HUGS 2021 Lectures on:
Experimental Meson Spectroscopy

LECTURE III. The Quark Model

IIIA. Charmonium Potential

IIIB. Radiative Transitions

IIIC. Color Factors

IIID. Doubly-Bottom Tetraquark

Ryan Mitchell

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IIIA. Charmonium Potential

Quark Model: Assume hadrons are made of quarks interacting via a potential.

One example:

PHYSICAL REVIEW D **72,** 054026 (2005)

Higher charmonia

T. Barnes,[1,*] S. Godfrey,[2,†] and E. S. Swanson[3,‡]

spin-spin (hyperfine)

$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\,\tilde{\delta}_\sigma(r)\vec{\mathrm{S}}_c\cdot\vec{\mathrm{S}}_{\bar{c}}$$

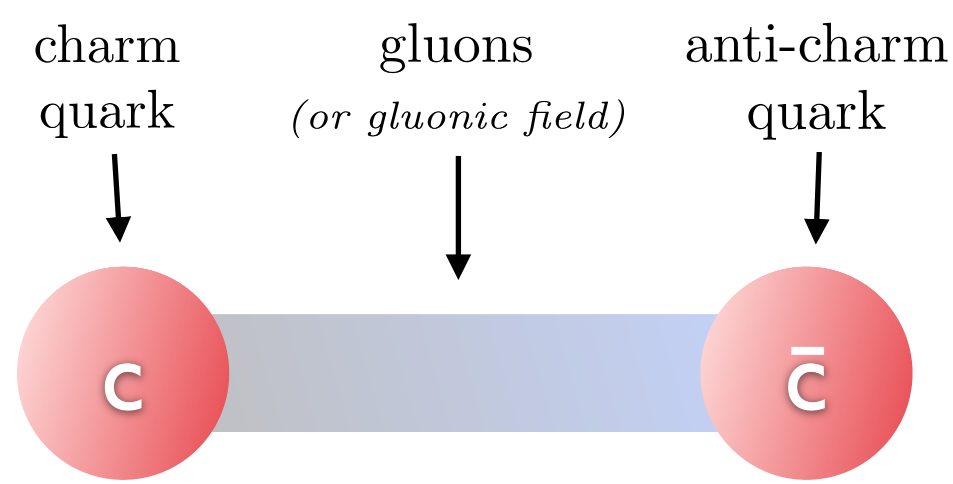
"Coulomb" confinement

$$\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$$

$$V_{\text{spin-dep}} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}} + \frac{4\alpha_s}{r^3}\mathrm{T}\right]$$

spin-orbit (fine) tensor (hyperfine)

$$\langle {}^3\mathrm{L_J}|\mathrm{T}|{}^3\mathrm{L_J}\rangle = \begin{cases} -\frac{\mathrm{L}}{6(2\mathrm{L}+3)}, & \mathrm{J} = \mathrm{L}+1 \\ +\frac{1}{6}, & \mathrm{J} = \mathrm{L} \\ -\frac{(\mathrm{L}+1)}{6(2\mathrm{L}-1)}, & \mathrm{J} = \mathrm{L}-1 \end{cases}$$



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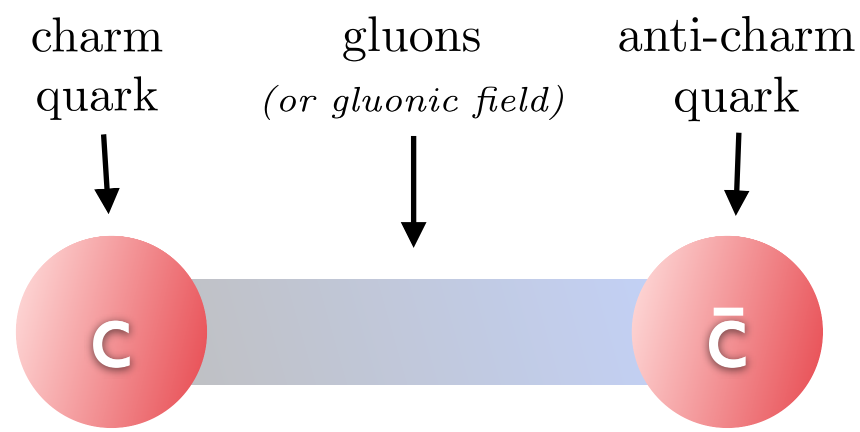
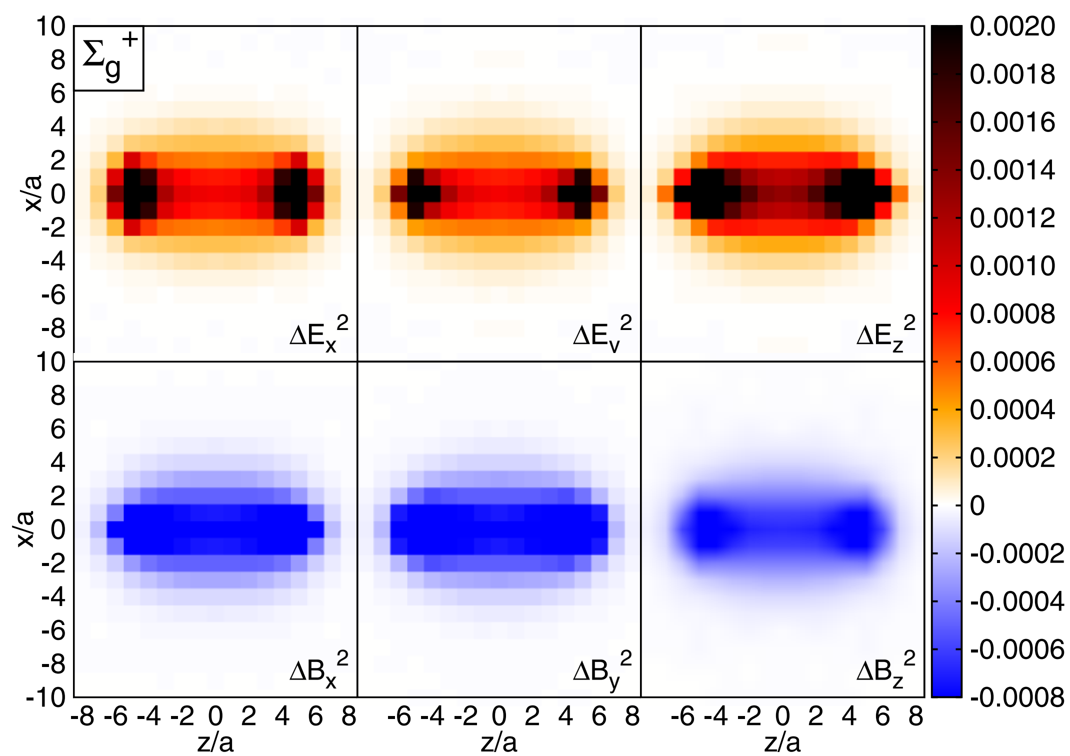


PHYSICAL REVIEW D **100,** 054503 (2019)

Hybrid static potential flux tubes from SU(2) and SU(3) lattice gauge theory



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$(\alpha_s, b, m_c, \sigma) = (0.5461, 0.1425\ \mathrm{GeV}^2, 1.4794\ \mathrm{GeV}, 1.0946\ \mathrm{GeV})$

Multiplet	State	Expt.	Input (NR)	Theor. NR	Theor. GI
1S	$J/\psi(1^3\mathrm{S}_1)$	3096.87 ± 0.04	3097	3090	3098
	$\eta_c(1^1\mathrm{S}_0)$	2979.2 ± 1.3	2979	2982	2975
2S	$\psi'(2^3\mathrm{S}_1)$	3685.96 ± 0.09	3686	3672	3676
	$\eta_c'(2^1\mathrm{S}_0)$	3637.7 ± 4.4	3638	3630	3623
3S	$\psi(3^3\mathrm{S}_1)$	4040 ± 10	4040	4072	4100
	$\eta_c(3^1\mathrm{S}_0)$			4043	4064
4S	$\psi(4^3\mathrm{S}_1)$	4415 ± 6	4415	4406	4450
	$\eta_c(4^1\mathrm{S}_0)$			4384	4425
1P	$\chi_2(1^3\mathrm{P}_2)$	3556.18 ± 0.13	3556	3556	3550
	$\chi_1(1^3\mathrm{P}_1)$	3510.51 ± 0.12	3511	3505	3510
	$\chi_0(1^3\mathrm{P}_0)$	3415.3 ± 0.4	3415	3424	3445
	$h_c(1^1\mathrm{P}_1)$	see text		3516	3517
2P	$\chi_2(2^3\mathrm{P}_2)$			3972	3979
	$\chi_1(2^3\mathrm{P}_1)$			3925	3953
	$\chi_0(2^3\mathrm{P}_0)$			3852	3916
	$h_c(2^1\mathrm{P}_1)$			3934	3956
3P	$\chi_2(3^3\mathrm{P}_2)$			4317	4337
	$\chi_1(3^3\mathrm{P}_1)$			4271	4317
	$\chi_0(3^3\mathrm{P}_0)$			4202	4292
	$h_c(3^1\mathrm{P}_1)$			4279	4318
1D	$\psi_3(1^3\mathrm{D}_3)$			3806	3849
	$\psi_2(1^3\mathrm{D}_2)$			3800	3838
	$\psi(1^3\mathrm{D}_1)$	3769.9 ± 2.5	3770	3785	3819
	$\eta_{c2}(1^1\mathrm{D}_2)$			3799	3837
2D	$\psi_3(2^3\mathrm{D}_3)$			4167	4217
	$\psi_2(2^3\mathrm{D}_2)$			4158	4208
	$\psi(2^3\mathrm{D}_1)$	4159 ± 20	4159	4142	4194
	$\eta_{c2}(2^1\mathrm{D}_2)$			4158	4208

+ 1F, 2F, 1G

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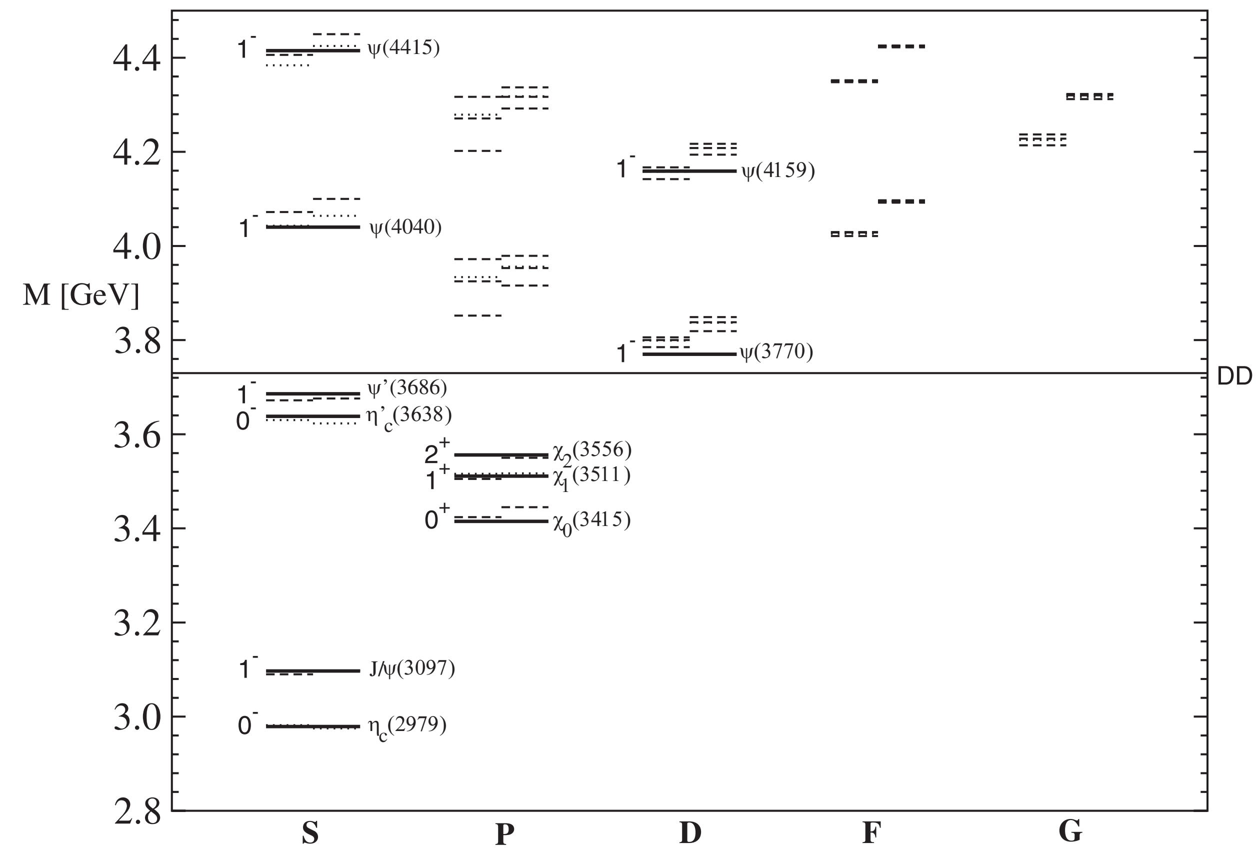

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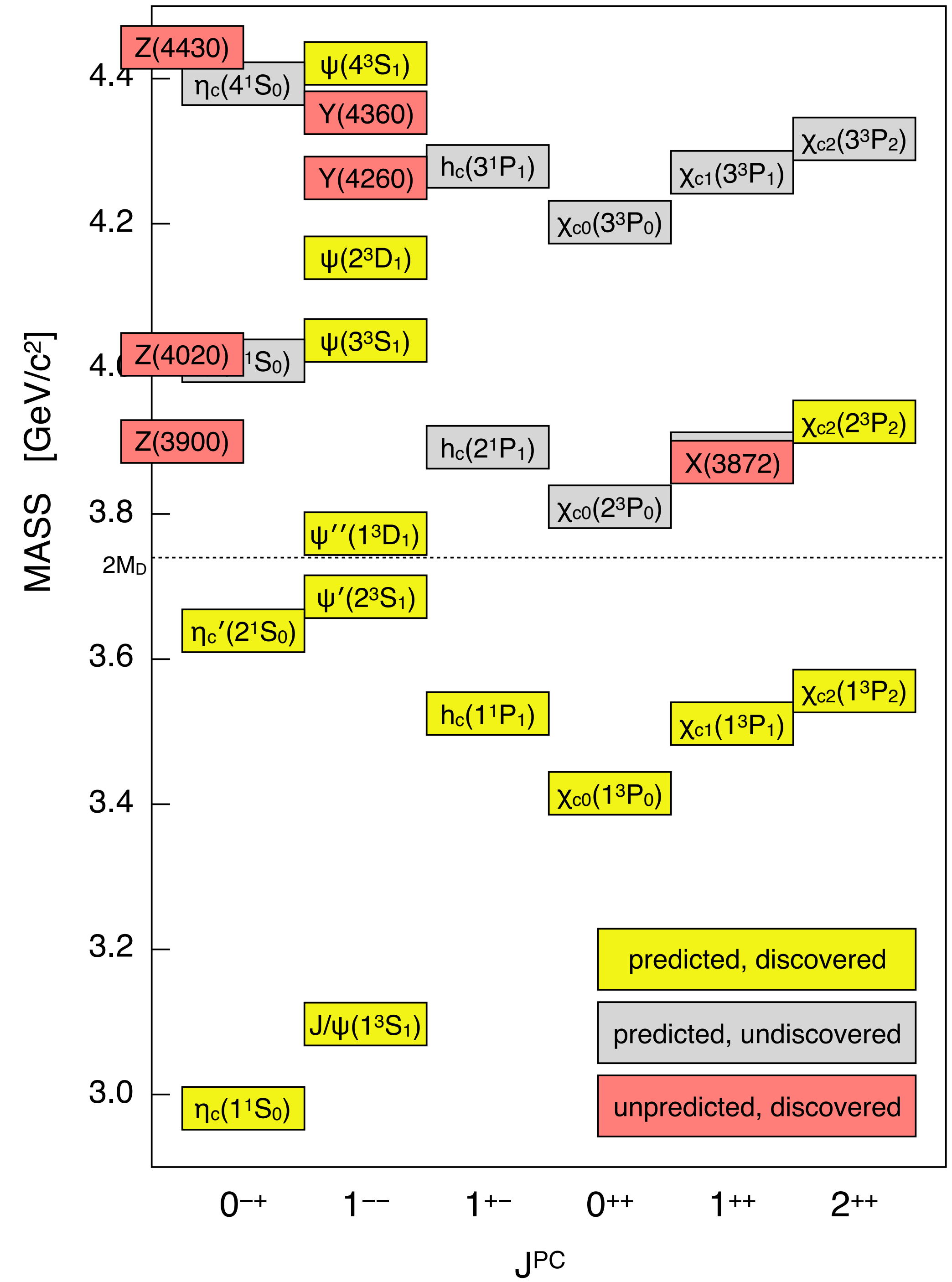

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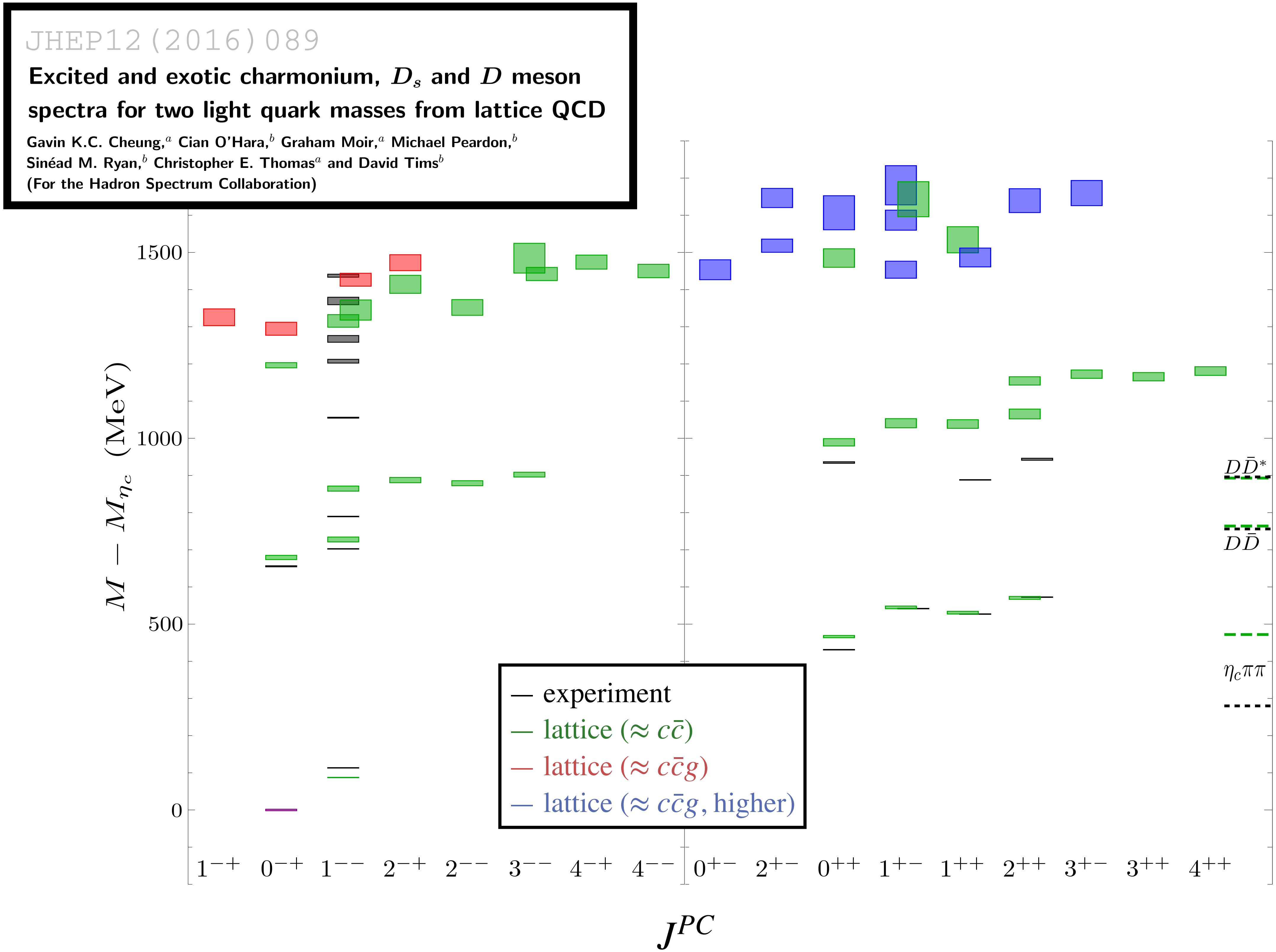
IIIA. Charmonium Potential

Also calculate the charmonium spectrum using lattice QCD.



