HUGS 2021 Lectures on:
Experimental Meson Spectroscopy

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LECTURE II. Meson Quantum Numbers

- IIA. Meson Naming Scheme
- IIB. J^{PC} (spin, parity, C-parity)
  - * from experiment
  - * from a $q\bar{q}$ model
- IIC. Flavor
  - * Strangeness, Charm, Bottomness
  - * Isospin
  - * G-Parity
  - * Flavor SU(3)

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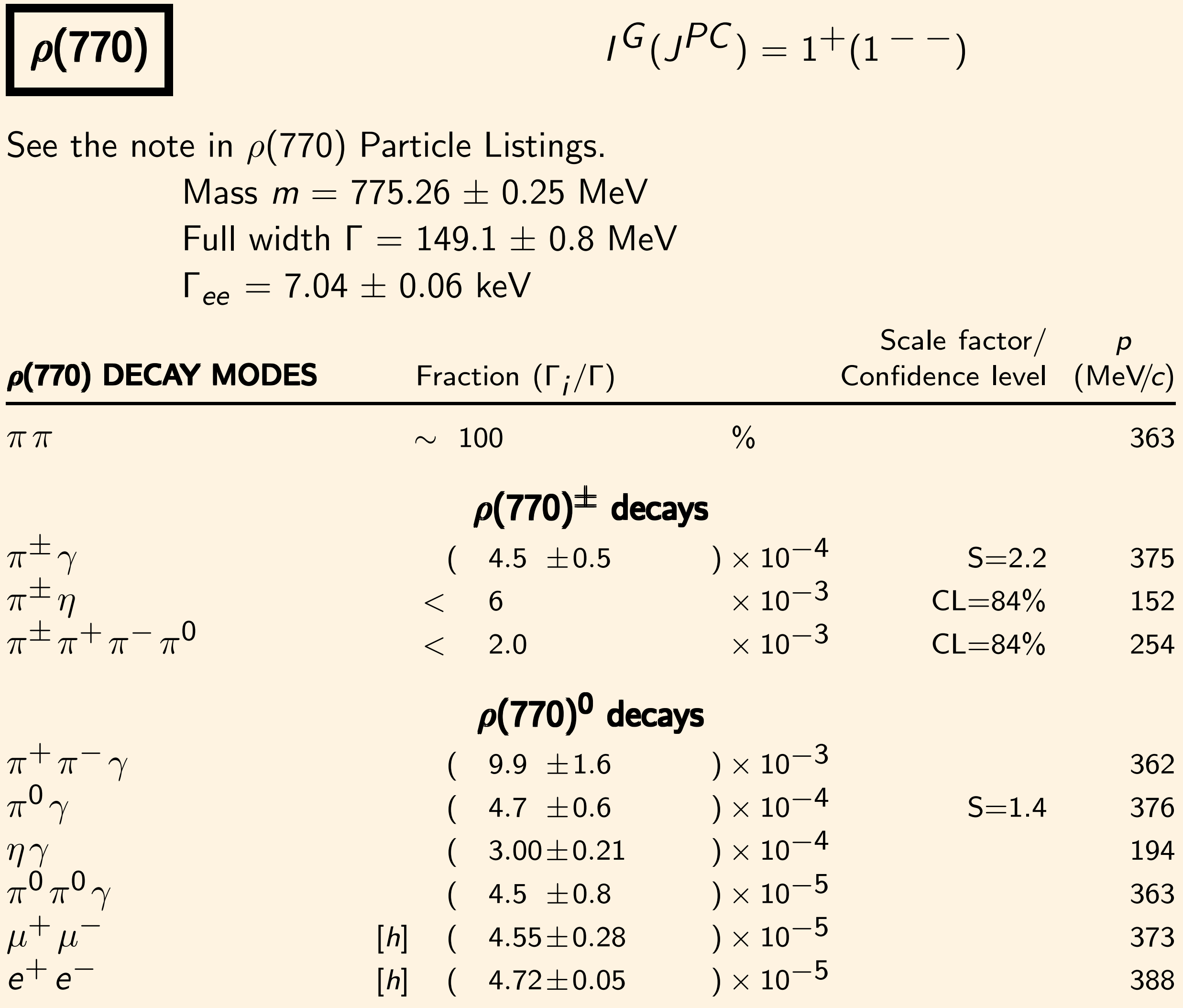


$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

See the note in $\rho(770)$ Particle Listings.

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

$\rho(770)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$\pi\pi$		$\sim$ 100 %		363
$\rho(770)^\pm$ decays				
$\pi^\pm\gamma$		$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2	375
$\pi^\pm\eta$		$< 6 \times 10^{-3}$	CL=84%	152
$\pi^\pm\pi^+\pi^-\pi^0$		$< 2.0 \times 10^{-3}$	CL=84%	254
$\rho(770)^0$ decays				
$\pi^+\pi^-\gamma$		$(9.9 \pm 1.6) \times 10^{-3}$		362
$\pi^0\gamma$		$(4.7 \pm 0.6) \times 10^{-4}$	S=1.4	376
$\eta\gamma$		$(3.00 \pm 0.21) \times 10^{-4}$		194
$\pi^0\pi^0\gamma$		$(4.5 \pm 0.8) \times 10^{-5}$		363
$\mu^+\mu^-$	[h]	$(4.55 \pm 0.28) \times 10^{-5}$		373
e^+e^-	[h]	$(4.72 \pm 0.05) \times 10^{-5}$		388

Particle Data Group (PDG) Summary Table Entry for $\rho(770)$

LECTURE II. Meson Quantum Numbers

IIA. Meson Naming Scheme

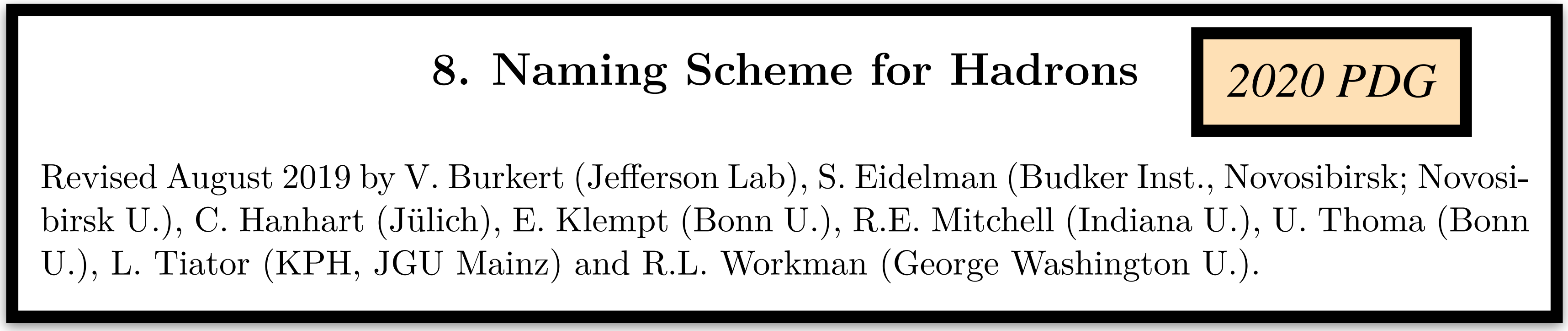
IIB. J^{PC} (spin, parity, C-parity)

- * from experiment
- * from a $q\bar{q}$ model

IIC. Flavor

- * Strangeness, Charm, Bottomness
- * Isospin
- * G-Parity
- * Flavor SU(3)

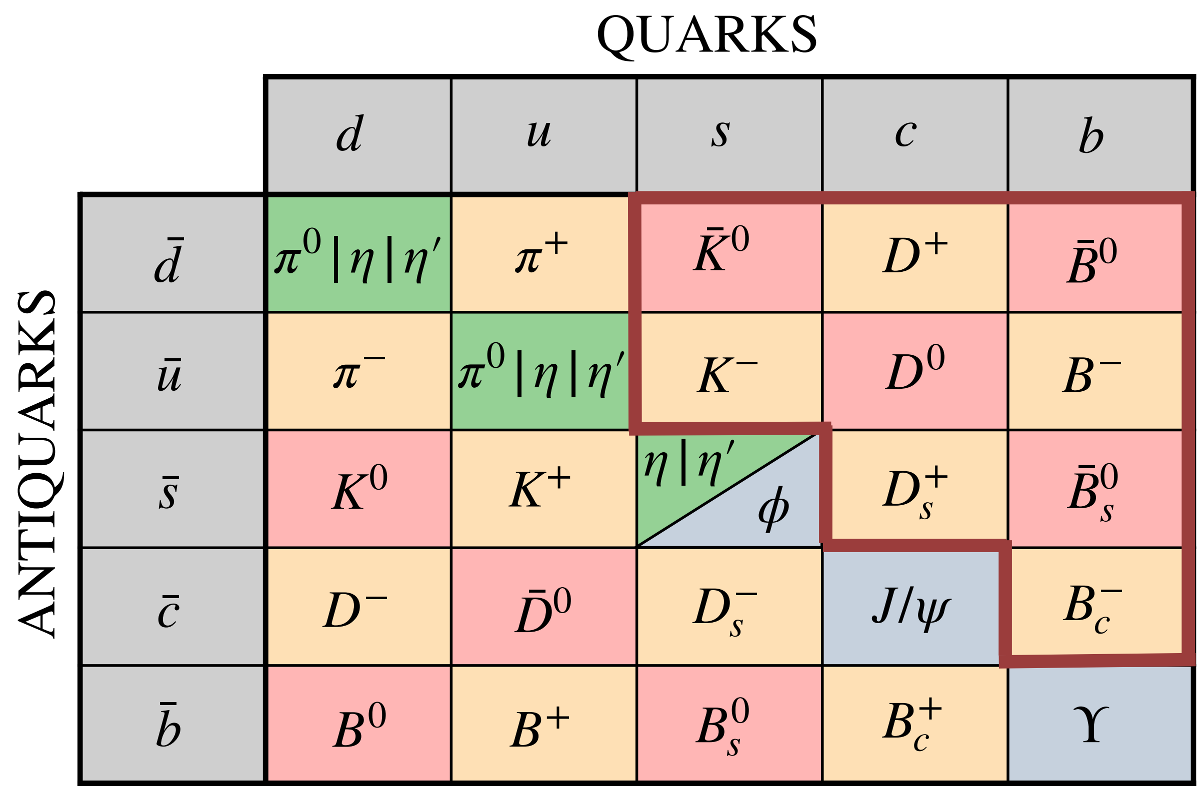
IIA. Meson Naming Scheme



8. Naming Scheme for Hadrons

2020 PDG

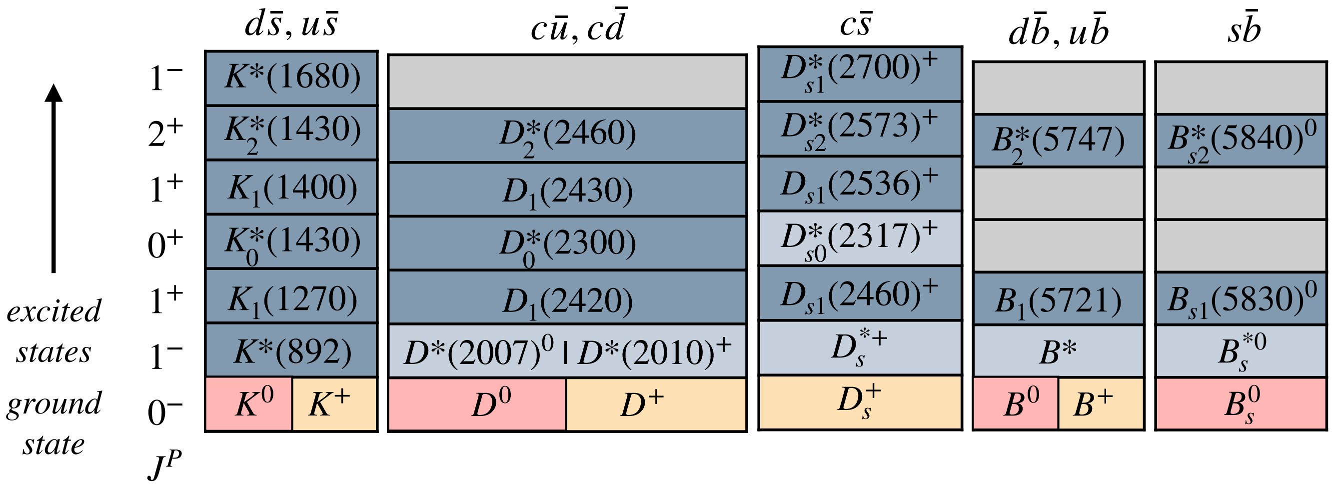
Revised August 2019 by V. Burkert (Jefferson Lab), S. Eidelman (Budker Inst., Novosibirsk; Novosibirsk U.), C. Hanhart (Jülich), E. Klempt (Bonn U.), R.E. Mitchell (Indiana U.), U. Thoma (Bonn U.), L. Tiator (KPH, JGU Mainz) and R.L. Workman (George Washington U.).



QUARKS

ANTIQUARKS

	d	u	s	c	b
$\bar{d}$	$\pi^0\,\vert\,\eta\,\vert\,\eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
$\bar{u}$	π^-	$\pi^0\,\vert\,\eta\,\vert\,\eta'$	K^-	D^0	B^-
$\bar{s}$	K^0	K^+	$\eta\,\vert\,\eta'$ ϕ	D_s^+	$\bar{B}_s^0$
$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ



	J^P	$d\bar{s}, u\bar{s}$	$c\bar{u}, c\bar{d}$	$c\bar{s}$	$d\bar{b}, u\bar{b}$	$s\bar{b}$
excited states	1^-	$K^*(1680)$		$D_{s1}^*(2700)^+$		
	2^+	$K_2^*(1430)$	$D_2^*(2460)$	$D_{s2}^*(2573)^+$	$B_2^*(5747)$	$B_{s2}^*(5840)^0$
	1^+	$K_1(1400)$	$D_1(2430)$	$D_{s1}(2536)^+$		
	0^+	$K_0^*(1430)$	$D_0^*(2300)$	$D_{s0}^*(2317)^+$		
	1^+	$K_1(1270)$	$D_1(2420)$	$D_{s1}(2460)^+$	$B_1(5721)$	$B_{s1}(5830)^0$
	1^-	$K^*(892)$	$D^*(2007)^0\,\vert\,D^*(2010)^+$	D_s^{*+}	B^*	B_s^{*0}
ground state	0^-	K^0 K^+	D^0 D^+	D_s^+	B^0 B^+	B_s^0

Separate mesons into "flavored" and "unflavored" ($S = 0, C = 0, B = 0$):

Flavored:

- * the name is based on the flavor of the heaviest quark (or antiquark)

  $\implies K, D, B$

- * a subscript is used for the flavor of the other quark (or antiquark), if there is another flavor

  $\implies D_s, B_s, B_c$

- * include a * for $J^P = 0^+,\ 1^-,\ 2^+,\ 3^-,\ \ldots$ and no * for $J^P = 0^-,\ 1^+,\ 2^-,\ 3^+,\ \ldots$

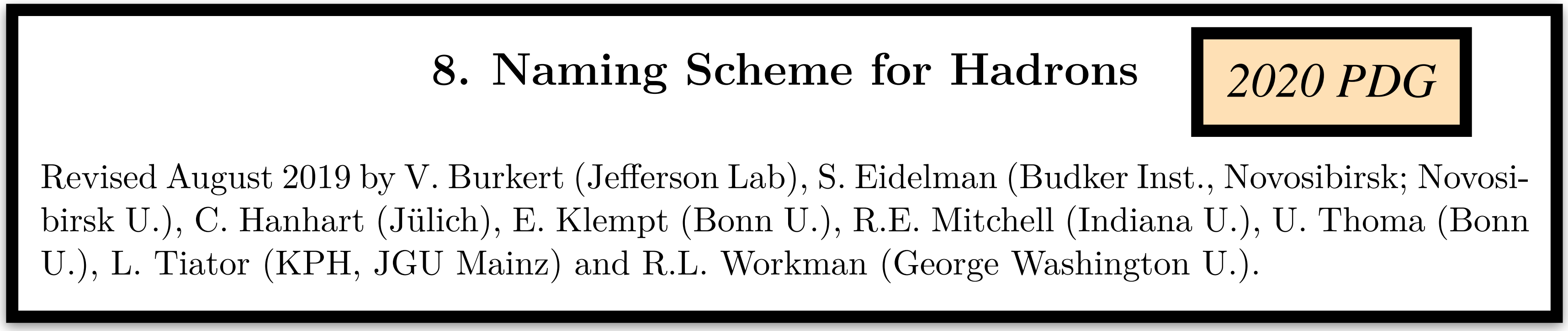
  $\implies K,\ K^*,\ \ldots$

- * the spin is another subscript, but is implied for $J^P = 0^-$ and 1^-

  $\implies K_1,\ K_2,\ K_2^*,\ \ldots$

Given the name, you know at least the flavor and J^P.

