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# Transverse momentum imaging

## Lecture 4

Hampton University Graduate School (e-HUGS) 2021

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# Plan of these lectures

1. **DIS and partons**
2. **From DIS to SIDIS**
3. **Symmetries and universality**
4. **Factorization, evolution, matching**
5. **Phenomenology**

# 4.1 TMD factorization

# Factorized cross section

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\,\bar{l} + X$$

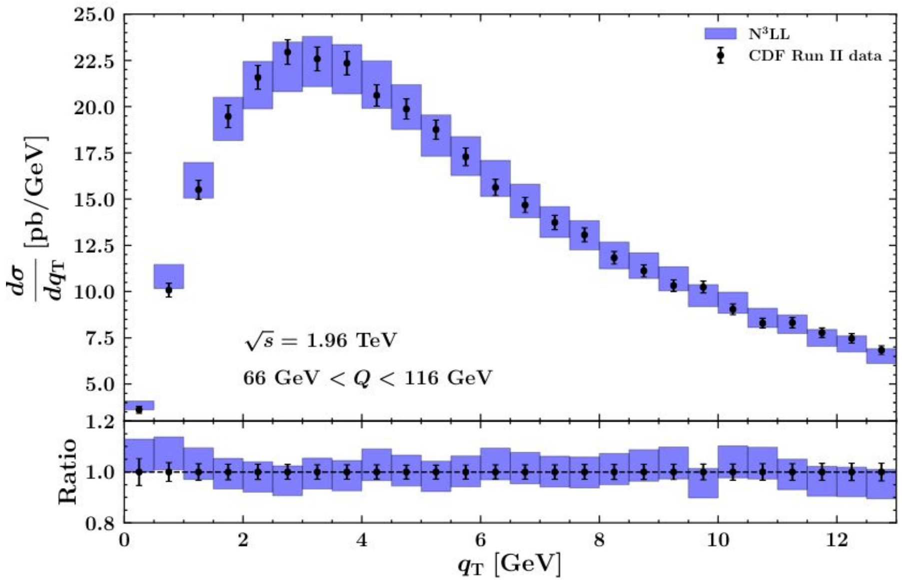
$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\left(q_T - k_{T\,a} - k_{T\,b}\right)$$

Renormalized TMDs

[TMD region, $q_T \ll Q$]

$$+\, \mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large $q_T$ and low $Q$ corrections]

Description of data: essential an approach with predictive power



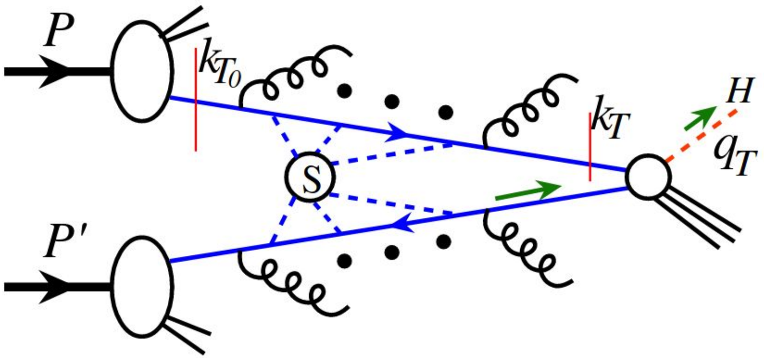

Factorization → renormalization (evolution) of TMDs

# Physical intuition

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\bar{l} + X$$

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}(q_T - k_{T\,a} - k_{T\,b})$$

- The TMDs reproduce the structure of the **IR poles** in the cross section (same non-perturbative physics)
- The **observed transverse momentum** is accounted for by the transverse momenta of **quarks**
- The quark transverse momentum has **radiative** (perturbative) and **intrinsic** (non-perturbative) components
- Renormalization = **evolution** equations tell us how to distinguish between the two



# TMD factorizable processes

Quark TMDs:

- Drell-Yan
- Electron-positron annihilation into two hadrons
- Semi-Inclusive DIS

Gluon TMDs:

- Pseudo-scalar quarkonium production in pp collision
- Higgs production in pp collisions

Processes with jets:

- jet-SIDIS, SIDIS with a identified hadron inside a jet, etc.
- Di-jet production
- e+e- into a jet and a hadron
- ...

