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Transverse momentum imaging

Lecture 3

Hampton University Graduate School (e-HUGS) 2021

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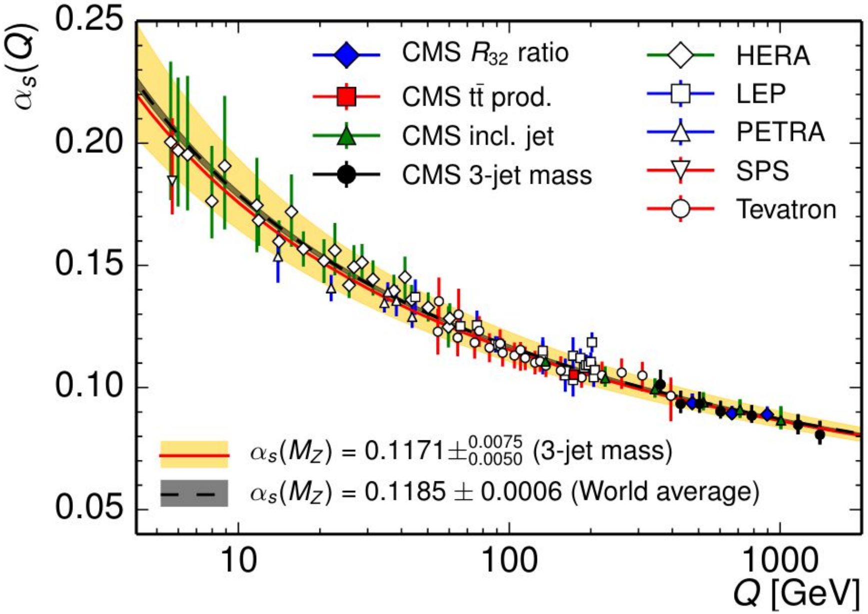
Recap++
of lectures 1, 2

How should we "use" QCD ?

Expansion of observable in powers of the coupling constant α:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} \ldots = ??$$

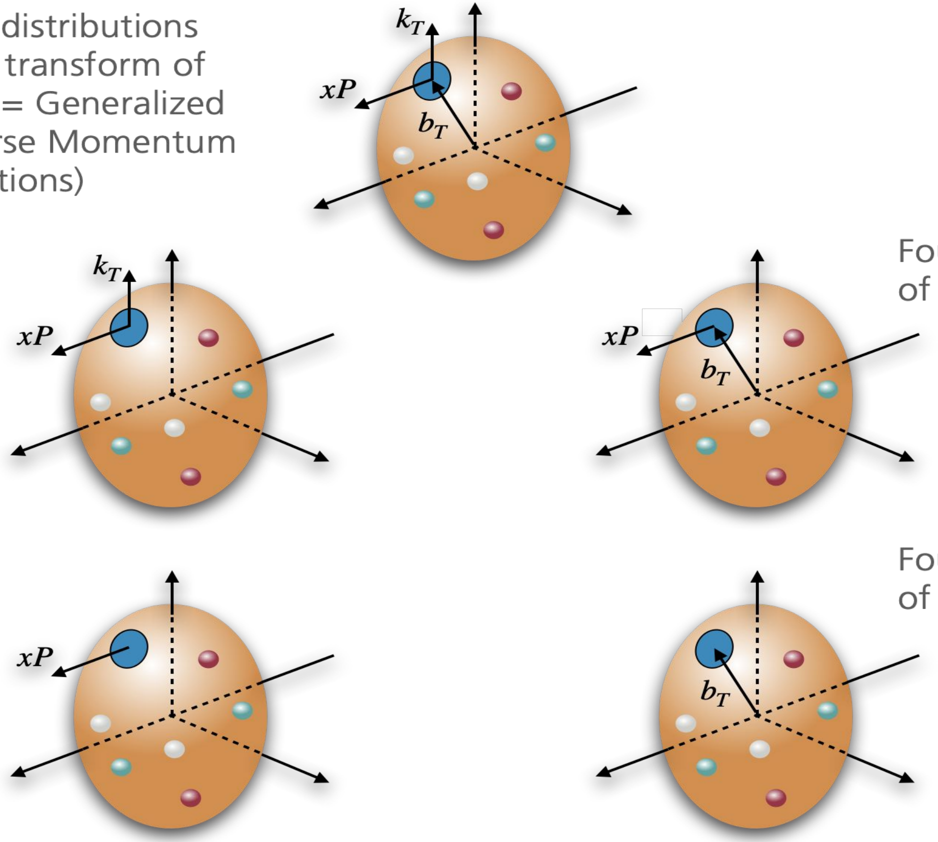


High energy → convergence
→ perturbative QCD

Low energy (hadronic scales)
→ non-perturbative QCD

need alternative techniques

The hadron structure landscape



Wigner distributions
(Fourier transform of
GTMDs = Generalized
Transverse Momentum
Distributions)

