Gluons in QCD

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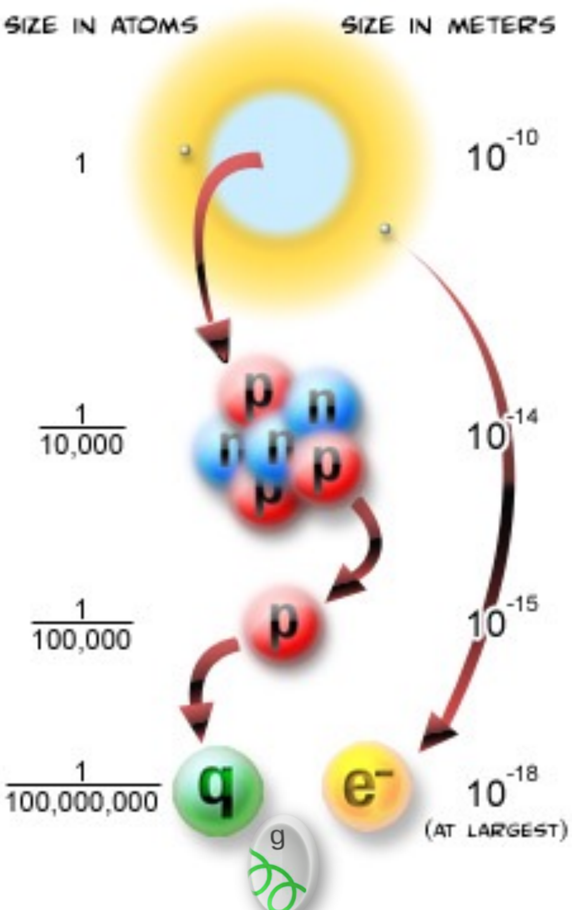
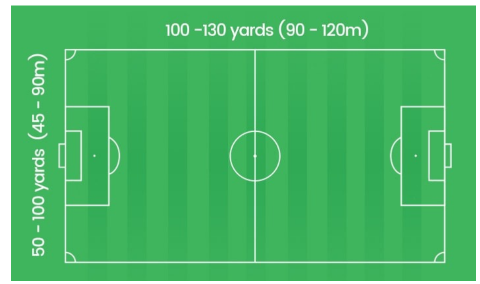
Setting the scale - The inner structure of the atom

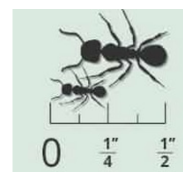
- The nuclei is 10.000 times smaller than the atom.
- Quarks, gluons, and electrons are 10.00 times smaller than the nuclei.



Source: Particle Adventure



Soccer field
~ 100 m

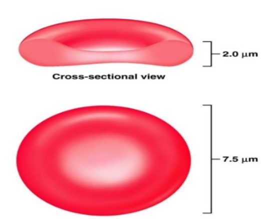


Carpenter ant
~ 1 cm

(30x0,65mm)

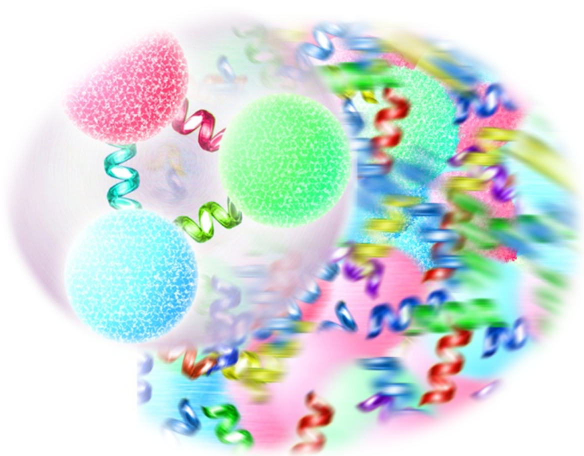
diameter of a pin head ~ 1 mm

Normal red cell

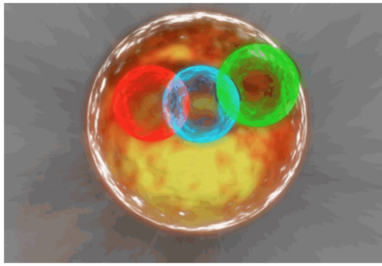


Size of red cell
~1 μm

What do we know about Quantum Chromodynamics QCD ?

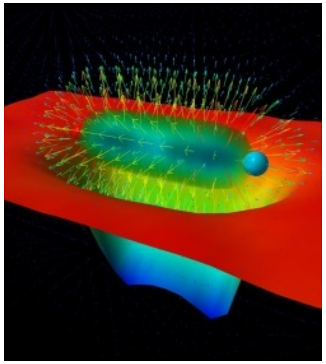


Quantum Chromodynamics - QCD

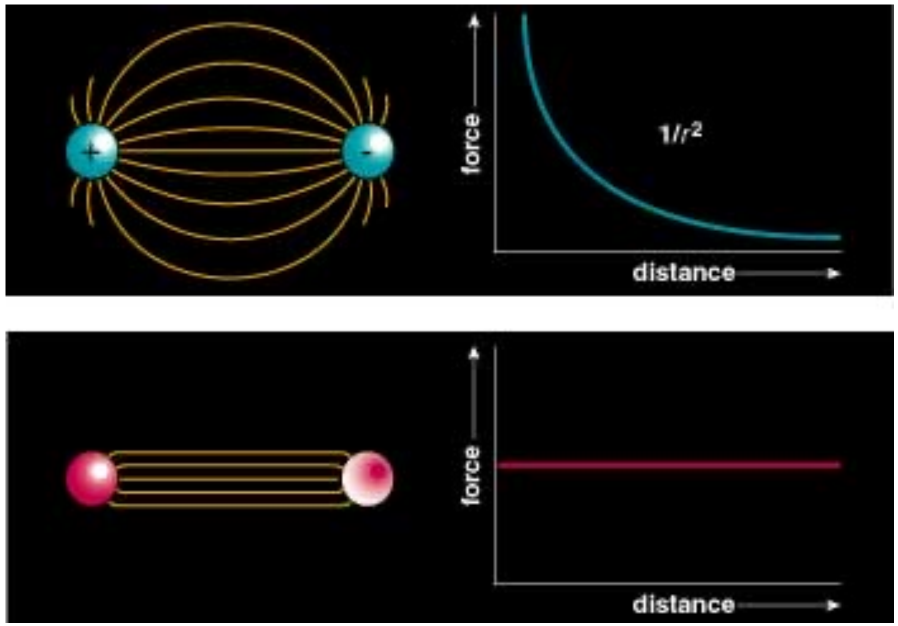


- QCD is the theory of the strong interaction, where the *quarks* and *gluons* are the fundamental degrees of freedom.
- Interactions are mediated by vector boson → gluon spin 1
- Quarks have masses and gluons are massless perturbatively.
- QCD is a renormalizable theory, and its energy range of validity goes from zero up to the Planck scale.
- Just need one observable to set the scale: $\Lambda_{\rm QCD} \simeq 300\,{\rm MeV}$
- QCD is not an effective theory is *the fundamental theory of strong interactions.*

- One of its challenges is to understand, from first principles, how quarks and gluons combine to create the hadrons we find in the nature → mesons, barions, glueballs…
- In the QCD IR region (strong regime) we have phenomena such as confinement and chiral symmetry.
- Both phenomena play a major role in the formation of bound states.
- For the above reasons, it is mandatory to explore the *strong regime of QCD.*

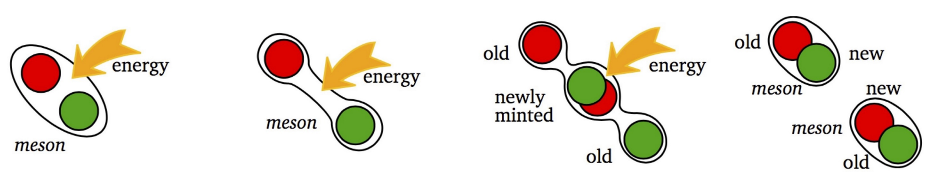


- We all know that, when we try to pull apart two eletric charges, the force generated is proportional to $1/r^2$ (Coulomb force)



- However in QCD, when two color charges (quarks) are separated, the force generated between them is constant (creation of a flux tube).

