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# Transverse momentum imaging
## Lecture 2

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# Plan of these lectures

1. **DIS and partons**
2. **From DIS to SIDIS**
3. **Symmetries and universality**
4. **Factorization, evolution, matching**
5. **Phenomenology**

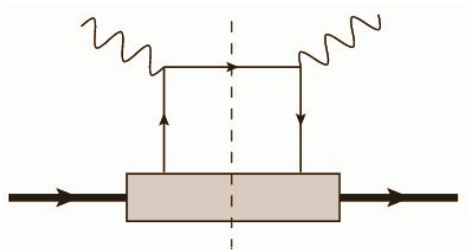
# 2. From DIS to SIDIS

# Where is transverse momentum?

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2 y}{2Q^4} L_{\mu\nu}(l, l', \lambda_e)\, 2MW^{\mu\nu}(q, P, S)$$

INCLUSIVE DIS → differential in xB

We need a process with an **“experimental handle” on transverse momentum**, for example **Semi Inclusive DIS**



quark-antiquark

$$2MW^{\mu\nu}(q, P, S) = \sum_q e_q^2 \frac{1}{2} \mathrm{Tr}\left[\Phi(x, S)\, \gamma^\mu \gamma^+ \gamma^\nu\right]$$

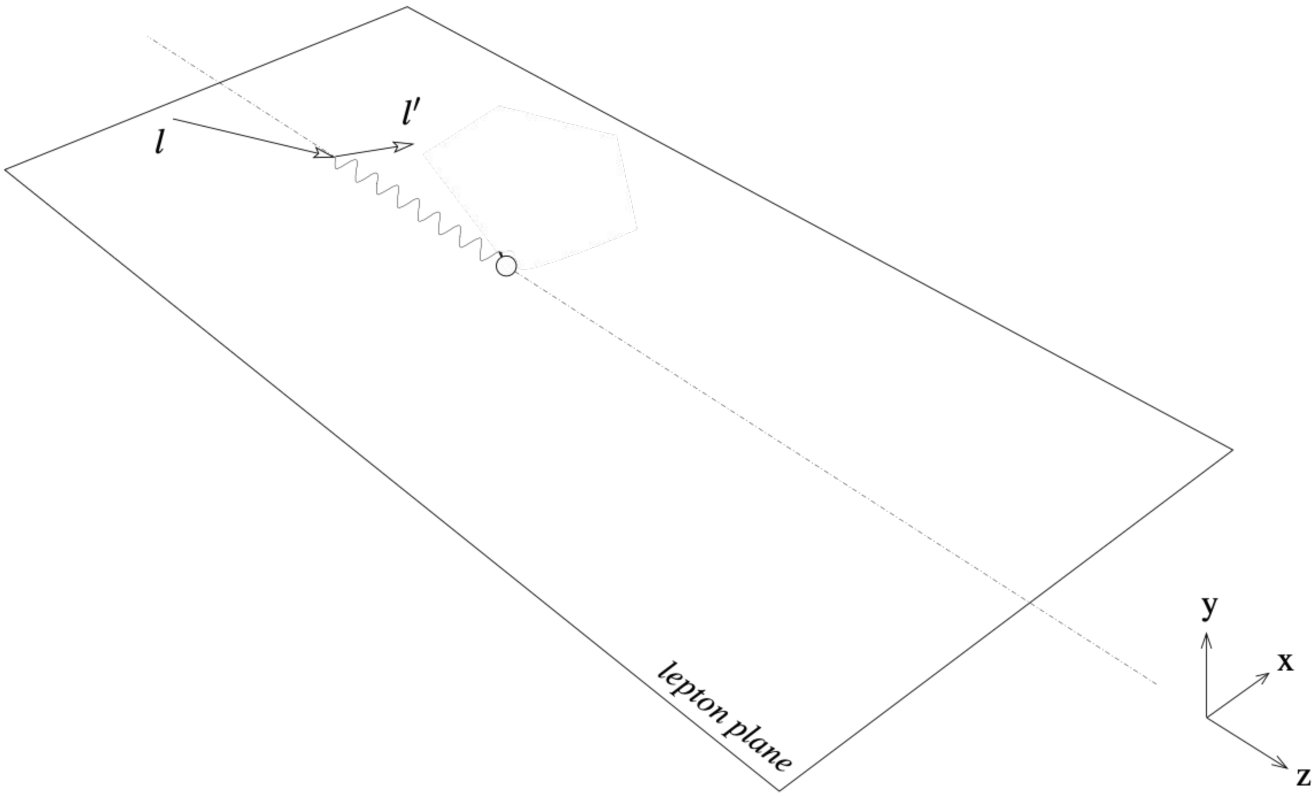
ϕ(x,S) : “collinear” quark correlator

$x_B \simeq x \equiv k^+/P^+$ → measure collinear parton dynamics

The quark transverse momentum is integrated out in DIS

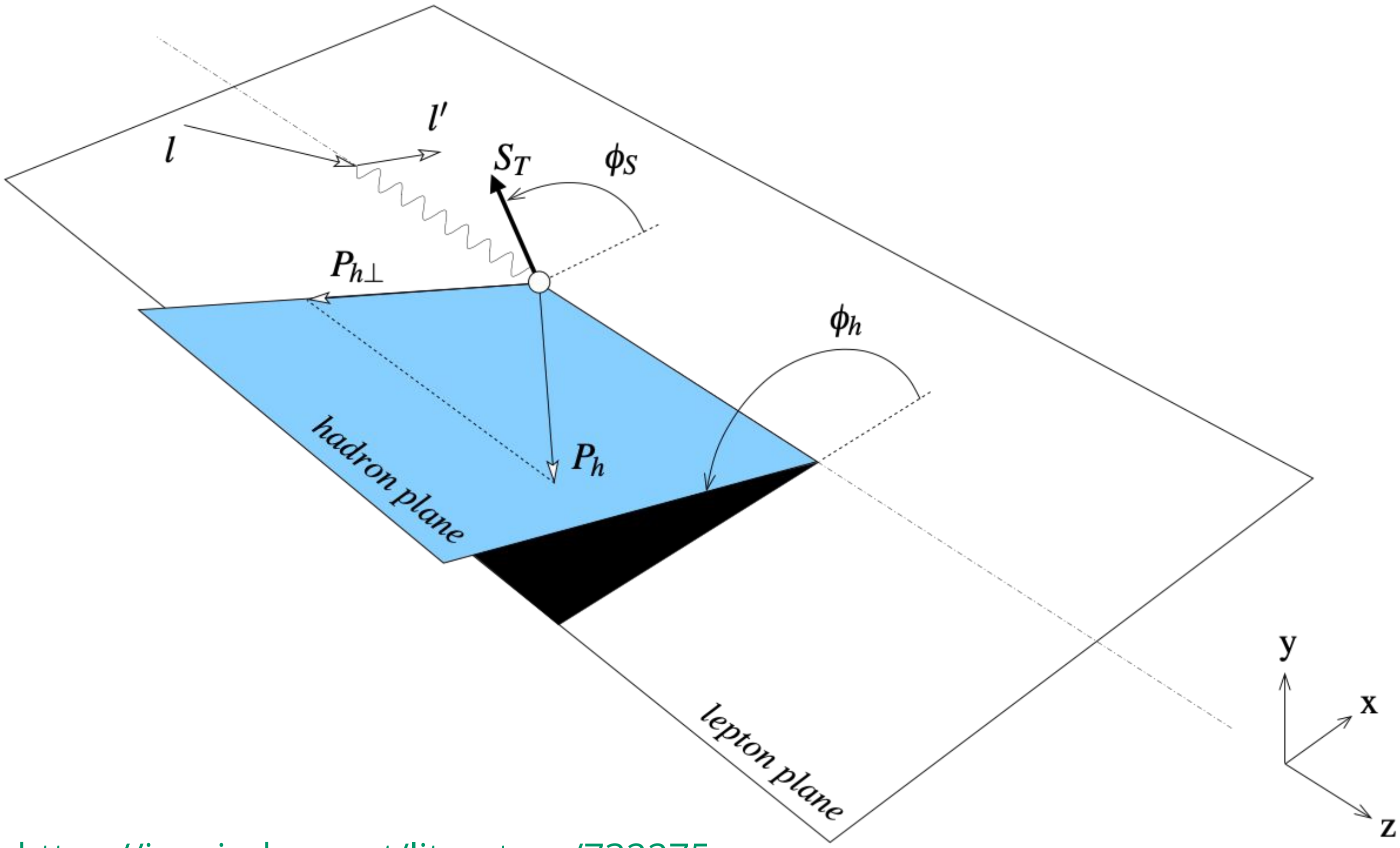
# Deep-inelastic scattering

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)$$