IIA. Meson Naming Scheme



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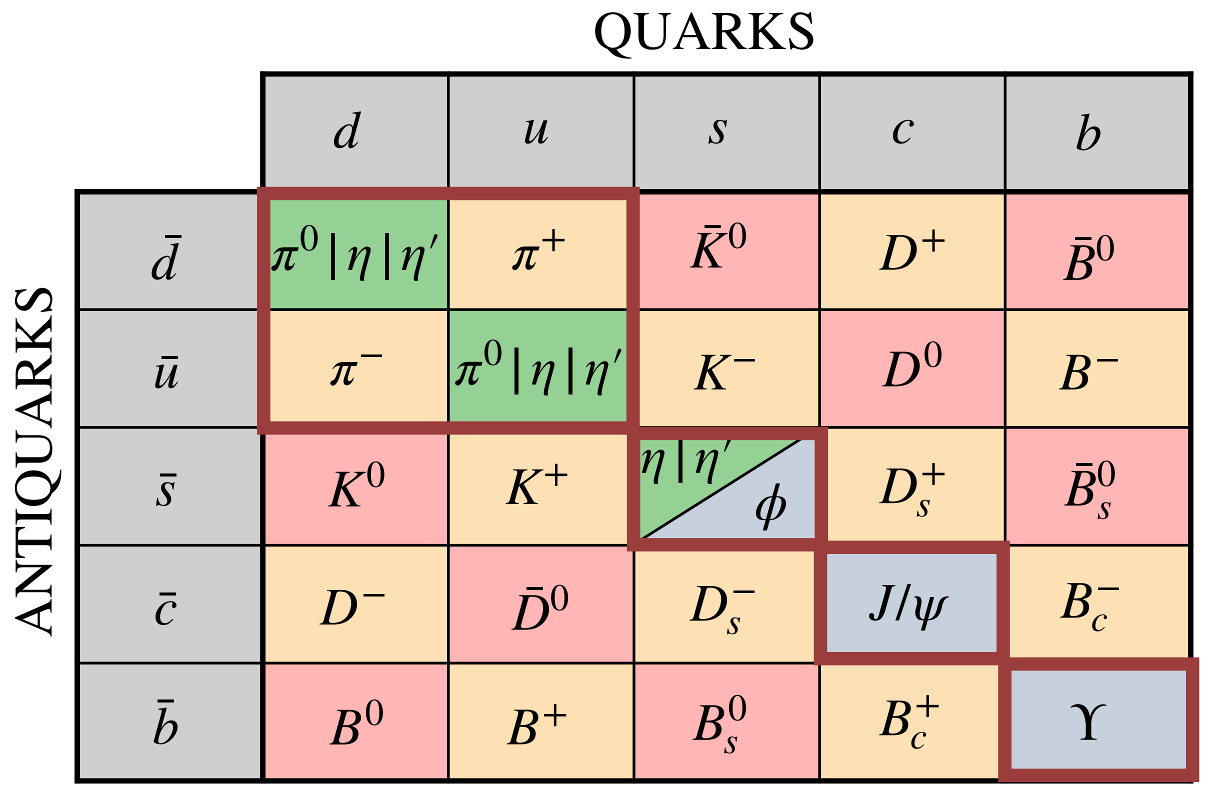
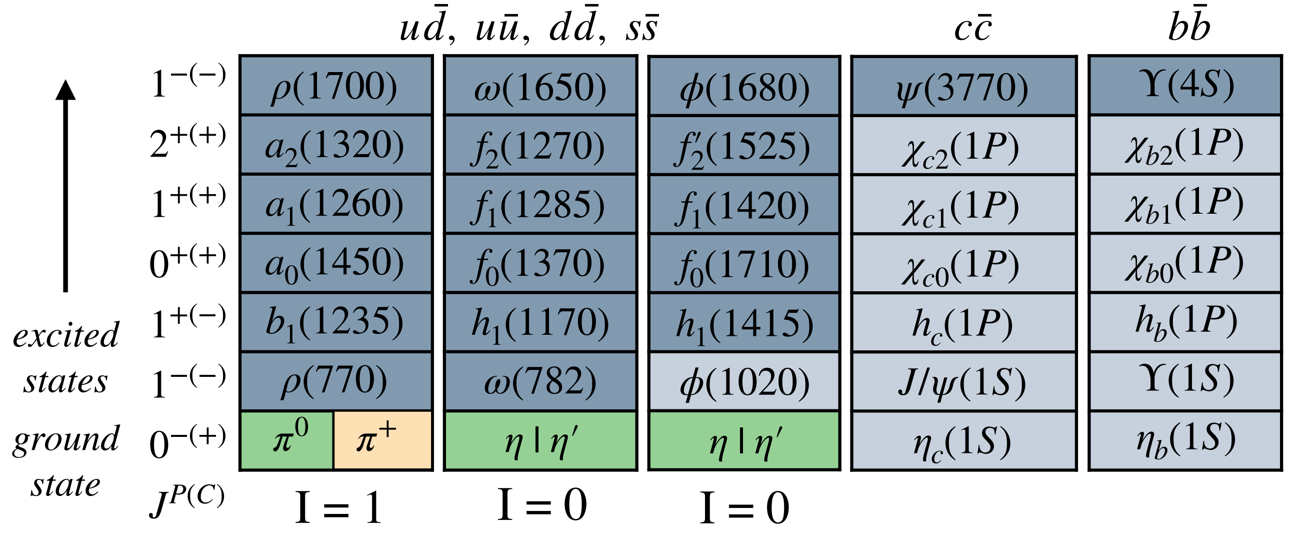


QUARKS

ANTIQUARKS	d	u	s	c	b
$\bar d$	$\pi^0 \mid \eta \mid \eta'$	π^+	$\bar K^0$	D^+	$\bar B^0$
$\bar u$	π^-	$\pi^0 \mid \eta \mid \eta'$	K^-	D^0	B^-
$\bar s$	K^0	K^+	$\eta \mid \eta'$ ϕ	D_s^+	$\bar B_s^0$
$\bar c$	D^-	$\bar D^0$	D_s^-	J/ψ	B_c^-
$\bar b$	B^0	B^+	B_s^0	B_c^+	Υ



	$u\bar d,\ u\bar u,\ d\bar d,\ s\bar s$			$c\bar c$	$b\bar b$
$1^{-(-)}$	$\rho(1700)$	$\omega(1650)$	$\phi(1680)$	$\psi(3770)$	$\Upsilon(4S)$
$2^{+(+)}$	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$
$1^{+(+)}$	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$
$0^{+(+)}$	$a_0(1450)$	$f_0(1370)$	$f_0(1710)$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$
$1^{+(-)}$	$b_1(1235)$	$h_1(1170)$	$h_1(1415)$	$h_c(1P)$	$h_b(1P)$
$1^{-(-)}$	$\rho(770)$	$\omega(782)$	$\phi(1020)$	$J/\psi(1S)$	$\Upsilon(1S)$
$0^{-(+)}$	π^0 π^+	$\eta \mid \eta'$	$\eta \mid \eta'$	$\eta_c(1S)$	$\eta_b(1S)$
$J^{P(C)}$	I = 1	I = 0	I = 0		

Separate mesons into "flavored" and "unflavored" ($S = 0, C = 0, B = 0$):

Unflavored:

$J^{PC} = \Big\{$	0^{-+}	1^{+-}	1^{--}	0^{++}
	2^{-+}	3^{+-}	2^{--}	1^{++}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Minimal quark content				
$u\bar d, u\bar u - d\bar d, d\bar u$ $(I = 1)$	π	b	ρ	a
$d\bar d + u\bar u$ and/or $s\bar s$ $(I = 0)$	η,η'	h,h'	ω,ϕ	f,f'
$c\bar c$	η_c	h_c	ψ	χ_c
$b\bar b$	η_b	h_b	Υ	χ_b
$I = 1$ with $c\bar c$	(Π_c)	Z_c	R_c	(W_c)
$I = 1$ with $b\bar b$	(Π_b)	Z_b	(R_b)	(W_b)

* again use subscripts for the spin, when not implied

* note that primes are ambiguous: f' is mostly $s\bar s$; the η' is not; sometimes $\rho' = \rho(1700)$, $\psi' = \psi(2S)$, etc.

* spectroscopic notation is sometimes used for states with two heavy quarks

Given the name, you know at least the isospin and J^{PC}.

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

Typically, determine the spin (J) of a meson using the angular distribution of its decay to a known final state.

(1) For a decay $A \to BC$, start with A at rest with spin J and spin projection M along $\hat{z}$:

CM 4-vector

$$|A\rangle = |JM\rangle\,|P_{\text{CM}}\rangle\,|\alpha_A\rangle$$

other quantum numbers
(e.g. PC, flavor)

spin and spin projection

(2) The final state, assuming a particular configuration of helicities λ_B and λ_C with $\lambda \equiv \lambda_B - \lambda_C$, is then:

spherical wave (s)

$$|BC\rangle = |JM\lambda_B\lambda_C\rangle_{\text{s}}\,|P_{\text{CM}}\rangle\,|\alpha_{BC}\rangle$$

plane wave (p)

$$= \int d\Omega\,|\theta\phi\lambda_B\lambda_C\rangle_{\text{p}}\,\langle\theta\phi\lambda_B\lambda_C|JM\lambda_B\lambda_C\rangle_{\text{s}}\,|P_{\text{CM}}\rangle\,|\alpha_{BC}\rangle$$

Wigner D-function

$$= \int d\Omega\sqrt{\frac{2J+1}{4\pi}}\,D^{J*}_{M,\lambda}(\phi,\theta,0)\,|\theta\phi\lambda_B\lambda_C\rangle_{\text{p}}\,|P_{\text{CM}}\rangle\,|\alpha_{BC}\rangle$$

(3) The probability to find particle B traveling in the (θ,ϕ) direction is:

$$I(\theta,\phi) \propto \left|D^{J*}_{M,\lambda}(\phi,\theta,0)\right|^2$$

Or if B and C both have spin-0:

spherical harmonic

$$I(\theta,\phi) \propto \left|Y^M_J(\theta,\phi)\right|^2$$

IIB. Mes

(details of a few missing steps)

spherical wave (s)

$$|BC\rangle = |JM\lambda_B\lambda_C\rangle_{\rm s}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

plane wave (p)

$$= \int d\Omega\,|\theta\phi\lambda_B\lambda_C\rangle_{\rm p}\,\langle\theta\phi\lambda_B\lambda_C|JM\lambda_B\lambda_C\rangle_{\rm s}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

rotation to $<\hat{n}| = <\theta,\phi|$ from $<\hat{z}| = <0,0|$

$$= \int d\Omega\,|\theta\phi\lambda_B\lambda_C\rangle_{\rm p}\,\langle 00\lambda_B\lambda_C|\hat{R}^\dagger(\phi,\theta,0)|JM\lambda_B\lambda_C\rangle_{\rm s}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

$$= \int d\Omega\,|\theta\phi\lambda_B\lambda_C\rangle_{\rm p}\sum_{J'M'}\underset{\Longrightarrow\ M'=\lambda}{\langle 00\lambda_B\lambda_C|J'M'\lambda_B\lambda_C\rangle_{\rm s}}\,\underset{\Longrightarrow\ J'=J}{\langle J'M'\lambda_B\lambda_C|\hat{R}^\dagger(\phi,\theta,0)|JM\lambda_B\lambda_C\rangle_{\rm s}}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

$$= \int d\Omega\,|\theta\phi\lambda_B\lambda_C\rangle_{\rm p}\,\langle 00\lambda_B\lambda_C|J\lambda\lambda_B\lambda_C\rangle_{\rm s}\,\langle J\lambda\lambda_B\lambda_C|\hat{R}^\dagger(\phi,\theta,0)|JM\lambda_B\lambda_C\rangle_{\rm s}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

Wigner D-function

$$= \int d\Omega\sqrt{\frac{2J+1}{4\pi}}\,D^{J*}_{M,\lambda}(\phi,\theta,0)\,|\theta\phi\lambda_B\lambda_C\rangle_{\rm p}\,|P_{\rm CM}\rangle\,|\alpha_{BC}\rangle$$

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(1) For a decay $A \rightarrow BC$

(2) The final state, assum

spherical wave (s)

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plane wave (p)

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(3) The probability to find particle B traveling in the (θ, ϕ) direction is:

$$I(\theta,\phi) \propto \left|D^{J*}_{M,\lambda}(\phi,\theta,0)\right|^2$$

Or if B and C both have spin-0:

spherical harmonic

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

In addition, determine the parity (P) and C-parity (C) using the conservation of P and C.

Example: Determine the J^{PC} of a meson A decaying to $\pi^+\pi^-$.

Start using known information about the pion. It has $J = 0$ and:

$$\hat{P}\,|\pi^{\pm}\rangle = -\,|\pi^{\pm}\rangle$$

$$\hat{C}\,|\pi^{\pm}\rangle = -\,|\pi^{\mp}\rangle$$

pion quantum numbers

(1) Put the $\pi^+\pi^-$ system into a definite state of J^{PC} to match the initial state, assuming A has been produced so that it has spin projection M along the $\hat{z}$-axis:

spherical wave

pion quantum numbers

$$|J^{PC}(\pi^+\pi^-)\rangle \equiv \frac{1}{\sqrt{2}}(1+\hat{P}_{12})\,|JM\rangle\,|P_{\rm CM}\rangle\,|\pi^+\pi^-\rangle$$

permutation symmetry

plane wave

CM 4-vector

$$= \frac{1}{\sqrt{2}}(1+\hat{P}_{12})\int d\Omega\,|\theta\phi\rangle\,\langle\theta\phi|JM\rangle\,|P_{\rm CM}\rangle\,|\pi^+\pi^-\rangle$$

$$= \frac{1}{\sqrt{2}}(1+\hat{P}_{12})\int d\Omega\,Y_J^M(\theta,\phi)\,|\theta\phi\rangle\,|P_{\rm CM}\rangle\,|\pi^+\pi^-\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|\pi^+\pi^-\rangle + (-1)^J\,|\pi^-\pi^+\rangle\right]\int d\Omega\,Y_J^M(\theta,\phi)\,|\theta\phi\rangle\,|P_{\rm CM}\rangle$$

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

In addition, determine the parity (P) and C-parity (C) using the conservation of P and C.

Example: Determine the J^{PC} of a meson A decaying to $\pi^+\pi^-$.

(1) Put the $\pi^+\pi^-$ system into a definite state of J^{PC} to match the initial state, assuming A has been produced so that it has spin projection M along the $\hat{z}$-axis:

$$|J^{PC}(\pi^+\pi^-)\rangle = \frac{1}{\sqrt{2}}\left[|\pi^+\pi^-\rangle + (-1)^J\,|\pi^-\pi^+\rangle\right]\int d\Omega\, Y_J^M(\theta,\phi)\,|\theta\phi\rangle\,|P_{\rm CM}\rangle$$

(2) Measure the $\pi^+\pi^-$ system in the state: $|\theta\phi\rangle\,|P_{\rm CM}\rangle\,|\pi^+\pi^-\rangle$

(3) Then the angular distribution will be:

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$

$J=0,\ M=0:\quad I \propto \text{constant}$
$J=1,\ M=\pm1:\quad I \propto \sin^2\theta$
$J=1,\ M=0:\quad I \propto \cos^2\theta$
$J=2,\ M=\pm2:\quad I \propto \sin^4\theta$
$J=2,\ M=\pm1:\quad I \propto \sin^2\theta\cos^2\theta$
$J=2,\ M=0:\quad I \propto 9\cos^4\theta - 6\cos^2\theta + 1$

(4) The parity and C-parity are:

$$\hat{P}\,|J^{PC}(\pi^+\pi^-)\rangle = (-1)^J\,|J^{PC}(\pi^+\pi^-)\rangle$$
$$\hat{C}\,|J^{PC}(\pi^+\pi^-)\rangle = (-1)^J\,|J^{PC}(\pi^+\pi^-)\rangle$$

(5) So the possible J^{PC} are: $0^{++},\ 1^{--},\ 2^{++},\ 3^{--},\ \ldots$

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

In addition, determine th … **using the conservation of *P* and *C*.**

Example: Determine … **g to $\pi^+\pi^-$.**

(1) Put the $\pi^+\pi^-$ system into … initial state, assuming *A* has been produced so that it has s

$$|J^{PC}(\pi^+\pi^-)\rangle = \quad {}^{+}\rangle] \int d\Omega\, Y_J^M(\theta,\phi)\, |\theta\phi\rangle\, |P_{\mathrm{CM}}\rangle$$

(2) Measure the $\pi^+\pi^-$ system … $\pi^-\rangle$

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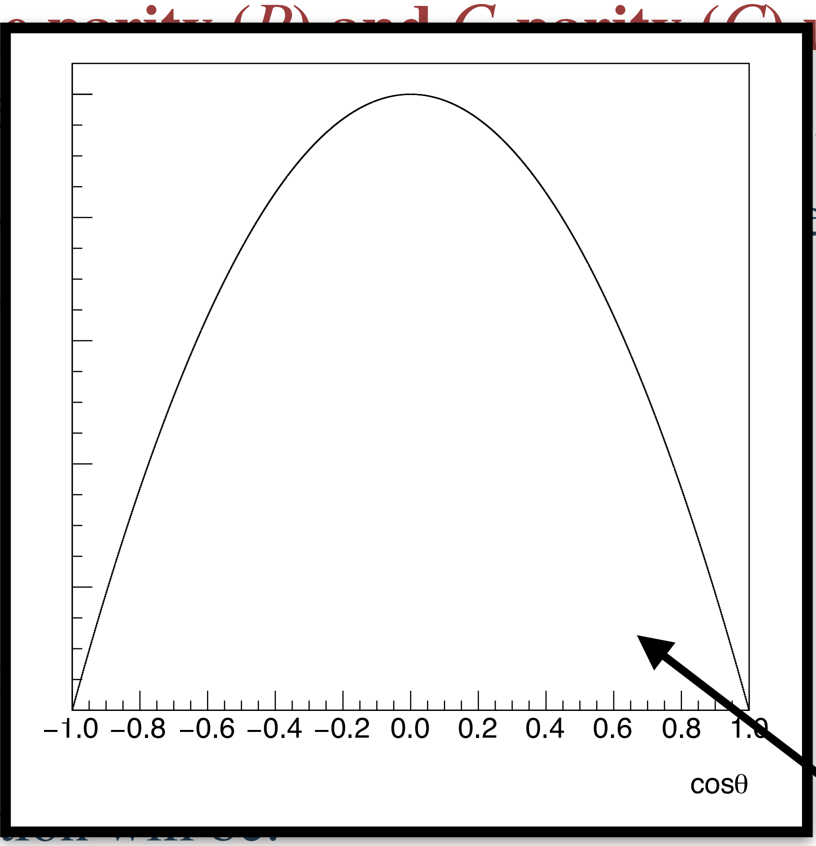

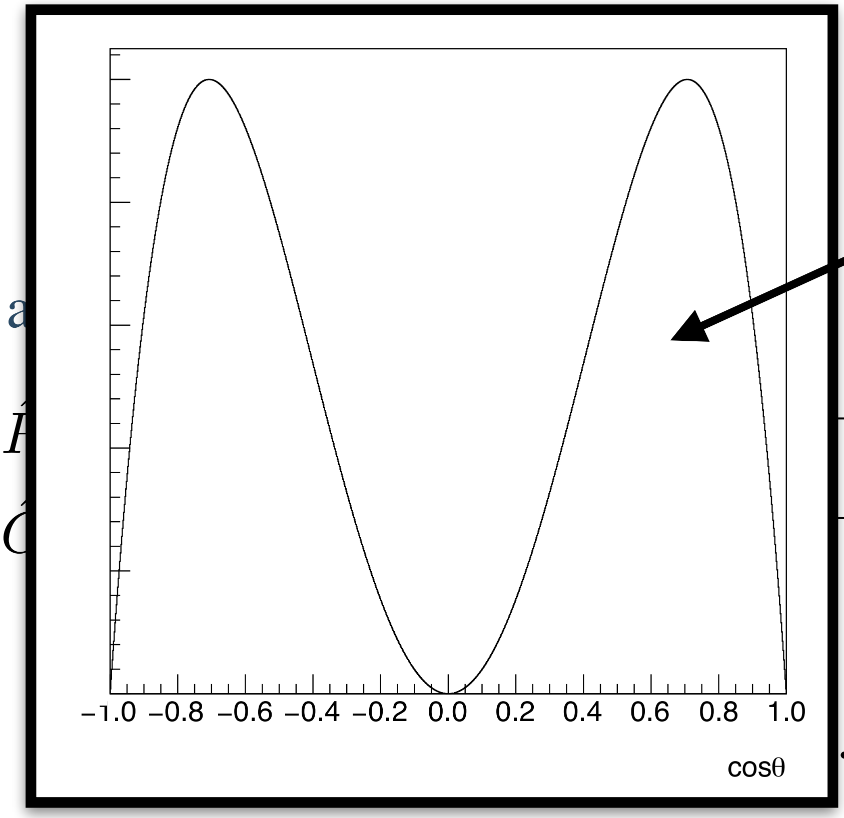

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(4) The parity and C-parity a

$\hat{P}$ … ${}^{+}\pi^-)\rangle$

$\hat{C}$ … ${}^{+}\pi^-)\rangle$

(5) So the possible J^{PC} are: … .