# Approximations

$$pp \longrightarrow \gamma^{\cdot} / Z \longrightarrow l\bar{l} + X$$

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}(q_T - k_{T\,a} - k_{T\,b})$$

[TMD region, $q_T \ll Q$]

$$+\,\mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large $q_T$ and low $Q$ corrections]

Large transverse momentum corrections: "Matching" to collinear factorization at large qT

Higher twist corrections: Multiparton correlations (?)

We will probably see an example in one of the recitation sessions

# Approximations

See https://inspirehep.net/literature/1732230 (recommended!)

Semi-Inclusive Deep-Inelastic Scattering: Nachtmann variable(s)

$$x_{\rm Bj} = \frac{Q^2}{2P\cdot q}\,, \qquad x_{\rm N} = -\frac{q^+}{P^+} = \frac{2x_{\rm Bj}}{1+\sqrt{1+\frac{4x_{\rm Bj}^2 M^2}{Q^2}}}\,,$$

Nachtmann and Bjorken-x are the same up to target mass corrections

$$z_{\rm N} = \frac{P_{\rm B}^-}{q^-}$$

$$z_{\rm N} = \frac{Q^4 x_{\rm N} z_{\rm h}\left(1 \pm \sqrt{1-\frac{4M^2 M_{\rm B}^2 x_{\rm Bj}^2 (Q^4 + x_{\rm N}^2 M^2 \mathbf{q}_{\rm T}^2)}{Q^8 z_{\rm h}^2}}\right)}{2x_{\rm Bj}(Q^4 + x_{\rm N}^2 M^2 \mathbf{q}_{\rm T}^2)} \overset{\text{Fixed } x_{\rm N}, z_{\rm h}, q_{\rm T}}{=} z_{\rm h}\left(1 + O\left(\frac{m^4}{Q^4}\right)\right)$$

# Approximations

See https://inspirehep.net/literature/1732230
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Semi-Inclusive Deep-Inelastic Scattering: Nachtmann variable(s)

$$x_{\mathrm{Bj}} = \frac{Q^2}{2P\cdot q}, \qquad x_{\mathrm{N}} = -\frac{q^+}{P^+} = \frac{2x_{\mathrm{Bj}}}{1+\sqrt{1+\frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}}},$$

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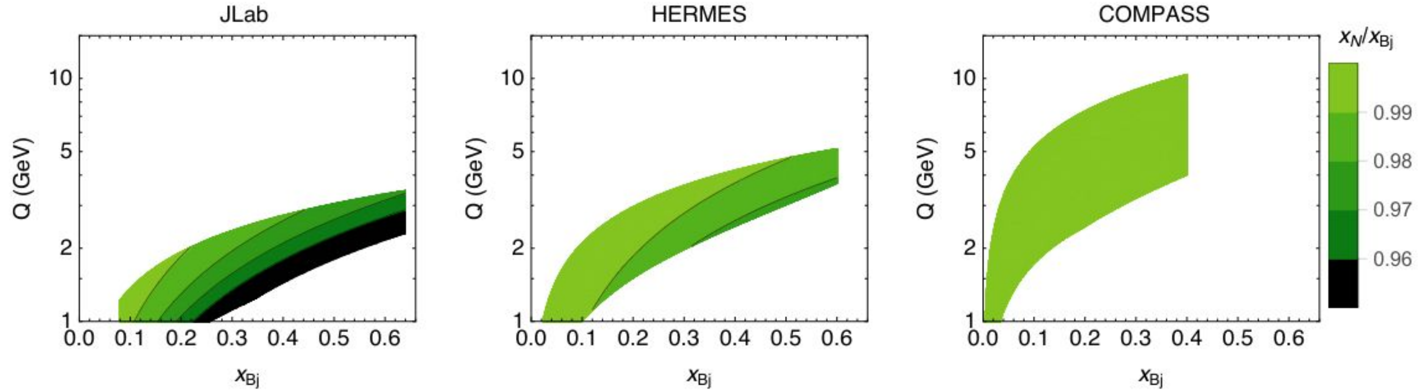