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JHEP12(2016)089

Excited and exotic charmonium, D_s and D meson spectra for two light quark masses from lattice QCD

Gavin K.C. Cheung,[a] Cian O'Hara,[b] Graham Moir,[a] Michael Peardon,[b] Sinéad M. Ryan,[b] Christopher E. Thomas[a] and David Tims[b]
(For the Hadron Spectrum Collaboration)

PHYSICAL REVIEW D **84,** 074023 (2011)

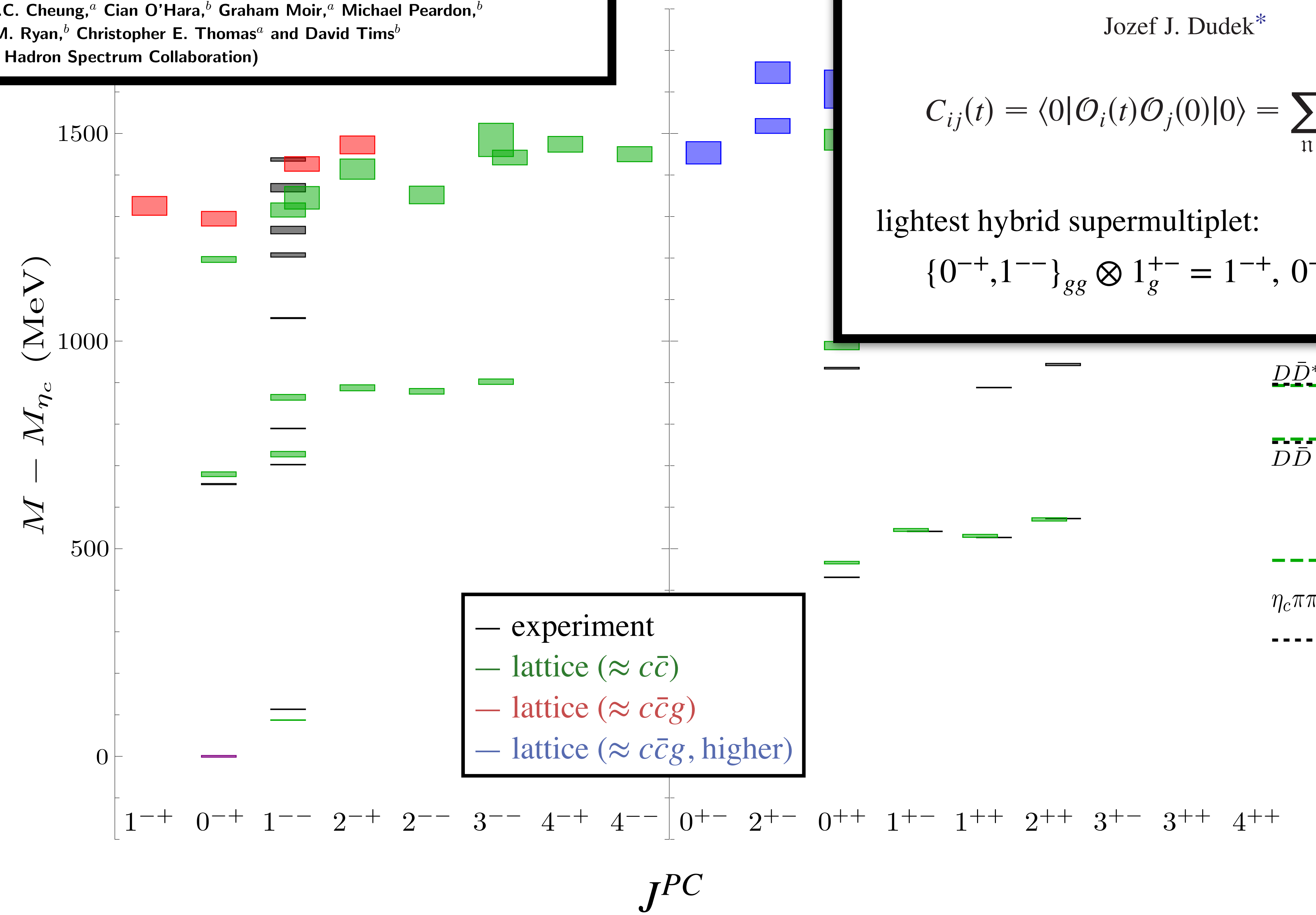
The lightest hybrid meson supermultiplet in QCD

Jozef J. Dudek*

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j(0)|0\rangle = \sum_{\mathfrak{n}} \frac{Z_i^{\mathfrak{n}} Z_j^{\mathfrak{n}}}{2m_{\mathfrak{n}}} e^{-m_{\mathfrak{n}}t}$$

lightest hybrid supermultiplet:

$$\{0^{-+},1^{--}\}_{gg} \otimes 1_g^{+-} = 1^{-+},\ 0^{-+},\ 1^{--},\ 2^{-+}$$

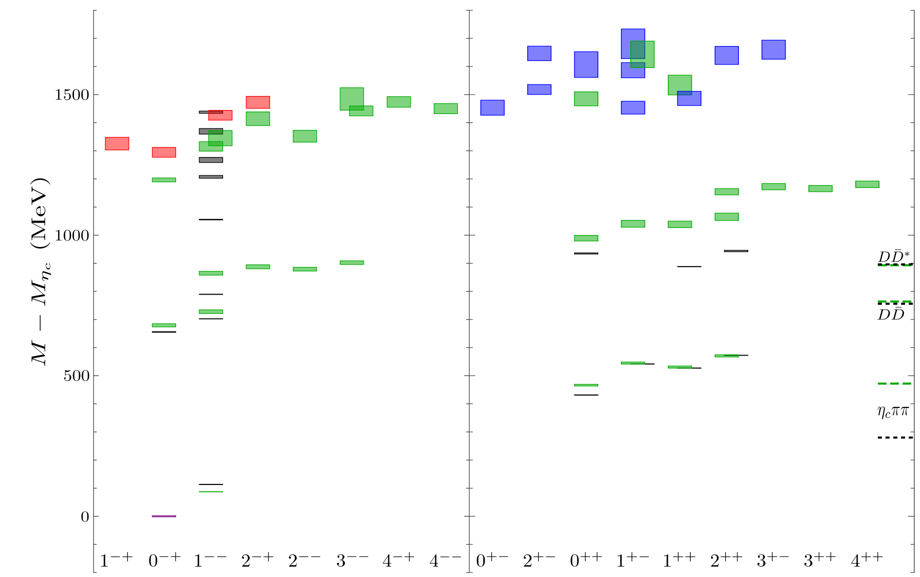


IIIA. Charmonium Potential

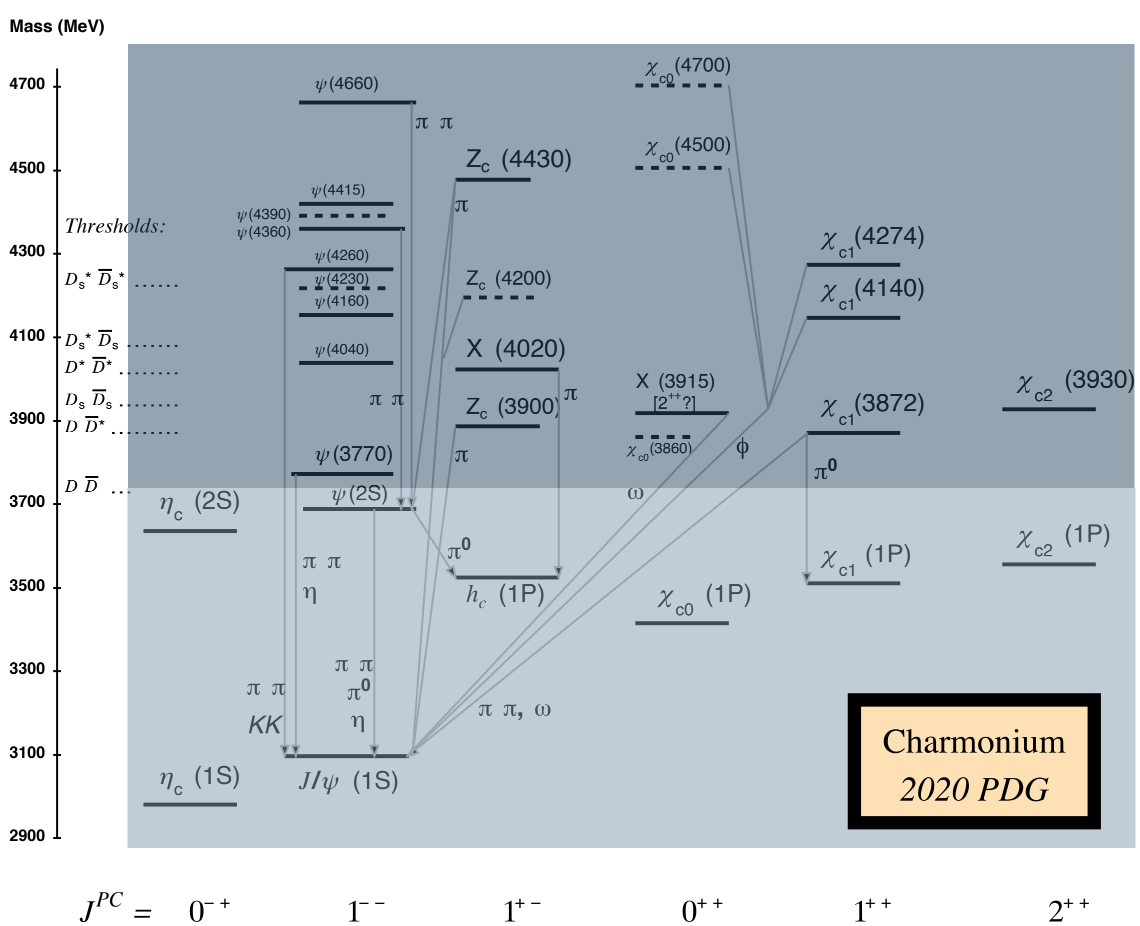
The goal of experimental meson (hadron) spectroscopy:

Uncover a broad set of physical phenomena
(including new meson states, their properties, decays patterns, etc.)
in order to build our understanding of the strong force.

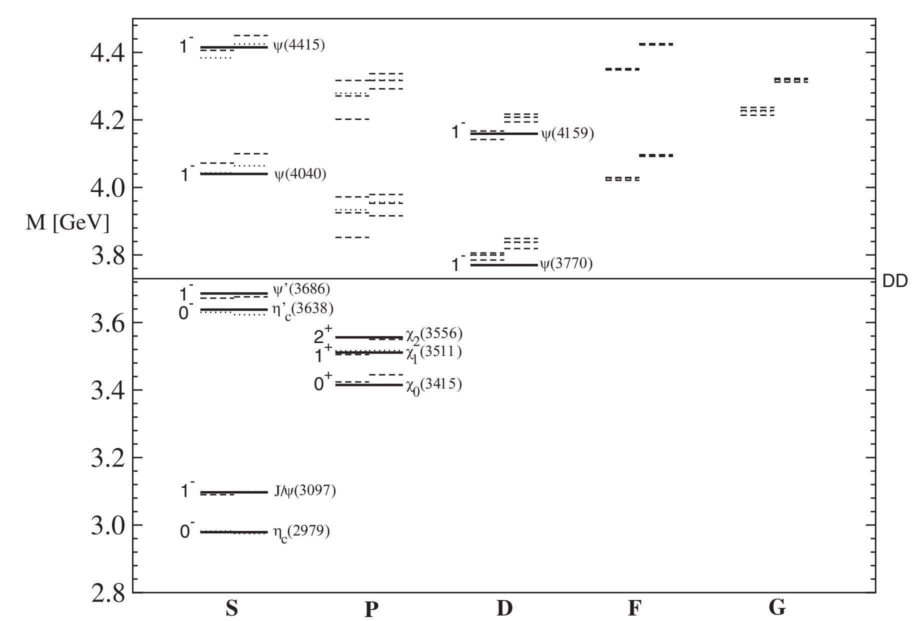
THEORY



EXPERIMENT



MODELS



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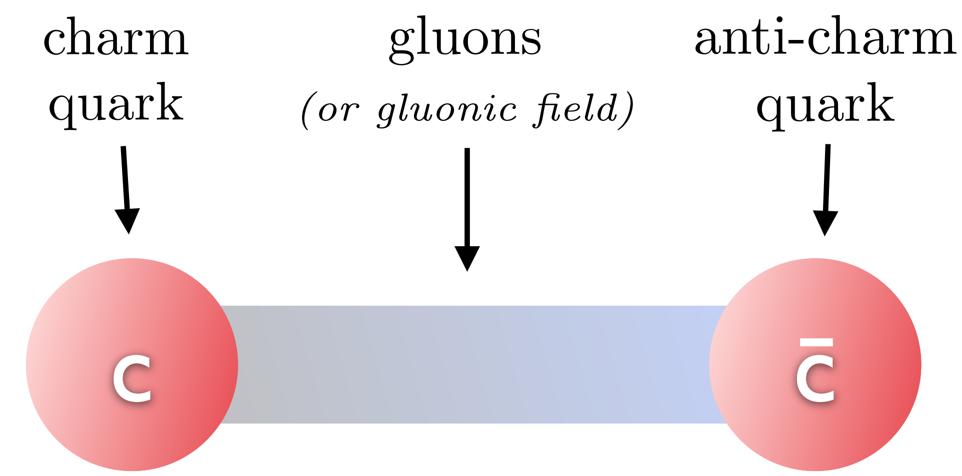

Use the resulting wavefunctions to calculate radiative transitions.

$$\Gamma_{\mathrm{E1}}(\mathrm{n}^{2\mathrm{S}+1}\mathrm{L}_\mathrm{J} \to \mathrm{n}'^{2\mathrm{S}'+1}\mathrm{L}'_{\mathrm{J}'} + \gamma) = \frac{4}{3} C_{fi}\delta_{\mathrm{SS}'} e_c^2 \alpha |\langle\psi_f|r|\psi_i\rangle|^2 \,\mathrm{E}_\gamma^3 \frac{\mathrm{E}_\mathrm{f}^{(c\bar{c})}}{\mathrm{M}_\mathrm{i}^{(c\bar{c})}}$$

$$C_{fi} = \max(\mathrm{L},\mathrm{L}')(2\mathrm{J}'+1)\begin{Bmatrix} \mathrm{L}' & \mathrm{J}' & \mathrm{S} \\ \mathrm{J} & \mathrm{L} & 1 \end{Bmatrix}^2$$

$$\Gamma_{\mathrm{M1}}(\mathrm{n}^{2\mathrm{S}+1}\mathrm{L}_\mathrm{J} \to \mathrm{n}'^{2\mathrm{S}'+1}\mathrm{L}'_{\mathrm{J}'} + \gamma) = \frac{4}{3}\frac{2J'+1}{2L+1}\delta_{\mathrm{LL}'}\delta_{\mathrm{S,S}'\pm 1} e_c^2 \frac{\alpha}{m_c^2}|\langle\psi_f|\psi_i\rangle|^2 \mathrm{E}_\gamma^3 \frac{\mathrm{E}_\mathrm{f}^{(c\bar{c})}}{\mathrm{M}_\mathrm{i}^{(c\bar{c})}}$$