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

Another Example: Determine the J^{PC} of a meson decaying to $\eta'\pi^0$.

We know the J^{PC} of the π^0 and η' are both 0^{-+}:

$$\hat{P}\,|\pi^0\rangle = -\,|\pi^0\rangle \qquad \hat{P}\,|\eta'\rangle = -\,|\eta'\rangle$$
$$\hat{C}\,|\pi^0\rangle = +\,|\pi^0\rangle \qquad \hat{C}\,|\eta'\rangle = +\,|\eta'\rangle$$

Since $J = L$ and:

$$\hat{P}\,|J^{PC}(\eta'\pi^0)\rangle = (-1)^J\,|J^{PC}(\eta'\pi^0)\rangle$$
$$\hat{C}\,|J^{PC}(\eta'\pi^0)\rangle = \qquad +\,|J^{PC}(\eta'\pi^0)\rangle$$

the total J^{PC} will be

0^{++} (S-wave)

1^{-+} (P-wave) (exotic)

2^{++} (D-wave)

3^{-+} (F-wave) (exotic)

etc.

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

Another Example: Determine the J^{PC} of a meson decaying to $\eta'\pi^0$.

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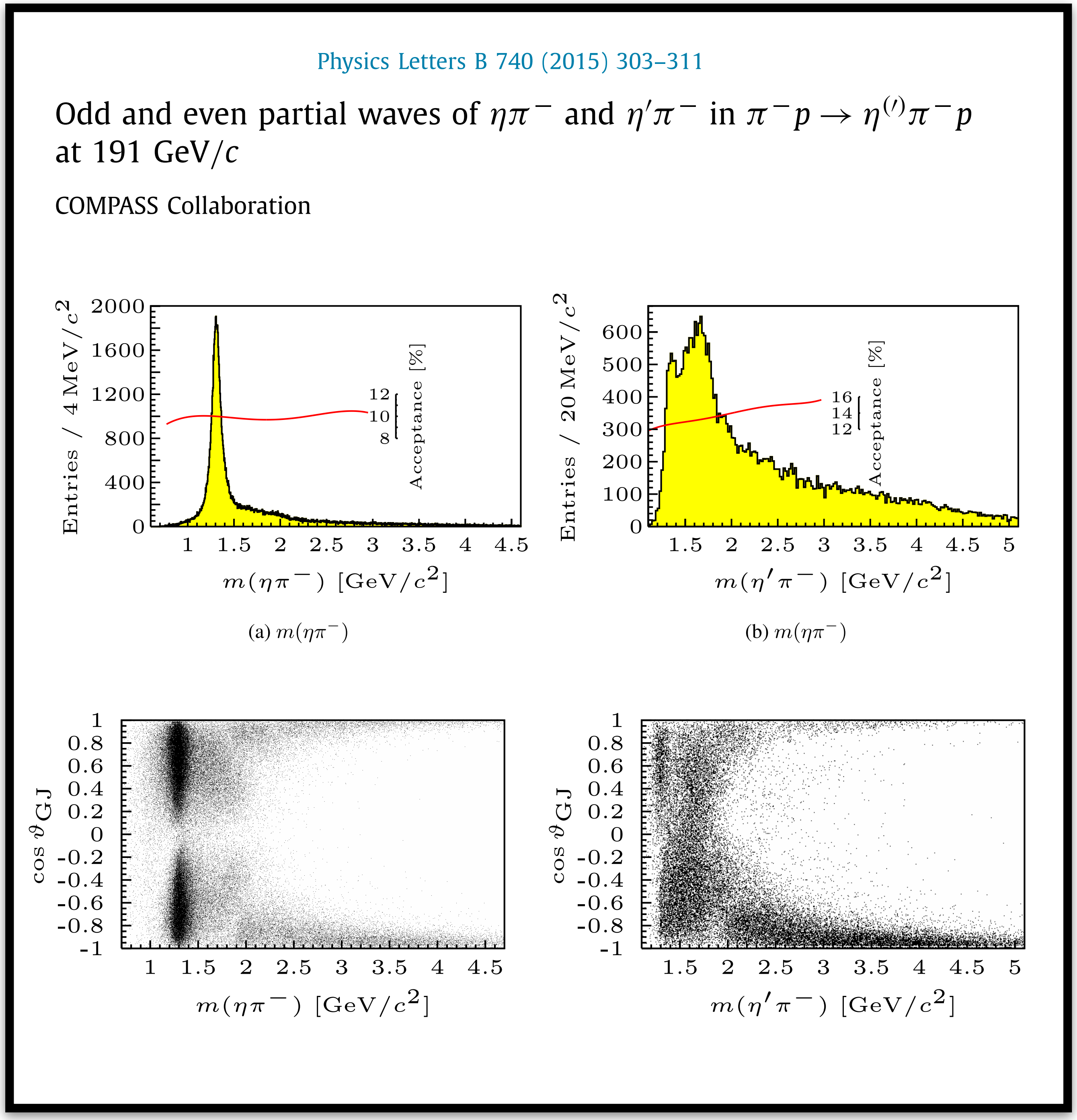

Physics Letters B 740 (2015) 303–311

Odd and even partial waves of $\eta\pi^-$ and $\eta'\pi^-$ in $\pi^- p \to \eta^{(\prime)}\pi^- p$ at 191 GeV/c

COMPASS Collaboration

(a) $m(\eta\pi^-)$

(b) $m(\eta\pi^-)$

IIB. Meson Quantum Numbers: J^{PC} *(experiment)*

Another Example: Determine the J^{PC} of a meson decaying to $\eta'\pi^0$.

We know the J^{PC} of the π^0 and η' are both 0^{-+}:

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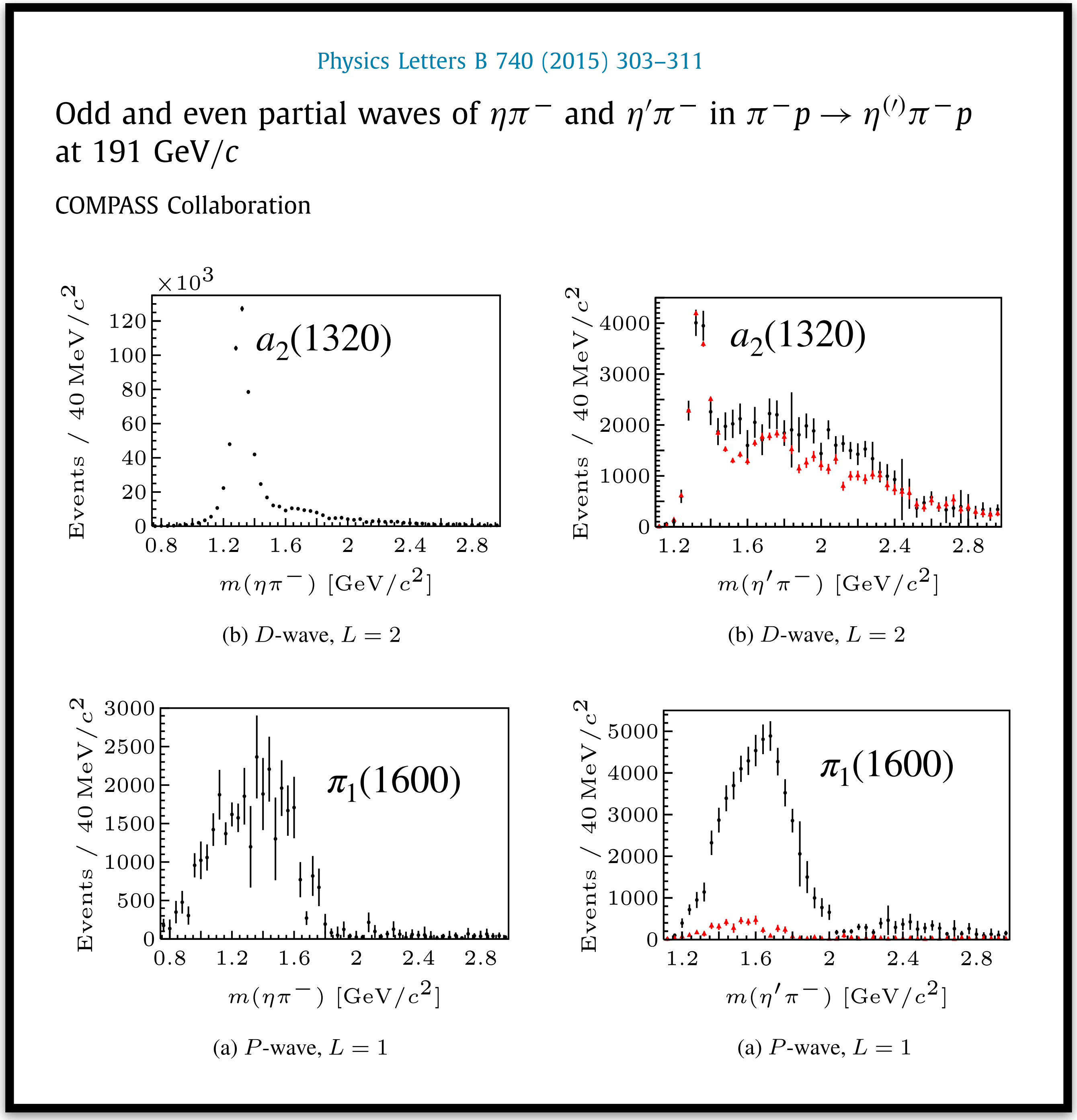
$$\hat{C}\,|\pi^0\rangle = +\,|\pi^0\rangle \qquad \hat{C}\,|\eta'\rangle = +\,|\eta'\rangle$$

Since $J = L$ and:

$$\hat{P}\,|J^{PC}(\eta'\pi^0)\rangle = (-1)^J\,|J^{PC}(\eta'\pi^0)\rangle$$
$$\hat{C}\,|J^{PC}(\eta'\pi^0)\rangle = \qquad +\,|J^{PC}(\eta'\pi^0)\rangle$$

the total J^{PC} will be

- 0^{++} (S-wave)
- 1^{-+} (P-wave) (exotic)
- 2^{++} (D-wave)
- 3^{-+} (F-wave) (exotic)
- etc.



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COMPASS Collaboration

(b) *D*-wave, $L = 2$

(b) *D*-wave, $L = 2$

(a) *P*-wave, $L = 1$

(a) *P*-wave, $L = 1$

IIB. Meson Quantum Numbers: J^{PC} *($q\bar{q}$ model)*

In the quark model, the J^{PC} of a meson can be related to the internal $q\bar{q}$ state:
$P = (-1)^{L+1}$ and $C = (-1)^{L+S}$.

(1) Start with a quark and an antiquark:

color

$$|s_1; s_{1z}\rangle\, |\vec{r}_1\rangle\, |q_1\rangle\, |c_1\rangle$$

anticolor

$$|s_2; s_{2z}\rangle\, |\vec{r}_2\rangle\, |\bar{q}_2\rangle\, |\bar{c}_2\rangle$$

spin state

position

flavor

(2) Combine them:

Clebsch-Gordan coefficients

$$|nLSJJ_z\rangle\, |q_1\bar{q}_2\rangle\, (|c\rangle) = \sum_{L_z S_z} \langle LS; L_z S_z | J J_z\rangle \sum_{s_{1z}s_{2z}} |s_1 s_2; s_{1z} s_{2z}\rangle\, \langle s_1 s_2; s_{1z} s_{2z} | S S_z\rangle$$

spin state

$$\times \int d^3 r\, |\vec{r}\rangle\, \langle \vec{r} | nLL_z\rangle\, |q_1\bar{q}_2\rangle\, \frac{1}{\sqrt{3}} \sum_i |c_{1i}\bar{c}_{2i}\rangle$$

color singlet

$$= \sum_{s_{1z}s_{2z}} \sum_{L_z S_z} C^{LS}_{JJ_z; L_z S_z}\, C^{s_1 s_2}_{SS_z; s_{1z} s_{2z}}\, |s_1 s_2; s_{1z} s_{2z}\rangle$$

$$\times \int d^3 r\, R_{nL}(r) Y_L^{L_z}(\hat{n})\, |\vec{r}\rangle\, |q_1\bar{q}_2\rangle\, \frac{1}{\sqrt{3}} \sum_i |c_{1i}\bar{c}_{2i}\rangle$$

radial wavefunction

spherical harmonic

IIB. Meson Quantum Numbers: J^{PC} *($q\bar{q}$ model)*

In the quark model, the J^{PC} of a meson can be related to the internal $q\bar{q}$ state:
$P = (-1)^{L+1}$ and $C = (-1)^{L+S}$.

(3) Antisymmetrize them:

exchange antisymmetry

$$|n^{2S+1}L_J(q_1\bar{q}_2)\rangle = \frac{1}{\sqrt{2}}(1 - \hat{P}_{12})\,|nLSJJ_z\rangle\,|q_1\bar{q}_2\rangle$$

spectroscopic notation

$$= \frac{1}{\sqrt{6}} \sum_{s_{1z}s_{2z}} \sum_{L_zS_z} C^{LS}_{JJ_z;L_zS_z}\, C^{s_1s_2}_{SS_z;s_{1z}s_{2z}}\, |s_1s_2;s_{1z}s_{2z}\rangle \int d^3r\, R_{nL}(r) Y_L^{L_z}(\hat{n})\,|\vec{r}\rangle$$

$$\times \Big[|q_1\bar{q}_2\rangle + (-1)^{L+S}\,|\bar{q}_2 q_1\rangle \Big]$$

(2) Combine them:

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spin state

$$\times \int d^3r\,|\vec{r}\rangle\,\langle\vec{r}|nLL_z\rangle\,|q_1\bar{q}_2\rangle\,\frac{1}{\sqrt{3}}\sum_i |c_{1i}\bar{c}_{2i}\rangle$$

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$$\times\left[|q_1\bar{q}_2\rangle + (-1)^{L+S}\,|\bar{q}_2q_1\rangle\right]$$

(4) Determine the parity and C-parity *(when $q_1 = q_2$)*:

using $P_q = -P_{\bar{q}}$

$$\hat{P}\,|n^{2S+1}L_J(q_1\bar{q}_2)\rangle = (-1)^{L+1}\,|n^{2S+1}L_J(q_1\bar{q}_2)\rangle$$

$$\hat{C}\,|n^{2S+1}L_J(q\bar{q})\rangle = (-1)^{L+S}\,|n^{2S+1}L_J(q\bar{q})\rangle$$

(5) Also write the state in terms of observable properties:

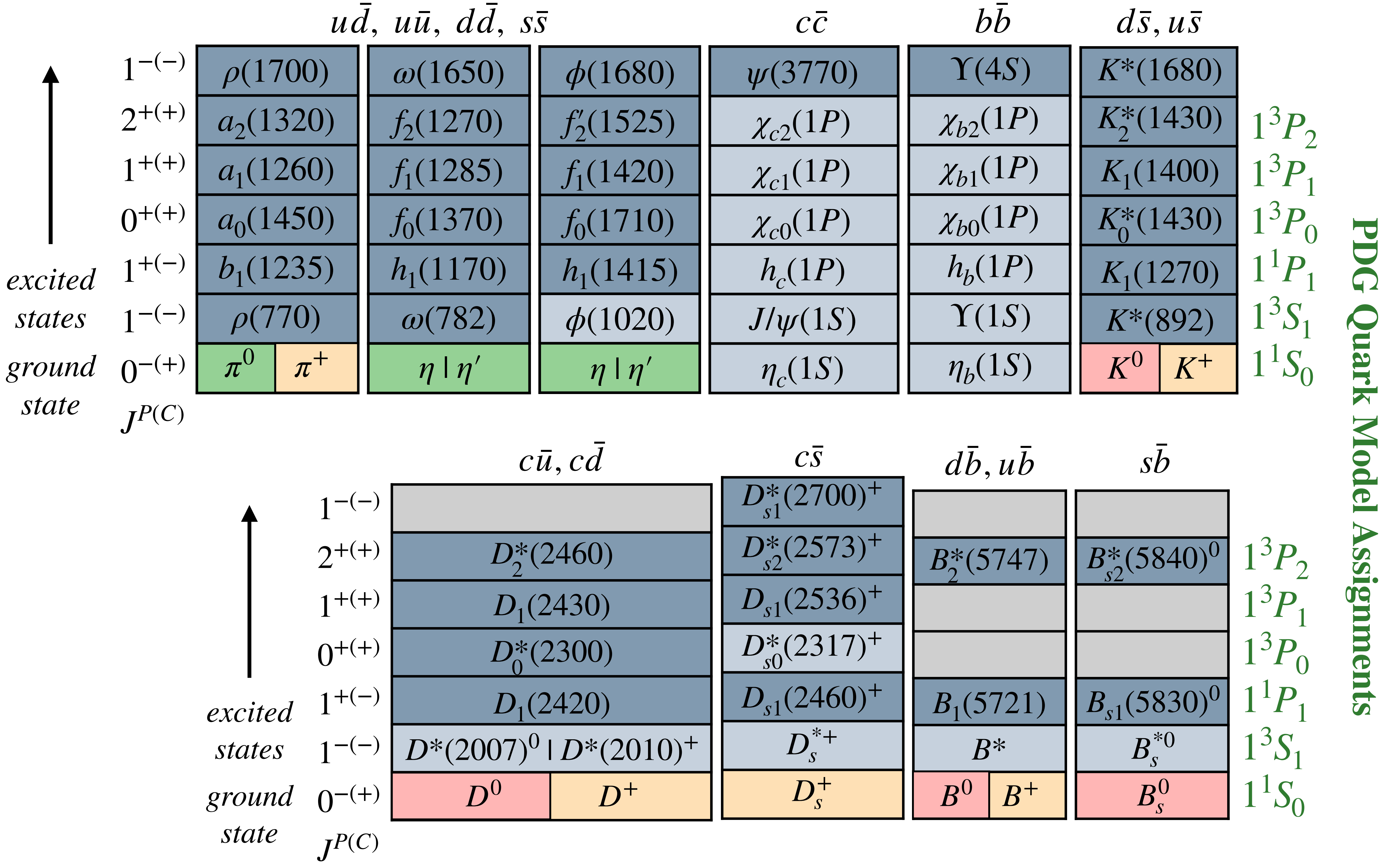
spin, parity, mass, etc.

flavor and C-parity

C is set by convention
when $q_1 \neq q_2$

$$|J^{P(C)}(q_1\bar{q}_2)(\text{mass})\rangle = \underline{|J^P(\text{mass})\rangle}\,\frac{1}{\sqrt{2}}\,\underline{\left[|q_1\bar{q}_2\rangle + C\,|\bar{q}_2q_1\rangle\right]}$$

IIB. Meson Quantum Numbers: J^{PC} *($q\bar{q}$ model)*



PDG Quark Model Assignments

excited states

ground state

$J^{P(C)}$	$u\bar{d},\ u\bar{u},\ d\bar{d},\ s\bar{s}$			$c\bar{c}$	$b\bar{b}$	$d\bar{s}, u\bar{s}$	
$1^{-(-)}$	$\rho(1700)$	$\omega(1650)$	$\phi(1680)$	$\psi(3770)$	$\Upsilon(4S)$	$K^*(1680)$	
$2^{+(+)}$	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	1^3P_2
$1^{+(+)}$	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	$K_1(1400)$	1^3P_1
$0^{+(+)}$	$a_0(1450)$	$f_0(1370)$	$f_0(1710)$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$	1^3P_0
$1^{+(-)}$	$b_1(1235)$	$h_1(1170)$	$h_1(1415)$	$h_c(1P)$	$h_b(1P)$	$K_1(1270)$	1^1P_1
$1^{-(-)}$	$\rho(770)$	$\omega(782)$	$\phi(1020)$	$J/\psi(1S)$	$\Upsilon(1S)$	$K^*(892)$	1^3S_1
$0^{-(+)}$	π^0 π^+	$\eta \mid \eta'$	$\eta \mid \eta'$	$\eta_c(1S)$	$\eta_b(1S)$	K^0 K^+	1^1S_0

excited states

ground state

$J^{P(C)}$	$c\bar{u}, c\bar{d}$	$c\bar{s}$	$d\bar{b}, u\bar{b}$	$s\bar{b}$	
$1^{-(-)}$		$D_{s1}^*(2700)^+$			
$2^{+(+)}$	$D_2^*(2460)$	$D_{s2}^*(2573)^+$	$B_2^*(5747)$	$B_{s2}^*(5840)^0$	1^3P_2
$1^{+(+)}$	$D_1(2430)$	$D_{s1}(2536)^+$			1^3P_1
$0^{+(+)}$	$D_0^*(2300)$	$D_{s0}^*(2317)^+$			1^3P_0
$1^{+(-)}$	$D_1(2420)$	$D_{s1}(2460)^+$	$B_1(5721)$	$B_{s1}(5830)^0$	1^1P_1
$1^{-(-)}$	$D^*(2007)^0 \mid D^*(2010)^+$	D_s^{*+}	B^*	B_s^{*0}	1^3S_1
$0^{-(+)}$	D^0 D^+	D_s^+	B^0 B^+	B_s^0	1^1S_0

Since $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, not all J^{PC} are allowed for $q\bar{q}$ states (e.g. 1^{-+}).

IIB. Meson Quantum Numbers: J^{PC} ($q\bar{q}$ *model*)

Examples in charmonium:

Connect J^{PC} to $n^{2S+1}L_J$ with $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$.

$\eta_c(1S)$ [ground state]

$n = 1,\ \ L = 0,\ \ S = 0$
$n^{2S+1}L_J = 1^1S_0$
$J^{PC} = 0^{-+}$

$J/\psi(1S)$

$n = 1,\ \ L = 0,\ \ S = 1$
$n^{2S+1}L_J = 1^3S_1$
$J^{PC} = 1^{--}$

$h_c(1P)$

$n = 1,\ \ L = 1,\ \ S = 0$
$n^{2S+1}L_J = 1^1P_1$
$J^{PC} = 1^{+-}$

$\chi_{c0}(1P)$

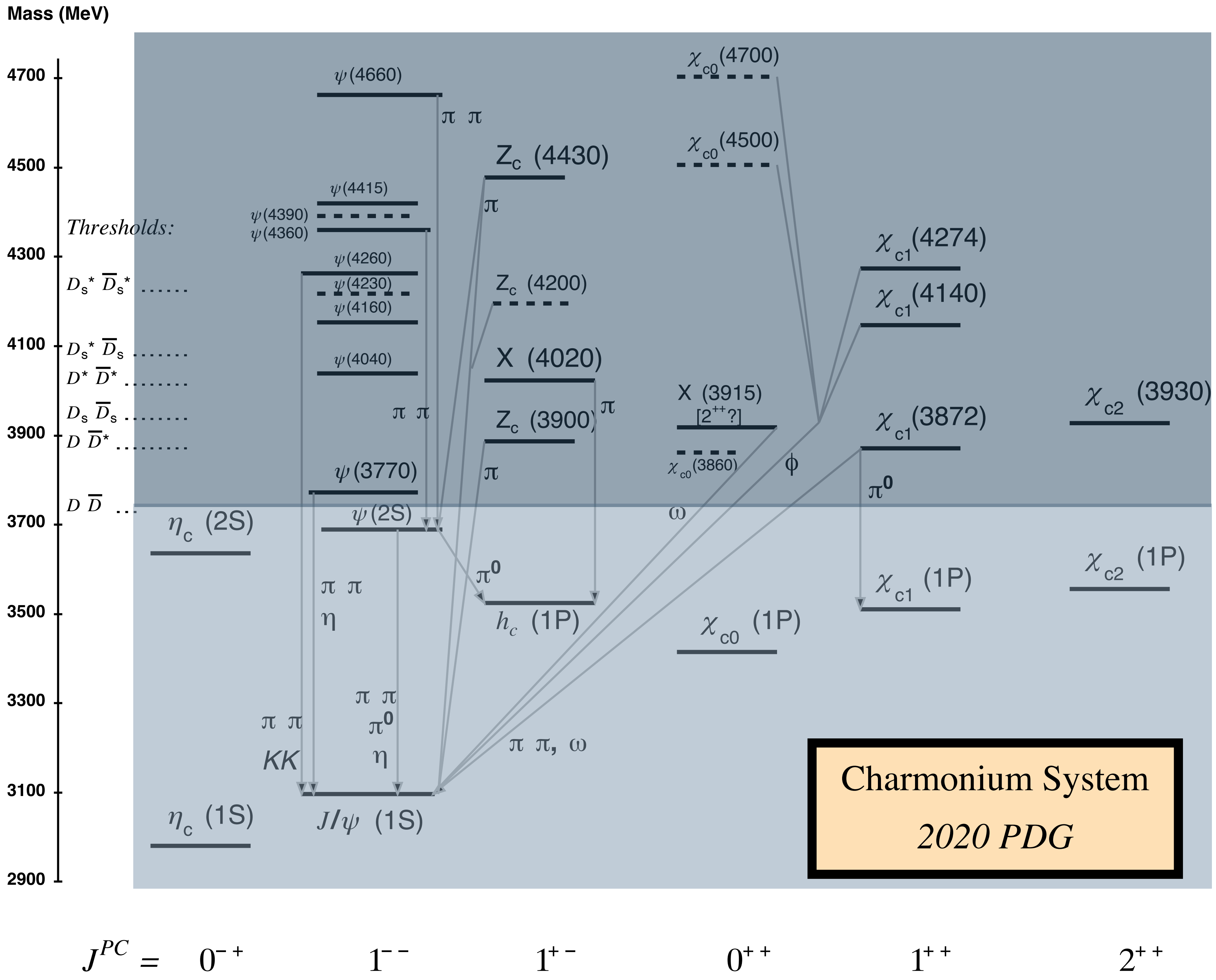
$n = 1,\ \ L = 1,\ \ S = 1$
$n^{2S+1}L_J = 1^3P_0$
$J^{PC} = 0^{++}$

$\chi_{c1}(1P)$

$n = 1,\ \ L = 1,\ \ S = 1$
$n^{2S+1}L_J = 1^3P_1$
$J^{PC} = 1^{++}$

$\chi_{c2}(1P)$

$n = 1,\ \ L = 1,\ \ S = 1$
$n^{2S+1}L_J = 1^3P_2$
$J^{PC} = 2^{++}$



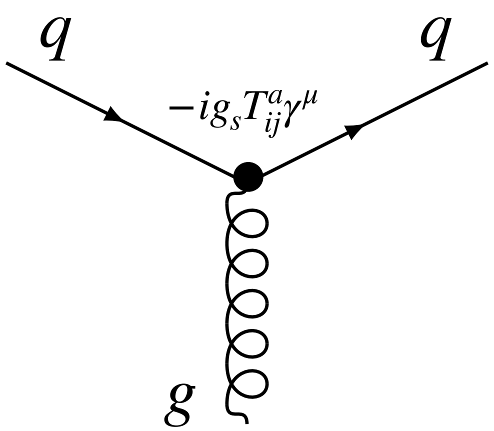
IIC. Meson Quantum Numbers: Flavor

QUARKS

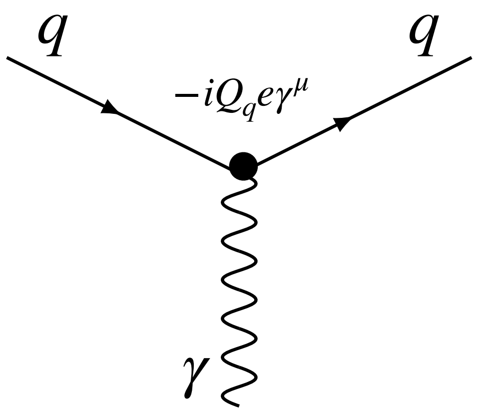
ANTIQUARKS

	d	u	s	c	b
$\bar{d}$	$\pi^0 \mid \eta \mid \eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
$\bar{u}$	π^-	$\pi^0 \mid \eta \mid \eta'$	K^-	D^0	B^-
$\bar{s}$	K^0	K^+	$\eta \mid \eta'$ ϕ	D_s^+	$\bar{B}_s^0$
$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

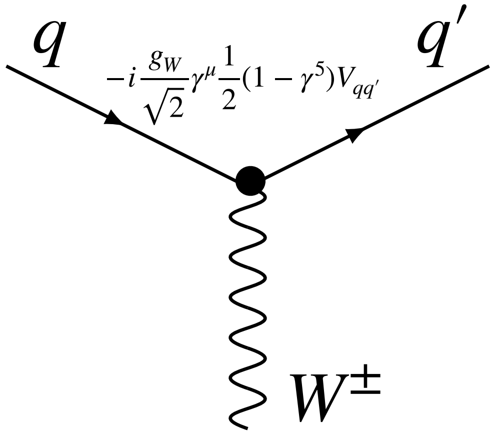
STRONG



ELECTROMAGNETIC



WEAK



Flavor quantum numbers:

$U \equiv N_u - N_{\bar{u}}$ ("upness")

$D \equiv N_{\bar{d}} - N_d$ ("downness")

$S \equiv N_{\bar{s}} - N_s$ (strangeness)

$C \equiv N_c - N_{\bar{c}}$ (charm)

$B \equiv N_{\bar{b}} - N_b$ (bottomness)

Also:

$$I_3 \equiv \frac{1}{2}U + \frac{1}{2}D$$

(third component of isospin)

$$Y \equiv \frac{1}{3}U - \frac{1}{3}D + \frac{2}{3}S$$

(hypercharge)

These are absolutely conserved by the strong and electromagnetic forces.

IIC. Meson Quantum Numbers: Flavor

QUARKS

ANTIQUARKS

	d	u	s	c	b
$\bar{d}$	$\pi^0\,|\,\eta\,|\,\eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
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$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

Examples:

$$K^*(892)^+ \to K^0\pi^+$$
$$(S = 1,\quad U = 1,\quad I_3 = \frac{1}{2})$$

$$B_{s2}^*(5840) \to B^+K^-$$
$$(B = 1,\quad S = -1)$$

$$X(2900)^0 \to D^-K^+$$
$$(C = -1,\quad D = -1,\quad S = 1,\quad U = 1,\quad I_3 = 0)$$

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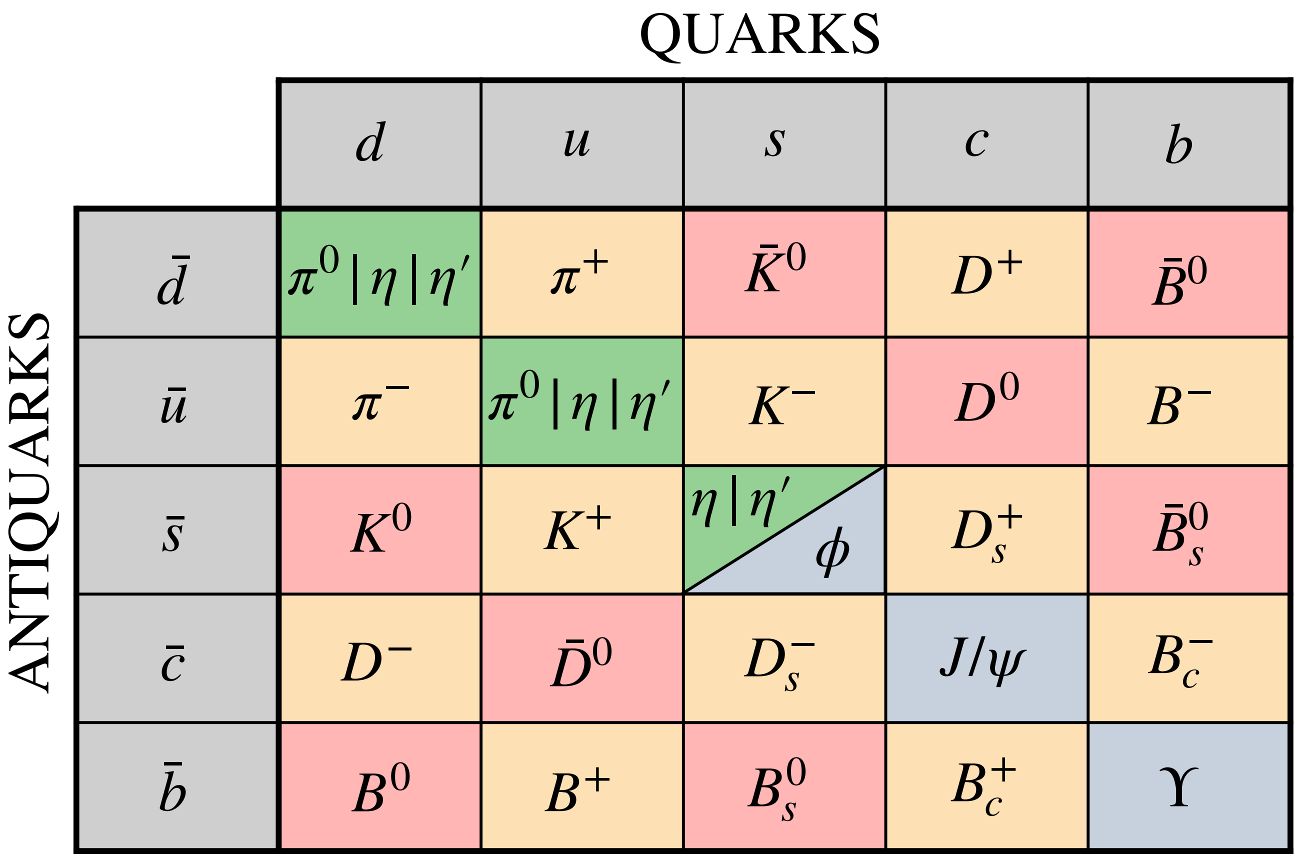

QUARKS

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$\bar c$	D^-	$\bar D^0$	D_s^-	J/ψ	B_c^-
$\bar b$	B^0	B^+	B_s^0	B_c^+	Υ

Off-diagonal combinations are eigenstates of $H_{\text{s,em}}$:

For example,

$$< q\bar q\,|\,H_{\text{s,em}}\,|\,u\bar d > \; = 0$$

whenever $q\bar q \neq u\bar d$

implies

$$H_{\text{s,em}}\,|\,u\bar d >$$

$$= \sum_{q\bar q} |\,q\bar q >\, < q\bar q\,|\,H_{\text{s,em}}\,|\,u\bar d >$$

$$= \, < u\bar d\,|\,H_{\text{s,em}}\,|\,u\bar d >\, |\,u\bar d >$$

$*$ *but note* $< u\bar u u\bar d\,|\,H_{\text{s,em}}\,|\,u\bar d > \; = \; ?$

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$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

On-diagonal combinations are not necessarily eigenstates of $H_{\text{s,em}}$:

For example,

$< u\bar{u} \,|\, H_{\text{s,em}} \,|\, d\bar{d} > \neq 0$

$\Longrightarrow \,|\, u\bar{u} >$ and $|\, d\bar{d} >$ are not eigenstates of $H_{\text{s,em}}$

$< u\bar{u} \,|\, H_{\text{s,em}} \,|\, s\bar{s} >$ small

$\Longrightarrow \,|\, s\bar{s} >$ might be nearly an eigenstate of $H_{\text{s,em}}$

$< u\bar{u} \,|\, H_{\text{s,em}} \,|\, c\bar{c} > \approx 0$

$\Longrightarrow \,|\, c\bar{c} >$ is more likely an eigenstate of $H_{\text{s,em}}$

* *but note* $< u\bar{u}c\bar{c} \,|\, H_{\text{s,em}} \,|\, c\bar{c} > = ?$

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These are absolutely conserved by the strong and electromagnetic forces.