## 4.2 TMD evolution

# Evolution equations

$$F_a(x, b_T^2; \mu, \zeta) = \int d^2\boldsymbol{k}_T \, e^{i\boldsymbol{k}_T \cdot \boldsymbol{b}_T} F_a(x, k_T^2; \mu, \zeta)$$

← Fourier transform of a TMD PDF
(bT conjugated to kT)

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \zeta} = -D\left(b_T \mu, \alpha_s(\mu)\right) ,$$

**rapidity**

$$\frac{d \ln F_a(x, b_T^2; \mu, \zeta)}{d \ln \mu} = \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right) ,$$

**UV**

$$\frac{d D\left(b_T \mu, \alpha_s(\mu)\right)}{d \ln \mu^2} = \frac{1}{2}\gamma_K\left(\alpha_s(\mu)\right) ,$$

**UV**

$$\frac{d \gamma_F\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right)}{d \ln \zeta} = -\gamma_K\left(\alpha_s(\mu)\right) .$$

**rapidity**


See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Evolved TMD distribution

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$ → TMD distribution at initial scales

x $$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$ → evolution in μ

x $$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T\mu_0, \alpha_s(\mu_0))}$$ → evolution in ζ

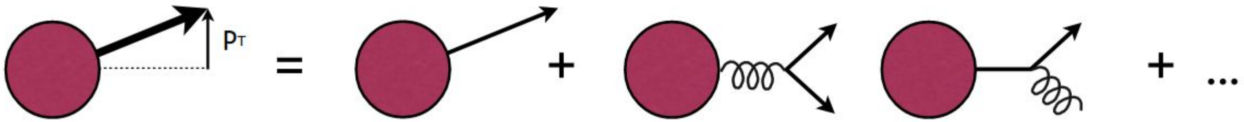
→ this solution is valid **at low bT**

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Matching with collinear PDFs

→ Also the input TMD contains perturbative information **AT LOW bT**:

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0)$$




Perturbative transverse momentum generated by the splitting into other partons

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Non perturbative components

$$F_a(x, b_T^2; \mu, \zeta) = F_a(x, b_T^2; \mu_0, \zeta_0)$$ → TMD distribution at initial scales

x $$\exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right)\right]$$ → evolution in μ

x $$\left(\frac{\zeta}{\zeta_0}\right)^{-D(b_T \mu_0, \alpha_s(\mu_0)) + g_K(b_T; \lambda)}$$ → evolution in ζ

**Non-pert. correction to evolution (large bT)**

$$F_a(x, b_T^2; \mu_0, \zeta_0) = \sum_b C_{a/b}(x, b_T^2, \mu_0, \zeta_0) \otimes f_b(x, \mu_0) F_{NP}(b_T; \lambda)$$

**Intrinsic contribution (large bT)**

See e.g. https://inspirehep.net/literature/1785810 for more details (but also JCC book, etc.)

# Predictive power

Small bT → **perturbative (radiation)** contributions to TMD PDF
Large bT → **non-perturbative (intrinsic)** contributions to TMD PDF

Exercise:

In which kinematic regions is the TMD PDF dominated by small / large bT contributions ?

(or, in which regions is the formalism ***predictive*** *or dominated by* ***non-perturbative*** *contributions*?)