- As the force between quarks does not decrease, this would require an infinite amount of energy to separate them → *Confinement*



QED - Quantum Electrodynamics

served as a prototype to develop

QCD- Quantum Chromodynamics

Electric charge ⟷ Color charge

Quantum Electrodynamics (QED)

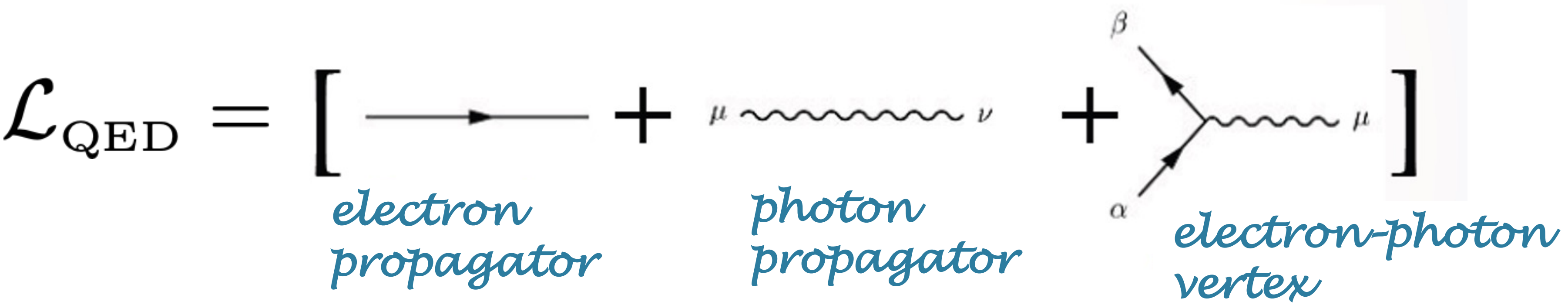
- Electrical charged particles interact through the exchanged of the photons.
- The strengh of the interaction is given by the fine structure $\alpha = 1/137$
- Quantum field theory which describes the eletromagnetism is the **Quantum Electrodynamics:QED**
- The most precise theory of the science!

- The QED Lagrangian is given by

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

field strength tensor



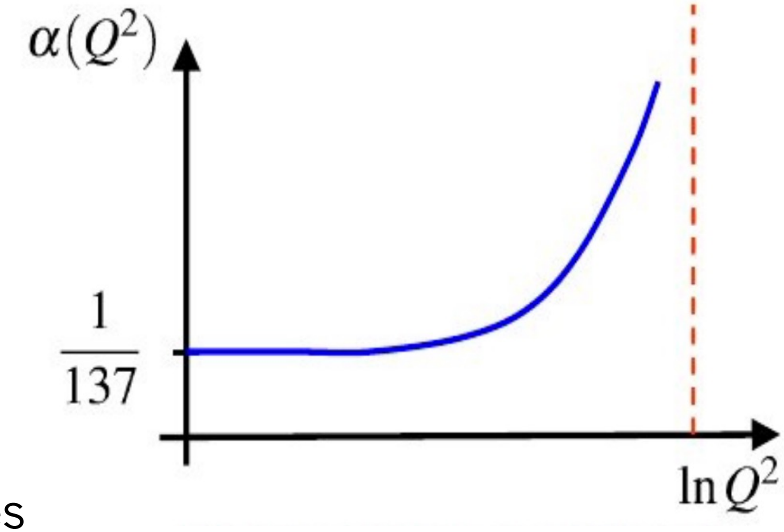
- In gauge theories *nothing is constant.*

- Couplings and masses acquire quantum corrections. Then, they will depend on the momenta scale.

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

Pay attention to this sign

increases as function of the momentum

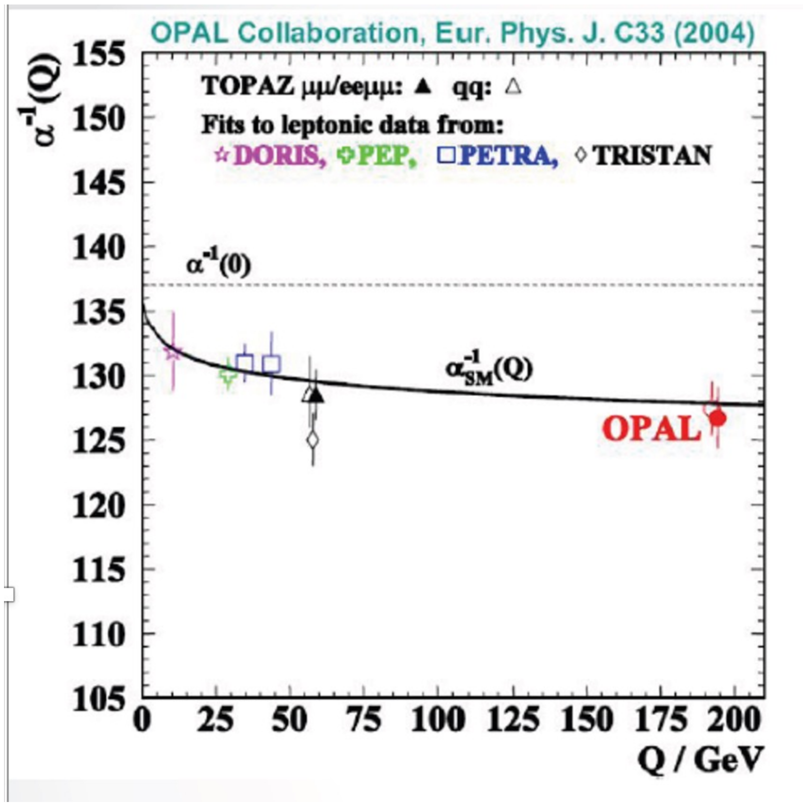


You might worry that the coupling becomes infinite at

$$\ln\left(\frac{Q^2}{\mu^2}\right) = \frac{3\pi}{1/137} \quad \Rightarrow \quad Q \approx 10^{26}\,\mathrm{GeV}$$

but at this scale quantum gravity effects are expected to dominate since Planck scale is much below this energy (10^{19} GeV) - highly unlikely that QED would be valid at this regime.

Inverse of the QED running coupling



In QED the running coupling increases (as function of the momentum) very slowly

- *Atomic physics:* $Q^2 \approx 0$

$$1/\alpha = 137.03599976(50)$$

- *High energy physics:*

$$1/\alpha(193\,\mathrm{GeV}) = 127.4 \pm 2.1$$

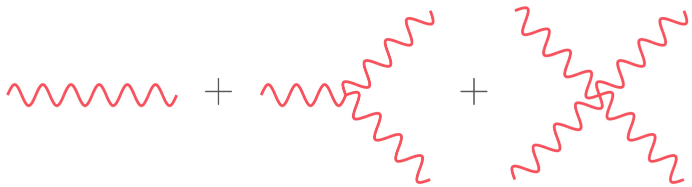
QCD Lagrangian

The QCD dynamics are governed by the Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{gluon}} + \mathcal{L}_{\mathrm{quarks}} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{ghost}}$$

where

$$\mathcal{L}_{\mathrm{gluon}} = -\frac{1}{4}(G^a_{\mu\nu})^2$$



Gluon self-interaction (Non-Abelian character)

Profound consequences!

$$G^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^{abc} A^b_\mu A^c_\nu$$

Gluonic field strength tensor

$$\mathcal{L}_{\mathrm{quarks}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + g A^a_\mu \bar{\psi}\gamma^\mu t_a \psi$$

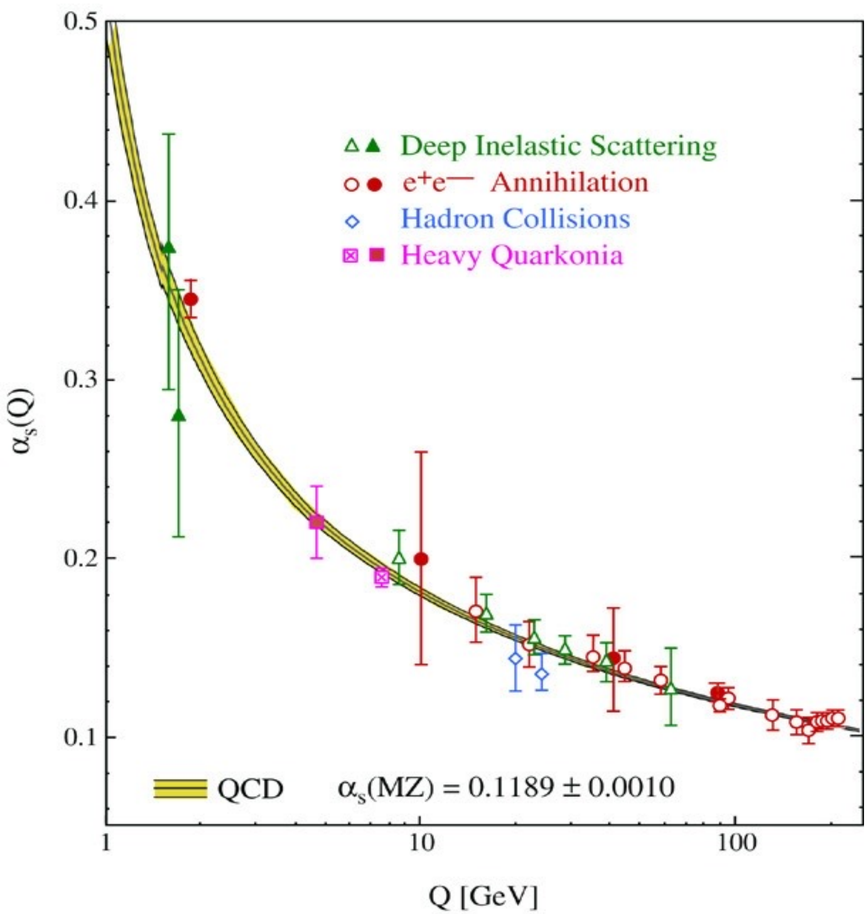
$$\mathcal{L}_{\mathrm{ghost}} = -\bar{c}^a \partial^2 c^a - g\bar{c}^a f^{abc} \partial^\mu A^b_\mu c^c$$

