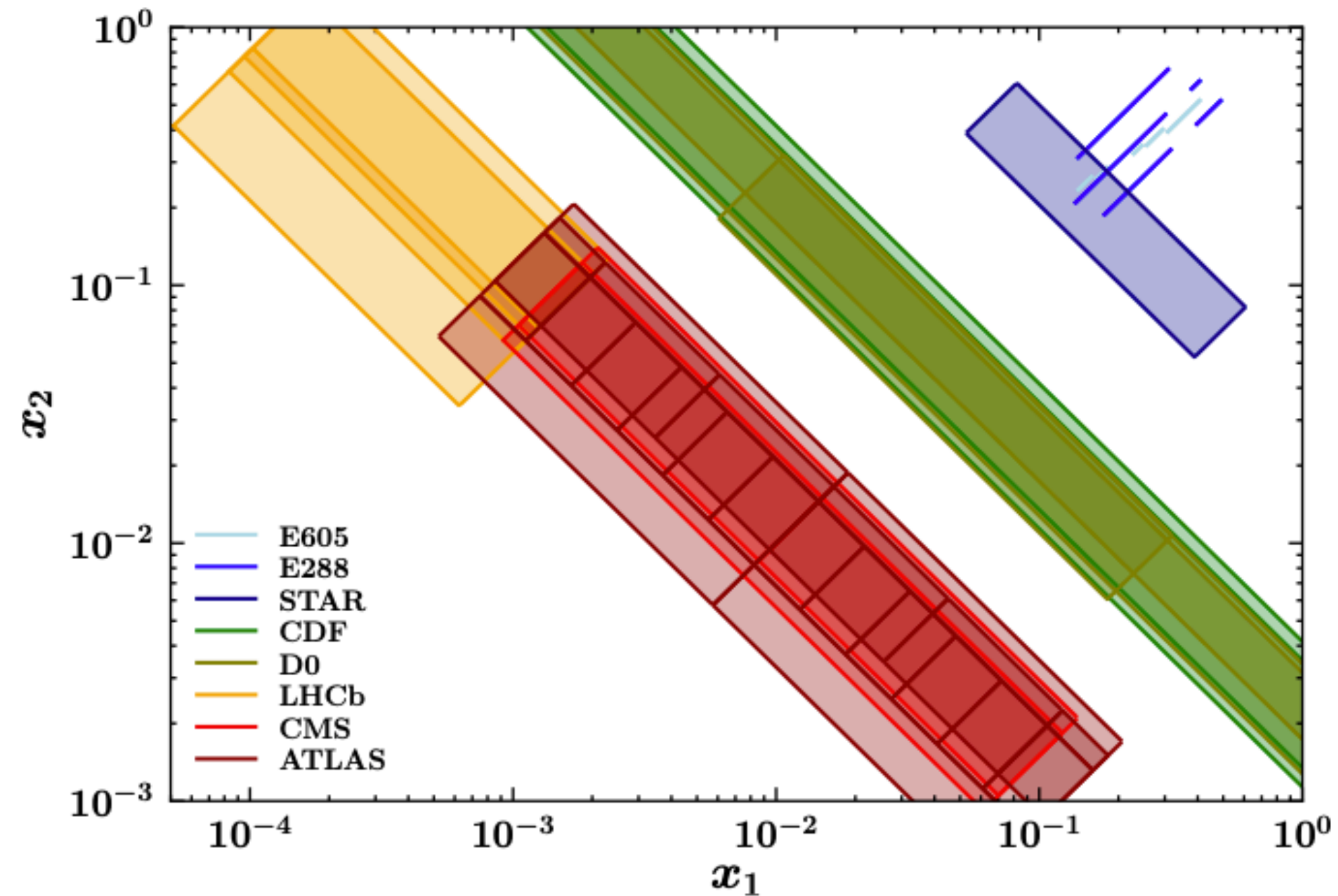
Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

Datasets:

only Drell-Yan

Total number of data:

353



Not so “global” but  
useful to know if we are  
in the right direction!

# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

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TMD parametrization:

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x) \frac{b_T^2}{4}\right) \right] \\ \times \exp\left[-(g_2 + g_{2B} b_T^2) \ln\left(\frac{\zeta}{Q_0^2}\right) \frac{b_T^2}{4}\right]$$

**b\*Gaussian + Gaussian**

**quadratic + quartic terms in b**

# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

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Gaussian widths:

$$g_1(x) = \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{x}{\alpha}\right)\right] \\ g_{1B}(x) = \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2} \ln^2\left(\frac{x}{\alpha_B}\right)\right]$$



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9 parameters to be fixed

7 -> TMD PDF

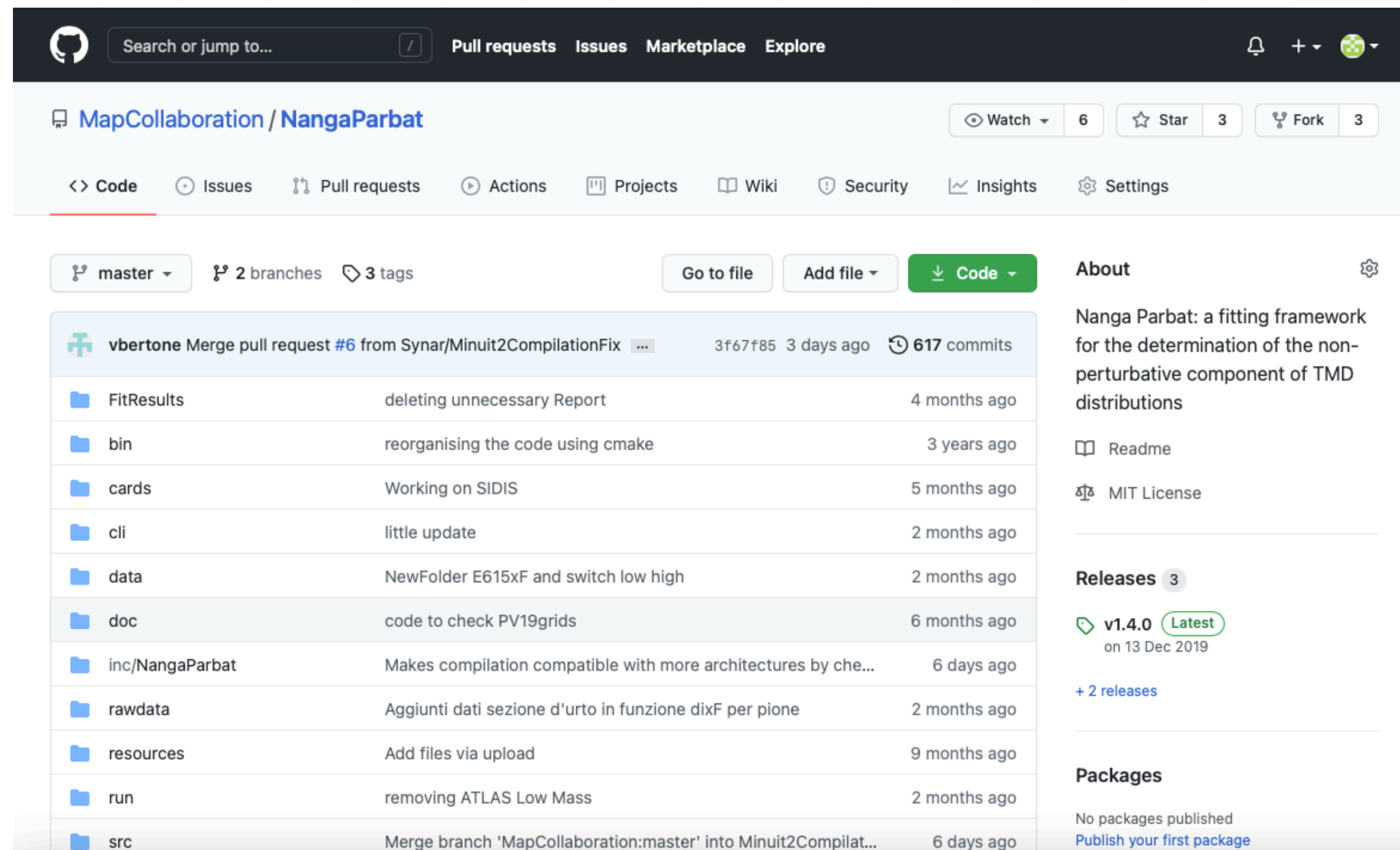
2 -> NP Evolution



# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

## Computational part: Nanga-Parbat fitting framework



The screenshot shows the GitHub repository page for `MapCollaboration/NangaParbat`. The repository has 6 watchers, 3 stars, and 3 forks. The main navigation includes Code, Issues, Pull requests, Actions, Projects, Wiki, Security, Insights, and Settings. The repository is currently on the `master` branch, with 2 