Hint: think about the shape of the TMD PDF in bT space and where it peaks

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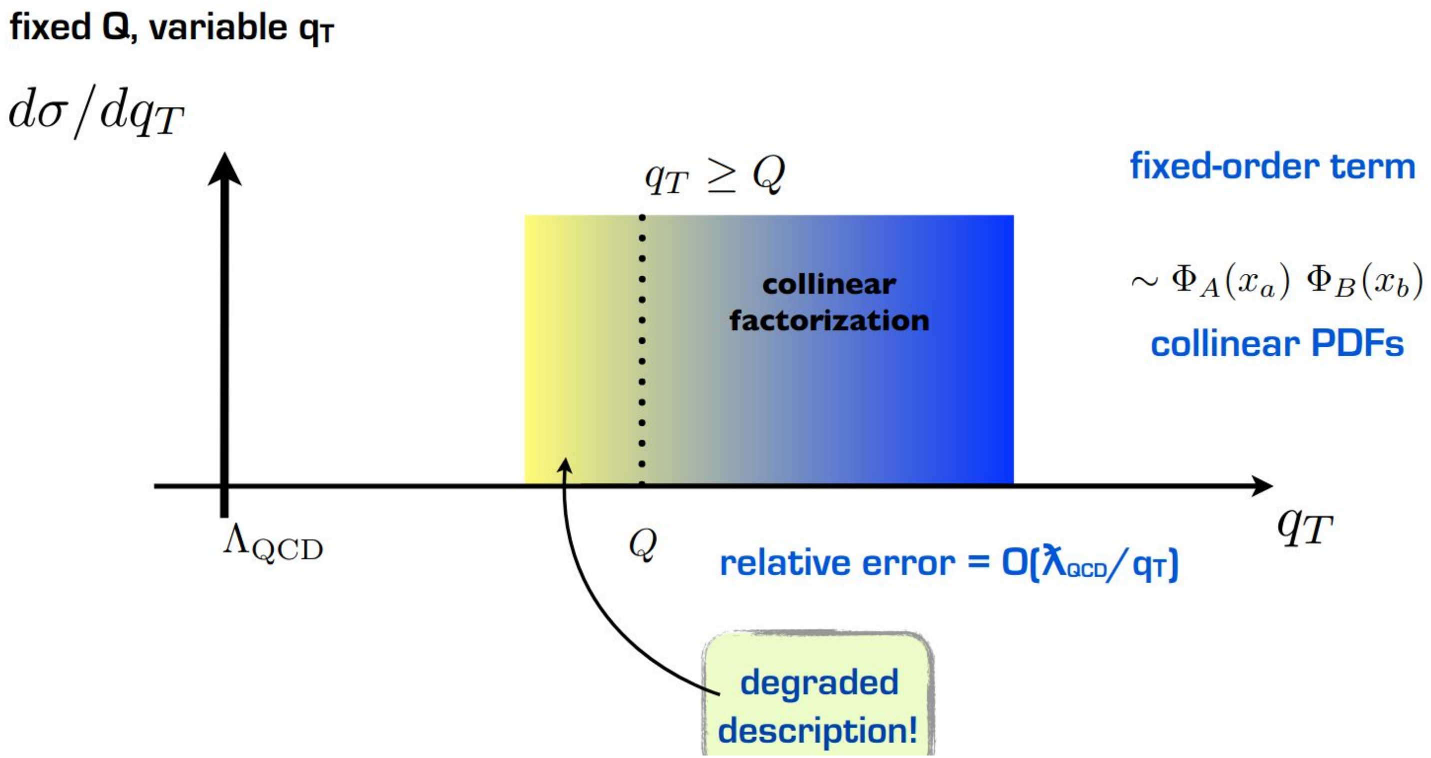
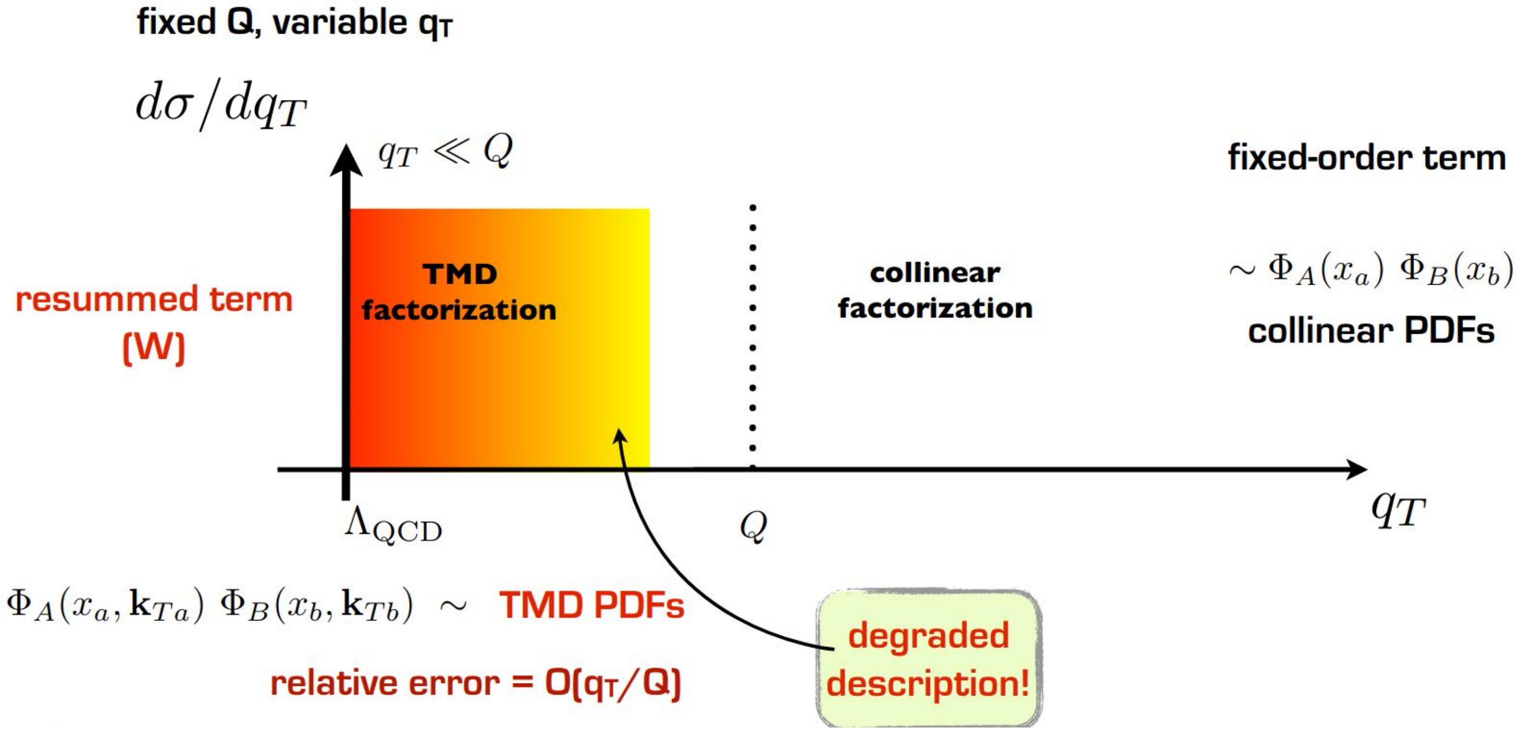
# 4.3 Matching