TMDs

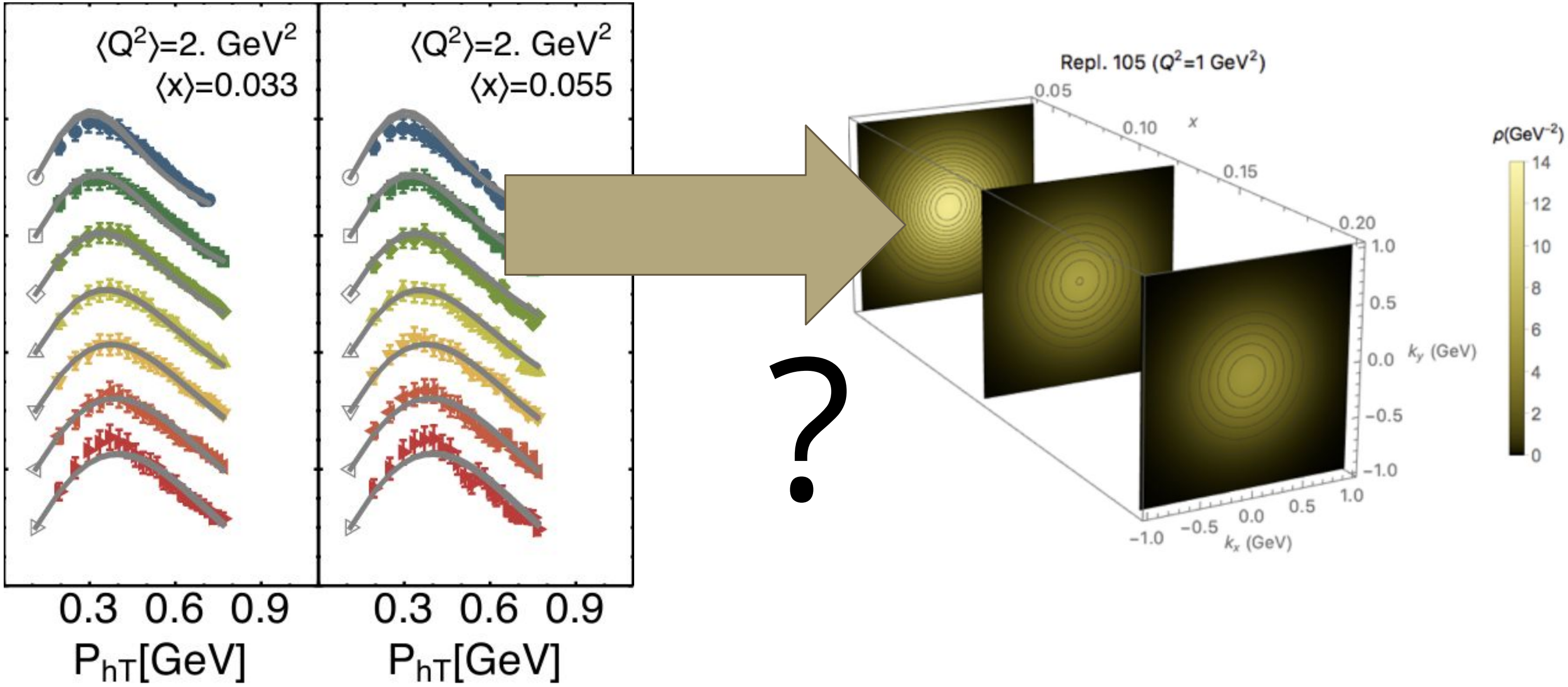
Fourier transform
of GPDs

PDFs

Fourier transform
of Form Factors

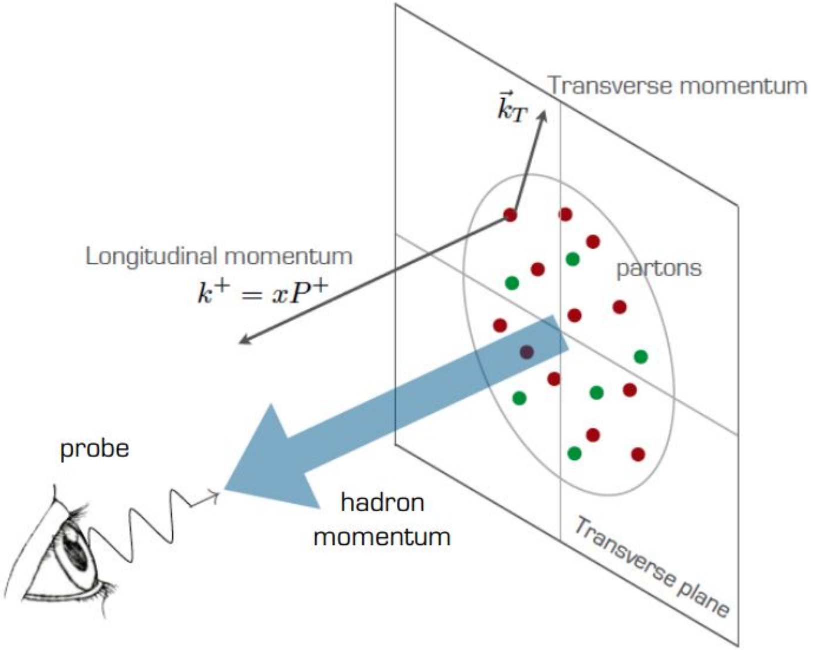
see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

Transverse momentum imaging



Parton distribution functions (PDFs)

"Maps" of hadron *structure* in momentum space



$$f_1(x)$$

1D structure
in momentum space

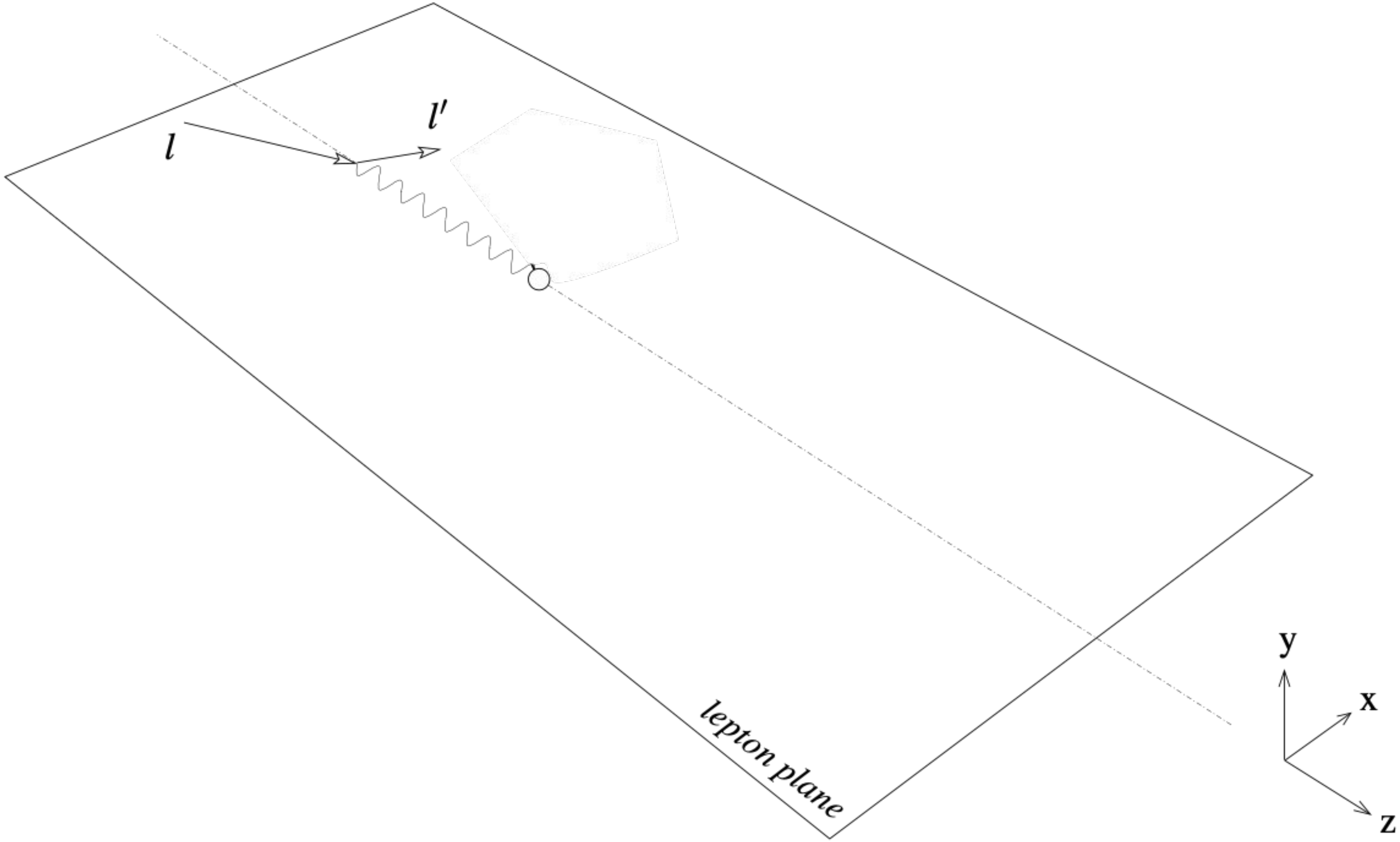
$$f_1(x, k_T^2)$$

3D structure
in momentum space

Credit picture: A. Bacchetta

Deep-inelastic scattering

$$l(\ell) + N(P) \rightarrow l'(\ell') + X(P_X)$$



Polarized case - spin 1/2

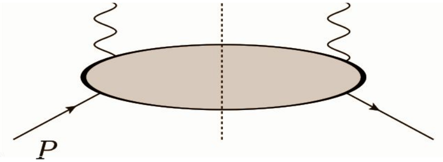
$$W^{\mu\nu}(q,P,S) \sim \; -g_{\perp}^{\mu\nu}\, F_{UU,T} + \hat{t}^{\mu}\hat{t}^{\nu}\, F_{UU,L}$$

$$+\, i\, S_L \epsilon_{\perp}^{\mu\nu} \boxed{F_{LL}} + i\left(\hat{t}^{\mu}\epsilon_{\perp}^{\nu\rho} - \hat{t}^{\nu}\epsilon_{\perp}^{\mu\rho}\right) S_{\rho} \boxed{F_{LT}^{\cos\phi}}$$

Two additional structure functions for the nucleon:
longitudinal and **transverse** target polarization → related to “standard” g1 and g2 functions

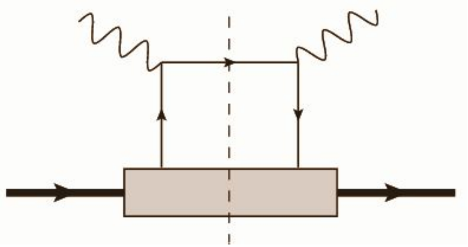
Transverse beam polarization is proportional to electron mass and thus suppressed

Partonic interpretation



$$2MW_{\mu\nu}(q,P,S) \;=\; \tfrac{1}{2\pi}\int d^4\xi\; e^{i\,q\cdot\xi}\left\langle PS\right|\left[J^\dagger_\mu(x),\,J_\nu(0)\right]\left|PS\right\rangle$$

$$J_\mu(\xi) \;=\; :\overline{\psi}(\xi)\,Q\,\gamma_\mu\,\psi(\xi):$$



quark-antiquark

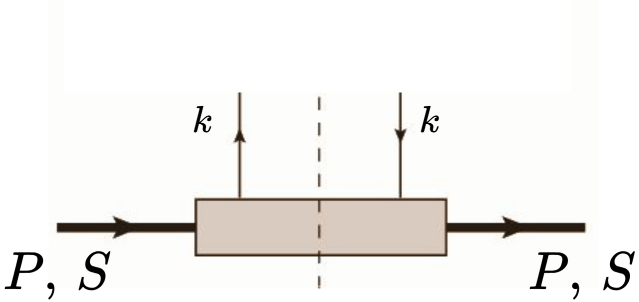
$$2MW^{\mu\nu}(q,P,S) \;=\; \textstyle\sum_q e_q^2\,\frac{1}{2}\,\mathrm{Tr}\left[\Phi(x,S)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right]$$