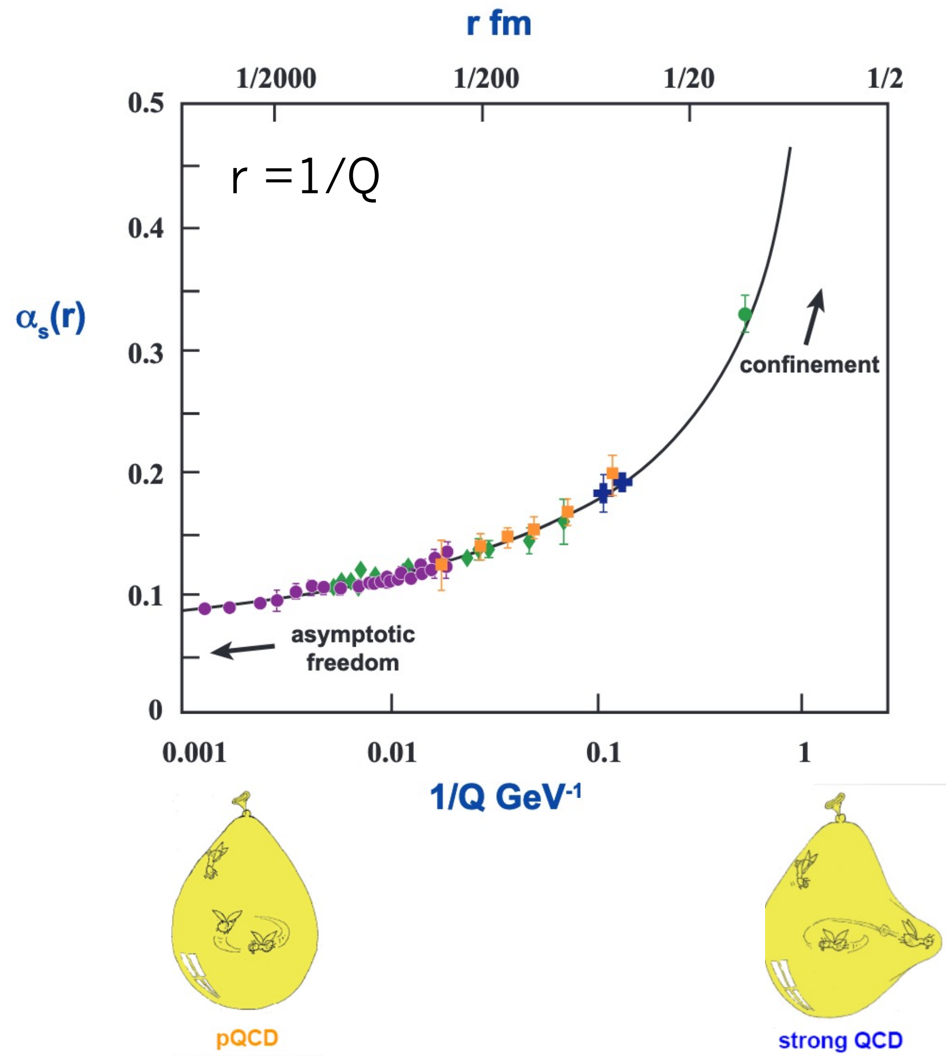
$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2$$

gauge fixing term → contributes to the gluon propagator

Strong interaction: QCD

Decreases as function of the momentum





- Asymptotically free→ Perturbation theory is valid for large values of Q^2
- *Essentially* nonperturbative around $Q^2 < 2$ GeV (~ 1 fermi)

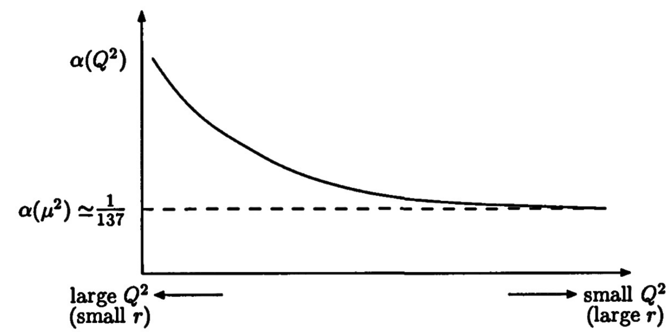
Comparison of the couplings

- Behavior of the QED and QCD the coupling constants depend on the distance (or momentum)

$r = 1/Q$

- In QED we have $Q^2 \gg \mu^2$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln\left(\frac{Q^2}{\mu^2}\right)}$$

