Pion parton distribution functions in the light-front formalism

Simone Venturini
Supervisor: Barbara Pasquini

Outline

1. Light-front formalism

2. Construction of hadron states

3. Pion parton distribution functions

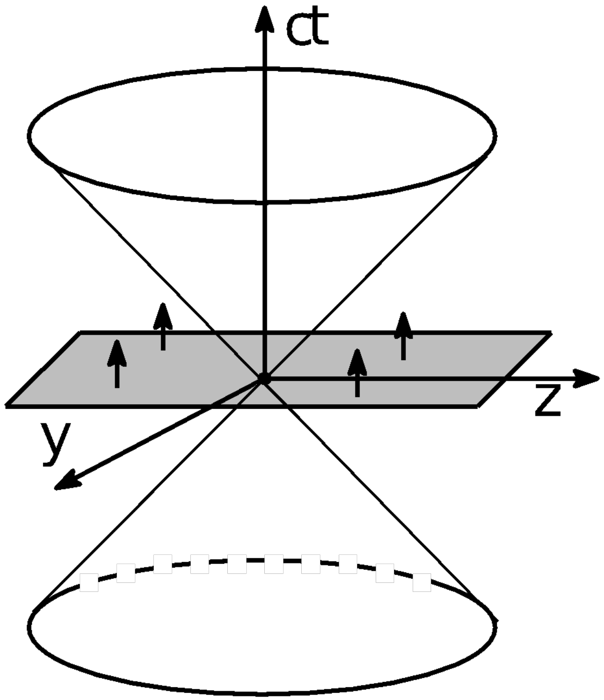
1. Light-front formalism

Canonical quantization

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x^\mu = (x^0, x^1, x^2, x^3)$$

Instant form



1. Light-front formalism

Change of spacetime parametrization

$$x^{\mu} = \left(x^0, x^1, x^2, x^3\right) \longrightarrow x'^{\mu}(x^{\mu}) = \left(x'^0, x'^1, x'^2, x'^3\right)(x^{\mu})$$

$$g'_{\mu\nu} = \left(\frac{\partial x^{\rho}}{\partial x'^{\mu}}\right) g_{\rho\sigma} \left(\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\right)$$

Light-front form

$$x'^{\mu}(x^{\mu}) = \left(x^+, x^-, \vec{x}_{\perp}\right)$$

$$x^{\pm} = \frac{1}{\sqrt{2}}\left(x^0 \pm x^3\right)$$

$$\vec{x}_{\perp} = \left(x^1, x^2\right)$$

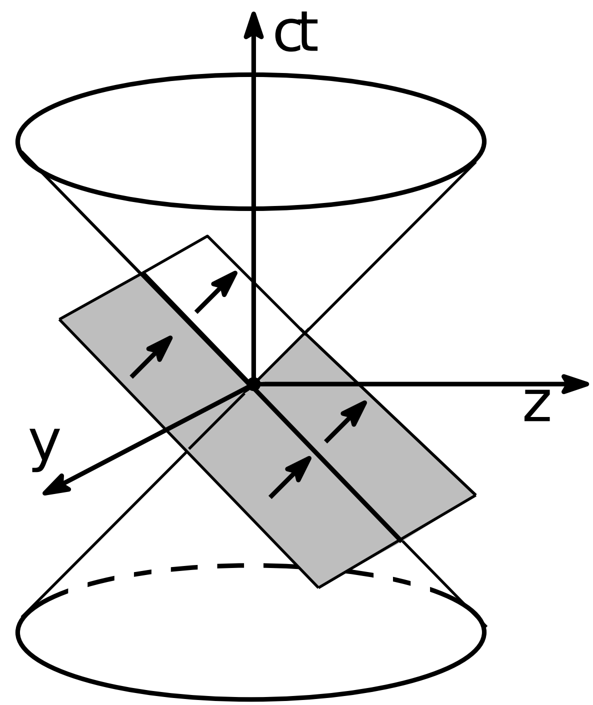
1. Light-front formalism

Light-front quantization

$$g_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x^\mu = (x^+, x^-, \vec{x}_\perp)$$

Light-front form



1. Light-front formalism

Canonical quantization

Bosons

$$[\phi_r(x_1), \pi_s(x_2)] = i\hbar\delta_{rs}\delta\left(\vec{x}_1 - \vec{x}_2\right)$$