$\phi(x,S)$: “collinear” quark correlator

$x_B \simeq x \equiv k^+/P^+$ → measure collinear parton dynamics

The quark transverse momentum is integrated out in DIS

Collinear parton distribution functions



$\Phi_{ij}(k,P,S)$: non-perturbative hadron structure matrix

$$\Phi(x,S) = \frac{1}{2}\boxed{f_1(x)}\,\not{n}_+ +$$

→ unpolarized PDF

$$\frac{1}{2}\boxed{g_1(x)}\,S_L\,\gamma_5\,\not{n}_+ +$$

→ longitudinally polarized PDF
(helicity)

$$\frac{1}{2}\boxed{h_1(x)}\,i\sigma_{\mu\nu}\,\gamma_5\,n_+^{\mu}\,S_T^{\nu}$$

→ transversely polarized PDF
(transversity)

“Leading twist”
approximation
for spin 1/2

DIS: structure functions and PDFs

$$
\begin{aligned}
F_T &= x_B \sum_q e_q^2 \boxed{f_1^q(x_B)}, \\
F_L &= 0, \\
F_{LL} &= x_B \sum_q e_q^2 \boxed{g_1^q(x_B)}, \\
F_{UT}^{\sin\phi_S} &= 0, \\
F_{LT}^{\cos\phi_S} &= -x_B \sum_q e_q^2 \boxed{\frac{2M}{Q}} \left(x_B \boxed{g_T^q(x_B)} + \frac{M_q - m_q}{M} \boxed{h_1^q(x_B)} \right)
\end{aligned}
$$

Leading twist PDFs

Higher twist PDF

LHS: measurable
RHS: partonic quantities

“Collinear” imaging

DIS on a spin ½ hadron: structure functions at leading order in perturbation theory
(at higher orders: convolution with perturbative coefficients)

For a summary see e.g. https://inspirehep.net/literature/1603092 and https://inspirehep.net/literature/732275

Higher twist PDFs

Twist 2

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + \lambda\, g_1(x)\,\gamma_5\,\not{n}_+ + h_1(x)\,\frac{\gamma_5\,[\not{S}_T, \not{n}_+]}{2} \right\}$$

Twist 3

$$+\frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not{S}_T + \lambda\, h_L(x)\,\frac{\gamma_5\,[\not{n}_+, \not{n}_-]}{2} \right\}$$

$$+\frac{M}{2P^+}\left\{ -\lambda\, e_L(x)\, i\gamma_5 - f_T(x)\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} + h(x)\,\frac{i\,[\not{n}_+, \not{n}_-]}{2} \right\}$$

Twist 4

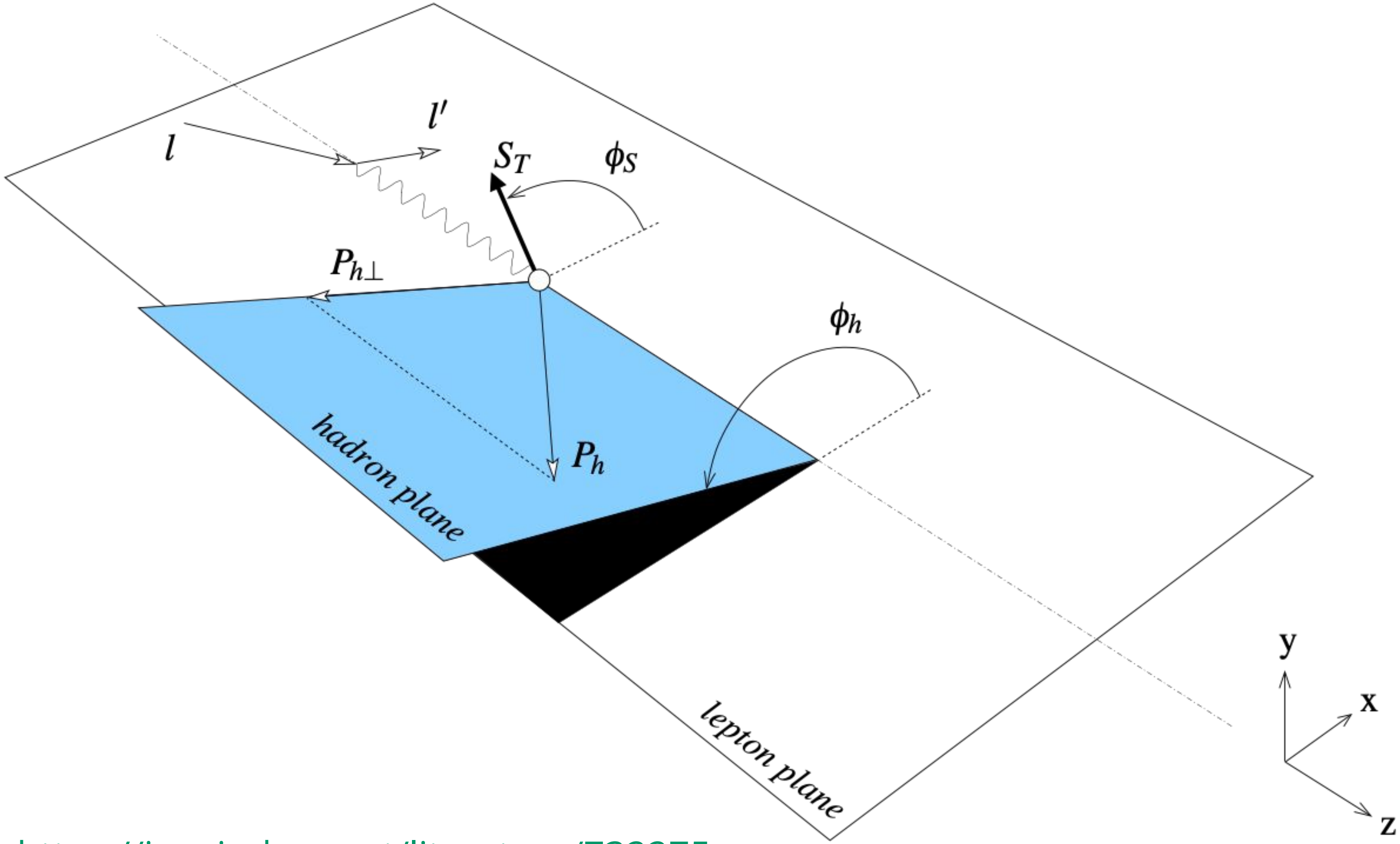
$$+\frac{M^2}{2(P^+)^2}\left\{ f_3(x)\,\not{n}_- + \lambda\, g_3(x)\,\gamma_5\not{n}_- + h_3(x)\,\frac{\gamma_5\,[\not{S}_T, \not{n}_-]}{2} \right\},$$

Twist t (operational definition): $\left(\frac{M}{P^+}\right)^{t\,-\,2}$

For more details on the definition(s) of twist, see Jaffe's “Erice” lecture notes

Semi-Inclusive DIS

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$



https://inspirehep.net/literature/732275

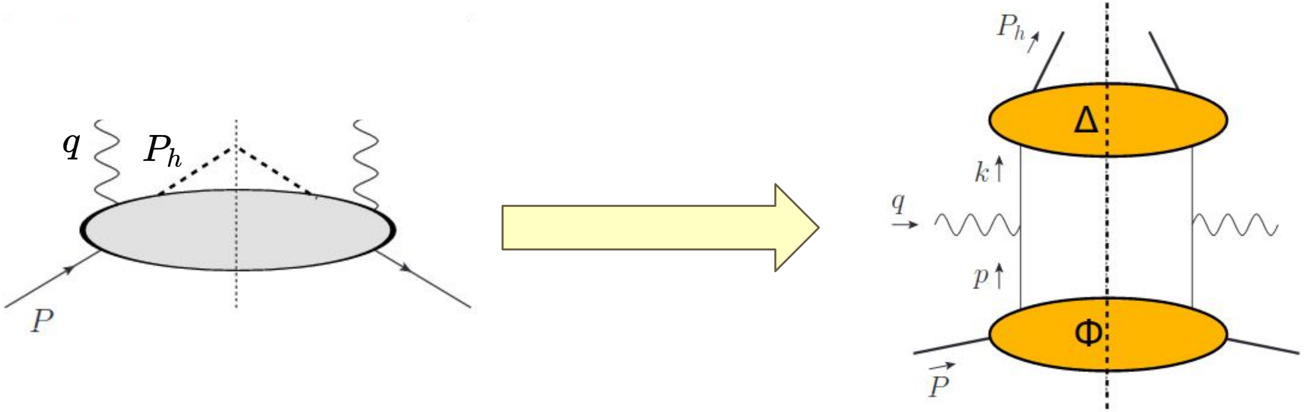
SIDIS cross section (polarized nucleon - spin 1/2)