IIC. Meson Quantum Numbers: Flavor **(Isospin)**

Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

Strong interactions are almost invariant under SU(2) rotations that mix the up and down quarks *(using the 2d fundamental representation* **2***)*:

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{I}}} \begin{pmatrix} u \\ d \end{pmatrix}$$

This is just like spin and leads to a new conserved quantum number called isospin.

Just like spin, we have states labeled by I and I_3 with the usual properties:

$$\begin{aligned} \hat{I}_3 \left|I, I_3\right\rangle &= I_3 \left|I, I_3\right\rangle \\ \hat{I}^2 \left|I, I_3\right\rangle &= I(I+1) \left|I, I_3\right\rangle \\ \hat{I}_\pm \left|I, I_3\right\rangle &= \sqrt{(I \mp I_3)(I \pm I_3 + 1)} \left|I, I_3 \pm 1\right\rangle \end{aligned}$$

Relabel the up and down quark flavors using $|u> \equiv |\frac{1}{2}, +\frac{1}{2}>$ and $|d> \equiv |\frac{1}{2}, -\frac{1}{2}>$.

IIC. Meson Quantum Numbers: Flavor (Isospin)

Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

The corresponding rotation for antiquarks is given by *(using the 2d conjugate representation* $\bar{\mathbf{2}}$*)*:

$$\begin{pmatrix} \bar{u}' \\ \bar{d}' \end{pmatrix} = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

This can be transformed to look like the quark rotations (*using the 2d fundamental representation* $\mathbf{2}$). Multiply left and right by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{u}' \\ \bar{d}' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d}' \\ \bar{u}' \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

note: using
the convention
$\hat{C}\,|\,u > \, = \, |\,\bar{u} >$
$\hat{C}\,|\,d > \, = \, |\,\bar{d} >$

Relabel the up and down antiquark flavors using $|\,\bar{u} > \, \equiv \, |\,\frac{1}{2}, -\frac{1}{2} >$ and $|\,\bar{d} > \, \equiv - \, |\,\frac{1}{2}, +\frac{1}{2} >$.

IIC. Meson Quantum Numbers: Flavor (Isospin)

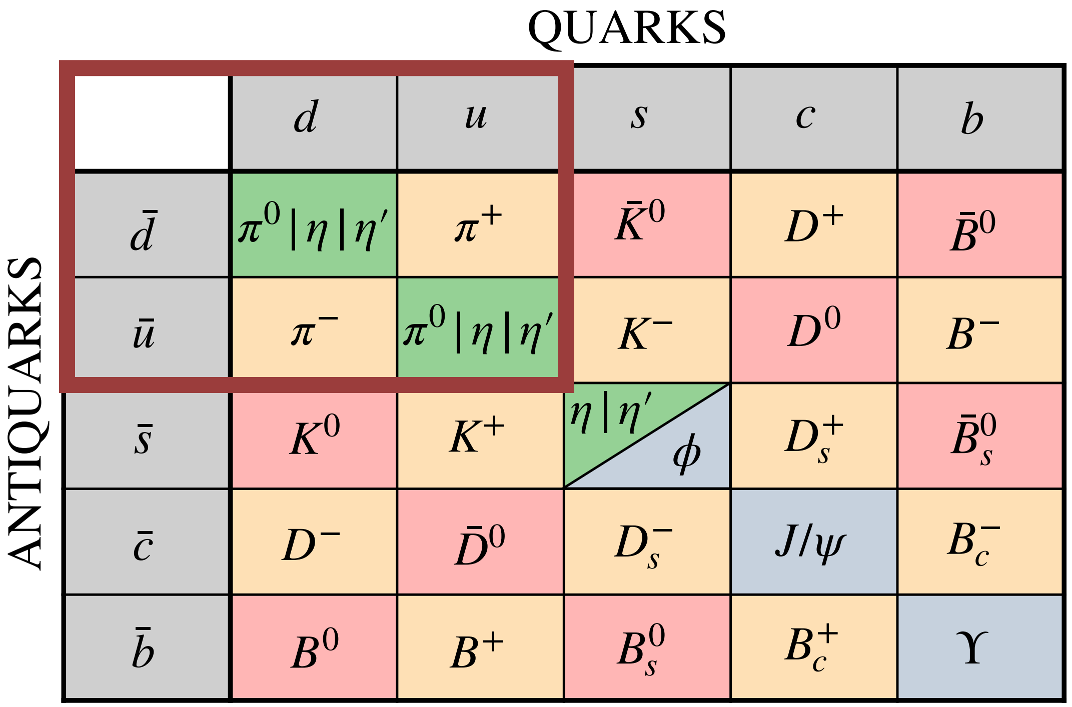
Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

Combine isodoublets into higher-order representations just like spin (*e.g.* $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$):

$$\begin{pmatrix} |u\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |d\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} \otimes \begin{pmatrix} -|\bar{d}\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |\bar{u}\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} = \begin{pmatrix} -|u\bar{d}\rangle = |1,\ +1\rangle \\ \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) = |1,\ 0\rangle \\ |d\bar{u}\rangle = |1,\ -1\rangle \end{pmatrix} \oplus \left(\frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right) = |0,\ 0\rangle\right)$$

($I = 0$ part will also contain $s\bar{s}$ in two different combinations)

$$I = \frac{1}{2} \qquad\qquad I = \frac{1}{2} \qquad\qquad I = 1 \qquad\qquad I = 0$$



QUARKS

ANTIQUARKS

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$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

With this notation:

$$|J^{P(C)}(q_1\bar{q}_2)(\text{mass})\rangle = |J^P(\text{mass})\rangle \frac{1}{\sqrt{2}}\left[|q_1\bar{q}_2\rangle + C\,|\bar{q}_2 q_1\rangle\right]$$

The pion is:

$$|\pi^+\rangle = |0^-(140)\rangle \frac{1}{\sqrt{2}}\left[-|u\bar{d}\rangle - |\bar{d}u\rangle\right]$$

(choose $C = +$ for $\pi^\pm$ to match π^0)

$$|\pi^0\rangle = |0^-(135)\rangle \frac{1}{2}\left[|u\bar{u}\rangle - |d\bar{d}\rangle + |\bar{u}u\rangle - |\bar{d}d\rangle\right]$$

$$|\pi^-\rangle = |0^-(140)\rangle \frac{1}{\sqrt{2}}\left[|d\bar{u}\rangle + |\bar{u}d\rangle\right]$$

note:
$\hat{C}\,|\pi^+> = -\,|\pi^->$

IIC. Meson Quantum Numbers: Flavor **(Isospin)**

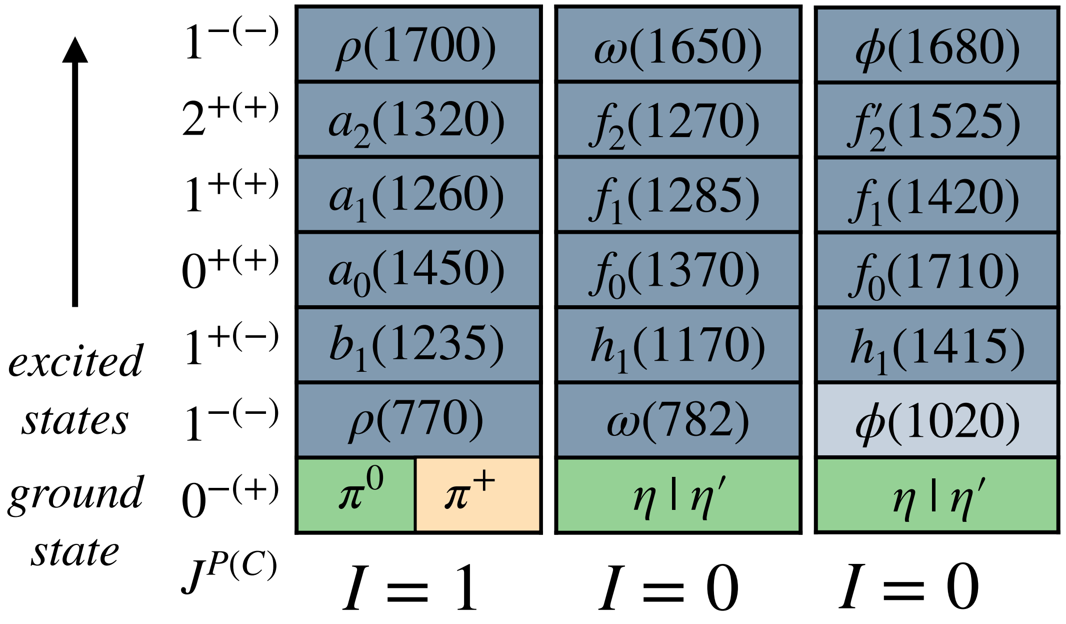
Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

Combine isodoublets into higher-order representations just like spin ($e.g.$ $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$):

$$\begin{pmatrix} |u\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |d\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} \otimes \begin{pmatrix} -|\bar{d}\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |\bar{u}\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} = \begin{pmatrix} -|u\bar{d}\rangle = |1,\ +1\rangle \\ \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) = |1,\ 0\rangle \\ |d\bar{u}\rangle = |1,\ -1\rangle \end{pmatrix} \oplus \left(\frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right) = |0,\ 0\rangle\right)$$

($I = 0$ part will also contain $s\bar{s}$ in two different combinations)

$$I = \frac{1}{2} \qquad I = \frac{1}{2} \qquad I = 1 \qquad I = 0$$



$1^{-(-)}$	$\rho(1700)$	$\omega(1650)$	$\phi(1680)$
$2^{+(+)}$	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$
$1^{+(+)}$	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$
$0^{+(+)}$	$a_0(1450)$	$f_0(1370)$	$f_0(1710)$
$1^{+(-)}$	$b_1(1235)$	$h_1(1170)$	$h_1(1415)$
$1^{-(-)}$	$\rho(770)$	$\omega(782)$	$\phi(1020)$
$0^{-(+)}$	π^0 π^+	$\eta \mid \eta'$	$\eta \mid \eta'$
$J^{P(C)}$	$I = 1$	$I = 0$	$I = 0$

With this notation:

$$|J^{P(C)}(q_1\bar{q}_2)(\text{mass})\rangle = |J^P(\text{mass})\rangle \frac{1}{\sqrt{2}}\left[|q_1\bar{q}_2\rangle + C\,|\bar{q}_2 q_1\rangle\right]$$

The pion is:

$$|\pi^+\rangle = |0^-(140)\rangle \frac{1}{\sqrt{2}}\left[-|u\bar{d}\rangle - |\bar{d}u\rangle\right]$$

(choose $C = +$ for $\pi^\pm$ to match π^0)

$$|\pi^0\rangle = |0^-(135)\rangle \frac{1}{2}\left[|u\bar{u}\rangle - |d\bar{d}\rangle + |\bar{u}u\rangle - |\bar{d}d\rangle\right]$$

$$|\pi^-\rangle = |0^-(140)\rangle \frac{1}{\sqrt{2}}\left[|d\bar{u}\rangle + |\bar{u}d\rangle\right]$$

note: $\hat{C}\,|\pi^+> = -\,|\pi^->$

IIC. Meson Quantum Numbers: Flavor **(Isospin)**

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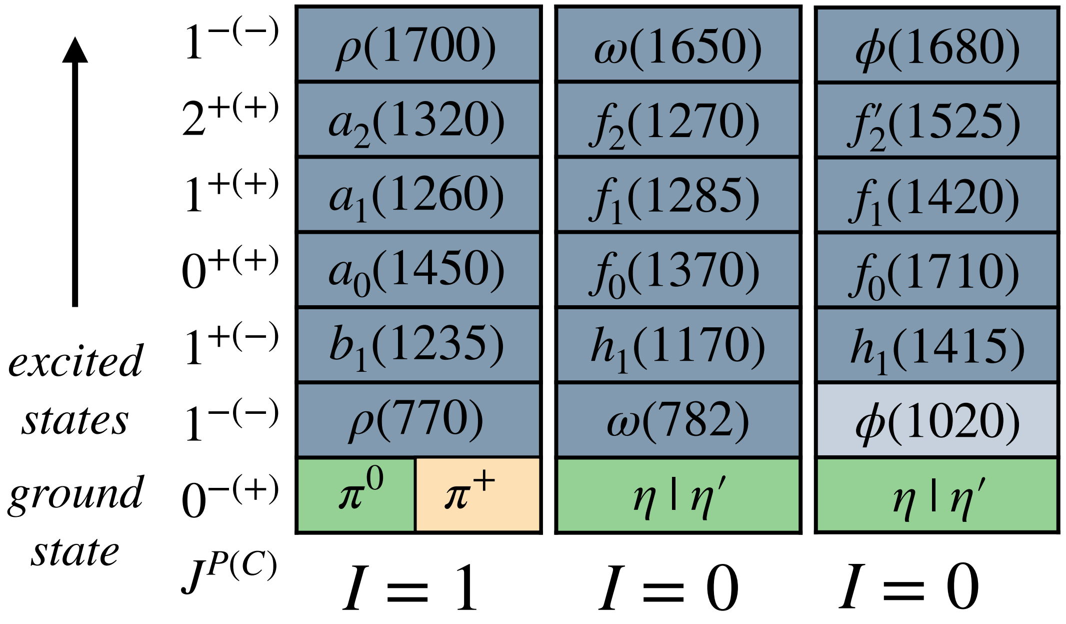

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With this notation:

$$|J^{P(C)}(q_1\bar{q}_2)(\text{mass})\rangle = |J^P(\text{mass})\rangle \, \frac{1}{\sqrt{2}} \left[|q_1\bar{q}_2\rangle + C\,|\bar{q}_2 q_1\rangle\right]$$

The $\rho(770)$ is:

$$|\rho^+\rangle = |1^-(770)\rangle \, \frac{1}{\sqrt{2}} \left[-|u\bar{d}\rangle + |\bar{d}u\rangle\right]$$

(choose $C = -$ for $\rho^\pm$ to match ρ^0)

$$|\rho^0\rangle = |1^-(770)\rangle \, \frac{1}{2} \left[|u\bar{u}\rangle - |d\bar{d}\rangle - |\bar{u}u\rangle + |\bar{d}d\rangle\right]$$

$$|\rho^-\rangle = |1^-(770)\rangle \, \frac{1}{\sqrt{2}} \left[|d\bar{u}\rangle - |\bar{u}d\rangle\right]$$

note: $\hat{C}\,|\,\rho^+ > = +\,|\,\rho^- >$

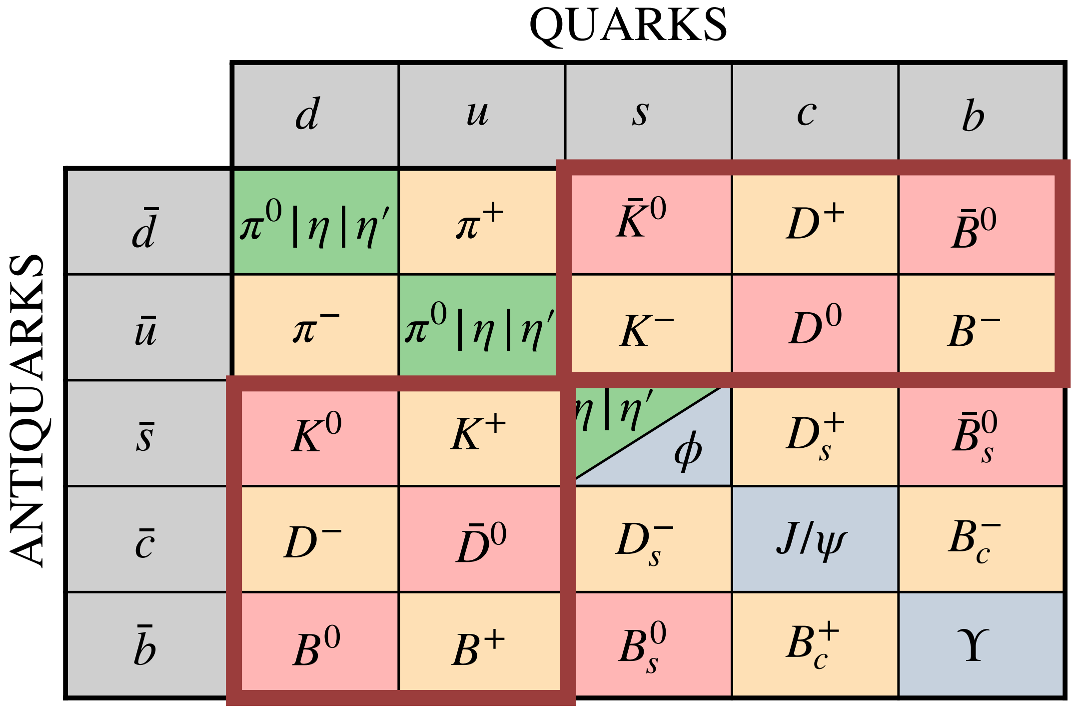
IIC. Meson Quantum Numbers: Flavor **(Isospin)**

Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

Also combine up and down quarks with other quarks to make isodoublets (**2** ⊗ **1** = **2**):

$$\begin{pmatrix} |u\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |d\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} \otimes \Big(|\bar{s}\rangle = |0,\ 0\rangle \Big) = \begin{pmatrix} |u\bar{s}\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |d\bar{s}\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix}$$

$$\begin{pmatrix} -|\bar{d}\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |\bar{u}\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix} \otimes \Big(|s\rangle = |0,\ 0\rangle \Big) = \begin{pmatrix} -|\bar{d}s\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |\bar{u}s\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix}$$



	d	u	s	c	b
$\bar{d}$	$\pi^0\,\vert\,\eta\,\vert\,\eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
$\bar{u}$	π^-	$\pi^0\,\vert\,\eta\,\vert\,\eta'$	K^-	D^0	B^-
$\bar{s}$	K^0	K^+	$\eta\,\vert\,\eta'$ ϕ	D_s^+	$\bar{B}_s^0$
$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

The kaon can then be identified as:

$$|K^+\rangle = |0^-(494)\rangle\,\frac{1}{\sqrt{2}}\left[|u\bar{s}\rangle + |\bar{s}u\rangle\right]$$

$$|K^0\rangle = |0^-(498)\rangle\,\frac{1}{\sqrt{2}}\left[|d\bar{s}\rangle + |\bar{s}d\rangle\right]$$

(choose $C = +$ using quark model $(-1)^{L+S}$)

$$|\bar{K}^0\rangle = |0^-(498)\rangle\,\frac{1}{\sqrt{2}}\left[-|\bar{d}s\rangle - |s\bar{d}\rangle\right]$$

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note:
$\hat{C}\,|\,K^+ > \; = +\,|\,K^- >$
$\hat{C}\,|\,K^0 > \; = -\,|\,\bar{K}^0 >$

IIC ers: Flavor (Isospin)

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Also com to make isodoublets (**2** ⊗ **1** = **2**):

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$$= \begin{pmatrix} -|\bar{d}s\rangle = |\tfrac{1}{2},\ +\tfrac{1}{2}\rangle \\ |\bar{u}s\rangle = |\tfrac{1}{2},\ -\tfrac{1}{2}\rangle \end{pmatrix}$$

The K_S^0 and K_L^0 are linear combinations of K^0 and $\bar{K}^0$:

$$|K_S^0 > \approx \frac{1}{\sqrt{2}}\left[|K^0 > + |\bar{K}^0 >\right]$$

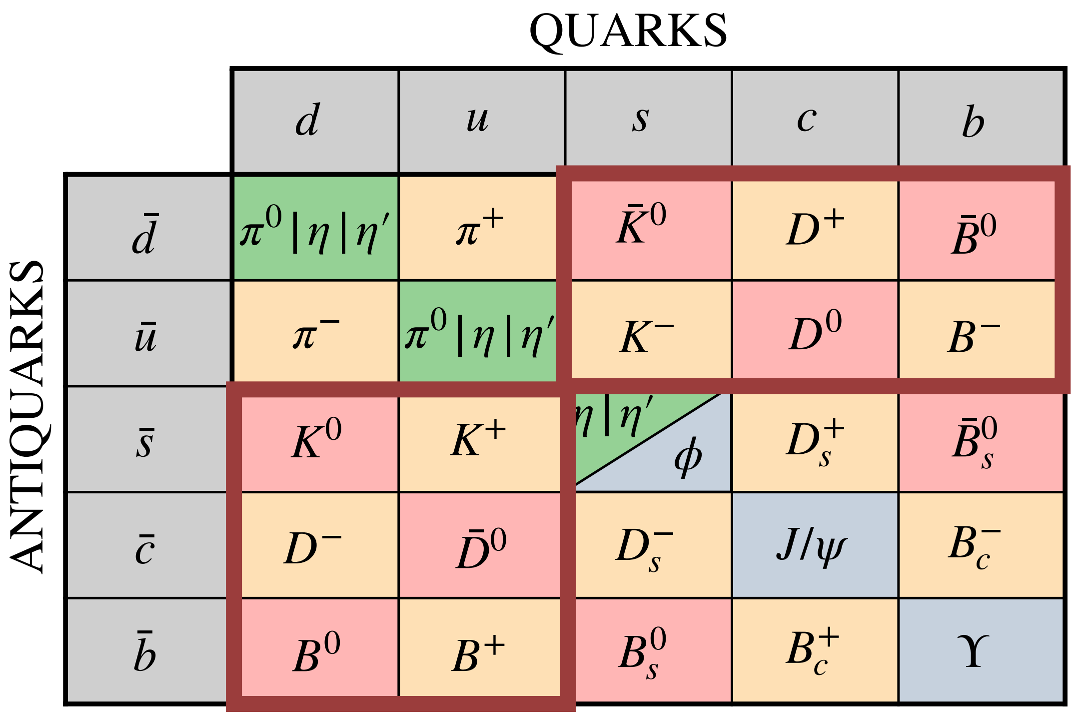
$$|K_L^0 > \approx \frac{1}{\sqrt{2}}\left[|K^0 > - |\bar{K}^0 >\right]$$

So that:

$$\hat{C}\hat{P}|K_S^0 > \approx +|K_S^0 >$$

$$\hat{C}\hat{P}|K_L^0 > \approx -|K_L^0 >$$

(and the J^{PC} of the K_S^0 is 0^{--})



QUARKS

ANTIQUARKS

	d	u	s	c	b
$\bar{d}$	$\pi^0 \mid \eta \mid \eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
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note:
$\hat{C}|K^+ > = +|K^- >$
$\hat{C}|K^0 > = -|\bar{K}^0 >$

IIC. Meson Quantum Numbers: Flavor **(Isospin)**

Since $m_u \approx m_d$, the strong force has an additional, *approximately conserved*, quantum number: **isosopin**.

Combine pion isotriplets using Clebsch-Gordon coefficients (**3** $\otimes$ **3** = **5** $\oplus$ **3** $\oplus$ **1**):

$$\underset{I=1}{\begin{pmatrix} |\pi^+\rangle = |1,1\rangle \\ |\pi^0\rangle = |1,0\rangle \\ |\pi^-\rangle = |1,-1\rangle \end{pmatrix}} \otimes \underset{I=1}{\begin{pmatrix} |\pi^+\rangle = |1,1\rangle \\ |\pi^0\rangle = |1,0\rangle \\ |\pi^-\rangle = |1,-1\rangle \end{pmatrix}} = \underset{I=2}{\begin{pmatrix} |2,2\rangle \\ |2,1\rangle \\ |2,0\rangle \\ |2,-1\rangle \\ |2,-2\rangle \end{pmatrix}} \oplus \underset{I=1}{\begin{pmatrix} \frac{1}{\sqrt{2}}|\pi^+\pi^0\rangle - \frac{1}{\sqrt{2}}|\pi^0\pi^+\rangle \\ \frac{1}{\sqrt{2}}|\pi^+\pi^-\rangle - \frac{1}{\sqrt{2}}|\pi^-\pi^+\rangle \\ \frac{1}{\sqrt{2}}|\pi^0\pi^-\rangle - \frac{1}{\sqrt{2}}|\pi^-\pi^0\rangle \end{pmatrix}} \oplus \underset{I=0}{\left(\frac{1}{\sqrt{3}}|\pi^+\pi^-\rangle - \frac{1}{\sqrt{3}}|\pi^0\pi^0\rangle + \frac{1}{\sqrt{3}}|\pi^-\pi^+\rangle\right)}$$

Comparing to the general $\pi^+\pi^-$ system in a definite state of J^{PC} *(first example, slide ≈ 8)*:

$$|J^{PC}(\pi^+\pi^-)\rangle = \frac{1}{\sqrt{2}}\left[|\pi^+\pi^-\rangle + (-1)^J|\pi^-\pi^+\rangle\right]\int d\Omega\, Y_J^M(\theta,\phi)\,|p\theta\phi\rangle\,|P_{\rm CM}\rangle$$

$$\begin{aligned} I = 0 \text{ for even } J\ \ (0^{++}, 2^{++}, \ldots &= f_0, f_2, \ldots) \\ I = 1 \text{ for odd } J\ \ (1^{--}, 3^{--}, \ldots &= \rho_1, \rho_3, \ldots) \end{aligned}$$

IIC. Meson Quantum Numbers: Flavor (G-Parity)

Combine isospin rotations and charge conjugation to define **G-Parity**.
Like isospin, G-parity is approximately conserved by the strong force.

$$\hat{G} \equiv \hat{C}\hat{R}_y(\pi) = \hat{R}_y(\pi)\hat{C} = \hat{C}e^{-i\pi\hat{I}_y}$$

The states $|\pi^+>$ and $|\rho^+>$, for example, are not eigenstates of $\hat{C}$, but are eigenstates of $\hat{G}$.

Starting with the **3** representation of $\hat{R}_y(\pi)$:

$$\hat{R}_y(\pi) = \begin{pmatrix} 0 & 0 & +1 \\ 0 & -1 & 0 \\ +1 & 0 & 0 \end{pmatrix}$$

$$\Longrightarrow \begin{array}{l} \hat{R}_y(\pi)\,|\pi^+\rangle = +|\pi^-\rangle \\ \hat{R}_y(\pi)\,|\pi^0\rangle = -|\pi^0\rangle \\ \hat{R}_y(\pi)\,|\pi^-\rangle = +|\pi^+\rangle \end{array} \Longrightarrow \begin{array}{l} \hat{C}\hat{R}_y(\pi)\,|\pi^+\rangle = -|\pi^+\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\pi^0\rangle = -|\pi^0\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\pi^-\rangle = -|\pi^-\rangle \end{array} \Longrightarrow \hat{G}|\pi> = -|\pi>$$

$$\Longrightarrow \begin{array}{l} \hat{R}_y(\pi)\,|\rho^+\rangle = +|\rho^-\rangle \\ \hat{R}_y(\pi)\,|\rho^0\rangle = -|\rho^0\rangle \\ \hat{R}_y(\pi)\,|\rho^-\rangle = +|\rho^+\rangle \end{array} \Longrightarrow \begin{array}{l} \hat{C}\hat{R}_y(\pi)\,|\rho^+\rangle = +|\rho^+\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\rho^0\rangle = +|\rho^0\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\rho^-\rangle = +|\rho^-\rangle \end{array} \Longrightarrow \hat{G}|\rho> = +|\rho>$$

In general, for eigenstates of $\hat{G}$:

$$G = C(-1)^I$$ *where C is given by the neutral member of the group*

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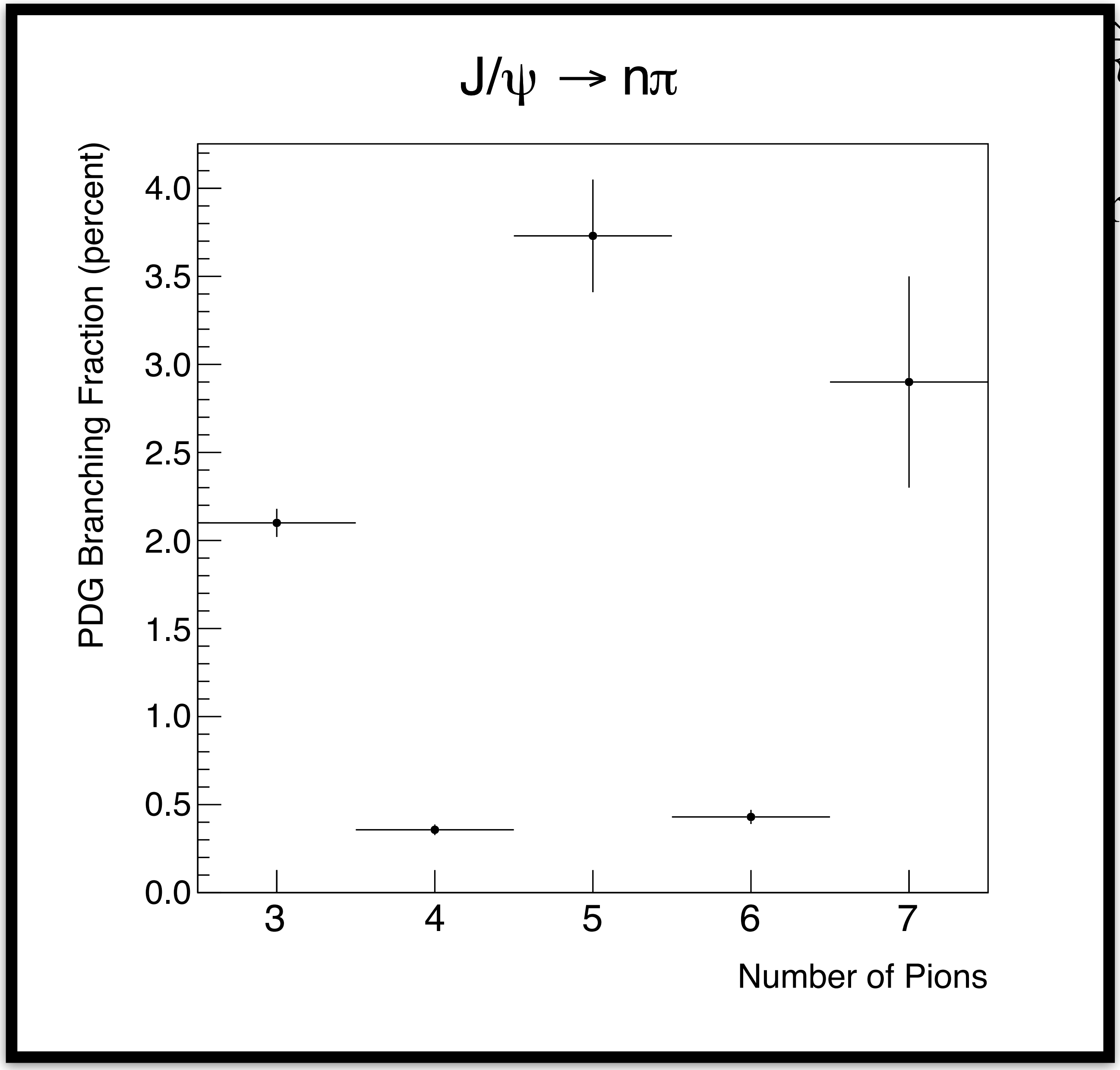

$$R_y(\pi)\hat{C} = \hat{C}e^{-i\pi\hat{I}_y}$$

re not eigenstates of $\hat{C}$, but are eigenstates of $\hat{G}$.

$$\Longrightarrow \quad \begin{aligned} \hat{C}\hat{R}_y(\pi)\,|\pi^+\rangle &= -\,|\pi^+\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\pi^0\rangle &= -\,|\pi^0\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\pi^-\rangle &= -\,|\pi^-\rangle \end{aligned} \quad \Longrightarrow \quad \hat{G}|\pi> = -|\pi>$$

$$\Longrightarrow \quad \begin{aligned} \hat{C}\hat{R}_y(\pi)\,|\rho^+\rangle &= +\,|\rho^+\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\rho^0\rangle &= +\,|\rho^0\rangle \\ \hat{C}\hat{R}_y(\pi)\,|\rho^-\rangle &= +\,|\rho^-\rangle \end{aligned} \quad \Longrightarrow \quad \hat{G}|\rho> = +|\rho>$$

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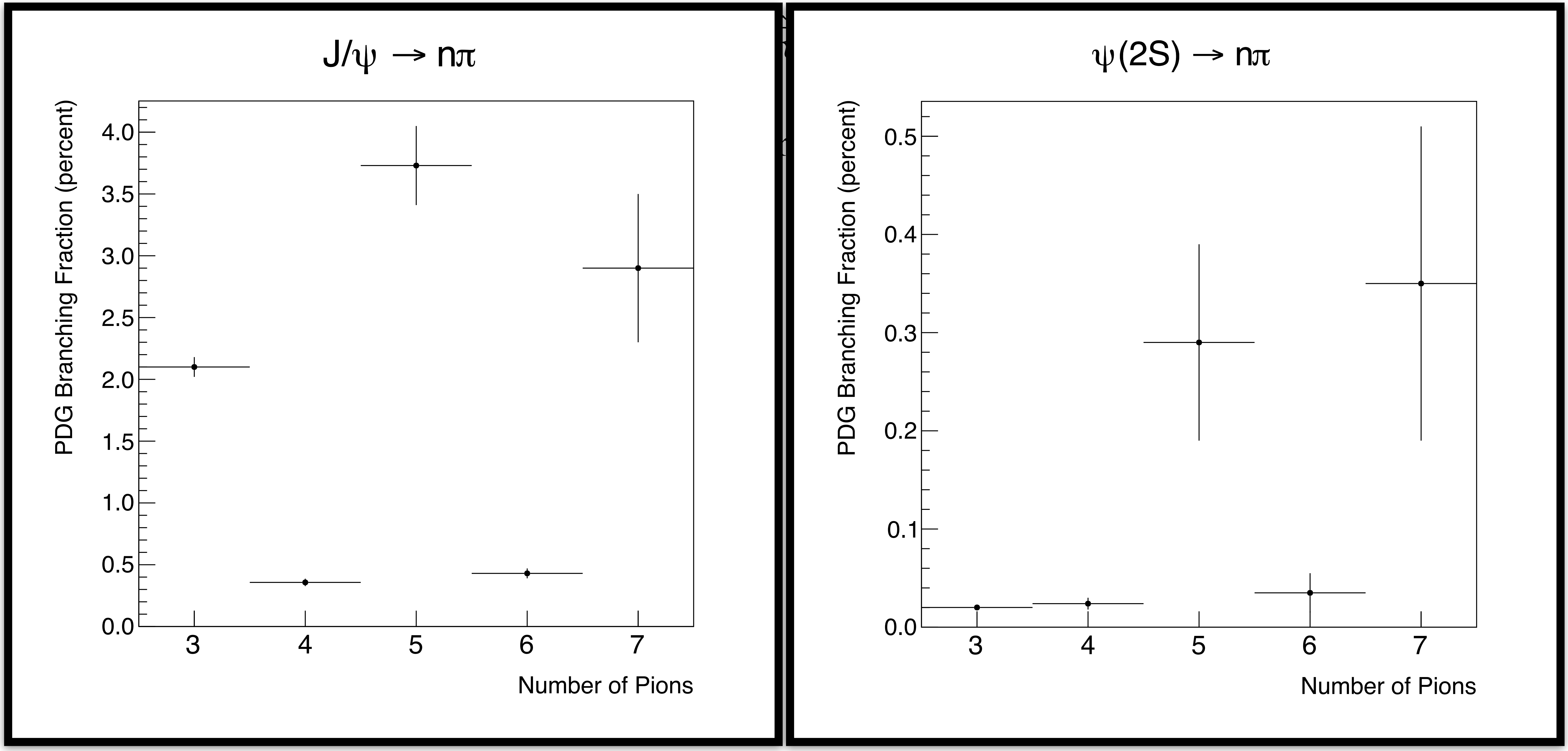

