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# Transverse momentum imaging

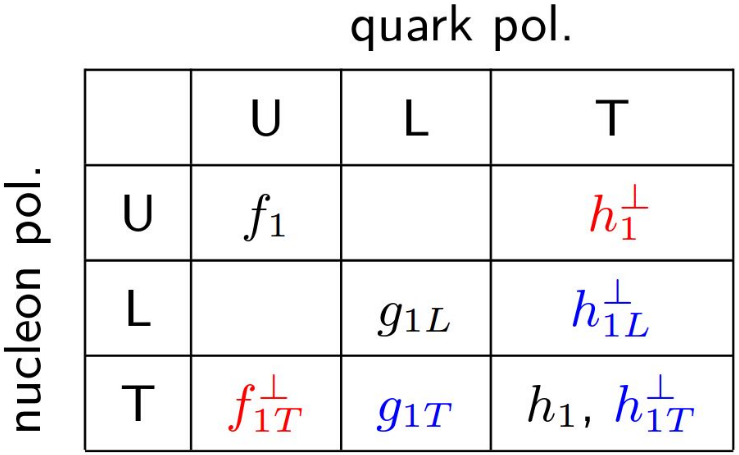
## Lecture 5

Hampton University Graduate School (e-HUGS) 2021

June 11, 2021

# Recap++
# of lectures 3,4

# Quark TMD PDFs (spin ½)



quark pol.

nucleon pol.

|  | U | L | T |
|---|---|---|---|
| U | $f_1$ |  | $h_1^\perp$ |
| L |  | $g_{1L}$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_1, h_{1T}^\perp$ |

At leading twist: 8 TMD PDFs

(similar classification for gluons)

The **symmetries of QCD** play a crucial role in this classification

- **Black**: time-reversal even AND collinear
- **Blue**: time-reversal even
- **Red**: time-reversal odd (*process dependence*)

Quark inside spin ½ hadron

# Quark TMD PDFs (spin ½)

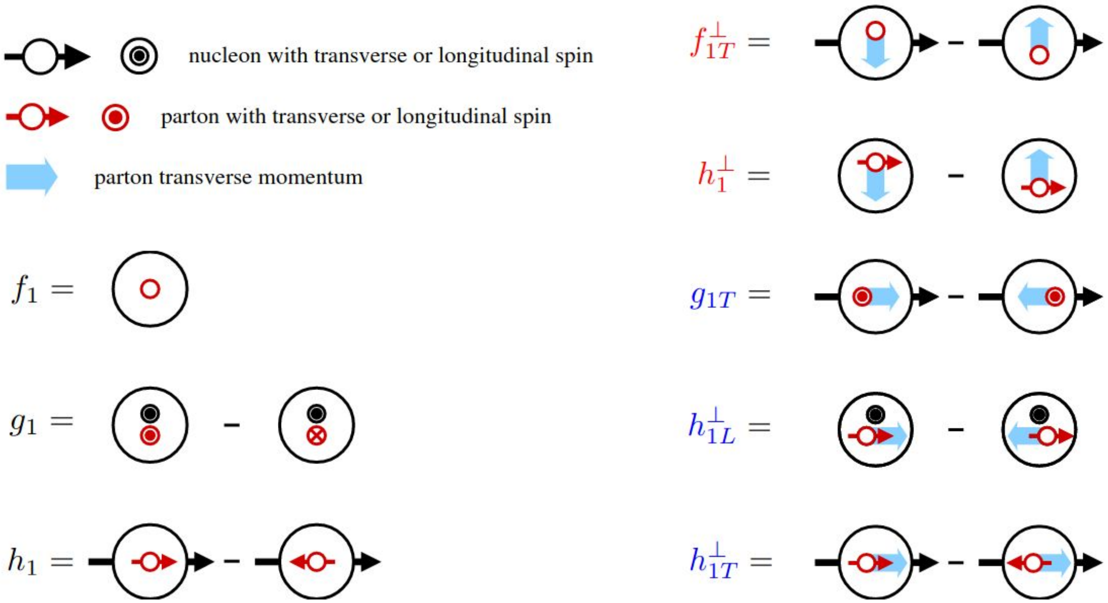
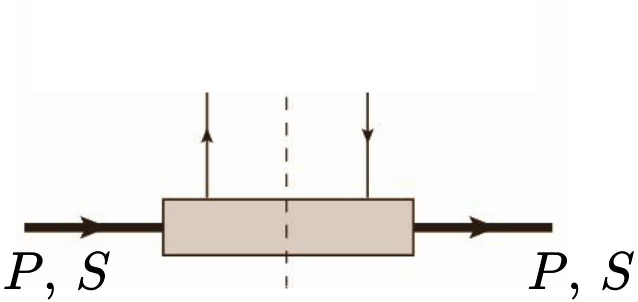


Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

From Bacchetta's “Trento” lecture notes

# Gauge invariant quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS| \boxed{\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)} |PS\rangle$$



**GAUGE INVARIANT!**

$$\mathcal{U}(x) = e^{i\,\alpha^a(x)\,t^a}$$

$$U(0,\xi) \;\rightarrow\; \mathcal{U}(0)\, U(0,\xi)\, \mathcal{U}^\dagger(\xi)$$

The Wilson line “bridges” the non-locality and makes the operator gauge invariant

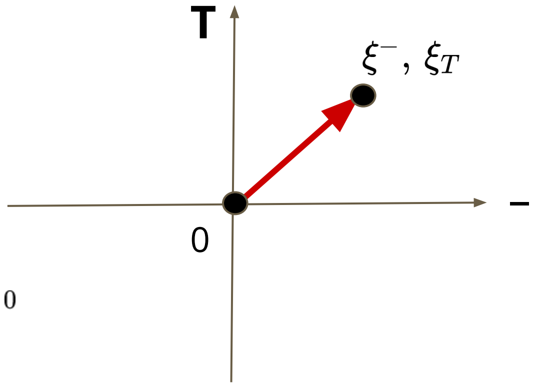
$$\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi) \;\rightarrow\; \overline{\psi}_j(0)\, \mathcal{U}^\dagger(0)\, \mathcal{U}(0)\, U(0,\xi)\, \mathcal{U}^\dagger(\xi)\, \mathcal{U}(\xi)\, \psi_i(\xi) \;=\; \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)$$

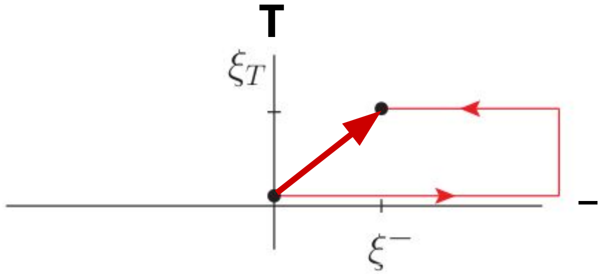
Eventually the correlator and the (TMD) PDFs **depend on the gauge link and its path** in spacetime

# Gauge links for TMD PDFs

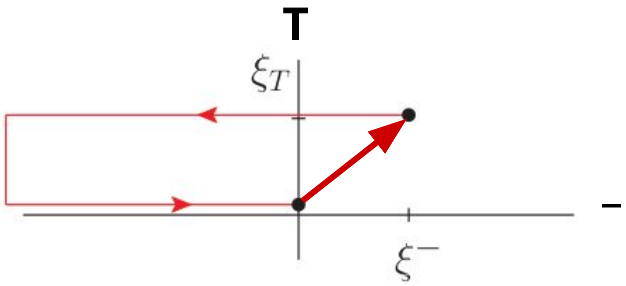
$$\Phi_{ij}^{[U]}(x,\, \mathbf{p}_T, S) \;=\; \int dp^+\, dp^-\, \delta\big(p^+ \;-xP^+\big)\,\Phi^{[U]}(p, P, S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\; e^{i\,p\cdot\xi}\, \langle PS\big|\,\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\,\big|PS\rangle_{\,\xi^+=0}$$





$U^{[+]}$ Future pointing (SIDIS)



$U^{[-]}$ Past pointing (Drell-Yan)

# Process dependence

The interplay between **time reversal** and **gauge symmetry** generates **relations** between the two configurations:

$$f_1^{a\ [+]}(x,k_T^2) = f_1^{a\ [-]}(x,k_T^2)$$

T-even distribution

striking consequence
of the symmetries of QCD

$$f_{1T}^{a\perp\ [+]}(x,k_T^2) = -f_{1T}^{a\perp\ [-]}(x,k_T^2)$$

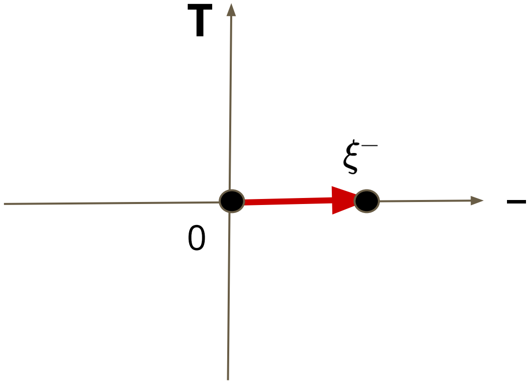
T-odd distribution

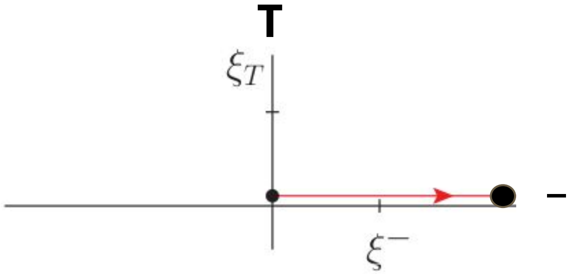
Sign-change relation for the Sivers function : not yet confirmed experimentally

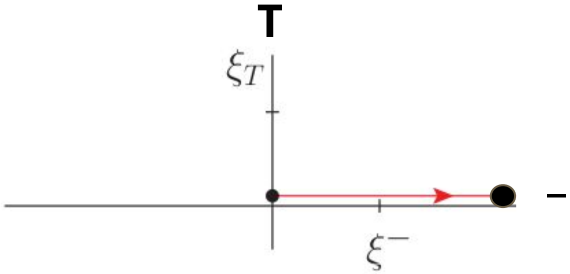
# Gauge links for collinear PDFs (simpler)

$$\Phi_{ij}^{[U]}(x,S) = \int dk^+\, dk^-\, d^2\mathbf{k}_T\, \delta\big(k^+ - xP^+\big)\Phi^{[U]}(k,P,S) =$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{i\,k\,\cdot\xi}\, \langle PS\big|\, \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\, \big|PS\rangle_{\,\xi^+ = \,\xi_T = 0}$$







In the collinear limit the two gauge links reduce to the same object

# Factorized cross section

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\bar{l} + X$$

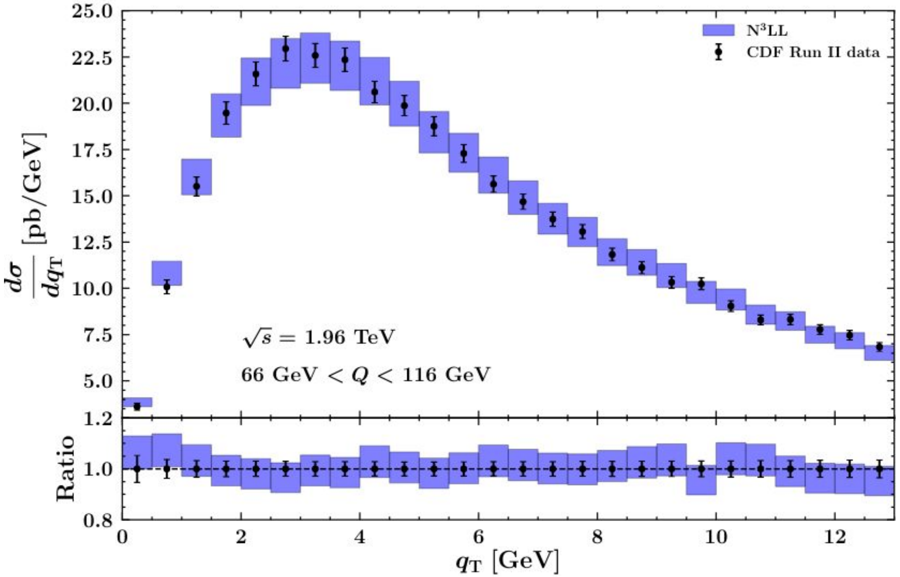
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\left(q_T - k_{T\,a} - k_{T\,b}\right)$$

Renormalized TMDs

[TMD region, $q_T \ll Q$]

$$+\,\mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large $q_T$ and low $Q$ corrections]

Description of data: essential an approach with predictive power



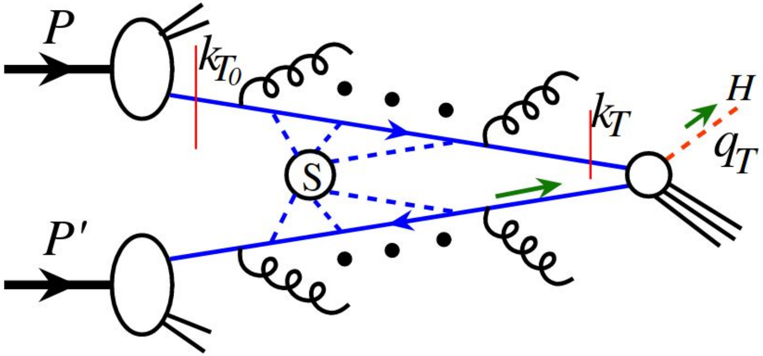
Factorization → renormalization (evolution) of TMDs