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

18 structure functions
for polarized nucleon target

For more details see https://inspirehep.net/literature/732275

Partonic interpretation

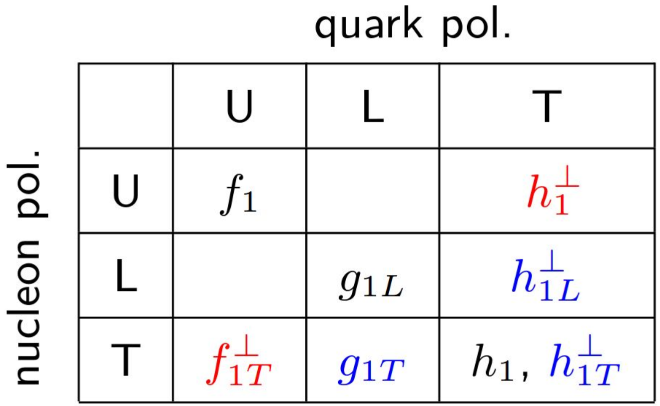


\+ higher twist (suppressed)

$$2MW^{\mu\nu}(q,P,S,P_h) = \frac{2z_h}{x_B}\,\mathcal{C}\Big[\mathrm{Tr}\big(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu\big)\Big]$$

$$\mathcal{C}\big[wfD\big] = \sum x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T,\boldsymbol{k}_T)\, f^a(x,p_T^2)\, D^a(z,k_T^2)$$

TMD PDFs and TMD FFs

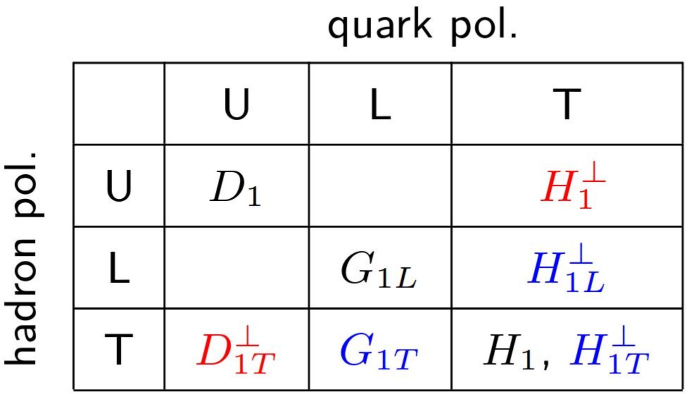


quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

8 TMD PDFs at leading twist



quark pol.

hadron pol.

	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_1,\ H_{1T}^\perp$

8 TMD FFs at leading twist

SIDIS: structure functions and TMDs

Leading twist TMDs

Higher twist TMDs

$$F_{UU,T} = \mathcal{C}\left[\boxed{f_1 D_1}\right],$$

“Collins effect”

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\boxed{h_1^\perp H_1^\perp}\right]$$

“Sivers effect”

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\boxed{f_{1T}^\perp D_1}\right],$$

$$F_{LU}^{\sin\phi_h} = \boxed{\frac{2M}{Q}}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(x\boxed{e}\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\boxed{\tilde{G}^\perp}}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(x\boxed{g^\perp}D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\boxed{\tilde{E}}}{z}\right)\right]$$

Etc. ...

SIDIS on a spin ½ hadron: structure functions at leading order in perturbation theory
(at higher orders: convolution with perturbative coefficients)

For a summary see https://inspirehep.net/literature/732275

Higher twist TMDs

Sivers effect:
correlation between transverse spin and momentum

Twist 2

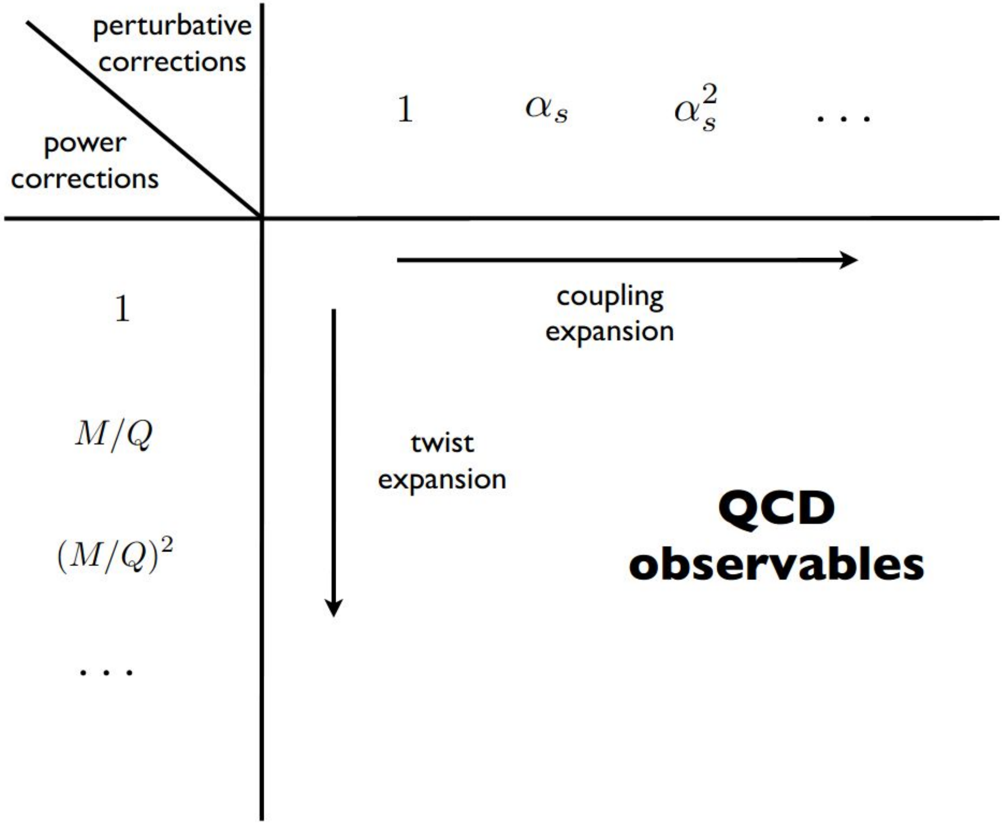
$$\begin{aligned}\Phi(x,\boldsymbol{k}_T) = \frac{1}{2}\Big\{ & f_1(x,\boldsymbol{k}_T)\,\slashed{n}_+ + \boxed{f_{1T}^{\perp}(x,\boldsymbol{k}_T)}\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu \boxed{k_T^\rho S_T^\sigma}}{M} + g_{1s}(x,\boldsymbol{k}_T)\,\gamma_5\,\slashed{n}_+ \\ & + h_{1T}(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\slashed{S}_T,\slashed{n}_+]}{2} + h_{1s}^{\perp}(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\slashed{k}_T,\slashed{n}_+]}{2M} + h_1^{\perp}(x,\boldsymbol{k}_T)\,\frac{i\,[\slashed{k}_T,\slashed{n}_+]}{2M}\Big\}\end{aligned}$$

Twist 3

$$\begin{aligned}+\frac{M}{2P^+}\Big\{ & e(x,\boldsymbol{k}_T) + f^{\perp}(x,\boldsymbol{k}_T)\,\frac{\slashed{k}_T}{M} - f_T(x,\boldsymbol{k}_T)\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} \\ & -\lambda\, f_L^{\perp}(x,\boldsymbol{k}_T)\,\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M} - e_s(x,\boldsymbol{k}_T)\,i\gamma_5 \\ & + g_T'(x,\boldsymbol{k}_T)\,\gamma_5\,\slashed{S}_T + g_s^{\perp}(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,\slashed{k}_T}{M} + h_T^{\perp}(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\slashed{S}_T,\slashed{k}_T]}{2M} \\ & + h_s(x,\boldsymbol{k}_T)\,\frac{\gamma_5\,[\slashed{n}_+,\slashed{n}_-]}{2} + h(x,\boldsymbol{k}_T)\,\frac{i\,[\slashed{n}_+,\slashed{n}_-]}{2}\Big\}.\end{aligned} \tag{3.44}$$