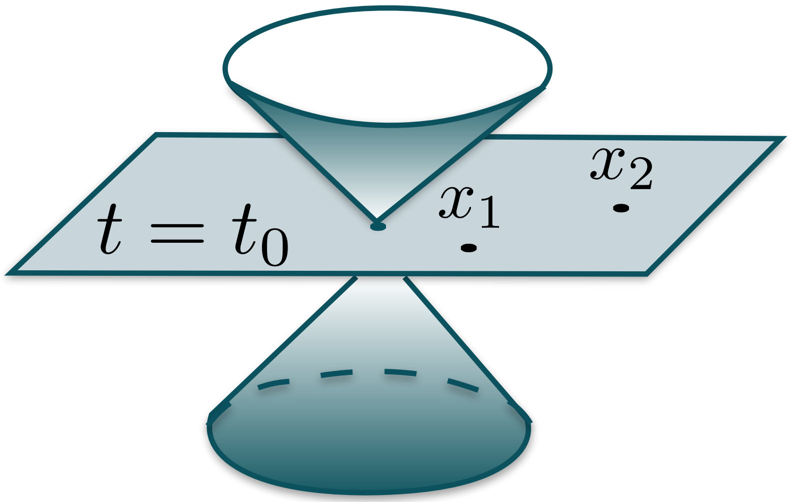
$$[\phi, \phi] = [\pi, \pi] = 0$$

Fermions

$$\{\phi_r(x_1), \pi_s(x_2)\} = i\hbar\delta_{rs}\delta\left(\vec{x}_1 - \vec{x}_2\right)$$

$$\{\phi, \phi\} = \{\pi, \pi\} = 0$$

$$t_1 = t_2 = t_0$$



1. Light-front formalism

Light-front quantization

$$x_1^+ = x_2^+ = 0$$

Bosons

$$[\phi_r(x_1), \pi_s(x_2)] = i\hbar\delta_{rs}\delta\left(\vec{x}_1 - \vec{x}_2\right)$$

$$[\phi, \phi] = [\pi, \pi] = 0$$

Fermions

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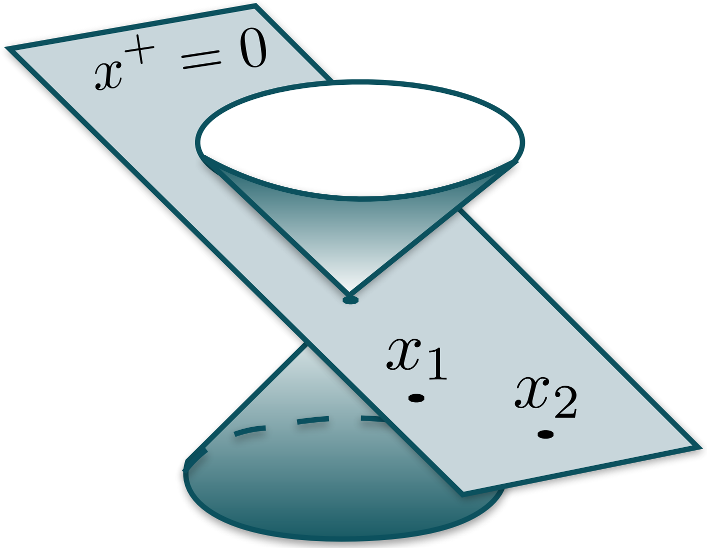
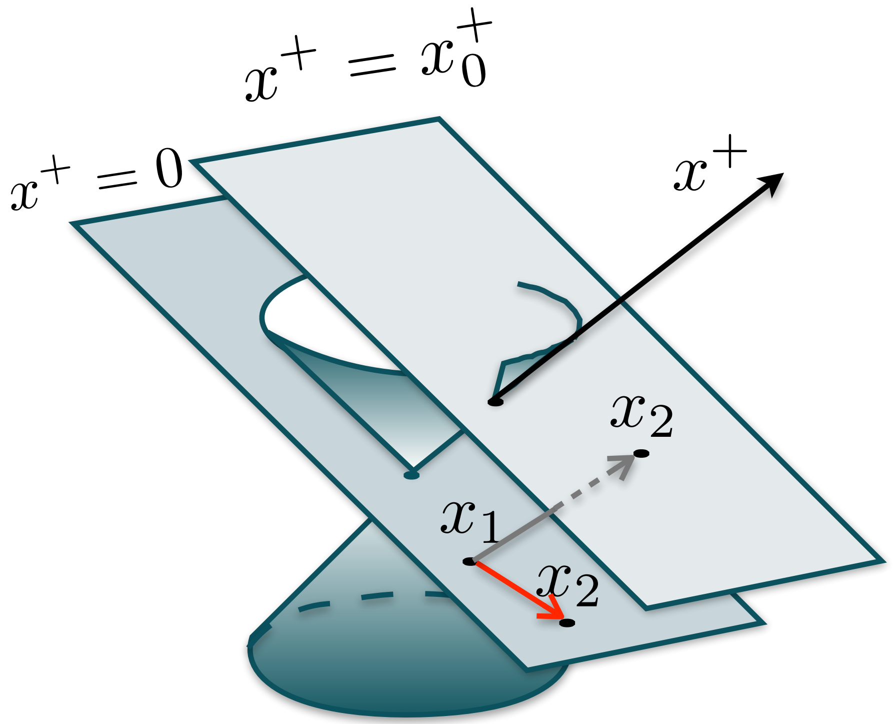