# Physical intuition

$pp \longrightarrow \gamma^{\cdot} \,/\, Z \longrightarrow l\,\bar{l} + X$

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\left(q_T - k_{T\,a} - k_{T\,b}\right)$$

- The TMDs reproduce the structure of the **IR poles** in the cross section (same non-perturbative physics)
- The **observed transverse momentum** is accounted for by the transverse momenta of **quarks**
- The quark transverse momentum has **radiative** (perturbative) and **intrinsic** (non-perturbative) components
- Renormalization = **evolution** equations tell us how to distinguish between the two



# Evolved TMD distribution

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

→ TMD distribution at initial scales

x $$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$

→ evolution in μ

x $$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T\mu_0, \alpha_s(\mu_0))}$$

→ evolution in ζ

→ this solution is valid **at low bT**

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Non perturbative components

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

→ TMD distribution at initial scales

$$\text{x} \quad \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$

→ evolution in μ

$$\text{x} \quad \left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T \mu_0, \alpha_s(\mu_0)) \boxed{+ g_K(b_T; \lambda)}}$$

→ evolution in ζ

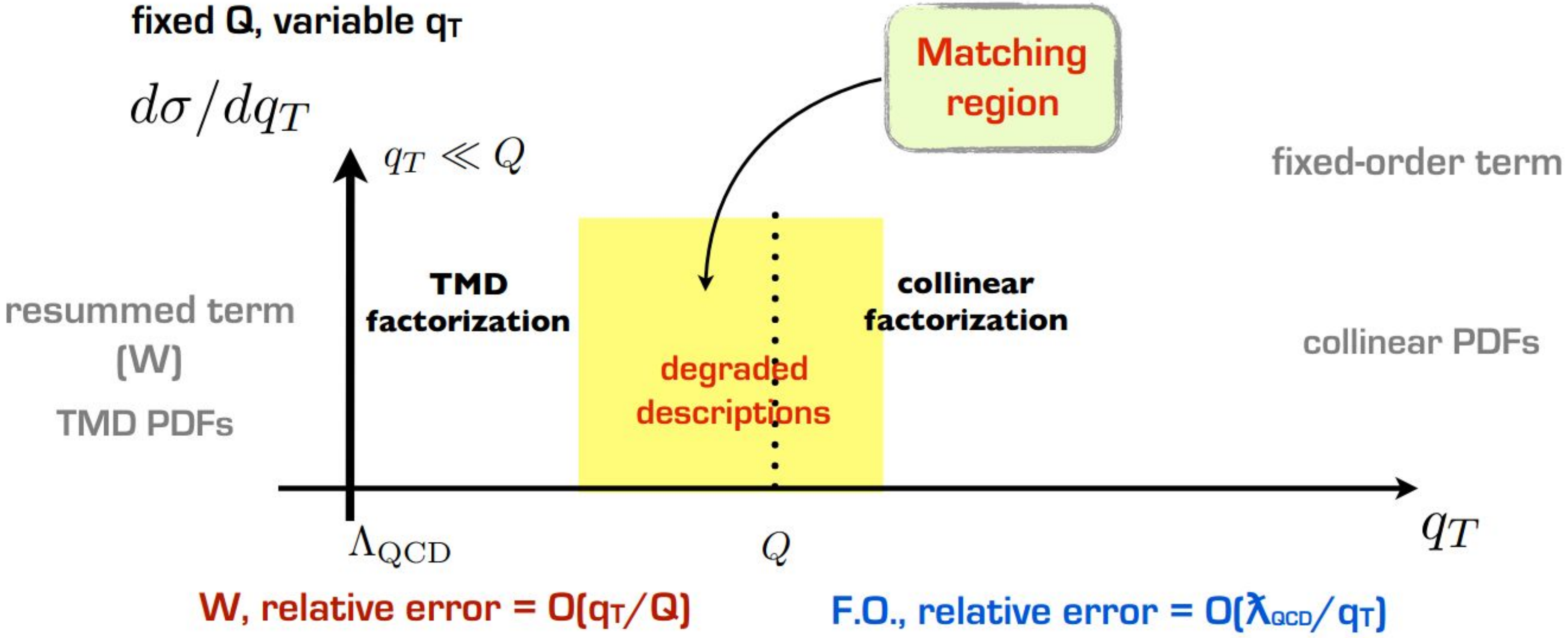
**Non-pert. correction to evolution (large bT)**

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0) \boxed{F_{NP}(b_T; \lambda)}$$

**Intrinsic contribution (large bT)**

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Matching TMD and collinear factorization

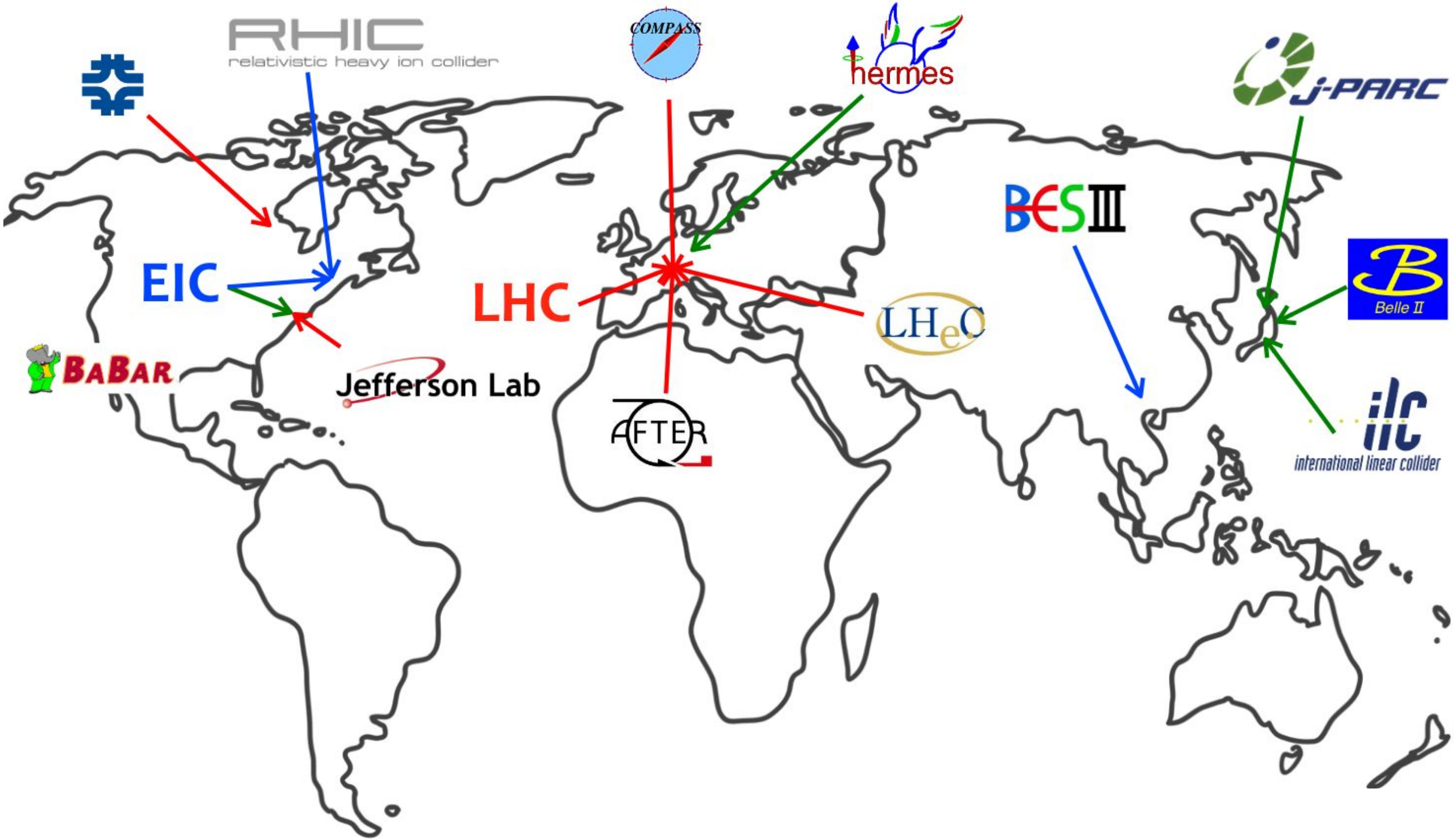


# Plan of these lectures

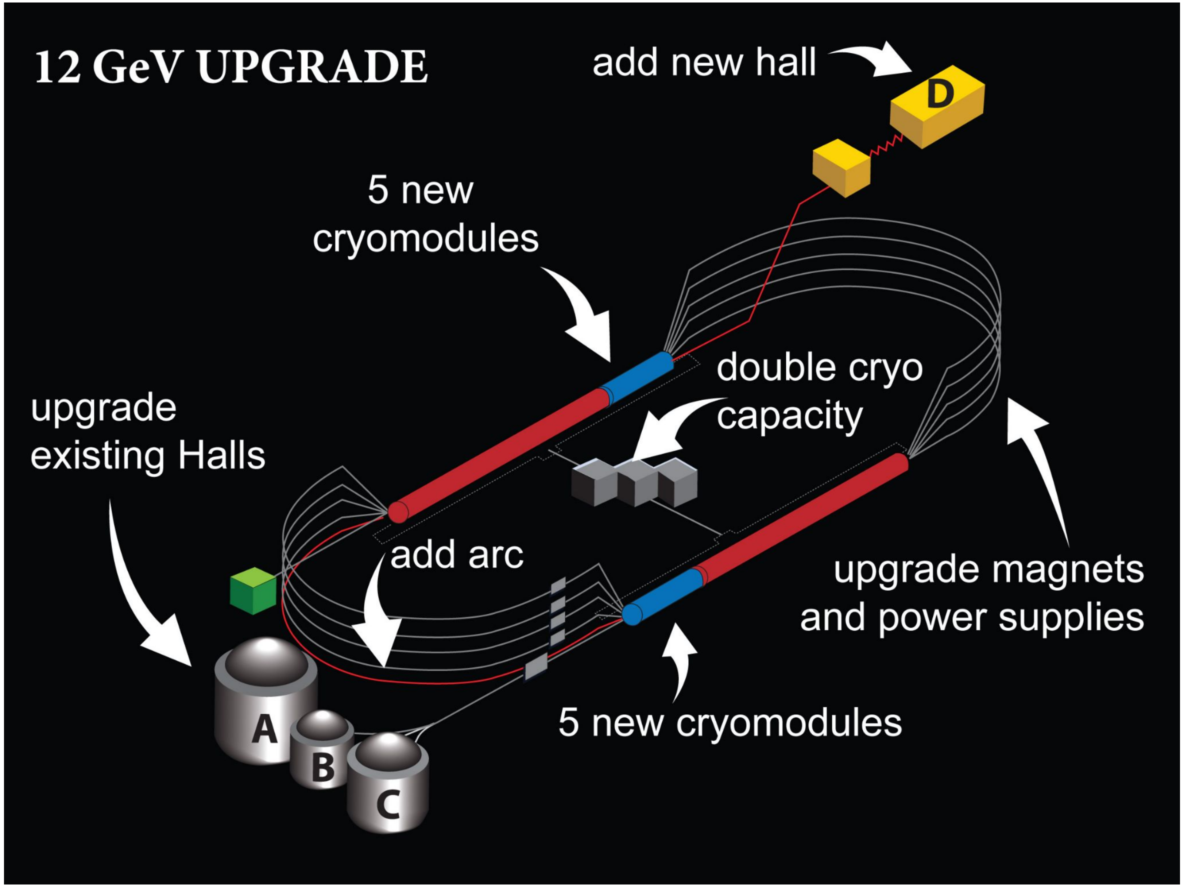
1. **DIS and partons**
2. **From DIS to SIDIS**
3. **Symmetries and universality**
4. **Factorization, evolution, matching**
5. **Phenomenology**

# 5.1 Experiments

(a short selection)


RHIC
relativistic heavy ion collider
COMPASS
hermes
J-PARC
EIC
LHC
BESIII
Belle II
LHeC
BaBar
Jefferson Lab
AFTER
ilc
international linear collider

# CEBAF at Jefferson Lab



CEBAF:

Continuous Electron Beam
Accelerator Facility

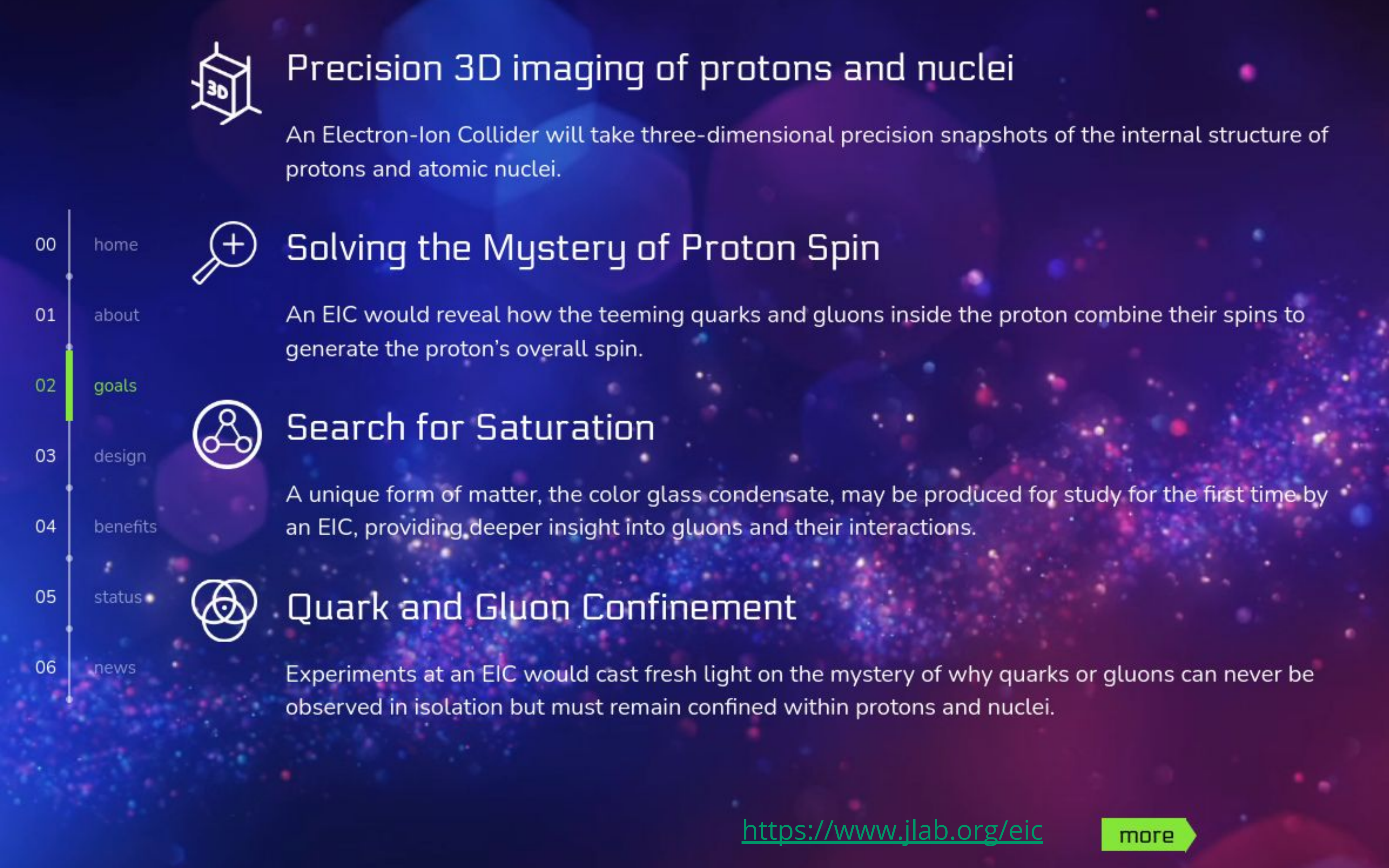
Built in 1984,
recently completed a major upgrade
from 6 GeV to 12 GeV
+ one new hall

- Hall A & C: hadron structure, high luminosity
- Hall B: hadron structure, $4\pi$ coverage
- Hall D: hadron spectroscopy

# The Electron-Ion Collider

The Electron-Ion Collider is a proposed machine for delving deeper than ever before into the building blocks of matter, so that we may better understand the matter within us and its role in the universe around us..

https://www.jlab.org/eic



## Precision 3D imaging of protons and nuclei

An Electron-Ion Collider will take three-dimensional precision snapshots of the internal structure of protons and atomic nuclei.

## Solving the Mystery of Proton Spin

An EIC would reveal how the teeming quarks and gluons inside the proton combine their spins to generate the proton's overall spin.

## Search for Saturation

A unique form of matter, the color glass condensate, may be produced for study for the first time by an EIC, providing deeper insight into gluons and their interactions.

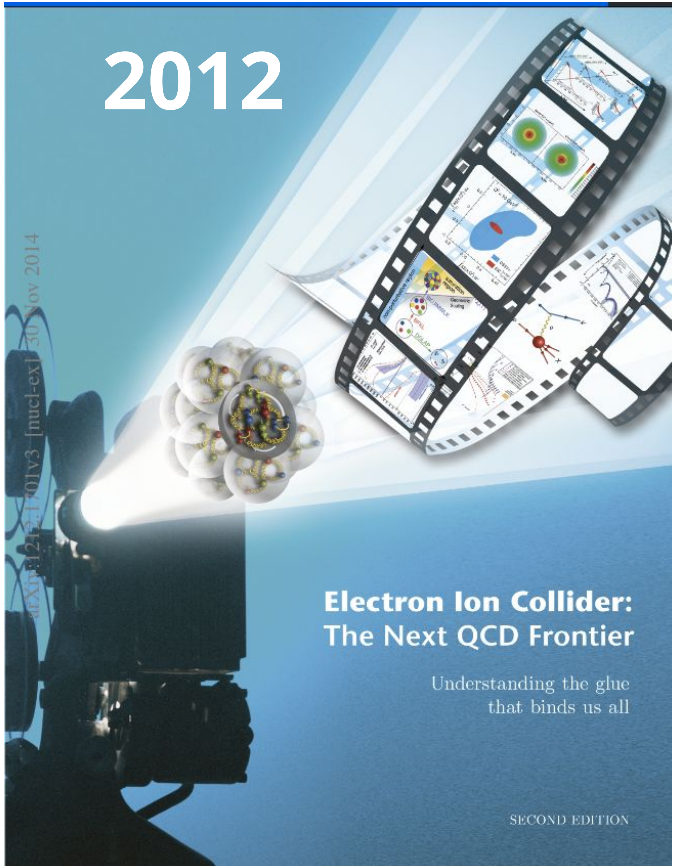
## Quark and Gluon Confinement

Experiments at an EIC would cast fresh light on the mystery of why quarks or gluons can never be observed in isolation but must remain confined within protons and nuclei.