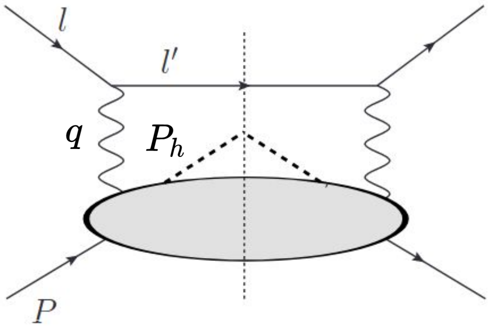
# Semi-Inclusive DIS

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X,$$




https://inspirehep.net/literature/732275

# Cross section DIS vs SIDIS

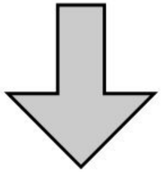



$$x_B = \frac{Q^2}{2\,P\cdot q}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q}.$$

"Handle" on collinear parton dynamics

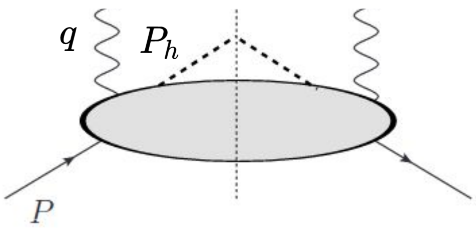
$$\frac{d^3\sigma}{dx_B\, dy\, d\phi_S} = \frac{\alpha^2\, y}{2\,Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$ DIS



"Handle" on transverse parton dynamics

$$\frac{d^6\sigma}{dx_B\, dy\, dz_h\, d\phi_S\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$ SIDIS

# Hadronic tensor (unpolarized)




Compared to DIS, there are five structure functions instead of two for unpolarized target

They depend on two extra variables

$$2MW^{\mu\nu}(q,P,S) = \frac{2z_h}{x_B}\Big[ -g_\perp^{\mu\nu} F_{UU,T}(x_B, z_h, P_{h\perp}^2, Q^2) + \hat{t}^\mu \hat{t}^\nu F_{UU,L}(x_B, z_h, P_{h\perp}^2, Q^2)$$
$$+ \left(\hat{t}^\mu \hat{h}^\nu + \hat{t}^\nu \hat{h}^\mu\right) F_{UU}^{\cos\phi_h}(x_B, z_h, P_{h\perp}^2, Q^2) + \left(\hat{h}^\mu \hat{h}^\nu + g_\perp^{\mu\nu}\right) F_{UU}^{\cos 2\phi_h}(x_B, z_h, P_{h\perp}^2, Q^2)$$
$$- \mathrm{i}\left(\hat{t}^\mu \hat{h}^\nu - \hat{t}^\nu \hat{h}^\mu\right) F_{LU}^{\sin\phi_h}(x_B, z_h, P_{h\perp}^2, Q^2)\Big],$$

$$\hat{h} = \frac{P_{h\perp}}{|P_{h\perp}|}$$

# SIDIS cross section (unpolarized)

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\Bigg\}$$

5 structure functions for unpolarized target

For more details see https://inspirehep.net/literature/732275

# SIDIS cross section (polarized nucleon - spin 1/2)

$$
\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\phi_S\, dz\, d\phi_h\, dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\, y\, Q^2}\, \frac{y^2}{2\,(1-\varepsilon)} \Bigg\{ F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \right] \\
&\quad + S_L\, \lambda_e \left[ \sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right] \\
&\quad + S_T \bigg[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\, \sin(3\phi_h - \phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \bigg] + S_T \lambda_e \bigg[ \sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \bigg] \Bigg\}
\end{aligned}
$$