1. Light-front formalism

Types of Operators



Kinematic operators

$$P^+, \quad J_z, \quad \vec{B}_\perp, \quad \vec{P}_\perp$$

Dynamic operators

$$P^-$$

2. Construction of hadron states

Hadron states

$$\mathscr{H}\,|\psi\rangle = \frac{M^2 + \vec{P}_\perp^2}{2P^+}\,|\psi\rangle$$

$$\mathscr{H} \longleftrightarrow P^-$$

$$|\psi\rangle \quad \begin{cases} m \\ \tilde{p} = (p^+, \vec{p}_\perp) \\ j \\ \Lambda \end{cases} \qquad |\psi\rangle \equiv |\tilde{p}, \Lambda\rangle$$

2. Construction of hadron states

Hadron states

It is possible to construct a hadron state in terms of its constituents using an expansion in Fock states:

$$|\psi\rangle \equiv |\tilde{p}, \Lambda\rangle = \sum_{n} \sum_{c_i} \sum_{\mu_i} \int [Du]_n \psi_n^{\Lambda}(r) \, |n, w_1^{c_1}, \ldots, w_n^{c_n}\rangle$$

$$|n, w_1^{c_1}, \ldots, w_n^{c_n}\rangle = \underbrace{\prod_{j=1}^{n_q} b^{\dagger}_{q_j^c}(w_j)}_{\text{quarks}} \underbrace{\prod_{l=1}^{n_{\bar{q}}} d^{\dagger}_{\bar{q}_l^{c'}}(w_l)}_{\text{antiquarks}} \underbrace{\prod_{m=1}^{n_{gl}} a^{\dagger}_{a}(w_m)}_{\text{gluons}} |0\rangle$$

2. Construction of hadron states

Hadron states

$$|\psi\rangle \equiv |\tilde{p}, \Lambda\rangle = \sum_{n} \sum_{c_i} \sum_{\mu_i} \int [Du]_n \psi_n^{\Lambda}(r) \, |n, w_1^{c_1}, \ldots, w_n^{c_n}\rangle$$

↕

Light-front wave functions

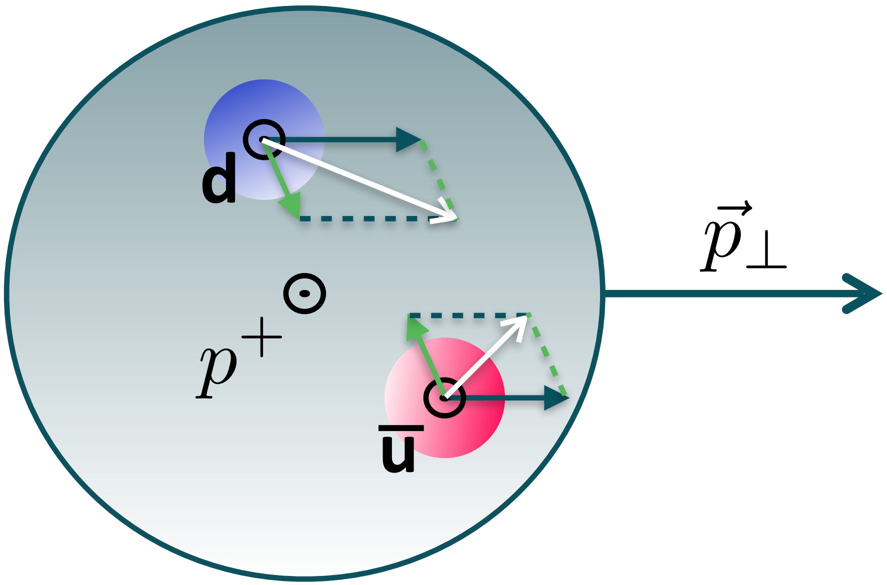
$$\psi_n^{\Lambda}(r) \equiv \langle \psi | n, w_1^{c_1}, \ldots, w_n^{c_n} \rangle$$

Probability amplitude

2. Construction of hadron states

Light-front wave functions' dependence

$$\psi_n^\Lambda(r) \begin{cases} r \equiv \{r_i\}_{i=1,\ldots,n} \\ r_i = \left(u_i, \vec{k}_{\perp i}, \mu_i, c_i\right) \end{cases}$$