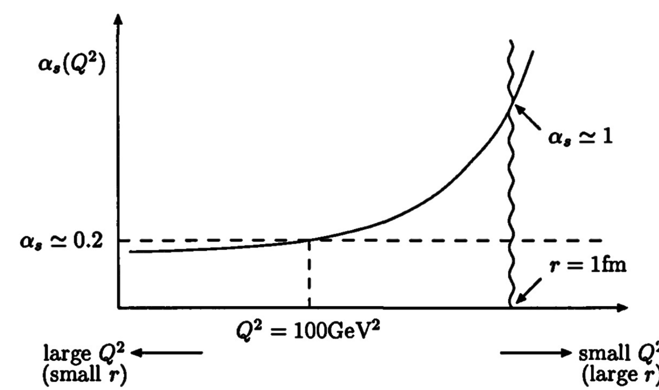


where $\alpha = \alpha(Q^2 \to 0) = e^2/4\pi = 1/137$

- The perturbative QCD coupling

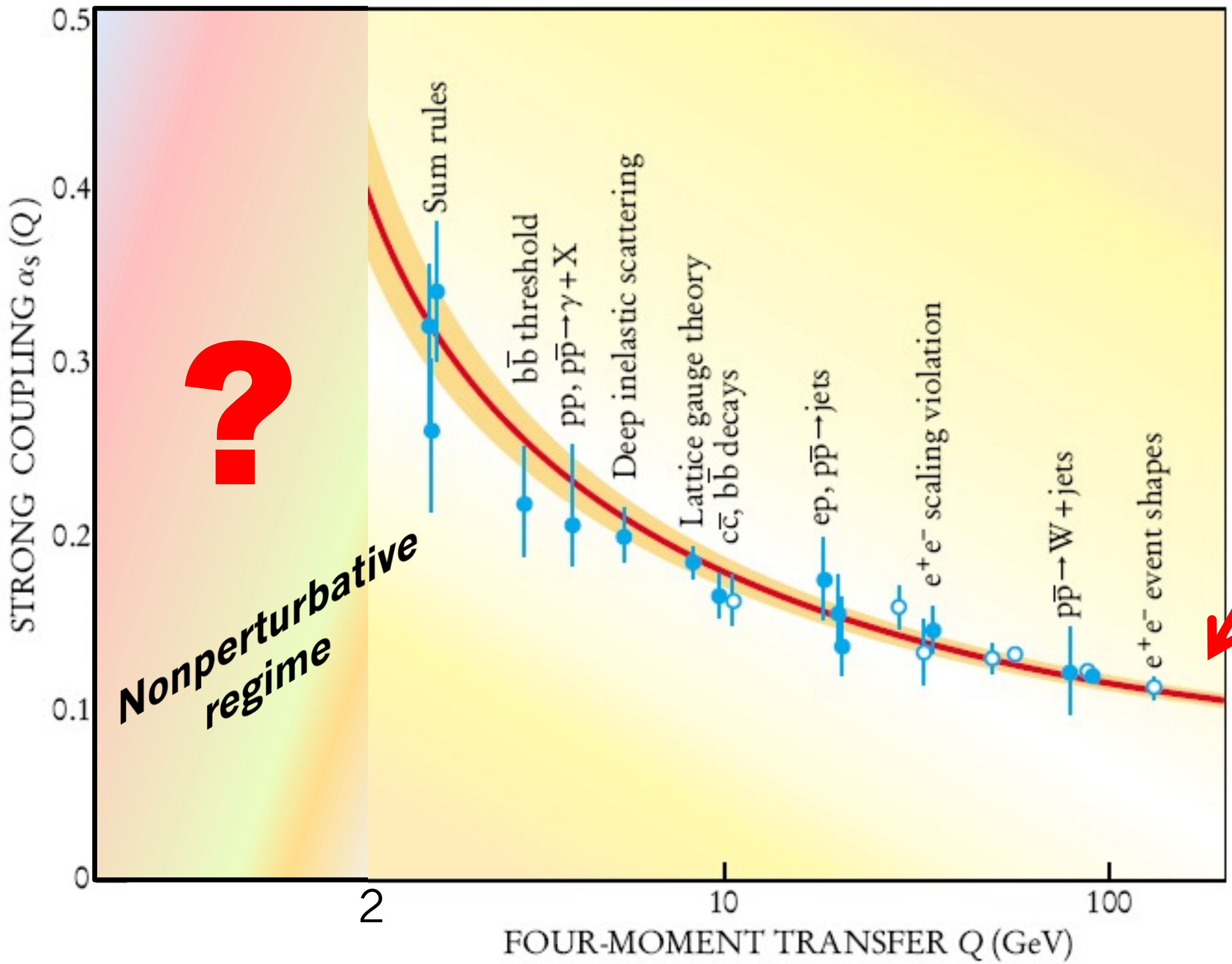
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{(11C_A - 2nf)}{12\pi} \ln\left(\frac{Q^2}{\mu^2}\right)}$$

quark loop



decreases for higher values of Q^2 if $n_f < 16$.

QCD coupling constant



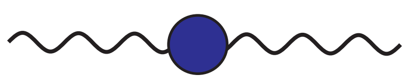
The strongest force in the nature turns off at large momentum values

Asymptotic fredoom

Objects of interest:
Green's functions

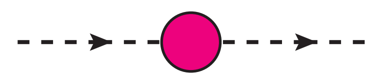
- Full propagators defined as vaccum expectation value of the fields

$$\langle\Omega|T\{A_\mu^a(x)A_\nu^b(y)\}|\Omega\rangle := -i\Delta_{\mu\nu}^{ab}(x-y)$$



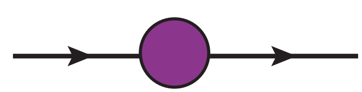
Gluon propagator

$$\langle\Omega|T\{c^a(x)\bar{c}^b(y)\}|\Omega\rangle := iD^{ab}(x-y)$$



Ghost propagator

$$\langle\Omega|T\{\psi(x)\overline{\psi}(y)\}|\Omega\rangle := iS(x-y)$$



Quark propagator

Off-shell QCD Green's functions

Green's functions:

Propagators and vertices

↓

Although they are:

- Gauge-dependent
- Renormalization point (μ) and scheme-dependent

However

- They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.
- When appropriately combined they give rise to physical observables.

Crucial pieces for completing the QCD puzzle

QCD

The nonperturbative QCD problems

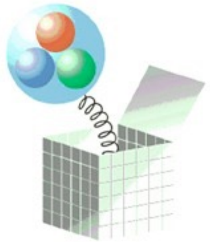
◎ The Green's functions are crucial for exploring the outstanding nonperturbative problems of QCD:

Bound states

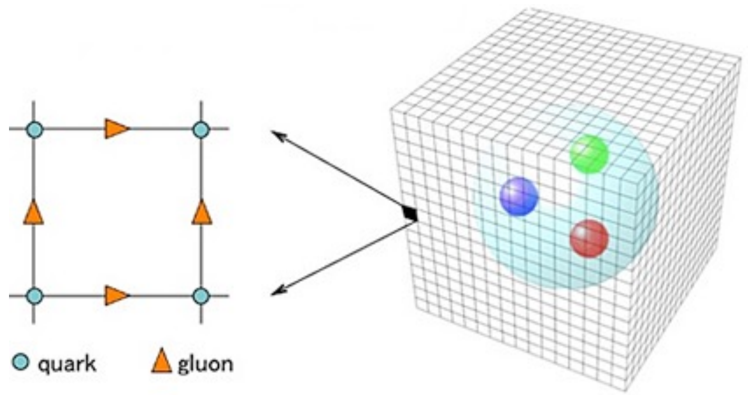
Mass generation

Confinement

Nonpertubative tools



- Non-perturbative physics requires special tools.
- For QCD we have (first principles):
- Lattice simulations

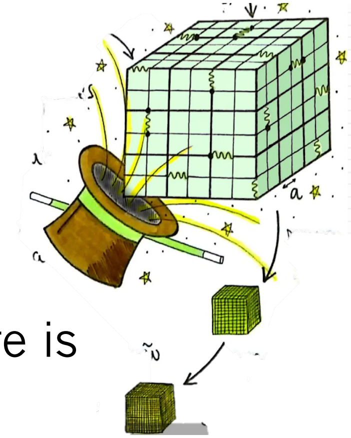


- Space-time is discretized;
- The precision depends on the lattice spacing parameter and volume.

Lattice

Suppose we wanted to study the Mona Lisa:

The first image is the original.



Source: Blog Coleção de Partículas - IFSC

The second image comes from putting the image on a lattice, you see that we lose details about small things (effects of the lattice space)

The third image comes from having a smaller canvas size so that we cannot see the big picture of the entire image (small volume)

If you're interested in only the broad features Mona Lisa's face , then the lattice isn't so bad. But, if you are a fine art critic...

Source: Quantum Diaries

Schwinger-Dyson equations - SDE

- Insightful computational framework.
- Equations of motion for off-shell Green's functions.
- It can be understood as the generalization of the Euler-Lagrange equation for a classical field $(\delta S/\delta\phi = 0)$.
- Derived formally from the generating functional.
- Infinite system of coupled nonlinear integral equations.
- Inherently non-perturbative, but at the same time captures the perturbative behavior → It accommodates the full range of physical momenta.

How to derive the SDE?

- Derived formally from the generating functional

$$\int \mathcal{D}\phi \, \frac{\delta}{\delta\phi(x)} e^{-S[\phi]+J\cdot\phi} = 0$$

which is equivalent to

$$\left(\frac{\delta S}{\delta \phi}\left[\frac{\delta}{\delta J(x)}\right] - J(x) \right) Z[J] = 0$$

- This equation is a *compact form of equations of motion for the Green's functions.*

- One has a tower of non-linear coupled integral equations.

Derivation using functional methods:

C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477-575 (1994)

R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001)

E. S. Swanson, AIP Conf. Proc. 1296, no.1, 75-121 (2010)

M. Q. Huber, Phys. Rept. 879, 1-92 (2020)

R.J. Rivers, Path Integral Methods in Quantum Field Theory, Cambridge University Press, New York (1990).

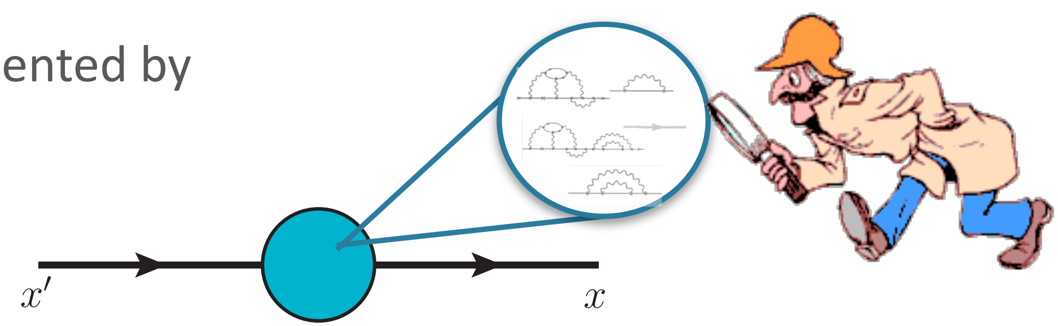
SDEs -Diagrammatic way

- Although the functional method is the formal way to derive the SDEs, it is quite abstract. Let us derive these equations in a diagrammatic way.
- First, let us do for QED which is easier than QCD.
- The full electron propagator is defined as

$$iS(x-x') := \langle \Omega | T\{\psi(x)\overline{\psi}(x')\} | \Omega \rangle$$

and diagramatically represented by

$$iS(x'-x) =$$



- The *full electron propagator* is *the sum all connected diagrams* which start and end with a electron leg.

