Cancellation of Infrared Divergences using Coherent States

HUGS 2021

Deepesh Bhamre

University of Mumbai

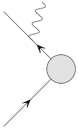
June 18, 2021

Outline

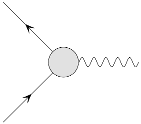
1. Origin of IR divergences
2. Coherent State Formalism
3. Some results in QED and QCD
4. Ongoing work in QCD

Origin of IR divergences

A QED example



$$= \left(\frac{d\sigma}{d\Omega}\right)_0 \left[\frac{\alpha}{\pi} log\left(\frac{-q^2}{m^2}\right) log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)\right] \quad (1)$$



$$= \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi} log\left(\frac{-q^2}{m^2}\right) log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)\right] \quad (2)$$

$\Longrightarrow$ Real emission + Virtual correction = IR finite!

(*But, at the cross-section level*)

KLN Theorem[1]: $P(i \rightarrow j)$ may be divergent in massless theories, but $\sum_{i,j \in D_\epsilon(E_0)} P(i \rightarrow j)$ is finite to all orders in PT

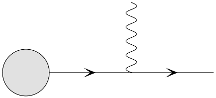
[1] T. Kinoshita *J. Math. Phys. 3, 650 (1962)*; T.D. Lee, M. Nauenberg *Phys. Rev. 133, 6B* (1964)

Origin of IR divergences

- Incorrect asymptotic states considered!
- Incoming and outgoing (Fock) states are usually considered to evolve according to the free Hamiltonian

$$|s(t)\rangle = e^{-itH_0} |s(0)\rangle \quad (3)$$

as $|t| \to \pm\infty$
- An *incorrect assumption* for theories involving long-range interactions



- What can be done about it?

Coherent State Formalism

Modifying the asymptotic states

- First proposed by Chung, Kulish-Faddeev[2]- New space of asymptotic states between which an IR-finite S-matrix can be defined
- Rectified assumption:

$$|s(t)\rangle = e^{-itH_A}|s(0)\rangle \tag{4}$$

as $|t| \to \pm\infty$

$$H_A = H_0 + H_s(\Delta) \tag{5}$$

$\to H_s(\Delta) \equiv H_\Delta$ takes into account all the long-range interactions (by taking the large-time limit of interaction Hamiltonian) of the theory.
$\to \Delta$ separates the hard and soft (with collinear) regions.

[2] V. Chung *Phys. Rev. 140, 4B* (1965); P.P. Kulish, L.D. Faddeev *Theor. Math. Phys. 4, 745 (1970)*; C. A. Nelson, Nucl. Phys. B 181, 141 (1981)

Evolution operator and coherent state

- Evolution operator in interaction picture:

$$U(t, t_0) = Texp\left(-i \int_{t_0}^{t} H_I(t')\, dt' \right) \tag{6}$$

- Asymptotic evolution operator ($|t| \to \pm\infty$):

$$U_\Delta(0, \mp\infty) \equiv \Omega_{A\pm} = Texp\left(-i \int_{\mp\infty}^{0} H_\Delta(t)\, dt \right) \tag{7}$$

- Define coherent state

$$|s : coh\rangle = \Omega_{A\pm} |s\rangle \tag{8}$$

A scheme for IR-finte S-matrix

- *Claim:* S-matrix elements defined between coherent states (instead of Fock states) i.e. $\langle f : coh | S | i : coh \rangle$ is IR-finite
  (At the amplitude level!)
- Coherent state expansion

$$|s : coh\rangle = |s\rangle + c\mathscr{O}\,|s\rangle + c^2\mathscr{O}_{inst}\,|s\rangle + c^2 T[\mathscr{O}(x_1^+)\mathscr{O}(x_2^+)]\,|s\rangle + \ldots \quad (9)$$

$\rightarrow \mathscr{O}$ contains the same operator-valued objects as H_I

- Perturbation expansion in Time-Ordered PT

$$\begin{aligned}\langle f|T|i\rangle =& c\,\langle f|H_I|i\rangle + c^2\,\langle f|H_I\frac{1}{p^- - H_0}H_I|i\rangle + c^2\,\langle f|H_{I_{inst}}|i\rangle \\ &+ c^3\,\langle f|H_I\frac{1}{p^- - H_0}H_I\frac{1}{p^- - H_0}H_I|i\rangle \\ &+ c^3\,\langle f|H_{I_{inst}}\frac{1}{p^- - H_0}H_I|i\rangle + c^3\,\langle f|H_I\frac{1}{p^- - H_0}H_{I_{inst}}|i\rangle + \ldots\end{aligned} \quad (10)$$