https://www.jlab.org/eic

# The Electron-Ion Collider (EIC)

https://www.bnl.gov/eic/

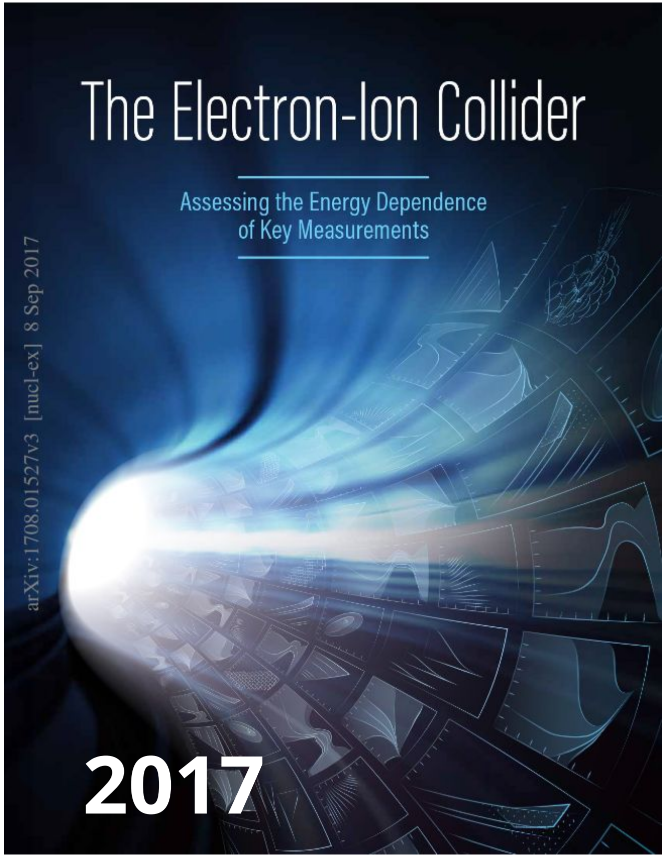


2012

**Electron Ion Collider:**
**The Next QCD Frontier**

Understanding the glue
that binds us all

SECOND EDITION



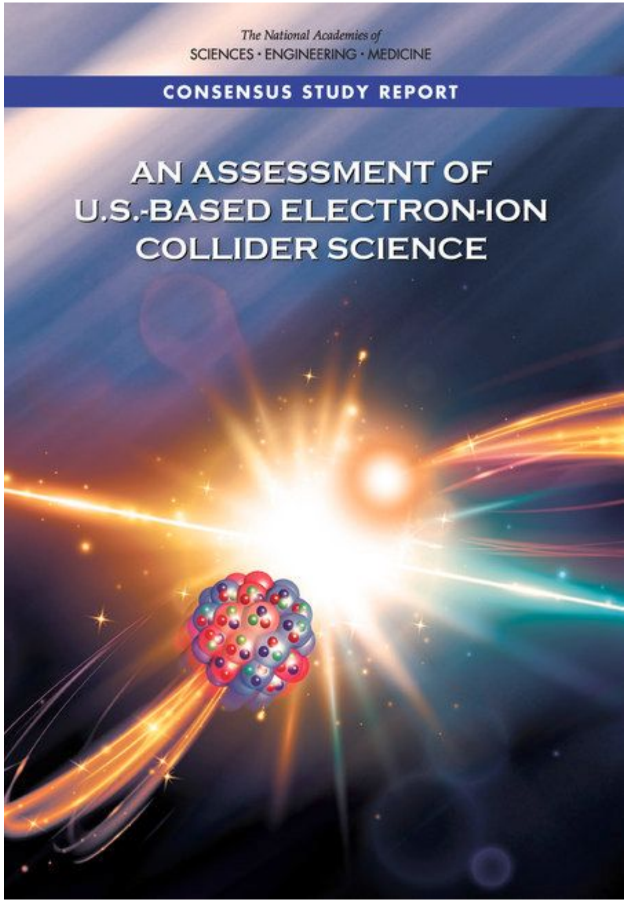
The Electron-Ion Collider

Assessing the Energy Dependence
of Key Measurements

2017

# The Electron-Ion Collider (EIC)

https://www.bnl.gov/eic/



The National Academies of SCIENCES · ENGINEERING · MEDICINE

CONSENSUS STUDY REPORT

AN ASSESSMENT OF U.S.-BASED ELECTRON-ION COLLIDER SCIENCE

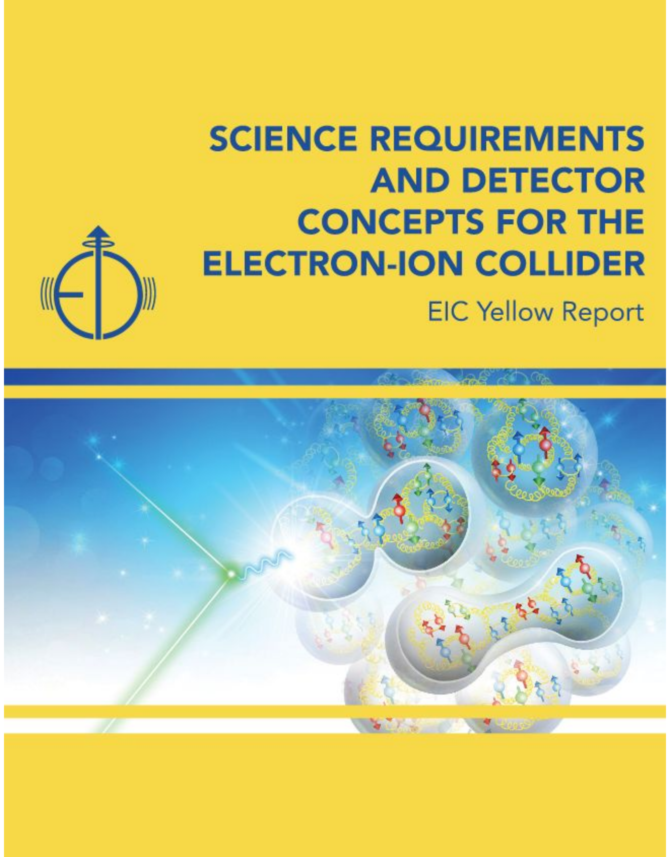
**2018**

**2019:** DoE critical decision 0

**2020:** site selection (BNL)

**>2030 (?):** operations

# The Electron-Ion Collider (EIC)



SCIENCE REQUIREMENTS AND DETECTOR CONCEPTS FOR THE ELECTRON-ION COLLIDER

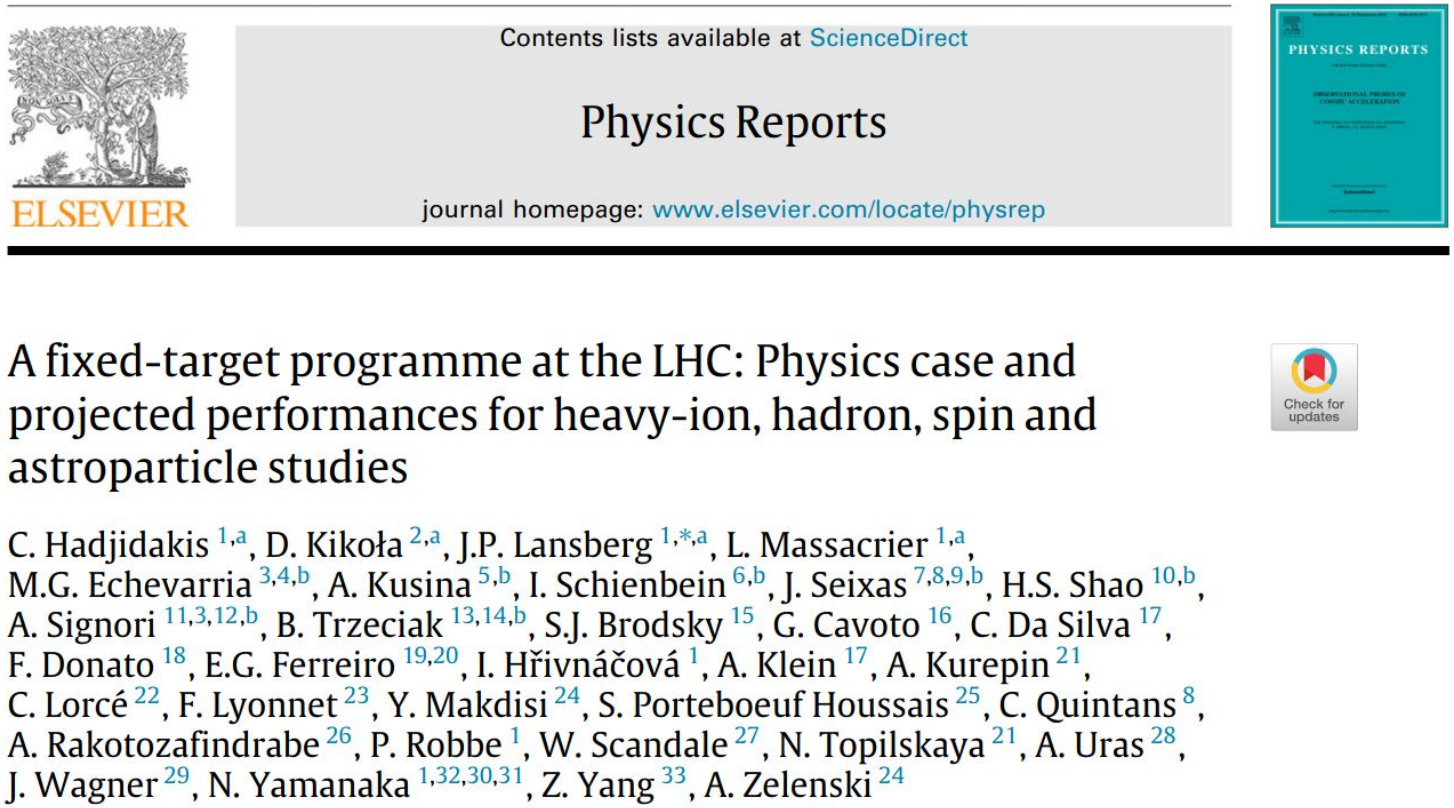
EIC Yellow Report

**2021**

The “EIC Yellow Report”
A community effort to line out the science requirements and detector concepts for the EIC

More details from the EIC Users Group website:
http://eicug.org/

# A fixed-target program at the LHC



Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

## A fixed-target programme at the LHC: Physics case and projected performances for heavy-ion, hadron, spin and astroparticle studies

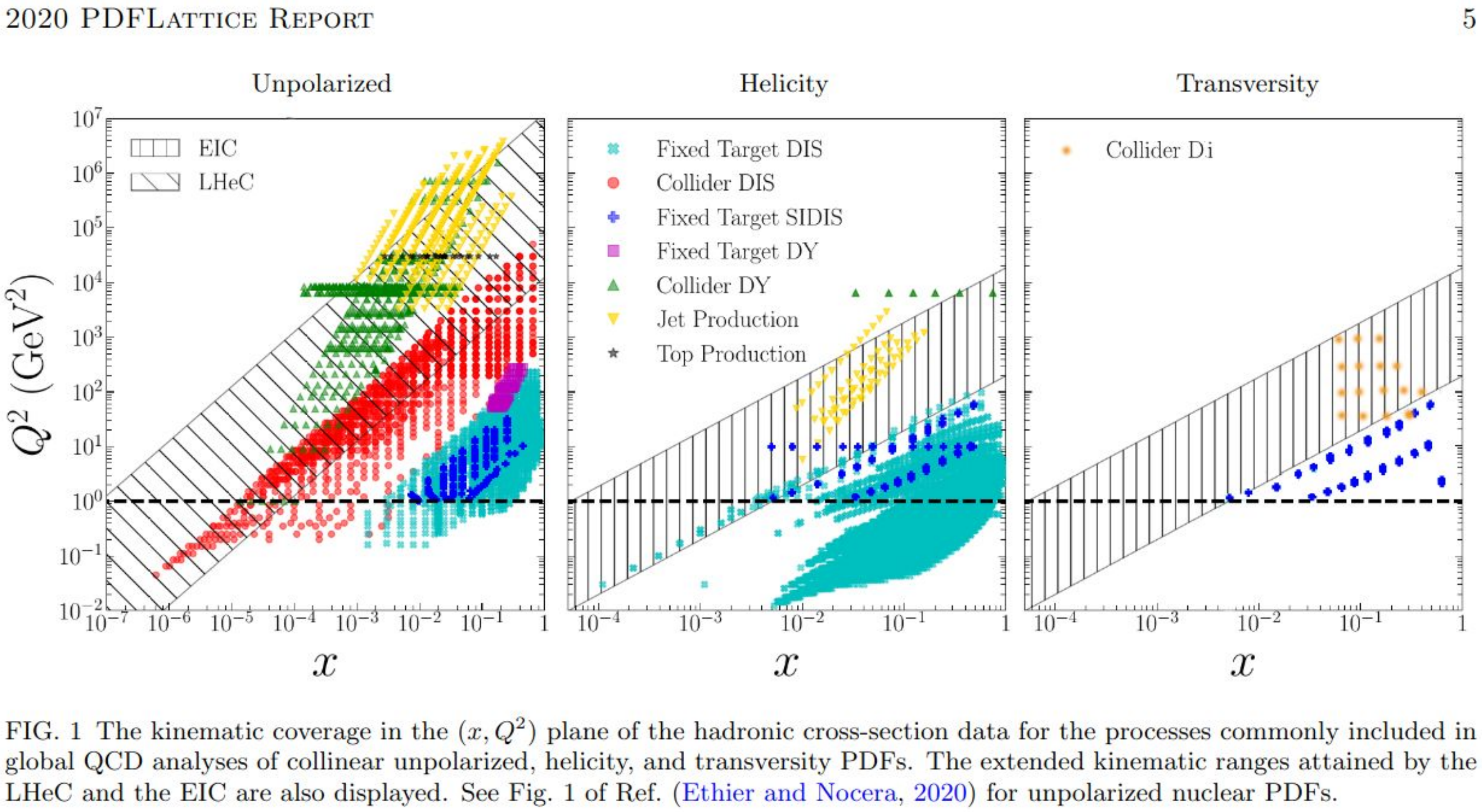
C. Hadjidakis [1,a], D. Kikoła [2,a], J.P. Lansberg [1,*,a], L. Massacrier [1,a], M.G. Echevarria [3,4,b], A. Kusina [5,b], I. Schienbein [6,b], J. Seixas [7,8,9,b], H.S. Shao [10,b], A. Signori [11,3,12,b], B. Trzeciak [13,14,b], S.J. Brodsky [15], G. Cavoto [16], C. Da Silva [17], F. Donato [18], E.G. Ferreiro [19,20], I. Hřivnáčová [1], A. Klein [17], A. Kurepin [21], C. Lorcé [22], F. Lyonnet [23], Y. Makdisi [24], S. Porteboeuf Houssais [25], C. Quintans [8], A. Rakotozafindrabe [26], P. Robbe [1], W. Scandale [27], N. Topilskaya [21], A. Uras [28], J. Wagner [29], N. Yamanaka [1,32,30,31], Z. Yang [33], A. Zelenski [24]

https://doi.org/10.1016/j.physrep.2021.01.002

# 5.2 Collinear PDFs

# PDFs: what do we know

See https://inspirehep.net/literature/1801417



2020 PDFLattice Report 5

FIG. 1 The kinematic coverage in the $(x, Q^2)$ plane of the hadronic cross-section data for the processes commonly included in global QCD analyses of collinear unpolarized, helicity, and transversity PDFs. The extended kinematic ranges attained by the LHeC and the EIC are also displayed. See Fig. 1 of Ref. (Ethier and Nocera, 2020) for unpolarized nuclear PDFs.

# PDFs: what do we know

See https://inspirehep.net/literature/1801417

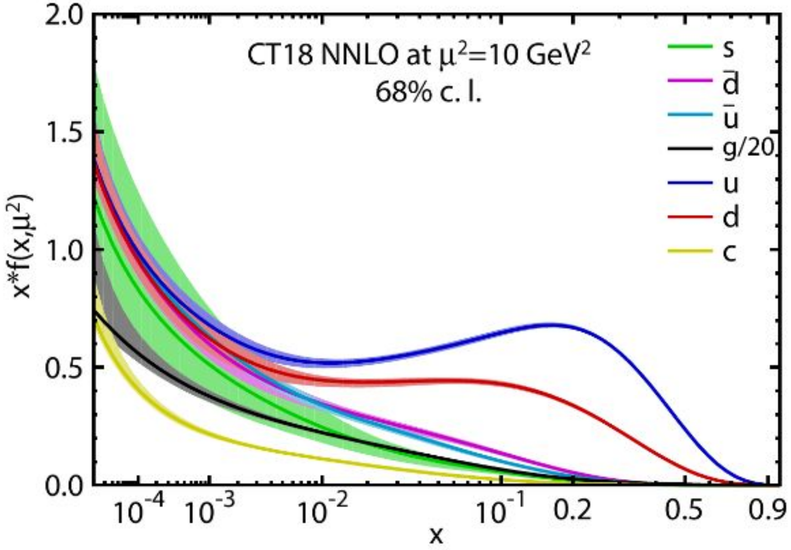


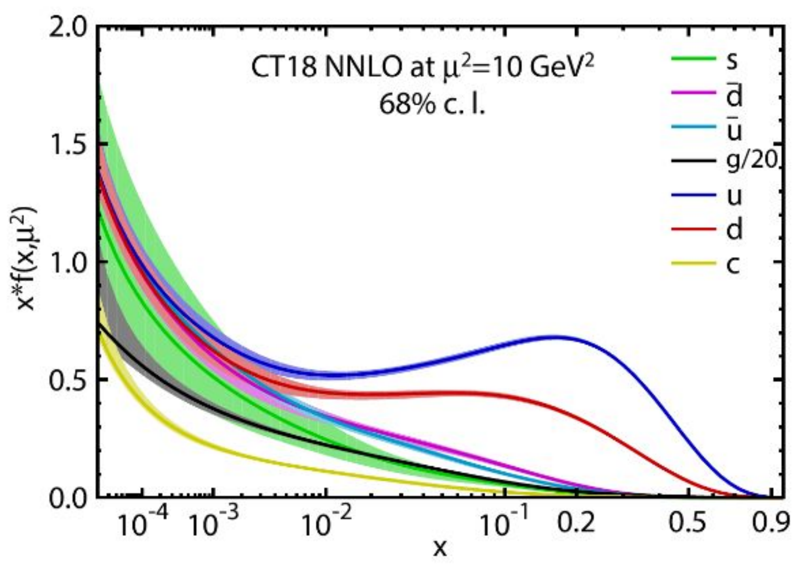
FIG. 2 The CT18 PDFs at $\mu^2 = 10$ GeV$^2$ for the $xu$, $x\bar{u}$, $xd$, $x\bar{d}$, $xs = x\bar{s}$, and $xg$ PDFs. Error bands correspond to the 68% confidence level. Figure from (Kovařík *et al.*, 2019).

Many extractions available
See e.g. the LHAPDF library
https://lhapdf.hepforge.org/

(and W. Melnitchouk’s talk)

# PDFs: what do we know

See https://inspirehep.net/literature/1801417



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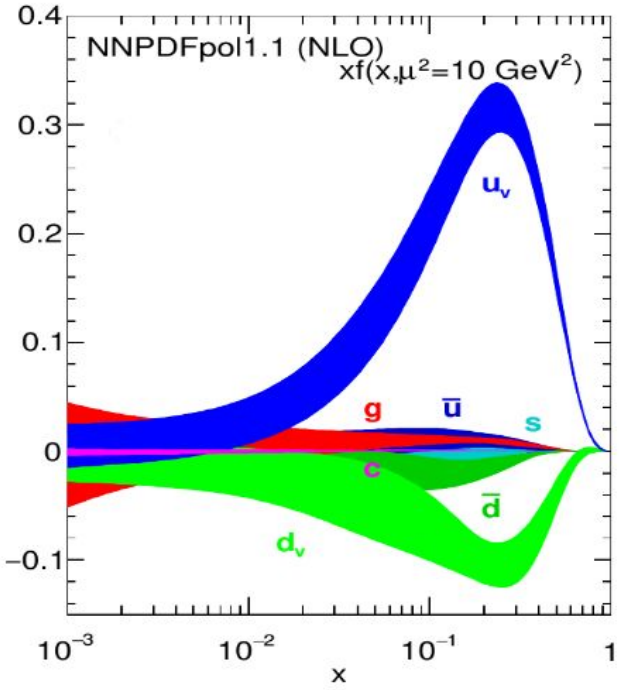
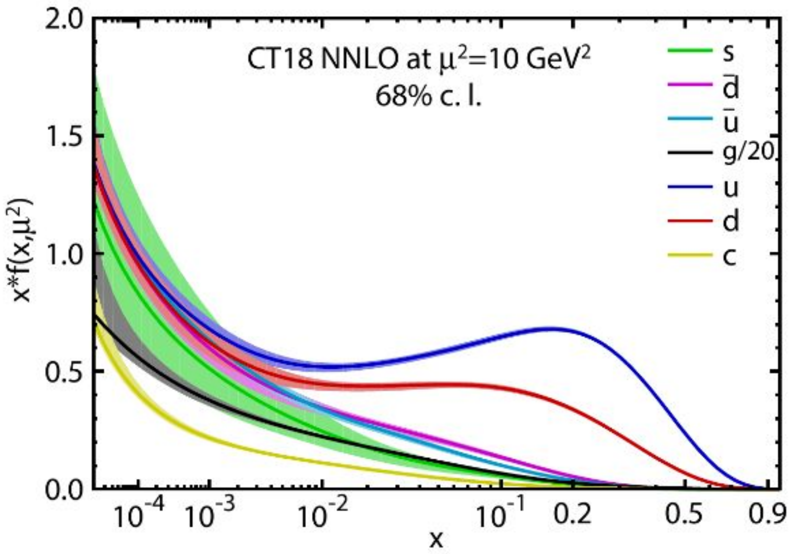


FIG. 5 The helicity PDFs from the NNPDFpol1.1 parton set at $\mu^2 = 10$ GeV$^2$. Figure from (Tanabashi *et al.*, 2018).