Formally derived within the “diagrammatic approach” :

no interpretation in TMD factorization (yet)



perturbative corrections

power corrections

1 α_s α_s^2 $\dots$

coupling expansion

1

M/Q

$(M/Q)^2$

$\dots$

twist expansion

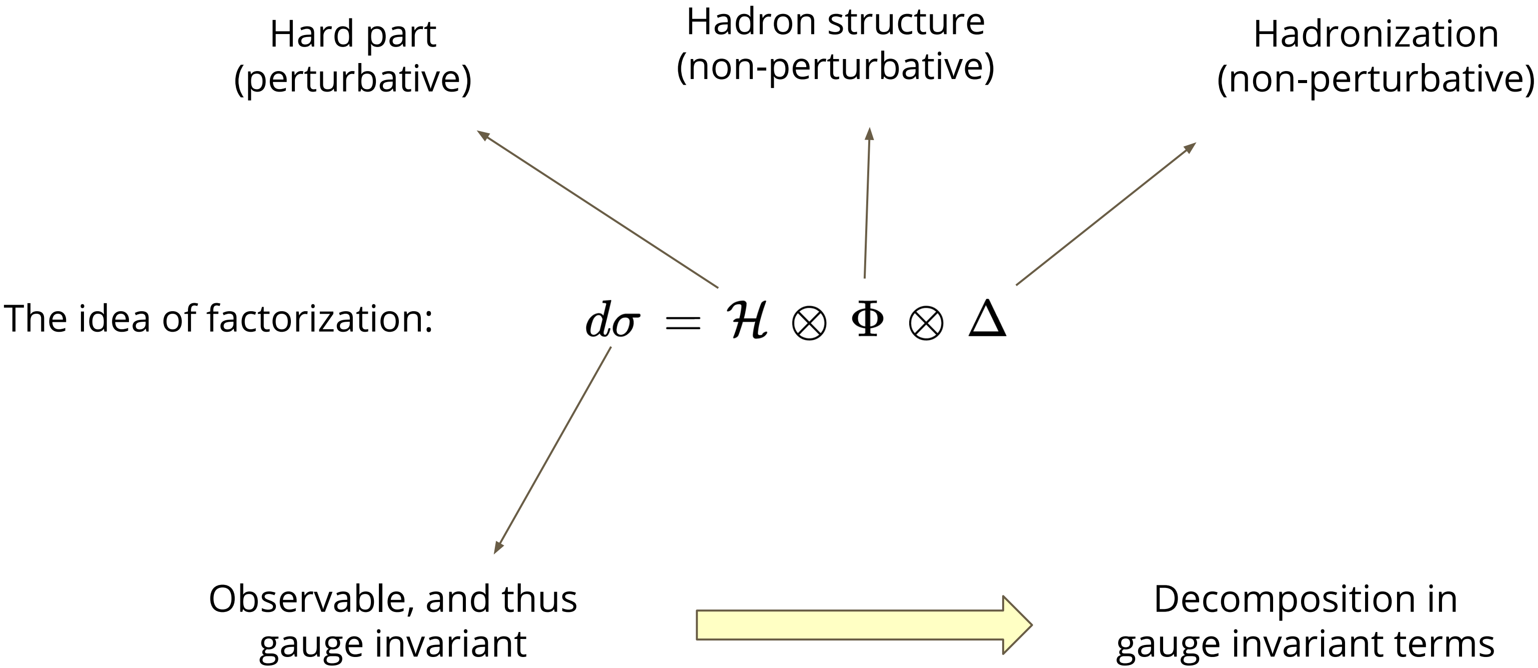
QCD observables

Plan of these lectures

1. **DIS and partons**
2. **From DIS to SIDIS**
3. **Symmetries and universality**
4. **Factorization, evolution, matching**
5. **Phenomenology**

3.1 Symmetries

Gauge symmetry



Hard part
(perturbative)

Hadron structure
(non-perturbative)

Hadronization
(non-perturbative)

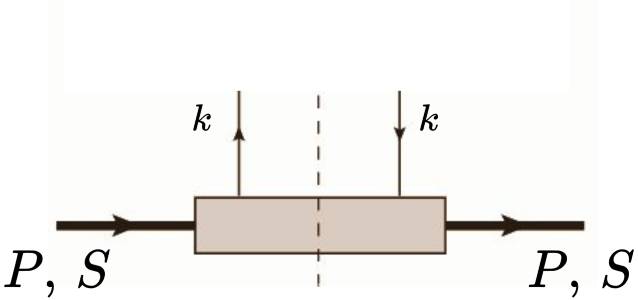
The idea of factorization: $d\sigma = \mathcal{H} \otimes \Phi \otimes \Delta$

Observable, and thus
gauge invariant

Decomposition in
gauge invariant terms

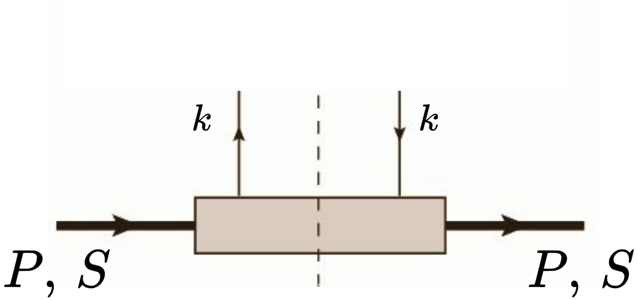
Quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS\big|\, \overline{\psi}_j(0)\,\psi_i(\xi)\big|PS\rangle$$



Quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\,\langle PS|\,\boxed{\overline{\psi}_j(0)\,\psi_i(\xi)}\,|PS\rangle$$



NOT GAUGE INVARIANT!

$$\mathcal{U}(x) = \ e^{i\,\alpha^a(x)\,t^a}$$

$$\overline{\psi}_j(0)\,\psi_i(\xi)\ \rightarrow\ \overline{\psi}_j(0)\,\mathcal{U}^\dagger(0)\,\mathcal{U}(\xi)\,\psi_i(\xi)$$

We need to “correct” the operator to make it gauge invariant

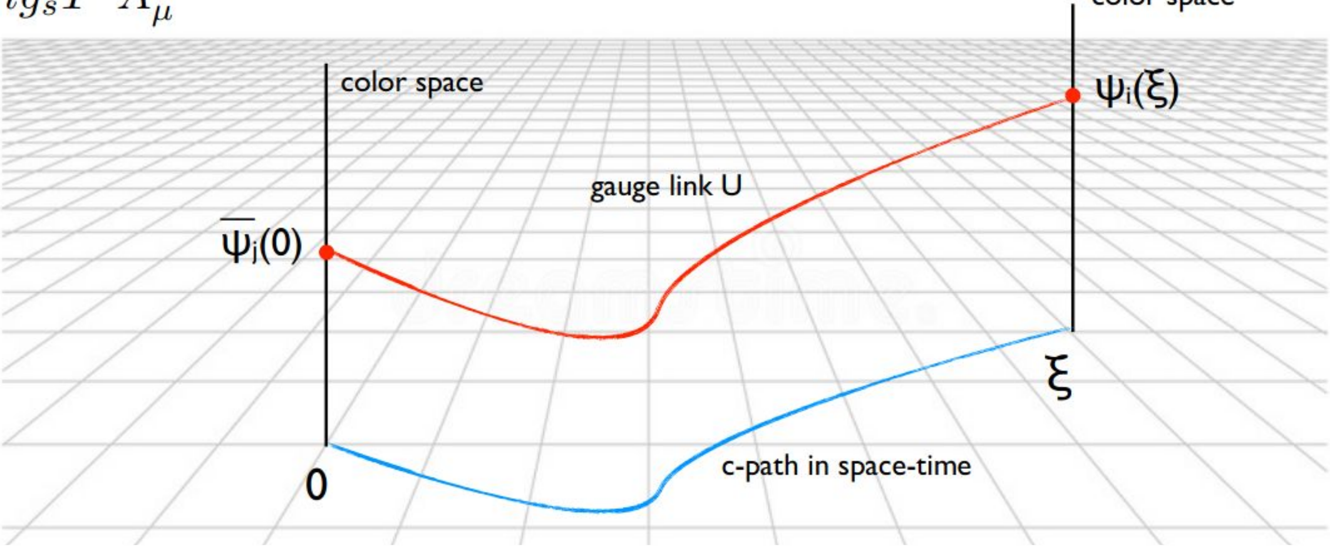
Close the non locality with a “gauge link” (or Wilson line)