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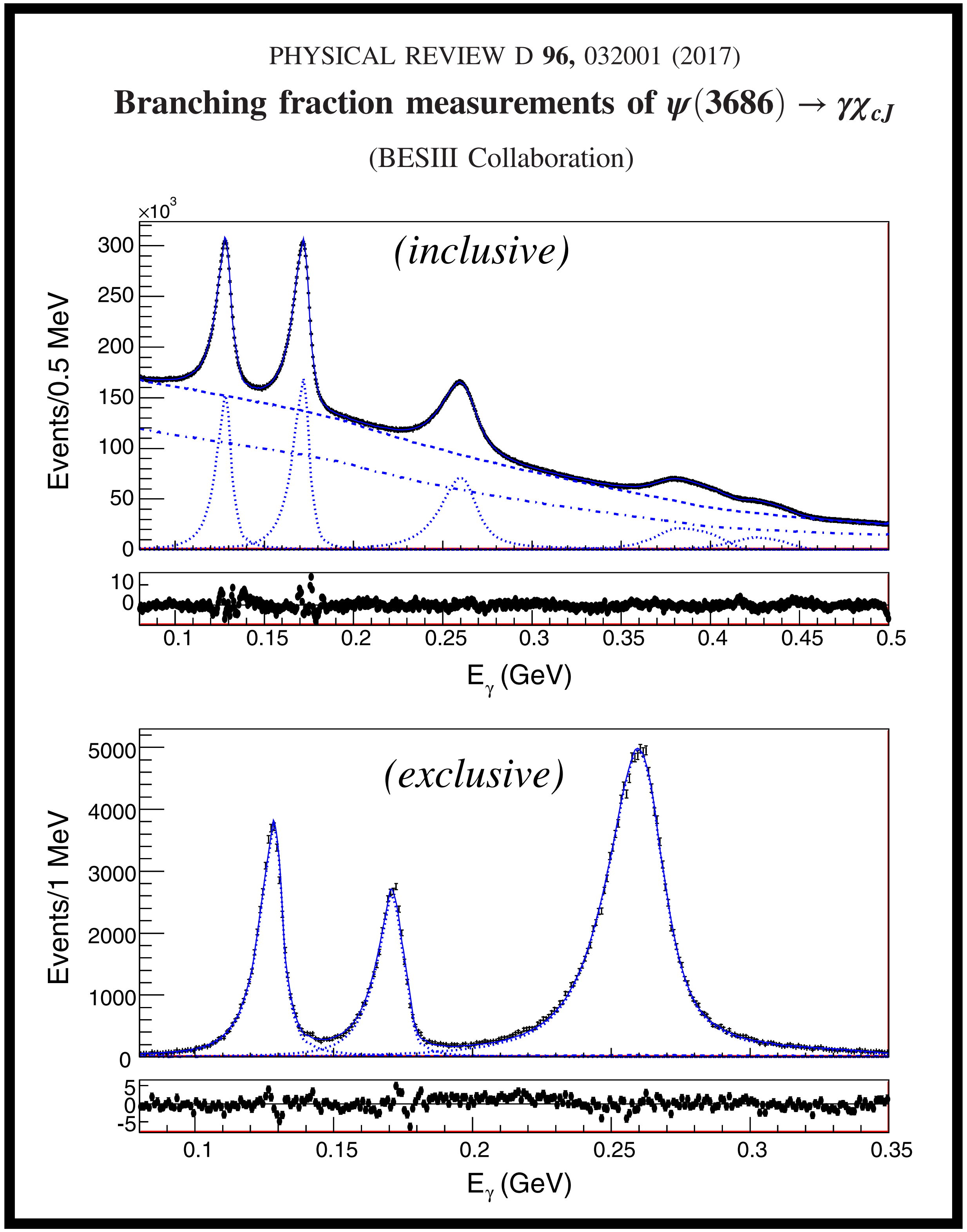
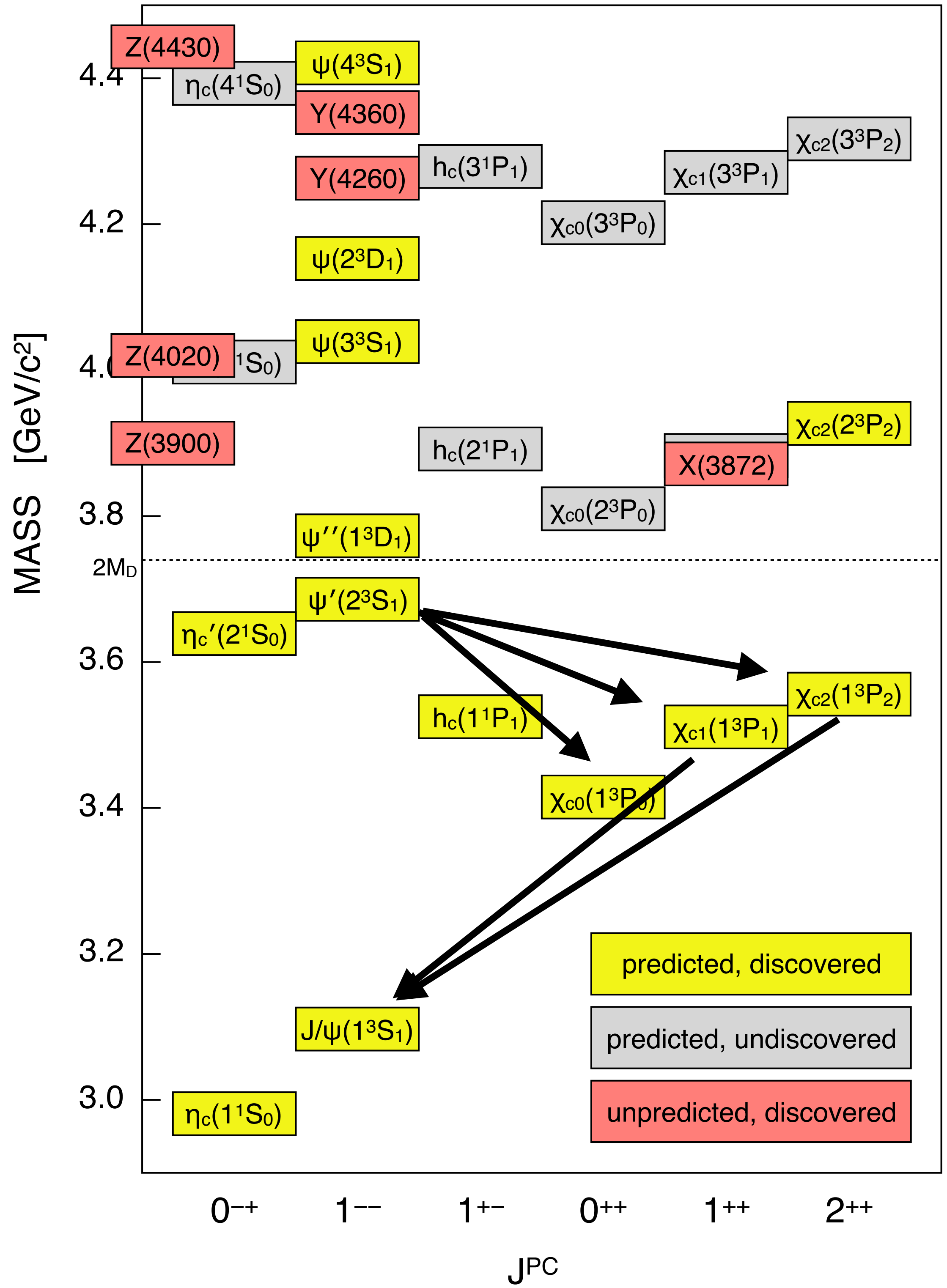
Radiative transitions can be studied at BESIII using $e^+e^- \to \psi(2S)$.





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		Γ_{E1} (keV)					Γ_{EM} (keV)		
Initial State	Final State	RQM [33]	NR/GI [34]	SNR$_{0/1}$ [35]	LP [8]	SP [8]	LP [8]	SP [8]	This Analysis
$\psi(3686)$	χ_{c0}	26.3	63/26	74/25	27	26	22	22	26.9 ± 1.8
	χ_{c1}	22.9	54/29	62/36	45	48	42	45	28.3 ± 1.9
	χ_{c2}	18.2	38/24	43/34	36	44	38	46	27.5 ± 1.7
χ_{c0}	J/ψ	121	152/114	167/117	141	146	172	179	
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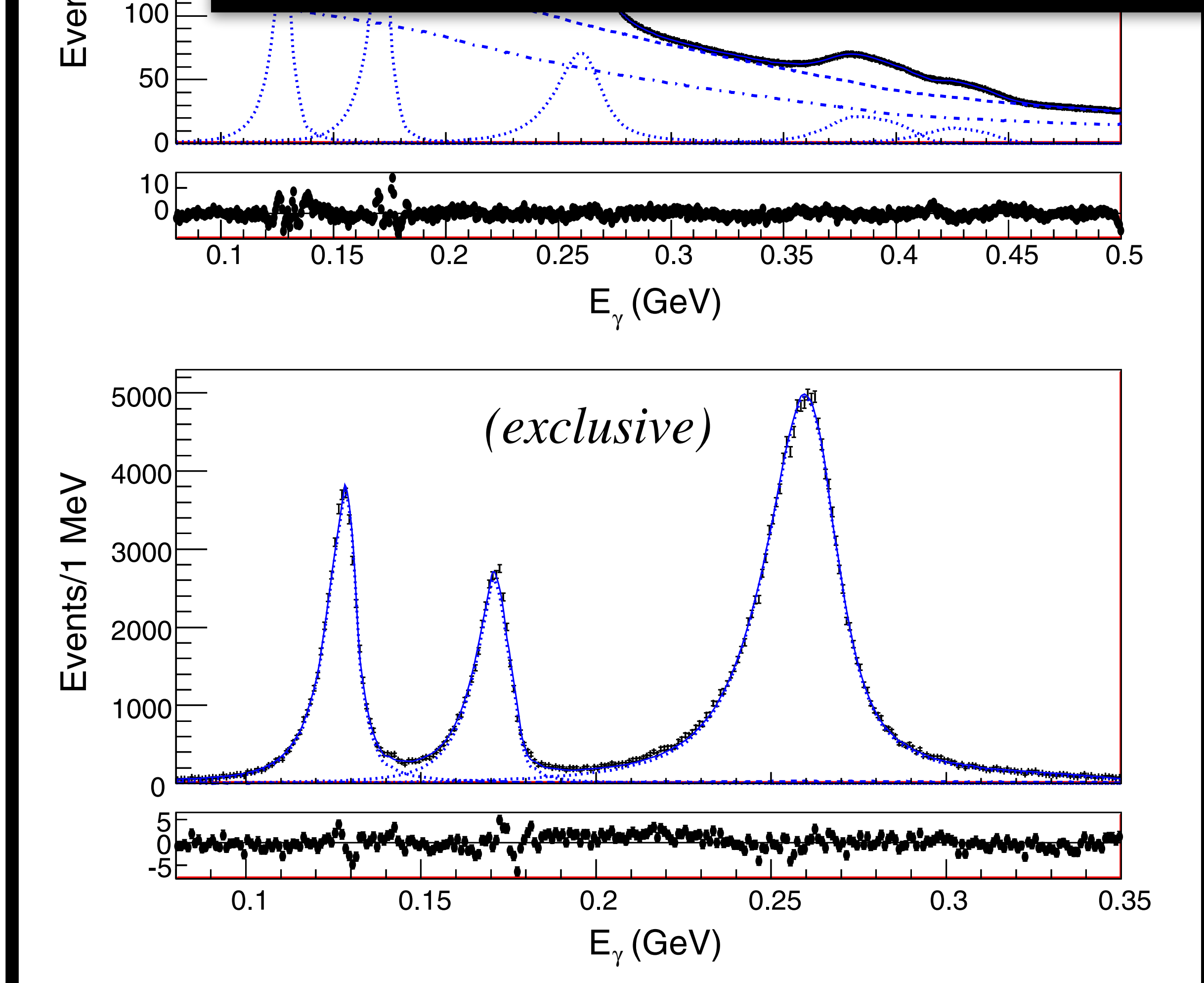
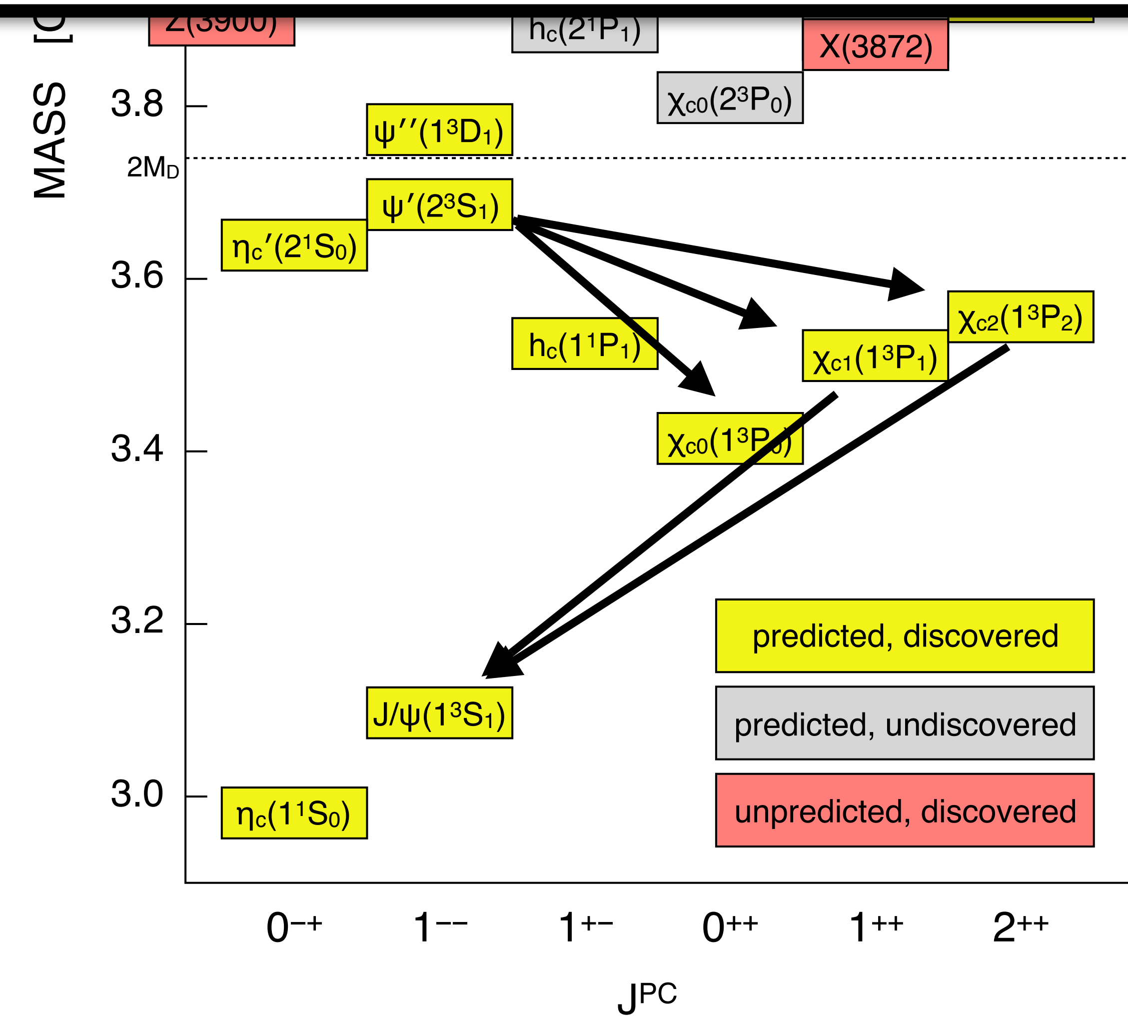
χ_{c2}		327	424/313	473/309	327	338	284	292	363 ± 41





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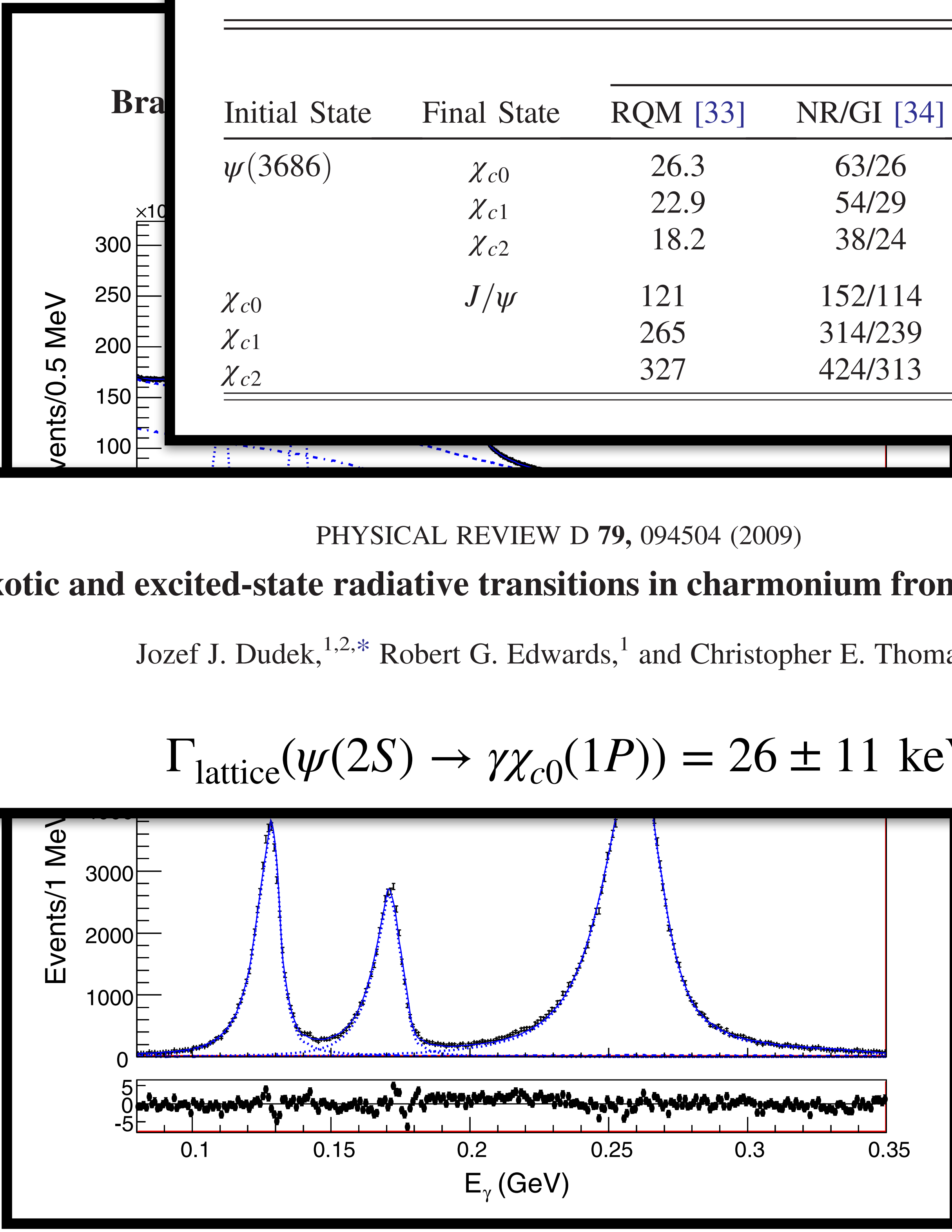
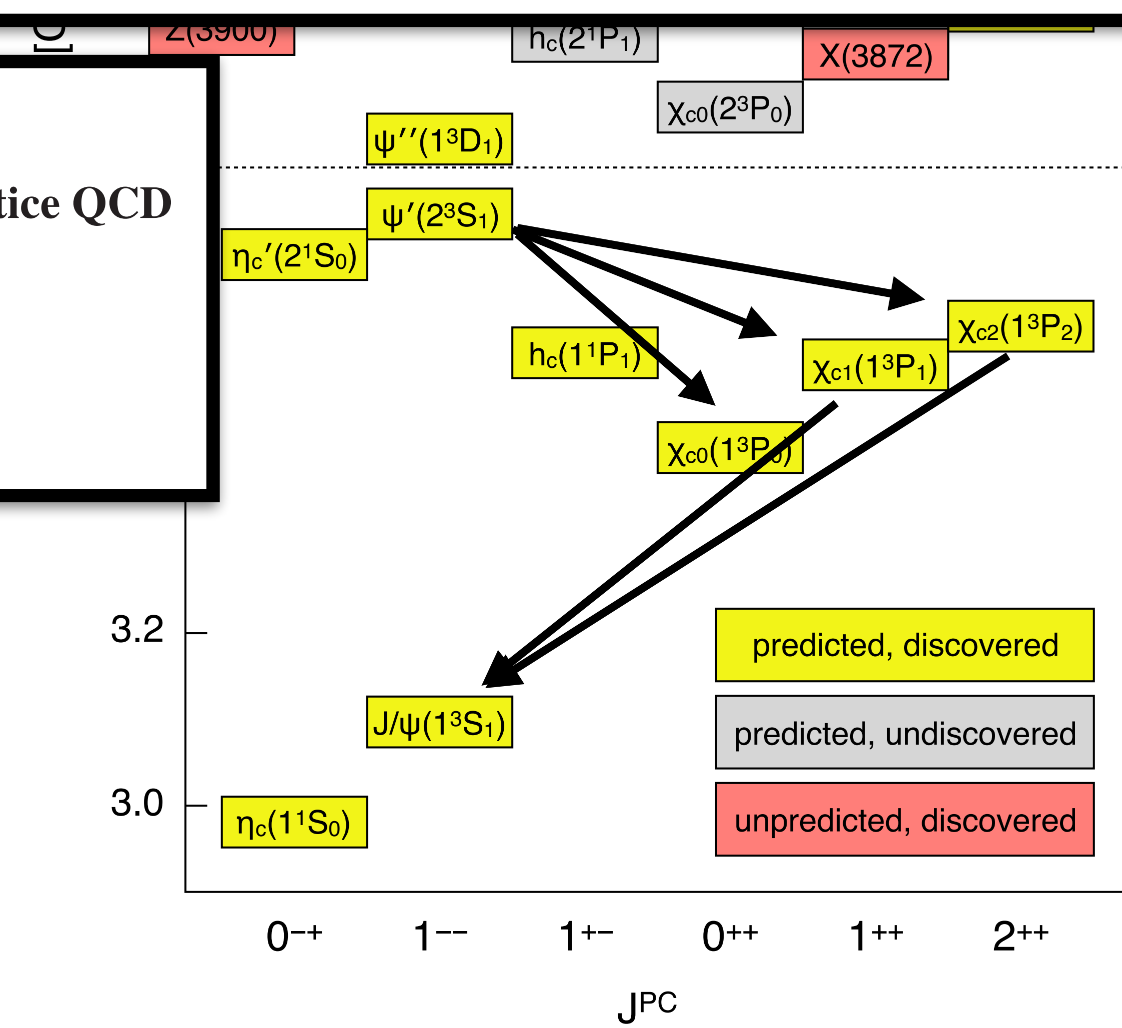
PHYSICAL REVIEW D **79,** 094504 (2009)