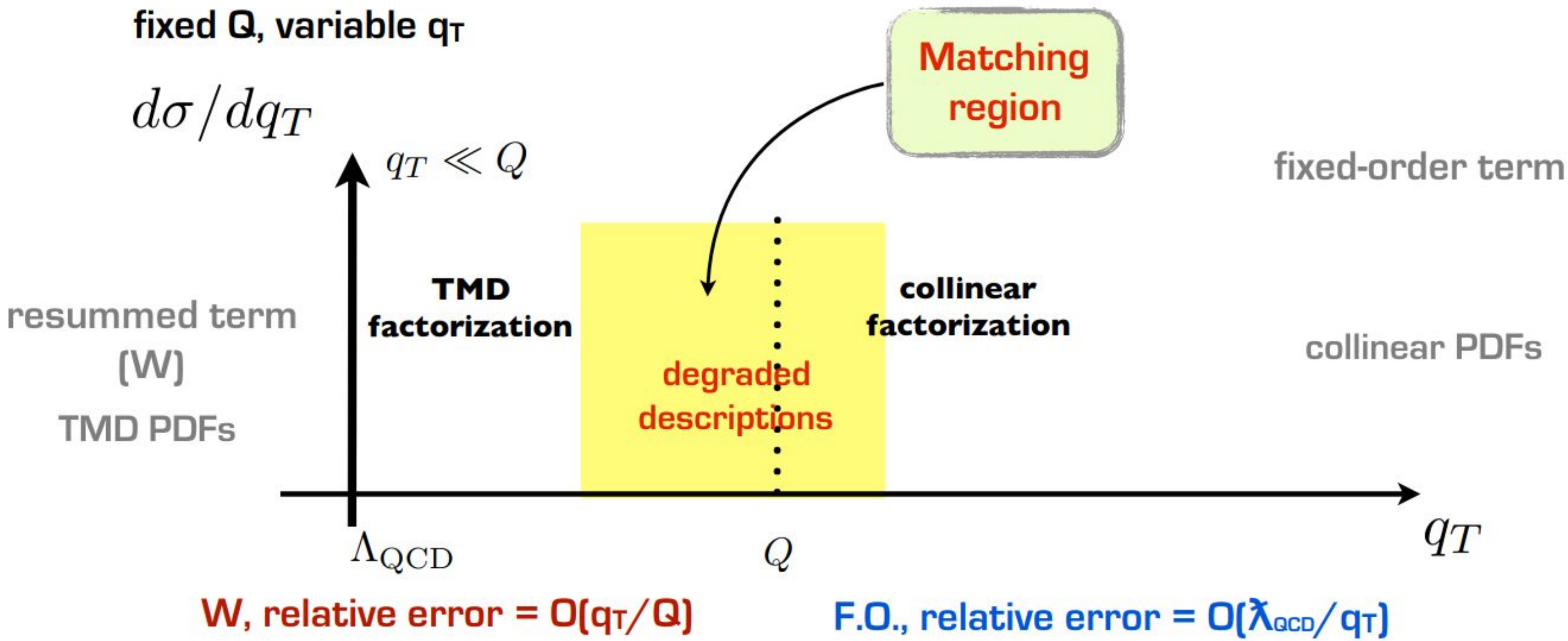
# Large transverse momentum



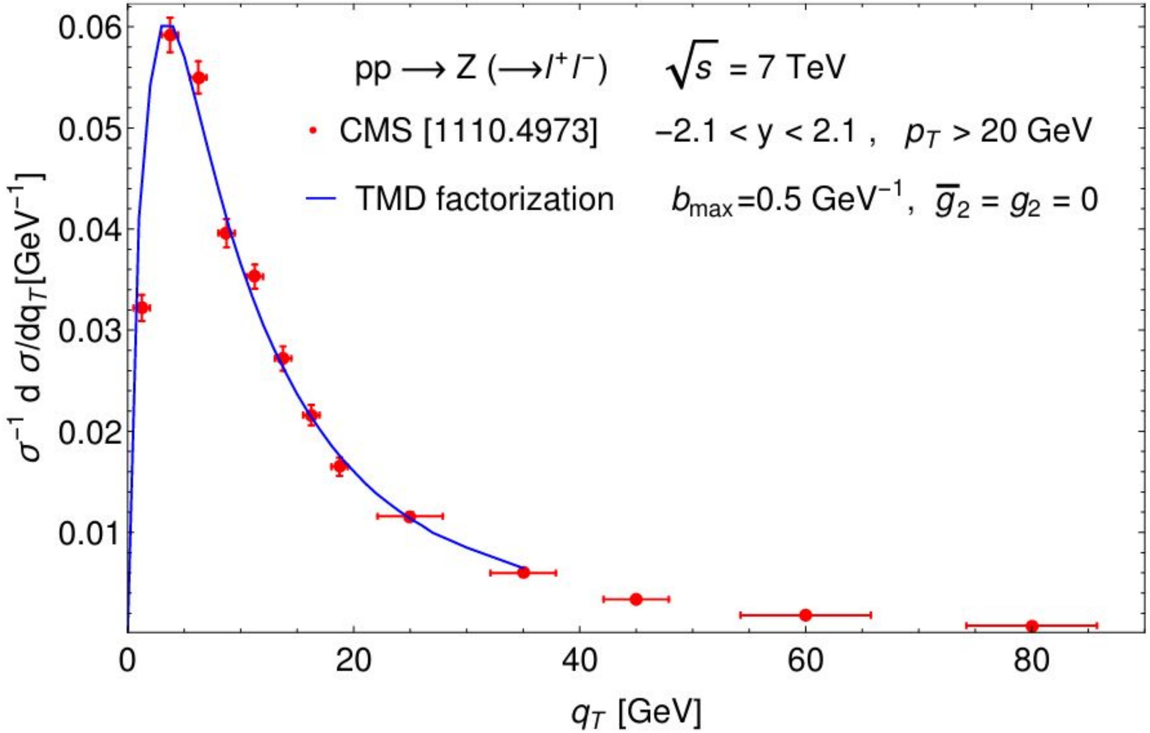
# Small transverse momentum



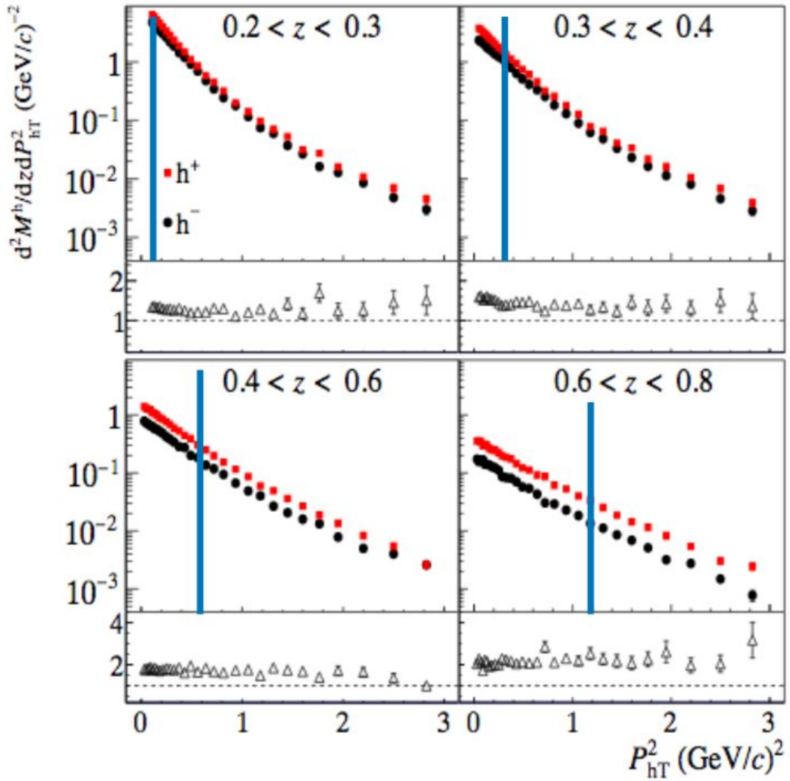
# Matching region



# Relevance for phenomenology



# Relevance for phenomenology




COMPASS unpolarized SIDIS multiplicities - arxiv 1709.07374

**SIDIS - TMD region**

$$P_{hT}^2/z^2 \ll Q^2$$

**Let's highlight**

$$P_{hT}^2/z^2 \sim 0.25\ Q^2$$

One of the bins with highest Q:

$$\langle Q^2 \rangle = 9.78\ \mathrm{GeV}^2$$
$$\langle x \rangle = 0.149$$