Geometric interpretation

$$D_{\dot{c}}\ \psi(x(t)) = 0\ , \quad t \in I \subset \mathbb{R}$$

$$D_\mu = \partial_\mu - i g_s T^a A^a_\mu$$

“Parallel transport” to close the non-locality



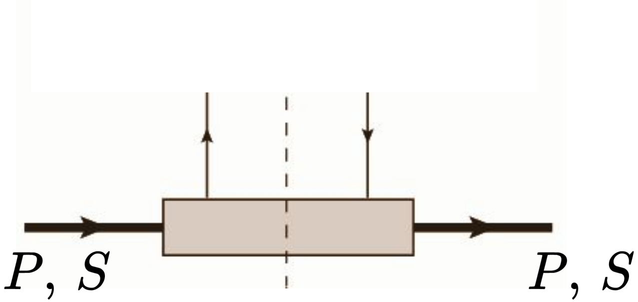
$$\psi'_\beta(x(t)) = \boxed{\mathbb{P}\exp\left\{-ig\int_0^t ds \frac{dx^\mu}{ds} A^a_\mu(x(s)) T^a_{\beta\alpha}\right\}}\ \psi_\alpha(x(0))$$

$$\doteq \boxed{U_{\beta\alpha}(x(t), x(0))}\ \psi_\alpha(x(0))\ .$$

Gauge link U

Gauge invariant quark correlator

$$\Phi_{ij}(k, P, S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\,\cdot\,\xi}\, \langle PS \big| \boxed{\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)} \big| PS\rangle$$



GAUGE INVARIANT!

$$\mathcal{U}(x) = e^{i\,\alpha^a(x)\,t^a}$$

$$U(0,\xi)\; \to\; \mathcal{U}(0)\, U(0,\xi)\, \mathcal{U}^\dagger(\xi)$$

The Wilson line “bridges” the non-locality and makes the operator gauge invariant

$$\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\; \to\; \overline{\psi}_j(0)\, \mathcal{U}^\dagger(0)\, \mathcal{U}(0)\, U(0,\xi)\, \mathcal{U}^\dagger(\xi)\, \mathcal{U}(\xi)\, \psi_i(\xi)\; =\; \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)$$

→ Eventually the correlator and the (TMD) PDFs **depend on the gauge link and its path** in spacetime

Discrete symmetries: parity

$a^\mu = \left(a^0,\, \vec{a}\right)\,, \qquad \tilde{a}^\mu = \left(a^0,\, -\vec{a}\right)$ ← let's consider this definition

$$
\begin{aligned}
&z^\mu \longrightarrow \tilde{z}^\mu \\
&P^\mu \longrightarrow \tilde{P}^\mu \\
&S^\mu \longrightarrow S^\mu \equiv -\tilde{S}^\mu \quad (\text{since } S^\mu = (0, \vec{S}) \ \text{ by definition}) \\
&n_\pm \longrightarrow n_\mp \\
&\psi(\xi) \longrightarrow \mathscr{P}\,\psi(\xi)\,\mathscr{P}^\dagger = \Lambda_{\mathscr{P}}\,\psi(\tilde{\xi})\,, \quad \Lambda_{\mathscr{P}} = \gamma^0 \\
&\gamma^\mu \longrightarrow \mathscr{P}\,\gamma^\mu\,\mathscr{P}^\dagger = \Lambda_{\mathscr{P}}\,\gamma^\mu\,\Lambda^\dagger_{\mathscr{P}}
\end{aligned}
$$

The action on the quark field is the one that leaves the QCD lagrangian invariant under parity transformation (symmetry)

Discrete symmetries: time reversal

$a^\mu = \left(a^0, \, \vec{a}\right) \, , \qquad \tilde{a}^\mu = \left(a^0, \, -\vec{a}\right)$ ← let's consider this definition

$$z^\mu \longrightarrow -\tilde{z}^\mu$$

$$P^\mu \longrightarrow \tilde{P}^\mu$$

$$S^\mu \longrightarrow \tilde{S}^\mu$$

$$n_\pm \longrightarrow -n_\mp$$

$$\psi(\xi) \longrightarrow \mathscr{T}\,\psi(\xi)\mathscr{T}^\dagger = \Lambda_\mathscr{T}\,\psi(-\tilde{\xi})\,, \quad \Lambda_\mathscr{T} = -i\gamma_5 C = i\gamma^1\gamma^3$$

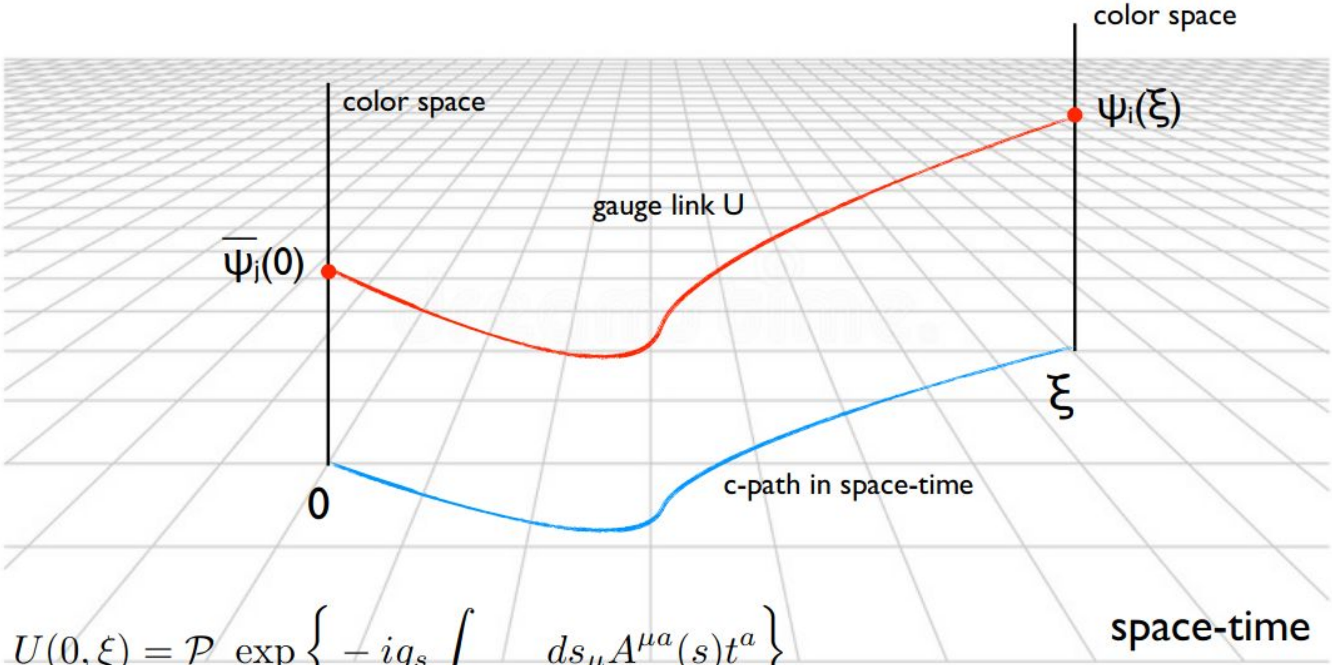
$$\gamma^\mu \longrightarrow \mathscr{T}\,\gamma^\mu\,\mathscr{T}^\dagger = \Lambda_\mathscr{T}\,\gamma^\mu\,\Lambda^\dagger_\mathscr{T} = \gamma^*_\mu$$

The action on the quark field is the one that leaves the QCD lagrangian invariant under time reversal transformation (symmetry)

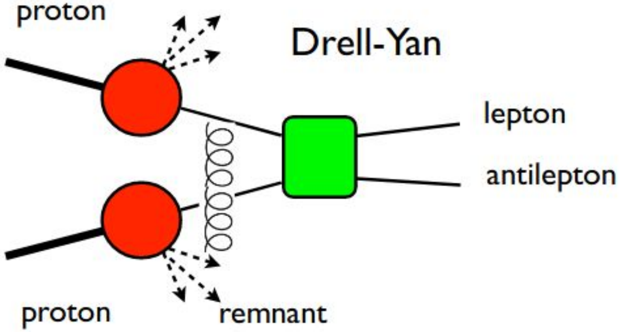
3.2 Universality ?