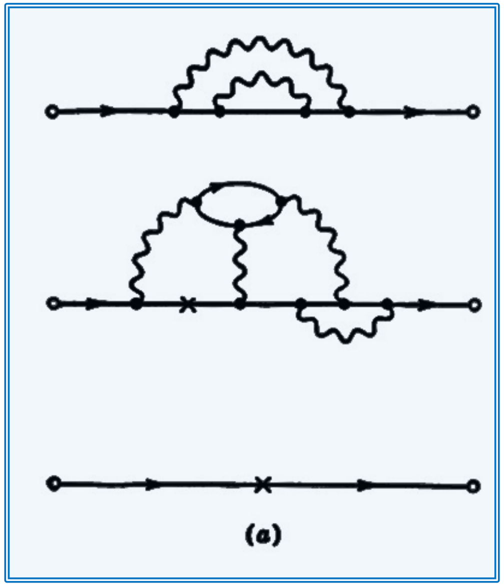
Based on:
J.D.Bjorken and S.D.Drell, “Relativistic quantum fields”, McGraw Hill Book Company, New York (1965).
M.R.Pennington, J. Phys. Conf. Ser. 18, 1-73 (2005).

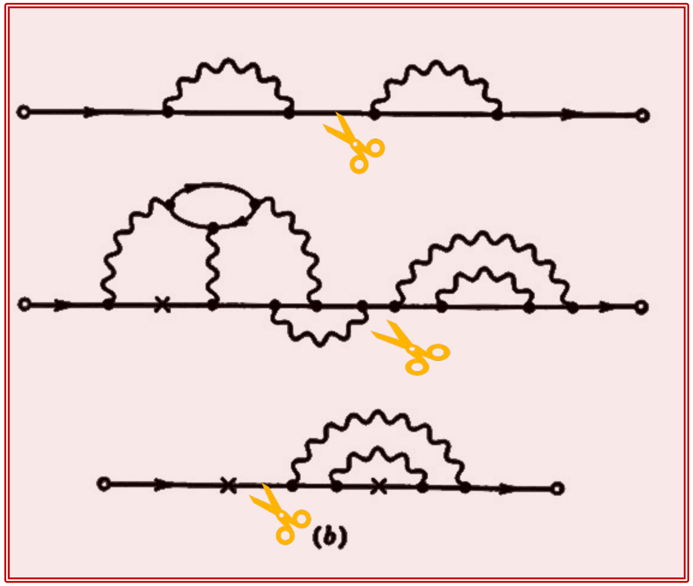
- The connected diagrams can be separated in two classes

Connected Diagrams

- ***Improper:*** CAN be split into two by removing a single line.
- ***Proper or one particle irreducible (1PI):*** CANNOT be split into two by removing a single line.

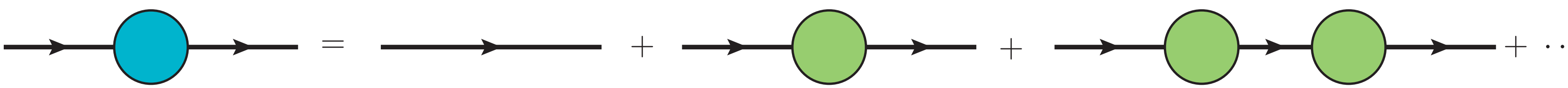
Examples:





- In the momentum space, we can write the full electron propagator, iS(p) as

$$iS(p) = iS_0(p) + iS_0(p)[-i\Sigma(p)]iS_0(p) + iS_0(p)[-i\Sigma(p)]iS_0(p)[-i\Sigma(p)]iS_0(p) + \cdots$$

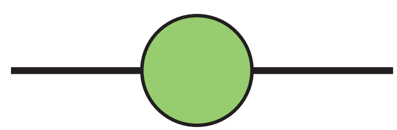


$$iS = iS_0 + iS_0[-i\Sigma]iS_0 + iS_0[-i\Sigma]iS_0[-i\Sigma]iS_0 + \cdots$$

where $$iS_0(p) = i(\not{p} - m)^{-1}$$

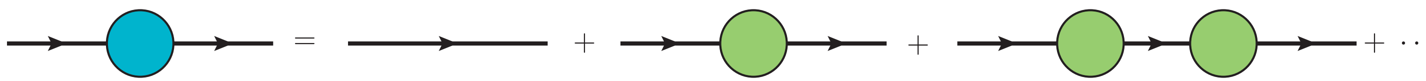
is the electron propagator at tree level.

In addition, $i\Sigma(p)$ represents the sum of all proper diagrams of one-electron with momentum p (the external legs removed) - *Electron self-energy*

$-i\Sigma(p) =$ 

- The series

$$iS(p) = iS_0(p) + iS_0(p)[-i\Sigma(p)]iS_0(p) + iS_0(p)[-i\Sigma(p)]iS_0(p)[-i\Sigma(p)]iS_0(p) + \cdots$$



$$iS = iS_0 + iS_0[-i\Sigma]iS_0 + iS_0[-i\Sigma]iS_0[-i\Sigma]iS_0 + \cdots$$

can be summed (Dyson sum), leading us to

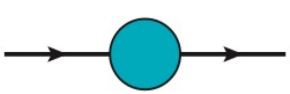
Remember that

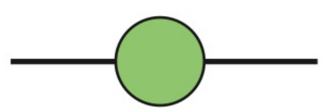
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$S(p) = \frac{1}{\not{p} - m - \Sigma(p)}$$

Full (complete, dressed)
Electron propagator

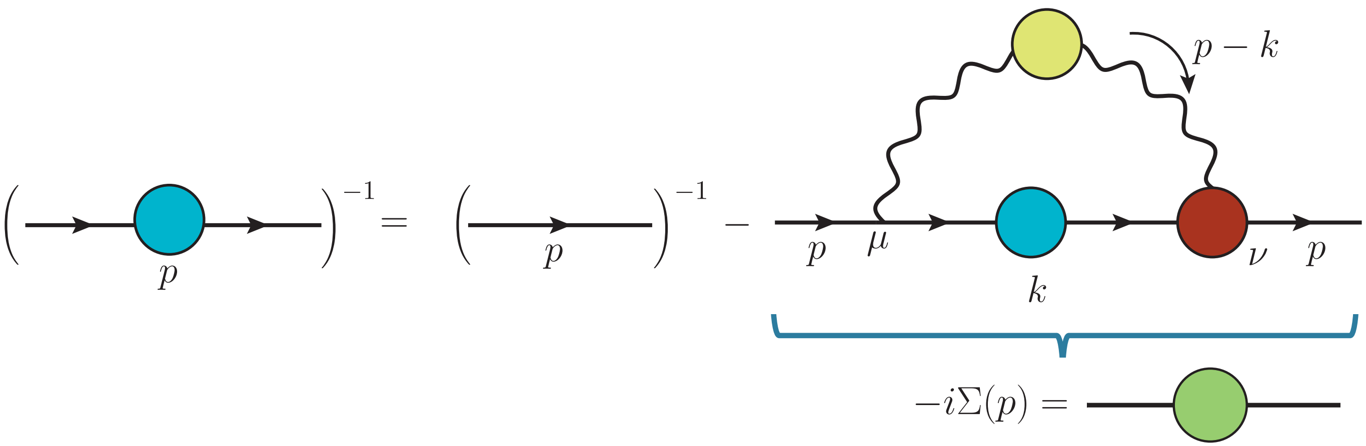
sum of all
1PI diagrams – full self-energy





How do we calculate the electron full self-energy?

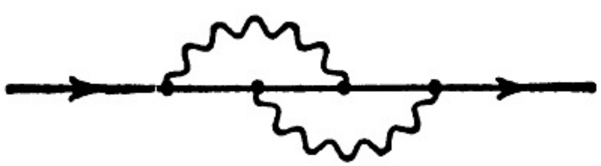
- The electron full self-energy is given by



Electron SDE

$$-i\Sigma(p) = (-ie_0)^2 \int \frac{d^4k}{(2\pi)^4} i\Delta_{\mu\nu}(p-k)\Gamma^{\nu}(p,k)iS(k)\gamma^{\mu}$$

- We would count twice the following diagram, if we have added another full vertex at μ.