18 structure functions
for polarized nucleon target

For more details see https://inspirehep.net/literature/732275

# SIDIS cross section (polarized deuteron - spin 1)

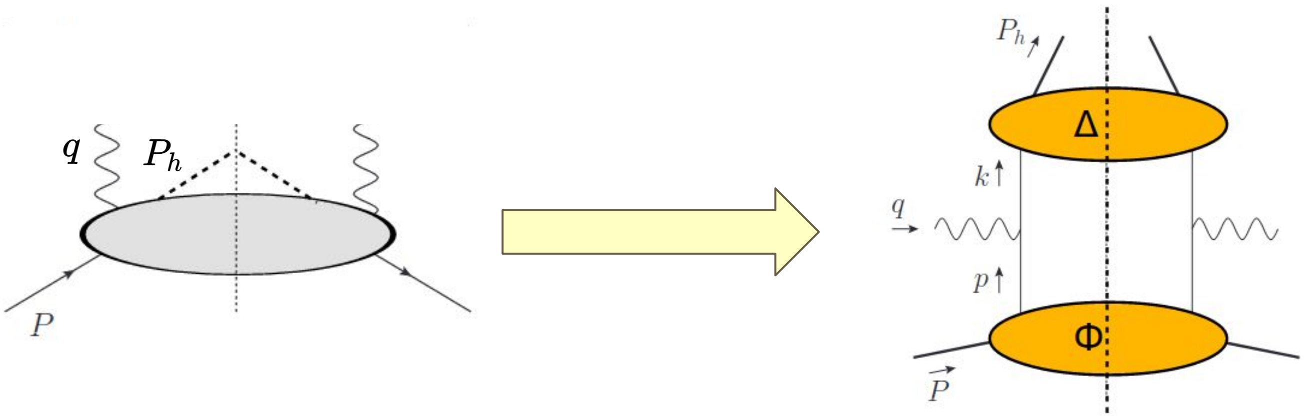
?

Again, up to now no partons ...

How do quarks and gluons emerge in this description?

# Partonic interpretation

$$2\,M\,W_{\mu\nu}(q,P,S,P_h) \;=\; \sum_X \int \frac{d^3P_X}{2E_X}\,\delta^4(P+q\;-\;P_X)\,\langle PS|\,J_\mu^\dagger(0)\,|P_h\,P_X\rangle\,\langle P_h\,P_X|\,J_\nu(0)|PS\rangle$$

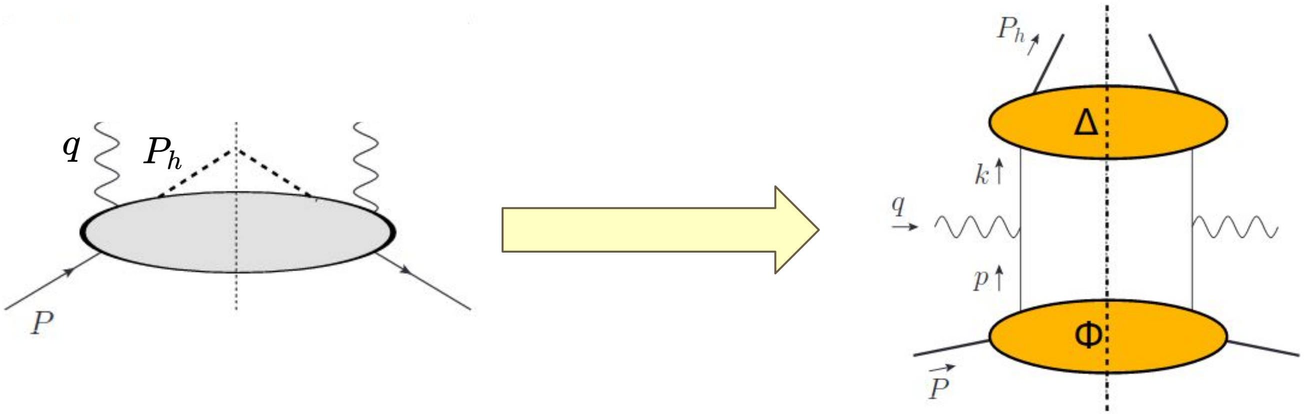



\+ higher twist (suppressed)

The presence of an identified hadron does not allow us to use the commutator form
→ **OPE not applicable**

Use "***diagrammatic approach***" → use quark correlation functions for hadron structure and formation : it corresponds to the result in **TMD factorization** (when there is one)

# Partonic interpretation



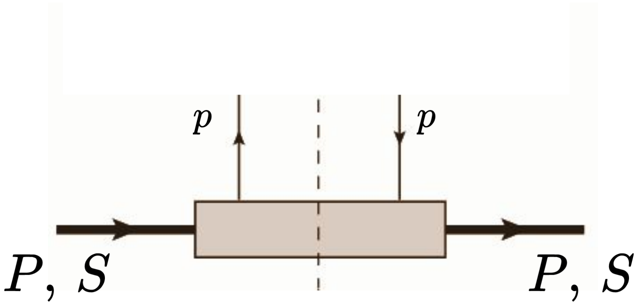

+ higher twist (suppressed)

$$2MW^{\mu\nu}(q,P,S,P_h) = \frac{2z_h}{x_B}\,\mathcal{C}\Big[\mathrm{Tr}\big(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu\big)\Big]$$

$$\mathcal{C}\big[wfD\big] = \sum x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{k}_T\,\delta^{(2)}\big(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{h\perp}/z\big)\,w(\boldsymbol{p}_T,\boldsymbol{k}_T)\,f^a(x,p_T^2)\,D^a(z,k_T^2)$$