Exotic and excited-state radiative transitions in charmonium from lattice QCD

Jozef J. Dudek,[1,2,*] Robert G. Edwards,[1] and Christopher E. Thomas[1]

$$\Gamma_{\rm lattice}(\psi(2S) \to \gamma\chi_{c0}(1P)) = 26 \pm 11\ \mathrm{keV}$$





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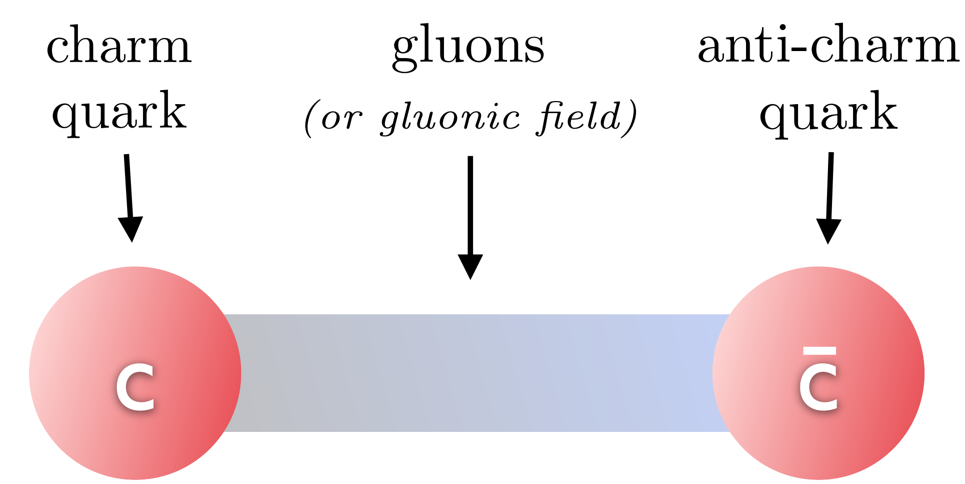
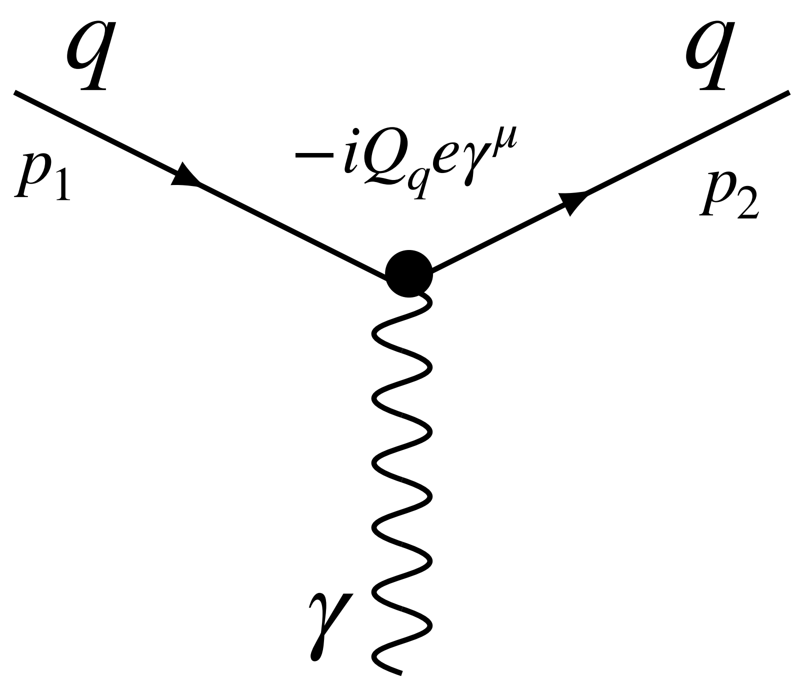
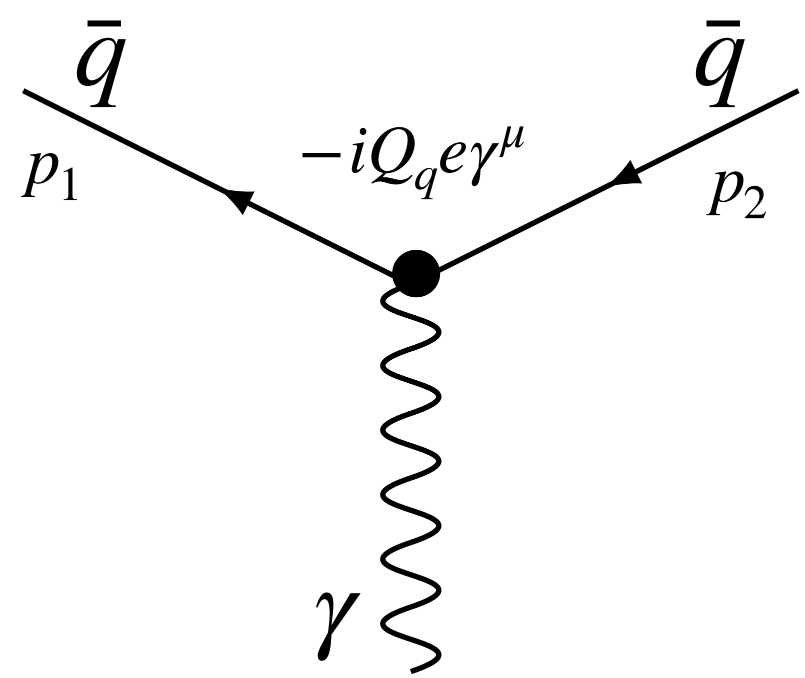


Solve the Schrödinger equation; fix parameters using experiment; predict masses of higher states.

The $q\bar{q}$ (or qq) potential depends on the configuration of colors.

$\boxed{\text{QED}}$





$$j_q^\mu = \bar{u}(p_2)\left[-iQ_q e\gamma^\mu\right] u(p_1) \qquad j_{\bar{q}}^\mu = \bar{\nu}(p_1)\left[-iQ_q e\gamma^\mu\right] \nu(p_2)$$

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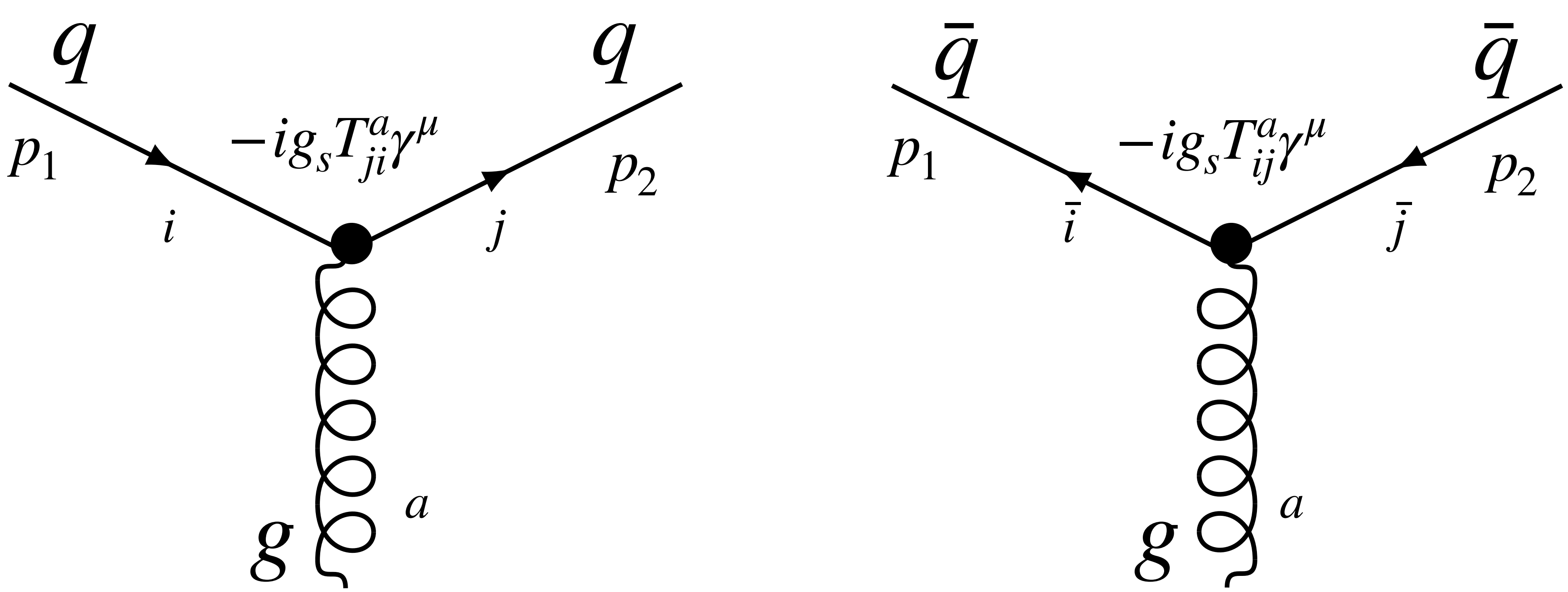
QCD



$$j^\mu_q = \bar{u}(p_2)\left[-ig_s T^a_{ji}\gamma^\mu\right]u(p_1) \qquad j^\mu_{\bar{q}} = \bar{\nu}(p_1)\left[-ig_s T^a_{ij}\gamma^\mu\right]\nu(p_2)$$

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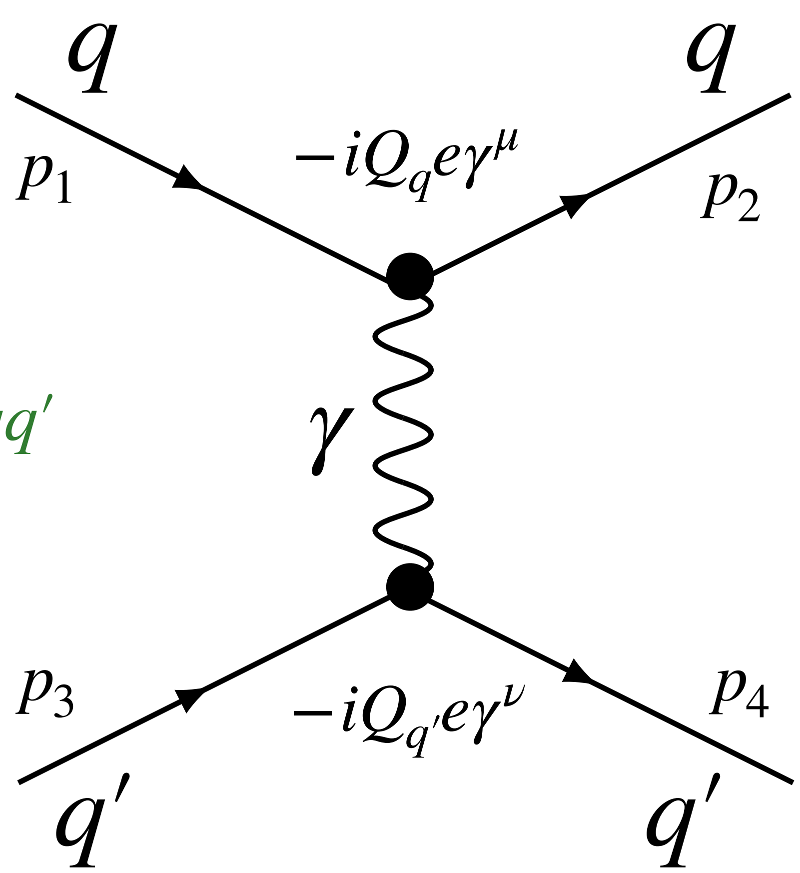
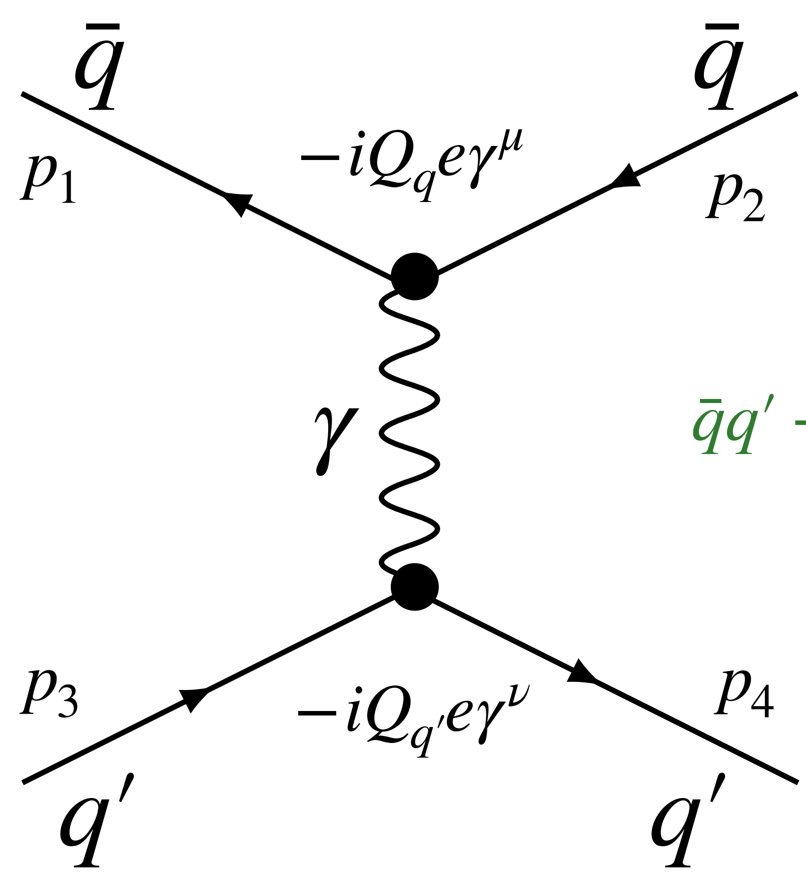
$qq' \to qq'$





$\bar{q}q' \to \bar{q}q'$

$$
\begin{aligned}
-i\mathcal{M} = \quad & \bar{u}(p_2)\left[-iQ_q e\gamma^\mu\right]u(p_1) \\
\times \quad & \left[\frac{-ig_{\mu\nu}}{q^2}\right] \\
\times \quad & \bar{u}(p_4)\left[-iQ_{q'} e\gamma^\nu\right]u(p_3)
\end{aligned}
$$

$$
\begin{aligned}
-i\mathcal{M} = \quad & \bar{v}(p_1)\left[-iQ_q e\gamma^\mu\right]v(p_2) \\
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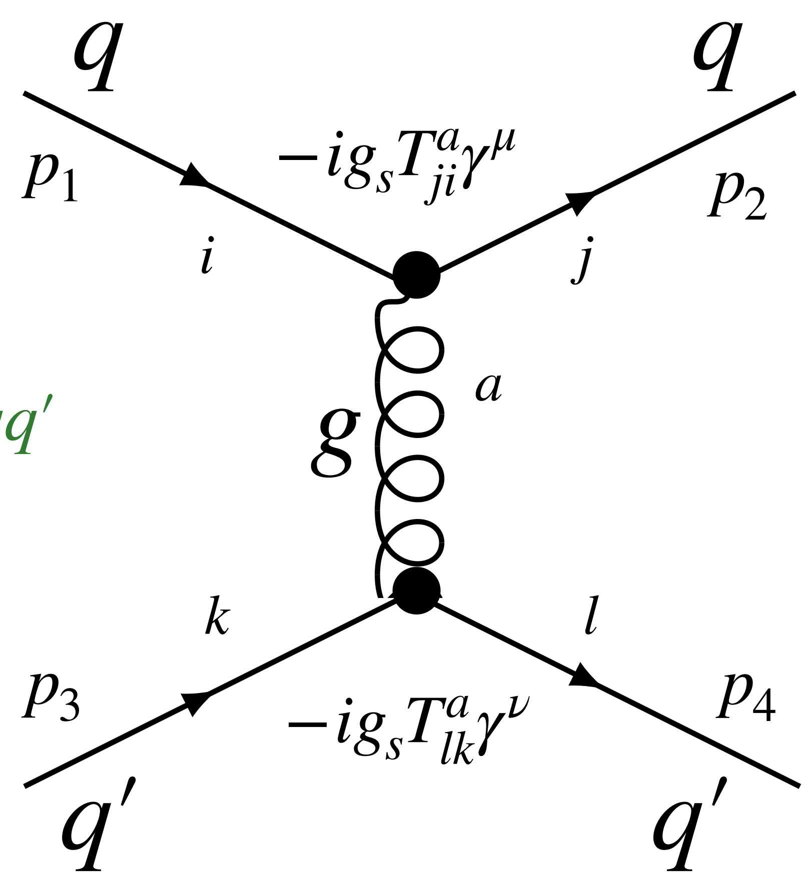
\times \quad & \bar{u}(p_4)\left[-iQ_{q'} e\gamma^\nu\right]u(p_3)
\end{aligned}
$$

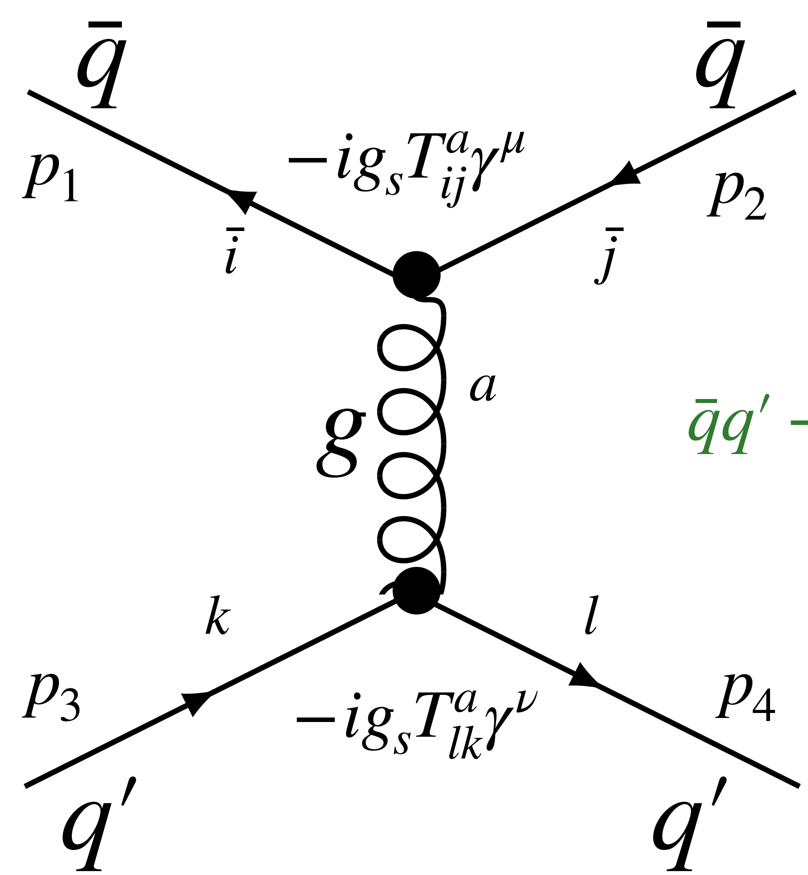
IIIC. Color Factors

The $q\bar{q}$ (or qq) potential depends on the configuration of colors.