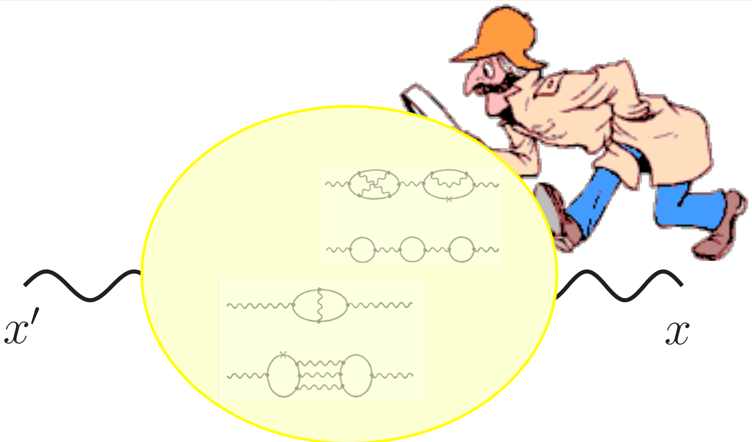


Photon SDE

- In a similar way, we can build the SDE for the photon propagator

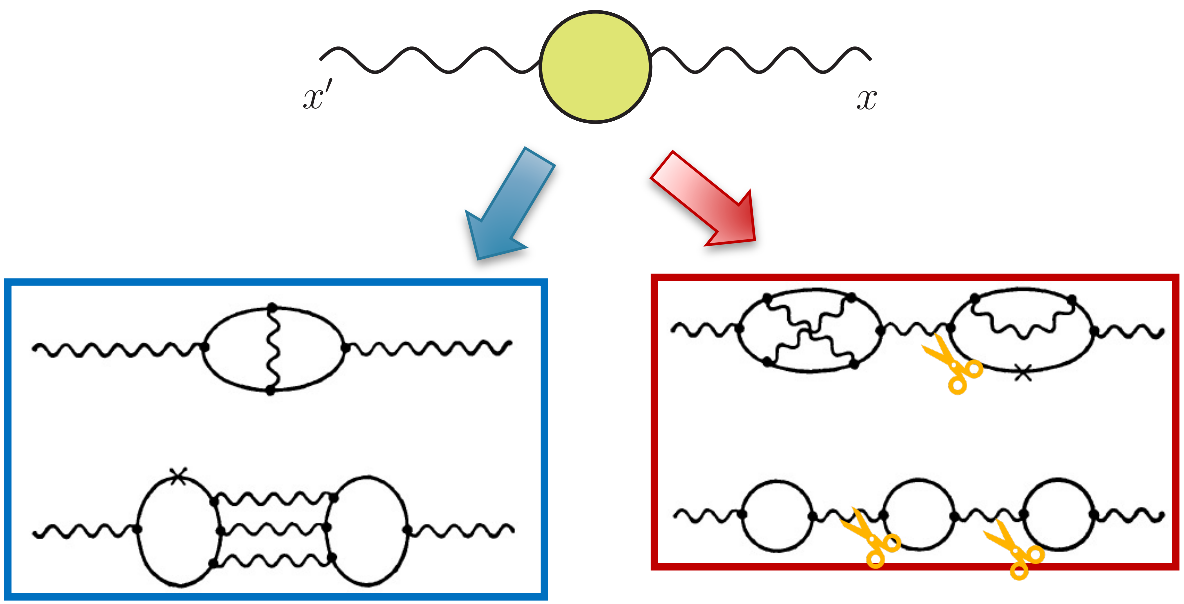
$$i\Delta^{\mu\nu}(x-x') = \langle \Omega | T\{A^{\mu}(x)A^{\nu}(x')\}|\Omega\rangle$$

$$i\Delta^{\mu\nu}(x-x') =$$



where the yellow circle represents the sum of all connected diagrams (proper and improper).

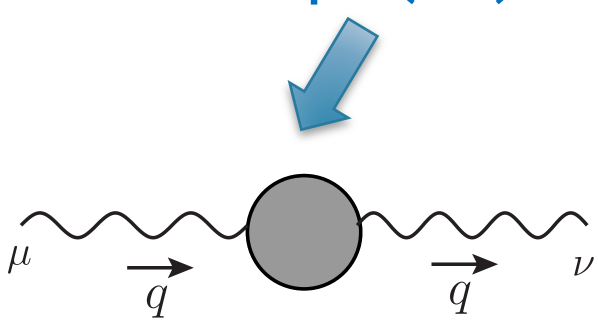
- Once again, we separate the proper from the improper ones.



Proper (1PI)

Improper

$$ie_0^2\Pi_{\mu\nu}(q) =$$



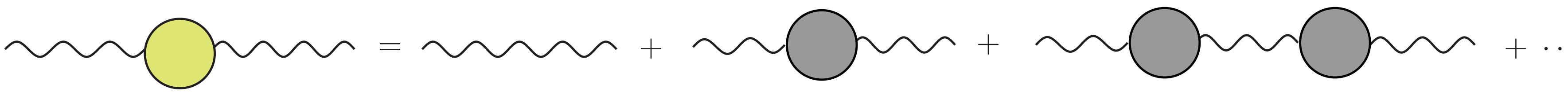
The sum of all 1PI diagrams is:

$$ie_0^2\Pi_{\mu\nu}(q)$$

vacuum polarization

- In analogy to the electron case, we can express the full photon propagator as

$$i\Delta^{\mu\nu}(q) = i\Delta_0^{\mu\nu}(q) + i\Delta_0^{\mu\lambda}(q)[ie_0^2\Pi_{\lambda\sigma}(q)]i\Delta_0^{\sigma\nu}(q) + \cdots$$
$$= \frac{-ig^{\mu\nu}}{q^2} - \frac{-ie_0^2}{q^2}[i\Pi^{\mu\nu}(q)]\frac{(-i)}{q^2} - \frac{-ie_0^4}{q^2}[i\Pi^{\mu\lambda}(q)]\frac{(-i)}{q^2}[i\Pi^{\nu}_{\lambda}(q)]\frac{(-i)}{q^2} + \cdots$$



$$i\Delta^{\mu\nu}(q) = i\Delta_0^{\mu\nu}(q) + i\Delta_0^{\mu\lambda}[ie_0^2\Pi_{\lambda\sigma}]i\Delta_0^{\sigma\nu} + i\Delta_0^{\mu\lambda}[ie_0^2\Pi_{\lambda\sigma}]i\Delta_0^{\sigma\alpha}[ie_0^2\Pi_{\alpha\tau}]i\Delta_0^{\tau\nu} + \cdots$$

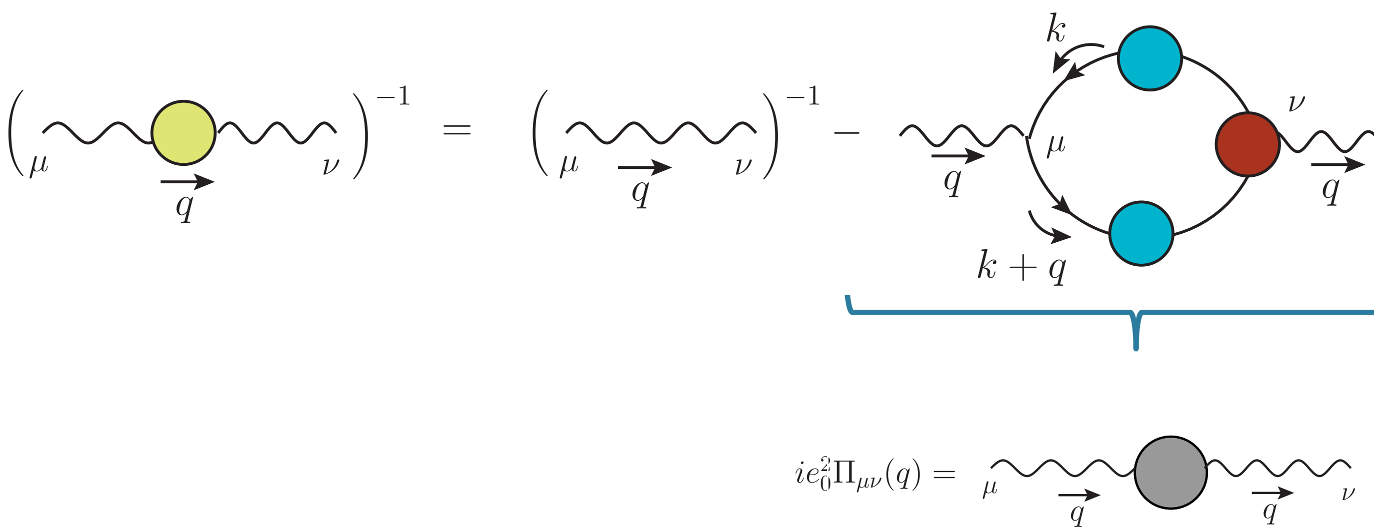
$$\Delta_{\mu\nu}(q) = -\frac{g^{\mu\nu}}{q^2} + \frac{e_0^2}{q^2}\Pi_{\mu\lambda}(q)\Delta^{\lambda}_{\nu}(q)$$

$$\left[q^2 g_{\mu\lambda} - e_0^2\Pi_{\mu\lambda}(q)\right]\Delta^{\lambda}_{\nu}(q) = -g^{\mu\nu}$$