# PDFs: what do we know

See https://inspirehep.net/literature/1801417



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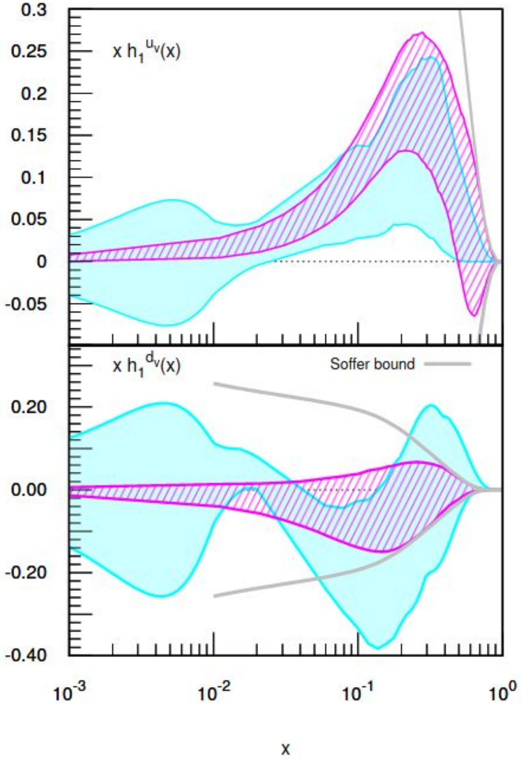


FIG. 6 The transversity $x\,h_1(x)$ at 90% CL. Upper (lower) plot for valence up (down) component. Gray lines represent the Soffer bound. Darker (pink) band for the PV18 global fit of (Radici and Bacchetta, 2018) at $Q^2 = 2.4$ GeV$^2$. Lighter (cyan) band for the MEX19 constrained analysis of (Benel *et al.*, 2020) at the average scale of the data.

# Impact studies

See https://inspirehep.net/literature/1851258

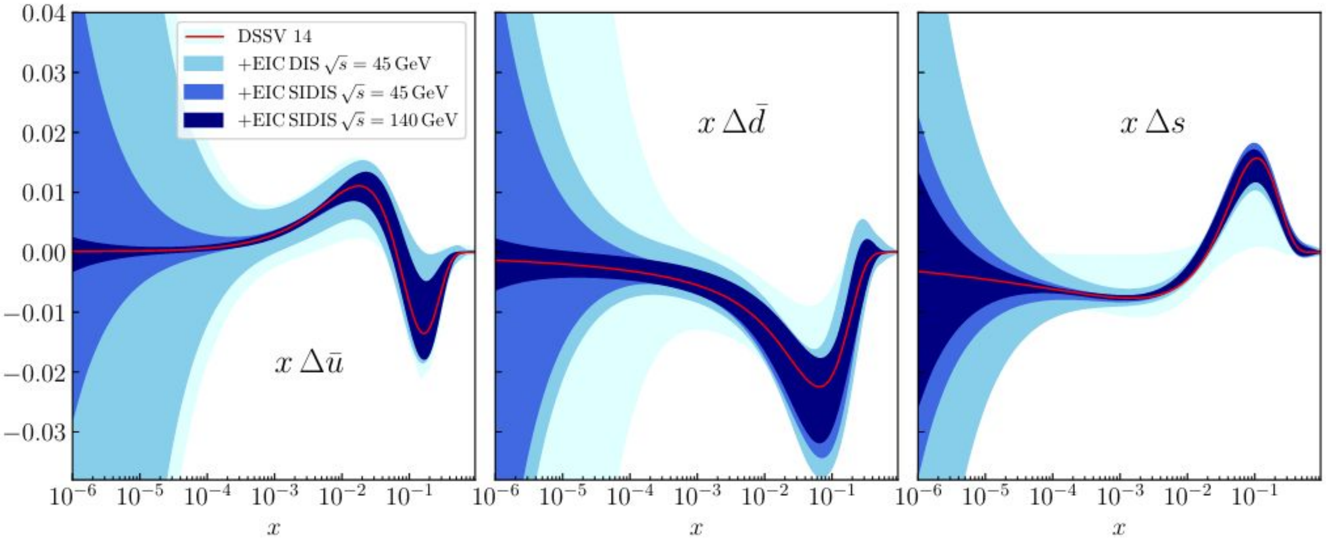
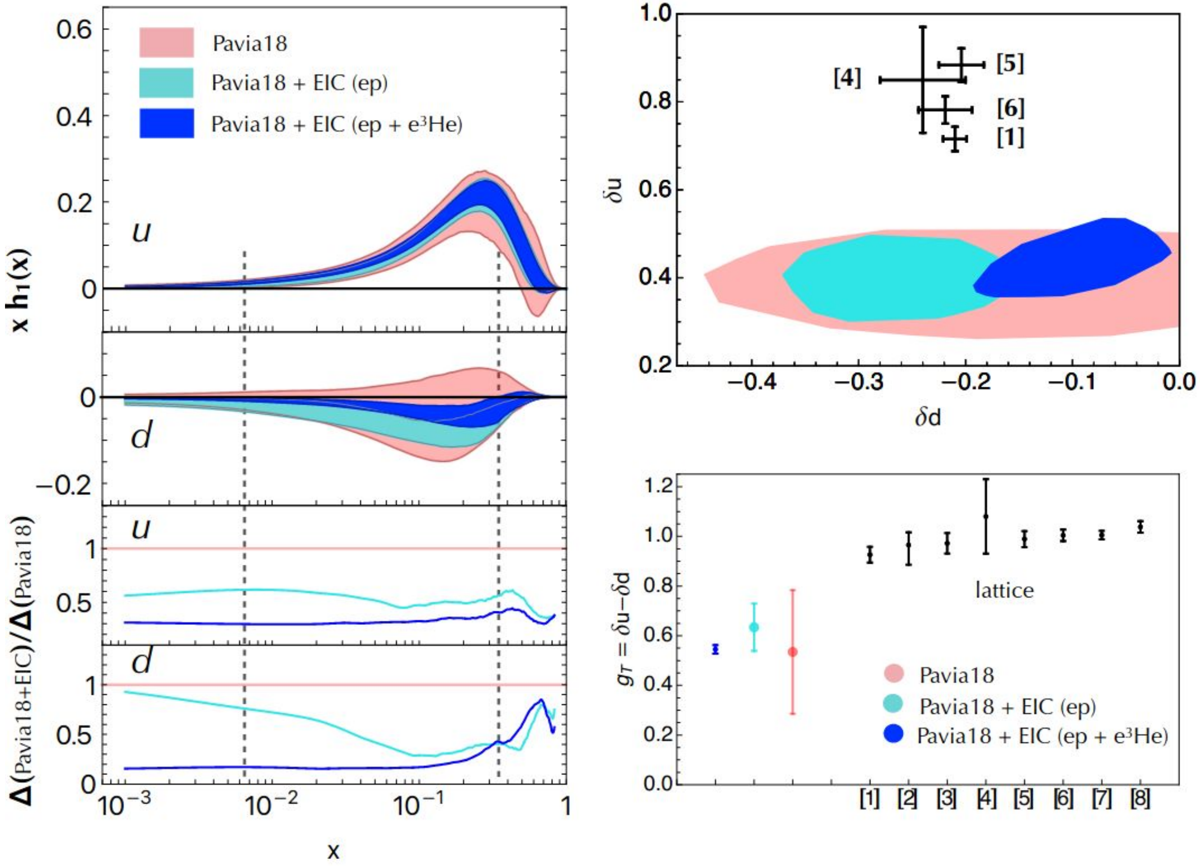


**Figure 7.19:** Impact of SIDIS measurements at the EIC on the sea quark helicities $x\Delta\bar{u}$, $x\Delta\bar{d}$ and $x\Delta s$ as a function of $x$ at $Q^2 = 10\ \text{GeV}^2$.

Helicity PDF

# Impact studies

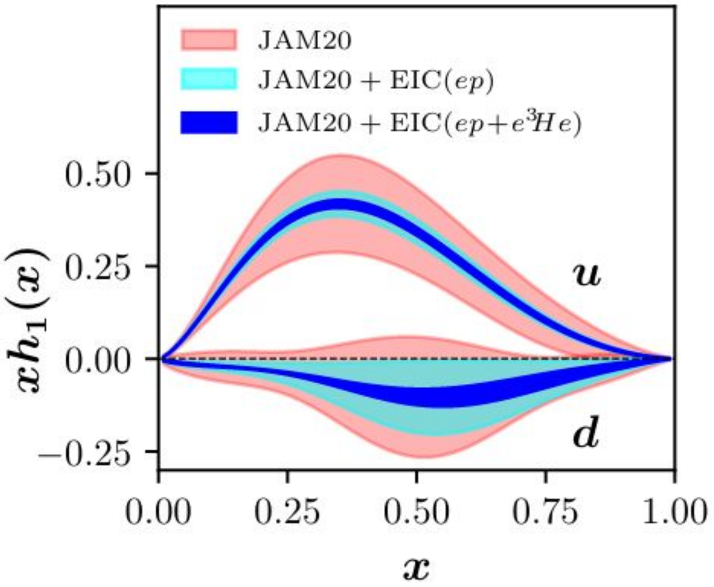
See https://inspirehep.net/literature/1851258