QCD

$qq' \to qq'$





$\bar{q}q' \to \bar{q}q'$

$$
\begin{aligned}
-i\mathcal{M} = \; & \bar{u}(p_2)\left[-ig_s T^a_{ji}\gamma^\mu\right] u(p_1) \\
\times \; & \left[\frac{-ig_{\mu\nu}}{q^2}\right] \\
\times \; & \bar{u}(p_4)\left[-ig_s T^a_{lk}\gamma^\nu\right] u(p_3)
\end{aligned}
$$

$$
\begin{aligned}
-i\mathcal{M} = \; & \bar{\nu}(p_1)\left[-ig_s T^a_{ij}\gamma^\mu\right] \nu(p_2) \\
\times \; & \left[\frac{-ig_{\mu\nu}}{q^2}\right] \\
\times \; & \bar{u}(p_4)\left[-ig_s T^a_{lk}\gamma^\nu\right] u(p_3)
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IIIC. Color Factors

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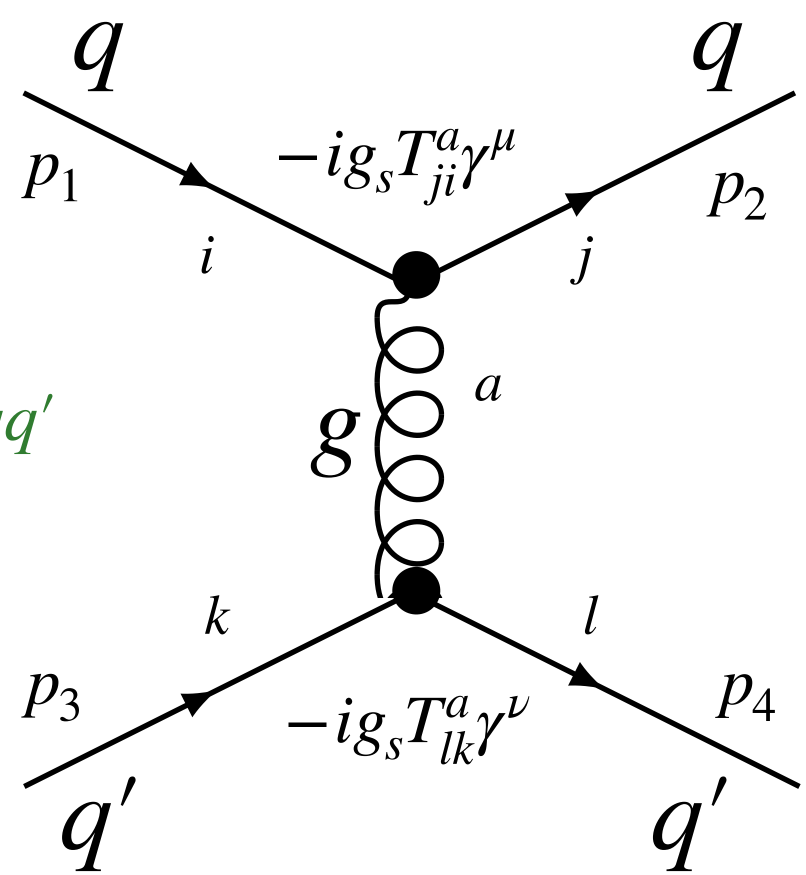

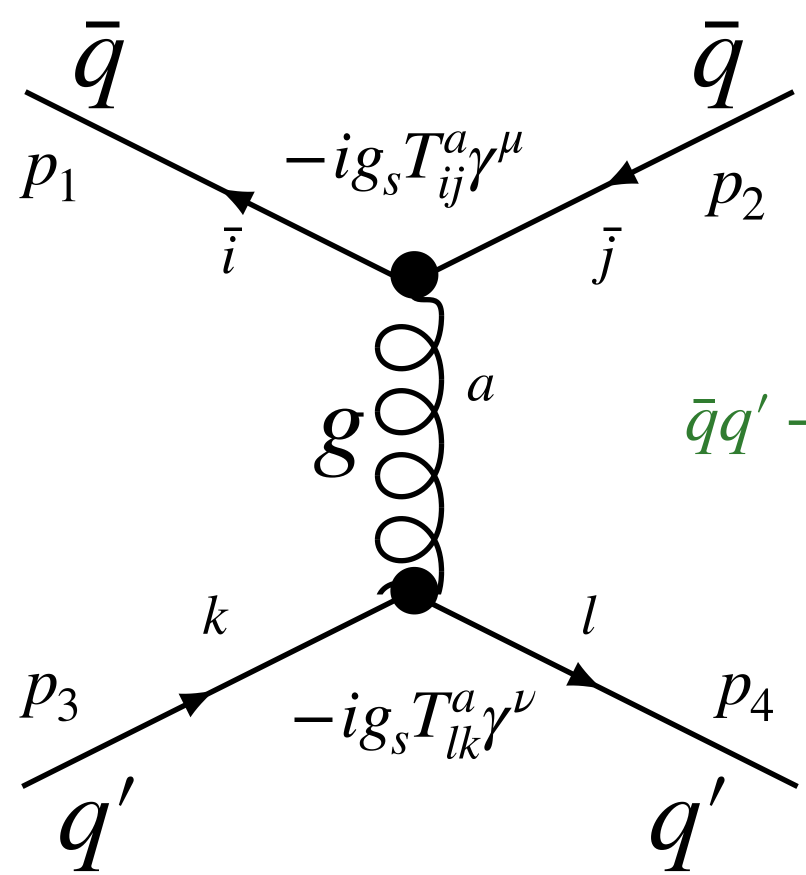

$\bar{q}q' \to \bar{q}q'$

QED $\to$ QCD

$$Q_q Q_{q'} e^2 \to \sum_{a=1}^{8} T^a_{ji} T^a_{lk} g_s^2 = C(ik \to jl) g_s^2 \qquad Q_q Q_{q'} e^2 \to \sum_{a=1}^{8} T^a_{ij} T^a_{lk} g_s^2 = C(\bar{i}k \to \bar{j}l) g_s^2$$

$$C(ik \to jl) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ji} \lambda^a_{lk} \qquad C(\bar{i}k \to \bar{j}l) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{lk}$$

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The $q\bar{q}$ (or qq) potential depends on the configuration of colors.

QED $\rightarrow$ QCD

$$Q_q Q_{q'} e^2 \rightarrow \sum_{a=1}^{8} T^a_{ji} T^a_{lk} g_s^2 = C(ik \rightarrow jl) g_s^2$$

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$$C(\bar{i}k \rightarrow \bar{j}l) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{lk}$$

qq potential:

$$V^{(\mathrm{EM})}_{qq'}(r) = Q_q Q_{q'} \frac{\alpha}{r} + \ldots$$

$$\rightarrow V^{(\mathrm{strong})}_{qq'}(r) = +C \frac{\alpha_s}{r} + \ldots$$

$q\bar{q}$ potential:

$$V^{(\mathrm{EM})}_{\bar{q}q'}(r) = -Q_q Q_{q'} \frac{\alpha}{r} + \ldots$$

$$\rightarrow V^{(\mathrm{strong})}_{\bar{q}q'}(r) = -C \frac{\alpha_s}{r} + \ldots$$

$\Longrightarrow$ The strong force between quarks is attractive or repulsive depending on the color factor C .

IIIC. Color Factors

The $q\bar{q}$ (or qq) potential depends on the configuration of colors.

For $q\bar{q}$ in a color singlet (like a meson):

$$|\mathbf{1}\rangle_{\text{color}} = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} |\bar{c}_i c_i\rangle$$

$$C = \frac{1}{3} \sum_{i,j=1}^{3} C(\bar{i}i \to \bar{j}j)$$

$$= \frac{1}{12} \sum_{i,j=1}^{3} \sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{ji} = \frac{1}{12}\left(3 \times \frac{4}{3} + 6 \times 2\right) = \frac{4}{3}$$

$$Q_q Q_{q'} e^2 \to \sum_{a=1}^{8} T^a_{ij} T^a_{lk} g_s^2 = C(\bar{i}k \to \bar{j}l) g_s^2$$

$$C(\bar{i}k \to \bar{j}l) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{lk}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & \cdot \\ 1 & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & \cdot \\ i & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & \cdot & 1 \\ \cdot & \cdot & \cdot \\ 1 & \cdot & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & \cdot & -i \\ \cdot & \cdot & \cdot \\ i & \cdot & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -2 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & 0 & 1 \\ \cdot & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & 0 & -i \\ \cdot & i & 0 \end{pmatrix}$$

$q\bar{q}$ potential:

$$V^{(\mathrm{EM})}_{\bar{q}q'}(r) = -Q_q Q_{q'} \frac{\alpha}{r} + \ldots$$

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$$Q_q Q_{q'} e^2 \to \sum_{a=1}^{8} T^a_{ij} T^a_{lk} g_s^2 = C(\bar{i}k \to \bar{j}l) g_s^2$$

$$C(\bar{i}k \to \bar{j}l) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{lk}$$

Relative strengths of $q\bar{q}$ and qq potentials:

$$\left(\begin{matrix} \text{for } q\bar{q}, \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \\ \text{for } qq, \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6} \end{matrix}\right)$$

state	color	size
$q\bar{q}$	$\mathbf{1}$	$-4/3$
$q\bar{q}$	$\mathbf{8}$	$+1/6$
qq	$\bar{\mathbf{3}}$	$-2/3$
qq	$\mathbf{6}$	$+1/3$

$q\bar{q}$ potential:

$$V^{(\text{EM})}_{\bar{q}q'}(r) = -Q_q Q_{q'} \frac{\alpha}{r} + \ldots$$

$$\to V^{(\text{strong})}_{\bar{q}q'}(r) = -C \frac{\alpha_s}{r} + \ldots$$

$\Longrightarrow$ The strong force between quarks is attractive or repulsive depending on the color factor C.

Quark Model: Assume hadrons are made of quarks interacting via a potential.

One example:

PHYSICAL REVIEW D **72,** 054026 (2005)

Higher charmonia

T. Barnes,[1,*] S. Godfrey,[2,†] and E. S. Swanson[3,‡]

For states with $L = 0$ and $n = 1$, mass splittings for different S can be modeled by the spin-spin term (compare $\eta_c(1S)$ and $J/\psi(1S)$).

spin-spin (hyperfine)

$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \boxed{\frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}_\sigma(r)\vec{\mathrm{S}}_c\cdot\vec{\mathrm{S}}_{\bar{c}}}$$

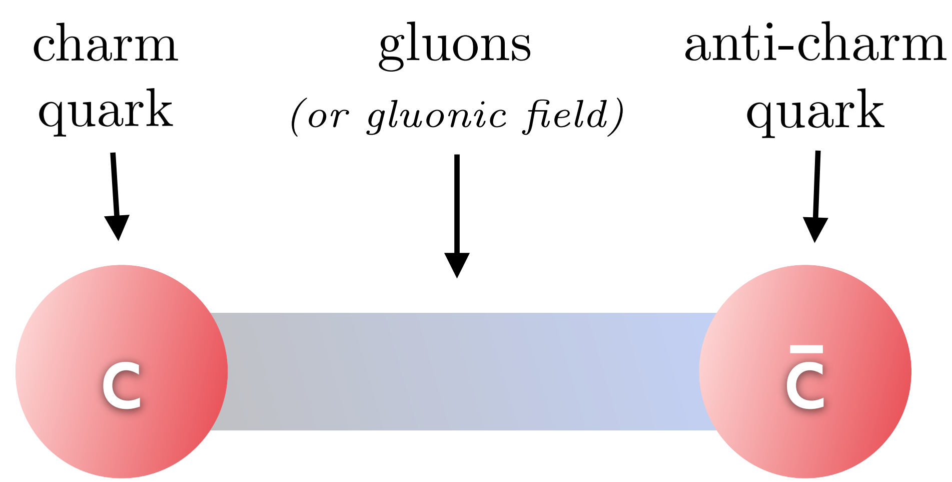
"Coulomb" confinement

$$\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$$

$$V_{\text{spin-dep}} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}} + \frac{4\alpha_s}{r^3}\mathrm{T}\right]$$

spin-orbit (fine) tensor (hyperfine)

$$\langle {}^3\mathrm{L_J}|\mathrm{T}|{}^3\mathrm{L_J}\rangle = \begin{cases} -\frac{\mathrm{L}}{6(2\mathrm{L}+3)}, & \mathrm{J} = \mathrm{L}+1 \\ +\frac{1}{6}, & \mathrm{J} = \mathrm{L} \\ -\frac{(\mathrm{L}+1)}{6(2\mathrm{L}-1)}, & \mathrm{J} = \mathrm{L}-1 \end{cases}$$



Solve the Schrödinger equation; fix parameters using experiment; predict masses of higher states.

IIID. Doubly-Bottom Tetraquark

Use the quark model to predict the mass of a doubly-bottom tetraquark ($bb\bar{u}\bar{d}$).

PRL **119**, 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

1. Use light quark mesons and baryons to calculate the effective masses of the up, down, and strange quarks.

2. Use open charm and open bottom mesons and baryons (D, D^*, B, B^*, Λ_b, Λ_c) to calculate the effective masses of charm and bottom quarks.

3. Use charmonium and bottomonium mesons ($\eta_c(1S)$, $J/\psi(1S)$, $\eta_b(1S)$, $\Upsilon(1S)$) to calculate $c\bar{c}$ and $b\bar{b}$ binding energies.

4. Use color factors to relate $c\bar{c}$ and $b\bar{b}$ binding energies ($\mathbf{1}$) to cc and bb binding energies ($\bar{\mathbf{3}}$).

5. Use the results above to predict the mass of the doubly charmed Ξ_{cc} baryon.

6. Use the same method to predict the mass of a doubly bottom tetraquark ($bb\bar{u}\bar{d}$).

IIID. Doubly-Bottom Tetraquark

1. Use light quark mesons and baryons to calculate the effective masses of the up, down, and strange quarks.

Hadron spectra and quarks

Stephen Gasiorowicz and Jonathan L. Rosner
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

Am. J. Phys. **49**(10), Oct. 1981

Table VIII. Meson masses with hyperfine splittings incorporated.

Meson	Coeff. of m_u or m_d	Coeff. of m_s	ΔE^{HFS}	Prediction (MeV/c^2)
$\pi(138)$	2	0	$-3a/m_u^2$	140
$K(496)$	1	1	$-3a/m_u m_s$	485
$\eta(549)$	2/3	4/3	$-a/m_u^2-2a/m_s^2$	559
$\rho(776)$, $\omega(783)$	2	0	a/m_u^2	780
$K^*(892)$	1	1	$a/m_u m_s$	896
$\phi(1020)$	0	2	a/m_s^2	1032

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$\Omega(1672)$	0	3	$3a'/m_s^2$	1682

For mesons:

$$M^{(m)} = m_1^{(m)} + m_2^{(m)} + 4a\frac{\langle \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2\rangle}{m_1^{(m)} m_2^{(m)}}$$

Use:

$$\langle \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2\rangle = \frac{1}{2}\left[\langle \hat{J}^2\rangle - \langle \hat{S}_1^2\rangle - \langle \hat{S}_2^2\rangle\right]$$

$$= \frac{1}{2}\left[J(J+1) - \frac{3}{2}\right]$$

$$= \begin{cases} -\frac{3}{4} & \text{for } J=0 \\ +\frac{1}{4} & \text{for } J=1 \end{cases}$$

And find a good match with:

$$m_u^{(m)} = m_d^{(m)} = 310 \text{ MeV}$$

$$m_s^{(m)} = 483 \text{ MeV}$$

$$\frac{a}{(m_u^{(m)})^2} = 160 \text{ MeV}$$

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For baryons:

$$M^{(b)} = m_1^{(b)} + m_2^{(b)} + m_3^{(b)} + 4a' \left[\frac{\langle \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2 \rangle}{m_1^{(b)} m_2^{(b)}} + \frac{\langle \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_3 \rangle}{m_1^{(b)} m_3^{(b)}} + \frac{\langle \hat{\vec{S}}_2 \cdot \hat{\vec{S}}_3 \rangle}{m_2^{(b)} m_3^{(b)}} \right]$$

For $\Xi(1318)$ (ssd or ssu) with $J = \dfrac{1}{2}$:

$$S_{ss} = 1 \implies \langle \hat{\vec{S}}_{s1} \cdot \hat{\vec{S}}_{s2} \rangle = +\frac{1}{4}$$

total ss wavefunction is antisymmetric
and ss color is antisymmetric ($\bar{\mathbf{3}}$)
and ss isospin is symmetric ($I = 0$)
$\Longrightarrow$ ss spin is symmetric $\Longrightarrow$ $S_{ss} = 1$

$$2\,\langle \hat{\vec{S}}_s \cdot \hat{\vec{S}}_u \rangle = \frac{1}{2}\left[\langle \hat{J}^2 \rangle - 2\,\langle \hat{S}_s^2 \rangle - \langle \hat{S}_u^2 \rangle - 2\,\langle \hat{\vec{S}}_{s1} \cdot \hat{\vec{S}}_{s2} \rangle \right] = \frac{1}{2}\left[J(J+1) - 3(\frac{3}{4}) - 2(\frac{1}{4}) \right] = -1$$

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For $\Lambda(1116)$ (uds) with $J = \dfrac{1}{2}$:

$$S_{ud} = 0 \implies \langle \hat{\vec{S}}_u \cdot \hat{\vec{S}}_d \rangle = -\frac{3}{4}$$

total ud wavefunction is antisymmetric
and ud color is antisymmetric ($\bar{\mathbf{3}}$)
and ud isospin is antisymmetric ($I = 0$)
$\implies$ ud spin is antisymmetric $\implies$ $S_{ud} = 0$

$$2\,\langle \hat{\vec{S}}_s \cdot \hat{\vec{S}}_u \rangle = \frac{1}{2}\left[\langle \hat{J}^2 \rangle - 2\,\langle \hat{S}_u^2 \rangle - \langle \hat{S}_s^2 \rangle - 2\,\langle \hat{\vec{S}}_u \cdot \hat{\vec{S}}_d \rangle\right]$$
$$= \frac{1}{2}\left[J(J+1) - 3(\frac{3}{4}) + 2(\frac{3}{4})\right] = 0$$

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And find a good match with:

$$m_u^{(b)} = m_d^{(b)} = 363 \text{ MeV}$$

$$m_s^{(b)} = 538 \text{ MeV}$$

$$\frac{a'}{(m_u^{(b)})^2} = 50 \text{ MeV}$$

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Am. J. Phys. **49**(10), Oct. 1981

PHYSICAL REVIEW D **90**, 094007 (2014)

Baryons with two heavy quarks: Masses, production, decays, and detection

Marek Karliner*
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Table VIII. Meson masses with hyperfine splittings incorporated.

Meson	Coeff. of m_u or m_d	Coeff. of m_s	ΔE^{HFS}	Prediction (MeV/c^2)
$\pi(138)$	2	0	$-3a/m_u^2$	140
$K(496)$	1	1	$-3a/m_u m_s$	485
$\eta(549)$	2/3	4/3	$-a/m_u^2 - 2a/m_s^2$	559
$\rho(776)$, $\omega(783)$	2	0	a/m_u^2	780
$K^*(892)$	1	1	$a/m_u m_s$	896
$\phi(1020)$	0	2	a/m_s^2	1032

Table IX. Baryon masses with hyperfine splittings incorporated.