The vacuum polarization

- The full vacuum polarization is given by the following equation

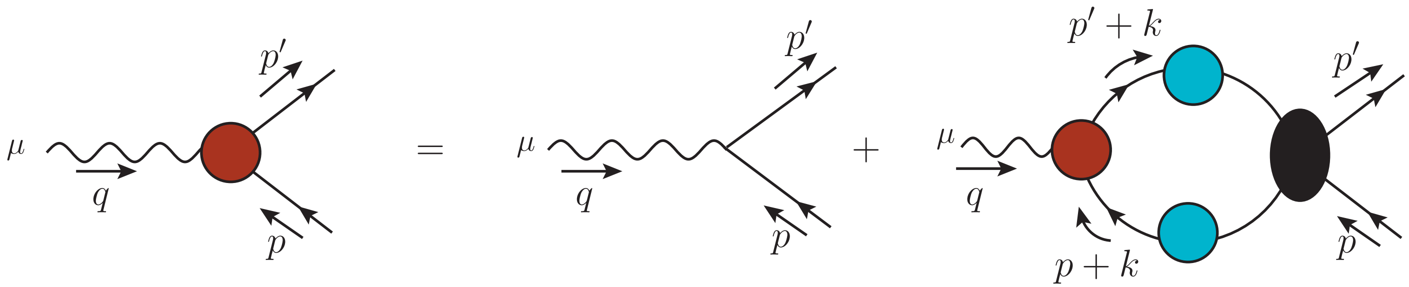
Photon SDE



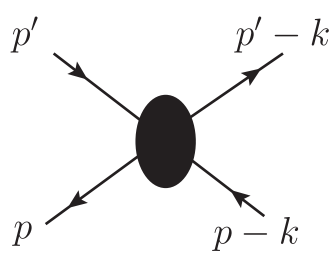
$$ie_0^2\Pi_{\mu\nu}(q) = (-ie_0)^2(-1)\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma_\mu iS(k)\Gamma_\nu(k,k+q)iS(k+q)\right]$$

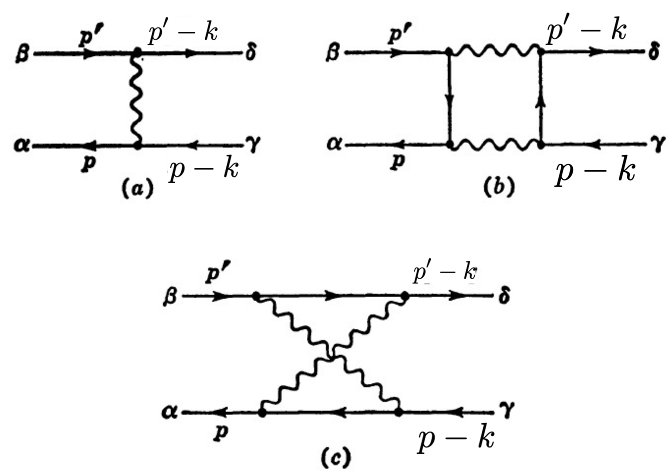
SDE for the electron-photon vertex

- Similarly, one can obtain the SDE for the electron-photon vertex



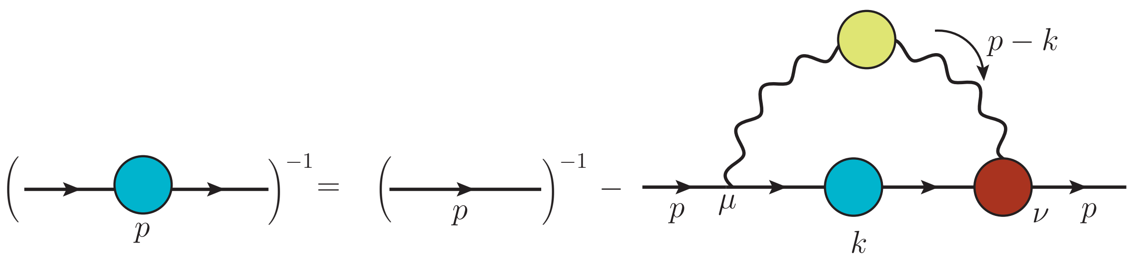
$$\Gamma_\mu(p',p) = \gamma_\mu + \int \frac{d^4k}{(2\pi)^4} iS(p+k)\Gamma_\mu(p'+k,p+k)iS(p+k)K(p+k,p'+k,k)$$



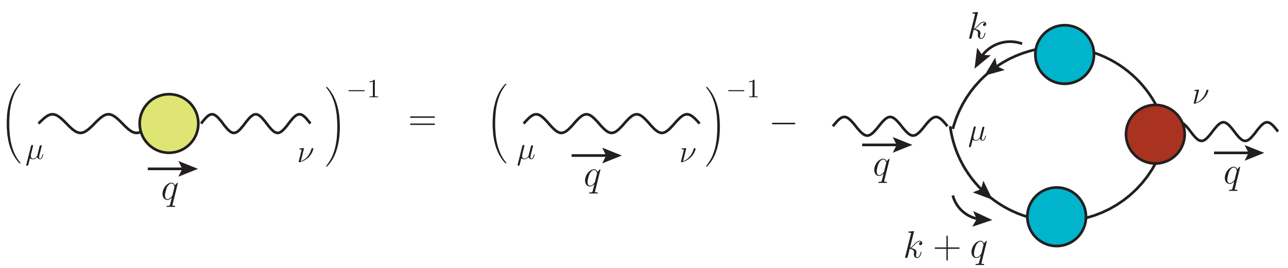


Scattering kernel – electron-positron

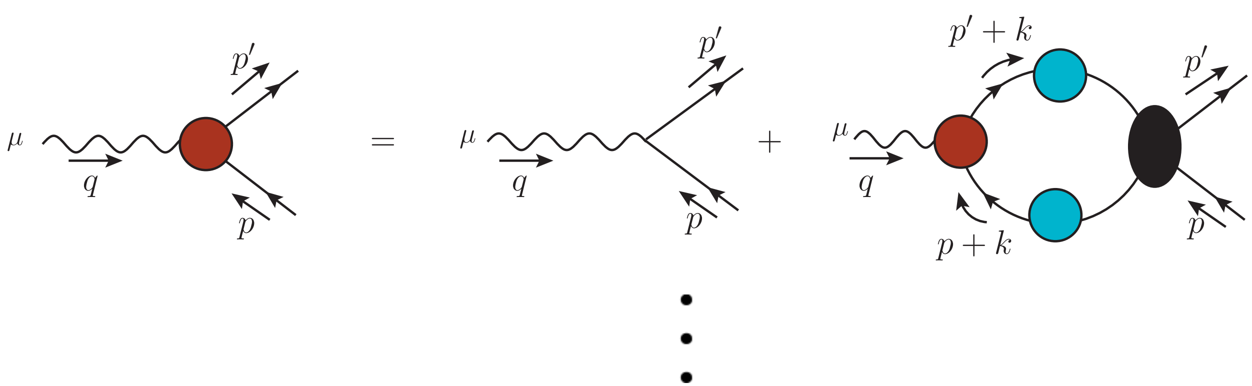
SDE tower for QED



Electron SDE



Photon SDE

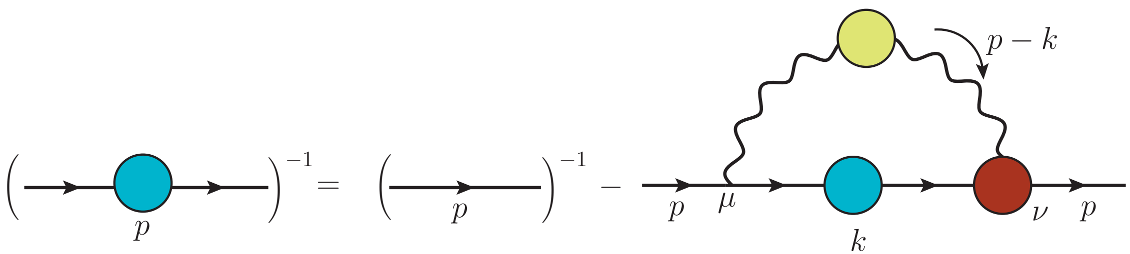


Vertex SDE

- *They form an infinite set of coupled integral equation!*

Need for a truncation scheme

- First, let us examine the SDE for the fermion in isolation



$$S^{-1}(p) = (\not{p} - m) + ie_0^2 \int \frac{d^4k}{(2\pi)^4} \Delta_{\mu\nu}(q) \Gamma^{\nu}(p,k) S(k) \gamma^{\mu}$$