$$
\begin{aligned}
\langle f:coh|T|i:coh\rangle =& c\,\langle f|H_I|i\rangle + c^2\,\langle f|H_I\frac{1}{p^- - H_0}H_I|i\rangle + c^2\,\langle f|H_{I_{inst}}|i\rangle \\
&+ c^3\,\langle f|H_I\frac{1}{p^- - H_0}H_I\frac{1}{p^- - H_0}H_I|i\rangle \\
&+ c^3\,\langle f|H_{I_{inst}}\frac{1}{p^- - H_0}H_I|i\rangle + c^3\,\langle f|H_I\frac{1}{p^- - H_0}H_{I_{inst}}|i\rangle \\
&+ \ldots \\
&+ c^2\,\langle f|H_I|\mathscr{O}i\rangle + c^2\,\left\langle f\mathscr{O}^\dagger\middle|H_I\middle|i\right\rangle + c^3\,\left\langle f\mathscr{O}^\dagger\middle|H_I\middle|\mathscr{O}i\right\rangle \\
&+ c^3\,\langle f|H_I\frac{1}{p^- - H_0}H_I|\mathscr{O}i\rangle + c^3\,\left\langle f\mathscr{O}^\dagger\middle|H_I\frac{1}{p^- - H_0}H_I\middle|i\right\rangle \\
&+ c^3\,\langle f|H_{I_{inst}}|\mathscr{O}i\rangle + c^3\,\left\langle f\mathscr{O}^\dagger\middle|H_{I_{inst}}\middle|i\right\rangle \\
&+ c^3\,\langle f|H_I|\mathscr{O}_{inst}i\rangle + c^3\,\left\langle f\mathscr{O}^\dagger{}_{inst}\middle|H_I\middle|i\right\rangle + \ldots
\end{aligned}
\tag{11}
$$

→ Cancellation of IR divergences in red terms with those in blue terms order by order in PT

Some results in QED and QCD

Using the Light-Front formulation

- QED vertex correction to one loop[3]
- QCD lowest order correction to qqg vertex[4]
- QED fermion self-energy to all orders[5]

Using regular co-ordinates

- $e^+e^- \to 2$ *jets* at next-to-leading order in α_s[6]

[3] A. Misra *Phys. Rev. D 50, 4088* (1994)
[4] A. Misra *Phys. Rev. D 62, 125017* (2000)
[5] J.D. More, A. Misra *Phys. Rev. D 89, 105021* (2014)
[6] D.A. Forde, A. Signer *Nuc. Phy. B 684, 125* (2004)

Ongoing work

Interaction Hamiltonian in LFQCD

In Light-Front QCD, the interaction Hamiltonian H_I is made up of the following vertices:

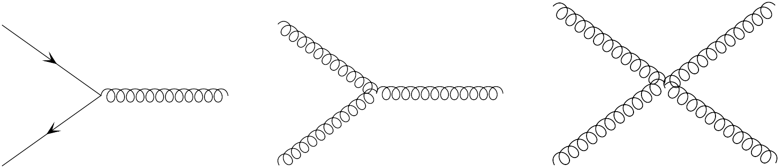


Figure: Regular vertices in LFQCD

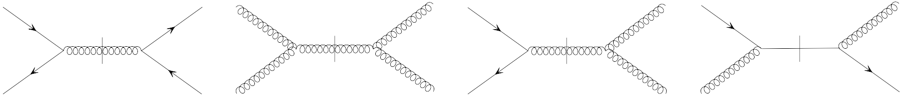


Figure: Instantaneous vertices in LFQCD

$e^+e^- \to q\bar{q}(g)$

- We consider lowest order in em coupling and higher orders in strong coupling for the process $e^+e^- \to q\bar{q}(g)$

$$\begin{aligned} &\left\langle q_{q1}\bar{q}_{q2}(g_{q3}) : coh\middle| T \middle| e^+_{p1} e^-_{p2} \right\rangle \Big|_{g^n, e^2} \\ &= \cancel{\left\langle q_{q1}\bar{q}_{q2}(g_{q3}) : coh\middle| V_{em} \middle| e^+_{p1} e^-_{p2} \right\rangle} \Big|_{g^n, e} \\ &+ \left\langle q_{q1}\bar{q}_{q2}(g_{q3}) : coh\middle| V_{em} \frac{1}{p_1^- + p_2^- - H^0_{em}} V_{em} \middle| e^+_{p1} e^-_{p2} \right\rangle \Big|_{g^n, e^2} \end{aligned} \tag{12}$$

- Insert complete sets of states - Only $|\gamma\rangle\langle\gamma|$ contribute at $\mathcal{O}(e^2)$
- Calculate $\langle q_{q1}\bar{q}_{q2}(g_{q3}) : coh| = \langle q_{q1}\bar{q}_{q2}(g_{q3})|\, \Omega^\dagger_{A-}$ and use in Eq.(12) to get IR-finite $e^+e^- \to q\bar{q}(g)$ amplitude.