In general, for eigenstates of $\hat{G}$:

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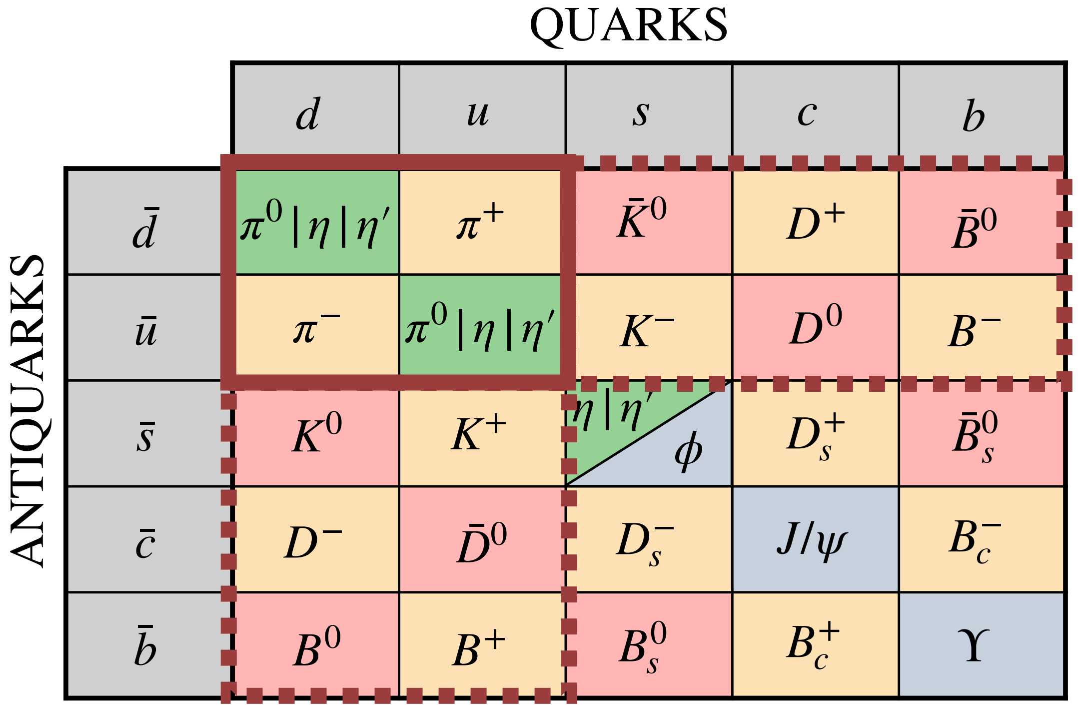
IIC. Meson Quantum Numbers: Flavor **SU(3)**

Extend SU(2) isospin symmetry to **SU(3) flavor symmetry** by including the strange quark. *(Since $m_u \approx m_d \ll m_s$, SU(3) flavor symmetry is less strict than isospin.)*

SU(2) Flavor Symmetry (isospin)

$$\begin{pmatrix}u'\\ d'\end{pmatrix} = \begin{pmatrix}a & b\\ -b^* & a^*\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{I}}}\begin{pmatrix}u\\ d\end{pmatrix}$$

3 generators: $\hat{I}_1, \hat{I}_2, \hat{I}_3$



QUARKS

ANTIQUARKS	d	u	s	c	b
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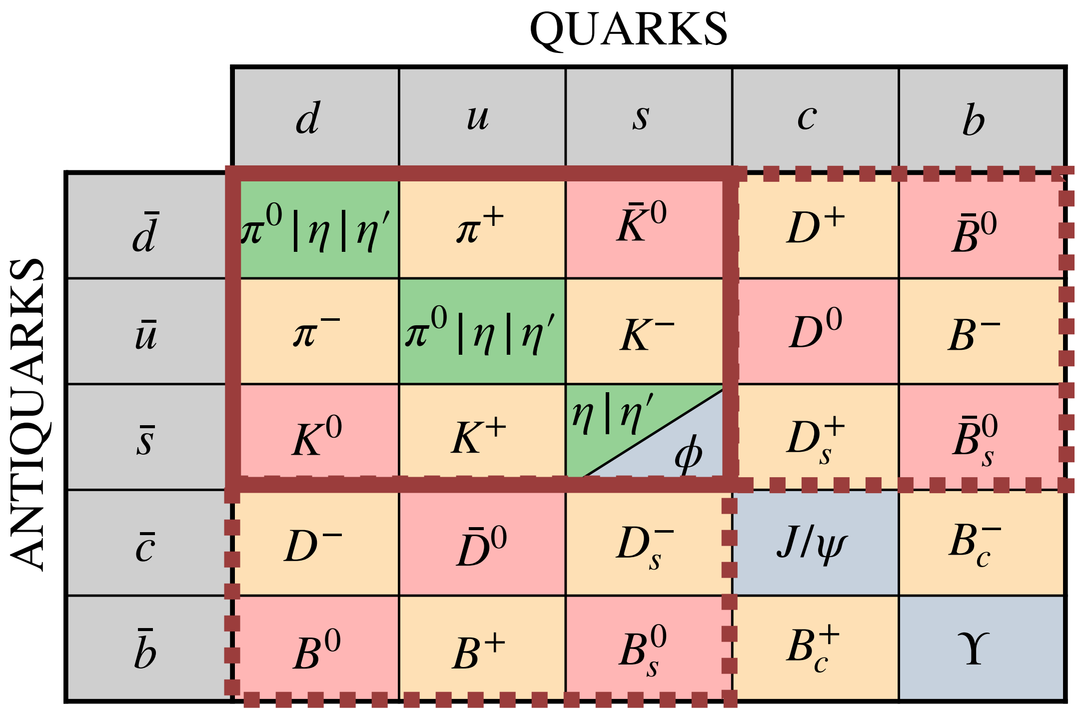
$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

SU(3) Flavor Symmetry

$$\begin{pmatrix}u'\\ d'\\ s'\end{pmatrix} = \begin{pmatrix}\mathbf{3}\end{pmatrix}\begin{pmatrix}u\\ d\\ s\end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{T}}}\begin{pmatrix}u\\ d\\ s\end{pmatrix}$$

8 generators: $\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4, \hat{T}_5, \hat{T}_6, \hat{T}_7, \hat{T}_8$



QUARKS

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$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

IIC. Meson Quantum Numbers: Flavor SU(3)

Extend SU(2) isospin symmetry to **SU(3) flavor symmetry** by including the strange quark. *(Since $m_u \approx m_d \ll m_s$, SU(3) flavor symmetry is less strict than isospin.)*

SU(2) Flavor Symmetry (isospin)

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{I}}} \begin{pmatrix} u \\ d \end{pmatrix}$$

3 generators: $\hat{I}_1, \hat{I}_2, \hat{I}_3$

label states by I, I_3

SU(3) Flavor Symmetry

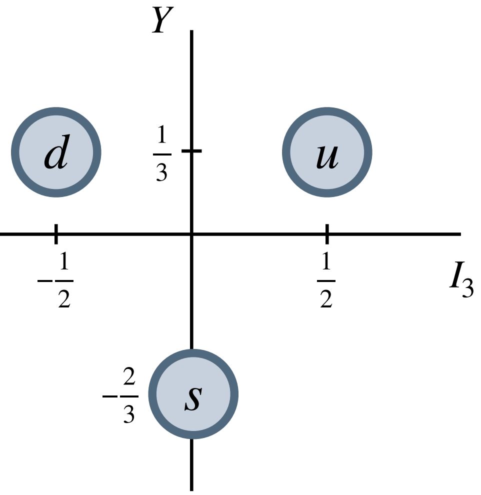
$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \left(\mathbf{3}\right) \begin{pmatrix} u \\ d \\ s \end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{T}}} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

8 generators: $\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4, \hat{T}_5, \hat{T}_6, \hat{T}_7, \hat{T}_8$

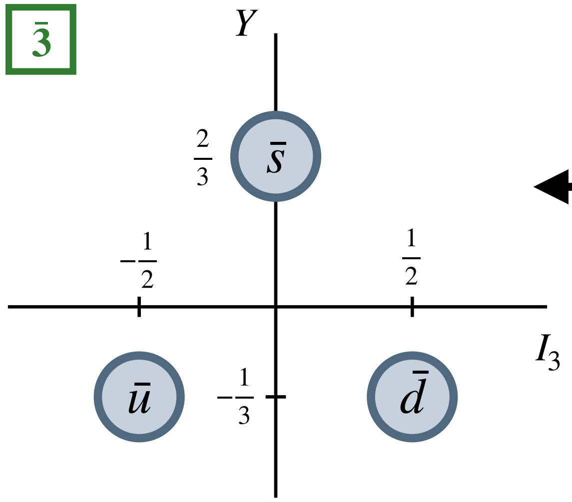
label states by T, I, I_3, Y (hypercharge)

(except instead of T use the dimension of the representation)

3



$\bar{3}$



Recall from earlier that I_3 and Y count quarks:

$$I_3 \equiv \frac{1}{2}U + \frac{1}{2}D$$

$$Y \equiv \frac{1}{3}U - \frac{1}{3}D + \frac{2}{3}S$$

IIC. Meson Quantum Numbers: Flavor SU(3)

Extend SU(2) isospin symmetry to **SU(3) flavor symmetry** by including the strange quark. *(Since $m_u \approx m_d \ll m_s$, SU(3) flavor symmetry is less strict than isospin.)*

The 8 SU(3) generators:

isospin subgroup

$$\hat{T}_1 = \hat{I}_1$$

$$\hat{T}_2 = \hat{I}_2$$

$$\hat{T}_3 = \hat{I}_3$$

$$\hat{T}_8 = \frac{\sqrt{3}}{2}\hat{Y}$$

hypercharge quantum number

raising and lowering operators

$$\hat{T}_1 \pm i\hat{T}_2 = \hat{I}_\pm$$

$$\hat{T}_4 \pm i\hat{T}_5 = \hat{V}_\pm$$

$$\hat{T}_6 \pm i\hat{T}_7 = \hat{U}_\pm$$

SU(3) Flavor Symmetry

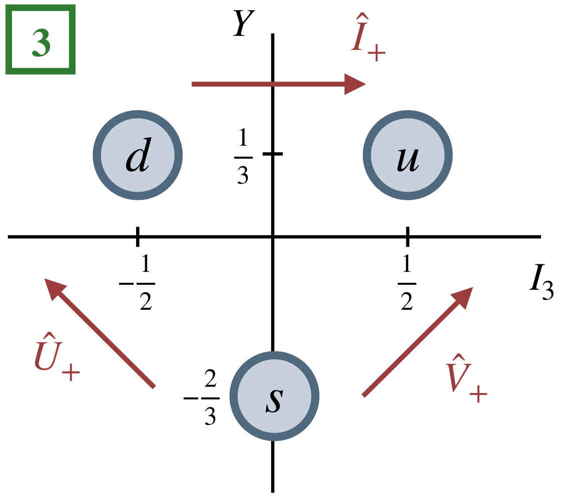
$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = (\mathbf{3}) \begin{pmatrix} u \\ d \\ s \end{pmatrix} = e^{i\boldsymbol{\alpha}\cdot\hat{\boldsymbol{T}}} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

8 generators: $\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4, \hat{T}_5, \hat{T}_6, \hat{T}_7, \hat{T}_8$

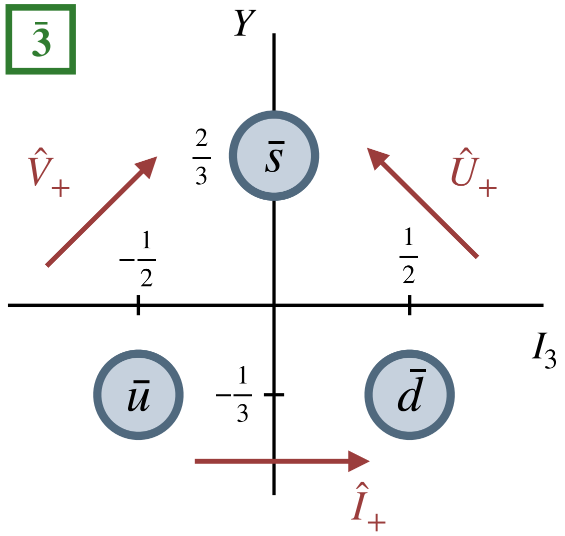
label states by T, I, I_3, Y (hypercharge)

(except instead of T use the dimension of the representation)

3



$\bar{3}$



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$$Y \equiv \frac{1}{3}U - \frac{1}{3}D + \frac{2}{3}S$$

IIC. Meson Quantum Numbers: Flavor SU(3)

Extend SU(2) isospin symmetry to **SU(3) flavor symmetry** by including the strange quark. *(Since $m_u \approx m_d \ll m_s$, SU(3) flavor symmetry is less strict than isospin.)*

The 8 SU(3) generators:

isospin subgroup

$$\hat{T}_1 = \hat{I}_1$$
$$\hat{T}_2 = \hat{I}_2$$
$$\hat{T}_3 = \hat{I}_3$$

$$\hat{T}_8 = \frac{\sqrt{3}}{2}\hat{Y}$$

hypercharge quantum number

raising and lowering operators

$$\hat{T}_1 \pm i\hat{T}_2 = \hat{I}_\pm$$
$$\hat{T}_4 \pm i\hat{T}_5 = \hat{V}_\pm$$
$$\hat{T}_6 \pm i\hat{T}_7 = \hat{U}_\pm$$

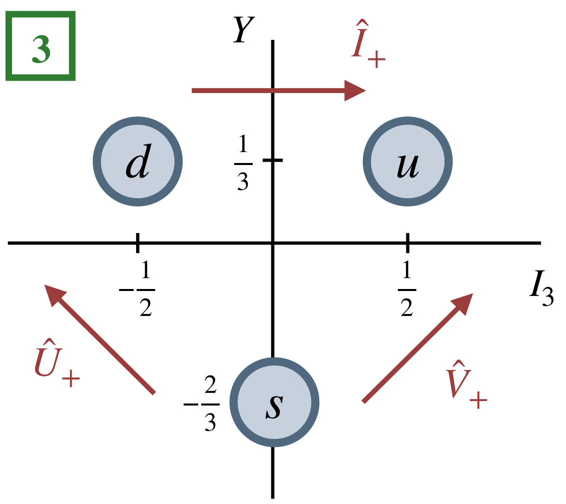
In the **3** representation, $\hat{T}_i = \frac{1}{2}\lambda_i$:

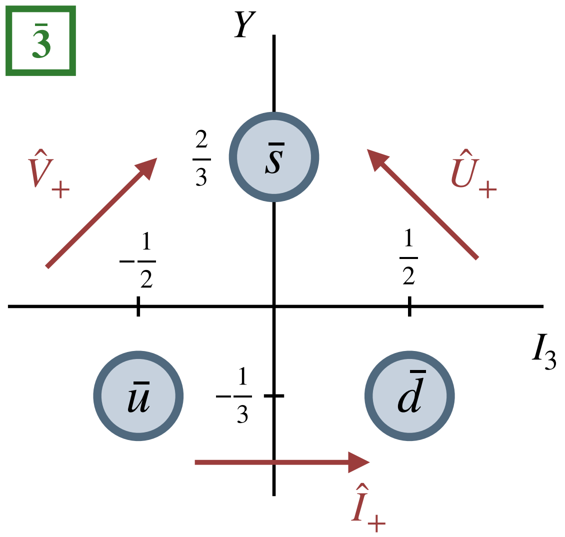
$$\lambda_1 = \begin{pmatrix} 0 & 1 & \cdot \\ 1 & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & \cdot \\ i & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & \cdot & 1 \\ \cdot & \cdot & \cdot \\ 1 & \cdot & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & \cdot & -i \\ \cdot & \cdot & \cdot \\ i & \cdot & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -2 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & 0 & 1 \\ \cdot & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & 0 & -i \\ \cdot & i & 0 \end{pmatrix}$$

(Gell-Mann matrices)