other branches and 3 tags. A recent pull request by `vbertone` is highlighted, titled "Merge pull request #6 from Synar/Minuit2CompilationFix". The file list shows various directories and files, including `FitResults`, `bin`, `cards`, `cli`, `data`, `doc`, `inc/NangaParbat`, `rawdata`, `resources`, `run`, and `src`. The right sidebar contains an "About" section describing the framework as a fitting framework for the determination of the non-perturbative component of TMD distributions, along with a "Releases" section showing version `v1.4.0` as the latest release on 13 Dec 2019.

File	Description	Last Update
FitResults	deleting unnecessary Report	4 months ago
bin	reorganising the code using cmake	3 years ago
cards	Working on SIDIS	5 months ago
cli	little update	2 months ago
data	NewFolder E615xF and switch low high	2 months ago
doc	code to check PV19grids	6 months ago
inc/NangaParbat	Makes compilation compatible with more architectures by che...	6 days ago
rawdata	Aggiunti dati sezione d'urto in funzione dixF per pione	2 months ago
resources	Add files via upload	9 months ago
run	removing ATLAS Low Mass	2 months ago
src	Merge branch 'MapCollaboration:master' into Minuit2Compilat...	6 days ago





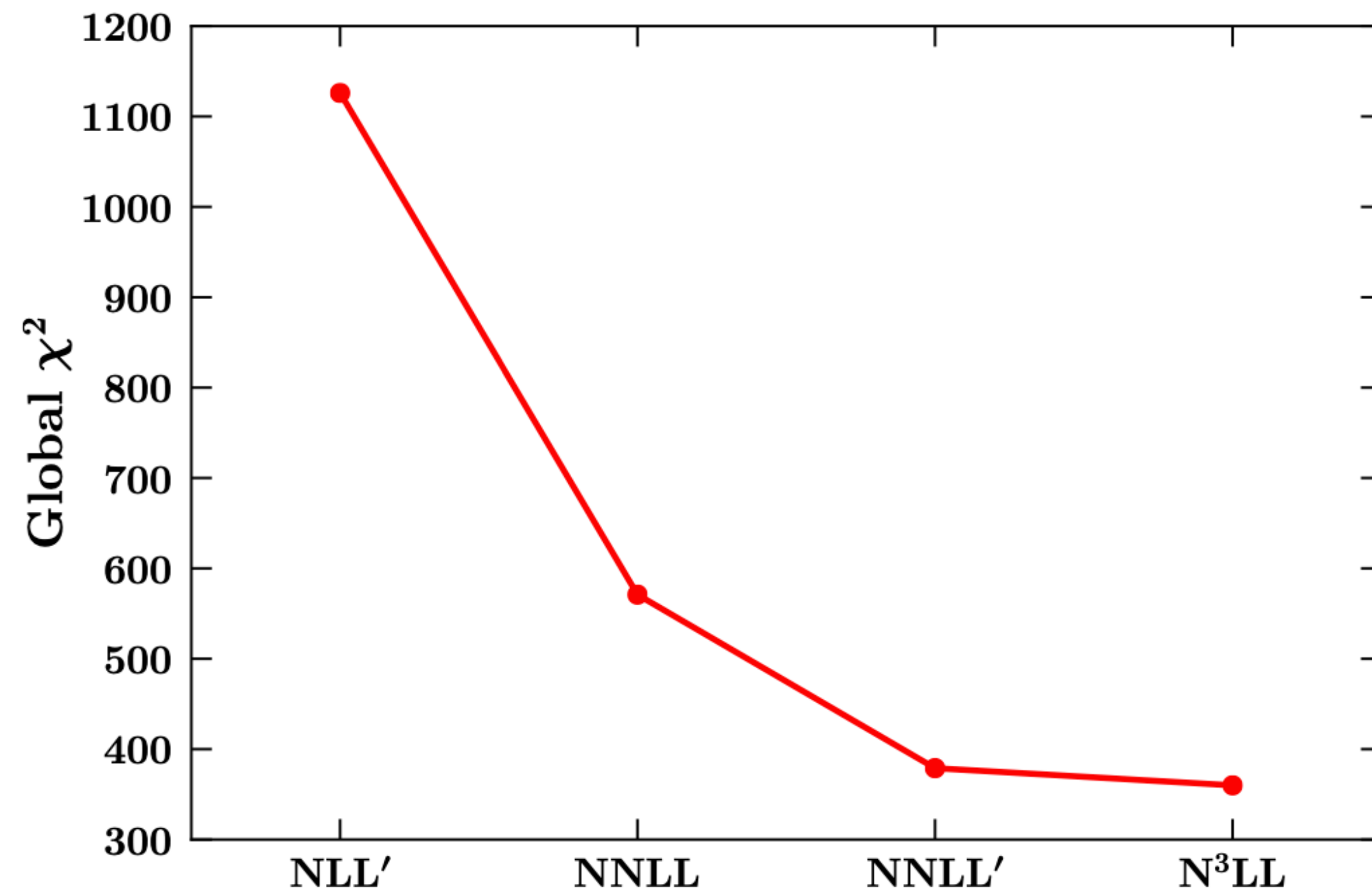
# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