π^-

$$\vec{p}_{\perp i} = \vec{k}_{\perp i} + u_i \vec{p}_\perp$$

$$p_i^+ = u_i p^+$$

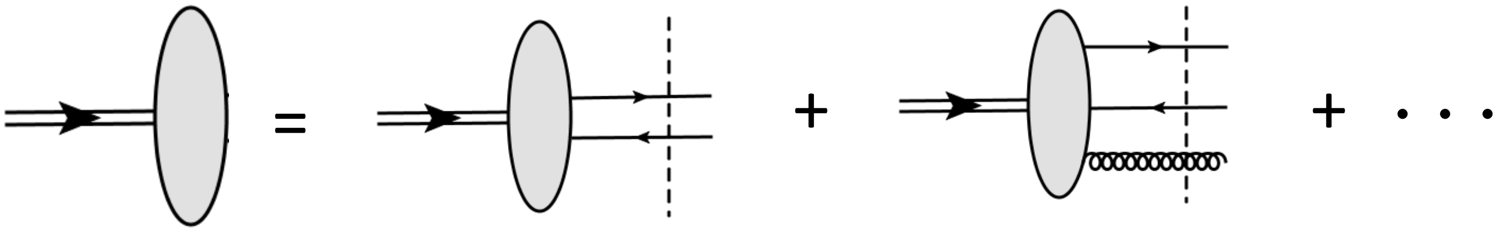
2. Construction of hadron states

Pion state

For theoretical calculations, the infinite expansion in the Fock state has to be stopped to a certain order.

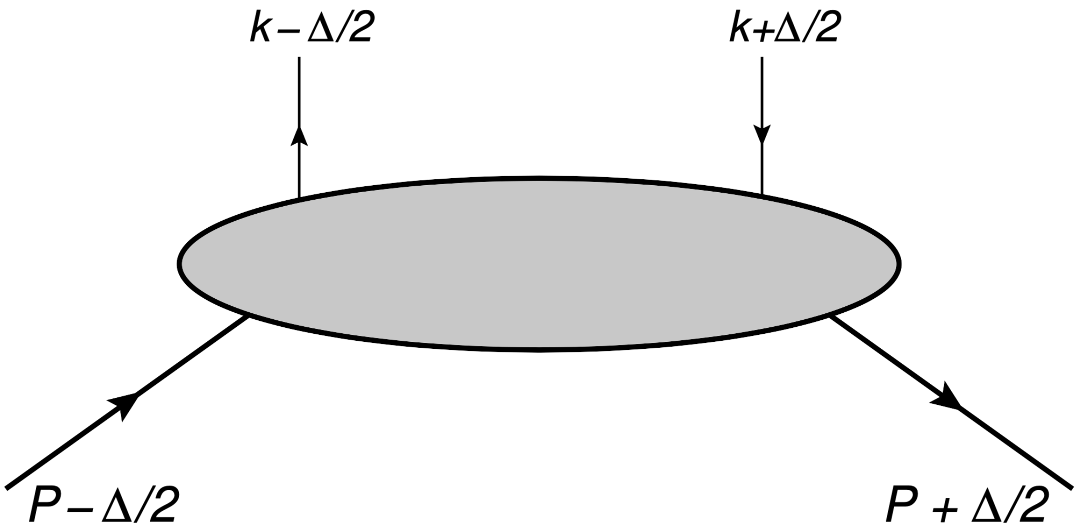
$$|\psi\rangle \equiv |\tilde{p}, \Lambda\rangle = \sum_{n} \sum_{c_i} \sum_{\mu_i} \int [Du]_n \psi_n^{\Lambda}(r) \, |n, w_1^{c_1}, \ldots, w_n^{c_n}\rangle$$

$$\big|\pi(p)\big\rangle \quad = \quad \big|\pi(p)\big\rangle_{q\bar{q}} \quad + \quad \big|\pi(p)\big\rangle_{q\bar{q}g} \quad + \quad \cdots$$



3. Pion parton distribution functions

Generalized parton correlator



$$\left[\Phi_{\Lambda\Lambda'}\right]_{ij}(k,\Delta;P) = \int \frac{d^4z}{(2\pi)^4} e^{ikz} \langle p',\Lambda'|\, \bar{q}_j(0)\mathcal{U}_{(0,z)}q_i(z)\, |p,\Lambda\rangle$$

$$\Phi_{\Lambda\Lambda'}(x,\vec{k}_\perp,\Delta;P) = \int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i\left(k^+z^- - \vec{z}_\perp\cdot\vec{k}_\perp\right)} \langle p',\Lambda'|\, \bar{q}(0)\mathcal{U}_{(0,z)}q(z)\, |p,\Lambda\rangle \Big|_{z^+=0}$$

3. Pion parton distribution functions

Other parton distribution functions