Baryon	Coeff. of m_u or m_d	Coeff. of m_s	ΔE^{HFS}	Prediction (MeV/c^2)
$N(939)$	3	0	$-3a'/m_u^2$	939
$\Lambda(1116)$	2	1	$-3a'/m_u^2$	1114
$\Sigma(1193)$	2	1	$a'/m_u^2 - 4a'/m_u m_s$	1179
$\Xi(1318)$	1	2	$a'/m_s^2 - 4a'/m_u m_s$	1327
$\Delta(1232)$	3	0	$3a'/m_u^2$	1239
$\Sigma^*(1384)$	2	1	$a'/m_u^2 + 2a'/m_u m_s$	1381
$\Xi^*(1533)$	1	2	$a'/m_s^2 + 2a'/m_u m_s$	1529
$\Omega(1672)$	0	3	$3a'/m_s^2$	1682

For mesons, use $D, D^*\ (c\bar{u}), B, B^*\ (b\bar{u})$:

$$m_c^{(m)} = \frac{1}{4}\left[3M(D^*) + M(D)\right] - m_u^{(m)} = 1663.3\ \mathrm{MeV}$$

$$m_b^{(m)} = \frac{1}{4}\left[3M(B^*) + 3M(B)\right] - m_u^{(m)} = 5003.8\ \mathrm{MeV}$$

For baryons, use $\Lambda_c\ (udc), \Lambda_b\ (udb)$:

$$m_c^{(b)} = M(\Lambda_c) - 2m_u^{(b)} + \frac{3a'}{(m_u^{(b)})^2} = 1710.5\ \mathrm{MeV}$$

$$m_b^{(b)} = M(\Lambda_b) - 2m_u^{(b)} + \frac{3a'}{(m_u^{(b)})^2} = 5043.5\ \mathrm{MeV}$$

IIID. Doubly-Bottom Tetraquark

3. Use charmonium and bottomonium mesons to calculate $c\bar{c}$ and $b\bar{b}$ binding energies.

For charmonium and bottomonium, allow for a tighter binding (due to the smaller radius) by adding an additional binding energy ($B_{c\bar{c}}$ and $B_{b\bar{b}}$) and hyperfine coupling ($a_{c\bar{c}}$ and $a_{b\bar{b}}$).

$$
\begin{aligned}
M_{c\bar{c}}^{(m)} &= B_{c\bar{c}} + 2m_c^{(m)} + 4a_{c\bar{c}}\frac{\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_{\bar{c}}\rangle}{(m_c^{(m)})^2} \\
&= B_{c\bar{c}} + 2m_c^{(m)} + \begin{cases} \frac{-3a_{c\bar{c}}}{(m_c^{(m)})^2} & \text{for } J=0 \qquad (\eta_c(1S), \eta_b(1S)) \\ \frac{+a_{c\bar{c}}}{(m_c^{(m)})^2} & \text{for } J=1 \qquad (J/\psi(1S), \Upsilon(1S)) \end{cases}
\end{aligned}
$$

For charmonium:

$$
\begin{aligned}
B_{c\bar{c}} &= \frac{1}{4}\left[3M(J/\psi) + M(\eta_c)\right] - 2m_c^{(m)} \\
&= -258.0 \text{ MeV}
\end{aligned}
$$

$$
\begin{aligned}
\frac{a_{c\bar{c}}}{(m_c^{(m)})^2} &= \frac{1}{4}\left[M(J/\psi) - M(\eta_c)\right] \\
&= 28.4 \text{ MeV}
\end{aligned}
$$

For bottomonium:

$$
\begin{aligned}
B_{b\bar{b}} &= \frac{1}{4}\left[3M(\Upsilon(1S)) + M(\eta_b)\right] - 2m_b^{(m)} \\
&= -562.8 \text{ MeV}
\end{aligned}
$$

$$
\begin{aligned}
\frac{a_{b\bar{b}}}{(m_b^{(m)})^2} &= \frac{1}{4}\left[M(\Upsilon(1S)) - M(\eta_b)\right] \\
&= 15.6 \text{ MeV}
\end{aligned}
$$

IIID. Doubly-Bottom Tetraquark

4. Use color factors to relate $c\bar{c}$ and $b\bar{b}$ binding energies ($\mathbf{1}$) to cc and bb binding energies ($\bar{\mathbf{3}}$).

Using $V^{\bar{\mathbf{3}}}_{qq} = \frac{1}{2} V^{\mathbf{1}}_{q\bar{q}}$ and neglecting the small differences between $m_c^{(m)}$ and $m_c^{(b)}$ and between $m_b^{(m)}$ and $m_b^{(b)}$:

$$B_{cc} = \frac{1}{2} B_{c\bar{c}} = -129.0 \text{ MeV}$$

$$\frac{a_{cc}}{(m_c^{(b)})^2} \approx \frac{1}{2} \frac{a_{c\bar{c}}}{(m_c^{(m)})^2} = 14.2 \text{ MeV}$$

$$B_{bb} = \frac{1}{2} B_{b\bar{b}} = -281.4 \text{ MeV}$$

$$\frac{a_{bb}}{(m_b^{(b)})^2} \approx \frac{1}{2} \frac{a_{b\bar{b}}}{(m_b^{(m)})^2} = 7.8 \text{ MeV}$$

For charmonium:

$$B_{c\bar{c}} = \frac{1}{4} [3M(J/\psi) + M(\eta_c)] - 2m_c^{(m)} = -258.0 \text{ MeV}$$

$$\frac{a_{c\bar{c}}}{(m_c^{(m)})^2} = \frac{1}{4} [M(J/\psi) - M(\eta_c)] = 28.4 \text{ MeV}$$

For bottomonium:

$$B_{b\bar{b}} = \frac{1}{4} [3M(\Upsilon(1S)) + M(\eta_b)] - 2m_b^{(m)} = -562.8 \text{ MeV}$$

$$\frac{a_{b\bar{b}}}{(m_b^{(m)})^2} = \frac{1}{4} [M(\Upsilon(1S)) - M(\eta_b)] = 15.6 \text{ MeV}$$

IIID. Doubly-Bottom Tetraquark

5. Use the previous results to predict the mass of the doubly charmed Ξ_{cc} baryon.

PHYSICAL REVIEW D **90,** 094007 (2014)

Baryons with two heavy quarks: Masses, production, decays, and detection

Marek Karliner*

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Jonathan L. Rosner†

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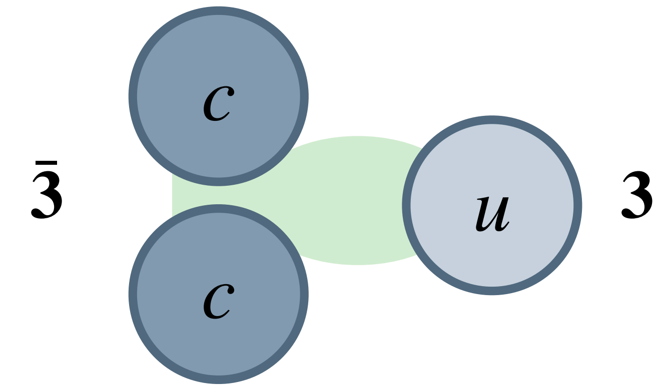
Ξ_{cc}^{++} baryon:



$$J^P = \frac{1}{2}^+ \qquad S_{cc} = 1 \qquad S_u = \frac{1}{2} \qquad \langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle = +\frac{1}{4} \qquad 2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle = -1$$

Combining all the pieces:

$$\begin{aligned} M(\Xi_{cc}^{++}) &= 2m_c^{(b)} + m_u^{(b)} + B_{cc} + 4a_{cc}\left[\frac{\langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle}{(m_c^{(b)})^2}\right] + 4a'\left[\frac{2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle}{m_c^{(b)} m_u^{(b)}}\right] \\ &= 2m_c^{(b)} + m_u^{(b)} + B_{cc} + \frac{a_{cc}}{(m_c^{(b)})^2} - \frac{4a'}{m_c^{(b)} m_u^{(b)}} \\ &= [2(1710.5) + 363 - 129 + 14.2 - 4(50)(363)/1710.5]\ \text{MeV} \end{aligned}$$

$$M(\Xi_{cc}^{++}) \ = 3627 \pm 12\ \text{MeV}$$

also calculate single-charm and single-bottom baryons with the same method and compare to experiment
$\Longrightarrow\ \approx 12$ MeV uncertainty

IIID. Doubly-Bottom Tetraquark

5. Use the previous results to predict the mass of the doubly charmed Ξ_{cc} baryon.

PHYSICAL REVIEW D **90**, 094007 (2014)

Baryons with two heavy quarks: Masses, production, decays, and detection

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Ξ_{cc}^{++} baryon:

$$S_{cc} = 1 \quad \Longrightarrow \quad \langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle = +\frac{1}{4}$$

total cc wavefunction is antisymmetric
and cc color is antisymmetric ($\bar{\mathbf{3}}$)
and cc isospin is symmetric ($I = 0$)
$\Longrightarrow$ cc spin is symmetric $\Longrightarrow$ $S_{cc} = 1$

$$2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle = \frac{1}{2}\left[\langle \hat{J}^2 \rangle - 2\,\langle \hat{S}_c^2 \rangle - \langle \hat{S}_u^2 \rangle - 2\,\langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle\right]$$

$$= \frac{1}{2}\left[J(J+1) - 3(\frac{3}{4}) - 2(\frac{1}{4})\right] = -1$$

Combining all the pieces:

$$M(\Xi_{cc}^{++}) = 2m_c^{(b)} + m_u^{(b)} + B_{cc} + 4a_{cc}\left[\frac{\langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle}{(m_c^{(b)})^2}\right] + 4a'\left[\frac{2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle}{m_c^{(b)} m_u^{(b)}}\right]$$

$$= 2m_c^{(b)} + m_u^{(b)} + B_{cc} + \frac{a_{cc}}{(m_c^{(b)})^2} - \frac{4a'}{m_c^{(b)} m_u^{(b)}}$$

$$= [2(1710.5) + 363 - 129 + 14.2 - 4(50)(363)/1710.5] \ \text{MeV}$$

$$M(\Xi_{cc}^{++}) \ = 3627 \pm 12 \ \text{MeV}$$

also calculate single-charm and single-bottom baryons
with the same method and compare to experiment
$\Longrightarrow$ $\approx$ 12 MeV uncertainty

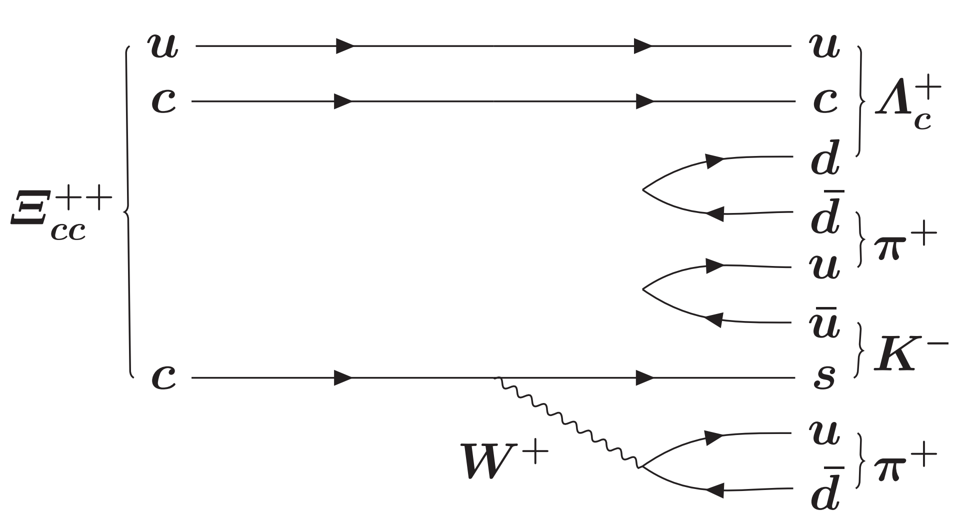
IIID. Doubly-Bottom Tetraquark

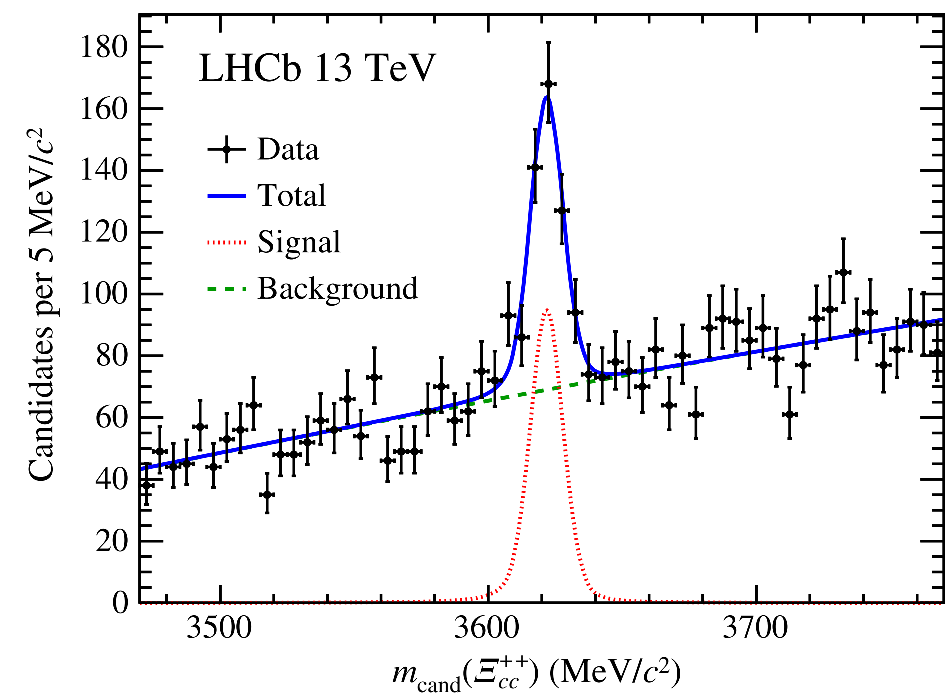
5. Use the previous results to predict the mass of the doubly charmed Ξ_{cc} baryon.

Selected for a Viewpoint in *Physics*
PRL **119,** 112001 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.**
(LHCb Collaboration)





$$M(\Xi_{cc}^{++}) = 3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}$$

$$M(\Xi_{cc}^{++}) = 3627 \pm 12\ \text{MeV}$$

also calculate single-charm and single-bottom baryons with the same method and compare to experiment
$\Longrightarrow\ \approx 12$ MeV uncertainty

IIID. Doubly-Bottom Tetraquark

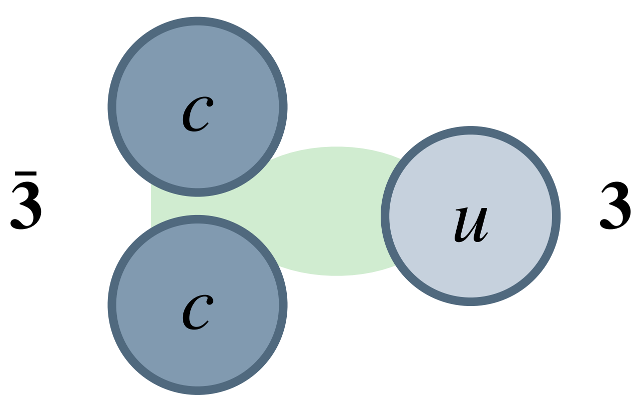
6. Use the same method to predict the mass of a doubly bottom tetraquark ($bb\bar{u}\bar{d}$).

PRL **119,** 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

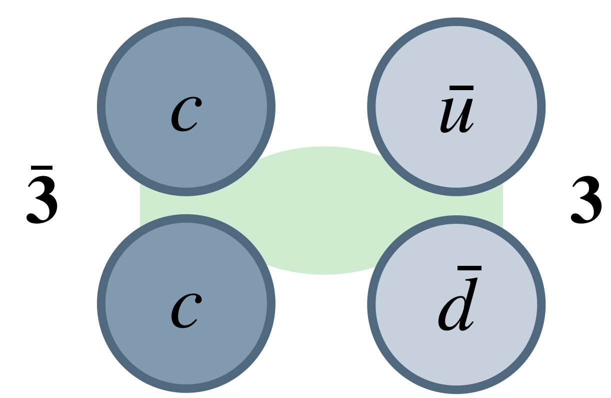
Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

Ξ_{cc}^{++} baryon:



$\Longrightarrow$

"T_{cc}^{+}" tetraquark (meson):



$$J^P = \frac{1}{2}^+ \qquad S_{cc} = 1 \qquad \langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle = +\frac{1}{4}$$

$$S_u = \frac{1}{2} \qquad 2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle = -1$$

$$J^P = 1^+ \qquad S_{cc} = 1 \qquad \langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle = +\frac{1}{4}$$

$$S_{\bar{u}\bar{d}} = 0 \qquad \langle \hat{\vec{S}}_{\bar{u}} \cdot \hat{\vec{S}}_{\bar{d}} \rangle = -\frac{3}{4}$$

$$\begin{aligned} M(\Xi_{cc}^{++}) &= 2m_c^{(b)} + m_u^{(b)} + B_{cc} + 4a_{cc}\left[\frac{\langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle}{(m_c^{(b)})^2}\right] + 4a'\left[\frac{2\,\langle \hat{\vec{S}}_c \cdot \hat{\vec{S}}_u \rangle}{m_c^{(b)} m_u^{(b)}}\right] \\ &= 2m_c^{(b)} + m_u^{(b)} + B_{cc} + \frac{a_{cc}}{(m_c^{(b)})^2} - \frac{4a'}{m_c^{(b)} m_u^{(b)}} \\ &= [2(1710.5) + 363 - 129 + 14.2 - 4(50)(363)/1710.5]\ \text{MeV} \end{aligned}$$

$$M(\Xi_{cc}^{++}) \;= 3627 \pm 12\ \text{MeV}$$

$$\begin{aligned} M(T_{cc}^{+}) &= 2m_c^{(b)} + 2m_u^{(b)} + B_{cc} + 4a_{cc}\left[\frac{\langle \hat{\vec{S}}_{c1} \cdot \hat{\vec{S}}_{c2} \rangle}{(m_c^{(b)})^2}\right] + 4a'\left[\frac{\langle \hat{\vec{S}}_{\bar{u}} \cdot \hat{\vec{S}}_{\bar{d}} \rangle}{(m_u^{(b)})^2}\right] \\ &= 2m_c^{(b)} + 2m_u^{(b)} + B_{cc} + \frac{a_{cc}}{(m_c^{(b)})^2} - \frac{3a'}{(m_u^{(b)})^2} \\ &= [2(1710.5) + 2(363) - 129 + 14.2 - 3(50)]\ \text{MeV} \end{aligned}$$

$$M(T_{cc}^{+}) = 3882 \pm 12\ \text{MeV}$$

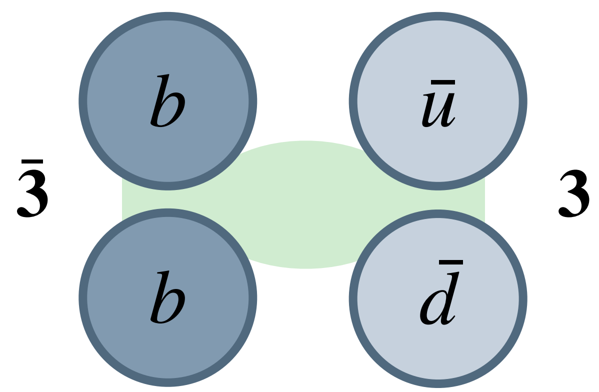
6. Use the same method to predict the mass of a doubly bottom tetraquark ($bb\bar{u}\bar{d}$).