Accuracy of the simulation: Next to Next to Next to Leading Log (N3LL)

resummation of  $\log^{2n-3} \frac{Q^2}{q_T^2}$

Results:



The higher the accuracy is, the better the fit is performed

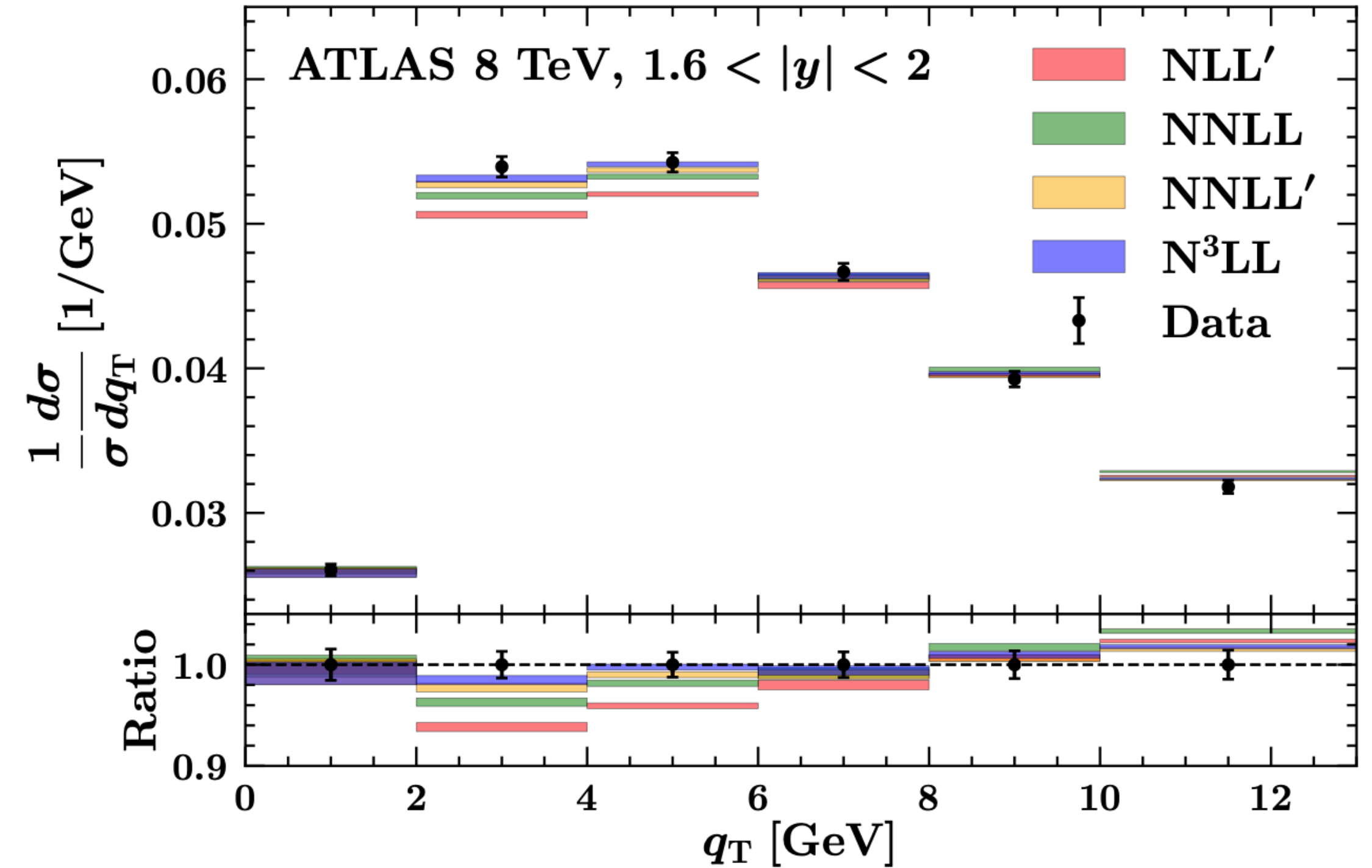
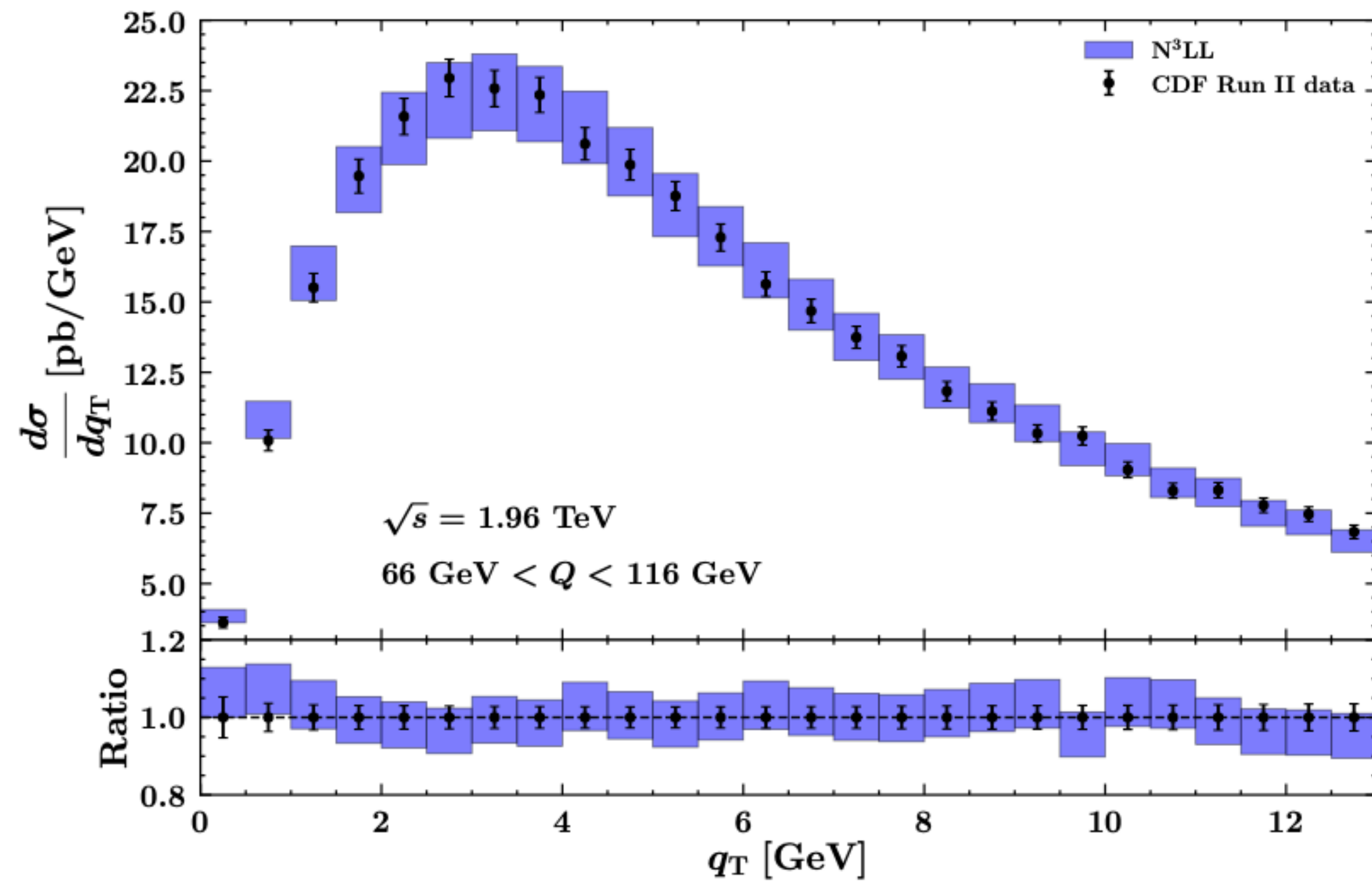
$$\chi_R^2 = 1.02$$



# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

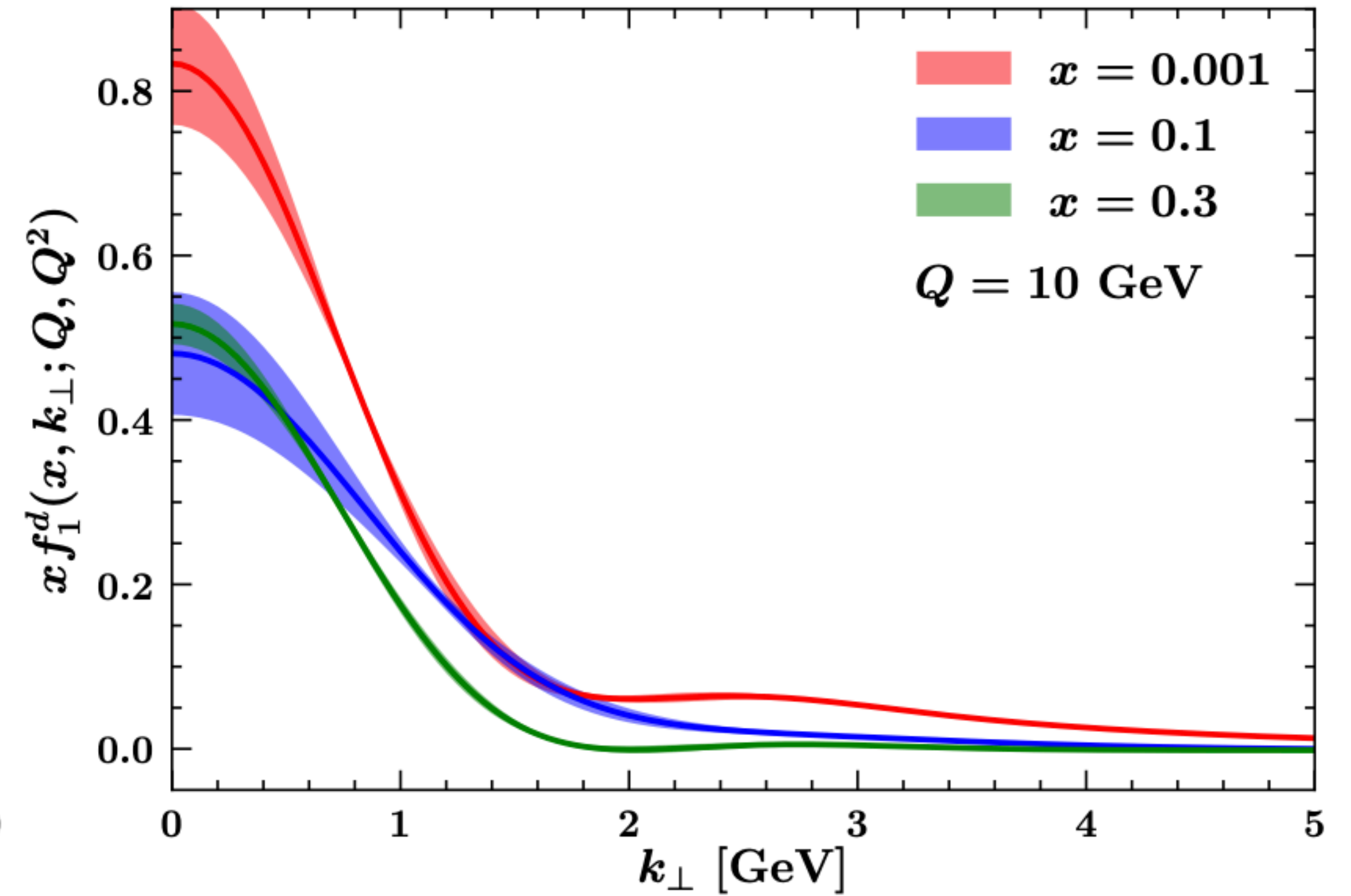
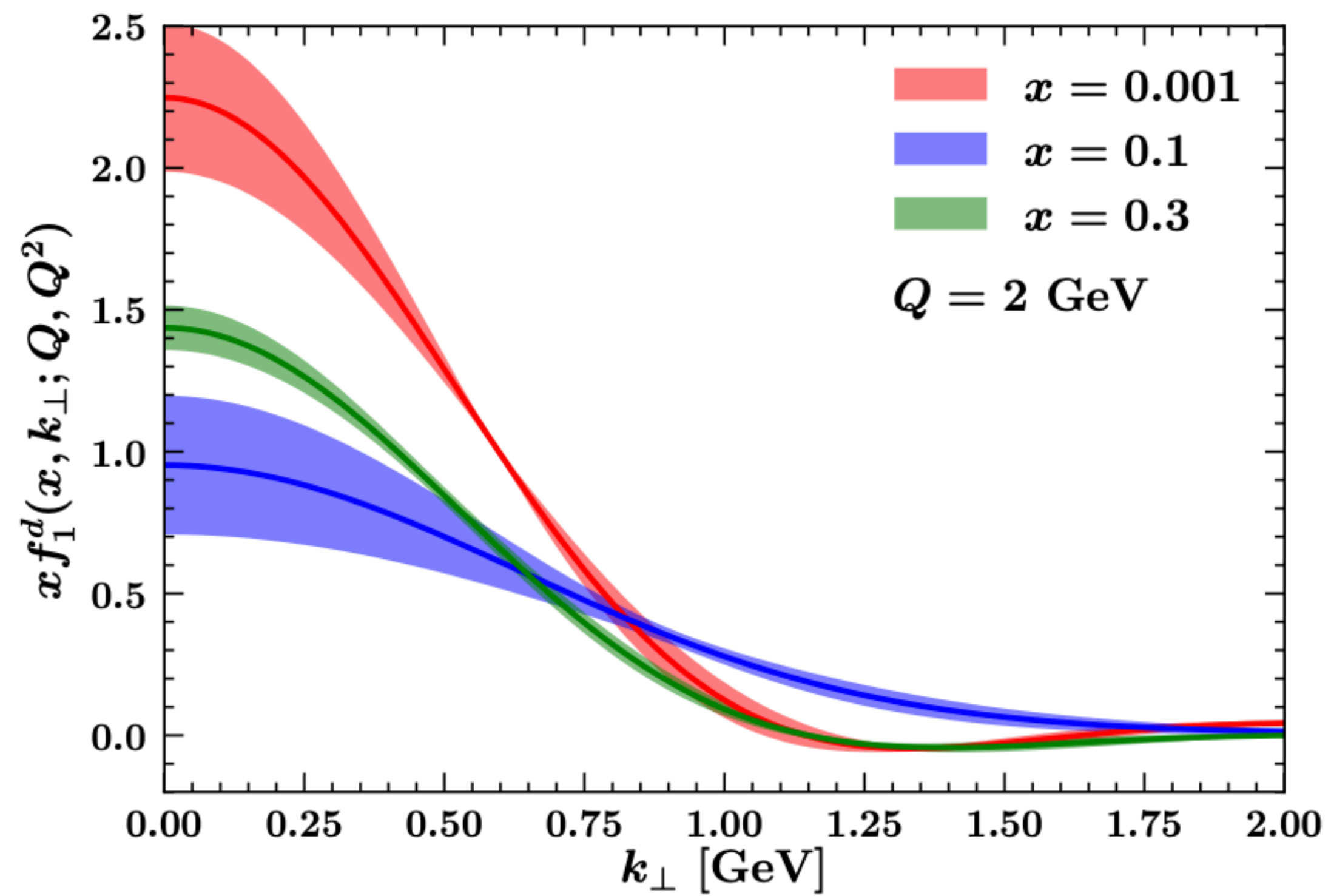
## Results:



# Global Fit: PV19

Bacchetta, Bertone, Bissolotti, et al. [arXiv: 1912.07550]

TMD PDF extraction:



**New global fit?**



# Global Fit: PV202?

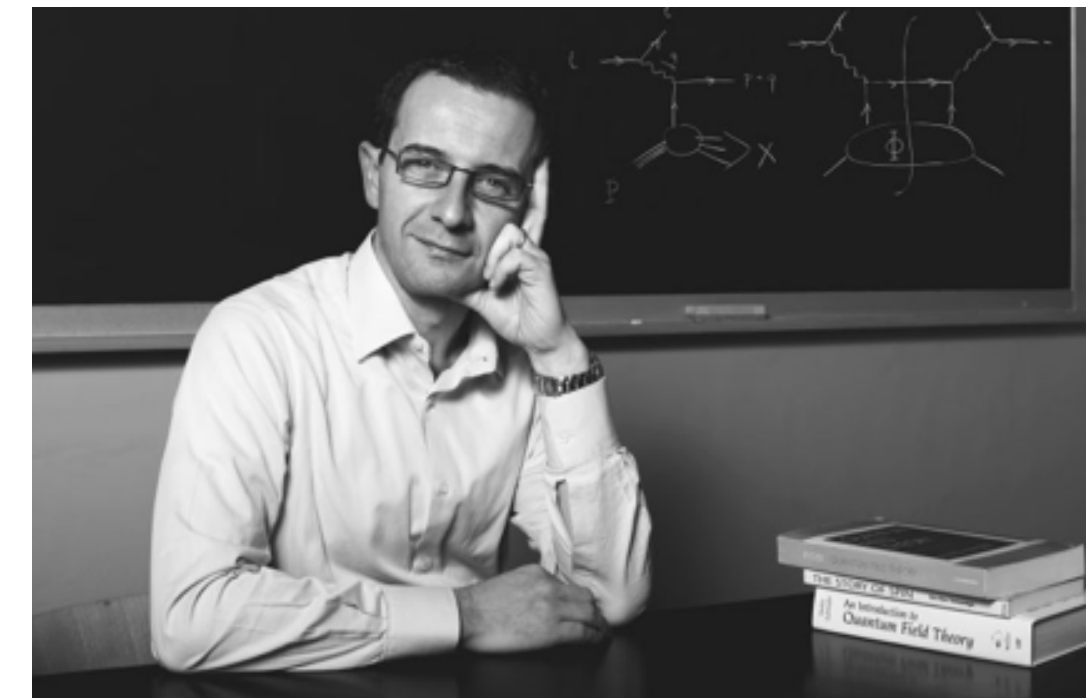
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Work still in progress!



<http://www.hadronicphysics.it/hasqcd/>

**Alessandro Bacchetta**



**Valerio Bertone**



**Chiara Biscolotti**



**Giuseppe Bozzi**



**Fulvio Piacenza**



**Andrea Signori**



**Marco Radici**





**STAY TUNED!**