Geometric structure

$$\Phi(k,P) = \text{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle \longrightarrow f_1^{a\ [U]}(x,k_T^2)\ \not{P} + \cdots$$

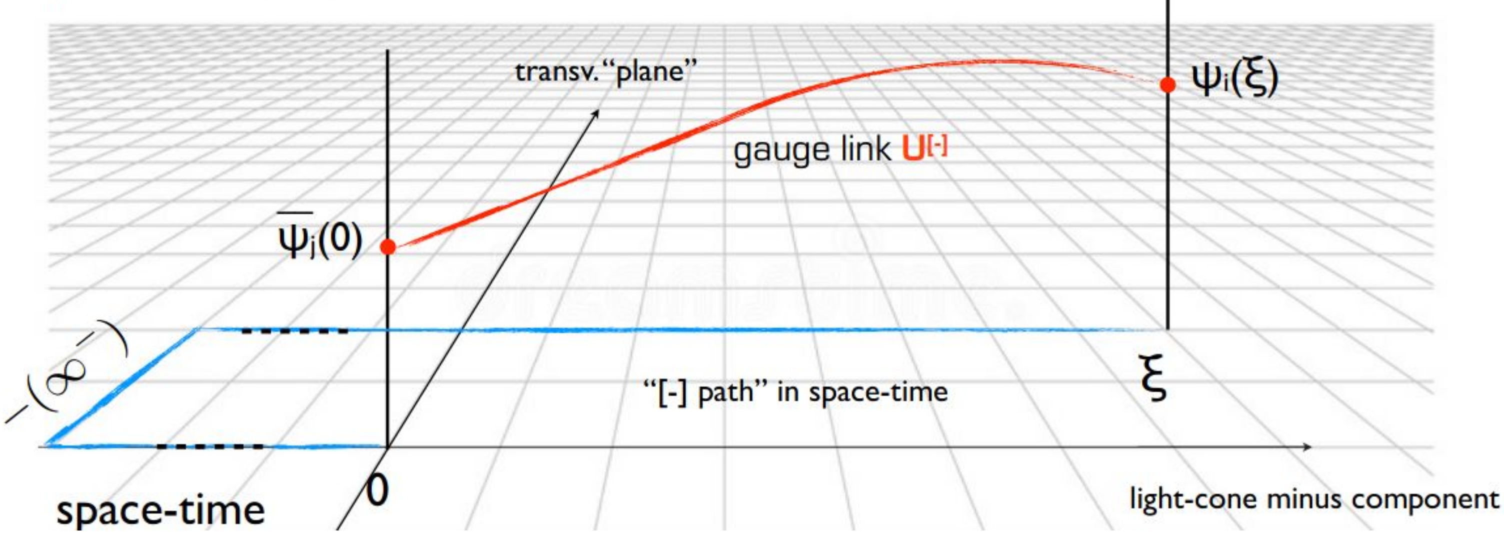


$$U(0,\xi) = \mathcal{P}\ \exp\left\{-ig_s\int_{c(0,\xi)} ds_\mu A^{\mu a}(s)t^a\right\}$$

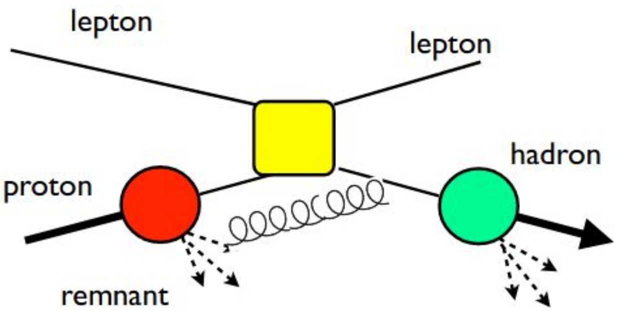


In **Drell-Yan** the **remnant** of the proton feels the color force of a **quark** in the **initial state**

$$\Phi(k,P) = \mathrm{F.T.}\langle P|\overline{\psi_j}(0)\; U^{[-]}(0,\xi)\; \psi_i(\xi)|P\rangle$$

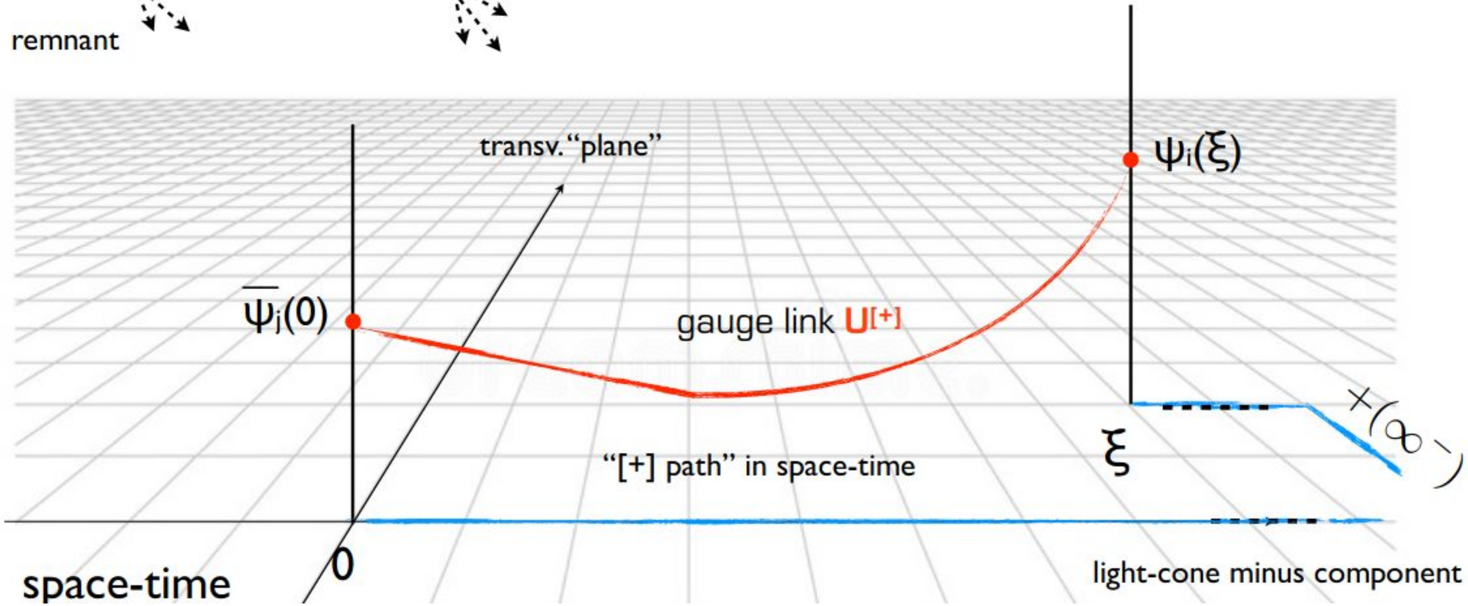


Distributions defined with **U^-** gauge link: $f_1^{[U^-]}\left(x, k_T^2\right)$



In **SIDIS** the **remnant** of the proton feels the color force of a **quark** in the **final state**

$$\Phi(k,P) = \text{F.T.}\langle P|\overline{\psi_j}(0)\; U^{[+]}(0,\xi)\; \psi_i(\xi)|P\rangle$$

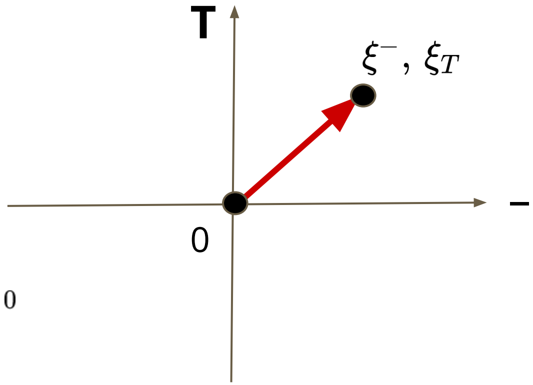


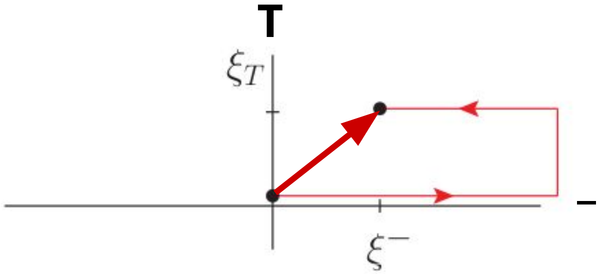
Distributions defined with ***U*⁺** gauge link: $f_1^{[U^+]}\left(x, k_T^2\right)$