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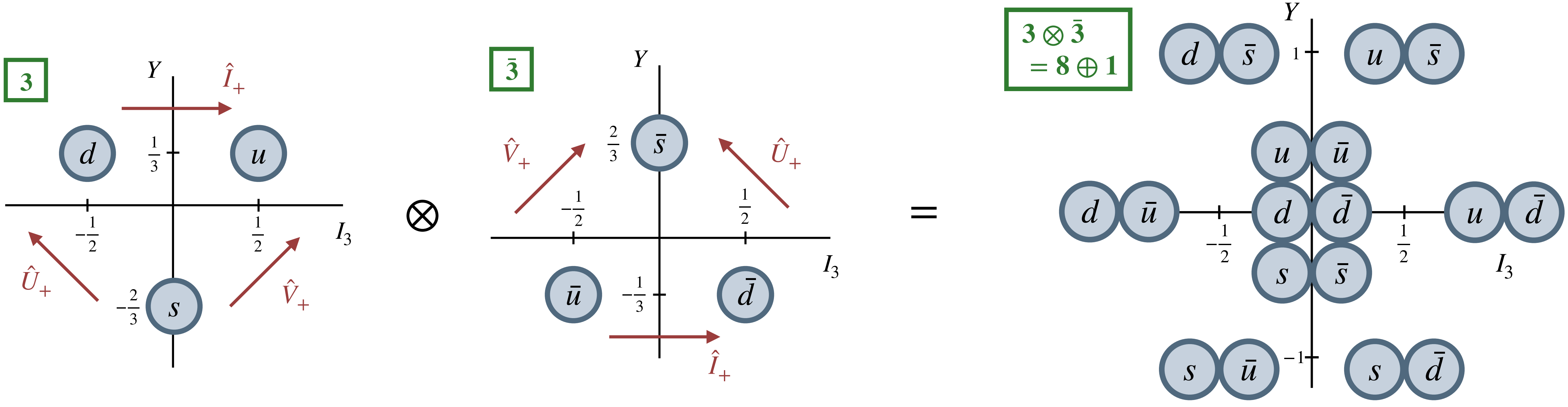
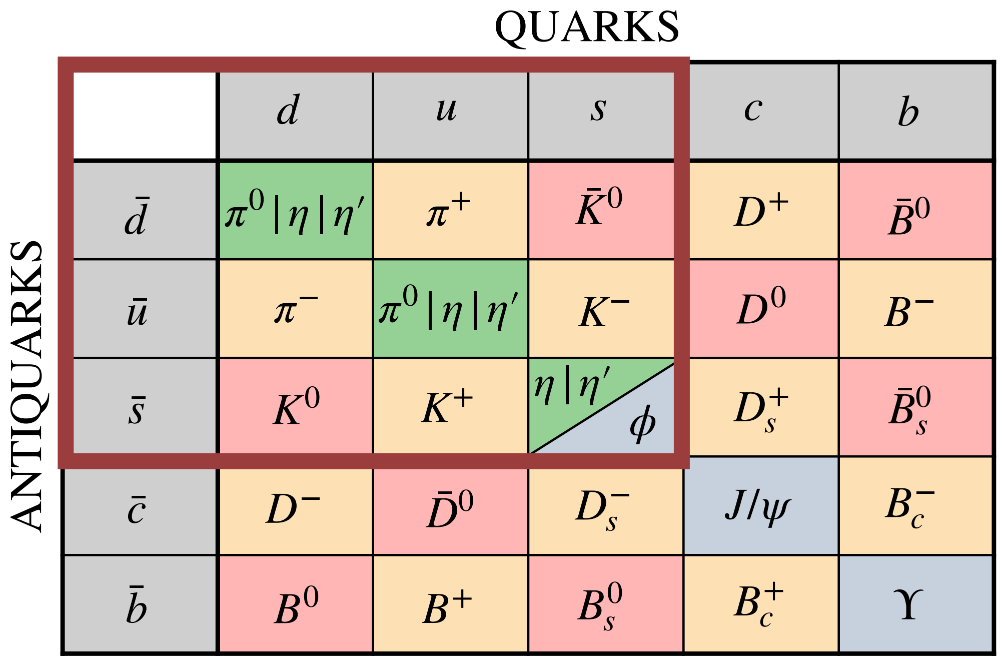


“nonet”

QUARKS

ANTIQUARKS



	d	u	s	c	b
$\bar{d}$	$\pi^0 \vert \eta \vert \eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
$\bar{u}$	π^-	$\pi^0 \vert \eta \vert \eta'$	K^-	D^0	B^-
$\bar{s}$	K^0	K^+	$\eta \vert \eta'$ ϕ	D_s^+	$\bar{B}_s^0$
$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

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IIC. Meson Quantum Numbers: Flavor **SU(3)**

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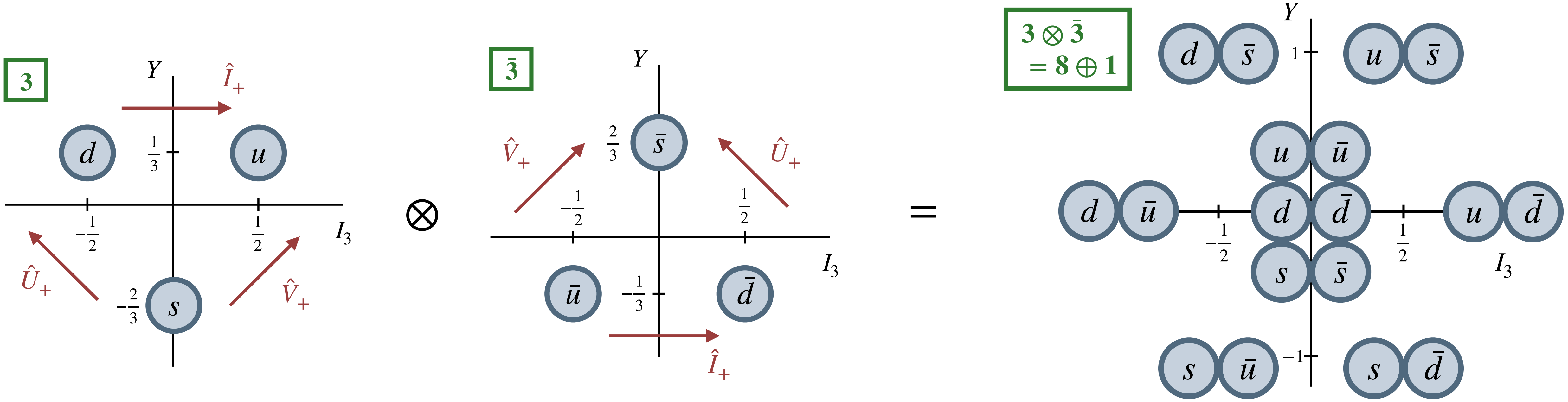

"nonet"

QUARKS

ANTIQUARKS

	d	u	s	c	b
$\bar{d}$	$\pi^0 \vert \eta \vert \eta'$	π^+	$\bar{K}^0$	D^+	$\bar{B}^0$
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$\bar{c}$	D^-	$\bar{D}^0$	D_s^-	J/ψ	B_c^-
$\bar{b}$	B^0	B^+	B_s^0	B_c^+	Υ

In the center of the nonet:

$$|\pi^0> = \frac{1}{\sqrt{2}}\left[|u\bar{u}> - |d\bar{d}>\right]$$

$m_\pi^0 \approx 135$ MeV

$$|\eta> \approx |\eta_8> = \frac{1}{\sqrt{6}}\left[|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>\right]$$

$m_\eta \approx 548$ MeV

$$|\eta'> \approx |\eta_1> = \frac{1}{\sqrt{3}}\left[|u\bar{u}> + |d\bar{d}> + |s\bar{s}>\right]$$

$m_{\eta'} \approx 958$ MeV

note: $m_{\eta'}$ is much larger than quark model expectations!

IIC. Meson Quantum Numbers: Flavor SU(3)

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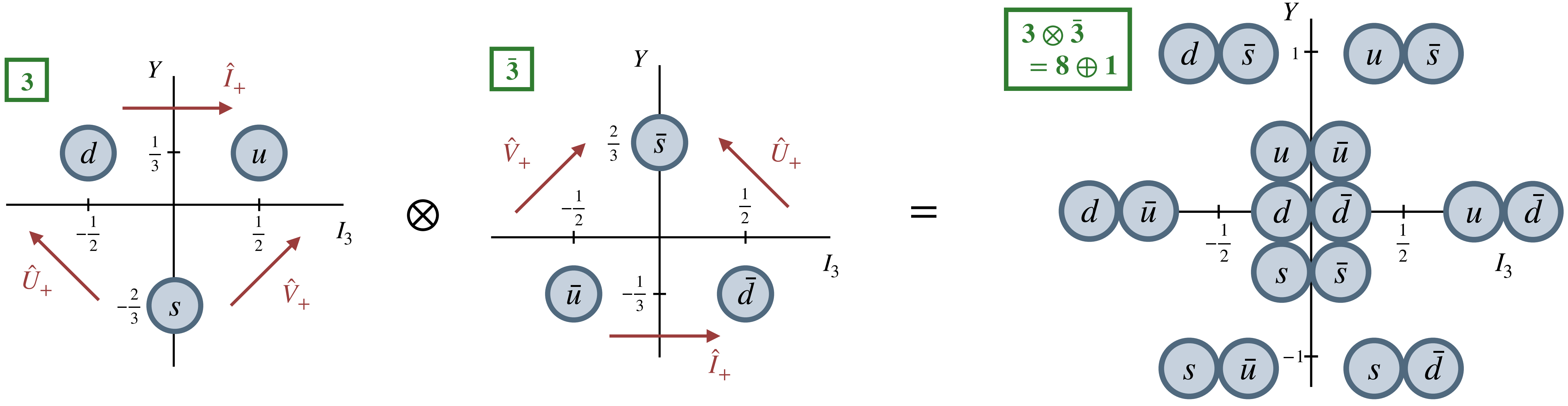

"nonet"

↑

excited states

ground state

$J^{P(C)}$	$I = 1$	$I = 0$	$I = 0$	$I = \frac{1}{2}$
$1^{-(-)}$	$\rho(1700)$	$\omega(1650)$	$\phi(1680)$	$K^*(1680)$
$2^{+(+)}$	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$	$K_2^*(1430)$
$1^{+(+)}$	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$	$K_1(1400)$
$0^{+(+)}$	$a_0(1450)$	$f_0(1370)$	$f_0(1710)$	$K_0^*(1430)$
$1^{+(-)}$	$b_1(1235)$	$h_1(1170)$	$h_1(1415)$	$K_1(1270)$
$1^{-(-)}$	$\rho(770)$	$\omega(782)$	$\phi(1020)$	$K^*(892)$
$0^{-(+)}$	π^0 π^+	$\eta \mid \eta'$	$\eta \mid \eta'$	K^0 K^+

Excited states have closer to "ideal mixing":

$$|\rho^0> = \frac{1}{\sqrt{2}}\left[|u\bar{u}> - |d\bar{d}>\right] \qquad m_\rho \approx 770 \text{ MeV}$$

$$|\omega> \approx \frac{1}{\sqrt{2}}\left[|u\bar{u}> + |d\bar{d}>\right] \qquad m_\omega \approx 782 \text{ MeV}$$

$m_{K^*} \approx 892$ MeV

$$|\phi> \approx |s\bar{s}> \qquad m_\phi \approx 1020 \text{ MeV}$$

note: $m_\rho \approx m_\omega$ *and* $m_\omega + m_\phi \approx 2m_{K^*}$ *as expected*

IIC. Meson Quantum Numbers: Flavor SU(3)

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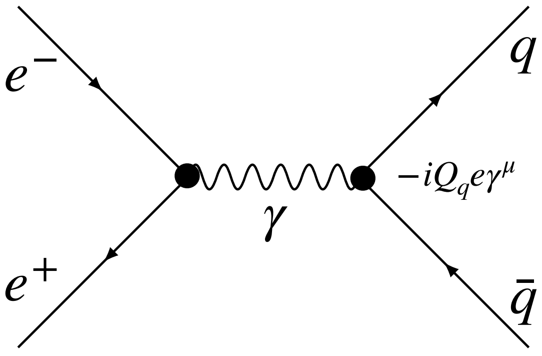
There are many ways to probe the quark content, e.g. Γ_{ee}:



$$\Gamma_{ee}(\rho) \propto \frac{1}{2}(Q_u - Q_d)^2 = \frac{1}{2}(1)^2 = \frac{1}{18}(9)$$

$$\Gamma_{ee}(\omega) \propto \frac{1}{2}(Q_u + Q_d)^2 = \frac{1}{2}(\frac{1}{3})^2 = \frac{1}{18}(1)$$

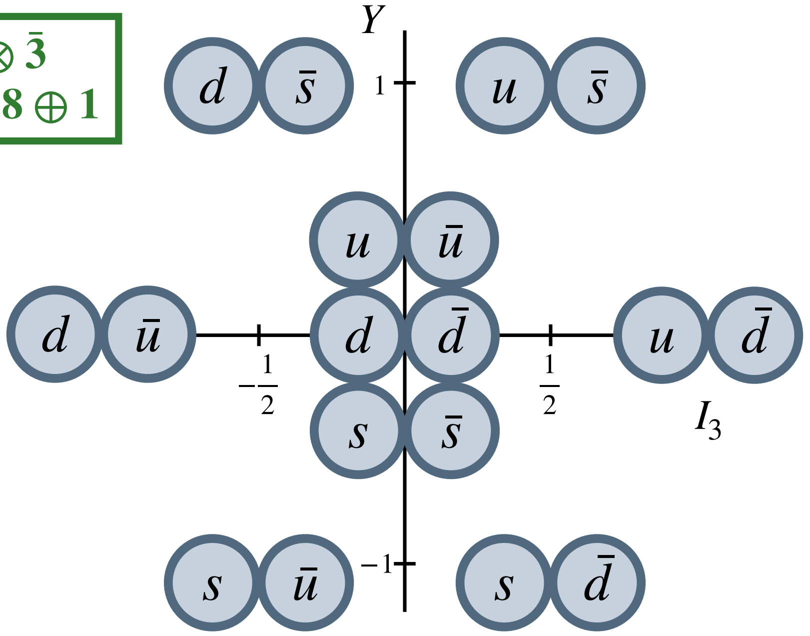
$$\Gamma_{ee}(\phi) \propto (Q_s)^2 = (-\frac{1}{3})^2 = \frac{1}{18}(2)$$

Experiment:

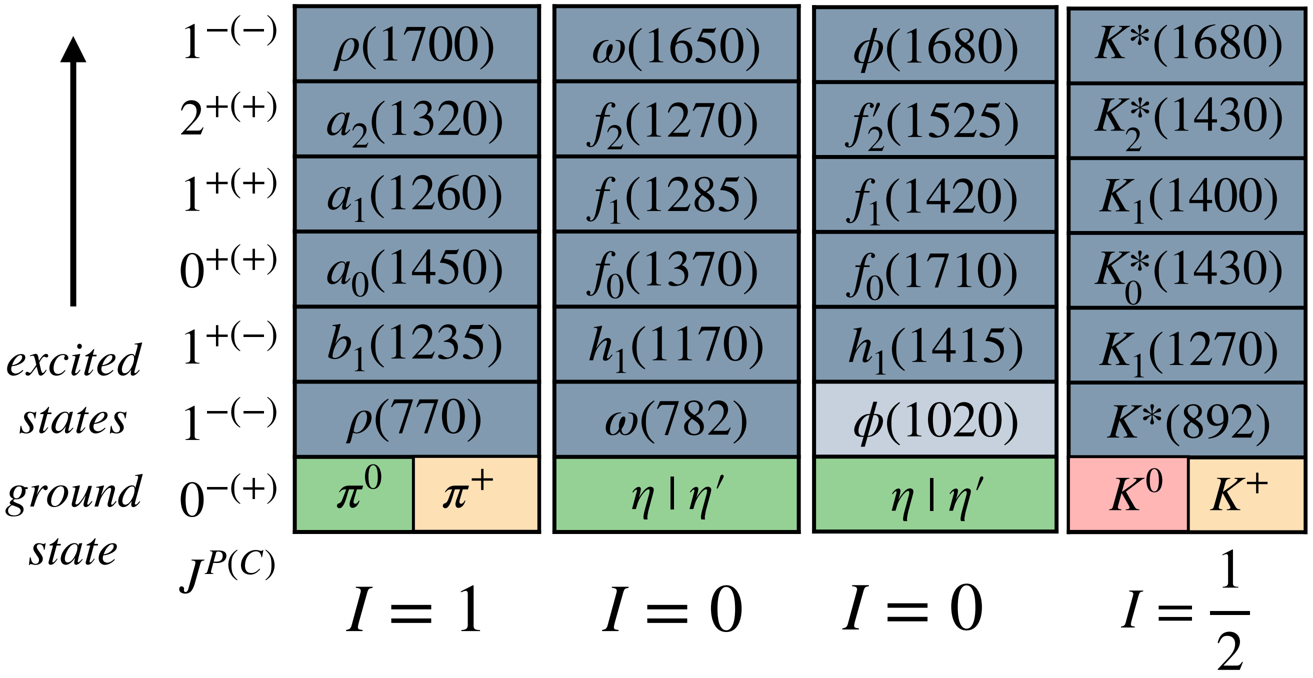
$\Gamma_{ee}(\rho) = 6.98$ keV, $\Gamma_{ee}(\omega) = 0.62$ keV, $\Gamma_{ee}(\phi) = 1.26$ keV *(with errors $\approx 1\,\%$)*

$$\Longrightarrow \Gamma_{ee}(\rho) : \Gamma_{ee}(\omega) : \Gamma_{ee}(\phi) = 11.2 : 1.0 : 2.0$$

$3 \otimes \bar{3}$
$= 8 \oplus 1$



"nonet"



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Experimental Meson Spectroscopy

Prologue: Definitions and Philosophy

I. A Field Guide to Meson Families

II. Meson Quantum Numbers

III. The Quark Model

IV. Exotic Mesons

V. Current and Future Experiments

LECTURE II. Meson Quantum Numbers

IIA. Meson Naming Scheme

IIB. J^{PC} (spin, parity, C-parity)

- * from experiment
- * from a $q\bar{q}$ model

IIC. Flavor

- * Strangeness, Charm, Bottomness
- * Isospin
- * G-Parity
- * Flavor SU(3)

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LECTURE II. Meson Quantum Numbers

Meson quantum numbers include J^{PC} and flavor, which can be determined experimentally.

The J^{PC} and flavor can also be mapped to a meson's quark content, but with ambiguities.

Given the name of a meson, you know its J^{PC} and flavor.