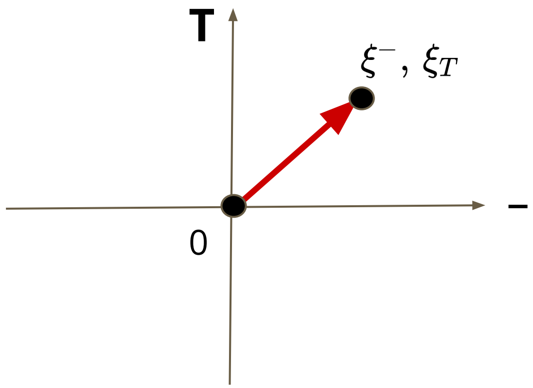
# Quark (distribution) correlator

$$\Phi_{ij}(p,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\, \psi_i(\xi) \big| PS\rangle$$



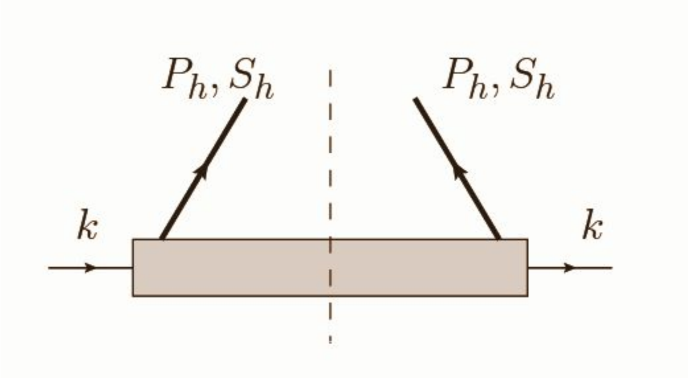

$$\Phi_{ij}(x,\mathbf{p}_T,S) = \int dp^+\, dp^-\, \delta\big(p^+ - xP^+\big)\Phi(p,P,S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\, e^{i\,p\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\, \psi_i(\xi)\, \big| PS\rangle_{\,\xi^+ = 0}$$



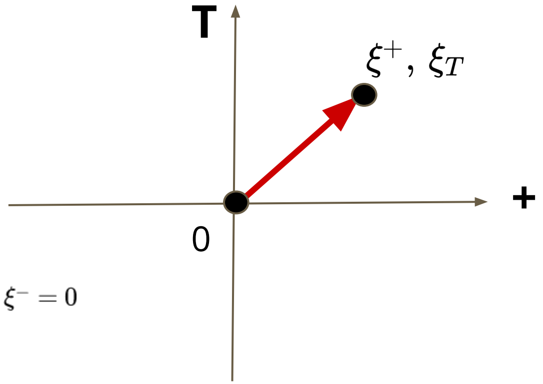
# Quark fragmentation correlator

$$\Delta_{ij}^{h}(k,P_h,S_h) \;=\; \sum_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\,\langle 0\big|\psi_i(\xi)|XP_hS_h\rangle\langle XP_hS_h|\overline{\psi}_j(0)\big|0\rangle$$




$$\Delta_{ij}(z,\,\mathbf{k}_T,S_h) \;=\; \int dk^-\,dk^+\,\delta\Big(k^- - \frac{1}{z}P_h^-\Big)\Delta_{ij}(k,P_h,S_h) =$$

$$= \sum_X \int \frac{d\xi^+\,d^2\xi_T}{2\pi}\; e^{i\,k\cdot\xi}\,\langle 0\big|\overline{\psi}_i(\xi)\big|XP_hS_h\rangle\langle XP_hS_h\big|\overline{\psi}_j(0)\big|0\rangle\,_{\xi^-=0}$$



# Quark TMD distribution functions (spin ½)

quark pol. / nucleon pol.

| nucleon pol. \ quark pol. | U | L | T |
|---|---|---|---|
| U | $f_1$ | | $h_1^\perp$ |
| L | | $g_{1L}$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_1,\ h_{1T}^\perp$ |

At leading twist: 8 TMD PDFs

(similar classification for gluons)