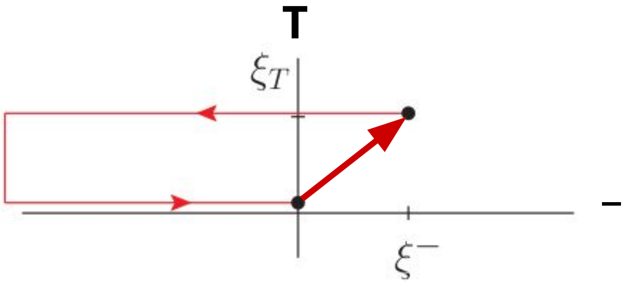
Gauge links for TMD PDFs

$$\Phi_{ij}^{[U]}(x,\, \mathbf{p}_T, S) \;=\; \int dp^+\, dp^-\, \delta\big(p^+ \;-xP^+\big)\,\Phi^{[U]}(p,P,S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\; e^{i\,p\cdot\xi}\,\langle PS\big|\,\overline{\psi}_j(0)\,U(0,\xi)\,\psi_i(\xi)\,\big|PS\rangle_{\,\xi^+=0}$$







$U^{[+]}$ Future pointing (SIDIS)

$U^{[-]}$ Past pointing (Drell-Yan)

Process dependence

The hard process determines the path of the link U,
and the **distributions are process dependent**.

What happens to the *universal* concept of hadron structure?

Process dependence

The interplay between **time reversal** and **gauge symmetry** generates **relations** between the two configurations:

$$f_1^{a\;[+]}(x, k_T^2) = f_1^{a\;[-]}(x, k_T^2)$$

T-even distribution

striking consequence
of the symmetries of QCD

$$f_{1T}^{a\perp\;[+]}(x, k_T^2) = -f_{1T}^{a\perp\;[-]}(x, k_T^2)$$

T-odd distribution

Sign-change relation for the Sivers function : not yet confirmed experimentally

Implications of discrete symmetries

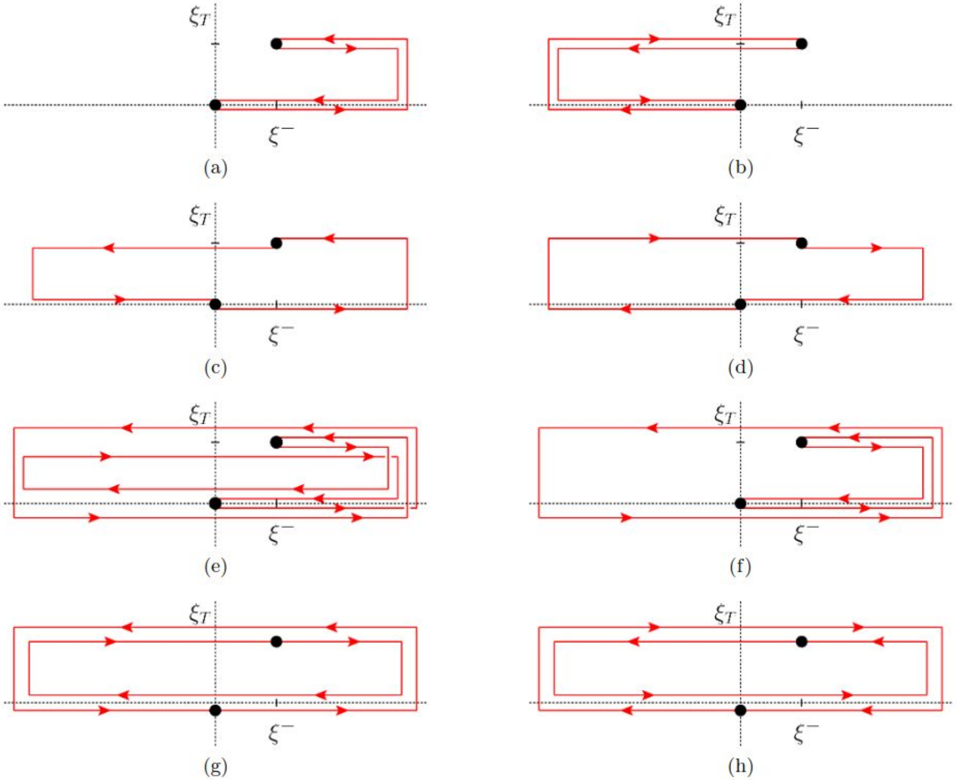
$$U_\pm(a,b)^\dagger = U_\pm(b,a)$$
$$\mathscr{P} U_\pm(a,b) \mathscr{P}^\dagger = U_\pm(\tilde{a},\tilde{b})$$
$$\mathscr{T} U_\pm(a,b) \mathscr{T}^\dagger = U_\mp(-\tilde{a},-\tilde{b})$$

We are going to **derive** these properties (together with the **sign change** of the T-odd Sivers function) during the **recitation sessions**

$$\begin{aligned}
\text{Hermiticity:} \quad & \Phi^{[\pm]\dagger}(k;P,S) = \gamma^0 \Phi^{[\pm]}(k;P,S)\gamma^0 \\
\text{Parity:} \quad & \Phi^{[\pm]}(k;P,S) = \gamma^0 \Phi^{[\pm]}(\tilde{k};\tilde{P},-\tilde{S})\gamma^0 \\
\text{Time reversal:} \quad & \Phi^{[\pm]*}(k;P,S) = i\gamma^1\gamma^3 \Phi^{[\mp]}(\tilde{k};\tilde{P},\tilde{S}) i\gamma^1\gamma^3
\end{aligned}$$

Gauge links for gluon TMDs (more complicated)

$F^{\mu\nu}(0)\,U(0,\xi)\,F^{\rho\sigma}(\xi)\,U'(\xi,\,0)$ ← more complicated operator with two gauge links



(a) (b) (c) (d) (e) (f) (g) (h)

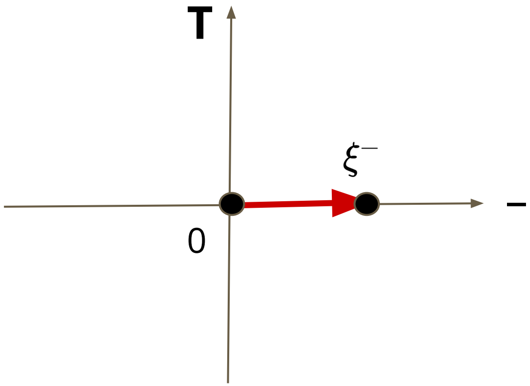
The process dependence for these TMDs amounts to more complicated relations than a minus sign (but still calculable!)

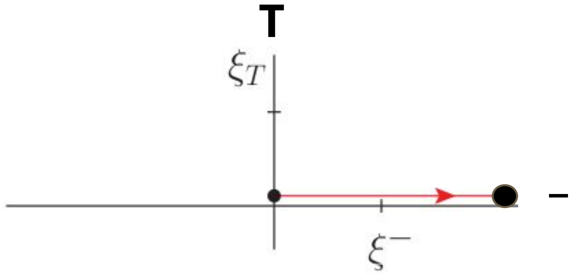
For more details see https://inspirehep.net/literature/1391461

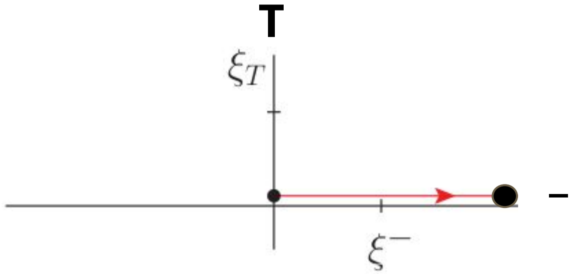
Gauge links for collinear PDFs (simpler)

$$\Phi_{ij}^{[U]}(x,S) = \int dk^+\, dk^-\, d^2\mathbf{k}_T\, \delta\big(k^+ - xP^+\big)\Phi^{[U]}(k,P,S) =$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{i\,k\cdot\xi}\, \langle PS\big|\, \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\, \big|PS\rangle_{\,\xi^+ = \,\xi_T = 0}$$







In the collinear limit the two gauge links reduce to the same object