- This equation is more complicated than it seems.
- The full electron propagator (containing all order corrections) can be written as

$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)\mathbb{I}$$

$$S_0^{-1}(p) = (\not{p} - m)$$

A and B are unknown functions

- Notice that at tree level $A(p^2) = 1$ and $B(p^2) = m$

$$\mathcal{M}(p) = \frac{B(p)}{A(p)}$$

- The pole of the propagator defines the ***mass of the particle.***

Dynamical mass

- The full photon propagator, in general covariant gauges, can be written as

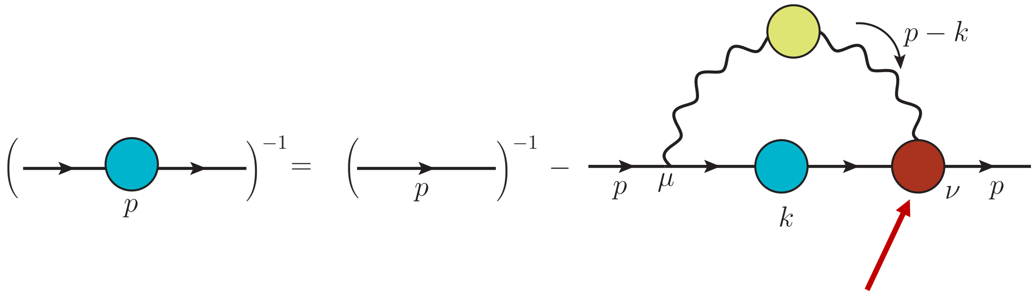
$$\Delta_{\mu\nu}(q) = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]\Delta(q^2) + \xi\frac{q_\mu q_\nu}{q^4}$$

- Here we will focus in the $\text{Landau gauge} \to \xi = 0$

$\Delta(q^2)$ → is the full (all-order) photon propagator
Unknown quantity determined from its own SDE

- At tree level, the photon propagator (in the Landau gauge) is given by

$$\Delta_0^{\mu\nu}(q) = \left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]\frac{1}{q^2} \quad \rightarrow \quad \Delta_0(q^2) = \frac{1}{q^2}$$

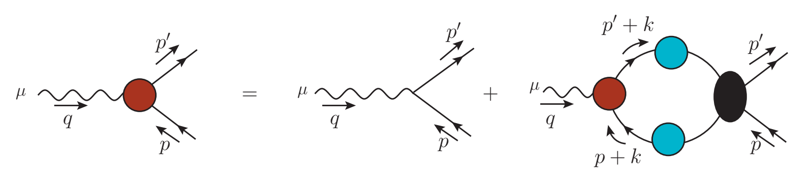


- The most general Lorentz structure of the full electron-photon is composed by 12 tensorial structures - [two momenta and a free Lorentz index]

$$\Gamma_\nu(p,k) = \gamma_\nu\Gamma_1 + p_\nu\Gamma_2 + k_\nu\Gamma_3 + \gamma_\nu \not{p}\Gamma_4 + \gamma_\nu \not{k}\Gamma_5 + p_\nu \not{p}\Gamma_6 + p_\nu \not{k}\Gamma_7 + k_\nu \not{p}\Gamma_8 + k_\nu \not{k}\Gamma_9 + \gamma_\nu \not{p}\not{k}\Gamma_{10} + p_\nu \not{p}\not{k}\Gamma_{11} + k_\nu \not{p}\not{k}\Gamma_{12}$$

where the form factors are unknown functions $\Gamma_i := \Gamma_i(p,k,p-k)$ which satisfy their own SDE.

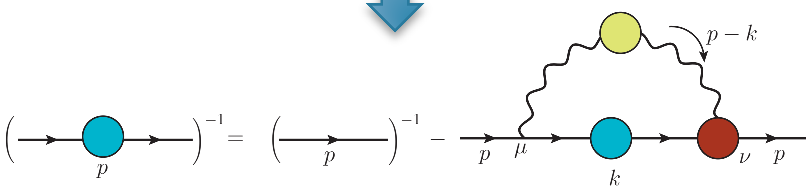
At tree level → $\Gamma_0^\nu(p,k) = \gamma^\nu$



- To sum up, we have:

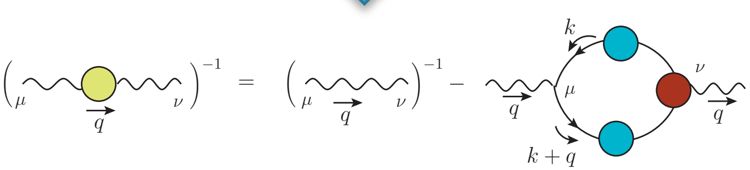
→ **2 unknowns functions**, A and B from full electron propagator, which

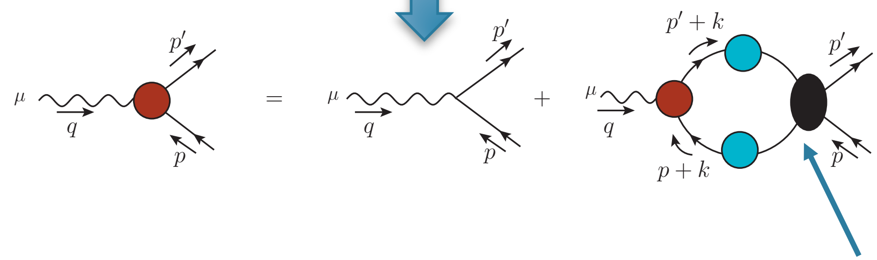
$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)\mathbb{I}$$



are coupled **1 (photon) + 12 (form factors of the vertex)** unknowns functions:

$$\Delta_{\mu\nu}(q) = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]\Delta(q^2) \quad + \quad \begin{aligned}\Gamma_\nu(p,k) = {} & \gamma_\nu\Gamma_1 + p_\nu\Gamma_2 + k_\nu\Gamma_3 + \gamma_\nu\not{p}\Gamma_4 + \gamma_\nu\not{k}\Gamma_5 + p_\nu\not{p}\Gamma_6 \\ & + p_\nu\not{k}\Gamma_7 + k_\nu\not{p}\Gamma_8 + k_\nu\not{k}\Gamma_9 + \gamma_\nu\not{p}\not{k}\Gamma_{10} + p_\nu\not{p}\not{k}\Gamma_{11} + k_\nu\not{p}\not{k}\Gamma_{12}\end{aligned}$$



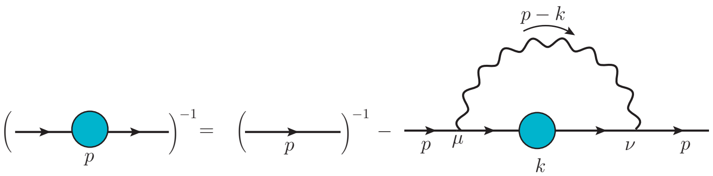


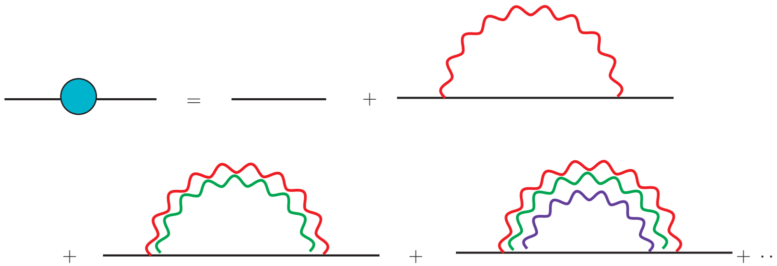
+ ...

- To understand the basic principles of the dynamical mass generation, it is not necessary to solve this intricate coupled system.
- Let's make some approximations to get the general idea of the problem.
- We will approximate the photon propagator and the vertex by their tree level values, i.e.

$$\Delta(q^2) \to \frac{1}{q^2} \qquad \Gamma^\nu(p,k) \to \gamma^\nu$$

- Then, only the electron is treated nonperturbatively. Diagrammatically we have





Rainbow approximation