PRL **119**, 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

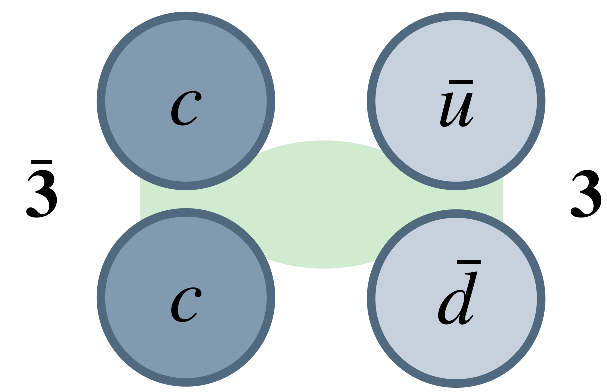
Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

"T_{bb}^{-}" tetraquark (meson):



$\Longleftarrow$

"T_{cc}^{+}" tetraquark (meson):



$$J^P = 1^+ \qquad \begin{aligned} S_{bb} &= 1 \\ S_{\bar{u}\bar{d}} &= 0 \end{aligned} \qquad \begin{aligned} \langle \hat{\vec{S}}_{b1}\cdot\hat{\vec{S}}_{b2}\rangle &= +\frac{1}{4} \\ \langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle &= -\frac{3}{4} \end{aligned}$$

$$J^P = 1^+ \qquad \begin{aligned} S_{cc} &= 1 \\ S_{\bar{u}\bar{d}} &= 0 \end{aligned} \qquad \begin{aligned} \langle \hat{\vec{S}}_{c1}\cdot\hat{\vec{S}}_{c2}\rangle &= +\frac{1}{4} \\ \langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle &= -\frac{3}{4} \end{aligned}$$

$$\begin{aligned} M(T_{bb}^{-}) &= 2m_b^{(b)} + 2m_u^{(b)} + B_{bb} + 4a_{bb}\left[\frac{\langle \hat{\vec{S}}_{b1}\cdot\hat{\vec{S}}_{b2}\rangle}{(m_b^{(b)})^2}\right] + 4a'\left[\frac{\langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle}{(m_u^{(b)})^2}\right] \\ &= 2m_b^{(b)} + 2m_u^{(b)} + B_{bb} + \frac{a_{bb}}{(m_b^{(b)})^2} - \frac{3a'}{(m_u^{(b)})^2} \\ &= [2(5043.5) + 2(363) - 281.4 + 7.8 - 3(50)]\ \text{MeV} \end{aligned}$$

$$M(T_{bb}^{-}) = 10389.4 \pm 12\ \text{MeV}$$

$$\begin{aligned} M(T_{cc}^{+}) &= 2m_c^{(b)} + 2m_u^{(b)} + B_{cc} + 4a_{cc}\left[\frac{\langle \hat{\vec{S}}_{c1}\cdot\hat{\vec{S}}_{c2}\rangle}{(m_c^{(b)})^2}\right] + 4a'\left[\frac{\langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle}{(m_u^{(b)})^2}\right] \\ &= 2m_c^{(b)} + 2m_u^{(b)} + B_{cc} + \frac{a_{cc}}{(m_c^{(b)})^2} - \frac{3a'}{(m_u^{(b)})^2} \\ &= [2(1710.5) + 2(363) - 129 + 14.2 - 3(50)]\ \text{MeV} \end{aligned}$$

$$M(T_{cc}^{+}) = 3882 \pm 12\ \text{MeV}$$

IIID. Doubly-Bottom Tetraquark

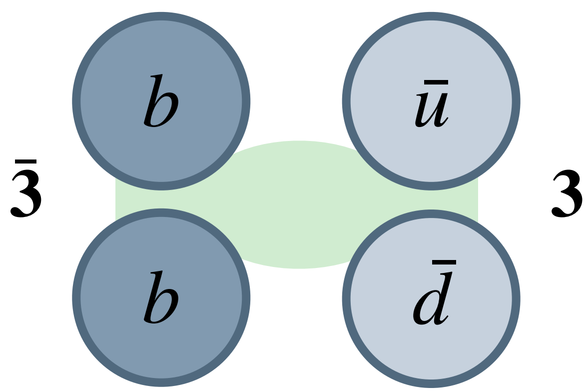
6. Use the same method to predict the mass of a doubly bottom tetraquark ($bb\bar{u}\bar{d}$).

PRL **119,** 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

"T_{bb}^{-}" tetraquark (meson):



$$J^P = 1^+ \qquad S_{bb} = 1 \quad \langle \hat{\vec{S}}_{b1}\cdot\hat{\vec{S}}_{b2}\rangle = +\frac{1}{4}$$
$$S_{\bar{u}\bar{d}} = 0 \quad \langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle = -\frac{3}{4}$$

$$M(T_{bb}^{-}) = 2m_b^{(b)} + 2m_u^{(b)} + B_{bb} + 4a_{bb}\left[\frac{\langle \hat{\vec{S}}_{b1}\cdot\hat{\vec{S}}_{b2}\rangle}{(m_b^{(b)})^2}\right] + 4a'\left[\frac{\langle \hat{\vec{S}}_{\bar{u}}\cdot\hat{\vec{S}}_{\bar{d}}\rangle}{(m_u^{(b)})^2}\right]$$
$$= 2m_b^{(b)} + 2m_u^{(b)} + B_{bb} + \frac{a_{bb}}{(m_b^{(b)})^2} - \frac{3a'}{(m_u^{(b)})^2}$$
$$= [2(5043.5) + 2(363) - 281.4 + 7.8 - 3(50)]\ \text{MeV}$$

$$M(T_{bb}^{-}) = 10389.4 \pm 12\ \text{MeV}$$

The lightest non-weak decays that conserves flavor and J^P are:

$$T_{bb}^{-} \to \gamma B^{-}\bar{B}^{0} \quad \text{(EM)}$$
$$T_{bb}^{-} \to B^{*-}\bar{B}^{0} \quad \text{(strong)}$$

but $M(B^-) + M(\bar{B}^0) = 10559$ MeV
and $M(B^{*-}) + M(\bar{B}^0) = 10604$ MeV

$\Longrightarrow$ T_{bb}^{-} would decay weakly

Lifetime would be ≈ 400 fs

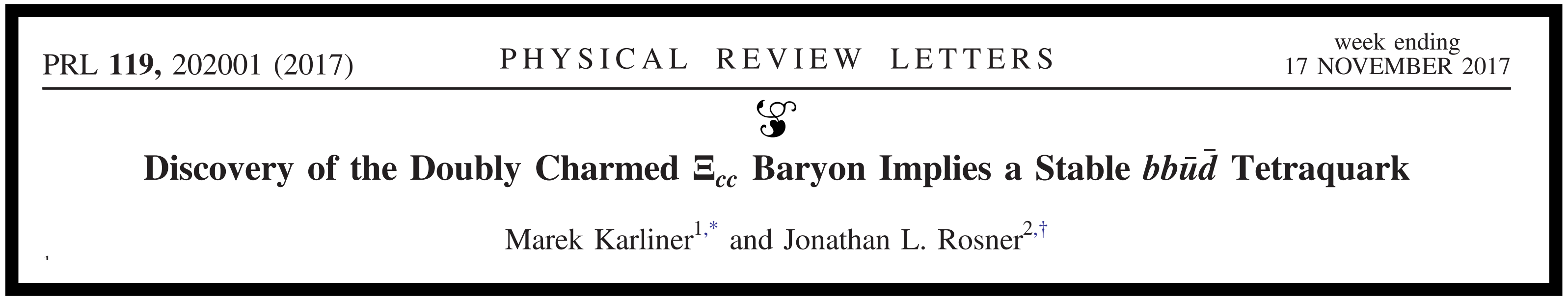
(compared to ≈ 1500 fs *for the B)*

Decay channels would include:

$(b \to c\bar{u}d)$: $D^0\bar{B}^0\pi^-$, $D^+B^-\pi^-$, …
$(b \to c\bar{c}s)$: $J/\psi K^-\bar{B}^0$, $B_c^- D^0\bar{K}^0$, …
$(b\bar{d} \to c\bar{u})$: D^0B^-, …

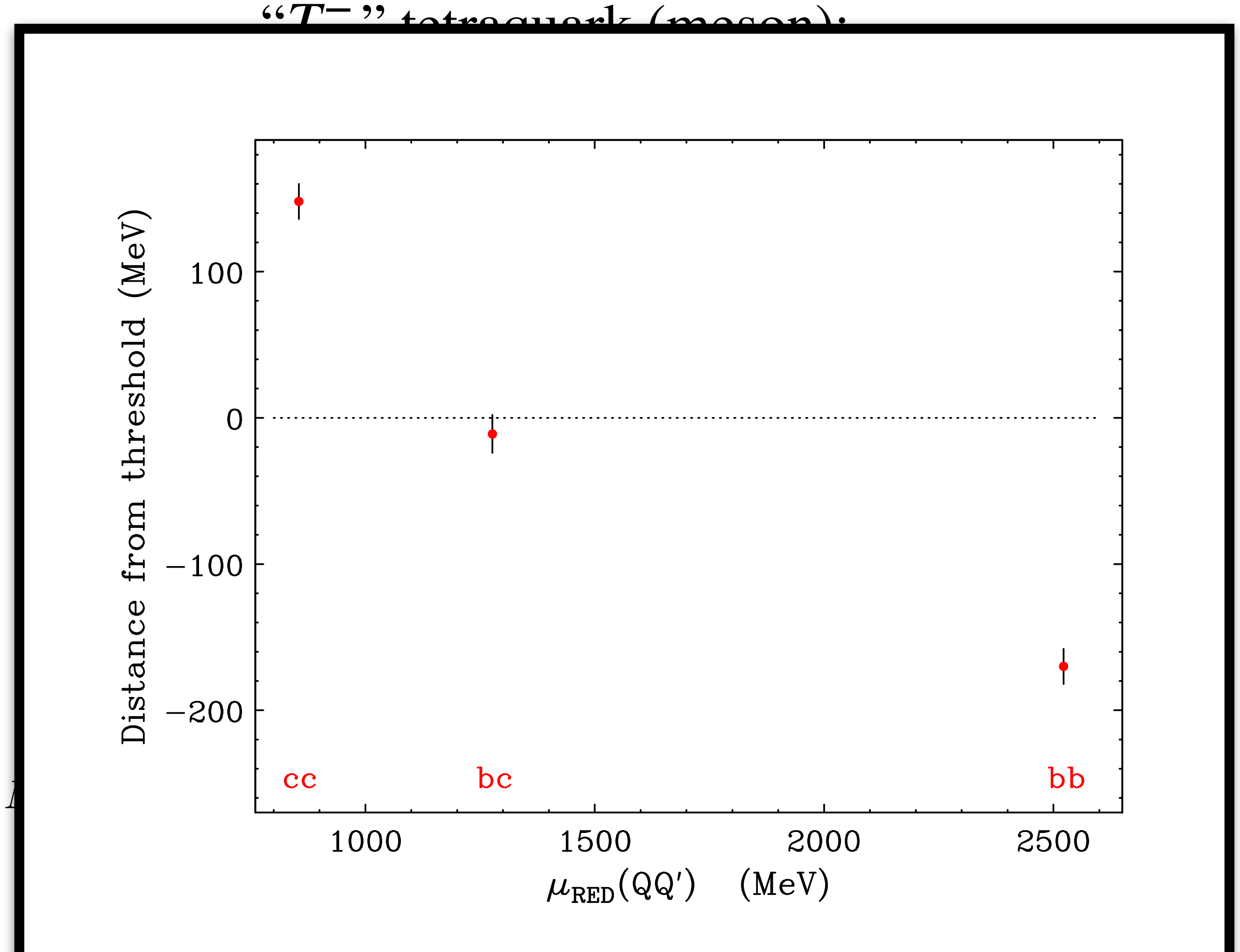
IIID. Doubly-Bottom Tetraquark

6. Use the same method to predict the mass of a doubly bottom tetraquark ($bb\bar{u}\bar{d}$).


PRL **119,** 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

Marek Karliner[1,*] and Jonathan L. Rosner[2,†]



$$= [2(5043.5) + 2(363) - 281.4 + 7.8 - 3(50)] \ \text{MeV}$$

$$M(T^-_{bb}) = 10389.4 \pm 12 \text{ MeV}$$

The lightest non-weak decays that conserves flavor and J^P are:

$$T^-_{bb} \to \gamma B^- \bar{B}^0 \quad \text{(EM)}$$
$$T^-_{bb} \to B^{*-} \bar{B}^0 \quad \text{(strong)}$$

but $M(B^-) + M(\bar{B}^0) = 10559$ MeV
and $M(B^{*-}) + M(\bar{B}^0) = 10604$ MeV

$\Longrightarrow$ T^-_{bb} would decay weakly

Lifetime would be ≈ 400 fs

(compared to ≈ 1500 fs *for the B)*

Decay channels would include:

$(b \to c\bar{u}d)$: $D^0 \bar{B}^0 \pi^-$, $D^+ B^- \pi^-$, …
$(b \to c\bar{c}s)$: $J/\psi K^- \bar{B}^0$, $B_c^- D^0 \bar{K}^0$, …
$(b\bar{d} \to c\bar{u})$: $D^0 B^-$, …

IIID. Doubly-Bottom Tetraquark

The simple quark model result is comparable to lattice QCD calculations.

PRL **119,** 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

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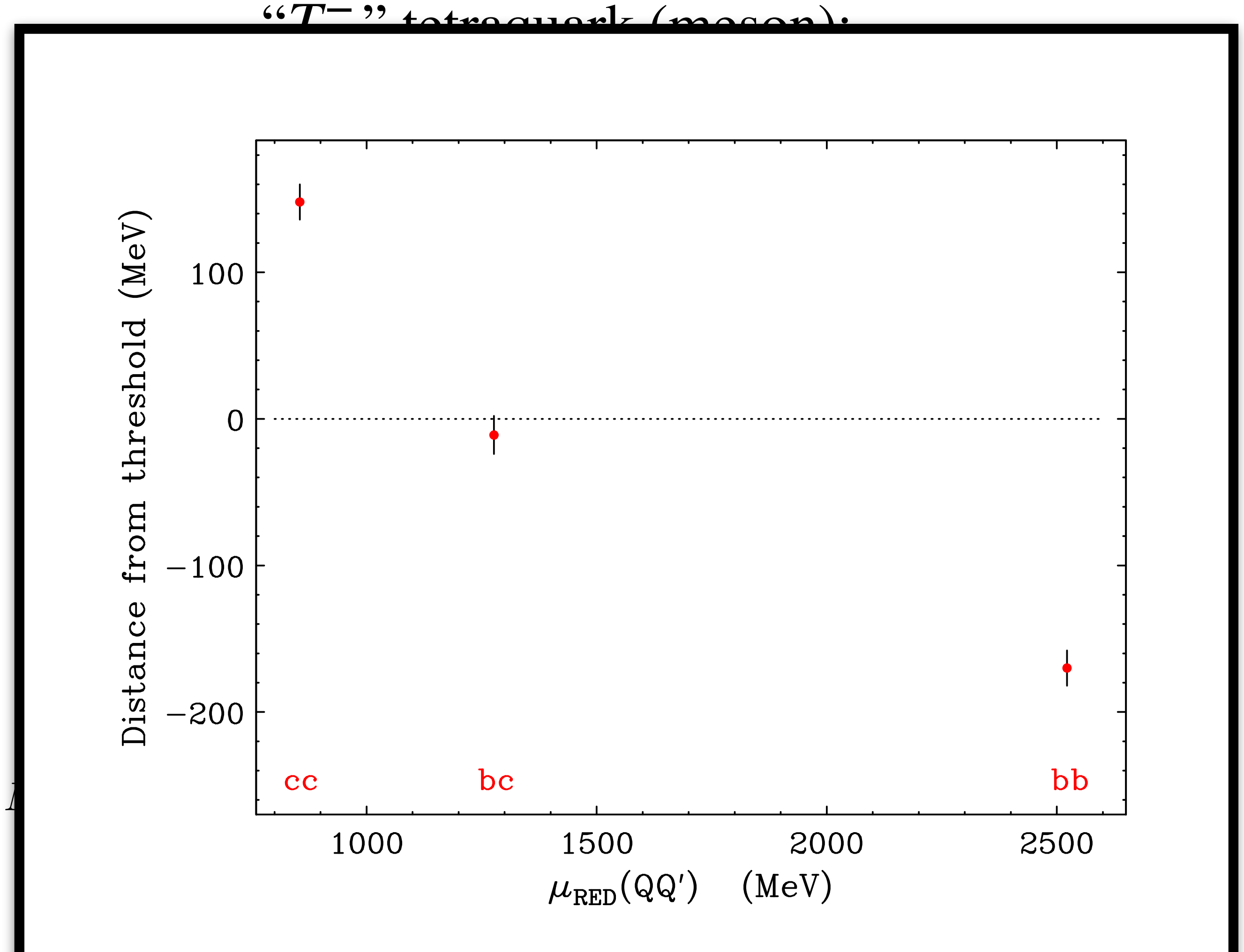


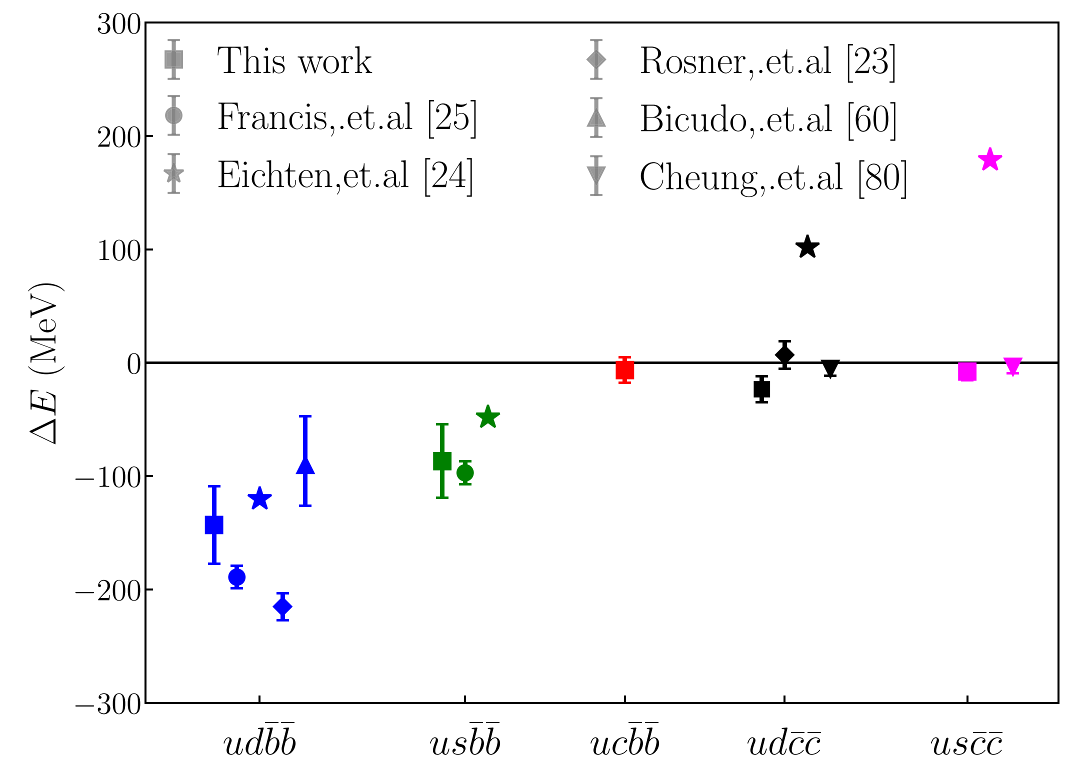
$$= [2(5043.5) + 2(363) - 281.4 + 7.8 - 3(50)]\ \text{MeV}$$

$$M(T^-_{bb}) = 10389.4 \pm 12\ \text{MeV}$$

PHYSICAL REVIEW D **99,** 034507 (2019)

Study of doubly heavy tetraquarks in lattice QCD

Parikshit Junnarkar,[1,*] Nilmani Mathur,[1,†] and M. Padmanath[2]



IIID. Doubly-Bottom Tetraquark

The simple quark model result is comparable to lattice QCD calculations.