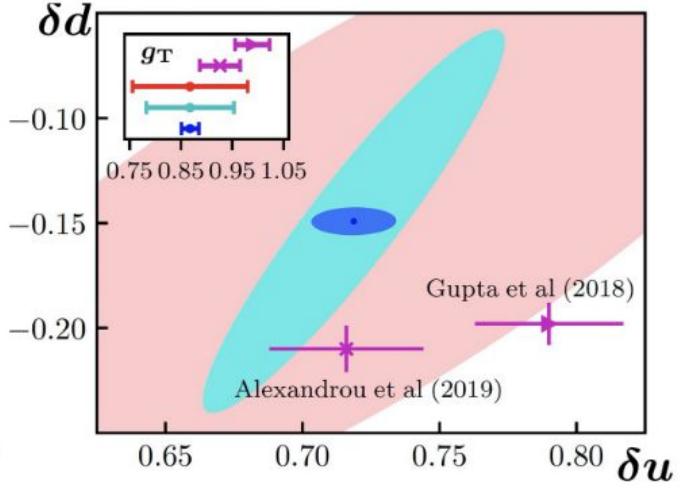


Transversity PDF and tensor charge

# Impact studies

See https://inspirehep.net/literature/1851258

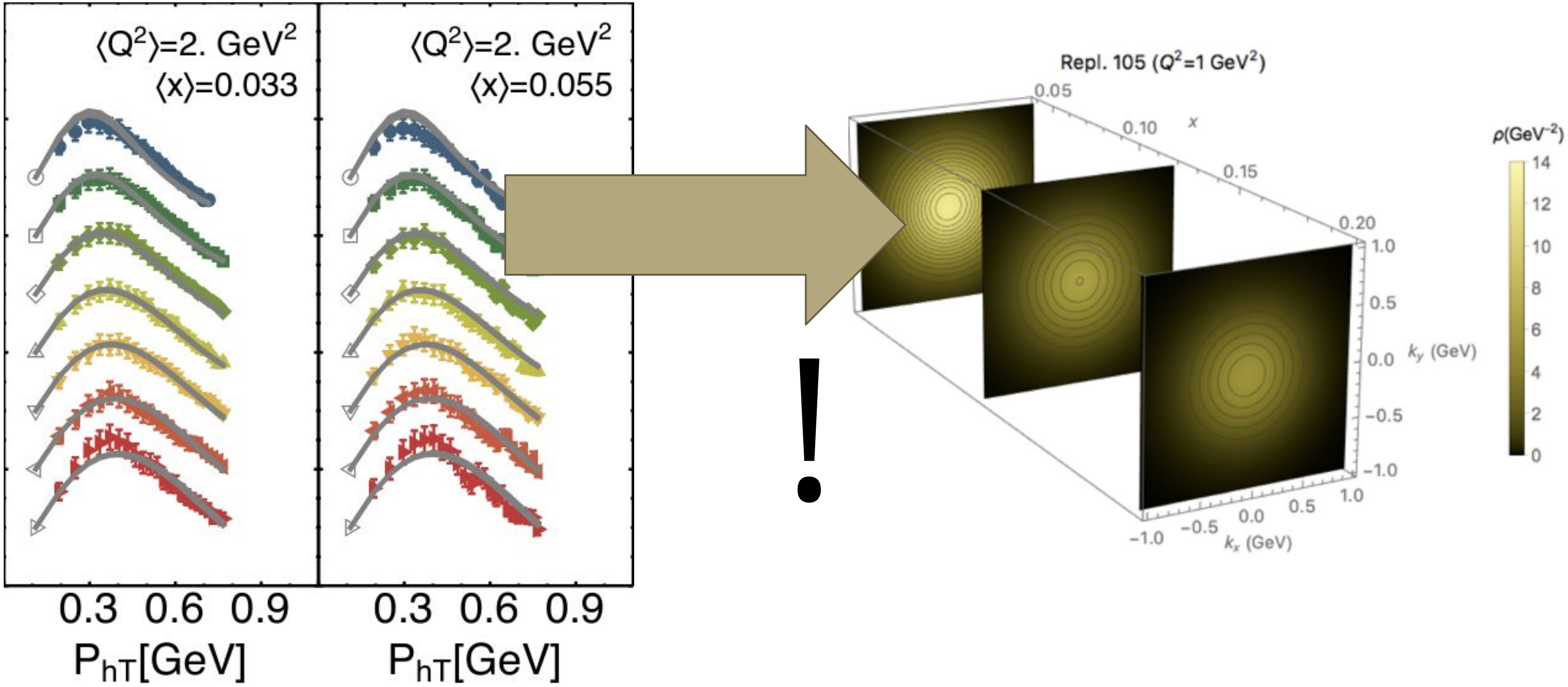




Transversity PDF and tensor charge

# 5.3 Unpolarized TMDs

# Transverse momentum imaging



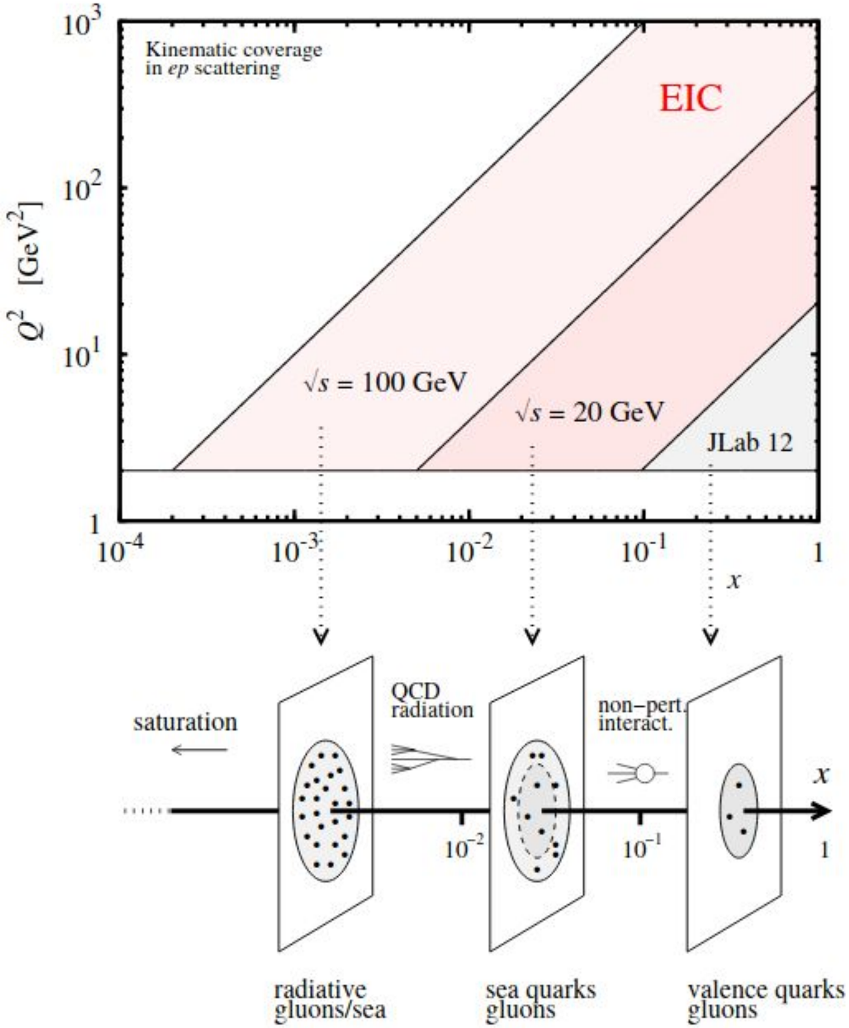
# Imaging from SIDIS

Importance of complementary experiments

from JLab 12 GeV, Hermes, Compass
to the EIC

**zooming** into hadron structure

Credit picture: C. Weiss



# Non perturbative components

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$

→ TMD distribution at initial scales

$$\times \quad \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \boxed{\gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)}\right]$$

→ evolution in μ

**Calculable in pQCD**

$$\times \quad \left(\frac{\zeta}{\zeta_0}\right)^{-\boxed{D(b_T\mu_0, \alpha_s(\mu_0))}\boxed{+g_K(b_T;\lambda)}}$$

→ evolution in ζ

**Non-pert. corrections (large bT)**

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b \boxed{C_{a/b}(x, b_T^2, \mu_0, \zeta_0)} \otimes \underline{f_b(x, \mu_0)}\, \boxed{F_{NP}(b_T; \lambda)}$$

**Prior knowledge assumed (?)**

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Separating small and large bT

One needs to “separate” the small (perturbative) bT region from the large (non-perturbative) bT region:

$$\alpha_s(\mu = \mu_b \sim 1/b_T) \longrightarrow b_T < b_{max}$$

Avoid the Landau pole of QCD

$$\int_{\mu_b \sim 1/b}^{Q} \gamma_F \;, \quad \mu_b < Q \longrightarrow b_T > b_{\min}$$

Otherwise gluon “absorption” instead of “emission”

For more details see https://inspirehep.net/literature/1520011

# Separating small and large bT

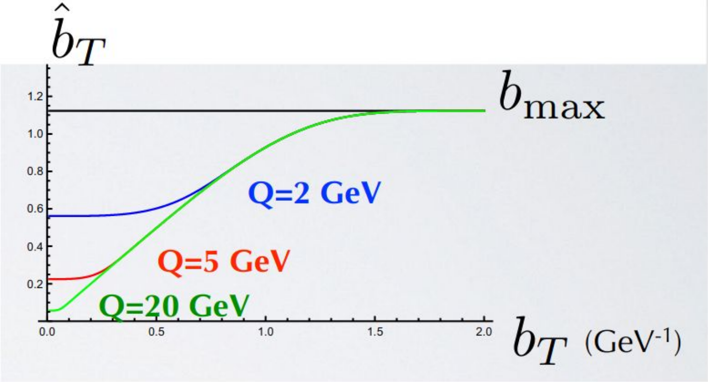
One needs to “separate” the small (perturbative) bT region from the large (non-perturbative) bT region:

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right) \begin{array}{l} \nearrow\ b_{\max}\ , \quad b_T \to +\infty \\ \\ \searrow\ b_{\min}\ , \quad b_T \to 0 \end{array}$$

$$b_{\max} = 2e^{-\gamma_E}$$

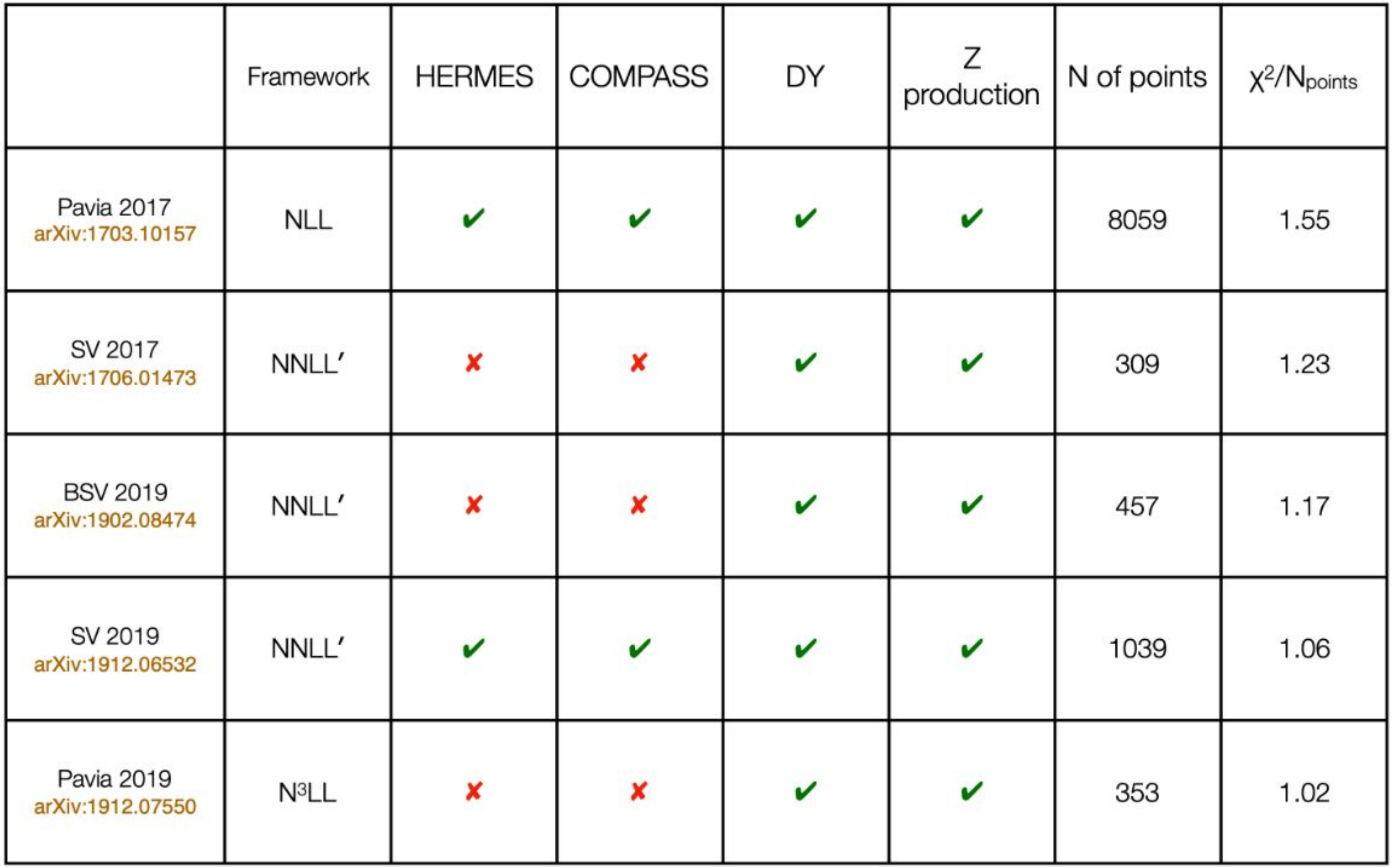
$$b_{\min} = 2e^{-\gamma_E}/Q$$

These choices guarantee that for Q=1 GeV the TMD coincides with the NP model



For more details see https://inspirehep.net/literature/1520011

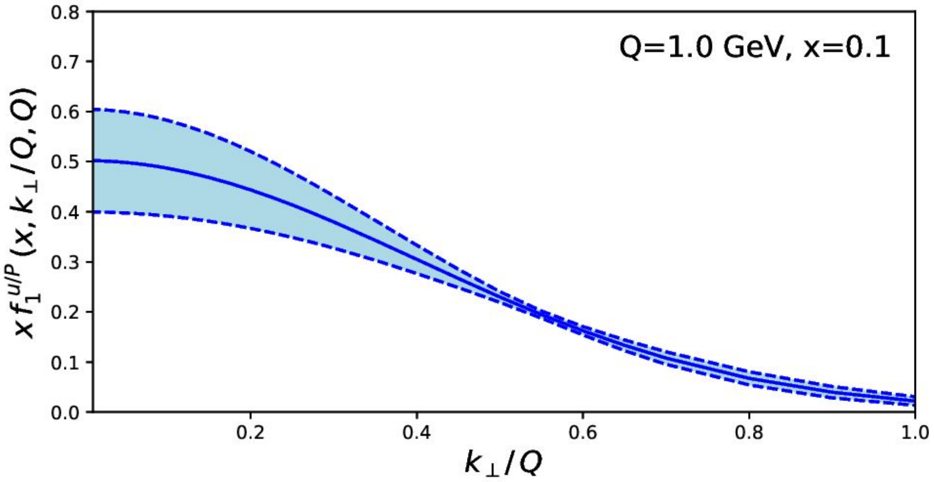
# Recent fits of unpolarized TMDs



| | Framework | HERMES | COMPASS | DY | Z production | N of points | $\chi^2/N_{points}$ |
|---|---|---|---|---|---|---|---|
| Pavia 2017<br>arXiv:1703.10157 | NLL | ✓ | ✓ | ✓ | ✓ | 8059 | 1.55 |
| SV 2017<br>arXiv:1706.01473 | NNLL′ | ✗ | ✗ | ✓ | ✓ | 309 | 1.23 |
| BSV 2019<br>arXiv:1902.08474 | NNLL′ | ✗ | ✗ | ✓ | ✓ | 457 | 1.17 |
| SV 2019<br>arXiv:1912.06532 | NNLL′ | ✓ | ✓ | ✓ | ✓ | 1039 | 1.06 |
| Pavia 2019<br>arXiv:1912.07550 | N$^3$LL | ✗ | ✗ | ✓ | ✓ | 353 | 1.02 |