The **symmetries of QCD** play a crucial role in this classification

- **Black**: time-reversal even AND collinear
- **Blue**: time-reversal even
- **Red**: time-reversal odd (*process dependence*)

Quark inside spin ½ hadron

# Quark TMD distribution functions (spin 1)

| **Quarks** | $\gamma^+$ | $\gamma^+\gamma^5$ | $i\sigma^{i+}\gamma^5$ |
|---|---|---|---|
| U | $\boldsymbol{f_1}$ | | $h_1^\perp$ |
| L | | $\boldsymbol{g_1}$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $\boldsymbol{h_1}$, $h_{1T}^\perp$ |
| LL | $\boldsymbol{f_{1LL}}$ | | $h_{1LL}^\perp$ |
| LT | $f_{1LT}$ | $g_{1LT}$ | $h_{1LT}$, $h_{1LT}^\perp$ |
| TT | $f_{1TT}$ | $g_{1TT}$ | $h_{1TT}$, $h_{1TT}^\perp$ |

At leading twist: **18 (!)** TMD PDFs

(similar classification for gluons)

The **symmetries of QCD** play a crucial role in this classification

Quark inside spin 1 hadron

# Quark TMD fragmentation functions

quark pol.

| hadron pol. | U | L | T |
|---|---|---|---|
| U | $D_1$ | | $H_1^{\perp}$ |
| L | | $G_{1L}$ | $H_{1L}^{\perp}$ |
| T | $D_{1T}^{\perp}$ | $G_{1T}$ | $H_1$, $H_{1T}^{\perp}$ |

At leading twist:
8 TMD FFs and
3 collinear FFs (diagonal)

The **symmetries of QCD** play
a crucial role in this classification

# Structure functions and TMDs

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\, h_1^\perp H_1^\perp\right]$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\, f_{1T}^\perp D_1\right],$$

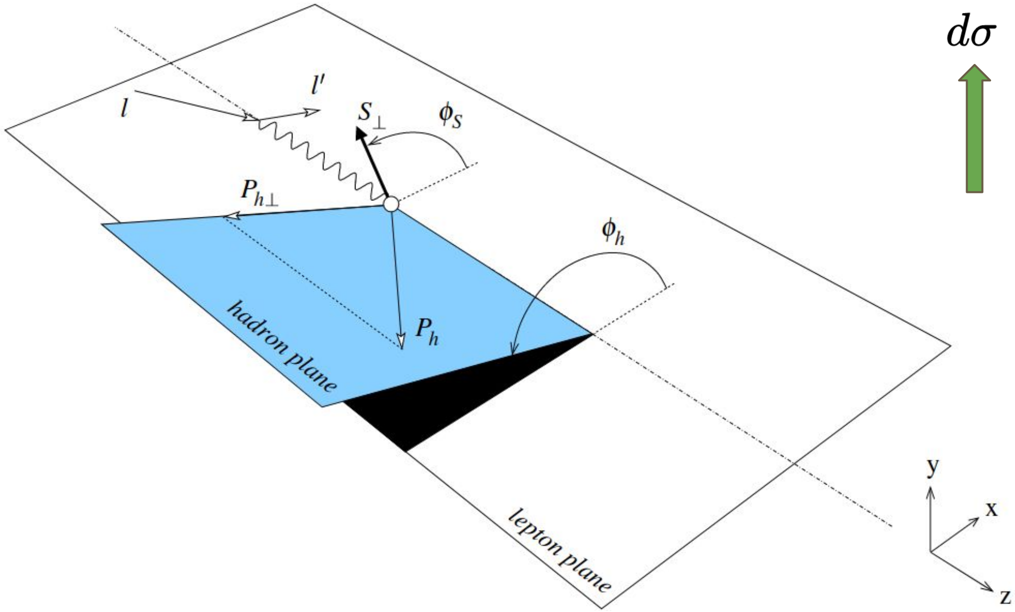
Etc. ...

The **LHS** of these equations can be **measured** and the **RHS** is expressed in terms of **partonic quantities (TMDs)**

→ **transverse momentum imaging**

$$\mathcal{C}\big[w f D\big] = \sum x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

# Transverse momentum imaging




Calculable
in perturbation theory

$$d\sigma \sim \mathcal{H} \; \boxed{\text{TMD PDF} \otimes \text{TMD FF}}$$

The partonic "maps", to be extracted from data

$$\ell \, N \to \ell' \, h \, X$$

hermes COMPASS JLab, EIC, ...