PRL 119, 202001 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Discovery of the Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

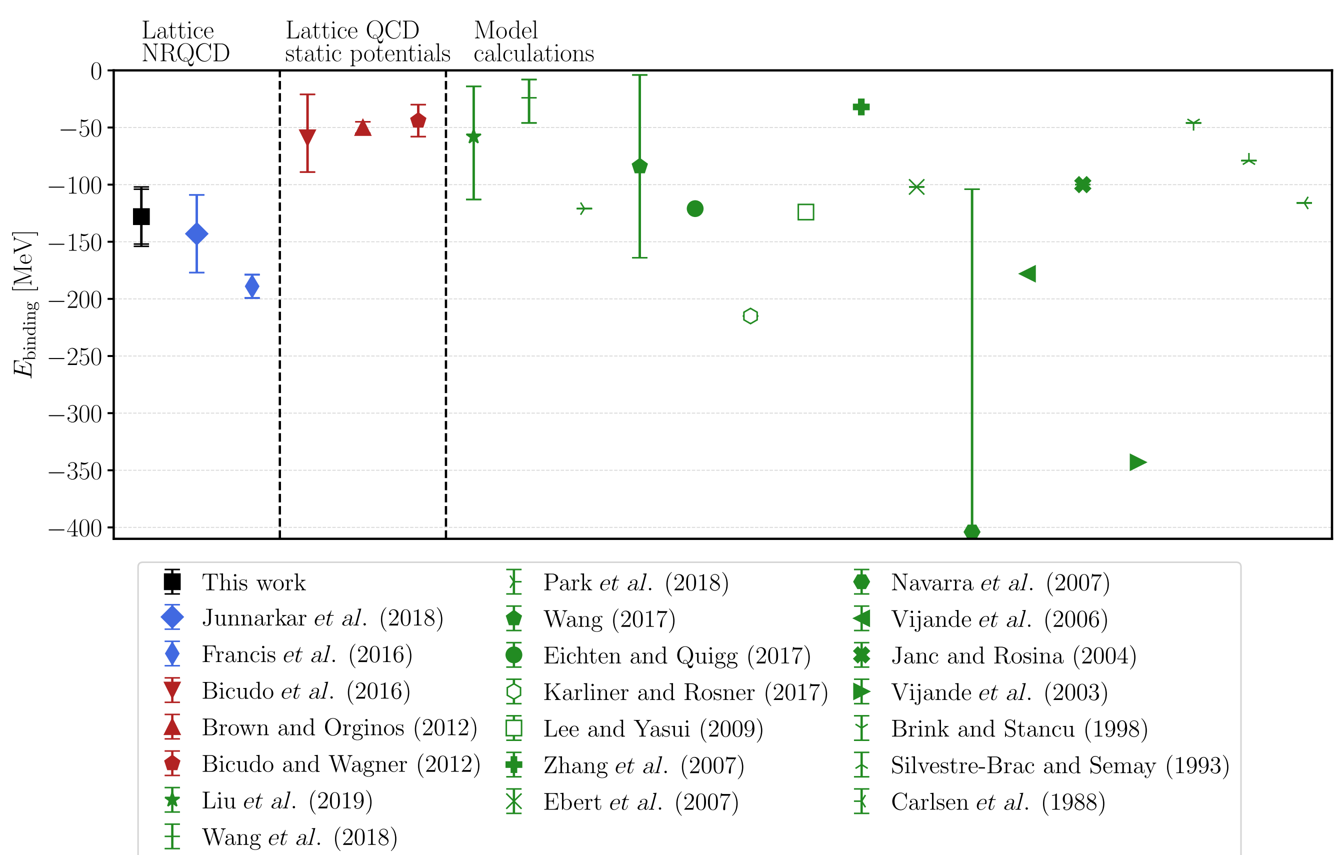
Marek Karliner[1,*] and Jonathan L. Rosner[2,†]

“T^-” tetraquark (meson):

PHYSICAL REVIEW D 100, 014503 (2019)

Lattice QCD investigation of a doubly-bottom $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$

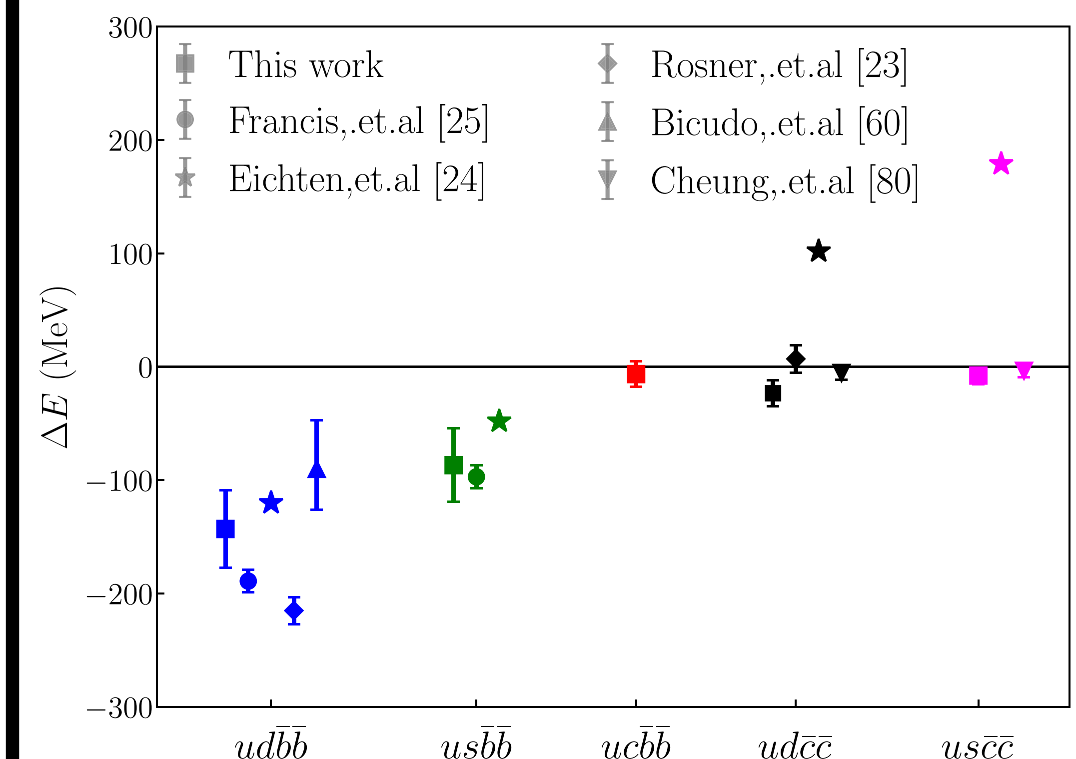
Luka Leskovec,[1] Stefan Meinel,[2,3] Martin Pflaumer,[4] and Marc Wagner[4]



PHYSICAL REVIEW D 99, 034507 (2019)

Study of doubly heavy tetraquarks in lattice QCD

Parikshit Junnarkar,[1,*] Nilmani Mathur,[1,†] and M. Padmanath[2]



IIID. Doubly-Bottom Tetraquark

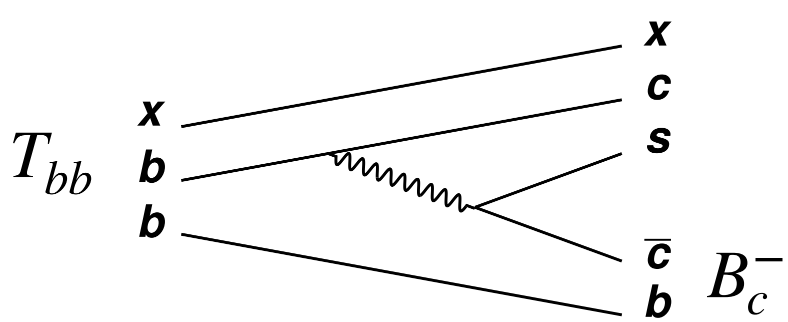
Finding a doubly-bottom tetraquark is a challenge for experiment.

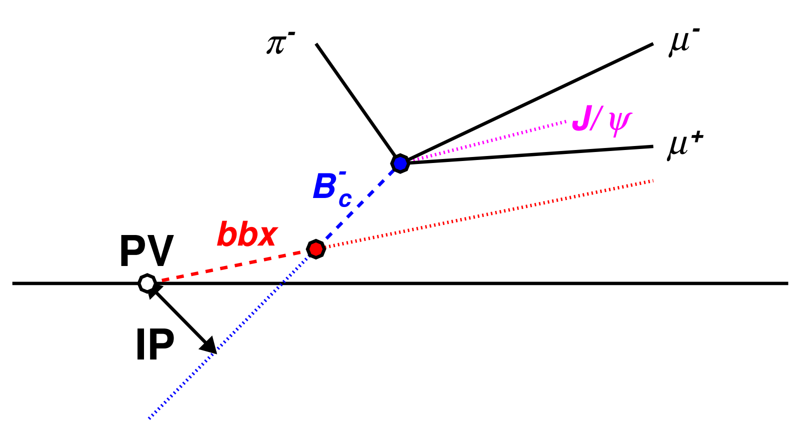
Displaced B_c^- mesons as an inclusive signature of weakly decaying double beauty hadrons

T. Gershon[a] and A. Poluektov[a,b]

JHEP01(2019)019

Perhaps use displaced B_c^- mesons as a signature at the LHC (esp. LHCb):



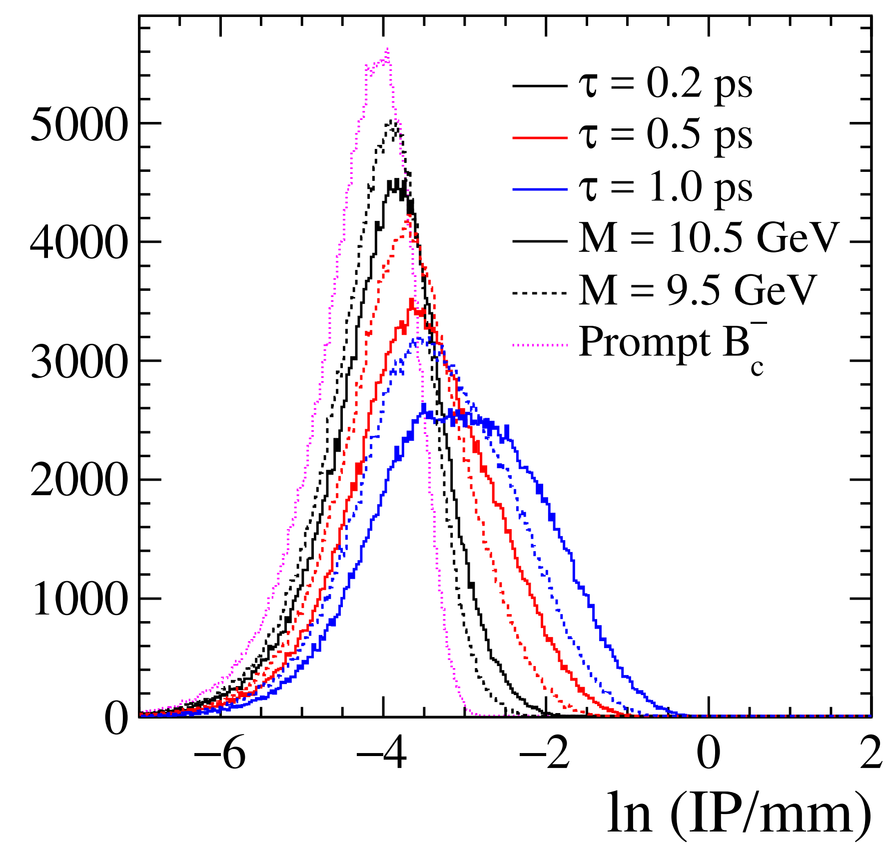


Rough estimate:

production cross section (~1 nb)
$\times$ branching fraction to B_c^- (~10%)
$\times$ $\mathscr{B}(B_c^- \to J/\psi\pi^-)$ (~ 2%)
$\times$ $\mathscr{B}(J/\psi \to \mu^+\mu^-)$ (6%)
$\times$ detection efficiency (~10%)
$\times$ integrated luminosity (1 fb^{-1})

— — — — — — — —

$\approx$ 10 detected displaced B_c^- in 1 fb^{-1} of data



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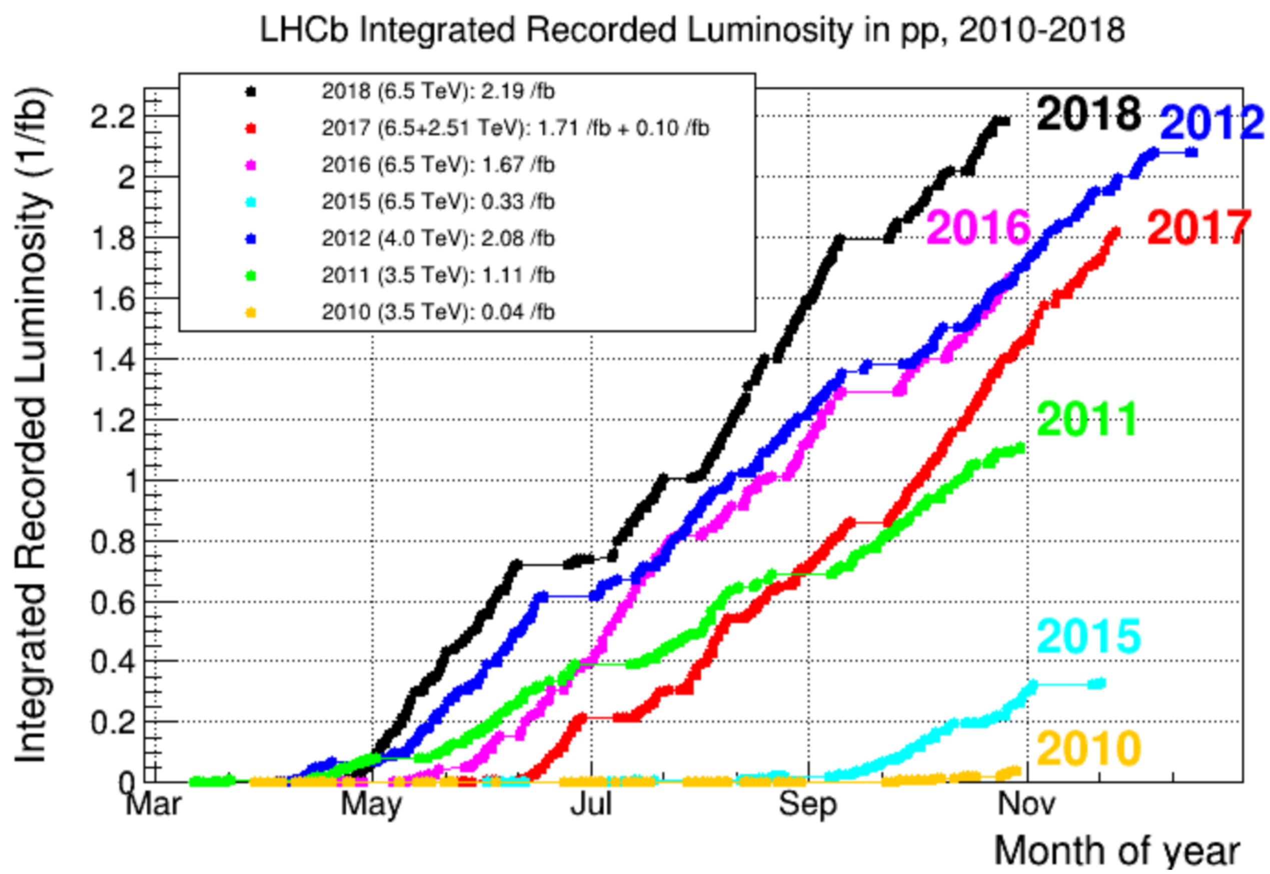

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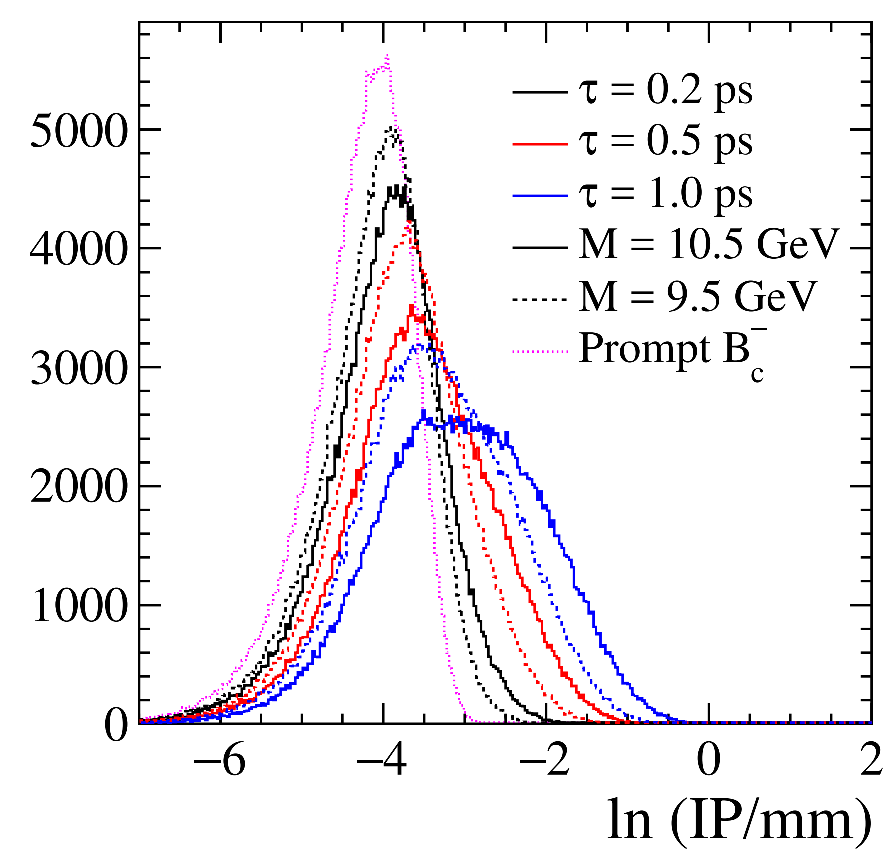

HUGS 2021 Lectures on:
Experimental Meson Spectroscopy

Prologue: Definitions and Philosophy

I. A Field Guide to Meson Families

II. Meson Quantum Numbers

III. The Quark Model

IV. Exotic Mesons

V. Current and Future Experiments

LECTURE III. The Quark Model

IIIA. Charmonium Potential

IIIB. Radiative Transitions

IIIC. Color Factors

IIID. Doubly-Bottom Tetraquark

HUGS 2021 Lectures on:
Experimental Meson Spectroscopy

LECTURE III. The Quark Model

In the quark model, describe hadrons as quarks and antiquarks bound by potentials ("QCD-inspired").

Potential models can describe the spectrum of mesons and their radiative transitions.

The strength of the potential is given by QCD color factors.

A stable doubly-bottom tetraquark appears to be a solid prediction of both potential models and lattice QCD.

Finding the doubly-bottom tetraquark experimentally appears difficult.