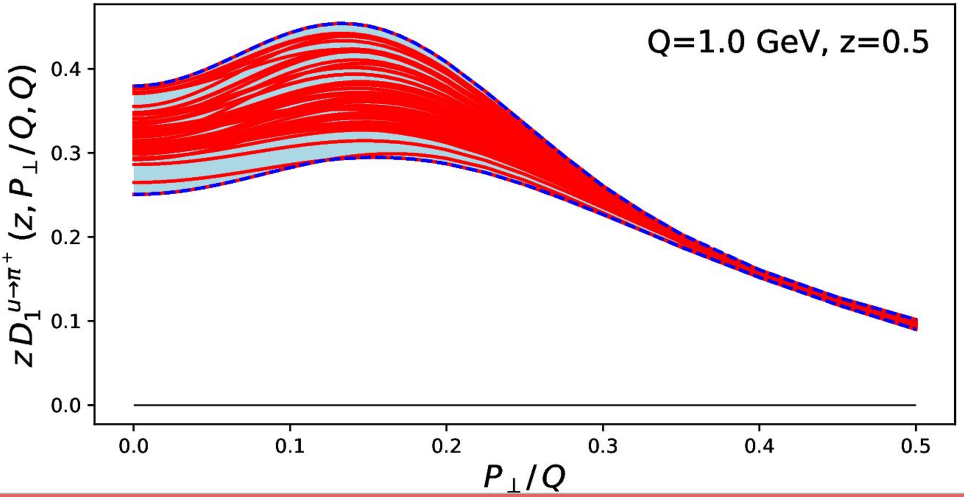
# Global fit of unpolarized TMDs (PV17)

Imaging from **SIDIS** data (Hermes and Compass) **and Drell-Yan** data (Fermilab, low energy and Z)



Unpolarized TMD PDF

Unpolarized TMD FF



Combining SIDIS and Drell-Yan:
Possibility to disentangle
hadron structure and formation

See https://inspirehep.net/literature/1520011

# Some open questions

A non-exhaustive *personal* list of open questions:

- deepen our understanding of **sea** quarks
- **flavor structure** of TMDs
- experimental confirmation of **sign change** relation
- **gluon** observables and **spin-1** effects
- what can **hadronization** teach us about **confinement**?
- interplay between **nuclear/hadron** and **high-energy** physics
- ..

# Predictive power

Small bT → **perturbative (radiation)** contributions to TMD PDF
Large bT → **non-perturbative (intrinsic)** contributions to TMD PDF

Exercise:

In which kinematic regions is the TMD PDF dominated by small / large bT contributions ?

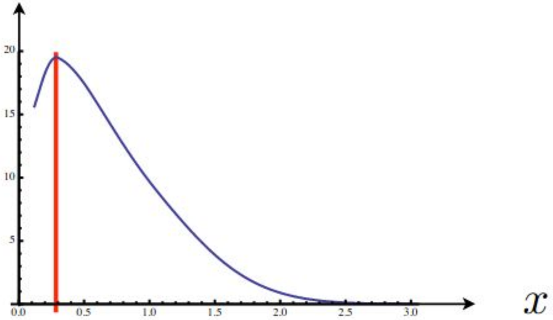
Hint: think about the shape of the TMD PDF in bT space and where it peaks

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Saddle-point approximation

Function f with a maximum “$x_0$” in (a,b)

$$I(x_0, A) = \int_a^b dx\; e^{Af(x)} = e^{Af(x_0)} \sqrt{\frac{2\pi}{A(-f''(x_0))}} \left(1 + \mathcal{O}\left(\frac{1}{A}\right)\right)$$



One can apply this approximation to the TMD PDF and plot the position of the peak in bT as a function of x and Q:

$$\frac{d}{db_T}\left\{ \ln\left[ b_T^2\, F_a(x, b_T^2; Q, Q^2) \right] \right\}_{b_T = b_T^{sp}} = 0$$

The TMD PDF is dominated by perturbative contributions at large Q and small x

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# 5.4 Polarized TMDs

# Spin asymmetries

$$A_{UT} \sim \frac{d\sigma(\uparrow) - d\sigma(\downarrow)}{d\sigma(\uparrow) + d\sigma(\downarrow)}$$

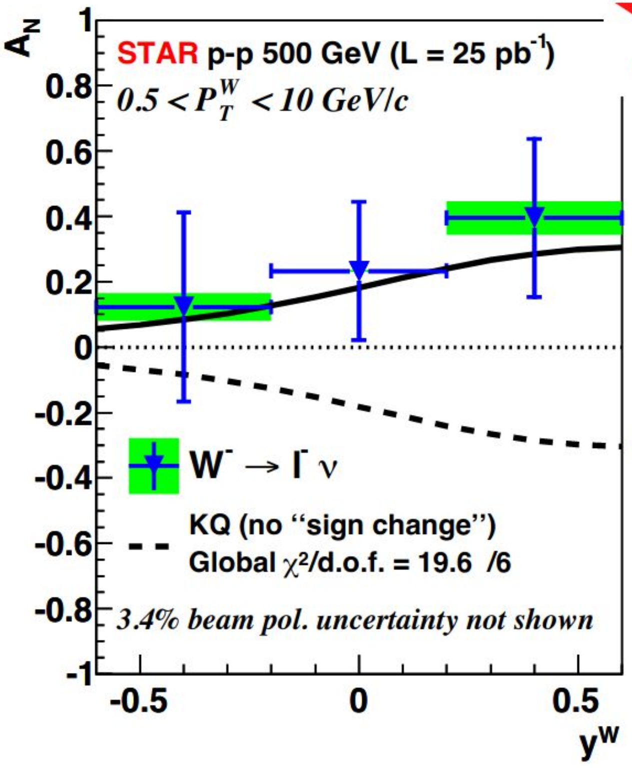
Polarized structure functions / unpolarized one

$$f_1 \otimes D_1$$

Asymmetries in general have the benefits to :

- Single out specific structure functions by using Fourier analysis
- Reduce the effect of systematic uncertainties common to denominator and numerator (e.g. acceptance effects)
- Knowledge of the unpolarized cross section (denominator) is required in order to study the numerator

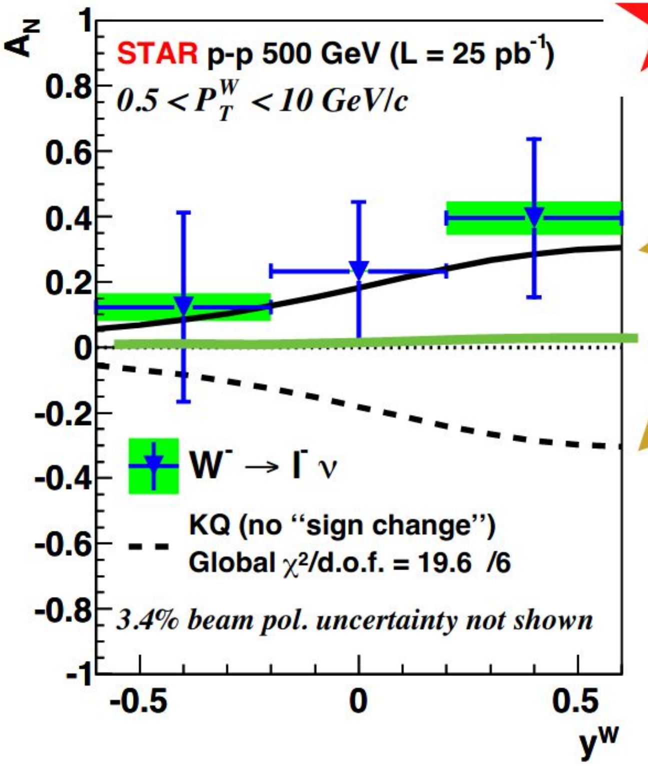
# Sign-change for Sivers function



first evidence
of sign change?

STAR Collab. arXiv:1511.06003

# Sign-change for Sivers function

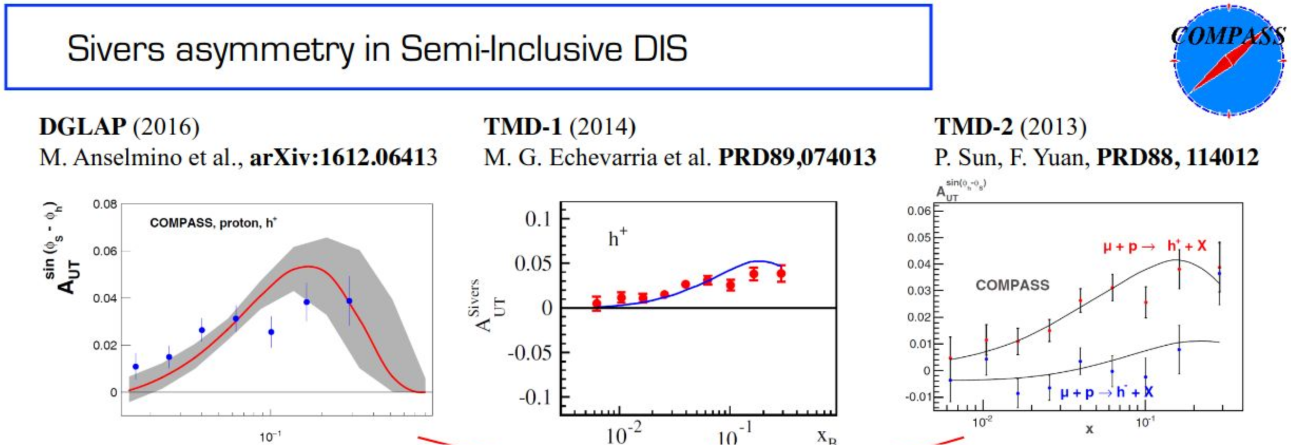


first evidence of sign change?

prediction with TMD evolution equations

STAR Collab. arXiv:1511.06003

# Sign-change for Sivers function



Sivers asymmetry in Semi-Inclusive DIS

**DGLAP** (2016)
M. Anselmino et al., **arXiv:1612.06413**

**TMD-1** (2014)
M. G. Echevarria et al. **PRD89,074013**

**TMD-2** (2013)
P. Sun, F. Yuan, **PRD88, 114012**

# Sign-change for Sivers function

Sivers asymmetry in Semi-Inclusive DIS

**DGLAP** (2016)
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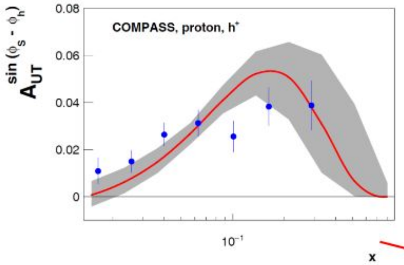
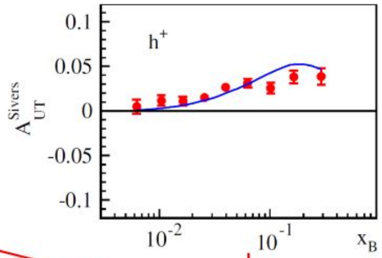
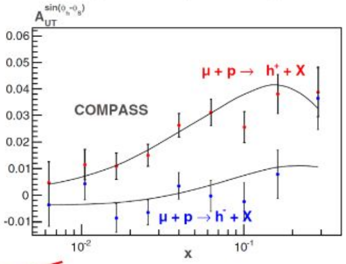


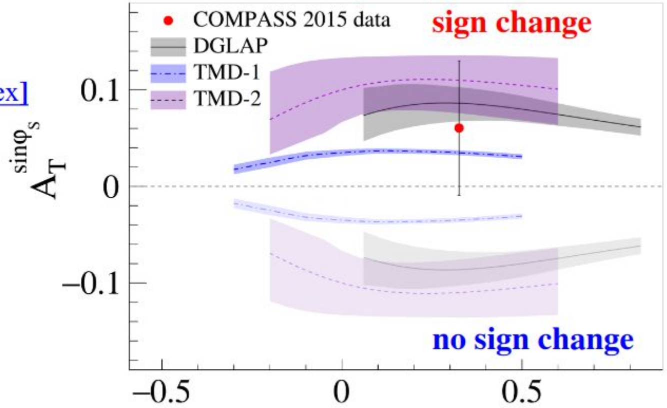
**TMD-1** (2014)
M. G. Echevarria et al. **PRD89,074013**



**TMD-2** (2013)
P. Sun, F. Yuan, **PRD88, 114012**



**New! 03 April 2017**
**COMPASS**
CERN-EP-2017-059
arXiv:1704.00488[hep-ex]



sign change

no sign change

Sivers asymmetry in Drell-Yan

courtesy B. Parsamyan

# Other TMDs from SIDIS

see https://inspirehep.net/literature/1806922

Transversity TMD PDF & Collins TMD FF

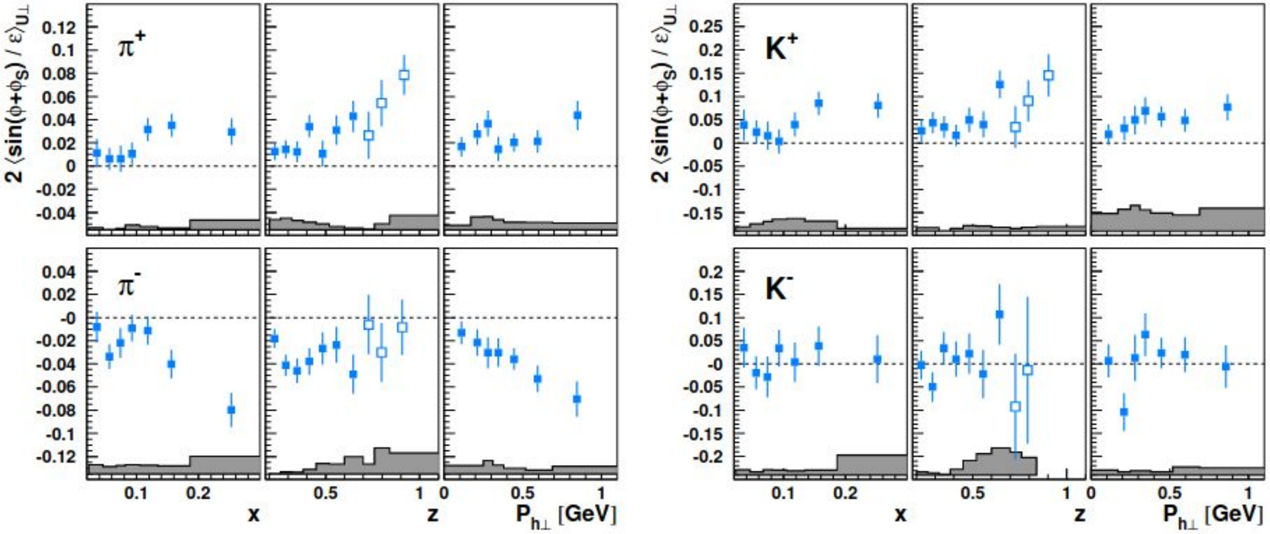


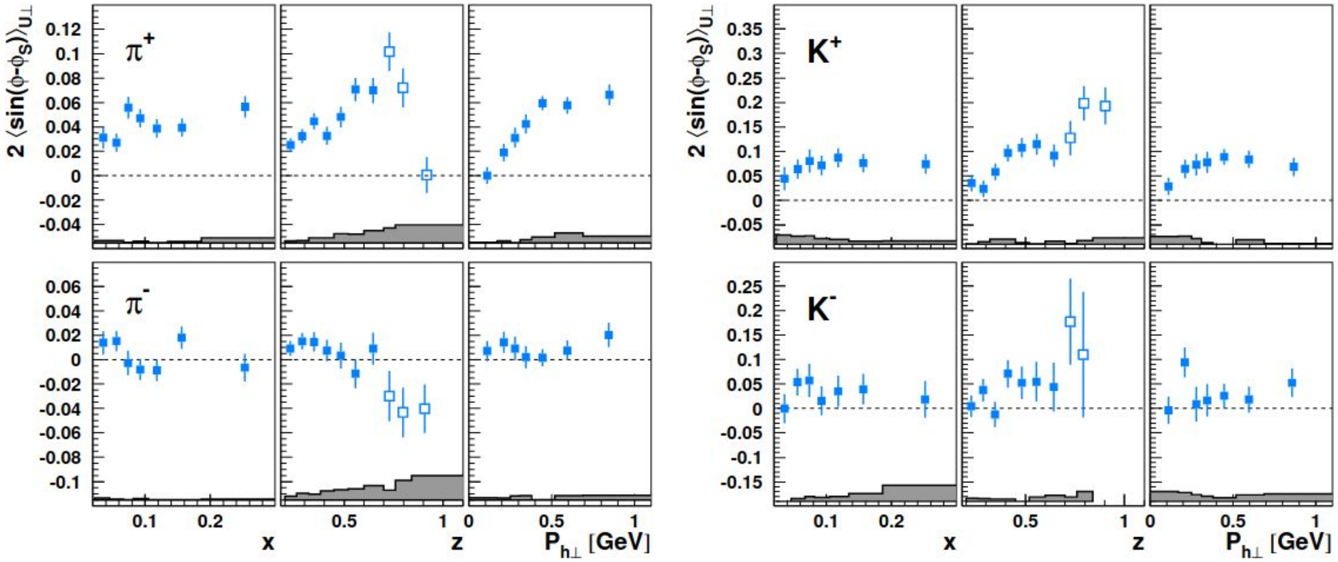
**Figure 8.** Collins SFA for charged mesons (left: pions; right: kaons) presented either in bins of $x$, $z$, or $P_{h\perp}$. Data at large values of $z$, marked by open points in the $z$ projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.

$$\frac{h_1 \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

# Other TMDs from SIDIS

see https://inspirehep.net/literature/1806922

Sivers TMD PDF



$$\frac{\boxed{f_{1T}^{\perp}} \otimes D_1}{f_1 \otimes D_1}$$

**Figure 12**. Sivers SFA for charged mesons (left: pions; right: kaons) presented either in bins of $x$, $z$, or $P_{h\perp}$. Data at large values of $z$, marked by open points in the $z$ projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.

hermes

# Other TMDs from SIDIS

see https://inspirehep.net/literature/1806922

Pretzelosity TMD PDF & Collins TMD FF (*vanishing signal ..?*)

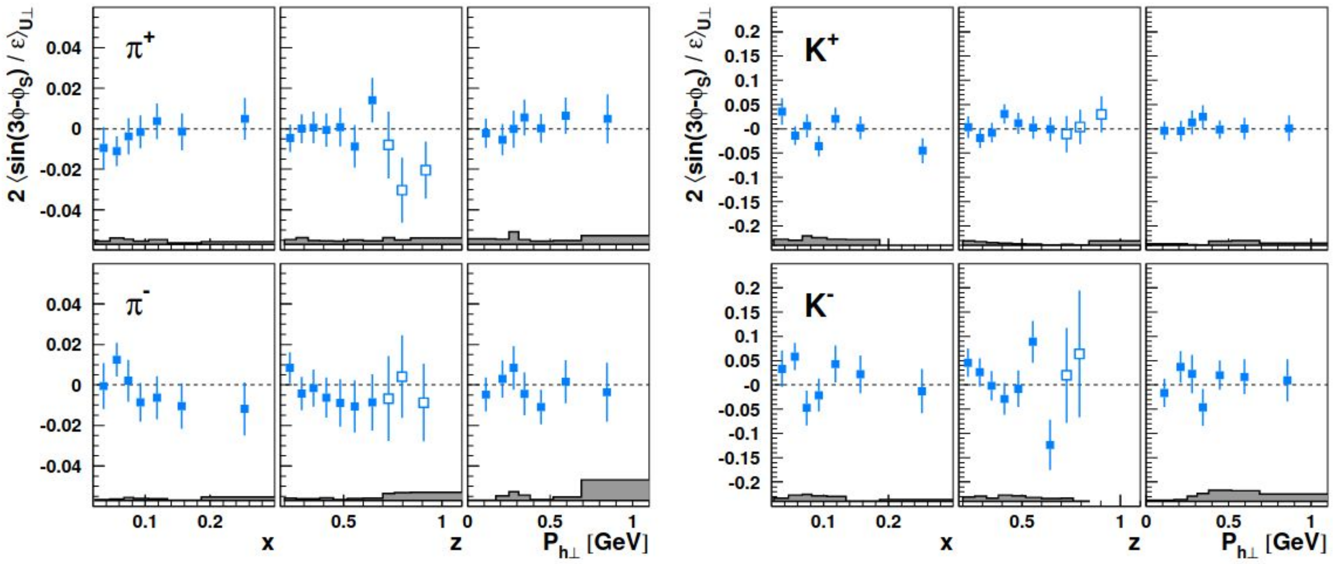


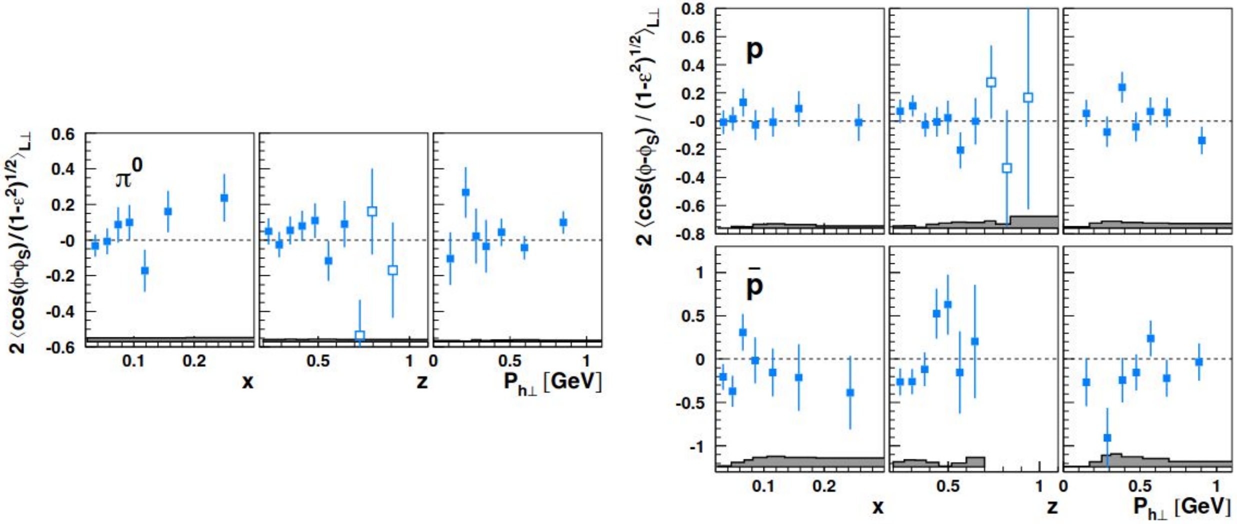
**Figure 19.** Pretzelosity SFA for charged mesons (left: pions; right: kaons) presented either in bins of $x$, $z$, or $P_{h\perp}$. Data at large values of $z$, marked by open points in the $z$ projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.

$$\frac{h_{1T}^{\perp} \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

# Other TMDs from SIDIS

see https://inspirehep.net/literature/1806922

Worm-gear TMD PDF



$$\frac{g_{1T} \otimes D_1}{f_1 \otimes D_1}$$

**Figure 22.** The $2\langle\cos(\phi-\phi_S)/\sqrt{1-\epsilon^2}\,\rangle^{h}_{\mathrm{L}\perp}$ amplitudes for $\pi^0$ (left), protons, and antiprotons (right) presented either in bins of $x$, $z$, or $P_{h\perp}$. Data at large values of $z$, marked by open points in the $z$ projection, are not included in the other projections (no such high-$z$ points are available for antiprotons due to a lack of precision). Systematic uncertainties are given as bands, not including the additional scale uncertainty of 8.0% due to the precision in the determination of the target and beam polarizations.

# Other TMDs from SIDIS

see https://inspirehep.net/literature/1806922

**Twist 3** TMD PDFs and TMD FFs

$$\frac{f_T \otimes D_1}{f_1 \otimes D_1} \qquad \frac{h_1 \otimes \tilde{H}}{f_1 \otimes D_1} \qquad \frac{h_T \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

$$\frac{g_{1T} \otimes \tilde{G}^{\perp}}{f_1 \otimes D_1}$$

$$\frac{h_T^{\perp} \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

$$\frac{f_{1T}^{\perp} \otimes \tilde{D}^{\perp}}{f_1 \otimes D_1}$$

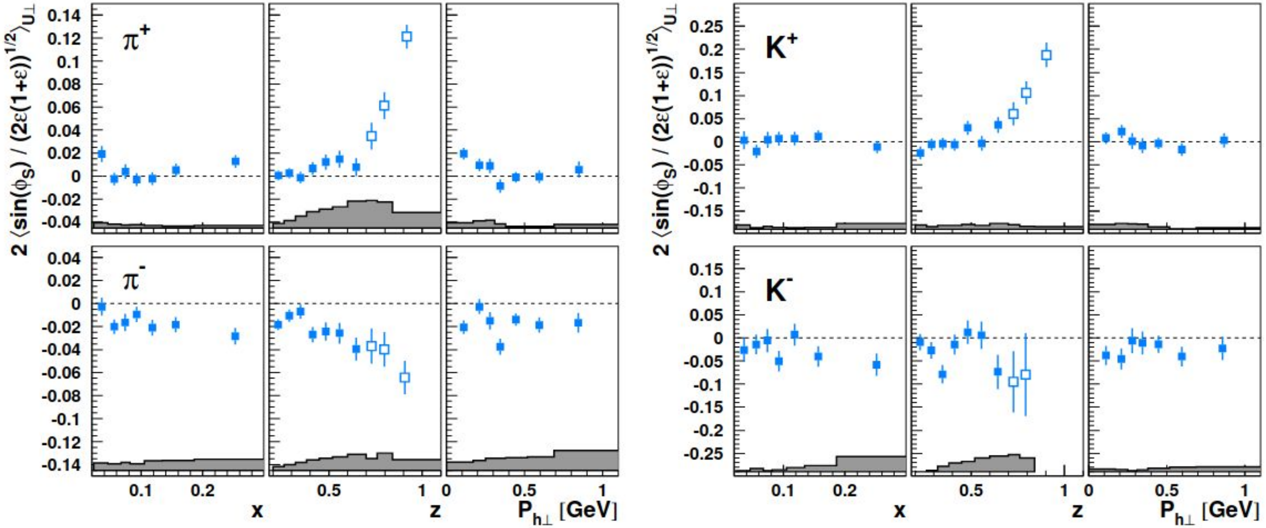


**Figure 25.** The $2\langle \sin(\phi_S)/\sqrt{2\epsilon(1+\epsilon)}\,\rangle^{h}_{\mathrm{U}\perp}$ amplitudes for charged mesons (left: pions; right: kaons) presented either in bins of $x$, $z$, or $P_{h\perp}$. Data at large values of $z$, marked by open points in the $z$ projection, are not included in the other projections. Systematic uncertainties are given as bands, not including the additional scale uncertainty of 7.3% due to the precision of the target-polarization determination.

hermes